Chiral Symmetry for an Accelerated and Rotated Observer

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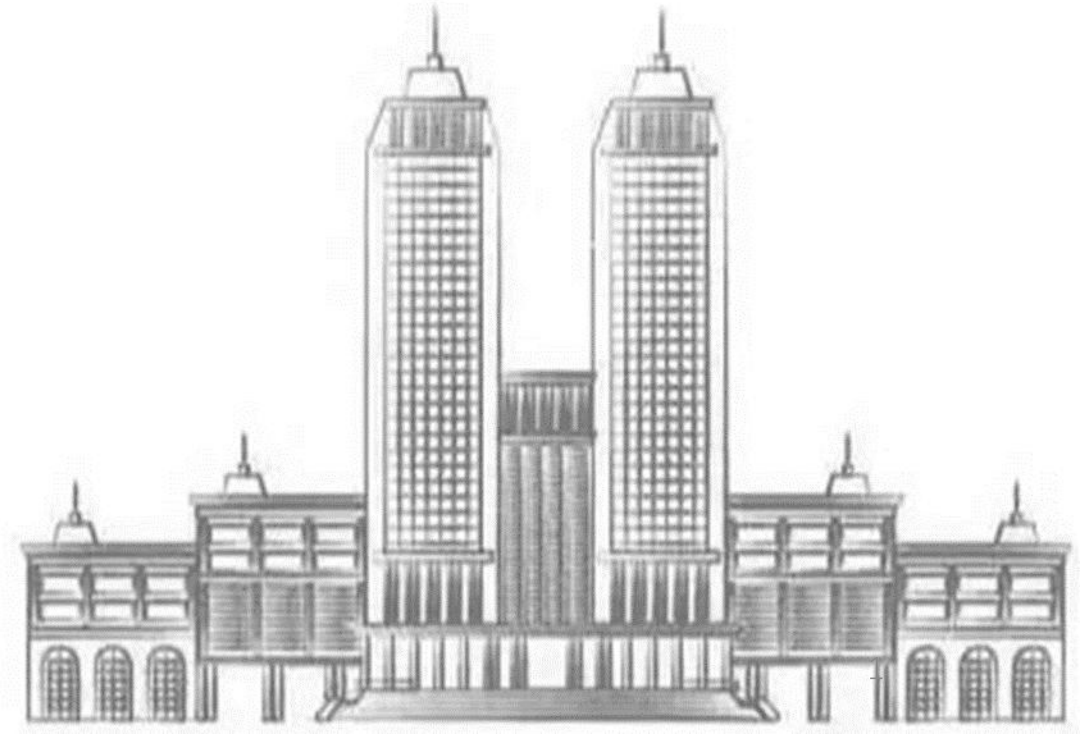


Motivation

Chiral phase transition under different condition have been widely studied(such as high temperature, electromagnetic field, rotation and so on). Especially, rotation has attract many attentions in the recent years as rapid rotation exist in non-central heavy ion collision ($\boldsymbol{\omega \sim 10^{21}\ s^{-1}}$) .

What about the acceleration?

Acceleration:

The famous effect induced from acceleration is **Unruh effect**. [Unruh, PRD 14 (1976) 870]

The Hawking–Unruh effect predicts that the accelerated observer sees Minkowski vacuum state as a thermal bath of particles with temperature $\boldsymbol{T_U = a/2\pi}$.

Particle in heavy ion collision may undergo a typical acceleration $\boldsymbol{a \sim 1\ GeV\ (T \sim 200\ MeV)}$

[D. Kharzeev Nucl. Phys. A753, 316 (2005)]

Formalism

$$\mathcal{L}_{NJL}=\bar{\psi}[i\gamma^{\mu}\nabla_{\mu}-m_0]\psi+\frac{G}{2}\left[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma^5\psi)^2\right]$$

$$\{\gamma_{\mu}(x),\gamma_{\nu}(x)\}=2g_{\mu\nu}(x),\{\gamma_{\hat{m}},\gamma_{\hat{n}}\}=2\eta_{\hat{m}\hat{n}}$$

$$g_{\mu\nu}g^{\nu\rho}=\delta_{\mu}^{\rho},g^{\mu\nu}(x)=e_{\hat{m}}^{\mu}(x)e^{\nu\hat{m}}(x),\gamma_{\mu}(x)=e_{\mu}^{\hat{m}}(x)\gamma_{\hat{m}}.$$

Covariant derivative : $\nabla_{\mu}=\partial_{\mu}+\Gamma_{\mu},\Gamma_{\mu}=-\frac{i}{4}\omega_{\mu\hat{i}\hat{j}}\sigma^{\hat{i}\hat{j}},\sigma^{ij}=\frac{i}{2}[\gamma^{\hat{i}},\gamma^{\hat{j}}],\omega_{\mu\hat{i}\hat{j}}=g_{ab}e_{\hat{i}}^{a}\nabla_{\mu}e_{\hat{j}}^{b}$

$$g_{\mu\nu}=\begin{pmatrix}(1+az)^2-\omega^2r^2 & \omega y & -\omega x & 0\\ \omega y & -1 & 0 & 0\\ -\omega x & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$$

$$\gamma^0(x)=\frac{1}{1+\boldsymbol{a}\cdot\boldsymbol{x}}\gamma^{\hat{0}},\gamma^{i}(x)=\frac{(\boldsymbol{\omega}\times\boldsymbol{x})^{i}}{1+\boldsymbol{a}\cdot\boldsymbol{x}}\gamma^{\hat{0}}+\gamma^{\hat{i}}$$

$$\Gamma_0=-\frac{i}{2}\boldsymbol{\omega}\cdot\sigma+\frac{1}{2}\boldsymbol{a}\cdot\boldsymbol{\alpha},$$

NJL model action in rotation and acceleration frame:

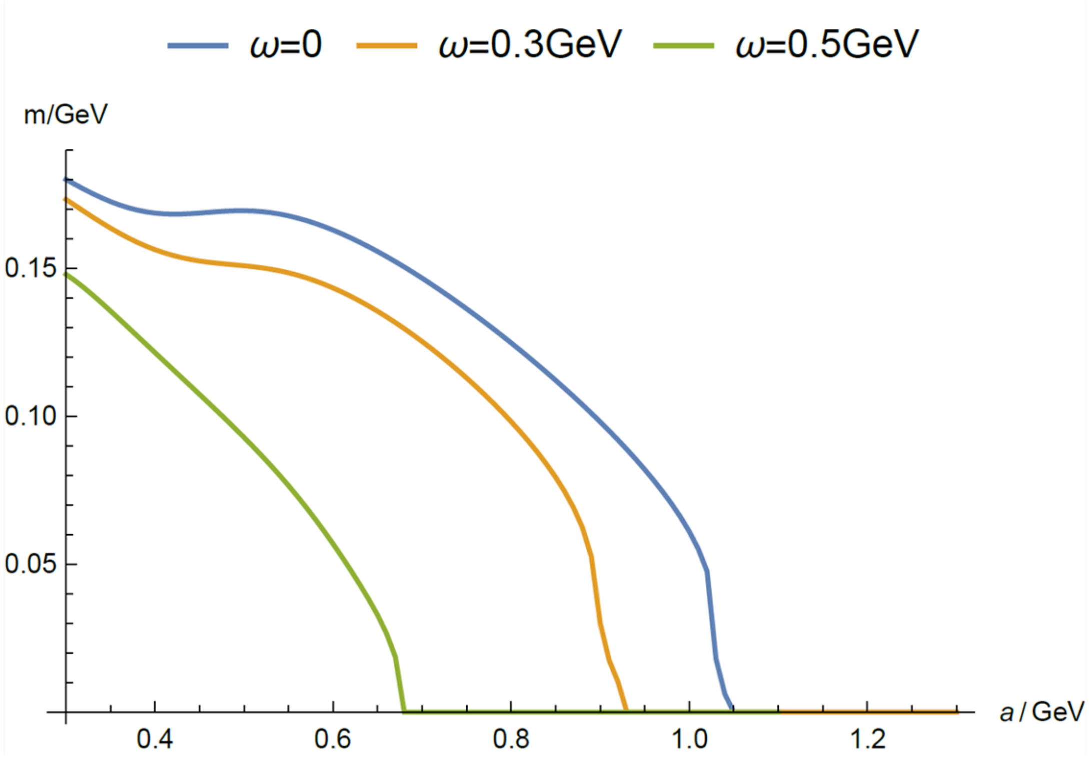
$$S=\int d^4x\left\{\bar{\psi}\left[i\gamma^{\hat{\mu}}\partial_{\mu}+ia\cdot x\gamma^{\hat{i}}\partial_{i}+\frac{i}{2}a\cdot\gamma+\gamma^{\hat{0}}\omega\cdot J-m_0\phi\right]\psi+\frac{G}{2}\phi\left[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\psi)^2\right]\right\}$$

Where $\phi=1+\boldsymbol{a}\cdot\boldsymbol{x}$

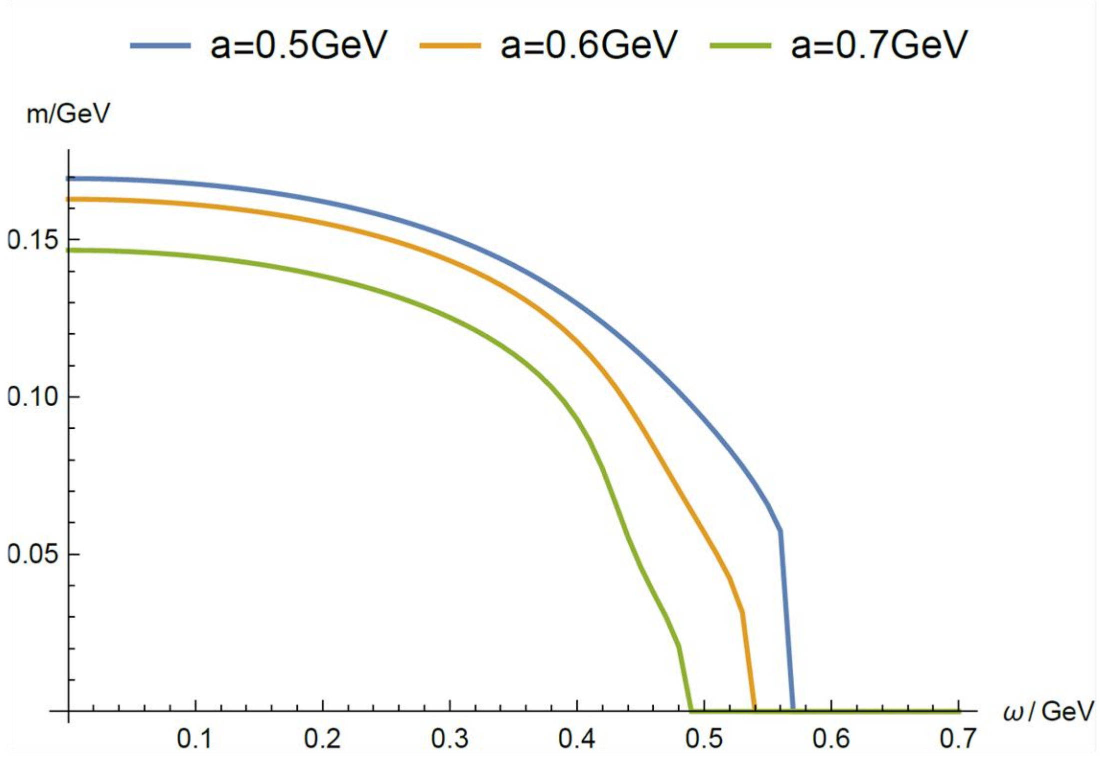
Gap Equation(most simple form), when $T=\frac{a}{2\pi}$

$$\frac{1}{G}=\sum_{l,k,s_1}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-is_1}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}\left\{\tanh\left(\frac{\Omega-\omega j}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}$$
$$\times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)+J_{l+1}^2(p_{l,k}r)\right]$$

Results



Chiral condensate as a function of acceleration

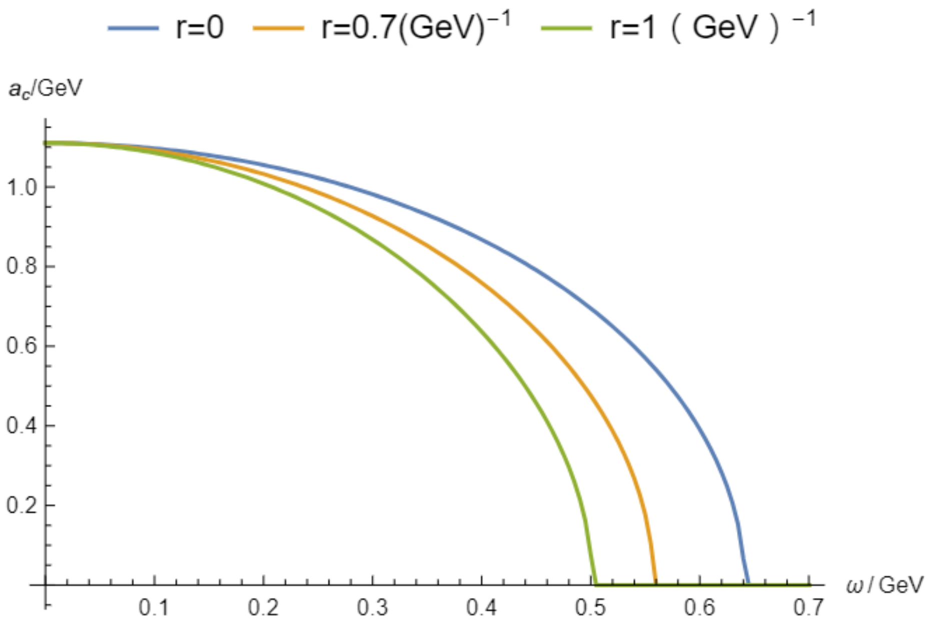


Chiral condensate as a function of rotation

When $T = T_U$, both acceleration and rotation reduce the chiral condensate, leading to the restoration of chiral symmetry.

Critical acceleration

We define critical acceleration a_c where the m reach to zero.



$$G\left(\frac{\Lambda^2}{2\pi^2} - \frac{a^2(r^4\omega^4+1) - r^2\omega^4 + 3\omega^2}{24\pi^2(r^2\omega^2-1)^2}\right) = 1$$

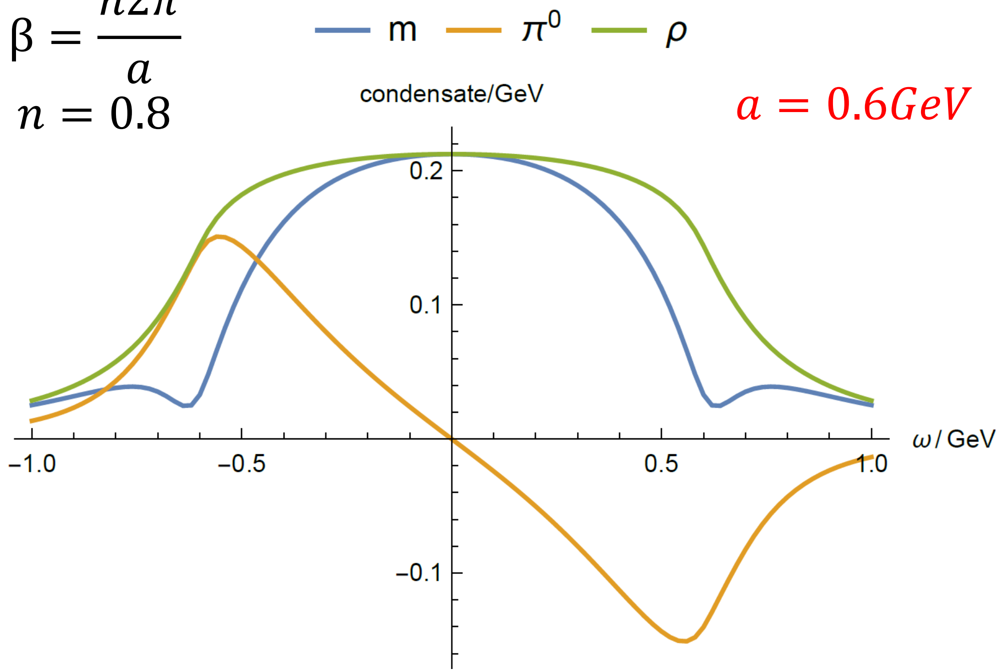
The critical acceleration (a_c) decreases as the angular velocity increases.
The effects of rotation become increasingly significant with increasing radius.
The rotation at $r = 0$ still has an effect because fermions have a spin of 1/2.

Result

Total condensate: $\rho = \left(m^2 + {\pi^0}^2\right)^{\frac{1}{2}}$

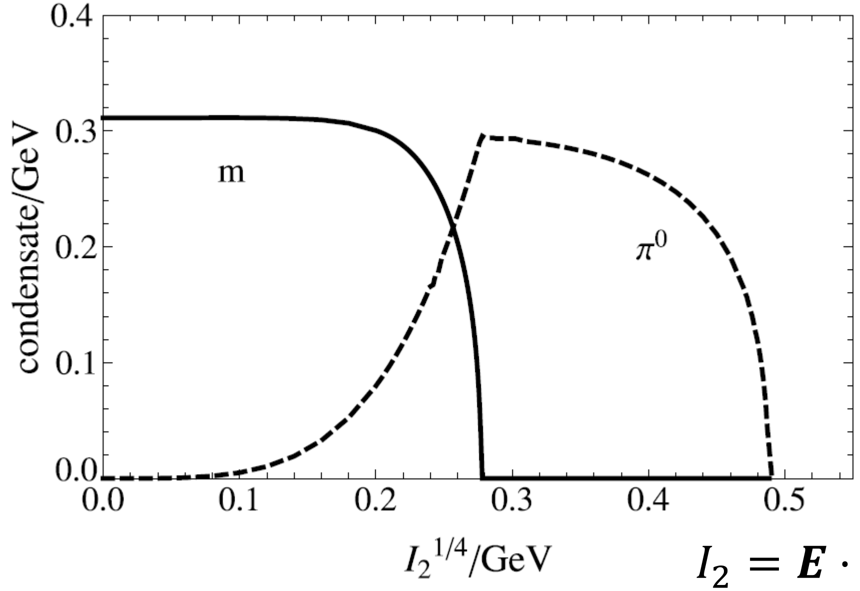
$\beta = \frac{n2\pi}{a}$

$n = 0.8$



$a = 0.6GeV$

Condensate as functions of rotation $\boldsymbol{\omega}$



$I_2 = \boldsymbol{E} \cdot \boldsymbol{B}$

The constitute quark mass m and $\boldsymbol{\pi}^0$ condensate as functions of $I^{\frac{1}{4}}$ (Cao, G., & Huang, X. G. (2016). Physics Letters B, 757, 1-5. arXiv:1509.06222)

- Acceleration and rotation induced the pion ($\boldsymbol{\pi}^0$) condensate when $\boldsymbol{T} \neq \boldsymbol{T_U}$
- The total condensate(ρ) rotate from the $\boldsymbol{\sigma}$-direction towards the $\boldsymbol{\pi}$-direction which can be also verified in the presence of parallel electromagnetic field($\boldsymbol{B} \cdot \boldsymbol{E}$)

Result

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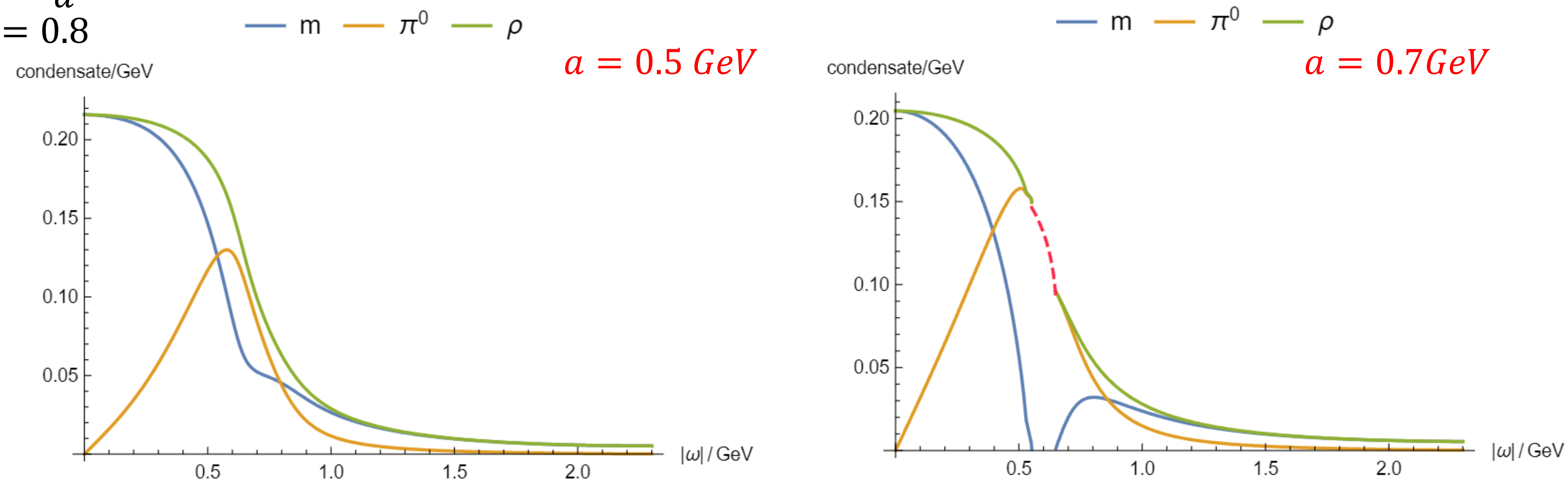

condensate as a function of rotation with different acceleration

- **With a larger a, the constitute quark mass will decrease to zero while the rotation increasing.**
- **There exist a region(red dash line) we cannot find solutions that satisfy both gap equations simultaneously.**
- **In the parallel electromagnetic study, they call this "chiral phase instability"** [Gaoqing Cao arXiv:2308.16448]

Summary

- When $T = T_U$, both acceleration and rotation reduce the chiral condensate, leading to the restoration of chiral symmetry. The critical acceleration (a_c) decreases as the angular velocity increases.
- In the presence current quark mass and $T \neq T_U$. There exist a non-zeros neutral pion condensate with the presence of $\boldsymbol{a} \cdot \boldsymbol{\omega}$. The total condensate($\rho$) rotate from the σ-direction towards the π-direction which can be also verified in the presence of parallel electromagnetic field($\boldsymbol{B} \cdot \boldsymbol{E}$)
- When the chiral phase instability show when the constituent quark mass reach zero. In this region, we cannot find solutions that satisfy both gap equations simultaneously. At this point, we only have one gap equation for the pion.

Thanks!

Formalism

Gap equation:
$$\frac{m-m_0}{G}=\sum_{l,k,s_1}\int d\Omega\frac{m}{2\pi}\frac{1}{N_{l,k}}\frac{-is_1}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)\right.$$
$$\left.+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)+J_l^2(p_{l+1,k}r)\right]+$$
$$\sum_{l,k,s_1}\int d\Omega\frac{-i\pi}{2\pi}\frac{1}{N_{l,k}}\frac{-i}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}\times\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)\right.$$
$$\left.+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)-J_{l+1}^2(p_{l,k}r)\right]$$

$$\frac{\pi}{G}=\sum_{l,k,s_1}\int d\Omega\frac{\pi}{2\pi}\frac{1}{N_{l,k}}\frac{-is_1}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)\right.$$
$$\left.+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)+J_l^2(p_{l+1,k}r)\right]+$$
$$im\sum_{l,k,s_1}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-i}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)\right.$$
$$\left.+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)-J_{l+1}^2(p_{l,k}r)\right]$$