A review of QCD phase diagram in functional QCD approaches

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fQCD collaboration :
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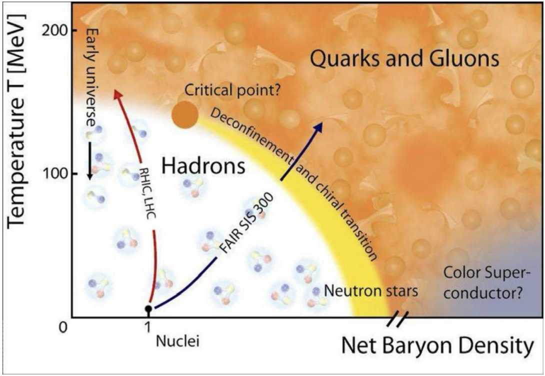
QCD in heavy ion collisions:

Roughly speaking, a transition between hadrons and asymptotic quarks/gluons.

- Transitions coming from the mass scale change of quark and gluon:
  Quark mass generation=Chiral PT
  Gluon mass generation ≈ Deconfinement
- The chiral phase transition dominates the phenomenology in HIC, and connected with crossover at low density by critical end point (CEP)
- Fine structures to classify phases like chiral spin symmetric phase, inhomogeneous phase/Moat regime, quarkyonic phase, color superconductivity phase



QCD in vacuum:

Cyrol, Mitter, Pawlowski, Strodhoff, PRD 97 (2018) 5, 054006.

Binosi, Chang, Papavassiliou, Qin, Roberts, PLB 742, (2015) 183

Williams, Fischer, Heupel, PRD 93, (2016)034026.

Mitter, Pawlowski, Strodthoff, PRD 91, (2015)054035.

Qin, Chang, Liu, Roberts, Schmidt, PLB 722 (2013) 384

Chang, Roberts, PRL 106 (2011) 072001 ...

Yang-Mills sector:

Eichmann, Pawlowski, Silva, PRD 104 (2021) 11, 114016

Aguilar, Ferreira, Papavassiliou, PRD 105 (2022) 1, 014030

Huber, PR 879, 1 (2020)

Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

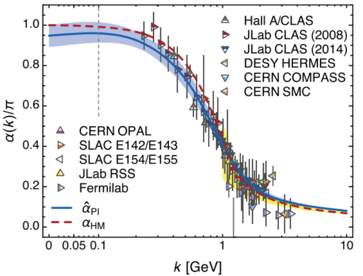
Aguilar, Binosi, Papavassiliou, PRD 86 (2012) 014032 ...

Phase Structure: Fu, Pawlowski, Renneke, PRD 101 (2020) 5, 054032; Gao, Chen, Liu, Roberts, Schmidt, PRD 93 (2016) 9, 094019; Fischer, PPNP 105,(2019)1;Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022 Isserstedt, Buballa, Fischer, PRD 100 (2019) 7, 074011; Qin, Chang, Chen, Liu, Roberts, PRL106 (2011) 172301...

The minimal requirements for a truncation scheme that describes QCD:

- *Describe the running mass of quark and gluon*
- *Describe the running of the coupling*

An infrared fixed point



A. Aguilar et al,PRD80,085018(2009), PRD96,054026(2017); A. Deur et al, PPNP 90,1(2016)

- Defines an "perturbative" expansion in infrared.
- Simplifies the running behavior of correlations that is simply described with scaling exponents.
- The higher order becomes irrelevant/regular.

The hints from the relation between fRG and holographic equation:

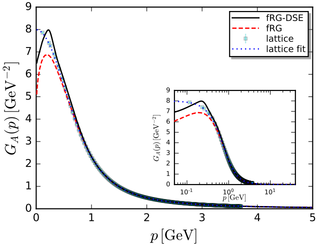
FG, Masatoshi Yamada, PRD 106, 126003(2022).

- Fixed point leads the effective potential/AdS potential independent of the regularization in critical region
- Promotes the Bulk wave function in the additional dimension to be physical.

The AdS/QFT correspondence/holography only works if QFT is put into a scaling region near a fixed point.

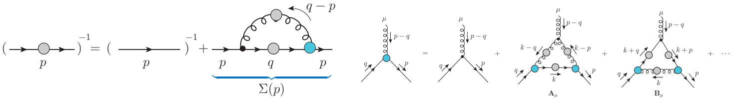
The minimal scheme

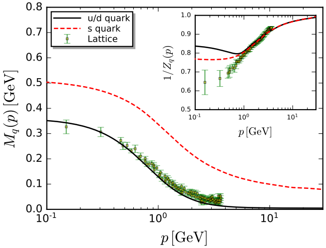
The Yang-Mills sector is relatively separable. One can apply the data in vacuum and compute the difference between finite T/μ and vacuum.



Lattice:
A. G. Duarte et al, PRD 94, 074502 (2016),
P. Boucaud et al, PRD 98, 114515 (2018),
S. Zafeiropoulos et al, PRL122, 162002 (2019)
fRG:
W.-j. Fu et al, PRD 101, 054032 (2020)
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

Solve the DSEs of quark propagator and quark gluon vertex:





lattice: P. O. Bowman et al, PRD71, 054507 (2005) **fRG**: W.-j. Fu et al, PRD 101, 054032 (2020) **DSE**: FG et al, PRD 103, 094013(2021)

A further simplification on the quark gluon vertex:

Quark gluon vertex In Landau gauge:

$$\Gamma^{\mu}(q,p) = \sum_{i=1}^{8} t_i(q,p) P^{\mu\nu}(q-p) \mathcal{T}_i^{\nu}(q,p)\,,$$

The dominant structures are Dirac and Pauli term:

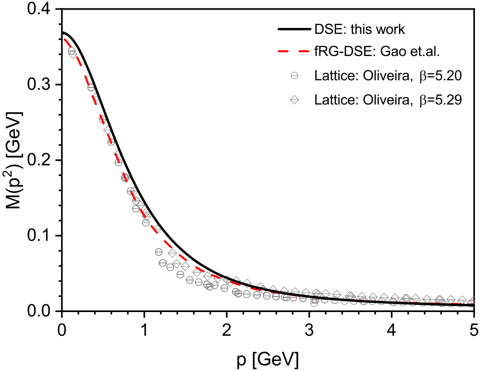
$$\mathcal{T}_1(p,q) = -i\gamma^{\mu}\,, \mathcal{T}_4^{\mu}(p,q) = \sigma_{\mu\nu}(p-q)^{\nu}\,,$$

$$t_1(p,q) = F(k^2)\frac{A(p^2)+A(q^2)}{2}$$

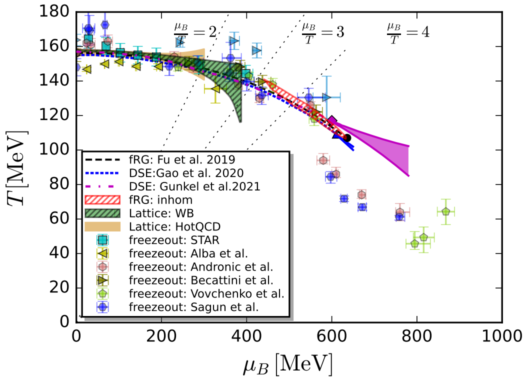
$$t_4(p,q) = \left[Z(k^2)\right]^{-1/2}\frac{B(p^2)-B(q^2)}{p^2-q^2}$$

FG, J. Papavassiliou, J. Pawlowski, PRD 103.094013 (2021).
Y, Lu, **FG**, YX Liu, J. Pawlowski, PRD 110, 014036 (2024)

All quantities are expressed by the running of two point functions.
The Quark Mass function:



Chiral Phase diagram for 2+1 flavour QCD can be directly obtained from quark propagator



The fQCD computations of chiral phase transition are converging:

- $T_c = 155\,\mathrm{MeV}$ and $\kappa \sim 0.016$
- Estimated range of CEP:
  $T \in (100, 110)$ MeV
  $\mu_B \in (600, 700)$ MeV
- $\sqrt{s_{NN}} \approx 3 - 5\mathrm{GeV}$

W.-j. Fu et al, PRD 101, 054032 (2020)

FG and J. Pawlowski, PRD 102, 034027 (2020)

FG and J. Pawlowski, PLB 820, 136584(2021)

P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

Update: a full computation beyond $O(4)$ symmetry approx. in vertex gives CEP closer to 700 MeV. Preliminary results, Y Lu, FG, J. Pawlowski in prep.

Lee Yang Edge (LYE) singularities are branch cuts of the partition functions on the complex plane of the μ:

- When the LYE singularities pinch the μ real axis, the singularities correspond to the critical end point;
- When the branch cut crosses the real axis, the first order phase transition occurs.

Yang, C. N., Lee T. D. Phys. Rev. 87, 404-409 (1952); Lee T. D. Yang, C. N., Phys. Rev. 87, 410-419 (1952)

Second order Phase transition and Scaling analysis:

- 3d O(4) scaling expanded by $(T, \mu_B) = (T_c^0 \sim 130 - 140, 0)MeV$.
- Z(2) scaling expanded by CEP of chiral PT.
- Roberge-Weiss scaling which is a scaling behavior in YM sector.

One may considering the general form of symmetry breaking pattern at finite temperature and magnetic field:

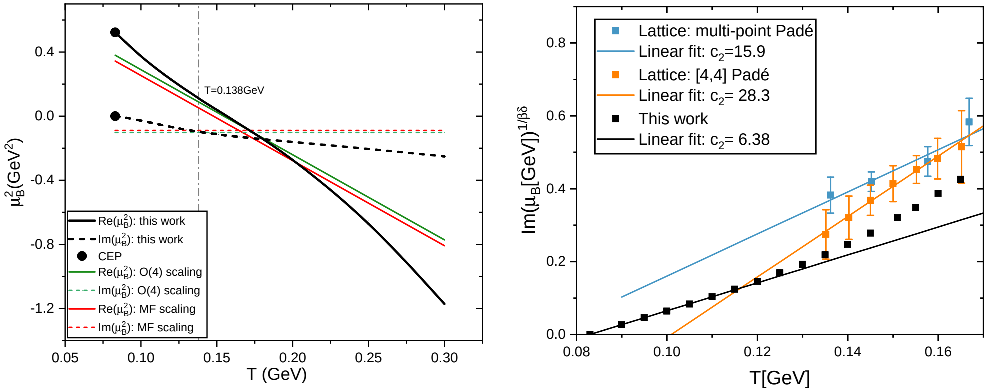
- Order parameter M: magnetization
- Two directions: t and h are the general scaling variables with t being the reduced temperature, and h being the reduced magnetic field.
- The LYE singularities locate at $z = t/h^{1/\beta\delta} = |z_c|e^{i\pi/2\beta\delta}$.

The $|z_c|$, β and δ are universal scaling parameters which can be determined in the universality class analysis:

	β	δ	$\lvert z_c \rvert$
$3d\ Z(2)$	0.33	4.79	2.43
$3d\ O(4)$	0.38	4.82	1.69
Mean field	0.5	3	1.89

A. Connelly, G. Johnson, F. Rennecke, V. Skokov, PRL 125(19), 191602 (2020);
G. Johnson, F. Rennecke, V. Skokov, PRD107(11), 116013 (2023);
F. Rennecke, V. Skokov, Annals Phys. 444, 169010 (2022)

Using imaginary chemical potential in quark gap equation:



- no 3d O(4) scaling, and CEP scaling works well
- in particular the slope c_2 is useful for extrapolating the CEP

Ziyan Wan, Y. Lu, FG, Yuxin Liu, Eur.Phys.J.C 84 (2024) 9, 899

CEP location within the scaling analysis

Considering the mapping between the Ising parameters to T and μ_B,

$$\begin{pmatrix} t \\ h \end{pmatrix} = \mathbb{M} \begin{pmatrix} T - T_{\rm cep} \\ \mu_B - \mu_{B,CEP} \end{pmatrix}, \quad \mathbb{M} = \begin{pmatrix} t_T & t_\mu \\ h_T & h_\mu \end{pmatrix},$$

$$\mu_{LYE} \sim \mu_{B,CEP} - c_1(T - T_{\rm cep}) + {\rm i}c_2(T - T_{\rm cep})^{\beta\delta}, c_1 = \frac{h_T}{h_\mu}, c_2 = x_{LY}\frac{{t_\mu}^{\beta\delta}}{h_\mu}\left(\frac{t_T}{t_\mu} - \frac{h_T}{h_\mu}\right)^{\beta\delta}$$

The specific mapping form together with the PT line(Yi Lu, FG, et al, PRD 109 (2024) 11, 114031):

$$\frac{\mu_B - \mu_{B,CEP}}{\mu_{B,CEP}} = -t\omega\rho\cos\alpha_1 - h\omega\cos\alpha_2, \frac{T - T_{\rm CEP}}{T_{\rm CEP}} = f_{\rm PT}(t) + h\omega\sin\alpha_2,$$

$$f_{\rm PT}(t) = \frac{\mu_{B,CEP}}{2T_{\rm CEP}}\left(2 - t\omega\rho\cos\alpha_1\right) t\omega\rho\sin\alpha_1$$

c_2 determines the CEP location completely:

$$\overline{c}_2 = x_{LY}\frac{(\frac{T_{CEP}}{\mu_{B,CEP}})^{\beta\delta}\omega\sin\alpha_1}{(\omega\rho\sin\alpha_1)^{\beta\delta}}\left(\frac{T_{CEP}}{\mu_{B,CEP}}\cot^2\alpha_1 + 1\right), \quad (T_{CEP}, \mu_{B,CEP}) = (118, 606)\ {\rm MeV}$$

A direct measurement of deconfinement is Polyakov loop:

- Reflects the $Z(N_c)$ center symmetry
- Stands for a nontrivial stationary point in gauge field potential.
- Related to the gluon mass scale (J Braun, H. Gies, J. Pawlowski, Phys.Lett.B684:262-267,2010).

However, there might be some different scenarios for deconfined phase:

- Quasi quarks that breaks center symmetry
- The quark is confined into colored bound states (Diquark) , with no asymptotic quarks but still breaks the symmetry(Partial deconfinement).

The diquark/quark pairing in the deconfined phase is an additional characteristic property.

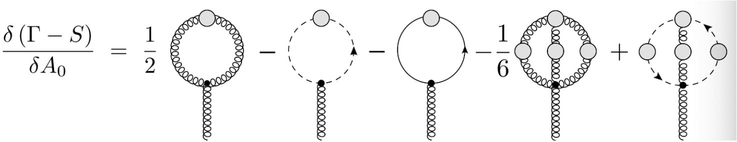
Deconfinement phase with Polyakov loop

Polyakov loop in background field approach is related to A_0^a condensate as:

$$\mathcal{L}(A_0) = \frac{1}{N_c} \mathrm{tr}\, \mathcal{P} e^{ig \int dx_0 A_0} = \frac{1}{3} \left[1 + 2 \cos\left(g\beta A_0/2\right)\right]$$

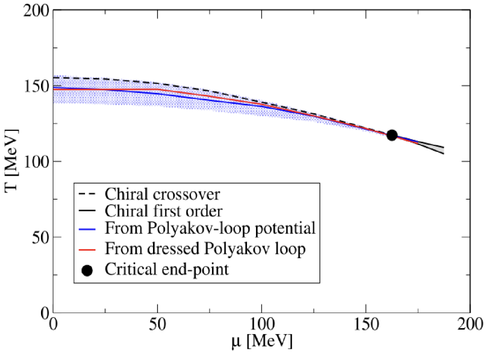
A_0^a condensate is equivalent to the colored imaginary chemical potential: Center symmetry is a symmetry for the shift in the imaginary time axis which is thus related to the pole structure of propagators.

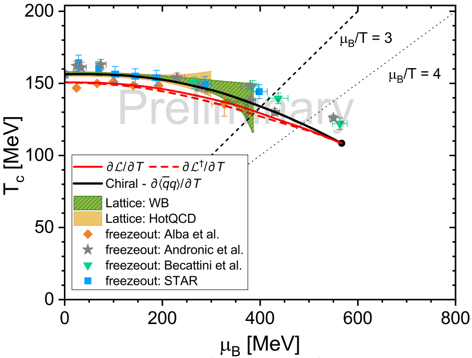
One can obtain A_0 condensate by solving the The DSE of A_0^a as $\frac{\delta(\Gamma - S_A)}{\delta A_0} = 0$.
The diagrammatic representation is:



Deconfinement phase diagram with Polyakov loop

The deconfinement characterized by Polyakov loop is in agreement with Chiral PT with the same CEP location.





The details of Polyakov loop computation and its impact on QCD EoS

Discussed in Yi Lu's talk:

C. S. Fischer et al, PLB 732, 273(2014); Yi Lu, FG, J. Pawlowski, Yuxin Liu, in preparation

The conventional CSC phase can only exist at high chemical potential and hence low temperature.

The Cooper pair Δ in conventional CSC is generated through the gap equation as:

$$\Delta = g^2 T \sum_m \int d^3\vec{q} \frac{\Delta}{q^2} G(p-q)$$

This type of propagator gives a gap that is proportional to chemical potential μ as $\Delta \sim \mu\, e^{-\frac{\mathrm{const}}{g}}$ in weak coupling limit. (D. Son, PRD 59, 094019 (1999); R. Pisarski, D. Rischke, PRD 61, 074017 (2000))

The conventional CSC in QCD is based on the Abelian approximation (bare or BC type vertex)and thus obtains the same type of pairing as in QED.

See Review: M. G. Alford, et al, RMP 80, 1455 (2008)

and Refs: A. Schmitt, Q. Wang, D. Rischke, PRL 91 (2003) 242301; M. Huang, P. Zhuang, W. Chao, PRD 67 (2003) 065015; L. He, M. Jin, P. Zhuang, PRD 71 (2005) 116001; D. Hou, Q. Wang, D. Rischke, PRD 69 (2004) 071501; I. Giannakis, D. Hou, H. Ren, D. Rischke PRL 93 (2004) 232301;D. Nickel, et al, PRD 73, 114028 (2006) D. Nickel, et al, PRD 73, 114028 (2006)

The DSEs in Nambu Gorkov basis

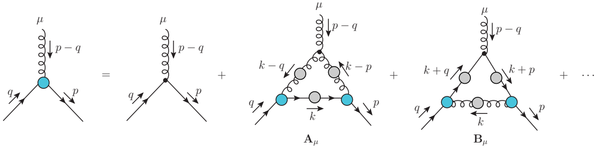
To study the quark pairing in QCD, one needs to compute the gap equation i.e. the quark propagator Schwinger-Dyson equation, in the Nambu-Gorkov basis. It is to extend the fermion field as:

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}, \qquad \overline{\Psi} = (\overline{\psi}, \overline{\psi}_C)\,,$$

$$\mathbf{S}^{-1}(p) = \mathbf{S}_0^{-1}(p) + \Sigma(p), \qquad \Sigma(p) = \int_q g^2 G^{aa'}_{\mu\nu}(q-p)[\Gamma^{(0)}_\mu]^a\,\mathbf{S}(q)\,\Gamma^{a'}_\nu(q,p)$$

The quark gluon vertex is essential input for the quark pairing gap equation.

In vertex DSE, diagram A is non Abelian diagram and diagram B is the Abelian diagram similar to QED.

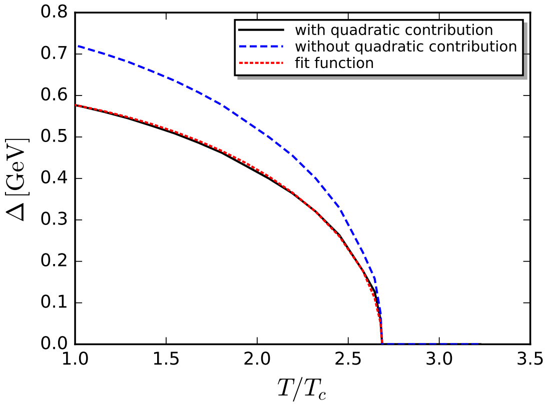


FG, Y Lu, YX Liu, arXiv:2403.16816

A new type of pairing at zero chemical potential

The pairing can be expanded as:

$$\Delta \propto \frac{3}{2}\langle g^2 A^2\rangle - \frac{3}{2}\langle g^2 \frac{k_4^+ p_4^+}{k_+^2}(G_L(\bar{k}^2) + 2G_T(\bar{k}^2))\rangle,$$

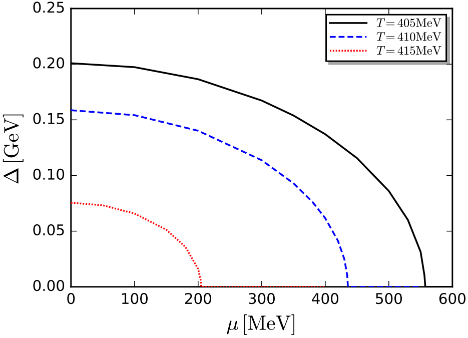


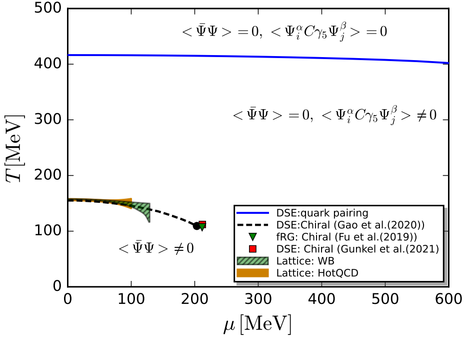
- The quark pairing gap is related to the dimensional 2 gluon condensate and thus dominant by the glue dynamics.
- A second order phase transition at temperature T_Δ, as one has $\Delta = 0$ above T_Δ, and near below T_Δ:

$$\Delta^2 \propto 1 - (T/T_\Delta)^a,$$

with the best fit as $a = 2.16$.

- The relation then yields a mean field critical exponent as $\beta = 1/2$.





The pairing phase in $T - \mu$ plane:

- Represents a color deconfined phase above the chiral phase transition;
- Quarks are confined into colored bound states as a partial deconfined phase;
- Temperature range $T \in [T_c, T_\Delta \approx 2 - 3\,T_c]$, overlaping with Chiral Spin Symmetric phase and the other conjectured strongly coupled states in sQGP.

Summary

The conclusions and discussions:

- The functional QCD approaches **calculate** the chiral PT and give the CEP located at $\mu_B \approx 600 - 700$ MeV/ $\sqrt{s_{\rm NN}} \approx 3 - 5$ GeV.
- The scaling analysis of LYE singularities also suggest similar results for CEP (3d O(4) scaling at physical mass is ruled out).
- The deconfinement characterized by the Polyakov loop **coincide** with chiral PT, especially with the same location of CEP.
- The quark pairing appears **near above** T_c which can possibly describes all the exotic properties of the rich phases like sQGP, quarkyonic, CSC matter in a unified way (No first order phase transition though).

Thank you!