Quantum simulations of non-perturbative effects in strong field QED

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08/07/2024

Outline

- Motivation
- Schwinger pair production with IBM’s 5-qubit quantum device
- Breit-Wheeler pair production with trapped ions (ongoing)
- Summary

Motivation

Quantum Simulation

- There are problems too hard to solve analytically,

e.g. quantum chromodynamics (QCD).

- There are limitations of classical computations,

e.g. the sign problem in lattice QCD.

- Quantum computers *can* simulate the real time dynamics of such highly entangled systems, hence solve the sign problem.

- Noisy intermediate-scale quantum (NISQ) era

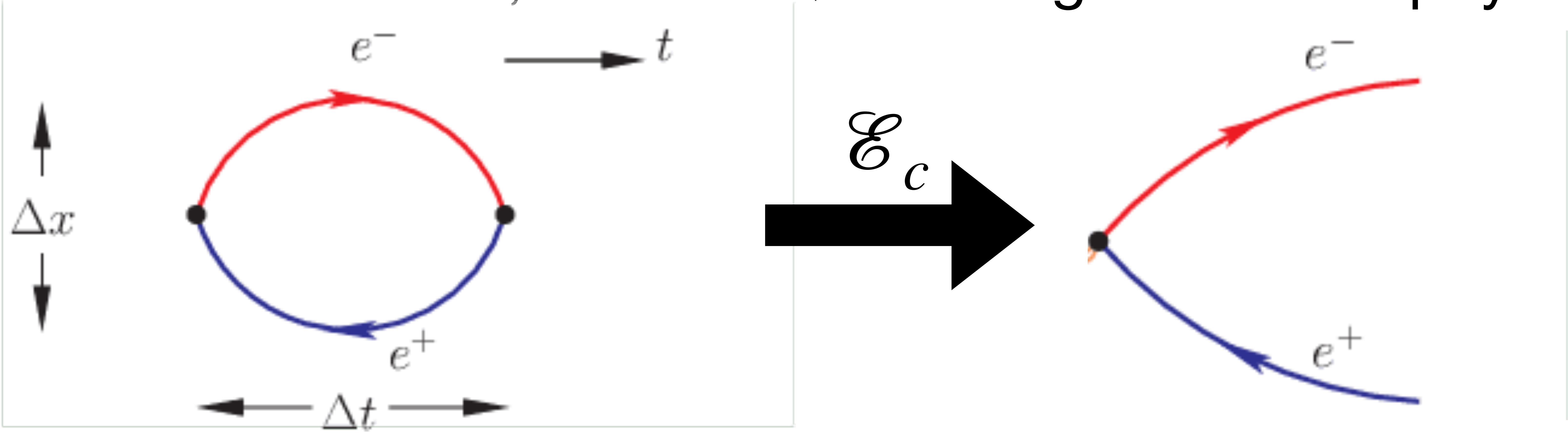
- Digital vs analog

Schwinger Pair Production

- The Schwinger effect is a very well understood phenomenon of quantum electrodynamics (QED) in which electron-positron pairs are spontaneously created in the presence of an electric field.

- It has never been directly observed due to the extremely strong electric-field strengths required. (The Schwinger limit: $\mathscr{E}_c = \dfrac{\pi c^3 m_e^2}{\hbar e} \simeq 10^{18} V/m$)

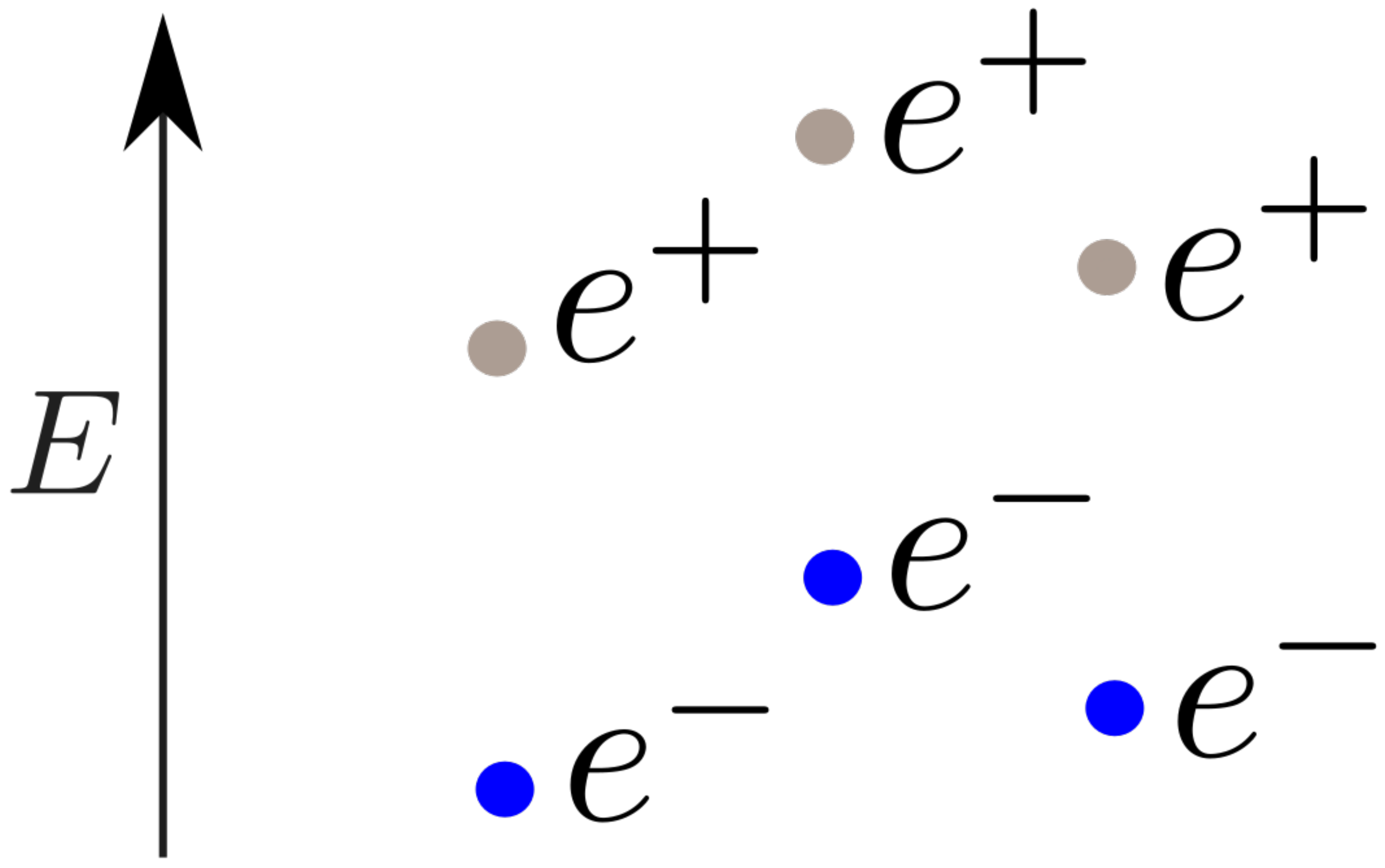
- It is a non-perturbative effect of vacuum decay. This makes this effect of great interest for other theories, such as QCD and gravitational physics.



Dimension reduction

- Homogeneous and isotropic in transverse
- Momentum and spin factorization

$$\psi(\mathbf{x}) = \int \frac{dp_x dp_y}{(2\pi)^2} e^{i(p_x x + p_y y)} \sum_s \psi_s(p_x, p_y, z)$$



- Hamiltonian

$$H = \int d^3x \bar{\psi}(-i\vec{\gamma} \cdot \nabla + m)\psi = \int \frac{dp_x dp_y}{(2\pi)^2} \sum_s H_s(p_x, p_y, z)$$

where $H_s(m; p_x, p_y, z) \simeq H_{1+1}(m'; z)$ with $m' = \sqrt{m^2 + p_x^2 + p_y^2}$

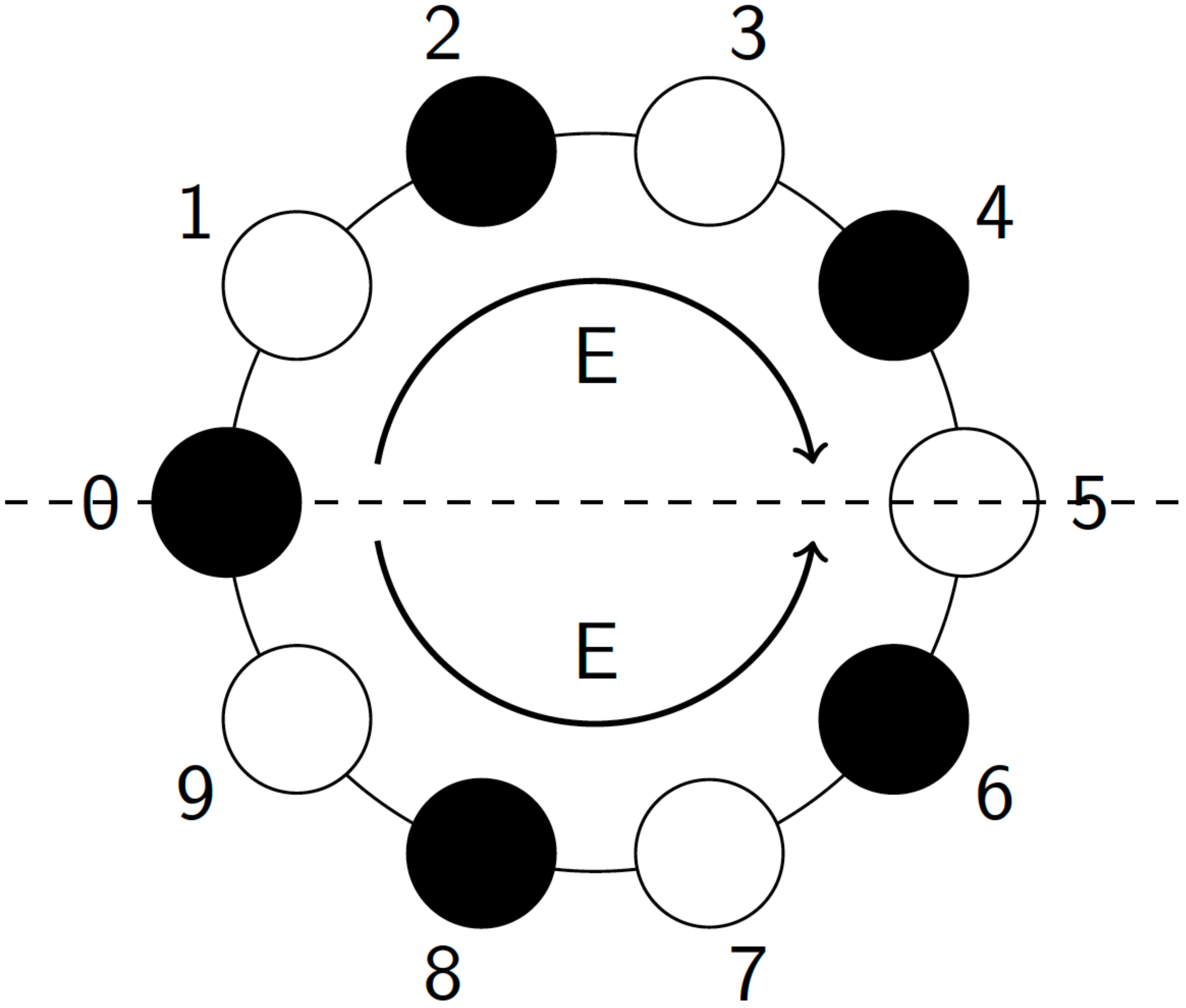
Lattice formulation

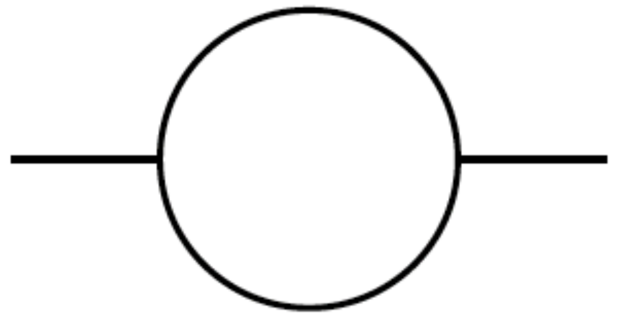
- discretize space $z \rightarrow z_n = na$, where $n \in Z_N$
- periodic boundary condition
- Staggered Fermion (the Kogut-Susskind formulation):

$\psi_{upper}(na) \rightarrow \phi(n)/\sqrt{a}$ for n even.

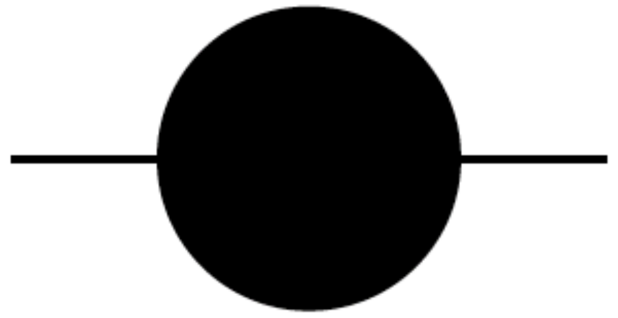
$\psi_{lower}(na) \rightarrow \phi(n)/\sqrt{a}$ for n odd.

Lattice formulation



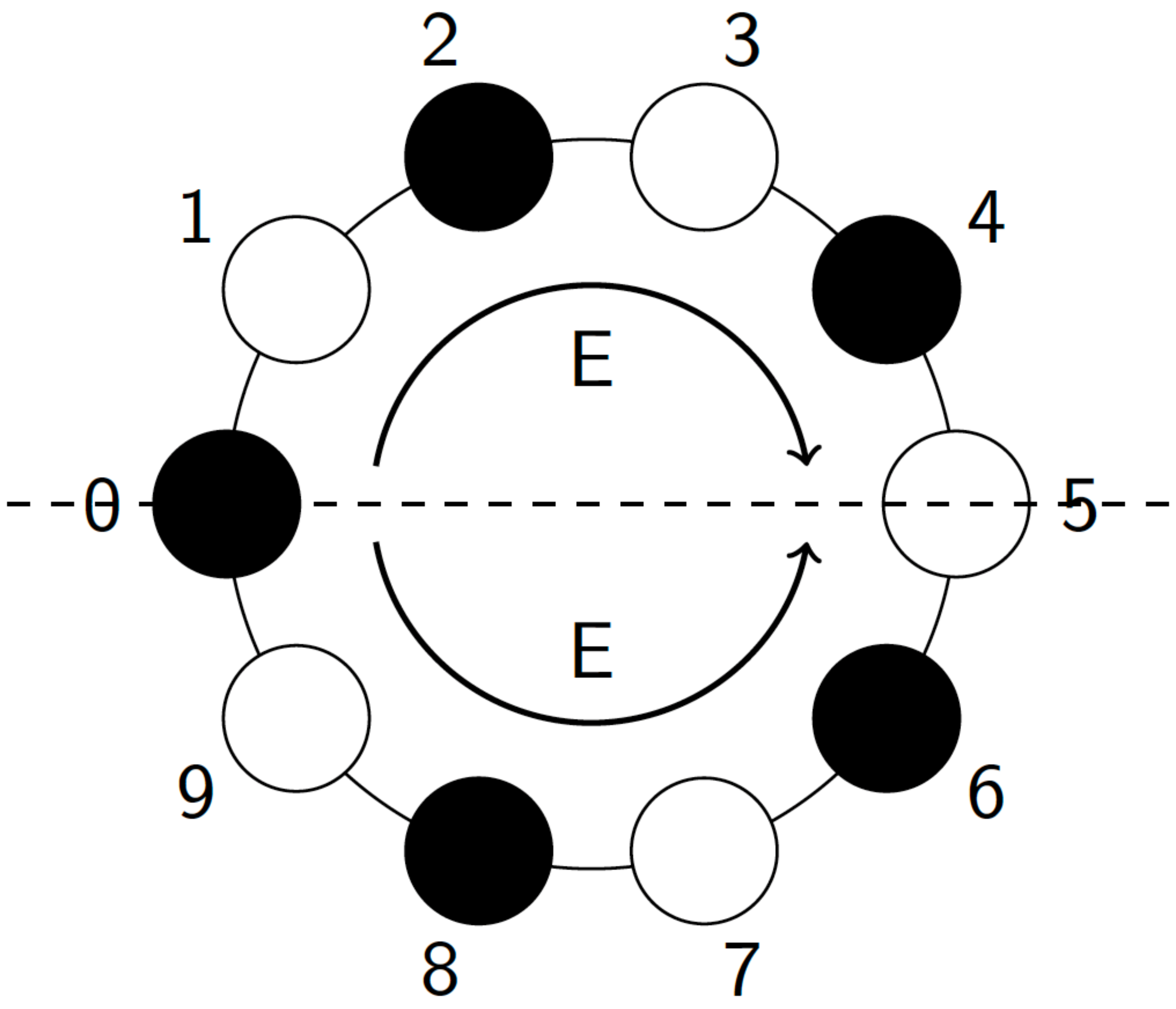
 $= |0\rangle$:

unoccupied electron (odd)
occupied positron (even)

 $= |1\rangle$:

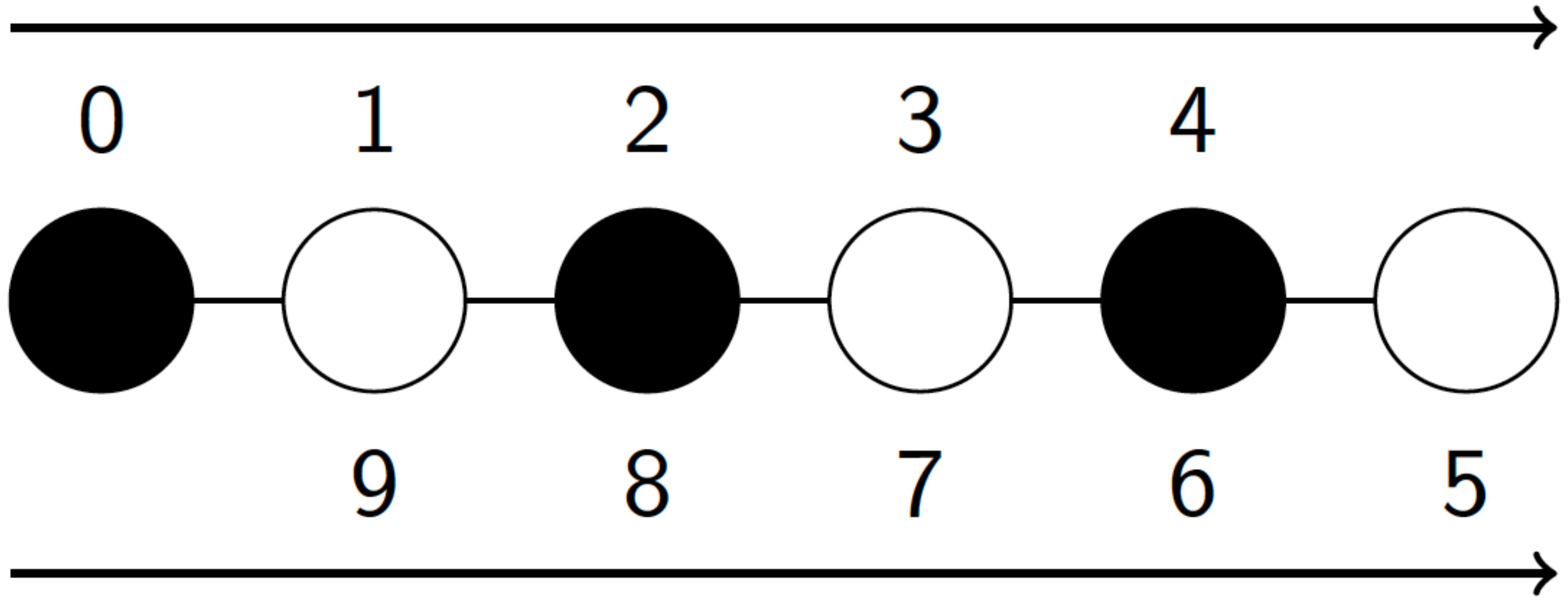
occupied electron (odd)
unoccupied positron (even)

Lattice formulation



The system H is invariant under the parity transformation:

$$P : \phi(m) \to (-1)^m \phi(N-m)$$



Define the parity even and odd fields

$$\phi_{\pm}(m) = \frac{\phi(m) \pm (-1)^m \phi(N-m)}{\sqrt{2}}$$

The Hamiltonian is further divided into two parts

$$H = H_+ + H_-$$

Lattice formulation

- Jordan-Wigner transformation

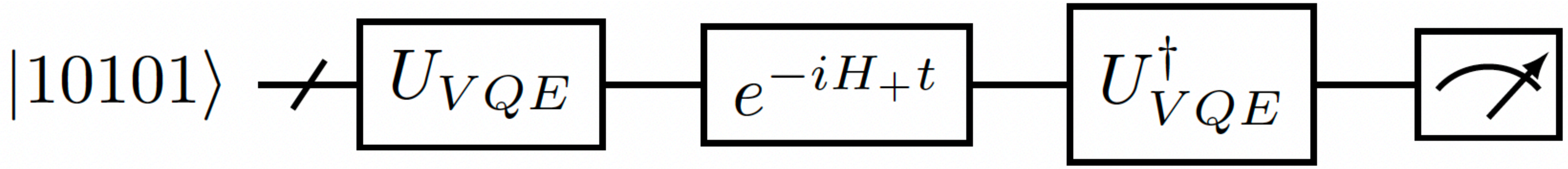
$$\phi(n) = \prod_{l<n} [i\sigma_z(l)]\sigma^-(n), \text{ where } \sigma^\pm(n) = [\sigma_x(n) \pm \sigma_y(n)]/2$$

- System Hamiltonian

$$H_0 = \frac{1}{2a}\sum_n [\sigma_x(n)\sigma_x(n+1) + \sigma_y(n)\sigma_y(n+1)] + \sum_n (-1)^n m \frac{\sigma_z(n)+1}{2}$$

$$H_{int} = \sum_n (-1)^n eEan \frac{\sigma_z(n)+1}{2}$$

Description of Algorithms



- Prepare the ground state of the Hamiltonian with both the electric field and the nearest-neighbor lattice-site interactions turned off.

- Adiabatic turn on the nearest-neighbor lattice site interactions or using the Variational Quantum Eigensolver (VQE) method to find the ground state of the free Hamiltonian.

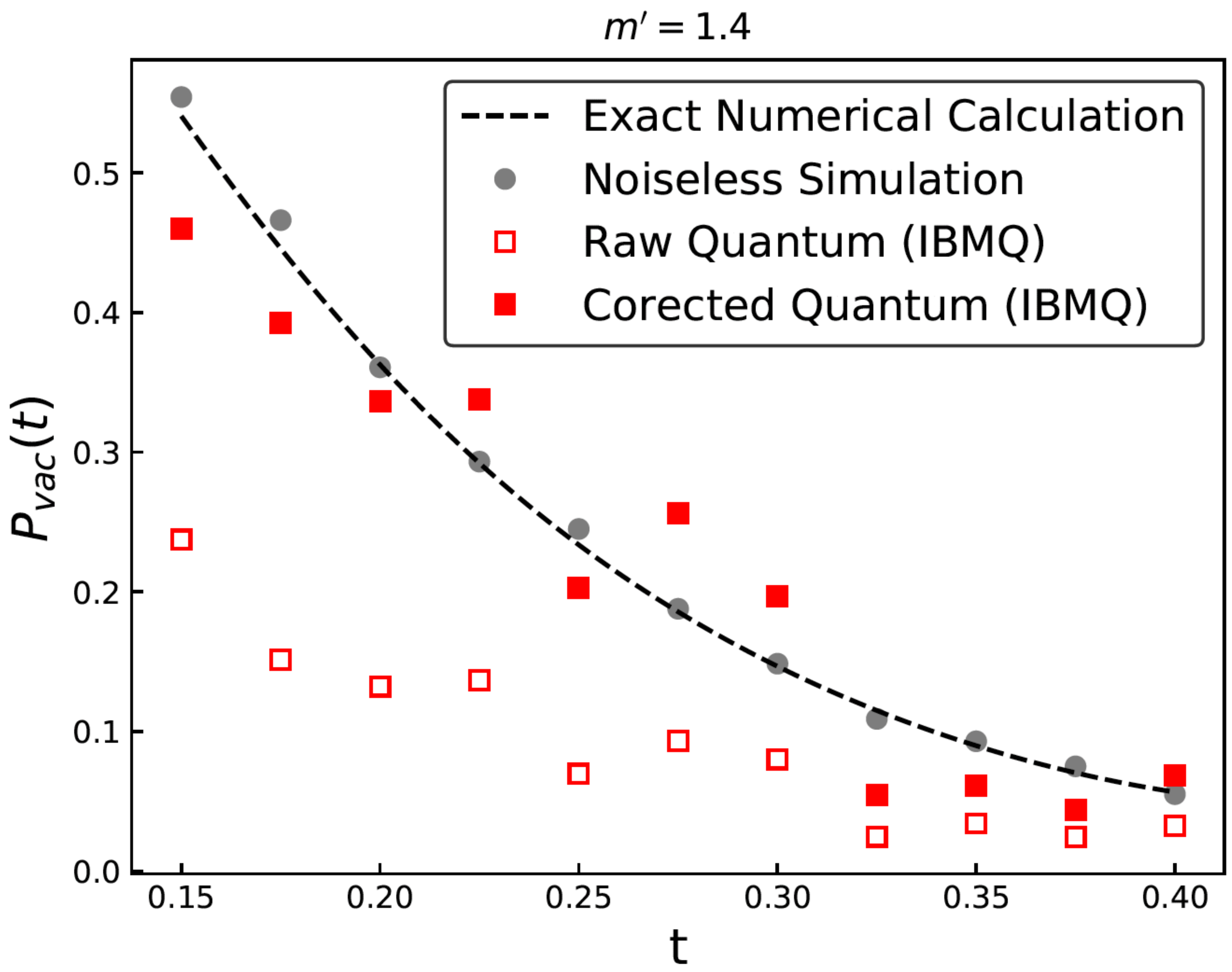
- Evolve in time, via Suzuki-Trotter formulae, according to the full Hamiltonian. It is during this time evolution that pair production may occur.

- Adiabatically turn off the nearest-neighbor lattice site interactions or apply the inverse VQE method.

- Measure the persistence probability of the ground state.

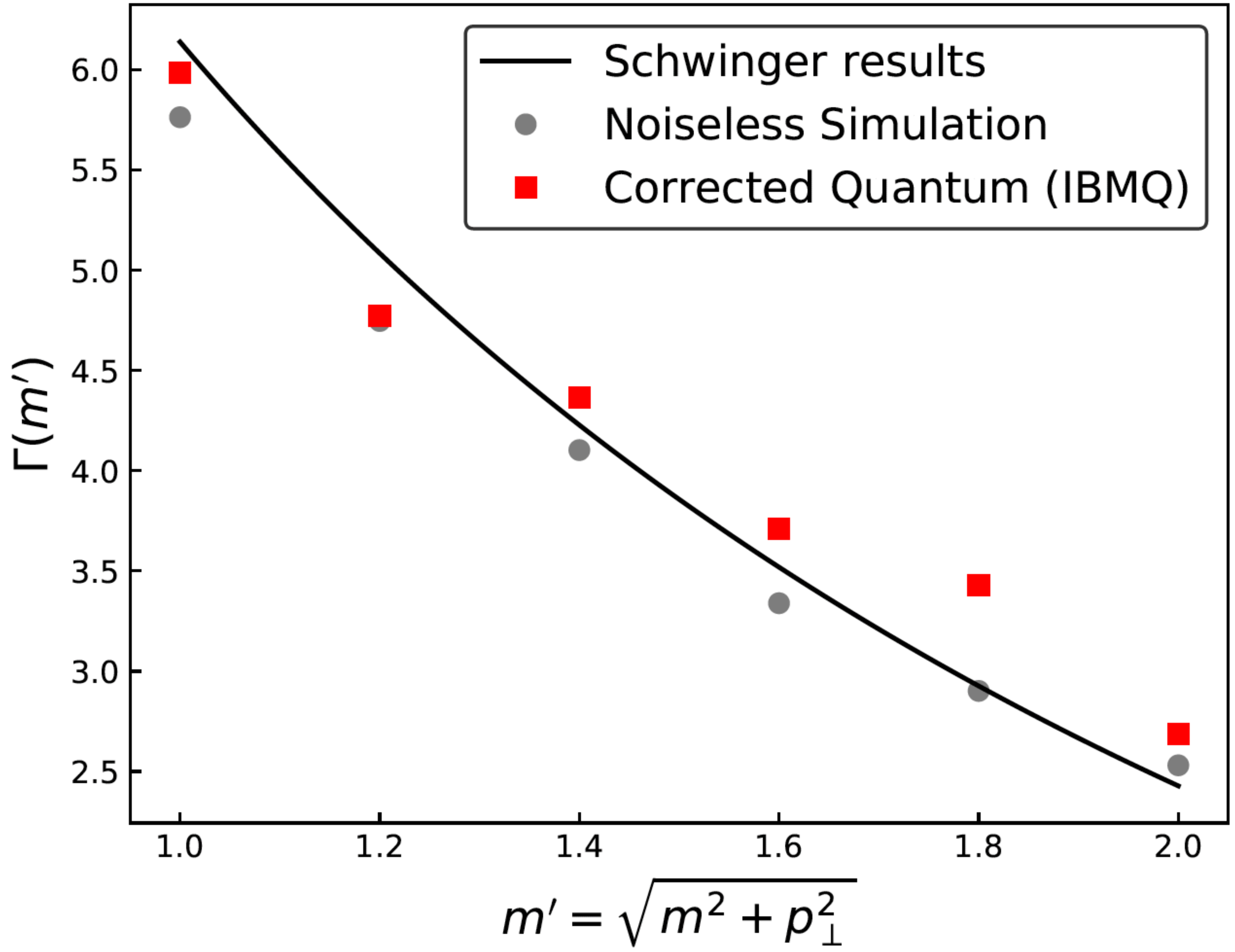
Experiment Results (1+1D)

- For a particular choice of the parameters (natural units): $m = 1, a = 0.45, eE = 20$, we get $\Gamma = 4.37$, while QED predicts $\Gamma = 4.22$



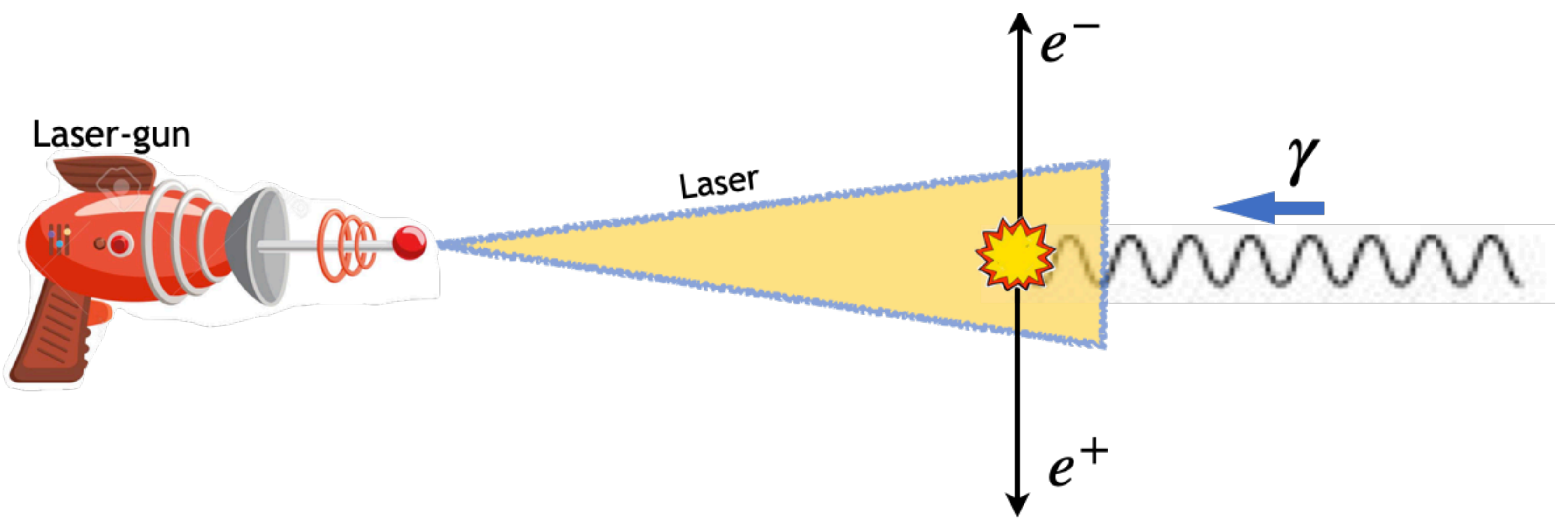
Experiment Results (3+1D)

- Integrating over the transverse momentum, theoretical predication of QED gives $\Gamma_{3+1} = 0.58$, while corrected quantum computer result gives $\Gamma_{3+1} = 0.60$

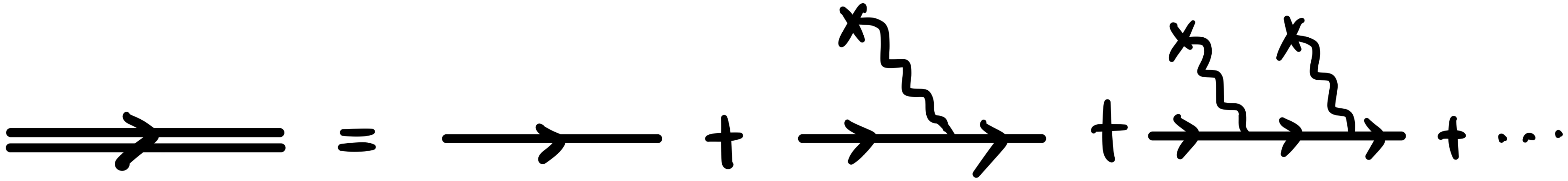


Breit-Wheeler pair production

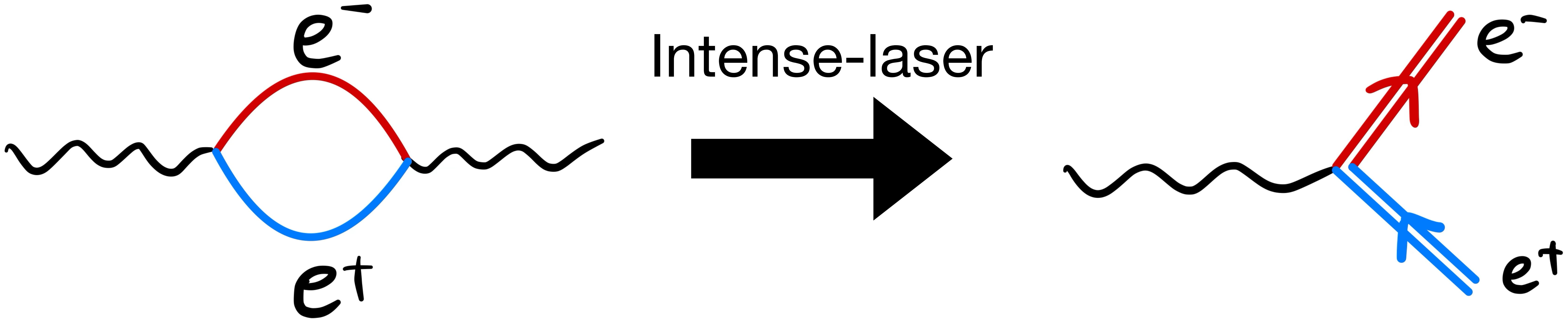
- New non-perturbative processes with intense laser



- Electrons in dressed (Volkov) states



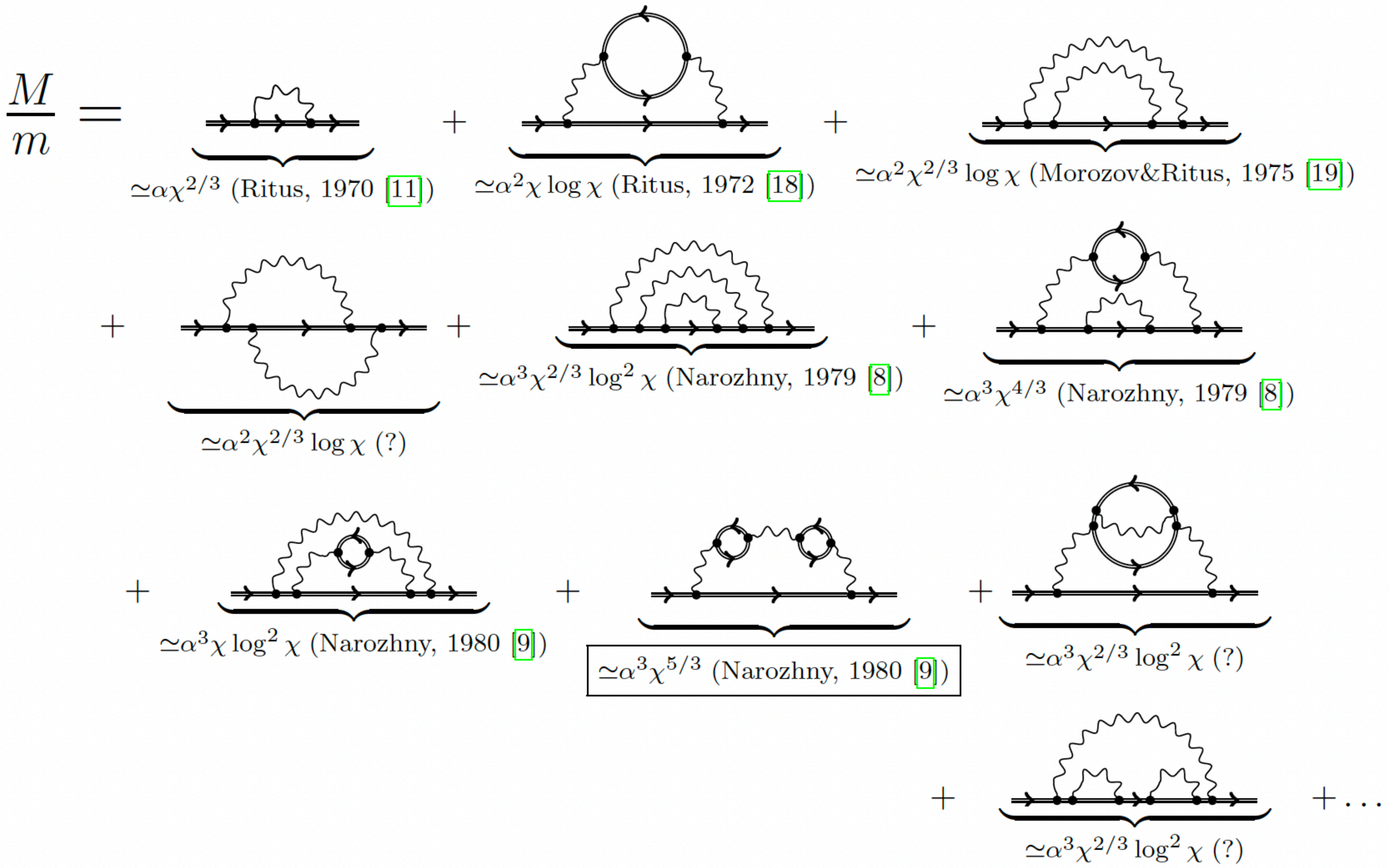
- Experimentally promising ($\chi = \dfrac{eE}{m_e\omega} > 1$)



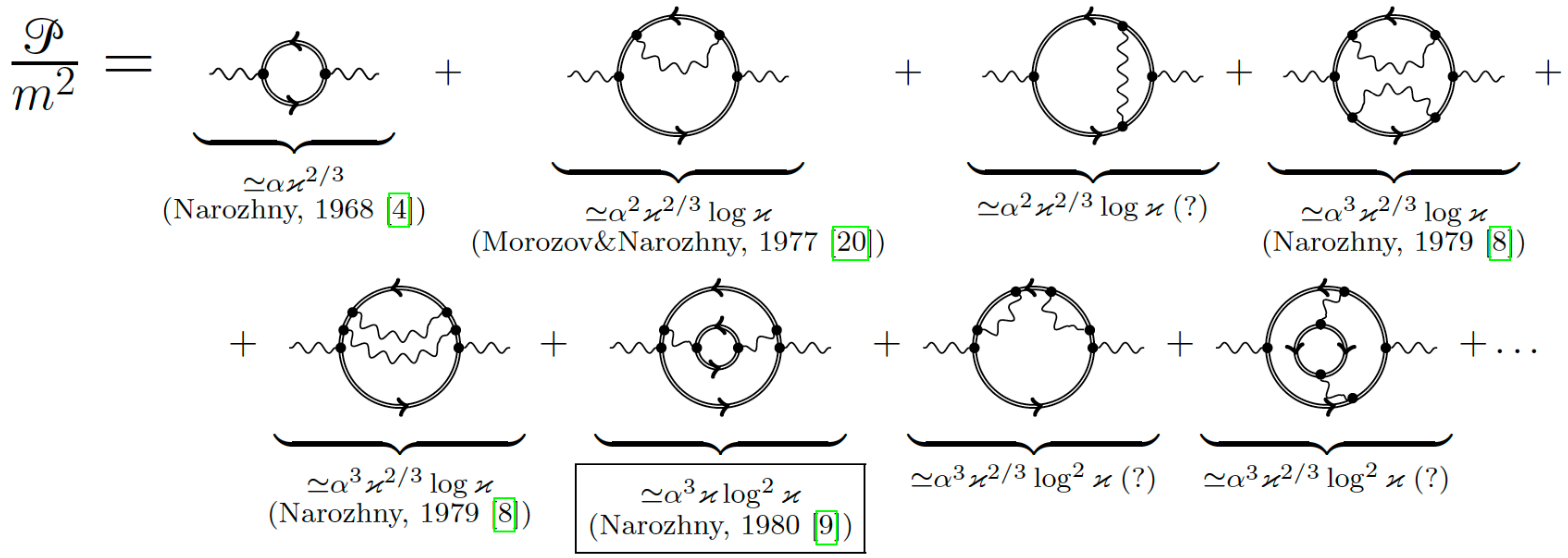
Ritus-Narozhny Conjecture

Furry picture (perturbative QED) breaks down when $\alpha\chi^{2/3} \gtrsim 1$

mass corrections:

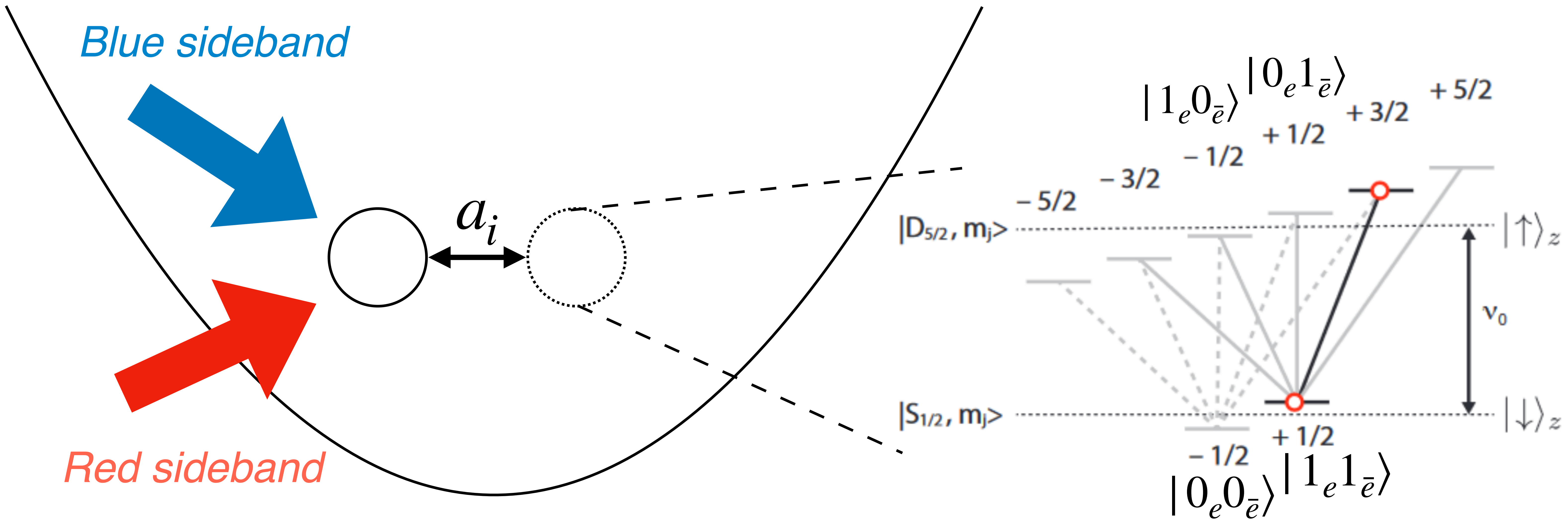


polarization corrections:



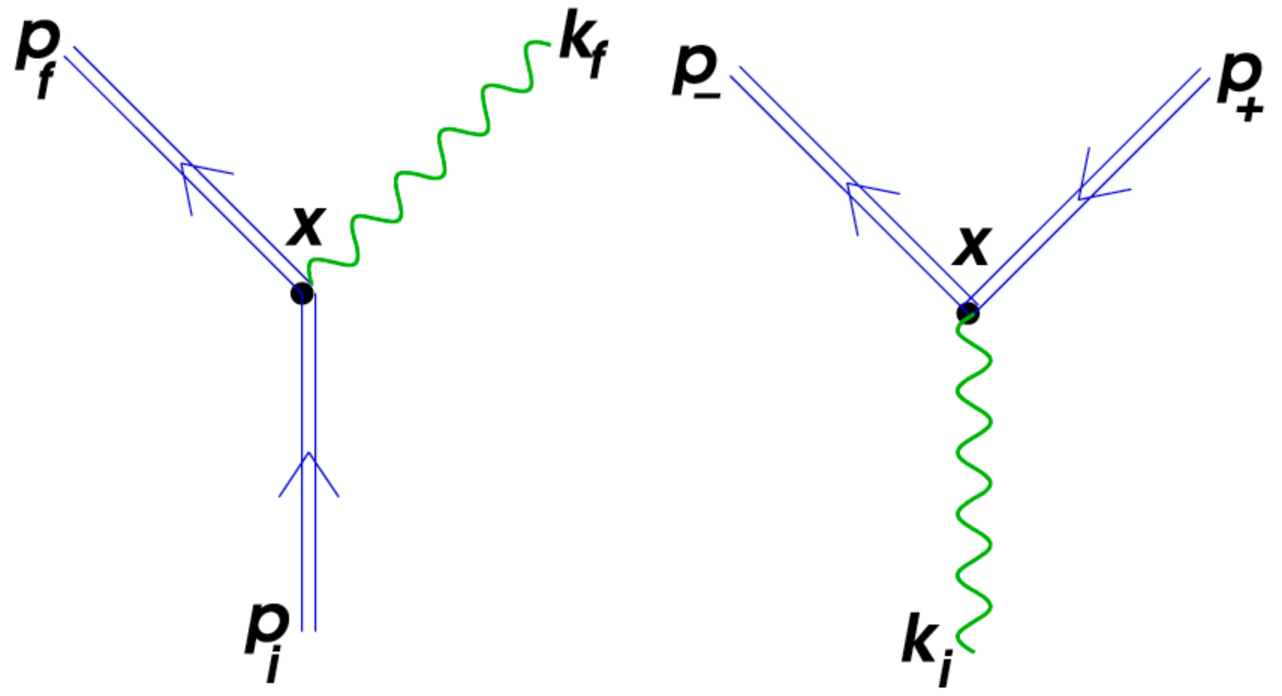
Trapped ion simulation

- Engineered Hamiltonian: $H = \sum |lm\rangle\langle l'm'| a e^{i\phi} + h.c.$
- Simulated Hamiltonian:
$H = g_1 e^{i\omega_0 t}(b^\dagger b a + d d^\dagger a) + g_2(e^{i\omega_1 t} b^\dagger d^\dagger a + e^{i\omega_2 t} d b a) + h.c.$

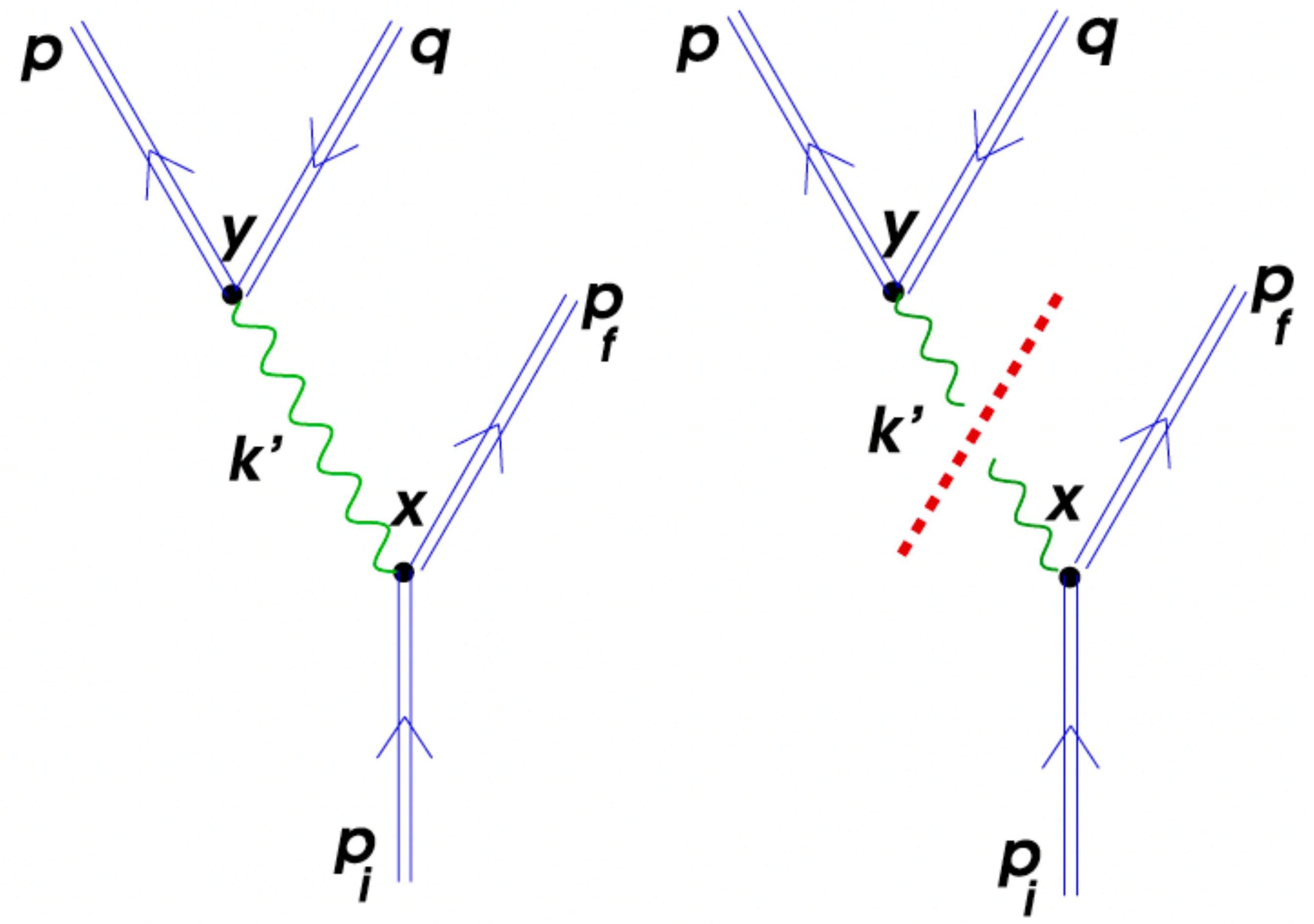


Interested processes

- One-vertex processes: non-linear Compton scattering/one photon pair production



- One-step/ two-step trident processes:



Summary

non-perturbative effects in strong field QED

- Schwinger Pair Production: digital, superconducting qubits, static electric field, coordinate basis

- Breit-Wheeler pair production: analog, trapped ions, laser field+free photon, momentum basis

Thanks for your attention!