Digitized Counterdiabatic Quantum Optimization Approach

Case Studies with Factorization and p-Spin Problem

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Background

Quantum hardware today is

- Beyond reach of classical simulation
- Noisy and unstable

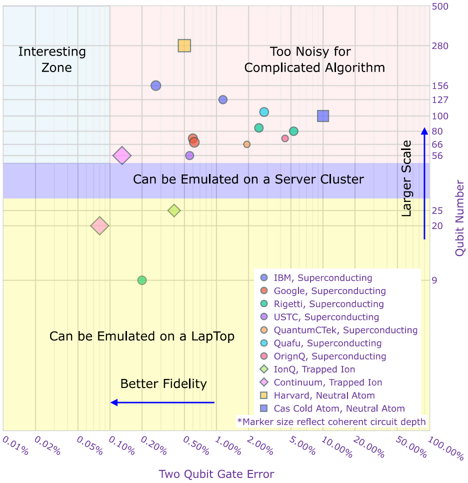

图: State of art quantum hardware scales and performance

Background

- High-impact commercially relevant application requires demanding quantum resources
- Needs savvy algorithm design to exploit the full power before errors drown out useful signals.

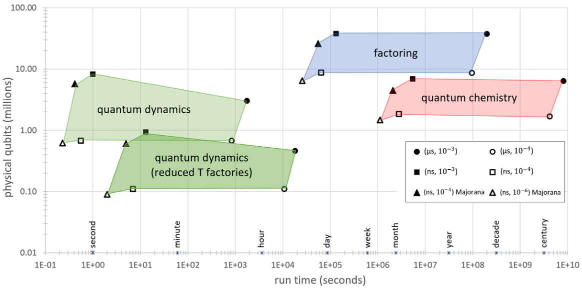

图: Resource Estimation for three types of Algorithms [a]

[a] Beverland, Michael E., et al. "Assessing requirements to scale to practical quantum advantage (2022). See also Azure Quantum Resource Estimator

Background

Variational Quantum Algorithms(VQA) are one of the promising algorithms for near-term applications with potential quantum advantages.

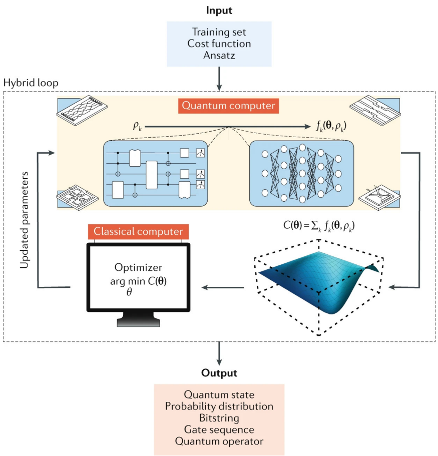

图: Illustration of Variational Quantum Algorithm workflow. Image taken from Cerezo, Marco, et al. "Variational quantum algorithms."

Background

Advantages of VQA

- Less sensitive to noise due to controlled circuit depth and parameter learning
- More flexible structures for different hardware design

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Advantages of VQA

- Less sensitive to noise due to controlled circuit depth and parameter learning
- More flexible structures for different hardware design

Limitations of VQA

- Quantum circuit architecture design is heuristic
- trade-off between expressibility and trainablity
- Low interpretability

Digitized Counterdiabatic Quantum Optimization(DCQO) is a type VQA with flexibility as well as physical intuition

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Principles of DCQO

- VQA is targeted at steering an initial state to a desired final state, i.e. state-transfer problem.
- Can be engineered by evolving from a ground state of $H_B(\sigma_x) = -\sum_i \sigma_x^i$ to the ground state of $H_C(\sigma_z)$ with

$$|\lambda\rangle = \mathcal{T}\left[e^{-i\int H_\lambda(t)\mathrm{d}t}\right]|+\rangle^{\otimes n} \qquad H_\lambda(t) = (1-\lambda(t))H_B + \lambda(t)H_C$$

- Excitations are caused by gauge terms in a time-dependent Hamiltonian, e.g. centrifugal force in a rotational frame.

$$\tilde{H}^{\text{eff}} = \tilde{H}_\lambda(t) - \dot{\lambda}\tilde{A}_\lambda,$$

Principles of DCQO

- There are two ways to suppress excitations.
  - reduce changing rate, e.g moving slowly, adiabatic evolution
  - actively compensating for gauge terms, e.g tilt plates, counterdiabatic evolution

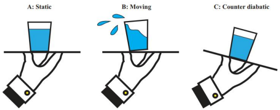

图: Ways for waiter to avoid spilling. Example and image taken from [1]

- Counterdiabatic driving(CD) has its origin in adiabatic control of atoms and molecules.
- By adding counterdiabatic terms proactively, the effective Hamiltonian becomes diagonal.

$$H_\lambda^{\mathrm{CD}} = H_\lambda(t) + \dot{\lambda} A_\lambda \qquad \tilde{H}_{\mathrm{CD}}^{\mathrm{eff}} = \tilde{H}_\lambda(t)$$

Ref[1]. Sels, D. and Polkovnikov, A., 2017. Minimizing irreversible losses in quantum systems by local counterdiabatic driving

Principles of DCQO

- Digitize counterdiabatic evolution leads to DCQO

$$|\psi\rangle_{\text{cd}} = \mathcal{T}\left[e^{-i\int_0^\tau H_\lambda(t)\mathrm{d}t + A_\lambda \mathrm{d}\lambda}\right]|+\rangle^{\otimes n} = \prod_{t_j=0}^{\tau}\left(e^{-iH_\lambda(t_j)\mathrm{d}t}e^{-iA_\lambda(t_j)\mathrm{d}\lambda}\right)|+\rangle^{\otimes n}$$

- In the quench limit($\tau \to 0$), CD terms dominates
- At $t = 0$, system state is eigenstate of $H_\lambda(t)$, evolution unitary is a pure phase
- At $t = \tau$, evolution unitary commutes with measurement, contribute a pure phase.

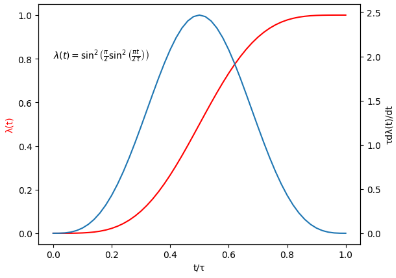

图: $\lambda(t)$ and $\mathrm{d}\lambda/\mathrm{d}t$

Principles of DCQO

- In simplest case with two trotter steps($\tau = 2dt$), quantum circuit has a single-layer structure

$$|\psi\rangle_{\mathrm{cd}} = e^{-i\Delta\lambda A_{\lambda=1/2}}|+\rangle^{\otimes}$$

- Variational CD formula allows for optimal CD terms that minimize overall transition rates for any CD ansatz.

$$G(A_\lambda) = \partial_\lambda \mathcal{H}(\lambda) + \frac{i}{\hbar}[A_\lambda, \mathcal{H}(\lambda)],$$
$$\mathcal{S}(A_\lambda) = \mathrm{Tr}[G^2(A_\lambda)]\,, \quad \frac{\delta S(A_\lambda)}{\delta A_\lambda} = 0$$

- Approximated CD term solves for the best solution within ansatzs, that account for physical constraint or to control complexity.

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Case Study: Factorization Problem

Factorization problems can be converted to an optimization problem from digit-wise constraint equations.[1]

	2^{11}	2^{10}	2^{9}	2^{8}	2^{7}	2^{6}	2^{5}	2^{4}	2^{3}	2^{2}	2^{1}	2^{0}
x						1	q_5	q_4	q_3	q_2	q_1	1
y									1	p_2	p_1	1
						1	q_5	q_4	q_3	q_2	q_1	1
					p_1	q_5p_1	q_4p_1	q_3p_1	q_2p_1	q_1p_1	p_1	
				p_2	q_5p_2	q_4p_2	q_3p_2	q_2p_2	q_1p_2	p_2		
			1	q_5	q_4	q_3	q_2	q_1	1			
carries	$c_{10,11}$	$c_{9,10}$	$c_{8,9}$	$c_{7,8}$	$c_{6,7}$	$c_{5,6}$	$c_{4,5}$	$c_{3,4}$	$c_{2,3}$	$c_{1,2}$		
		$c_{8,10}$	$c_{7,9}$	$c_{6,8}$	$c_{5,7}$	$c_{4,6}$	$c_{3,5}$	$c_{2,4}$	$c_{1,3}$			
$x \times y = 1261$	0	1	0	0	1	1	1	0	1	1	0	1

表: Multiplication Table for factoring $1261 = x \times y$

[1] Here number of digits is taken as a pre-assumption to reduce the number of variables, which should be removed with abundant quantum resources.

Case Study: Factorization Problem

$$
\begin{aligned}
q_1 + p_1 &= 2c_{1,2} \\
q_2 + q_1 p_1 + p_2 + c_{1,2} &= 1 + 2c_{2,3} + 4c_{2,4} \\
&\vdots \\
p_2 + q_5 + c_{7,8} + c_{6,8} &= 2c_{8,9} + 4c_{8,10} \\
1 + c_{8,9} + c_{7,9} &= 2c_{9,10} \\
c_{9,10} + c_{8,10} &= 1 + 2c_{10,11} \\
c_{10,11} &= 0
\end{aligned}
$$

Remove Trivial Solutions →

$$
\begin{aligned}
q_1 = p_1 &= c_{1,2} \\
c_{9,10} &= 1 \\
c_{8,9} &= 1 - c_{7,9} \\
c_{8,10} &= 0 \\
c_{9,10} &= 1 \\
c_{10,11} &= 0
\end{aligned}
$$

Case Study: Factorization Problem

$$
\begin{aligned}
q_1 + p_1 &= 2c_{1,2} \\
q_2 + q_1 p_1 + p_2 + c_{1,2} &= 1 + 2c_{2,3} + 4c_{2,4} \\
&\vdots \\
p_2 + q_5 + c_{7,8} + c_{6,8} &= 2c_{8,9} + 4c_{8,10} \\
1 + c_{8,9} + c_{7,9} &= 2c_{9,10} \\
c_{9,10} + c_{8,10} &= 1 + 2c_{10,11} \\
c_{10,11} &= 0
\end{aligned}
$$

Remove Trivial Solutions

$$
\begin{aligned}
q_1 = p_1 &= c_{1,2} \\
c_{9,10} &= 1 \\
c_{8,9} &= 1 - c_{7,9} \\
c_{8,10} &= 0 \\
c_{9,10} &= 1 \\
c_{10,11} &= 0
\end{aligned}
$$

Construct Cost Function Based on Violation of Constraint

$$(q_2 + 2q_1 + p_2 - 1 - 2c_{2,3})^2 + \ldots + (p_2 + q_5 + c_{7,8} + c_{6,8} - 2c_{8,9})^2 = 0$$

Case Study: Factorization Problem

$$
\begin{aligned}
q_1 + p_1 &= 2c_{1,2}\\
q_2 + q_1 p_1 + p_2 + c_{1,2} &= 1 + 2c_{2,3} + 4c_{2,4}\\
&\vdots\\
p_2 + q_5 + c_{7,8} + c_{6,8} &= 2c_{8,9} + 4c_{8,10}\\
1 + c_{8,9} + c_{7,9} &= 2c_{9,10}\\
c_{9,10} + c_{8,10} &= 1 + 2c_{10,11}\\
c_{10,11} &= 0
\end{aligned}
$$

Remove Trivial Solutions →

$$
\begin{aligned}
q_1 = p_1 &= c_{1,2}\\
c_{9,10} &= 1\\
c_{8,9} &= 1 - c_{7,9}\\
c_{8,10} &= 0\\
c_{9,10} &= 1\\
c_{10,11} &= 0
\end{aligned}
$$

Construct Cost Function Based on Violation of Constraint →

$$(q_2 + 2q_1 + p_2 - 1 - 2c_{2,3})^2 + \ldots + (p_2 + q_5 + c_{7,8} + c_{6,8} - 2c_{8,9})^2 = 0$$

Map Binary Bits to Spin ↓

$$
\begin{aligned}
H^{5q} = {} & \frac{23}{4} - \frac{5\sigma_z^{(0)}}{4} - -\frac{\sigma_z^{(0)}\sigma_z^{(1)}}{4} + \frac{3\sigma_z^{(0)}\sigma_z^{(2)}}{4} - \frac{3\sigma_z^{(0)}\sigma_z^{(3)}}{4} - \sigma_z^{(0)}\sigma_z^{(4)}\\
& - \frac{\sigma_z^{(1)}\sigma_z^{(2)}}{4} + \frac{5\sigma_z^{(1)}\sigma_z^{(3)}}{4} - \sigma_z^{(2)}\sigma_z^{(3)} - \sigma_z^{(3)}\sigma_z^{(4)} + \frac{\sigma_z^{(0)}\sigma_z^{(1)}\sigma_z^{(2)}}{4} - \frac{\sigma_z^{(0)}\sigma_z^{(2)}\sigma_z^{(3)}}{4}\\
& + \frac{\sigma_z^{(1)}\sigma_z^{(2)}\sigma_z^{(3)}}{4} - \frac{\sigma_z^{(1)}\sigma_z^{(2)}\sigma_z^{(4)}}{2},
\end{aligned}
$$

Case Study: Factorization Problem

- Factorization problem can be converted to spin problem upto 4th order interaction
- Solutions are encoded into ground state spin configuration.
- Ground state can be fabricated by adiabatically evolving the Ising system from a transverse field to the problem Hamiltonian for large τ

$$|\psi\rangle_{\text{ad}} = \mathcal{T}\left[e^{-i\int_0^\tau H_\lambda(t)\mathrm{d}t}\right]|+\rangle$$

$$H_\lambda(t) = (1-\lambda(t))\sum_i(-\sigma_x^i) + \lambda(t)H \quad \lambda(t) = \sin^2\left(\pi/2\sin^2\left(\frac{\pi t}{2\tau}\right)\right)$$

- Or from accelerated evolution for short τ with

$$|\psi\rangle_{\text{cd}} = \mathcal{T}\left[e^{-i\int_0^\tau (H_\lambda(t)+\dot{\lambda}A_\lambda)\mathrm{d}t}\right] = \mathcal{T}\left[e^{-i\int_0^\tau H_\lambda(t)\mathrm{d}t + A_\lambda \mathrm{d}\lambda}\right]$$

Case Study: Factorization Problem

$$|\psi\rangle_{\text{ad}} = \mathcal{T}\left[e^{-i\int_0^{\tau} H_\lambda(t)\mathrm{d}t}\right]|+\rangle \quad |\psi\rangle_{\text{cd}} = \mathcal{T}\left[e^{-i\int_0^{\tau} H_\lambda(t)\mathrm{d}t + A_\lambda \mathrm{d}\lambda}\right]$$

$$H_\lambda(t) = (1-\lambda(t))\sum_i(-\sigma_x^i) + \lambda(t)H \quad \lambda(t) = \sin^2\left(\pi/2\sin^2\left(\frac{\pi t}{2\tau}\right)\right)$$

- Decompose adiabatic unitary into discrete time step, we recover QAOA as

$$|\psi\rangle_{\text{ad}} = \prod_i\left(e^{-i\gamma_i H_\lambda} e^{-i\beta_i \sum_j \sigma_x^j}\right)|+\rangle^{\otimes n}$$

$$\gamma_i = \lambda(t)\mathrm{d}t \quad \beta_i = (\lambda(t)-1)\mathrm{d}t$$

- Decompose counterdiabatic unitary into discrete time step, we obtain DCQO as

$$|\psi\rangle_{\text{cd}} = \prod_{t_j=0}^{\tau}\left(e^{-i(\lambda(t)-1)(\sum_j \sigma_x^j)\mathrm{d}t} e^{-i\lambda(t)H_\lambda \mathrm{d}t} e^{-iA_\lambda(t_j)\mathrm{d}\lambda}\right)|+\rangle^{\otimes n}$$

Case Study: Factorization Problem

$$|\psi\rangle_{\mathrm{ad}} = \mathcal{T}\left[e^{-i\int_0^{\mathcal{T}} H_\lambda(t)\mathrm{d}t}\right]|+\rangle \quad |\psi\rangle_{\mathrm{cd}} = \mathcal{T}\left[e^{-i\int_0^{\mathcal{T}} H_\lambda(t)\mathrm{d}t + A_\lambda \mathrm{d}\lambda}\right]$$

$$H_\lambda(t) = (1-\lambda(t))\sum_i(-\sigma_x^i) + \lambda(t)H \quad \lambda(t) = \sin^2\left(\pi/2\sin^2\left(\frac{\pi t}{2\mathcal{T}}\right)\right)$$

- Decompose adiabatic unitary into discrete time step, we recover QAOA as

$$|\psi\rangle_{\mathrm{ad}} = \prod_i \left(e^{-i\gamma_i H_\lambda} e^{-i\beta_i \sum_j \sigma_x^j}\right)|+\rangle^{\otimes n}$$

$$\gamma_i = \lambda(t)\mathrm{d}t \quad \beta_i = (\lambda(t)-1)\mathrm{d}t$$

- Decompose counterdiabatic unitary into discrete time step, we obtain DQCO as

$$|\psi\rangle_{\mathrm{cd}} = \prod_{t_j=0}^{\mathcal{T}} \left(e^{-i(\lambda(t)-1)(\sum_j \sigma_x^j)\mathrm{d}t} e^{-i\lambda(t)H_\lambda \mathrm{d}t} e^{-iA_\lambda(t_j)\mathrm{d}\lambda}\right)|+\rangle^{\otimes n}$$

Case Study: Factorization Problem

DCQO solution with single layer structure

$$|\psi\rangle_{\mathrm{cd}} = e^{-iA_\lambda(t)\mathrm{d}\lambda}|+\rangle^{\otimes n}$$

User defined CD terms with optimized Coefficients

- $(\mathrm{Y}+\mathrm{YZ_u})$-type $\quad A_\lambda = \sum_i \alpha_i J_i \sigma_y^i + \beta \sum_{i<j} J_{ij}\sigma_y^i\sigma_z^j$
- $(\mathrm{Y}+\mathrm{YZ_u}+\mathrm{ZY_u})$-type $\quad A_\lambda = \sum_i \alpha_i J_i \sigma_y^i + \beta \sum_{i<j} J_{ij}\sigma_y^i\sigma_z^j + \gamma \sum_{i<j} J_{ij}\sigma_z^i\sigma_y^j$
- $(\mathrm{Y}+\mathrm{YZ})$-type $\quad A_\lambda = \sum_i \alpha_i J_i \sigma_y^i + \sum_{i<j} \beta_{ij} J_{ij}\sigma_y^i\sigma_z^j$
- $(\mathrm{Y}+\mathrm{YZ}+\mathrm{ZY})$-type $\quad A_\lambda = \sum_i \alpha_i J_i \sigma_y^i + \sum_{i<j} \beta_{ij} J_{ij}\sigma_y^i\sigma_z^j + \sum_{i<j} \gamma_{ij} J_{ij}\sigma_z^i\sigma_y^j$

Case Study: Factorization Problem

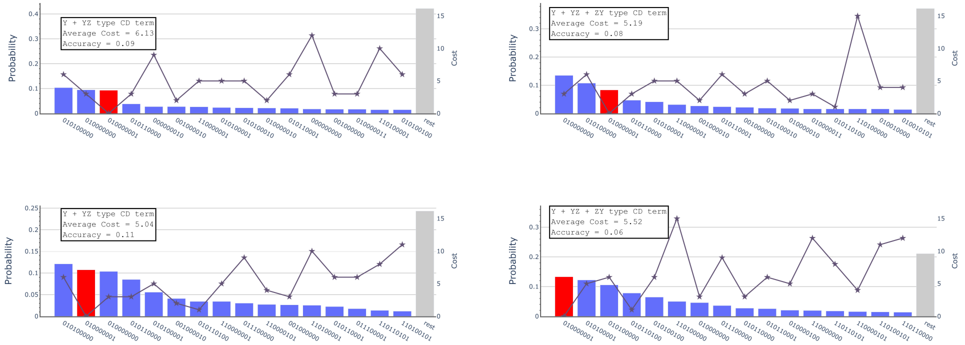

图: Performance of single-layer DCQO with various local CD ansatz

Case Study: Factorization Problem

DCQO can also be used as VQA with warm-started parameter optimization. Knowledge from DCQO helps prevent trapping to local minimums.

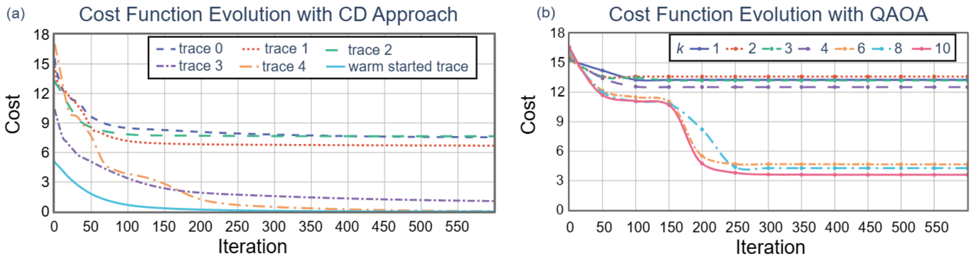

图: :Comparison of convergent curve for variational DCQO and QAOA. DCQO circuit has one-layer structure and QAOA has varying *k*-layer structure

Case Study: Factorization Problem

Experiments on 'Xiaohong' superconducting quantum computing chips confirms efficency of the algorithm. Circuit is taken as single layer with ($\mathrm{Y} + \mathrm{YZ_u}$)-type CD ansatz. Parameters are taken as the optimal coefficients from warm-started optimization solution on a simulator.

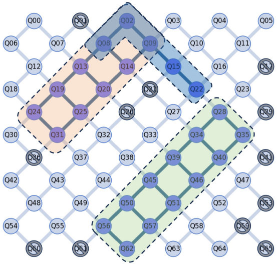

图: Quantum processor layout diagram. Bad qubits are marked by a cross-out sign. Qubits used in the H^{5q}, H^{9q}, and H^{12q} experiments are highlighted and shaped in blue, orange, and green colors respectively.

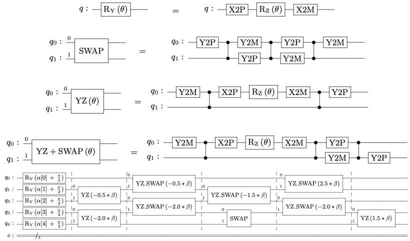

图: Quantum circuit diagram for DCQO circuit for H^{5q}.

Case Study: Factorization Problem

DCQO Algorithm
Huijie Guan
Background
Principles
Factorization
p-Spin problem
Summary & Outlook

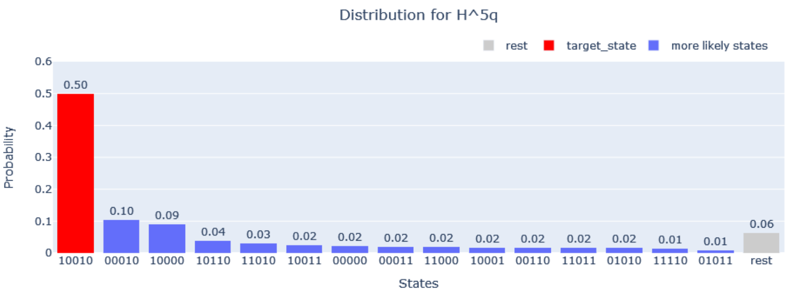

图: Final state of DCQO's solution to H^{5q}, correponding to factorization of 1261

label	Q08	Q02	Q09	Q15	Q22
Single Qubit Gate Error (%)	0.27	0.18	0.16	0.23	0.22
T1 (μs)	20.71	28.47	28.12	20.60	17.29
T2 *(μs)	12.73	3.94	6.38	2.29	8.03
Readout Error (%)	1.22	0.16	0.25	1.36	1.03

Label	Q02-Q08	Q02-Q09	Q09-Q15	Q15-Q22
CZ XEB Error (%)	2.338964	2.194998	2.81408	2.959614

Case Study: Factorization Problem

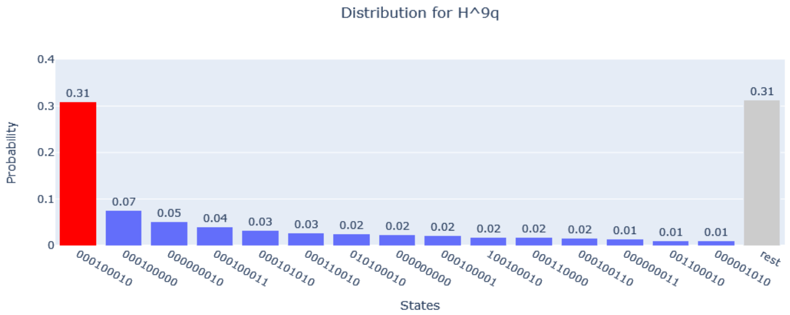

图: Final state of DCQO's solution to H^{9q}, correponding to factorization of 767

Label	Q02	Q08	Q13	Q19	Q24	Q09
Single Qubit Gate Error (%)	0.18	0.27	0.42	0.21	0.31	0.16
T1 (μs)	28.47	20.71	16.08	25.29	17.83	28.12
T2* (μs)	3.94	12.73	3.58	1.58	2.47	6.38
Readout Error (%)	0.16	1.22	0.75	0.88	1.71	0.25
Label	Q14	Q20	Q25	Q31		
Single Qubit Gate Error	0.21	0.55	0.33	0.15		
T1	26.83	20.71	9.71	25.65		
T2*	4.46	7.39	3.24	11.17		
Readout Error	1.93	2.58	0.25	1.21		

Label	Q02-Q08	Q02-Q09	Q08-Q13	Q08-Q14	Q09-Q14
CZ XEB Error (%)	2.34	2.19	2.95	4.11	4.12
Label	Q13-Q19	Q13-Q20	Q14-Q20	Q19-Q24	Q19-Q25
CZ XEB Error (%)	2.36	2.35	2.88	2.82	2.77
Label	Q20-Q25	Q24-Q31	Q25-Q31		
CZ XEB Error (%)	2.57	2.9	4.01		

Case Study: Factorization Problem

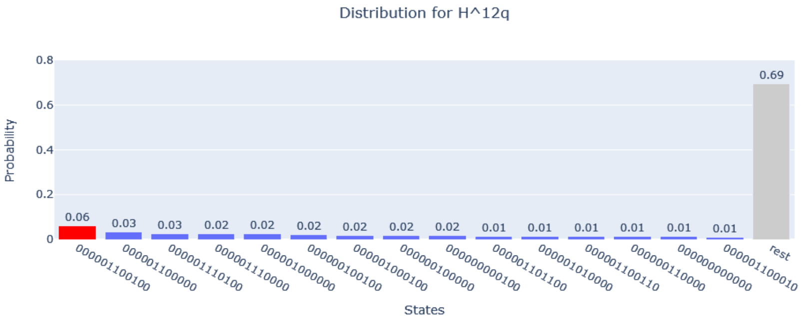

图: Final state of DCQO's solution to H^{12q}, correponding to factorization of 9983

Label	Q28	Q34	Q39	Q45	Q50	Q56	Q35	Q40
Single Qubit Gate Error (%)	0.21	0.22	0.16	0.28	0.19	0.15	0.44	0.23
T1	25.77	20.39	28.92	24.85	22.98	28.75	18.93	29.20
T2*	6.66	1.48	2.11	1.88	0.77	2.08	7.57	8.11
Readout Error (%)	0.27	0.56	1.53	0.19	1.20	0.61	0.19	0.57
Label	Q46	Q51	Q57	Q62				
Single Qubit Gate Error (%)	0.59	0.21	0.21	0.27				
T1 (μs)	7.58	27.70	26.49	17.78				
T2* (μs)	7.27	2.90	2.61	5.18				
Readout Error (%)	0.46	1.44	0.29	22.81				

Label	Q28-Q34	Q28-Q35	Q34-Q39	Q34-Q40	Q35-Q40	Q39-Q45
CZ XEB Error (%)	2.24	2.38	2.92	1.94	3.12	2.69
Label	Q39-Q46	Q40-Q46	Q45-Q50	Q45-Q51	Q46-Q51	Q50-Q56
CZ XEB Error (%)	3.96	2.75	1.5	2.44	3.51	2.97
Label	Q50-Q57	Q51-Q57	Q56-Q62	Q57-Q62		
CZ XEB Error (%)	2.85	2.83	3.34	3.2		

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p-Spin Problem

- p-Spin problem aims at solving ground state of

$$H_{p\text{-spin}} = \frac{1}{N^{(p-1)/2}} \sum_{k=1}^{p} J_{i_1 \dots i_k} \sigma^z_{i_1} \dots \sigma^z_{i_k} \qquad J_{i_1 \dots i_k} \sim \mathcal{N}(0,1)$$

- DCQO workflow
  - Choose an CD ansatz. E.g. $A_\lambda = \sum_i \alpha_i J_i \sigma^i_y + \sum_{i<j} \beta_{ij} J_{ij} \sigma^i_y \sigma^j_z$
  - Solve for optimal parameters J_i, J_{ij} from variational CD formula
  - Construct VQA

$$|\psi\rangle_{\text{cd}} = e^{-iA_\lambda(t)\mathrm{d}\lambda} |+\rangle^{\otimes n}$$

  - Optimize parameters

p-Spin Problem

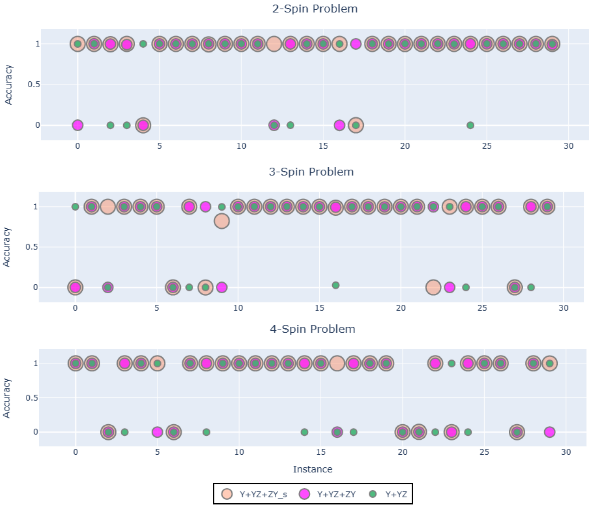

图: Accuracy of DCQO with various CD ansatz for 2-Spin, 3-Spin and 4-Spin Problem.

- For 2-spin problems, at least one CD ansatz leads to true ground state
- There are trade-offs between expressibility and trainability
- For 3-spin and 4-spin, there are instances cannot be solved by DCQO with 2-local interaction.

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Summary and Outlook

- Quantum technology has entered NISQ beyond classical simulatable
- VQA has the potential for quantum advantage with limited Gate fidelity and coherence time
- DCQO provides a framework to construct circuit with more efficiency and physical intuition
- DCQO-based VQA presents better trainability with warm-started parameter initialization
- Near-term application may benefit more from control protocols like STA by making VQA more process-aware

Thank you for your attention

Backup Slides

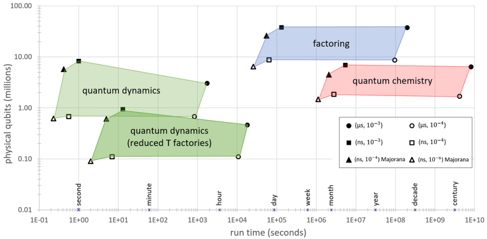

图: Resource Estimation for three types of Algorithms

qubit parameter examples	operation times		error rates	
	gate	measurement	Clifford	non-Clifford
(μs, 10^{-3}) qubit	100 μs	100 μs	10^{-3}	10^{-6}
(μs, 10^{-4}) qubit	100 μs	100 μs	10^{-4}	10^{-6}
(ns, 10^{-3}) qubit	50 ns	100 ns	10^{-3}	10^{-3}
(ns, 10^{-4}) qubit	50 ns	100 ns	10^{-4}	10^{-4}
(ns, 10^{-4}) Majorana qubit	100 ns	100 ns	10^{-4}	0.05
(ns, 10^{-6}) Majorana qubit	100 ns	100 ns	10^{-6}	0.01

图: Examples of qubit parameters. 1-2 row relevant for trapped ion, 3-4 row relevant for superconducting and spin system, 5-6 for Majorana qubits

application	algorithm execution accuracy $1-\epsilon$	quantum executable parameters			quality requirements	
		Q	$C_{\min}$	M	max P	max P_T
quantum dynamics	0.999	230	$1.5 \cdot 10^5$	$2.4 \cdot 10^6$	$9.7 \cdot 10^{-12}$	$1.4 \cdot 10^{-10}$
quantum chemistry	0.99	2740	$4.1 \cdot 10^{11}$	$5.4 \cdot 10^{11}$	$3.0 \cdot 10^{-17}$	$6.1 \cdot 10^{-15}$
factoring	0.667	25481	$1.2 \cdot 10^{10}$	$1.5 \cdot 10^{10}$	$3.5 \cdot 10^{-16}$	$7.4 \cdot 10^{-12}$

图: Algorithm Details, Q number of logical qubit, C logical timesteps, M T gate counts, P logical error rate, P_T logical error rate for distilled T states

(Reference: Beverland, Michael E., et al. "Assessing requirements to scale to practical quantum advantage (2022)." arXiv preprint arXiv:2211.07629 (2022). See also Azure Quantum Resource Estimator: https://learn.microsoft.com/en-gb/azure/quantum/intro-to-resource-estimation)