Scattering amplitude from quantum computing with reduction formula

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- Quantum algorithm for reduction formula
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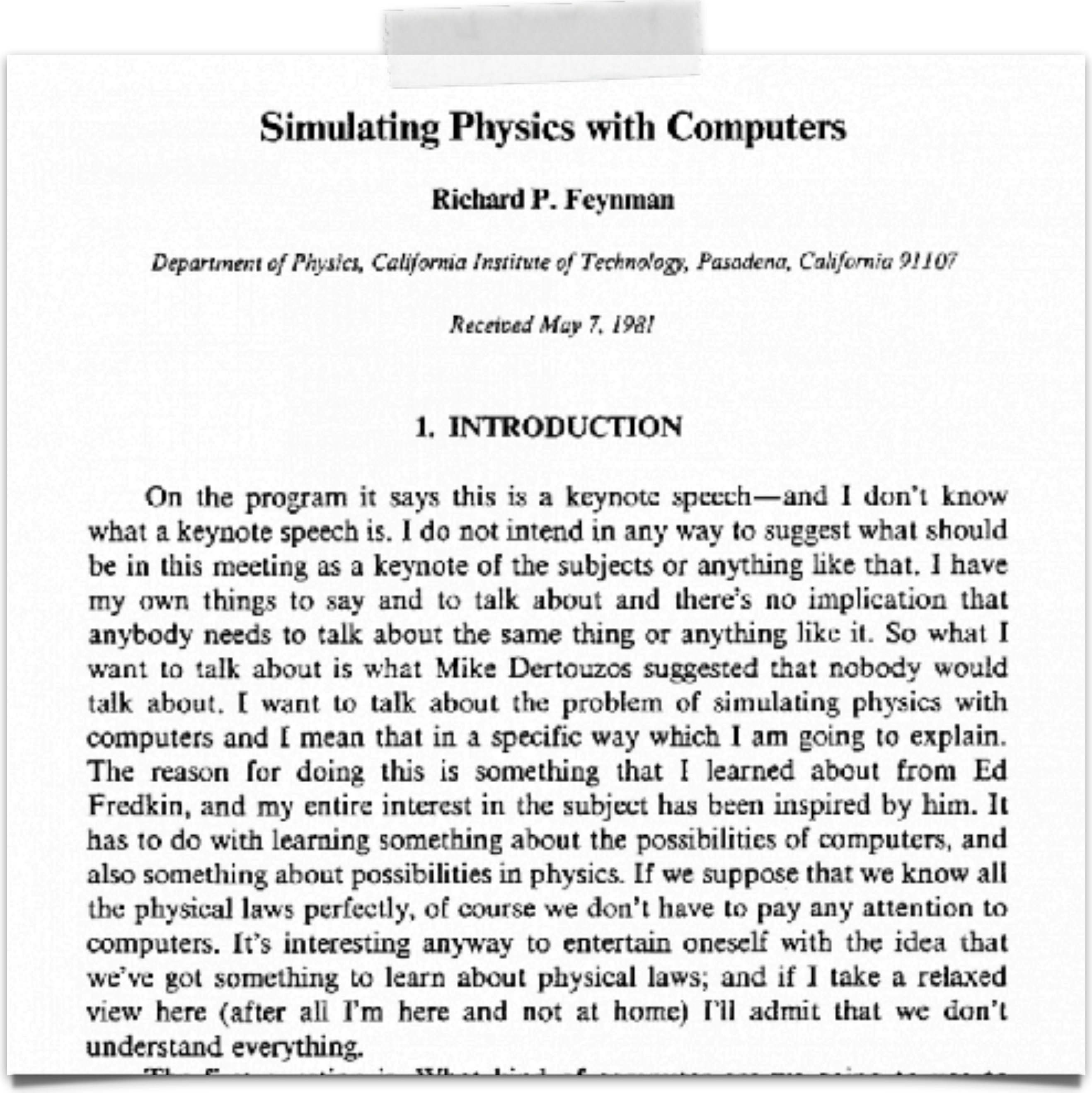
Quantum Computing

- Computing with quantum bits (“qubits”)
  - Hardware: How to build a quantum computer
  - Algorithm: How to “run” a quantum computer
    - Quantum Simulation: Simulate systems with a quantum computer
- Advantage: entanglement, superposition, ...



Simulating Physics with Computers

Richard P. Feynman

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Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have my own things to say and to talk about and there's no implication that anybody needs to talk about the same thing or anything like it. So what I want to talk about is what Mike Dertouzos suggested that nobody would talk about. I want to talk about the problem of simulating physics with computers and I mean that in a specific way which I am going to explain. The reason for doing this is something that I learned about from Ed Fredkin, and my entire interest in the subject has been inspired by him. It has to do with learning something about the possibilities of computers, and also something about possibilities in physics. If we suppose that we know all the physical laws perfectly, of course we don't have to pay any attention to computers. It's interesting anyway to entertain oneself with the idea that we've got something to learn about physical laws; and if I take a relaxed view here (after all I'm here and not at home) I'll admit that we don't understand everything.

> “... and if you want to make a simulation of nature, you'd better make it quantum mechanical, ...”
>
> —Feynman

Basic Elements of Quantum Computing

- Operator (gate): unitary
  - Single qubit: X(σ_x), Y(σ_y), Z(σ_z), Rx(θ)($e^{i\theta\sigma_x}$), ...
  - $X_n = I \otimes I \otimes \cdots \otimes X \otimes \cdots$
  - Two(Multi) qubits: CNOT($\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$)
- Measurements: Hermitian
  - X, Y, Z
- Quantum circuits: series of gates

QC in High Energy Physics

- Parton structure
  - H. Lamm, S. Lawrence, Y. Yamauchi, Phys. Rev. Res. 2(2020), 013272
  - N. Mueller, A. Tarasov, R. Venugopalan, Phys. Rev. D 102(2020), 016007
- Jet production / fragmentation
  - A. Florio et al., arXiv:2301.11991
- Phase transition
  - A. M. Czajka, Z.-B. Kang, Y. Tee, F. Zhao, arXiv:2210.03062
  - A. Thompson, G. Siopsis, arXiv:2303.02425
  - K. Ikeda, D. E. Kharzeev, R. Meyer, S. Shi, arXiv:2305.00996
- Simulation of gauge theory
  - Z. Davoudi, A. F. Shaw, J. R. Stryker, arXiv:2212.14030
  - R. C. Farrell et al., Phys. Rev. D 107(2023), 054513
- And many more!

QC in High Energy Physics

- Challenges
  - Space complexity
    - Limited number of qubits
    - Classical simulations: exponentially difficult
  - Time complexity
    - Noises
  - Gauge field
    - Analog simulation
    - Digital simulation
  - …

What Can We Do Now?

- What system can we simulate?
  - Low dimensional (1+1D,1+2D)
  - Fermionic (NJL, 1D gauge field)
- But our framework could be:
  - Readily expendable
  - General
  - First principle

Scattering Amplitude

- Scattering amplitudes are important, especially in high-energy physics
  - Non-perturbative
  - Bounded states
- Current framework (Science 336, 1130; Quant. Inf, Comput. 14, 1014)
  - General
  - High lattice size requirement
  - Difficulty with bounded states
- Related to correlation functions (with on-shell particles).

LSZ Reduction Formula

$$i\mathcal{M} = R^{n/2} \lim_{p_i^2 \to m^2, k_j^2 \to m^2} G(\{p_i\}, \{k_i\}) \times \left(\sum_{r=1}^{n_{out}} K^{-1}(p_r) \right) \left(\sum_{s=1}^{n_{in}} K^{-1}(k_s) \right)$$

$$G(\{p_i\}, \{k_j\}) = \left(\sum_{i=1}^{n_{out}} \int d^4 x_i e^{ip_i \cdot x_i} \right) \left(\sum_{j=1}^{n_{in}} \int d^4 x_j e^{ik_j \cdot y_j} \right) \langle \Omega | \phi(x_1) \cdots \phi(x_{n_{out}}) \phi^\dagger(y_1) \cdots \phi^\dagger(y_{n_{in}-1}) \phi^\dagger(0) | \Omega \rangle$$

$$K(p) = \int d^4 x e^{p \cdot x} \langle \Omega | T\{\phi(x) \phi^\dagger(0)\} | \Omega \rangle$$

$$R = | \langle \Omega | \phi(0) | h(p = 0) \rangle |^2$$

- Scattering amplitude is a n-point correlation function divided by propagators of incoming and outgoing particles.

Calculation of Correlation Function

$$\langle \Omega | \phi(x_1) \cdots \phi(x_{n_{out}}) \phi^\dagger(y_1) \cdots \phi^\dagger(y_{n_{in}-1}) \phi^\dagger(0) | \Omega \rangle$$

- Prepare the vacuum or hadronic state
- Measure the desired operator on the state

QAOA

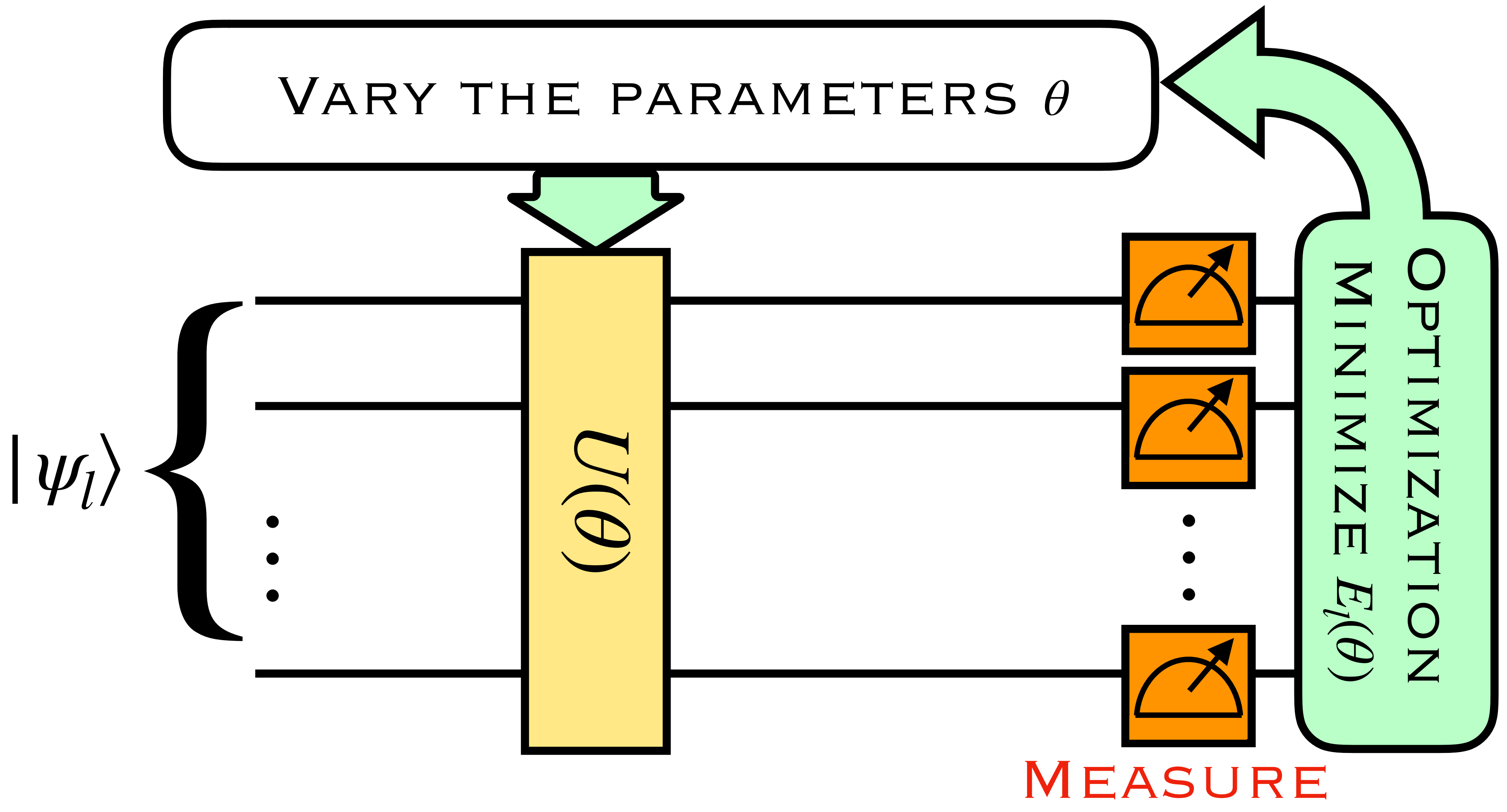
- Divide the Hamiltonian

$$H = H_1 + H_2 + \cdots + H_k$$

  - $[H_i, H_{i+1}] \neq 0$
  - Each H_i preserves the same symmetry as H
- Parameterized symmetry-preserving operator

$$U(\theta) \equiv \prod_{i=1}^{p} \prod_{j=1}^{k} \exp(i\,\theta_{ij} H_j)$$

- The m-th state with quantum number l is $|\psi_{lm}(\theta)\rangle = U(\theta)|\psi_{lm}\rangle$.



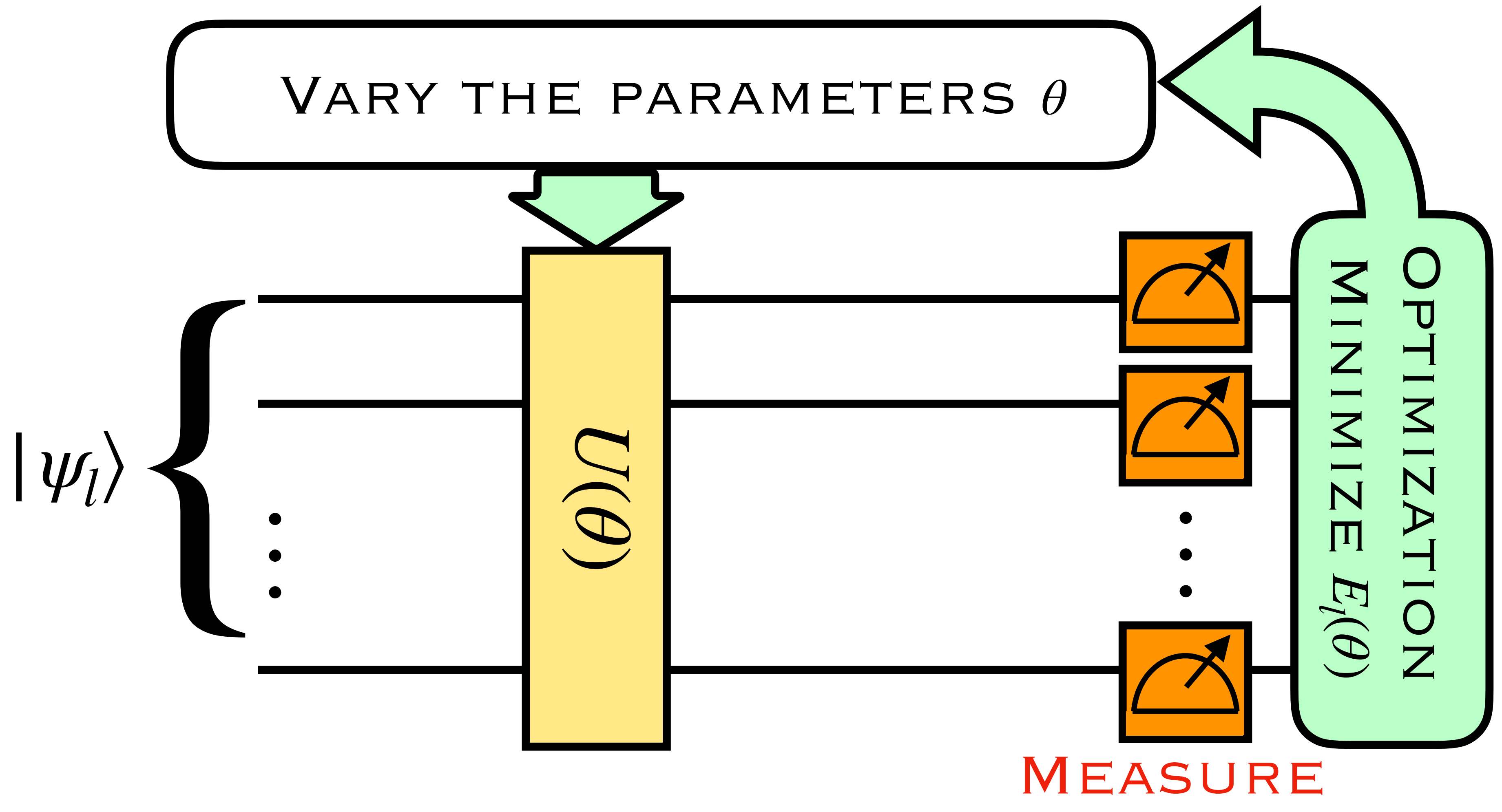
QAOA

- Cost function: weighted combination of the energy expectations [J. S. Pedernales, R. D. Candia, I. L. Egusquiza, J. Casanova, E. Solano, Phys. Rev. Lett. 113(2014), 020505]

$$E_l(\theta) = \sum_{i=1}^{k} w_{li} \langle \psi_{li}(\theta) | H | \psi_{li}(\theta) \rangle$$

$$\omega_{l1} > \omega_{l2} > \cdots > \omega_{lk}$$

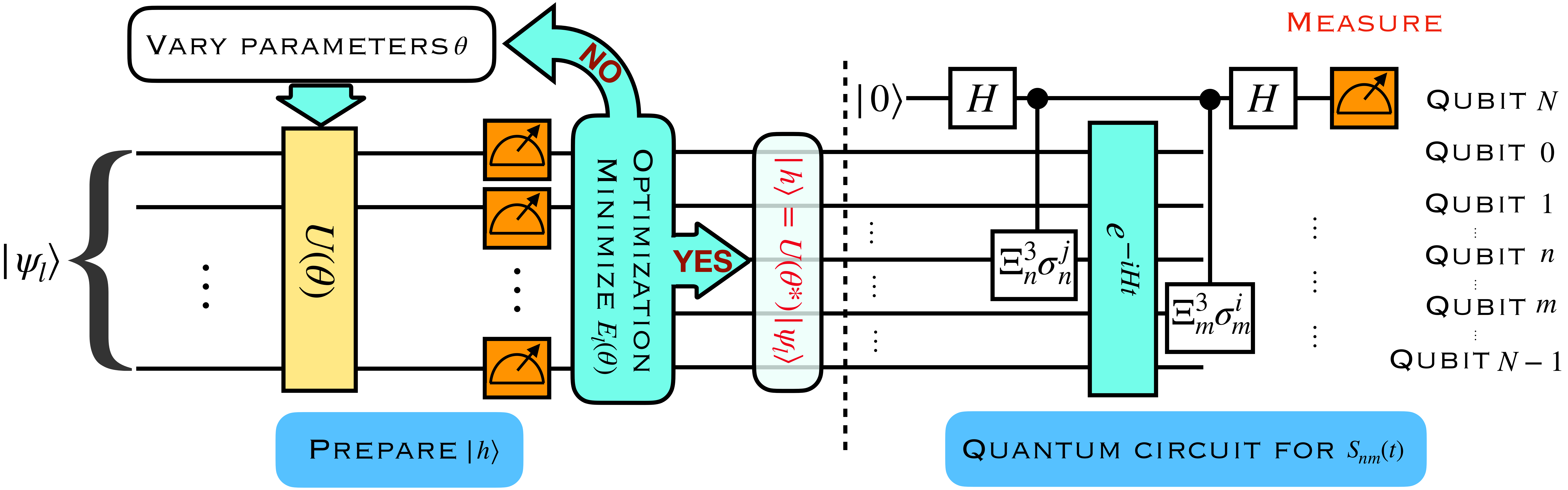
- The eigenstates are obtained by minimizing E_l.
- The minimizing is done by a classical computer.
- Hybrid algorithm



Dynamical Correlation Function

- What we measure: $S_{mn}(t) = \langle h | \hat{O} | h \rangle$, where $\hat{O}$ is unitary
- A unitary measurement on the quantum computer is achieved by one auxiliary qubit and the controlled gates. [A. Francis, J. K. Fredrick's A. F. Kemper, Phys. Rev. B 101(2020), 014411]
- $|\alpha\rangle_a |0\rangle_b \rightarrow \frac{\sqrt{2}}{2} |\alpha\rangle_a (|0\rangle_b + |1\rangle_b) \rightarrow |\phi\rangle \equiv \frac{\sqrt{2}}{2} (|\alpha\rangle_a |0\rangle_b + \hat{O} |\alpha\rangle_a |1\rangle_b)$
- $\langle \phi | I_a \otimes X_b | \phi \rangle = \frac{1}{2} + Re(\langle \alpha | \hat{O} | \alpha \rangle)$
- $\langle \phi | I_a \otimes Y_b | \phi \rangle = \frac{1}{2} - Im(\langle \alpha | \hat{O} | \alpha \rangle)$
- In more complicated cases, $\hat{O}$ can be separated into a series of unitary operators.

Quantum Circuit for Correlation Functions



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1 flavor G-N Model

- Lagrangian:

$$\mathscr{L} = \bar{\psi}_\alpha(i\gamma^\mu\partial_\mu - m_\alpha)\psi_\alpha + g(\bar{\psi}_\alpha\psi_\alpha)^2$$

- Discretization: staggered fermion
- Hamiltonian:

$$H = \sum_{\alpha,n}[-\frac{i}{2a}(\phi^\dagger_{\alpha,n}\phi_{\alpha,n+1} - h.c) + (-1)^n m_\alpha \phi^\dagger_{\alpha,n}\phi_{\alpha,n}]$$
$$-g\sum_{\alpha,n=even}[\phi^\dagger_{\alpha,n}\phi_{\alpha,n} + \phi^\dagger_{\alpha,n+1}\phi_{\alpha,n+1} - 2\phi^\dagger_{\alpha,n}\phi_{\alpha,n}\phi^\dagger_{\alpha,n+1}\phi_{\alpha,n+1}]$$

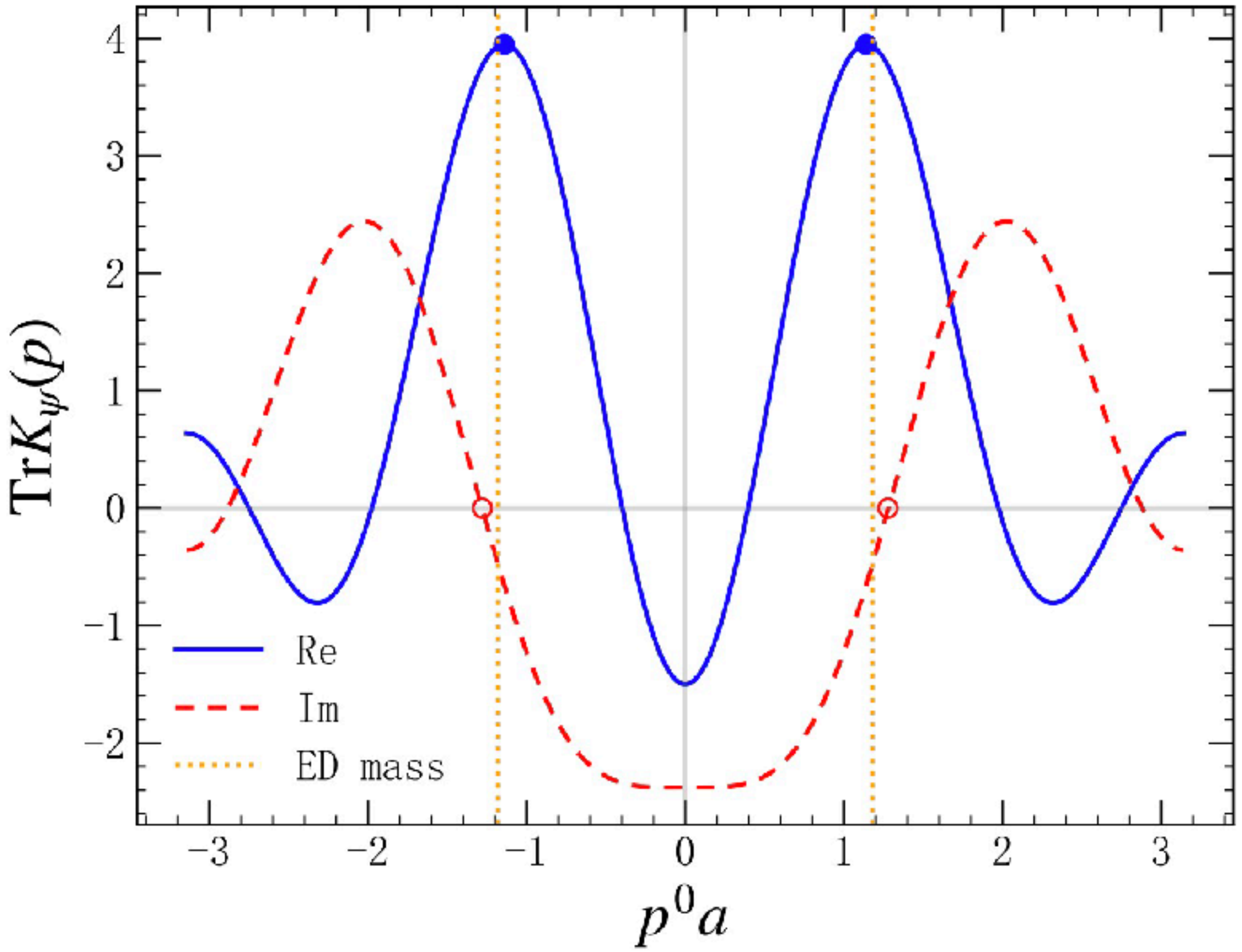
- Mapping to quantum gates: Jordan-Wigner transformation

Quark Propogator

- The pole structure is essential to LSZ reduction formula.
- Quark propagator:

$$K_\psi(p) = \int d^2x e^{ip\cdot x} \langle \Omega | T\{\psi(x)\bar{\psi}(0)\,|\,\Omega\rangle$$

- Pole can be identified by maximum of $Re(K_\psi)$ or zero of $Im(K_\psi)$.
- p_1 set to zero in the simulation.
- Vertical lines show exact diagonation results.

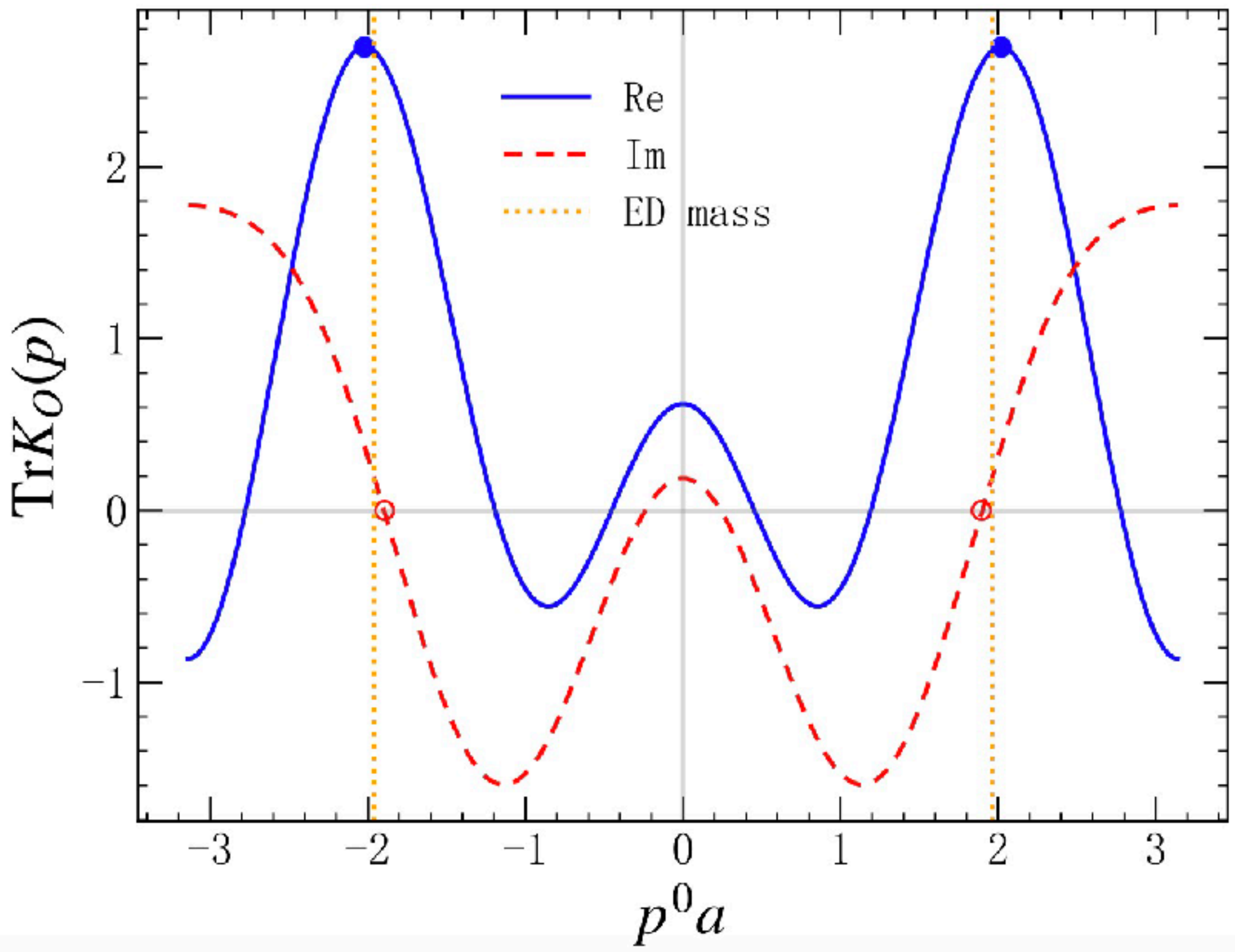


Meson Propagator

$$K_O(p) = \int d^2x e^{ip\cdot x} \langle \Omega | T\{O(x)O(0)\} | \Omega \rangle$$

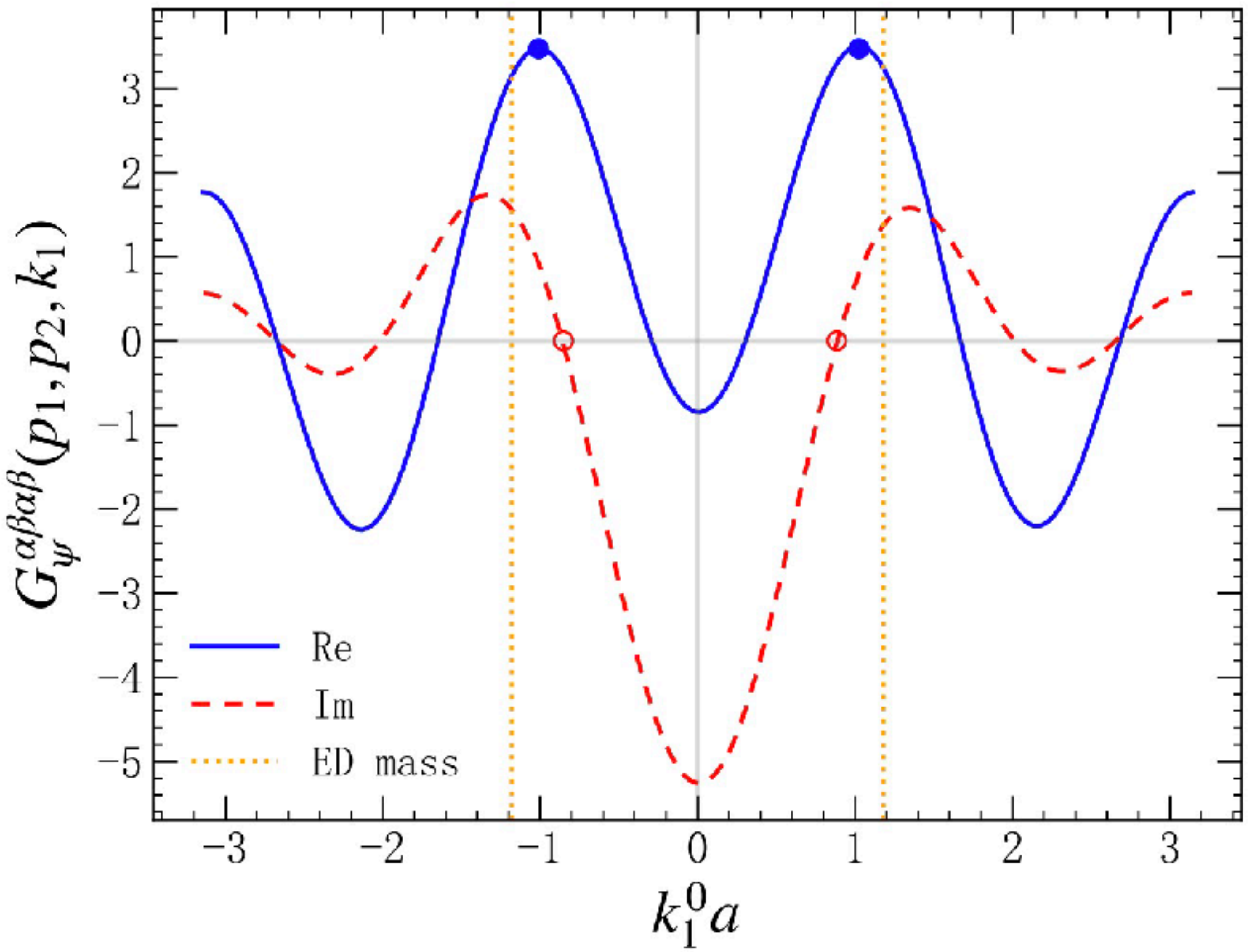
$$O(x) = \bar{\psi}(x)\psi(x)$$

- The LSZ reduction formula can produce the correct pole structure.



Quark 2 by 2 Scattering

$$G_\psi^{\alpha\beta\gamma\delta}(p_1, p_2, k_1)$$
$$= \int d^2x_1 d^2x_2 d^2y_1 e^{i(p_1\cdot x_1 + p_2\cdot x_2 - k_1\cdot y_1)}$$
$$\times \langle \Omega \,|\, \psi^\alpha(x_1)\bar{\psi}^\beta(x_2)\bar{\psi}^\gamma(y_1)\psi^\delta(0) \,|\, \Omega \rangle$$
$$p_1 = (0,0), p_2 = (k_1^0, \pi/a), k_1 = (k_1^0, 0)$$



Summary

- With LSZ reduction forumla, we extend our framework of simulating correlation functions to scattering amplitudes.
- Quantum algorithm agrees well with exact diagonalization.
- Results qualitatively reasonable.
- The framework is expendable to more complicated models.

Thank you!