



Studies on the Symmetry Energy of Strong-Interacting Matter

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Definition of E_{sym}

energy per nucleon in asymmetric nuclear matter (EOS)

$$E(\rho, \delta) \approx E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + E_{\text{sym}}^{(4)}(\rho)\delta^4 + \dots$$

SNM EOS

nucleon density $\rho = \rho_n + \rho_p$ isospin asymmetry $\delta = \frac{\rho_n - \rho_p}{\rho}$

expansion around saturation density ρ_0

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L \left(\frac{\rho - \rho_0}{3\rho_0} \right) + \frac{K_{\text{sym}}}{2!} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \frac{Q_{\text{sym}}}{3!} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^3 + \dots$$

slope

$$L = 3\rho_0 \left. \frac{dE_{\text{sym}}}{d\rho} \right|_{\rho_0}$$

curvature

$$K_{\text{sym}} = 9\rho_0^2 \left. \frac{d^2 E_{\text{sym}}}{d\rho^2} \right|_{\rho_0}$$

skewness

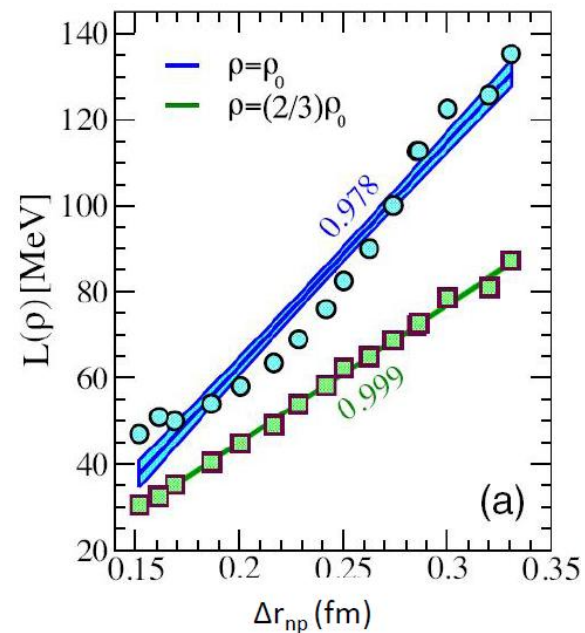
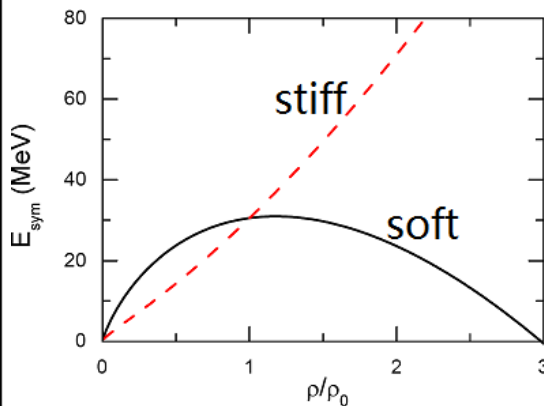
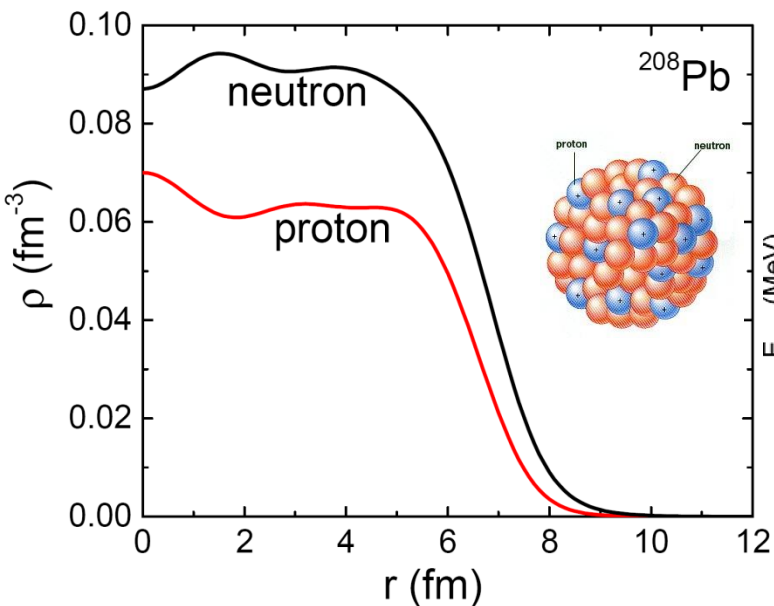
$$Q_{\text{sym}} = 27\rho_0^3 \left. \frac{d^3 E_{\text{sym}}}{d\rho^3} \right|_{\rho_0}$$

mean-field potential

$$U_{n/p}(\rho, \delta) \approx U_0(\rho) \pm U_{\text{sym}}(\rho)\delta + U_{\text{sym}}^{(2)}(\rho)\delta^2 + \dots$$

symmetry potential

E_{sym} and neutron skin



Neutron-Skin Thickness:

$$\Delta r_{np} = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle} \quad (\text{fm})$$

from parity-violating electron scatterings

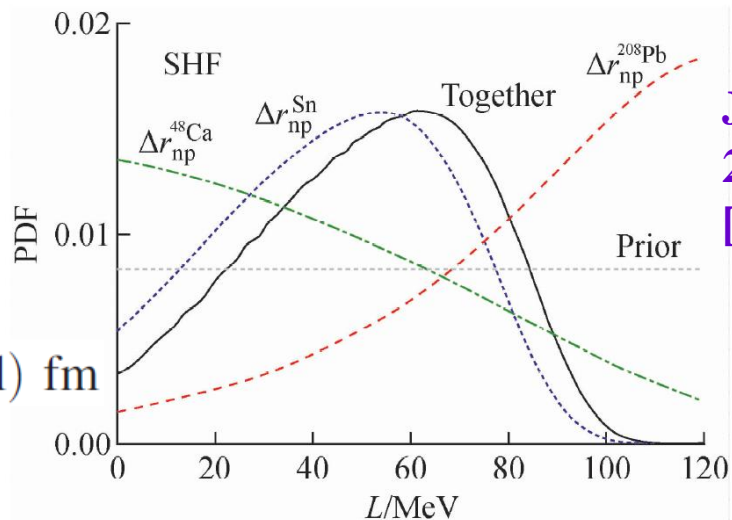
^{208}Pb (PREXII)

$$\Delta r_{np} = 0.283 \pm 0.071 \text{ fm}$$

^{48}Ca (CREX)

$$\Delta r_{np} = 0.121 \pm 0.026(\text{exp}) \pm 0.024(\text{model}) \text{ fm}$$

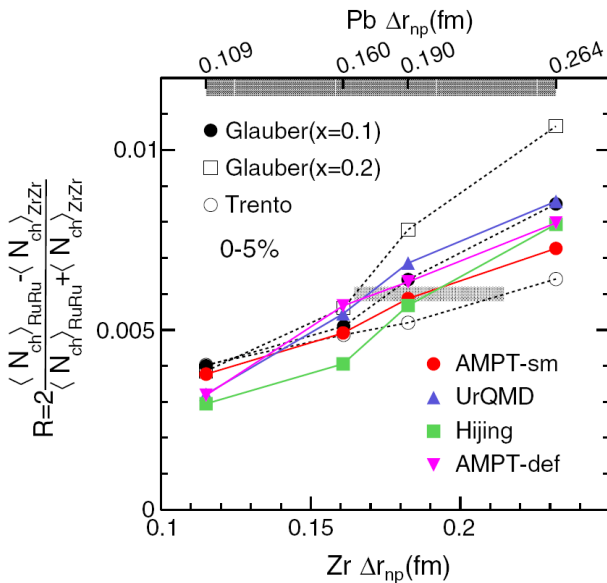
Posterior PDF of L from Bayesian inference



JX,
2301.07884
[nucl-th]

Probing Δr_{np} with relativistic HIC

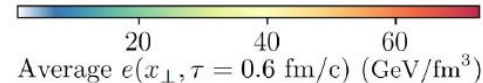
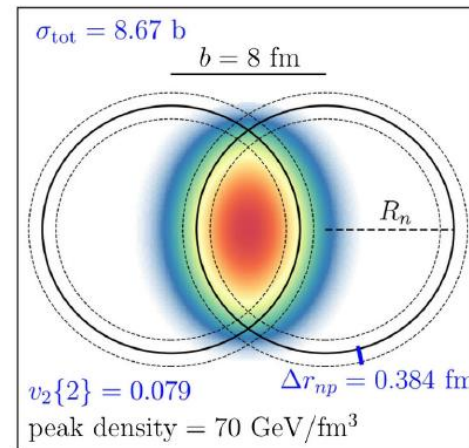
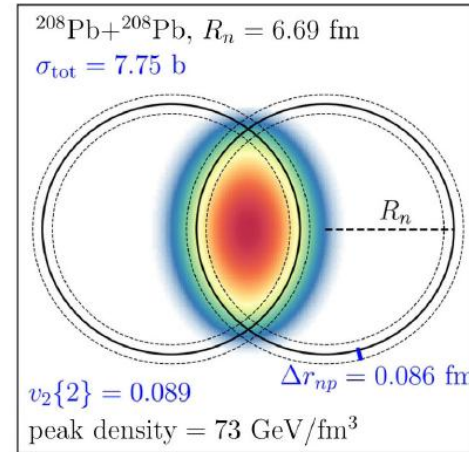
Charged-particle multiplicity



H. L. Li, H. J. Xu, Y. Zhou, X. B. Wang, J. Zhao, L. W. Chen, and F. Q. Wang, PRL (2020)

using observables at midrapidities

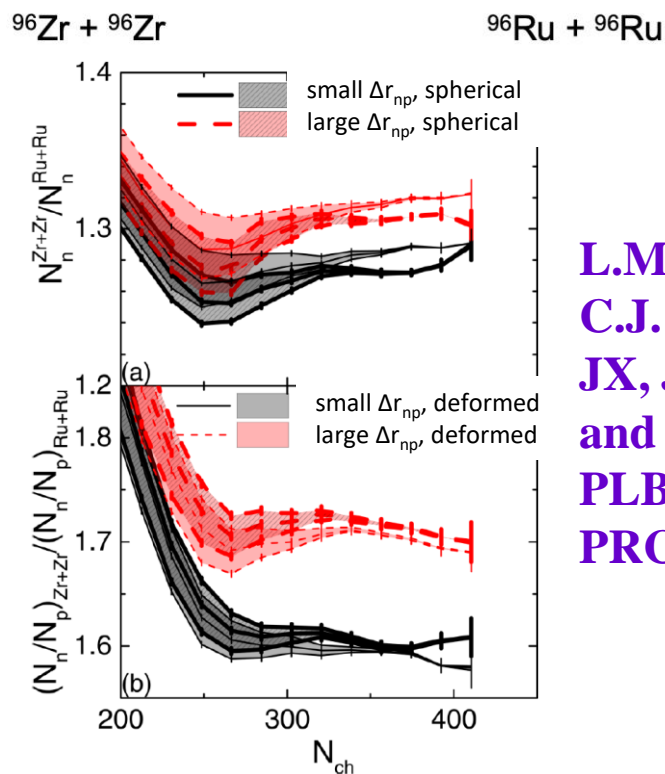
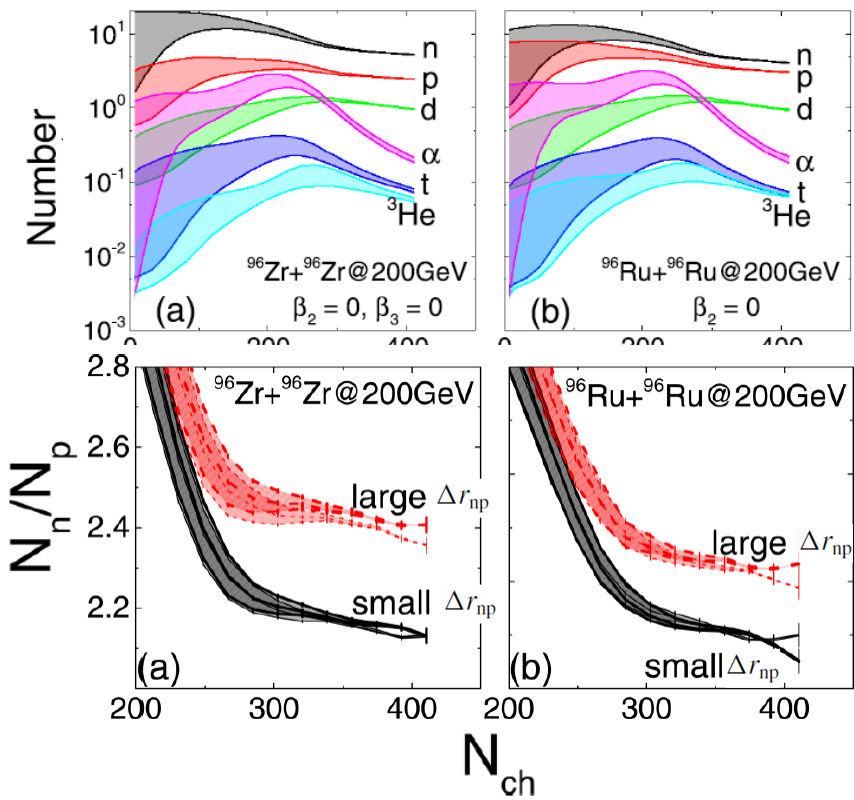
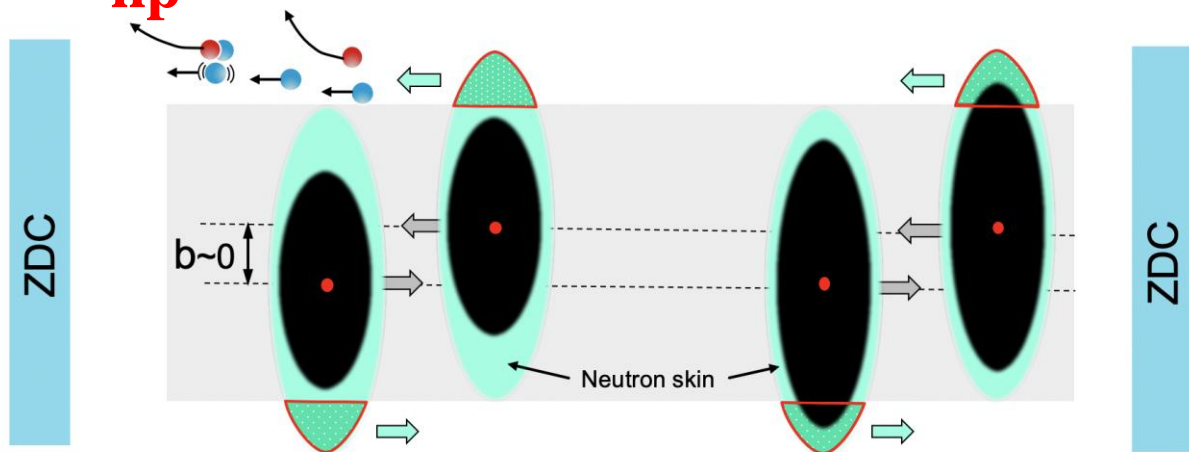
v_2 and $\langle p_T \rangle$



G. Giacalone, G. Nijs, and W. van der Schee, PRL (2023)

Probing Δr_{np} with relativistic HIC

using spectator
particle yield
at beam rapidity

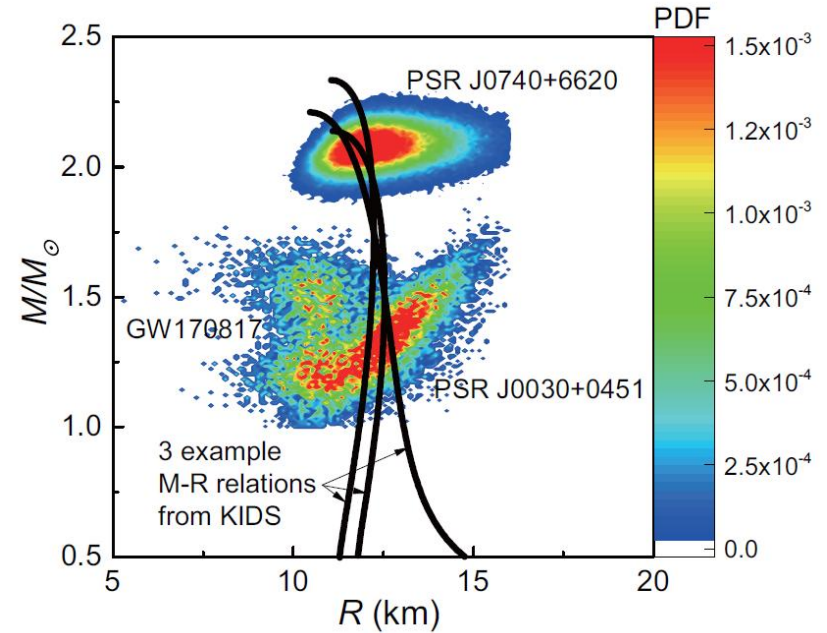


L.M. Liu,
C.J. Zhang,
JX, J.Y. Jia,
and G.X. Peng,
PLB (2022);
PRC (2022)

E_{sym} and neutron stars

3 energy-density functionals:

- Skyrme-Hartree-Fock (SHF)
- Korea-IBS-Daegu-SKKU (KIDS)
- Relativistic Mean-Field (RMF)



nucleonic DOF EOS $P(\epsilon)$

TOV equation

M-R relation

model coefficients

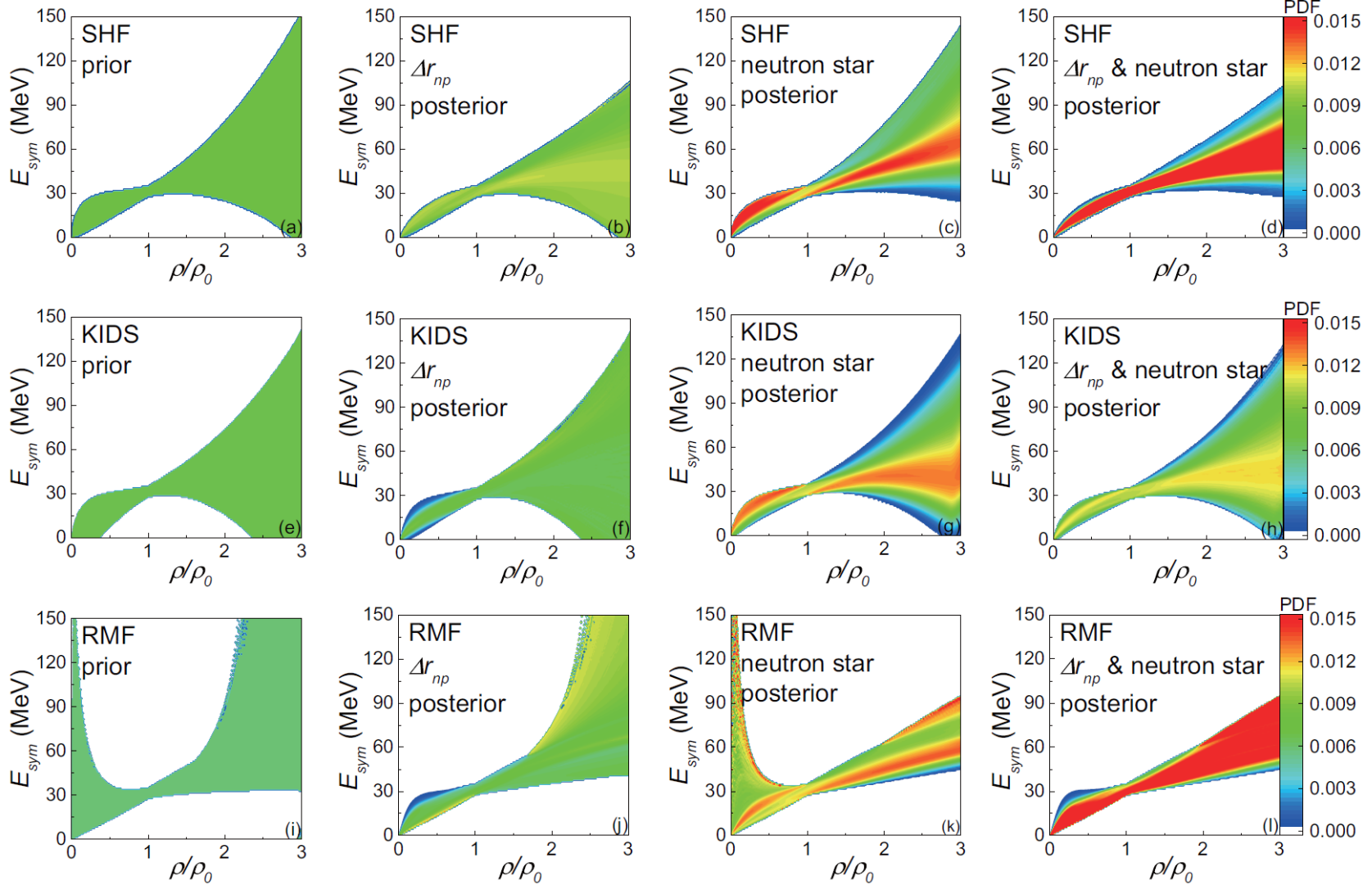
physics quantities

Bayesian analysis

Table 1 Prior ranges of model parameters in the standard SHF, KIDS, and RMF models for the Bayesian analysis in the present study

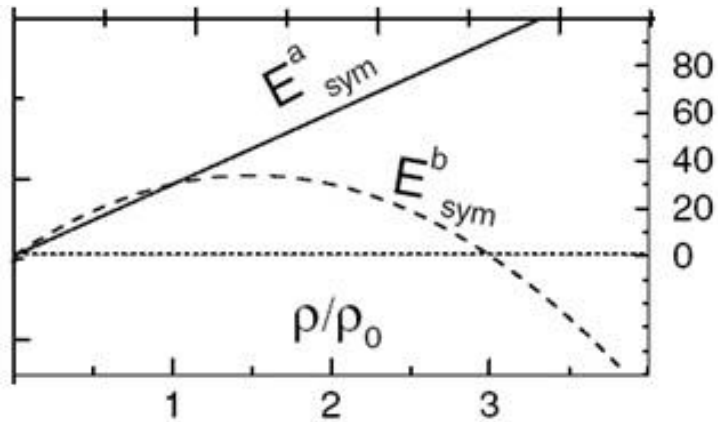
Parameters	SHF	KIDS	RMF
K_0 (MeV)	220-260	220-260	220-260
Q_0 (MeV)	–	–800-400	–800-400
E_{sym}^0 (MeV)	28.5-34.9	28.5-34.9	28.5-34.9
L (MeV)	30-90	30-90	30-90
K_{sym} (MeV)	–	–400-100	–
Q_{sym} (MeV)	–	–200-800	–
m_s^*/m	0.5-0.9	0.5-0.9	0.5-0.9
m_v^*/m	0.5-0.9	0.5-0.9	–

E_{sym} from both neutron stars and Δr_{np}

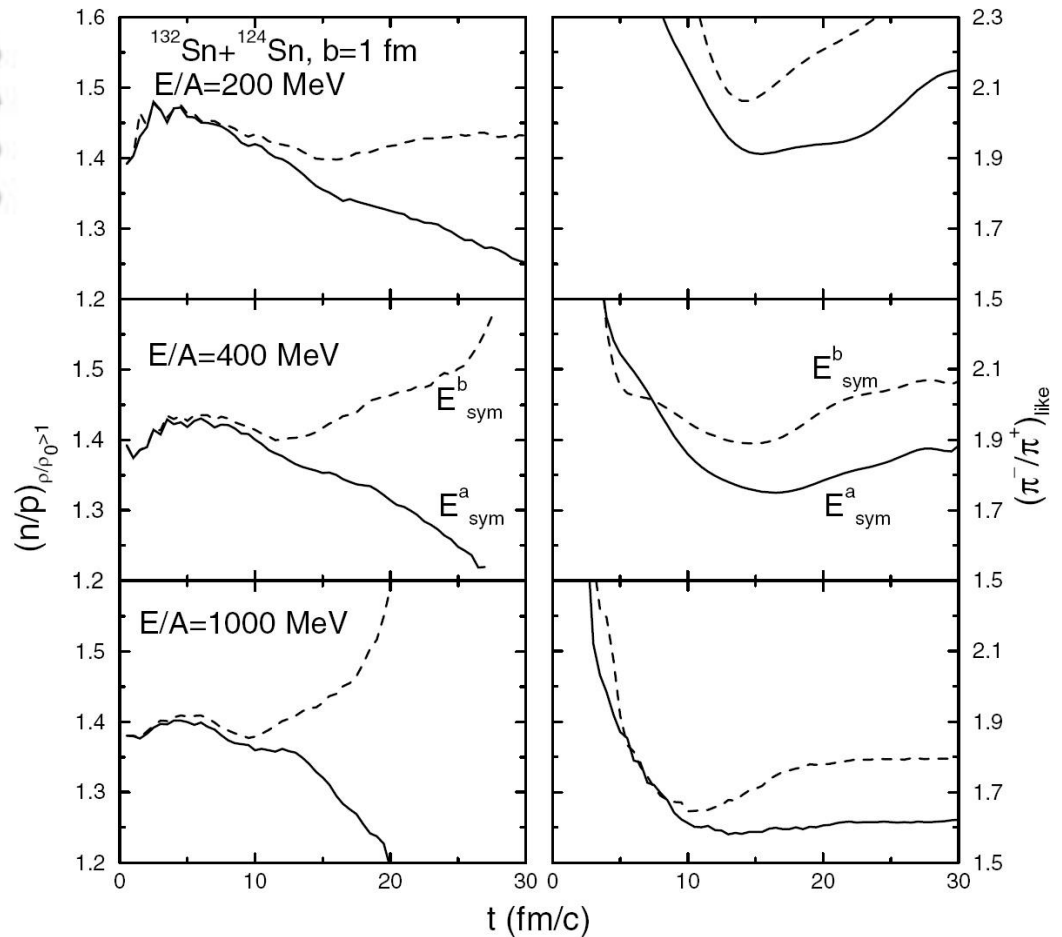


J. Zhou and JX, *Sci. China-Phys. Mech. Astron.* (2024)

E_{sym} and π^-/π^+ ratio in HIC



B.A. Li, PRL (2002)



Divergence of E_{sym} from FOPI π^-/π^+ data

LQMD, Feng/Jin, PLB (2010)

TuQMD

Cozma/Trautmann/Li,
PRC (2013)

Pion s&p-wave
from thermal model
Xu/Ko/Oh, PRC (2010)

Xu/Chen/Li/Ko/Ma,
PRC (2013)

Pion s-wave from pBUU
Hong/Pawel, PRC (2014)

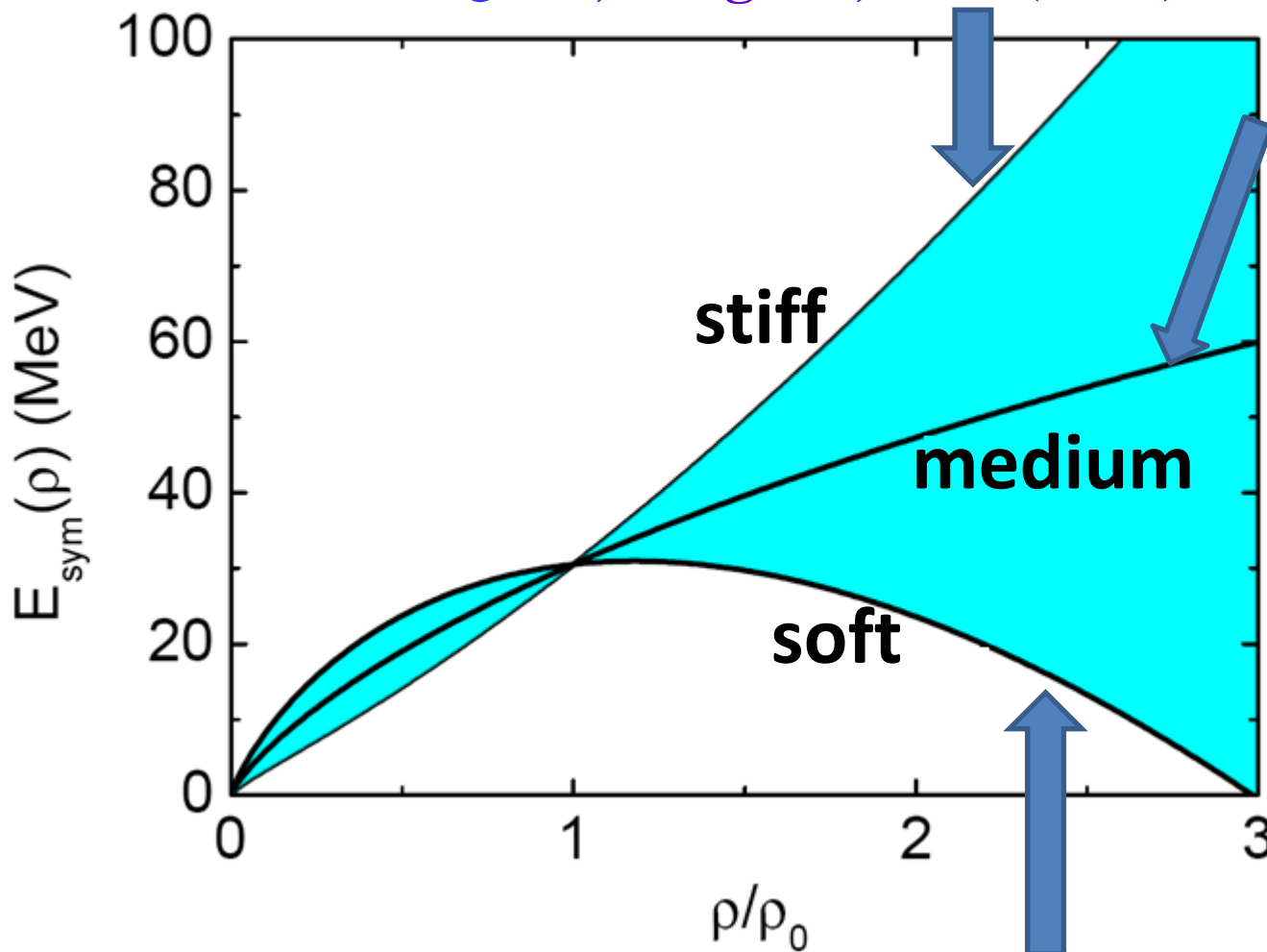
Pion threshold effect
from RVUU
Song/Ko, PRC (2015)

Energy conservation
from TuQMD
Cozma, PLB (2016)

Clustering and PB
from JAM/AMD
Ikeno/Ono/Nara/Ohnishi,
PRC (2016,2019)

Pion s&p-wave from RVUU
Zhang/Ko (2017,2018)

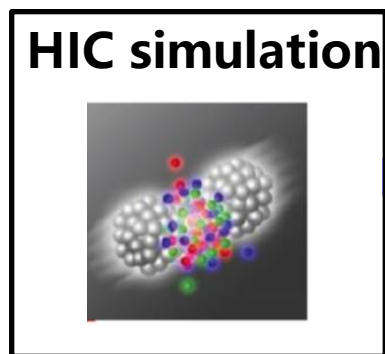
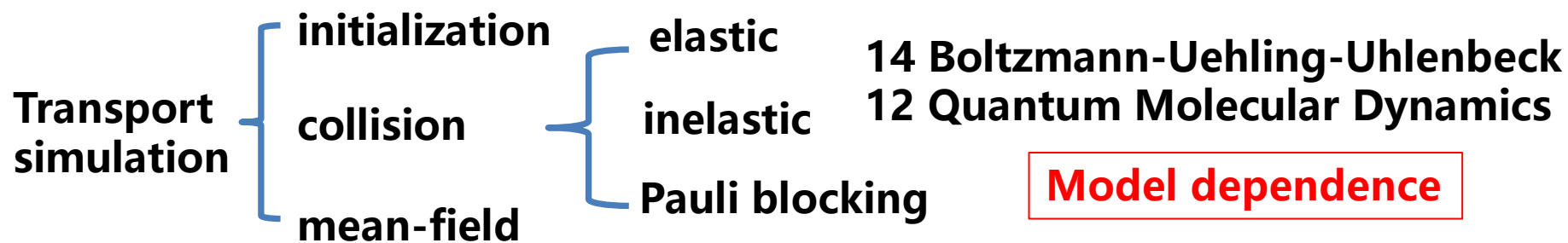
...



IBUU04, Xiao/Chen/Li/Zhang, PRL, (2009)

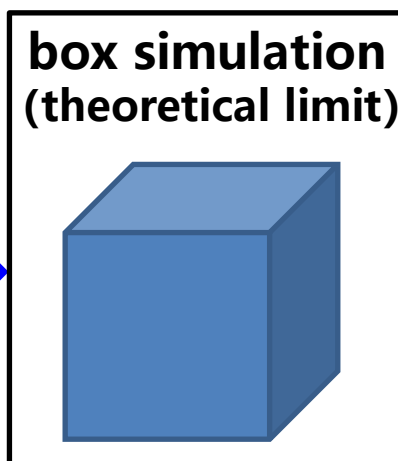
IBL, Xie/Su/Zhu/Zhang, PLB, (2013)

Reduce uncertainty of transport simulation (TMEP)



JX, et al.,
PRC (2016)

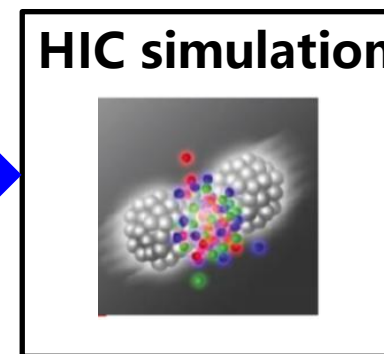
initialization;
theoretical error
of directed flow



Y.X. Zhang, et al., PRC (2018)
elastic collisions, Pauli blocking

A. Ono, JX, et al., PRC (2019)
inelastic collisions, π production

M. Colonna, et al., PRC (2021)
mean-field evolution



JX, et al.,
PRC (2024)

reduce uncertainty of
 π^-/π^+ from 5.2%
to 1.6%

main uncertainties:
mean-field, Pauli blocking

Quark matter E_{sym}

strange quark matter EOS

$$E(\rho_B, \delta, \rho_s) = E_0(\rho_B, \rho_s) + E_{\text{sym}}(\rho_B, \rho_s)\delta^2 + \vartheta(\delta^4)$$

baryon density $\rho_B = (\rho_u + \rho_d + \rho_s)/3$

isospin asymmetry $\delta = 3 \frac{\rho_d - \rho_u}{\rho_d + \rho_u}$

$$E_{\text{sym}}(\rho_B, \rho_s) = \frac{1}{2!} \left. \frac{\partial^2 E(\rho_B, \delta, \rho_s)}{\partial \delta^2} \right|_{\delta=0}$$

Nambu-Jona-Lasinio model

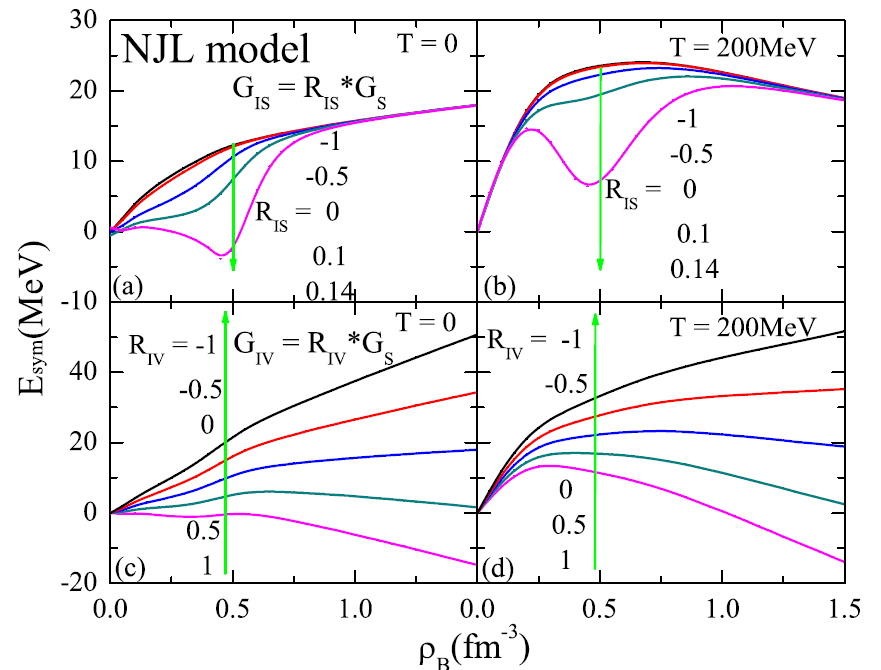
$$\mathcal{L}_{\text{NJL}} = \bar{q}(i\partial - \hat{m})q + \frac{G_S}{2} \sum_{a=0}^8 [(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2]$$

$$+ \frac{G_V}{2} \sum_{a=0}^8 [(\bar{q}\gamma_\mu\lambda_a q)^2 + (\bar{q}\gamma_5\gamma_\mu\lambda_a q)^2]$$

$$- K \{ \det[\bar{q}(1 + \gamma_5)q] + \det[\bar{q}(1 - \gamma_5)q] \}$$

$$+ G_{IS} \sum_{a=1}^3 [(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2]$$

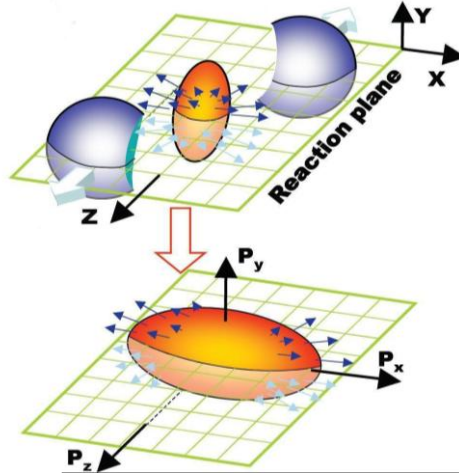
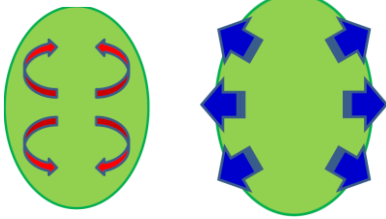
$$+ G_{IV} \sum_{a=1}^3 [(\bar{q}\gamma_\mu\lambda_a q)^2 + (\bar{q}\gamma_5\gamma_\mu\lambda_a q)^2],$$



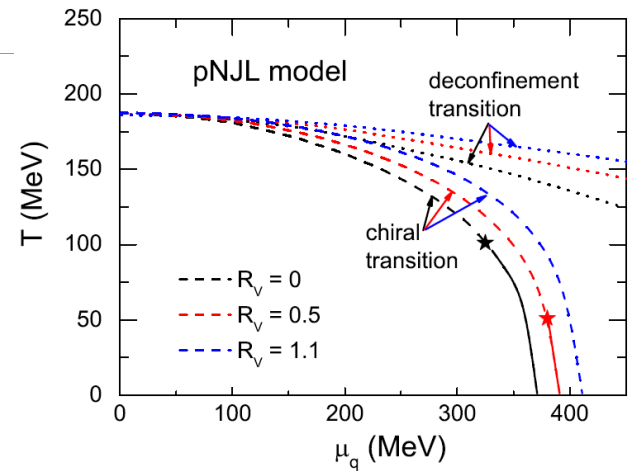
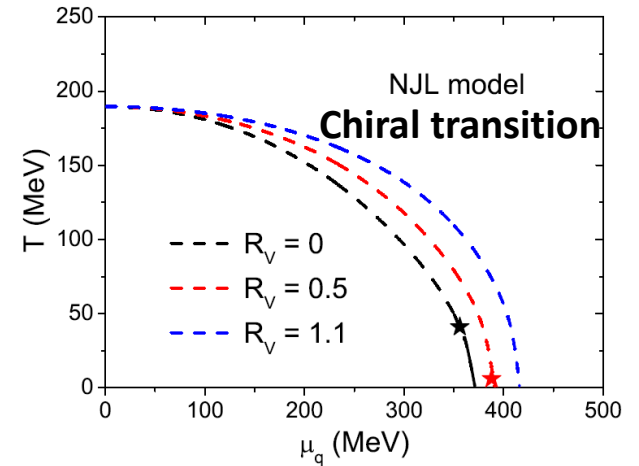
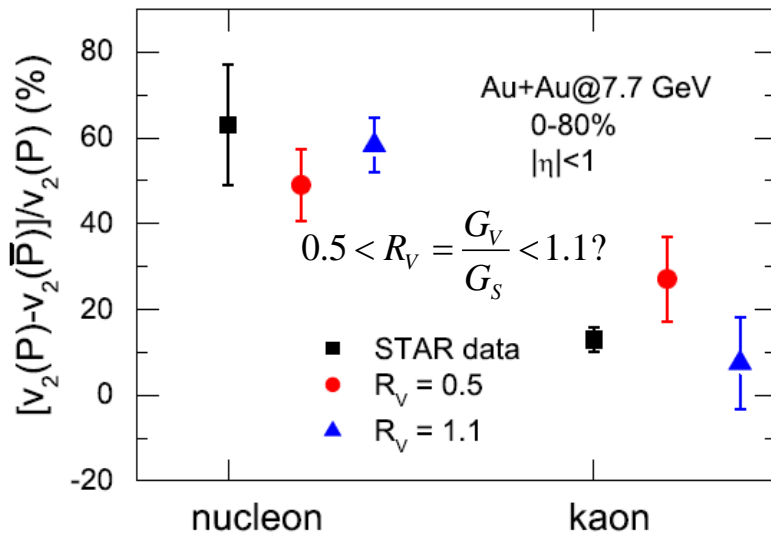
Baryon-antibaryon “ E_{sym} ”

$$v_2 = \langle (p_x^2 - p_y^2) / p_T^2 \rangle$$

attractive repulsive



$$H_i = \sqrt{M_i^2 + p_i^{*2}} \pm \frac{2}{3} G_V (\rho_u^0 + \rho_d^0 + \rho_s^0)$$

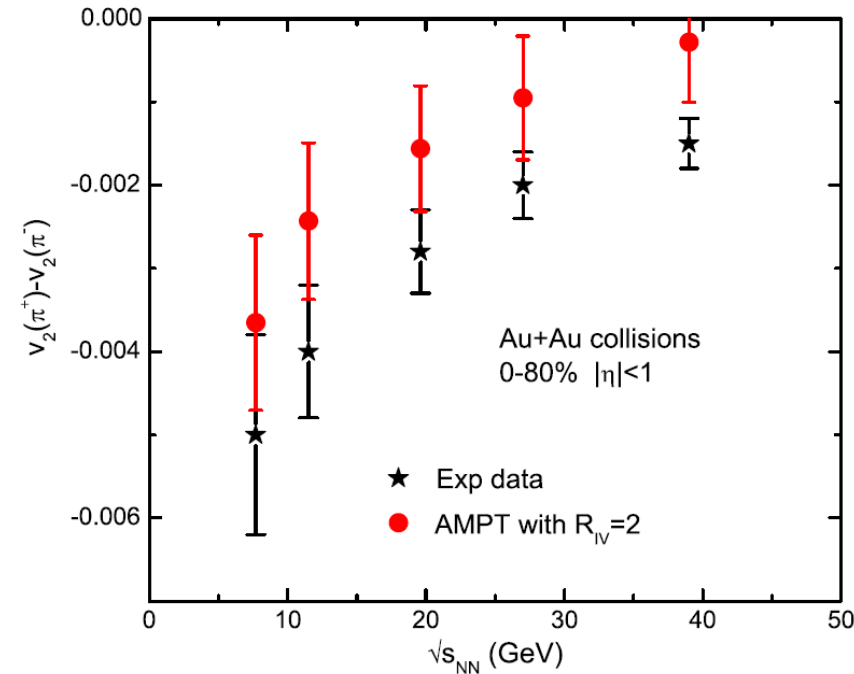


JX, T. Song, C.M. Ko, and F. Li, PRL (2014)

u-d quark E_{sym}

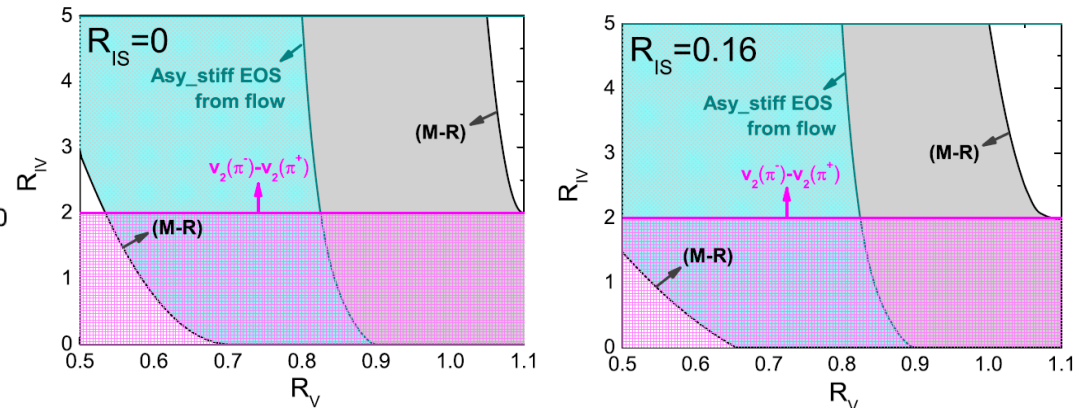
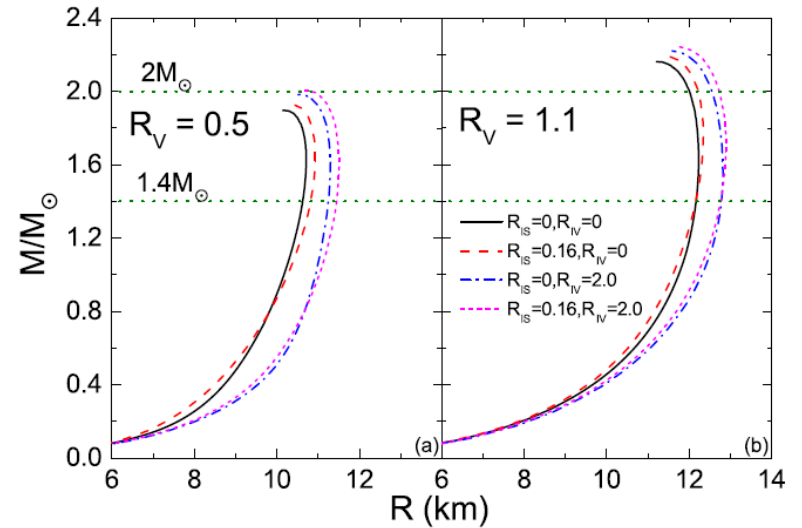
$$H_i = \sqrt{M_i^2 + p_i^{*2}} \pm \frac{2}{3} G_V (\rho_u^0 + \rho_d^0 + \rho_s^0) \pm G_{IV} \tau_{3i} (\rho_u^0 - \rho_d^0)$$

$$R_V = G_V / G_S \quad R_{IV} = G_{IV} / G_S \quad R_{IS} = G_{IS} / G_S$$



H. Liu, F.T. Wang, K.J. Sun,
JX, and C.M. Ko, PLB (2019)

M-R relation for strange quark stars



H. Liu, JX, and C.M. Ko, PLB (2020)

A few remarks

- Our knowledge on asymmetric matter is still lacking compared with that on symmetric matter
- Constraints on E_{sym} generally need accurate and reliable theoretical models
- Different probes are needed to constrain E_{sym} at various densities/temperatures (Bayesian analysis)

Thank you!

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