Higher-order Skyrme pseudopotential for transport model and neutron stars

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What is higher-order pseudopotential?

The two-body part of the standard Skyrme effective interaction:

$$
\begin{aligned}
v_c =& t_0\left(1+x_0\hat{P}_\sigma\right)\delta\left(\vec{r}_1-\vec{r}_2\right) \\
&+t_1^{[2]}\left(1+x_1^{[2]}\hat{P}_\sigma\right)\underset{\hat{A}}{\boxed{\frac{1}{2}\left[\hat{\vec{k}}'^2\delta\left(\vec{r}_1-\vec{r}_2\right)+\delta\left(\vec{r}_1-\vec{r}_2\right)\hat{\vec{k}}^2\right]}}+t_2^{[2]}\left(1+x_2^{[2]}\hat{P}_\sigma\right)\underset{\hat{B}}{\boxed{\hat{\vec{k}}'\cdot\delta\left(\vec{r}_1-\vec{r}_2\right)\hat{\vec{k}}}}
\end{aligned}
$$

(p^2,Skyrme pseudopotential up to N1LO)

$$
\begin{aligned}
&+t_2^{[4]}\left(1+x_2^{[4]}\hat{P}_\sigma\right)\left(\hat{\vec{k}}'\cdot\hat{\vec{k}}\right)\left(\hat{\vec{k}}'^2+\hat{\vec{k}}^2\right)+t_1^{[6]}\left(1+x_1^{[6]}\hat{P}_\sigma\right)\left(\hat{\vec{k}}'^2+\hat{\vec{k}}^2\right)\left[\frac{1}{2}\left(\hat{\vec{k}}'^2+\hat{\vec{k}}^2\right)^2+6\left(\hat{\vec{k}}'\cdot\hat{\vec{k}}\right)^2\right] \\
&+t_2^{[6]}\left(1+x_2^{[6]}\hat{P}_\sigma\right)\left(\hat{\vec{k}}'\cdot\hat{\vec{k}}\right)\left[3\left(\hat{\vec{k}}'^2+\hat{\vec{k}}^2\right)^2+4\left(\hat{\vec{k}}'\cdot\hat{\vec{k}}\right)^2\right]
\end{aligned}
$$

(p^6,Skyrme pseudopotential up to N3LO)

T.H.R Skyrme,
Nucl. Phys. 9, 615 (1959)

B.G. Carlsson et al. PRC78,044326 (2008)
F.Raimondi et al. PRC83,054311 (2011)

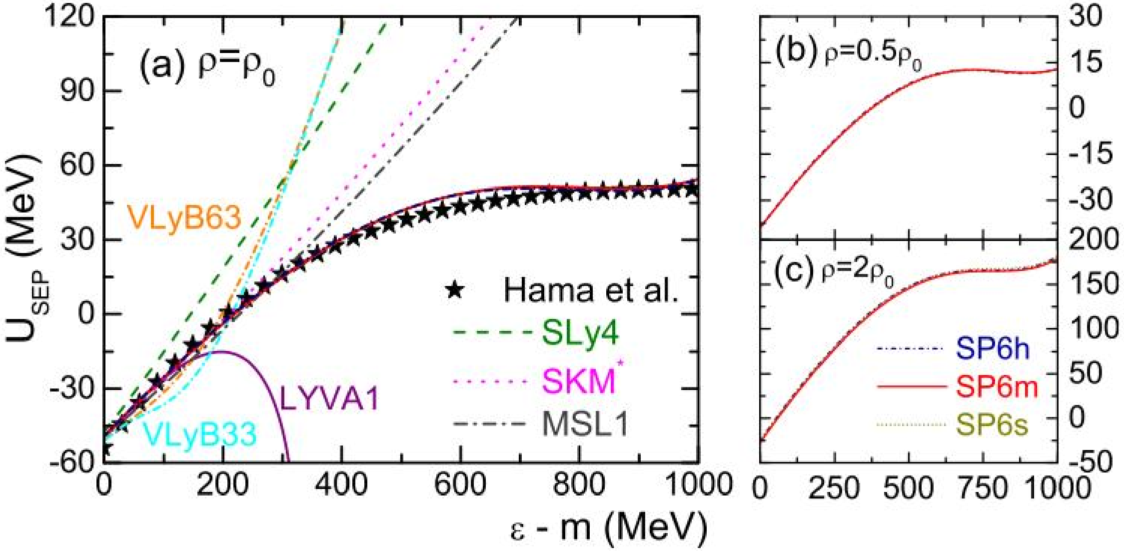
$$\left(\hat{A}+\hat{B}\right)^n$$

(p^{2n})

Why do we need higher-orders ?

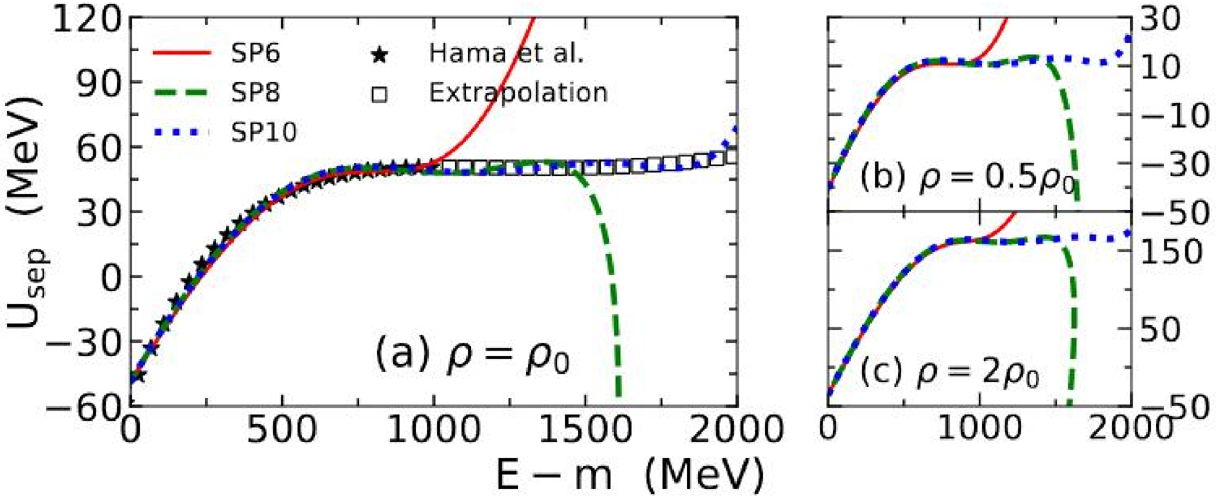
The mean-field potential (single-nucleon potential) is a basic input of BUU equation:

$$\frac{\partial f(\vec{r},\vec{p},t)}{\partial t} + \vec{\nabla}_p U(\vec{r},\vec{p}) \cdot \vec{\nabla}_r f - \vec{\nabla}_r U(\vec{r},\vec{p}) \cdot \vec{\nabla}_p f = I_c\left(f,\sigma_{NN}\right) \qquad U_\tau(\vec{r},\vec{p}) = \frac{\delta \mathrm{H}^{\mathrm{pot}}}{\delta n_\tau(\vec{r},\vec{p})}$$



standard Skyrme(p^2) vs N3LO(p^6)

300 MeV **1 GeV**



N3LO(p^6) vs N4LO(p^8) vs N5LO(p^{10})

1 GeV **1.5 GeV** **2 GeV**

R.Wang et al. PRC 98,054618 (2018)

This work (in preparation)

The Symmetry Energy

The density-dependent term

$$V_N^{\mathrm{DD}} = \sum_{n=1}^{N} \frac{1}{6} t_3^{[2n-1]} \left(1 + x_3^{[2n-1]} \hat{P}_\sigma\right) \rho^{\frac{2n-1}{3}} (\vec{R})$$

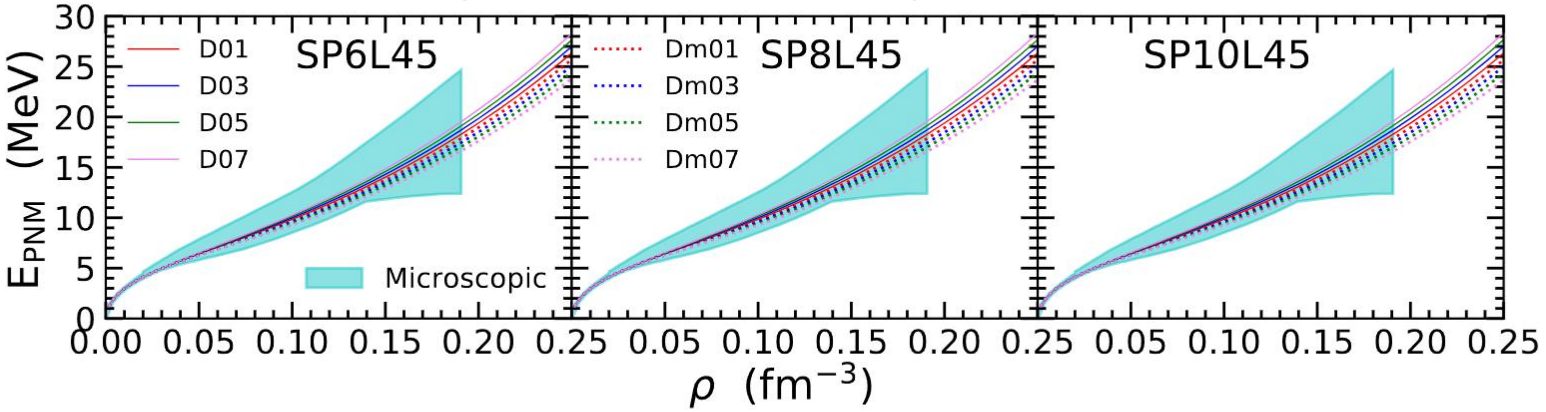
$$E(\rho, \delta) = E_0(\rho) + E_{\mathrm{sym}}(\rho)\delta^2 + E_{\mathrm{sym},4}(\rho)\delta^4 + \mathcal{O}\left(\delta^6\right)$$

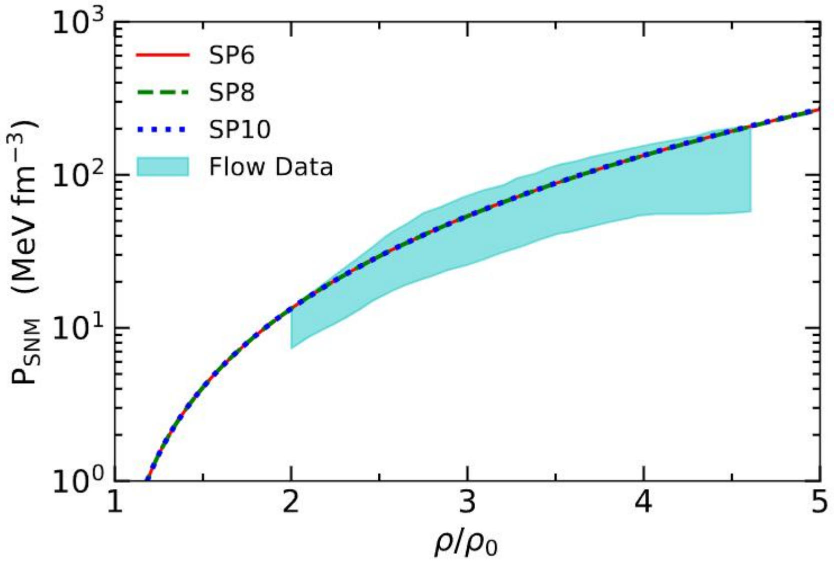
$$E_{\mathrm{sym}}(\rho) = E_{\mathrm{sym}}\left(\rho_0\right) + L\chi + \frac{K_{\mathrm{sym}}}{2!}\chi^2 \qquad \chi \equiv \frac{\rho - \rho_0}{3\rho_0}$$

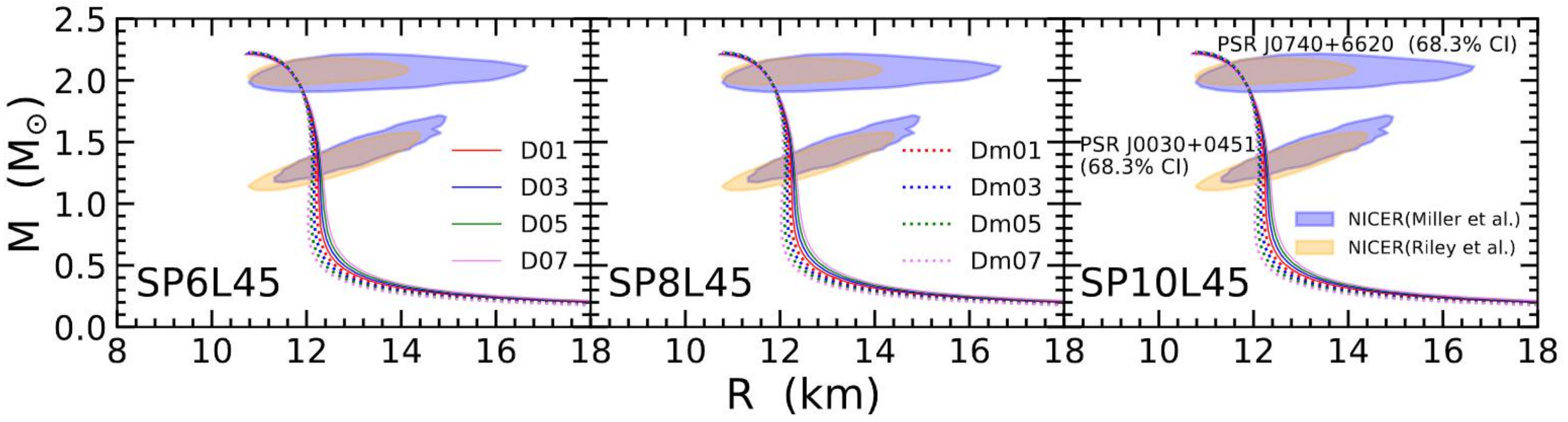
$$+ \frac{J_{\mathrm{sym}}}{3!}\chi^3 + \frac{I_{\mathrm{sym}}}{4!}\chi^4 + \frac{H_{\mathrm{sym}}}{5!}\chi^5 + \mathcal{O}\left(\chi^6\right)$$

Fermi momentum expansion

SPW et al. PRC 109,054623 (2024)







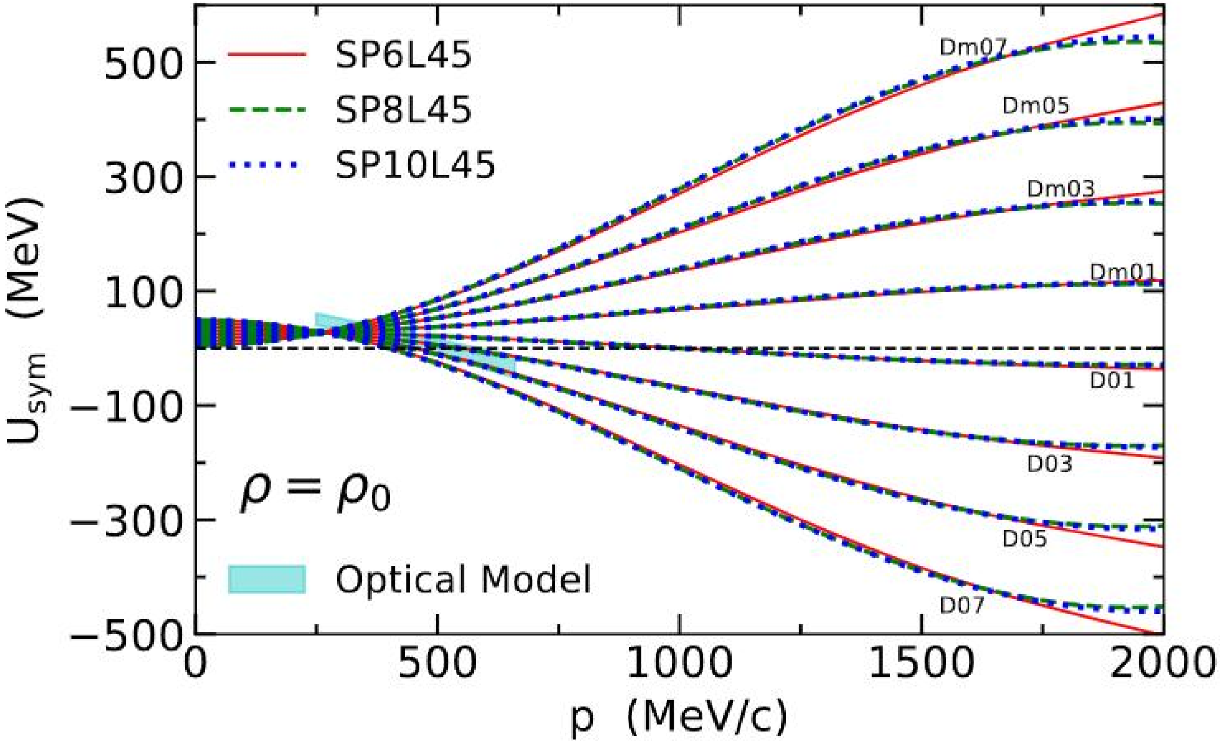
The symmetry potential

$$U_\tau(\rho,\delta,p) = U_0(\rho,p) + \sum_i U_{\mathrm{sym},i}(\rho,p)(\tau\delta)^i = U_0(\rho,p) + U_{\mathrm{sym},1}(\rho,p)(\tau\delta) + U_{\mathrm{sym},2}(\rho,p)(\tau\delta)^2 + \cdots$$

$$U_\tau(\rho,\delta,p) \approx U_0(\rho,p) + U_{\mathrm{sym}}(\rho,p)(\tau\delta)$$ **(Lane potential)**

$$E_{\mathrm{sym}}(\rho) = \frac{1}{3}\frac{p_F^2}{2m_s^*} + \frac{1}{2}U_{\mathrm{sym}}(\rho,p_F)$$

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k^2}{2m_0^*}\bigg|_{k_F} - \frac{1}{6}\left(\frac{\hbar^2 k^3}{m_0^{*2}}\frac{\partial m_0^*}{\partial k}\right)\bigg|_{k_F} + \frac{3}{2}U_{sym,1}(\rho,k_F) + \frac{\partial U_{sym,1}}{\partial k}\bigg|_{k_F}\cdot k_F + 3U_{sym,2}(\rho,k_F)$$



C.Xu et al. PRC82,054607 (2010)
C.Xu et al. NPA865,1 (2011)
R.Chen et al. PRC85,024305 (2012)

comparison to HADES data

Phase-space distribution functions $f(\vec{r},\vec{p},t)$ satisfy the BUU equation:

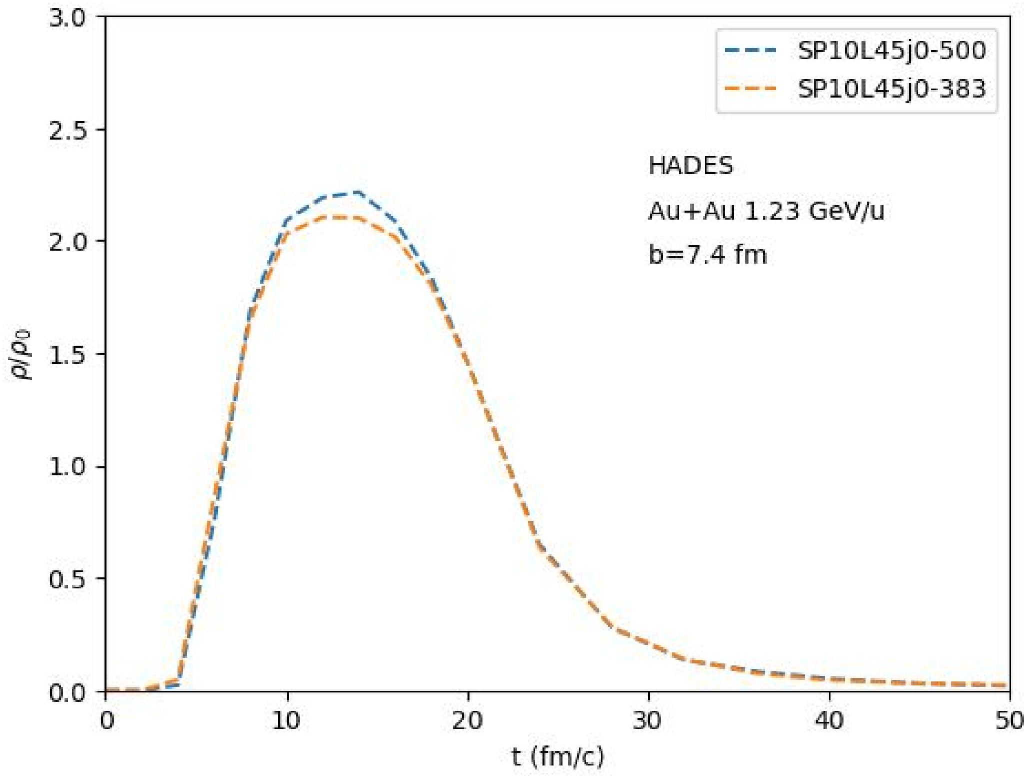
$$\frac{\partial f(\vec{r},\vec{p},t)}{\partial t}+\vec{\nabla}_p U(\vec{r},\vec{p})\cdot\vec{\nabla}_r f-\vec{\nabla}_r U(\vec{r},\vec{p})\cdot\vec{\nabla}_p f=I_c\left(f,\sigma_{NN}\right)$$

Lattice Hamilton method **R.Wang et al. PRC99,044607 (2019)**
Stochastic approach **R.Wang et al. PLB807,135532 (2020)**

HADES:
Fixed-target Au+Au collision at
$E_{\text{beam}} = 1.23A$GeV ($\sqrt{s_{NN}} = 2.4$GeV)

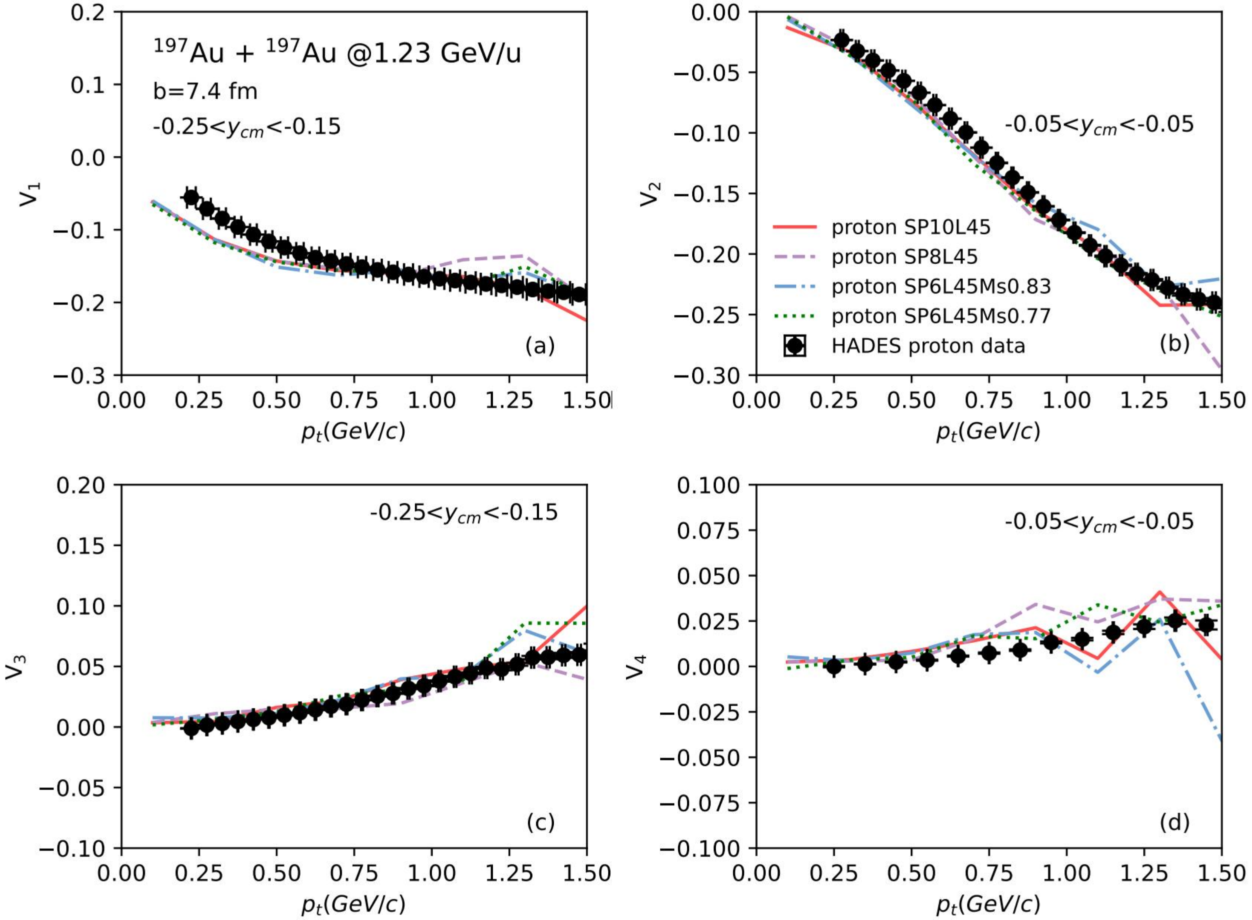
- b=7.4fm for 20%-30% centrality
- N_E= 50000
- Thomas-Fermi initialization
- $\Delta(1950)$ $NN \leftrightarrow N\Delta$ $\Delta \leftrightarrow N\pi$

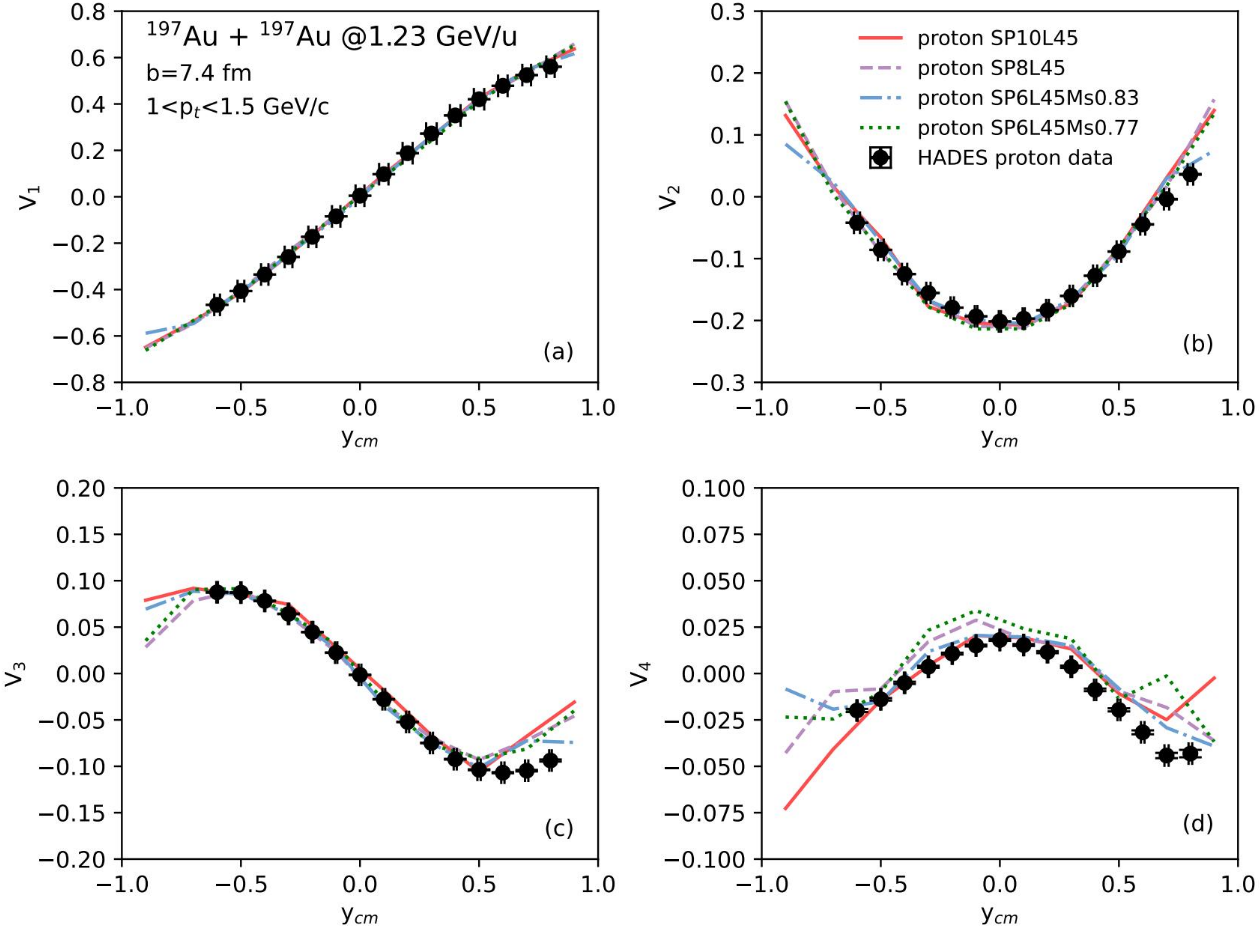
$$\rho > 2\rho_0$$



comparison to HADES data

Flows of protons from v1 to v4
preliminary results:





Conclusions

1. General method to construct higher-order Skyrme pseudopotential (2GeV/A,N5LO)

2. with lattice BUU method, the predicted flows of protons conform to the data measured by HADES collaboration

Thank you！

Introduction

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