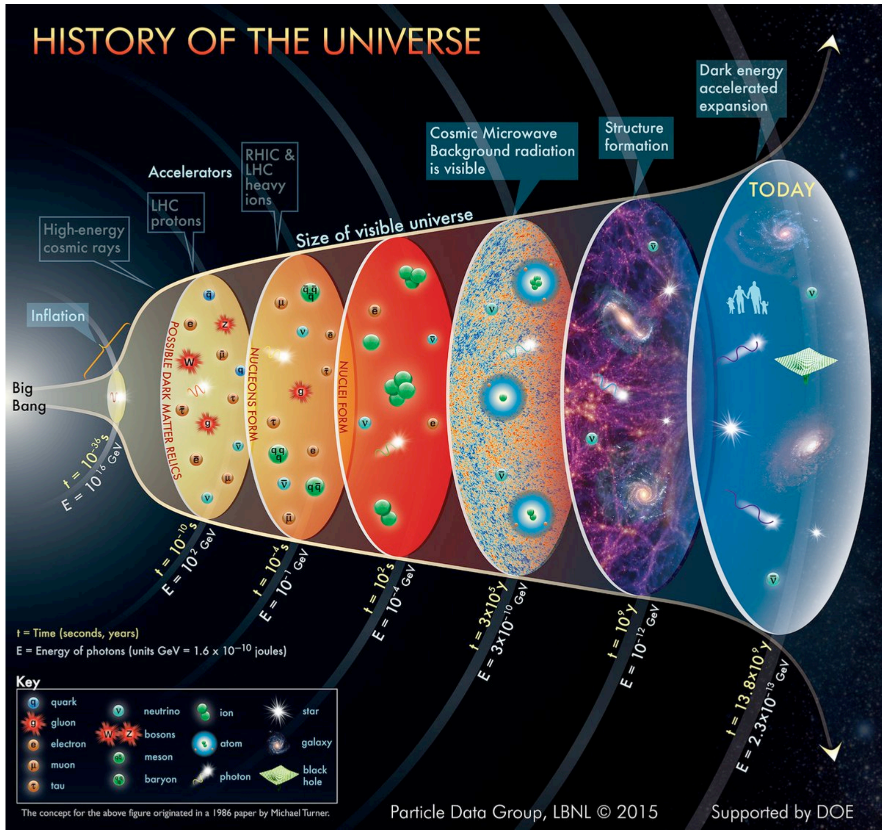
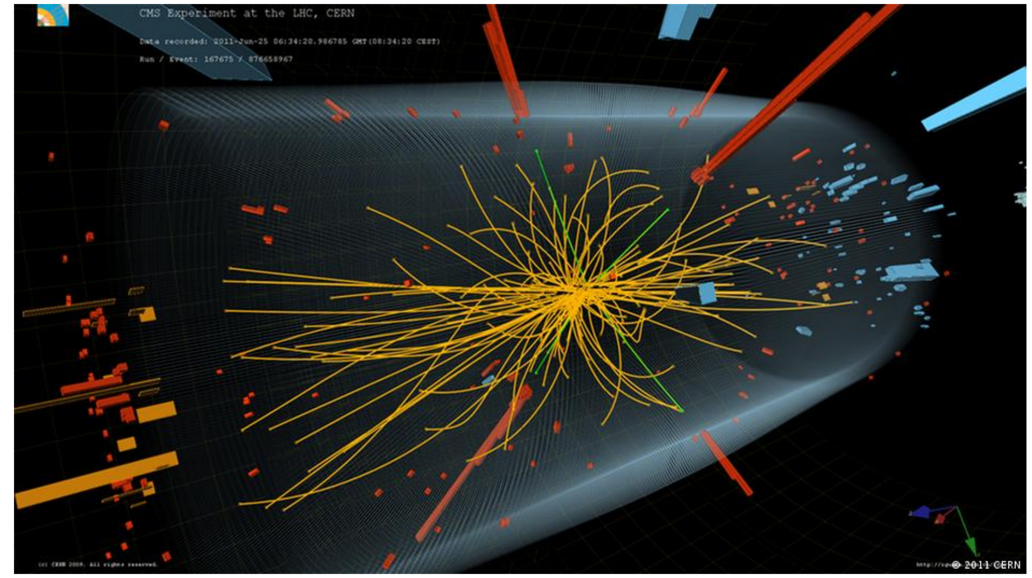
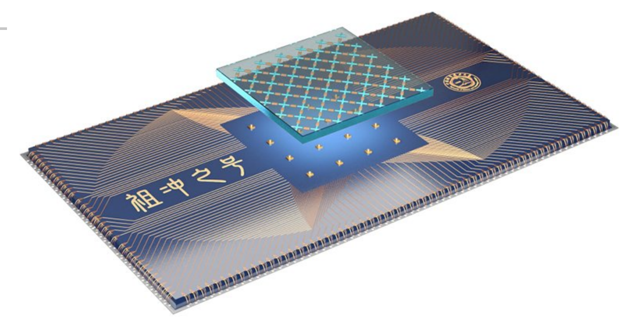
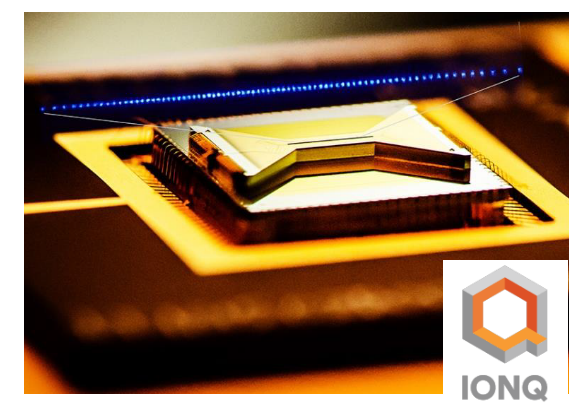








中国科学技术大学
University of Science and Technology of China

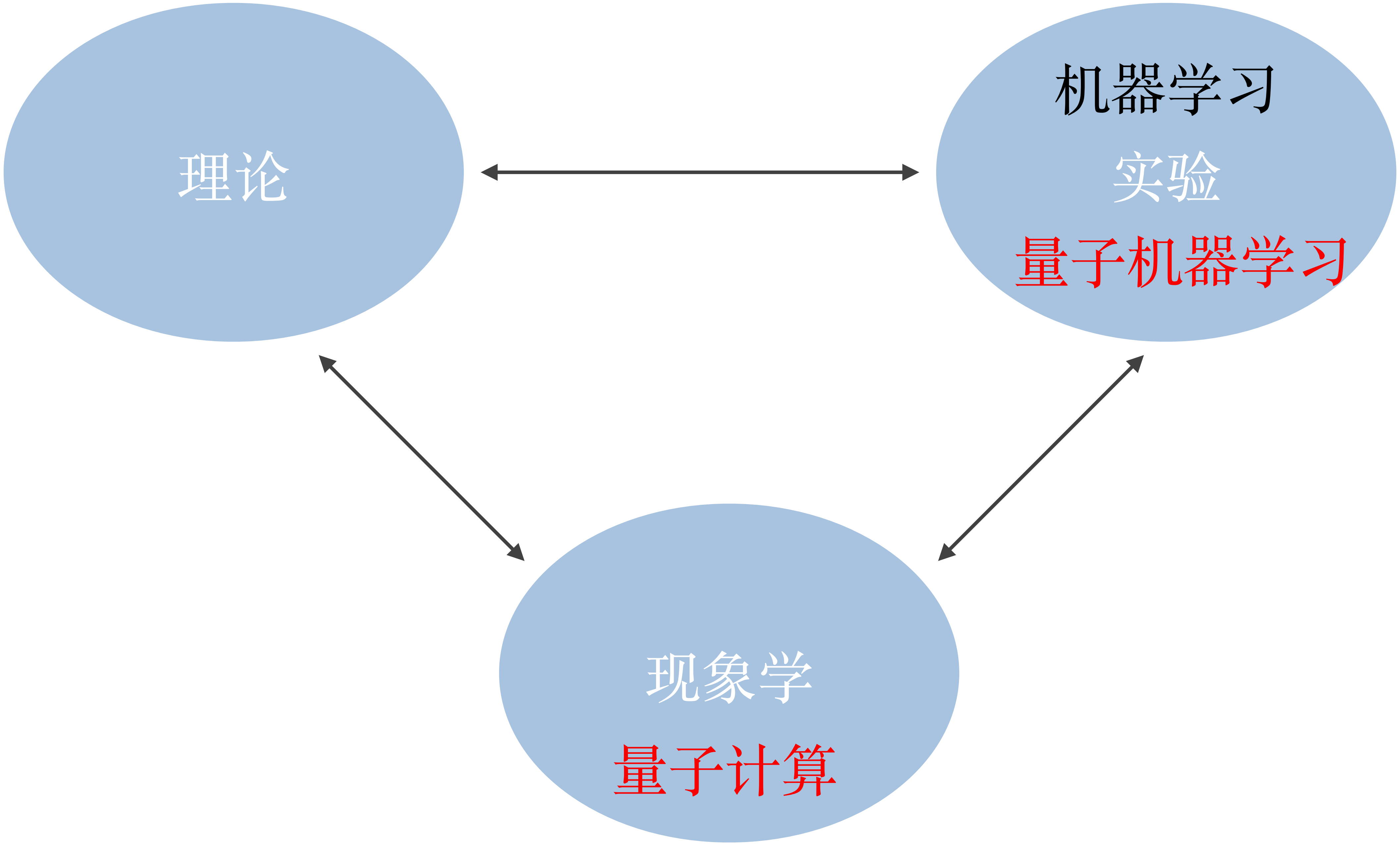
Ying-Ying Li

yingyingli@ustc.edu.cn

粒子物理计算前沿之量子计算

August, 2024

粒子物理



现象学

量子计算

Objectives

- Quantum computing for first-principle calculations in High Energy Physics
- The framework for this calculation: resources we need to simulate QFT
- Simulating dynamics of gauge theories on quantum computer
- 1+1d Schwinger models on quantum computer

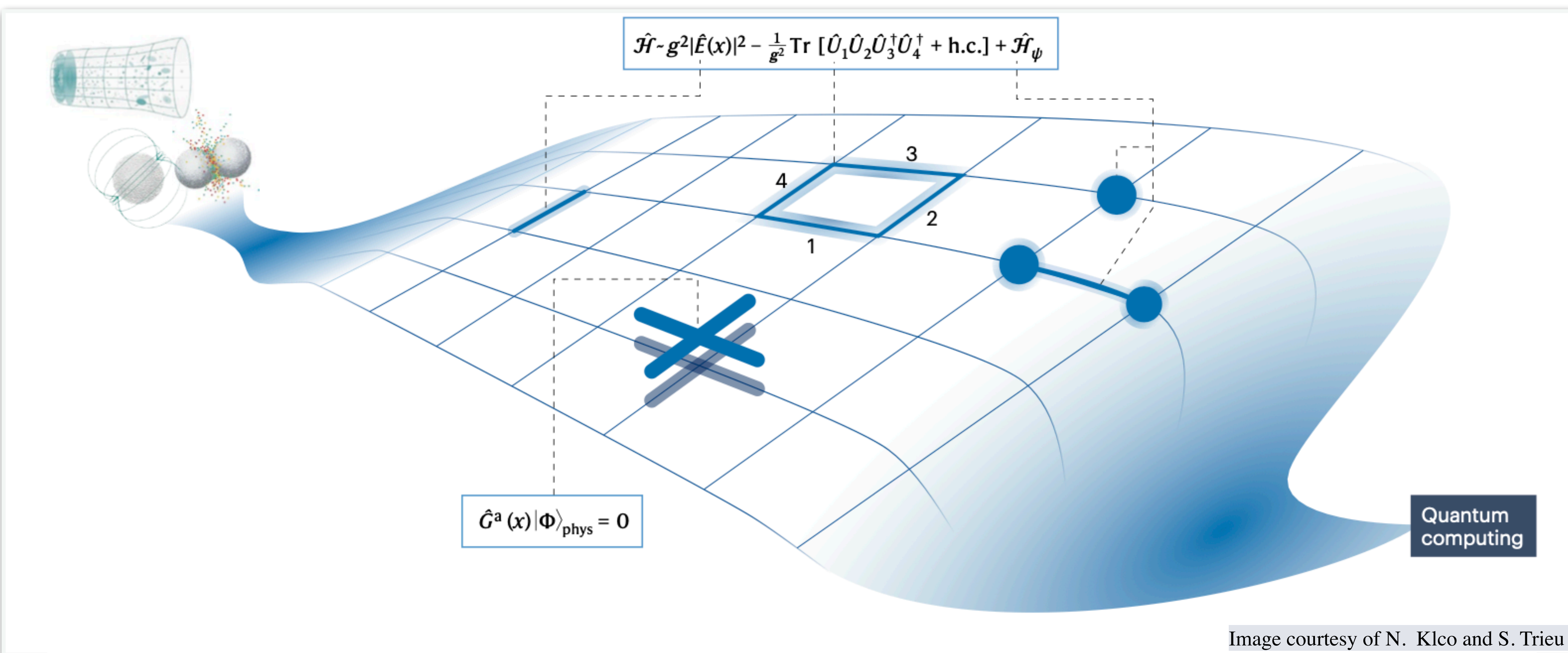
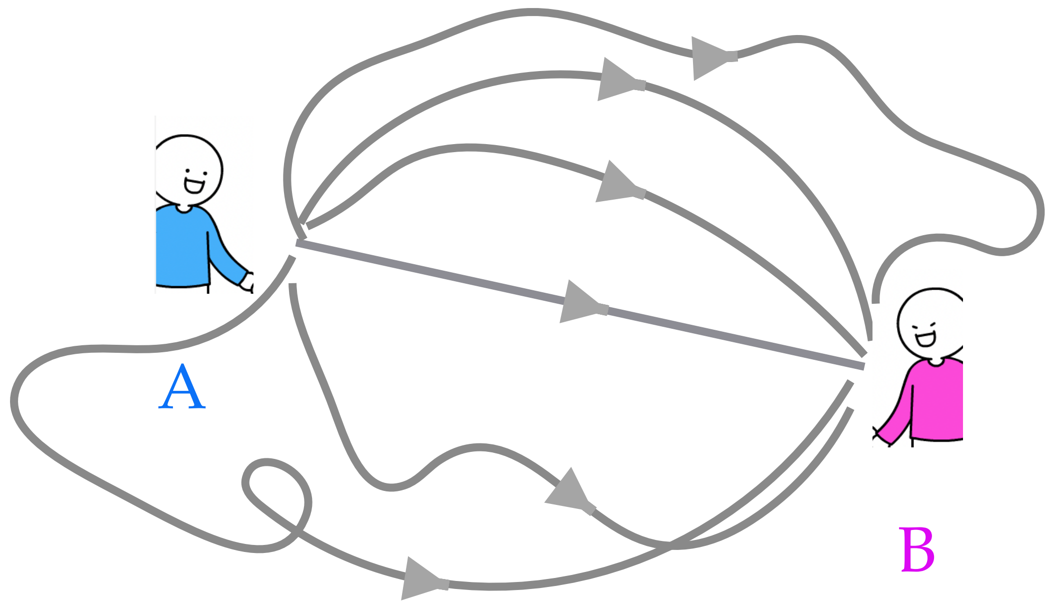
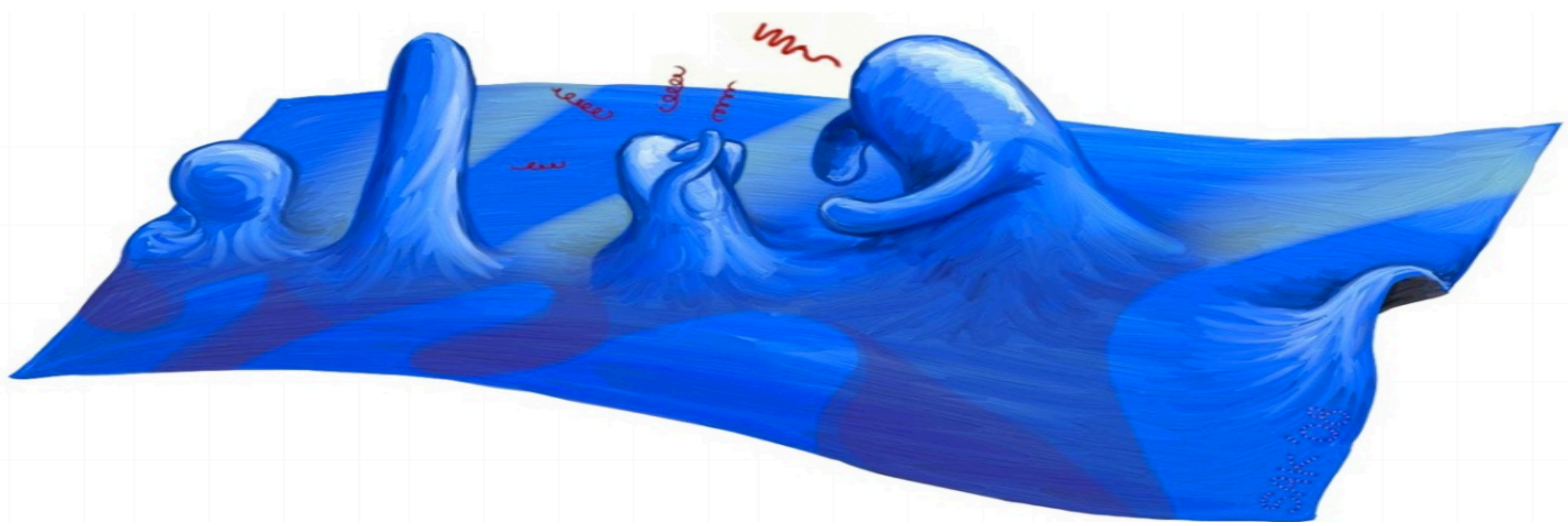


Image courtesy of N. Klco and S. Trieu

Simulating the Theory

Relativistic Quantum Mechanics → Quantum Field Theory (QFT)

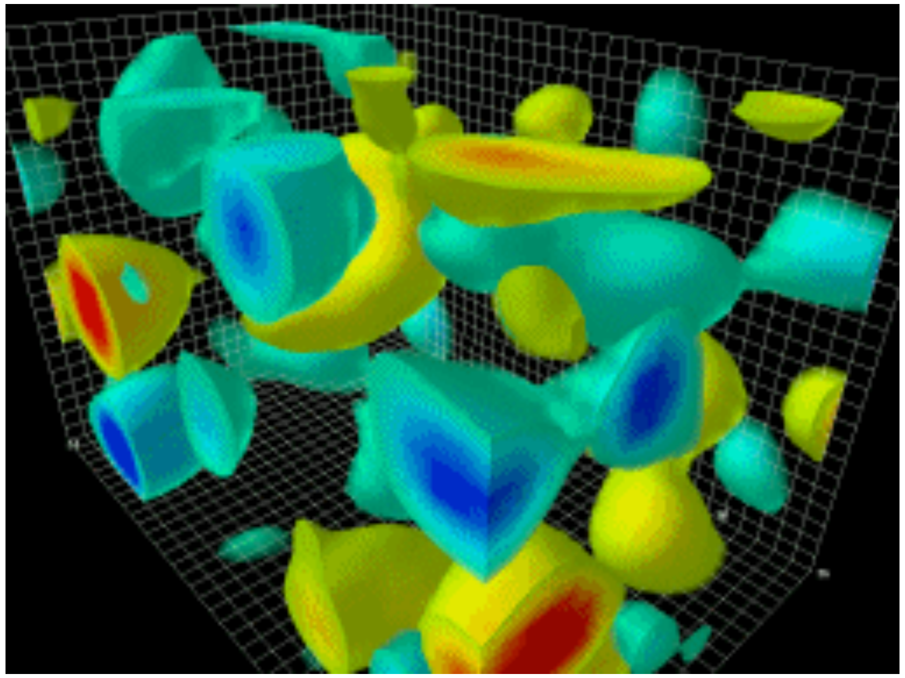




path integral on the background of field configurations

Lattice QCD - Euclidean Spacetime

remains the only tool for
precise, controllable,
first principle calculations



field configurations
$\mathcal{C}$ on lattice

path integral in
Euclidean spacetime

Monte Carlo
sampling of lattice
field configurations

→

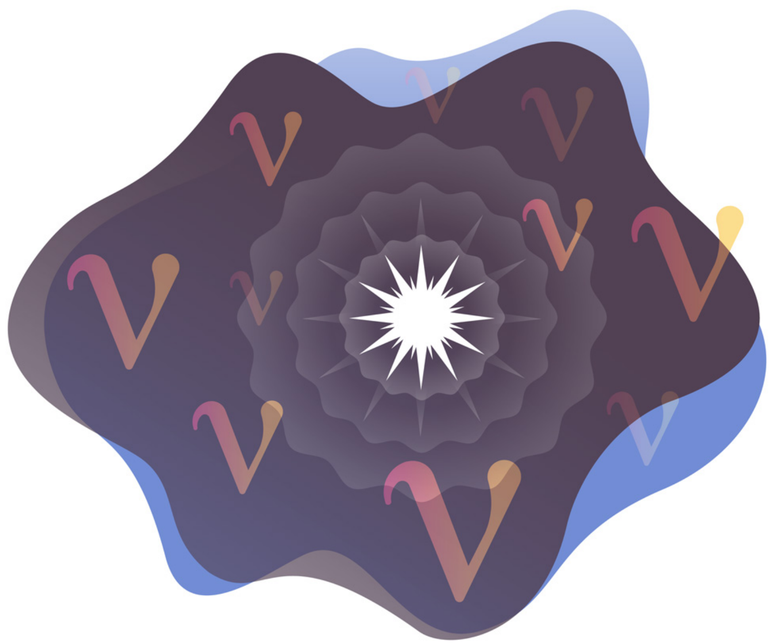
Euclidean
correlations and
physical observables

$$W(\mathcal{C}) \sim \exp(-S(\mathcal{C}))$$

$$\langle O \rangle = \frac{\sum_{\mathcal{C}} O(\mathcal{C}) W(\mathcal{C})}{\sum_{\mathcal{C}} W(\mathcal{C})}$$

Lattice QCD - Euclidean Spacetime - first principle calculations

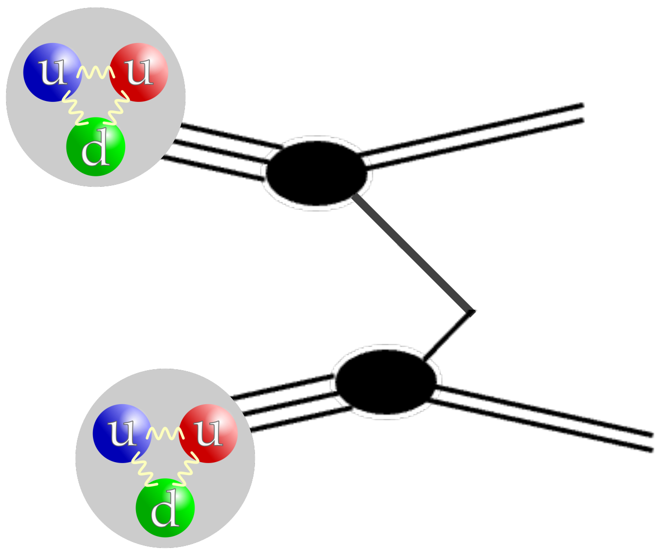
finite density



$$S \to S + iS_1$$

real-time dynamics

PDF



$$\int \mathcal{D}\phi e^{iS}$$

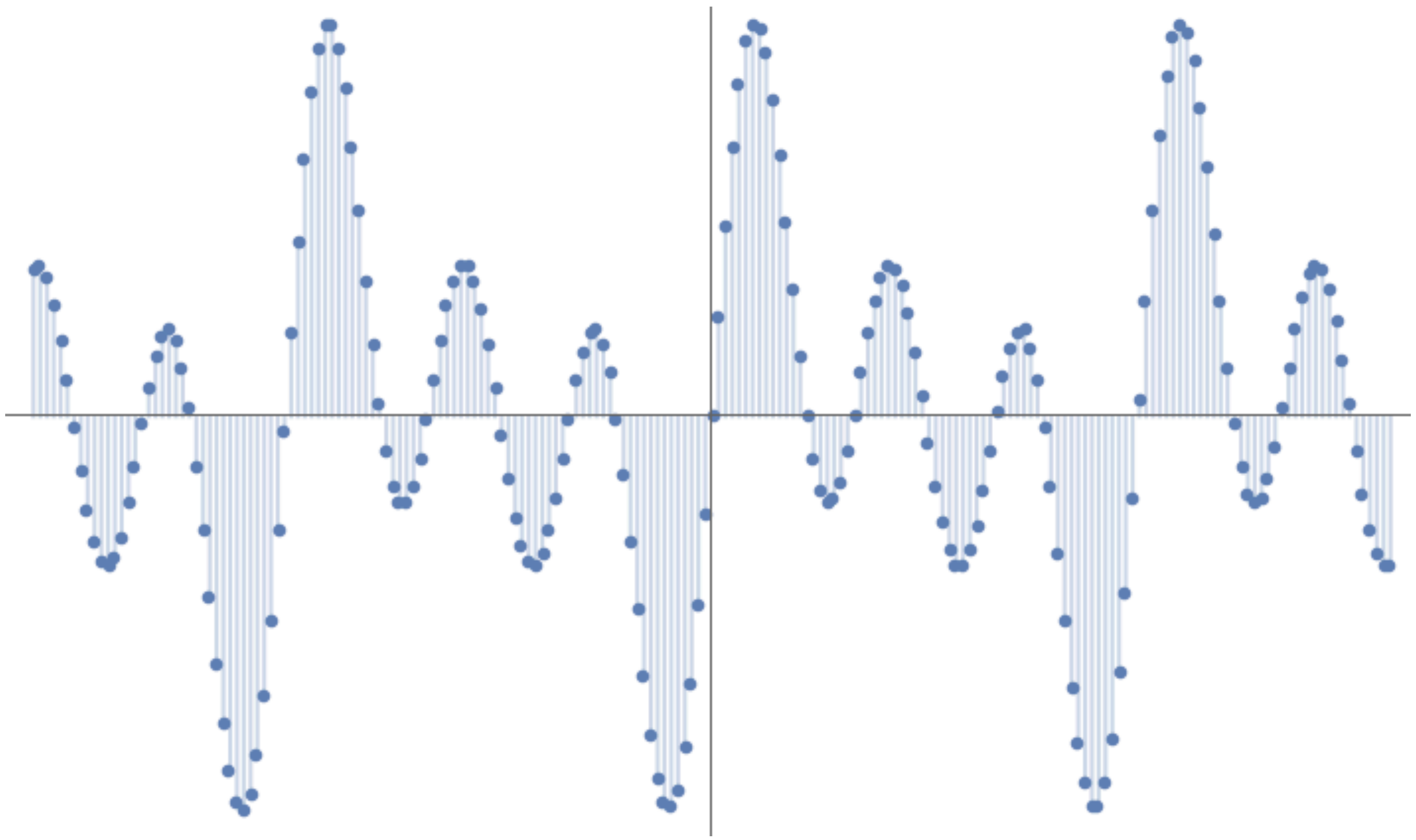
complex $S(\mathcal{C})$

Sign Problem

complex $S(\mathcal{C})$

$$W(\mathcal{C}) \sim \exp(-S(\mathcal{C}))$$

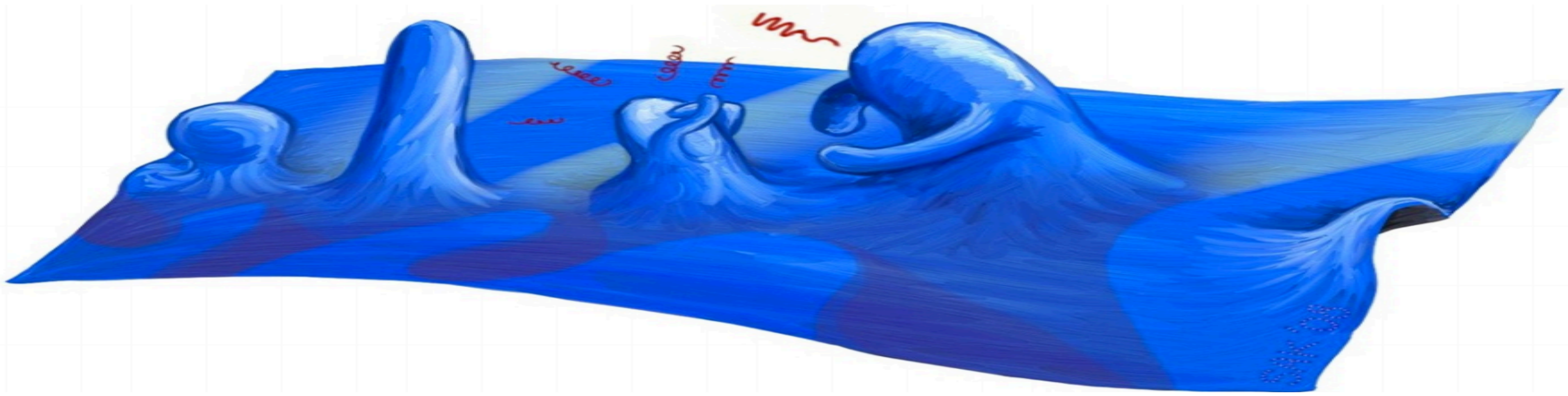
$$\langle O \rangle = \frac{\sum_{\mathcal{C}} O(\mathcal{C}) W(\mathcal{C})}{\sum_{\mathcal{C}} W(\mathcal{C})}$$



configuration space $\mathcal{C}$ is exponentially large in system size

system size N_V : number of lattice sites

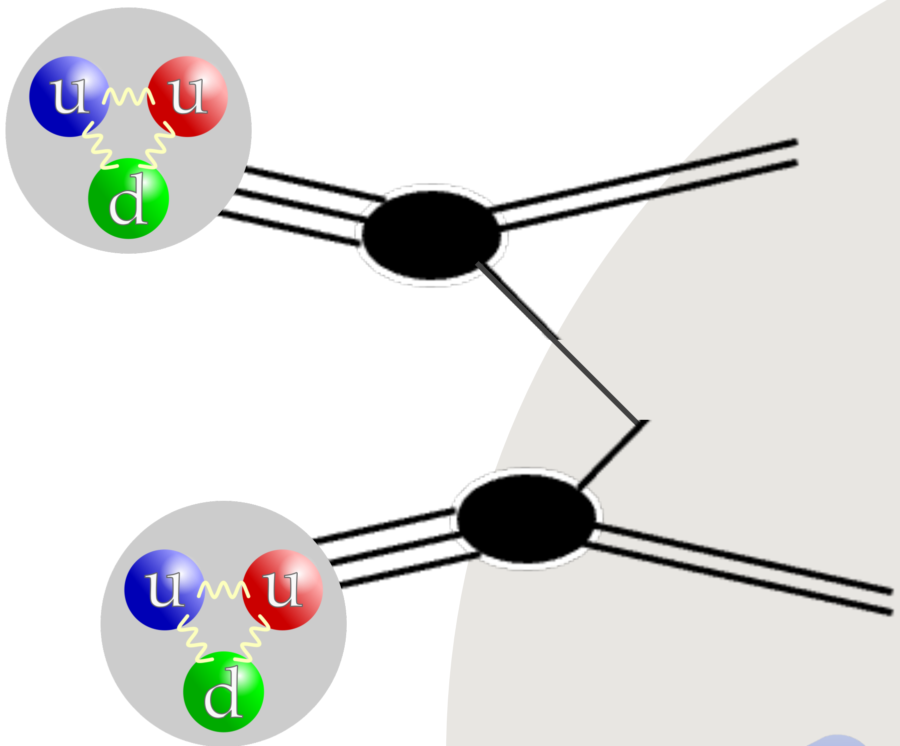
$$\int \mathcal{D}\phi e^{iS} = \langle x| \, e^{-iHt} \, |y\rangle$$



$$\dim H \propto |G|^{N_V}$$

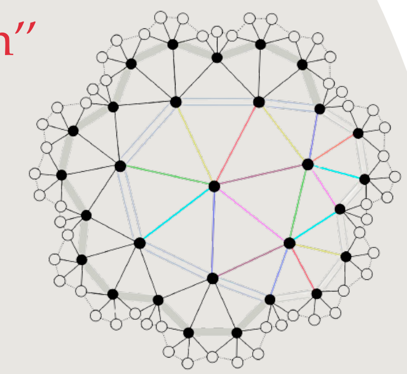
system size N_V : number of lattice sites

exponentially large number
of classical bits in system size

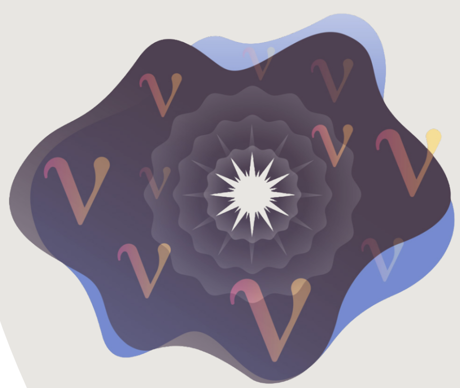


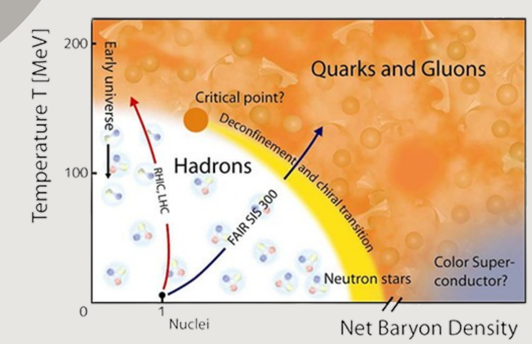
High Energy Physics

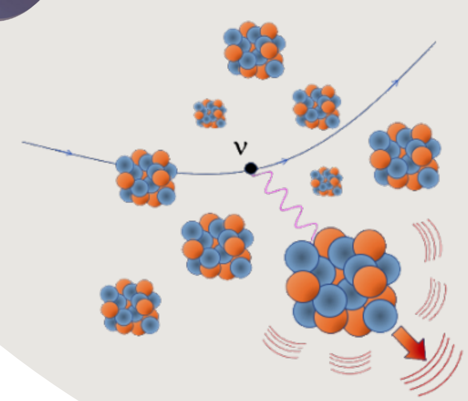
real-time dynamics
finite density
quantum interference
out-of equilibrium
``strongly interacting many-body system"

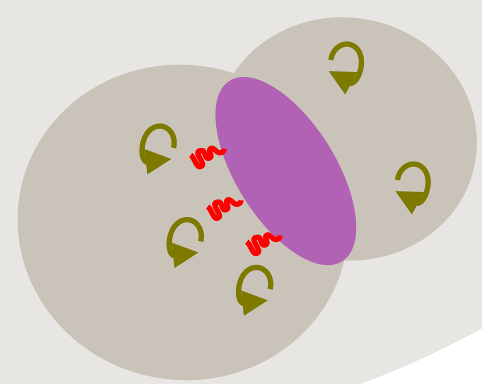


CLASSICAL
EASY
polynomial time









``a computing system that scales well with the system size?"

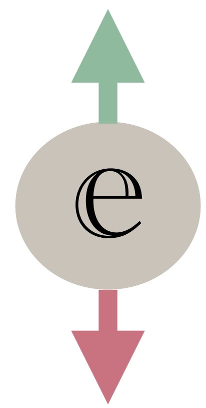
Quantum Computing

1982 “nature isn’t classical”

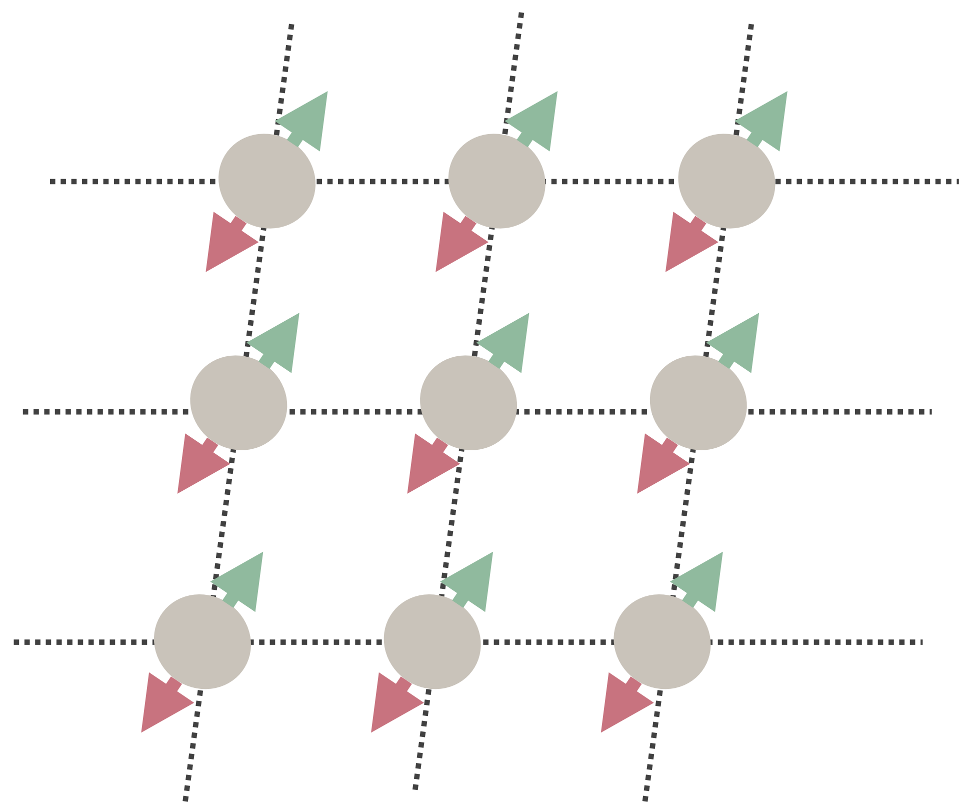
R. P. Feynman

“ if you want to make a simulation of Nature, you’d better make it quantum mechanical”

Quantum Computing



$$|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$$



$$N_q = 300$$

2^{300} classical bits are needed to describe the system of 300 qubits

``a computer that uses qubits”

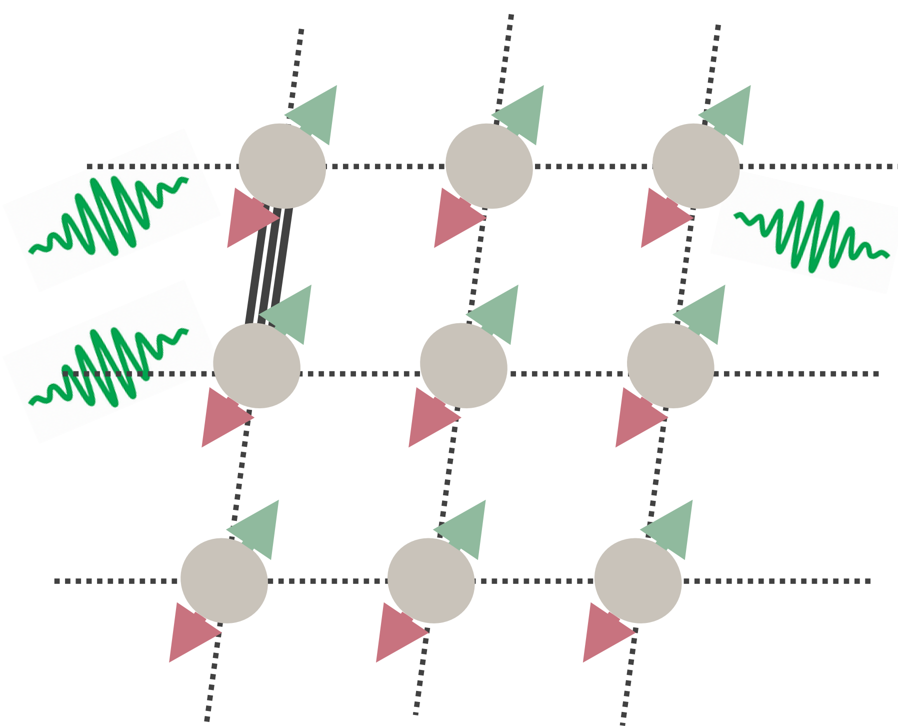
Quantum Computing

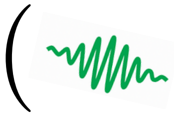
1996 - Seth LIoyd: efficient simulation of LOCAL Hamiltonians

Universal Quantum Simulators

Seth Lloyd

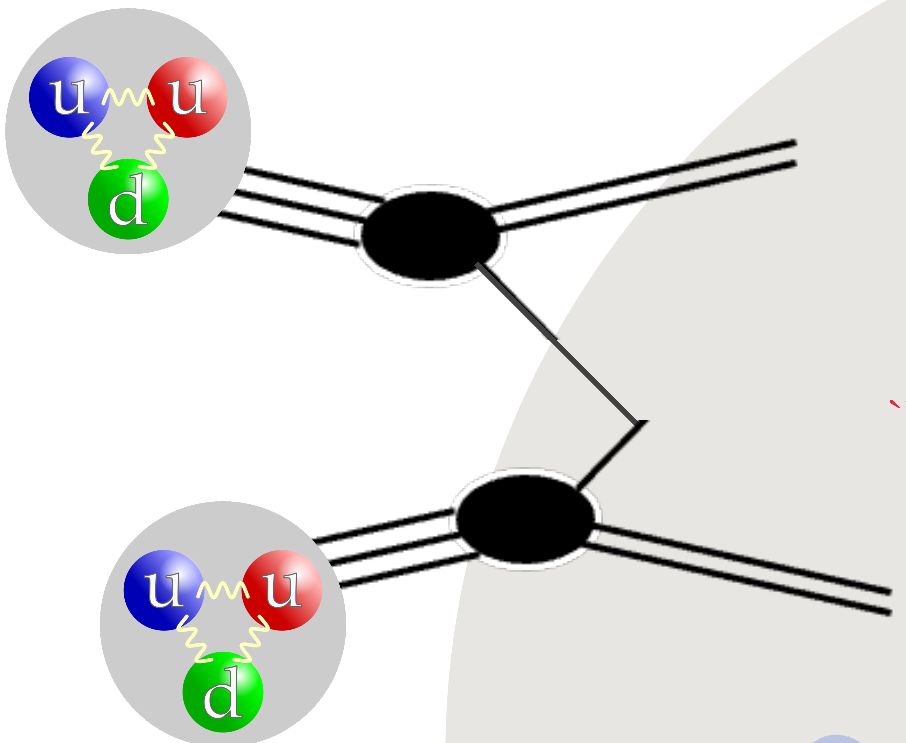
Feynman's 1982 conjecture, that quantum computers can be programmed to simulate any local quantum system, is shown to be correct.



$N($  $) \propto N_q^m$

Polynomial Time Complexity

How Powerful It is?



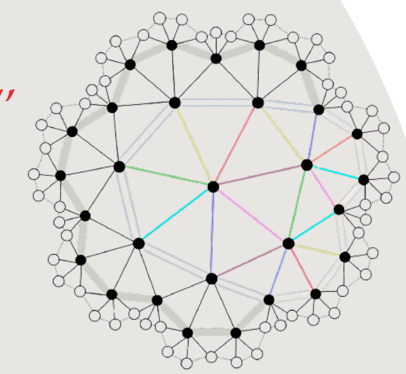
QUANTUM EASY

High Energy Physics

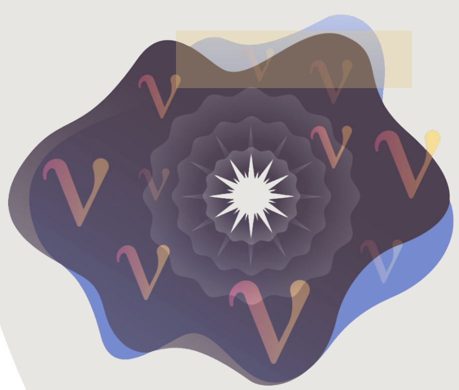
real-time dynamics
finite density
quantum interference
out-of equilibrium
``strongly interacting many-body system''

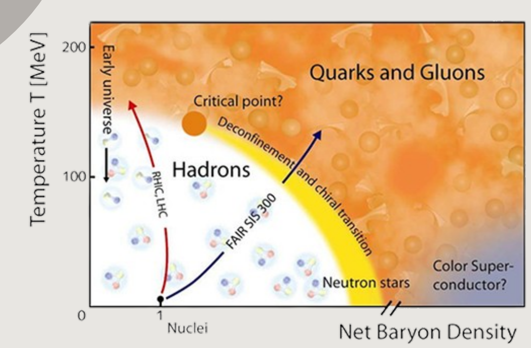
QUANTUM HARD

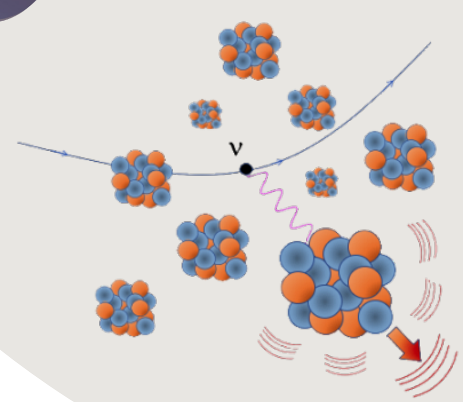
e.g. traveling salesmen problem

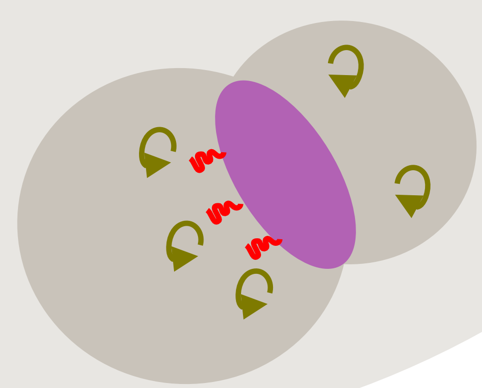


CLASSICAL EASY





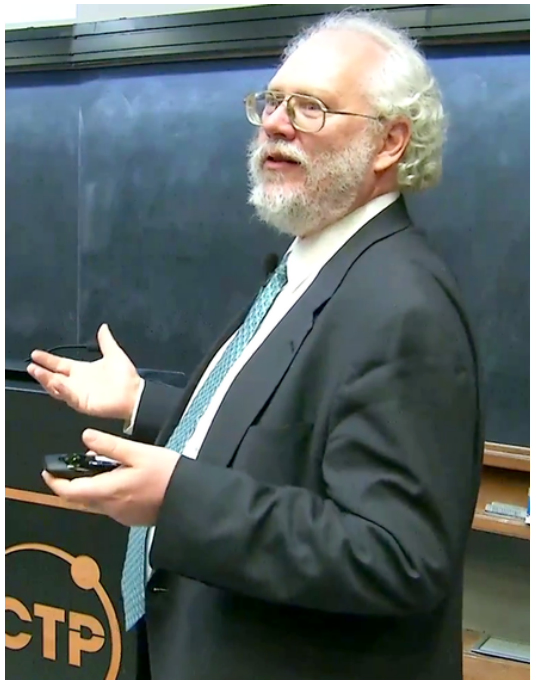




Quantum Computing

- 1990s - error-correcting codes and fault-tolerant methods

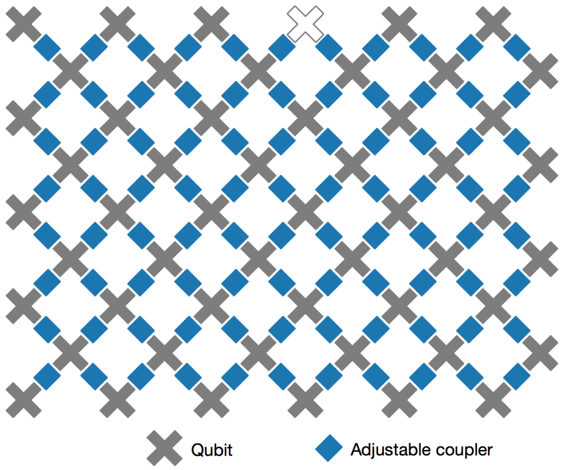
  Quantum Threshold Theorem



P. Shor

- 2019 - Google: Quantum Computational Supremacy

  The quantum hardware is producing meaningful results

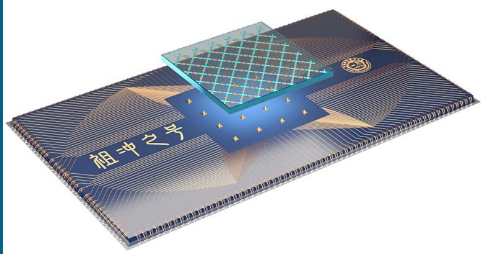


Quantum Computing

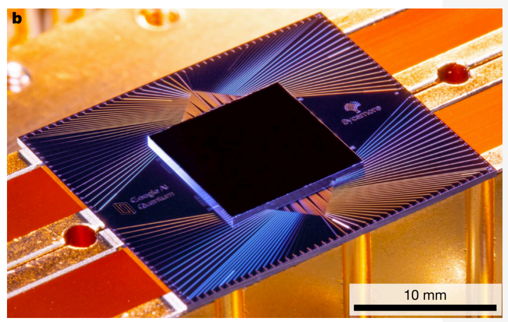
Now - Noisy Intermediate Scale Quantum (NISQ) era

more than 50 well controlled qubits, not error-corrected yet

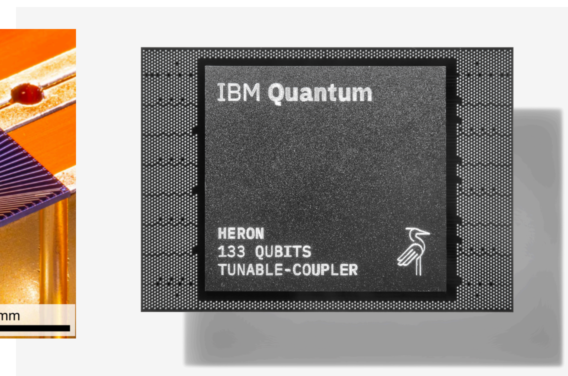
superconducting processor



176 qubits

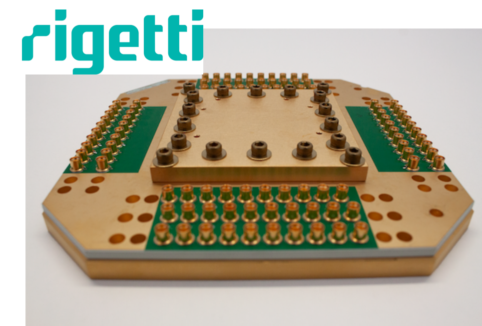


54 qubits



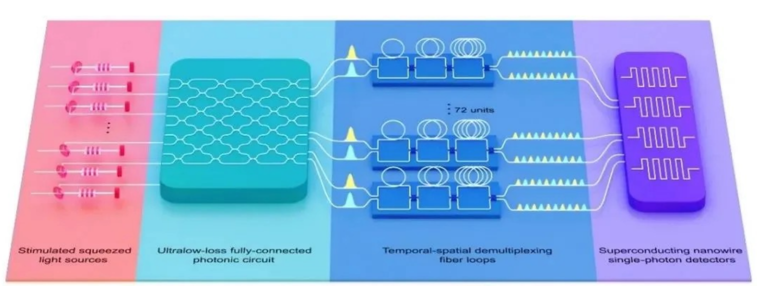
1121 qubits
access to 133 qubits

multi-chip quantum processor



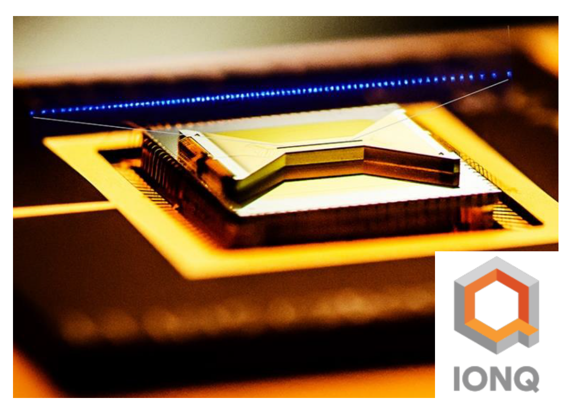
80 qubits

photon qubits



Jiuzhang - 255 qubits

trapped ion qubits

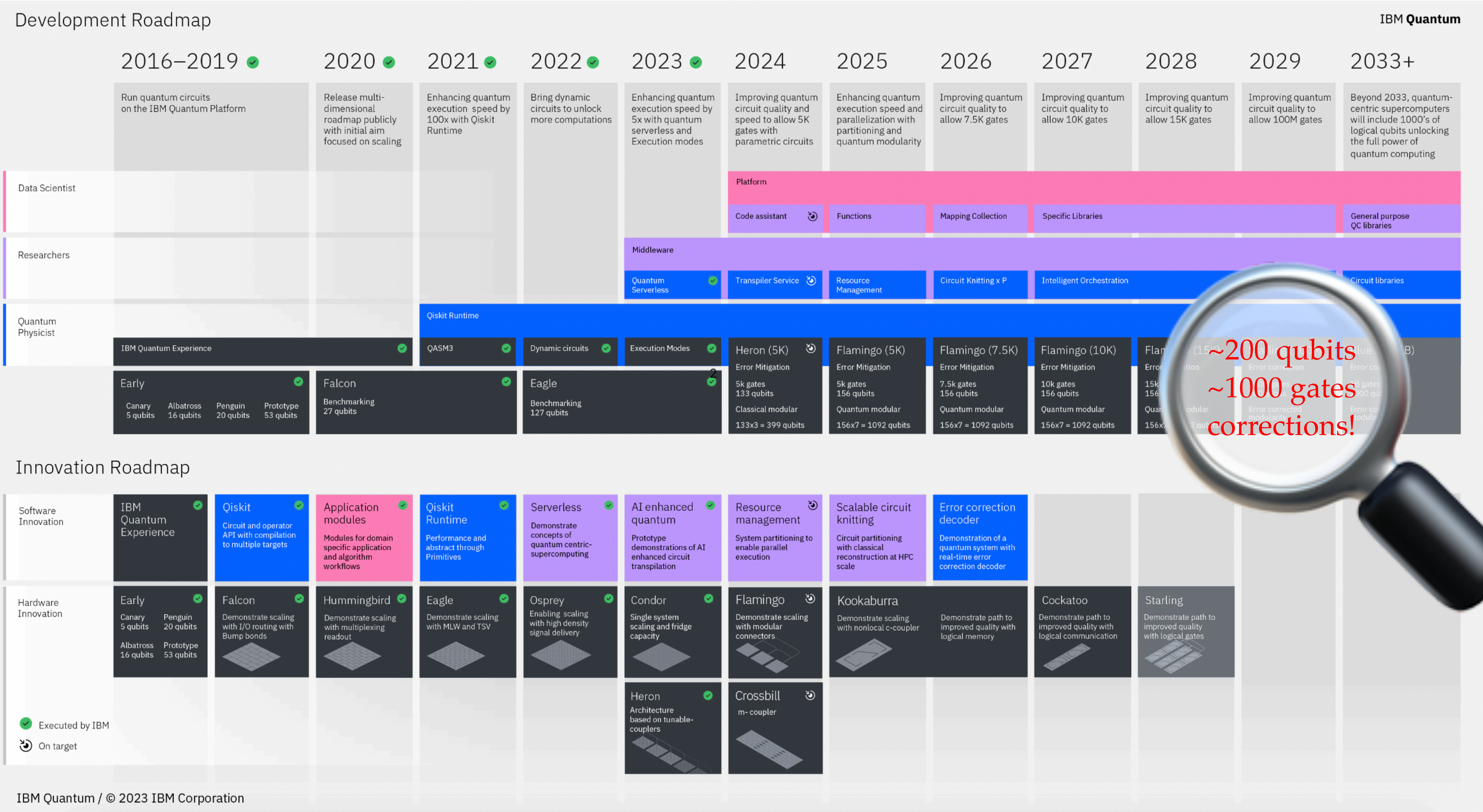


22 qubits

48 logical qubits

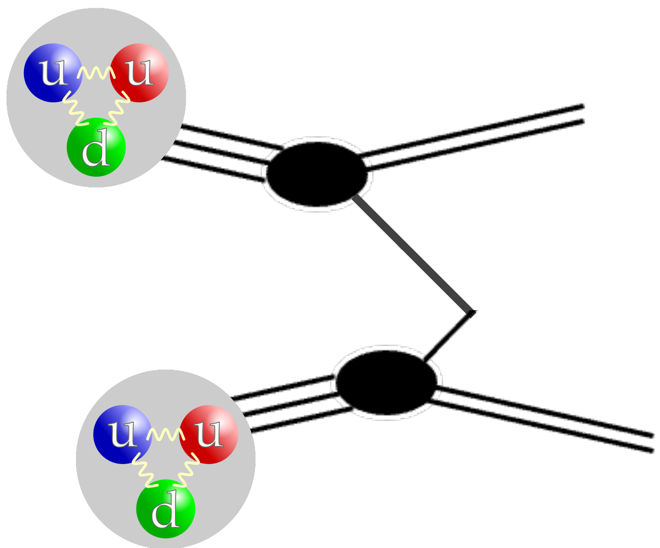
Quantum Computing

Next decades



Quantum Computing for HEP

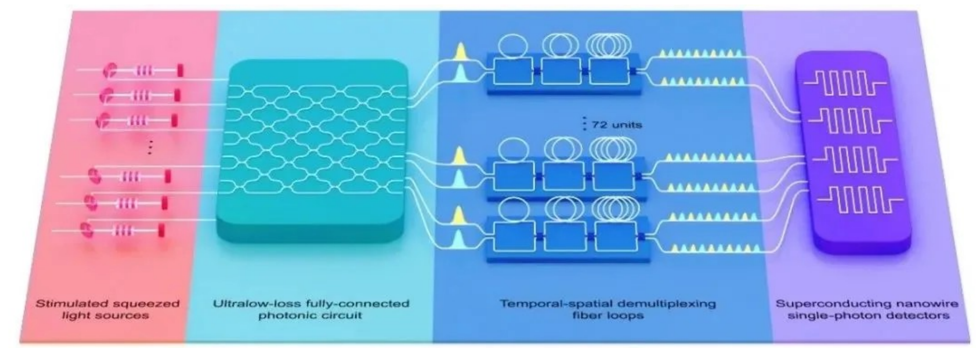
$$\int \mathcal{D}\phi e^{iS} = \langle x| \, e^{-iHt} \, |y\rangle$$

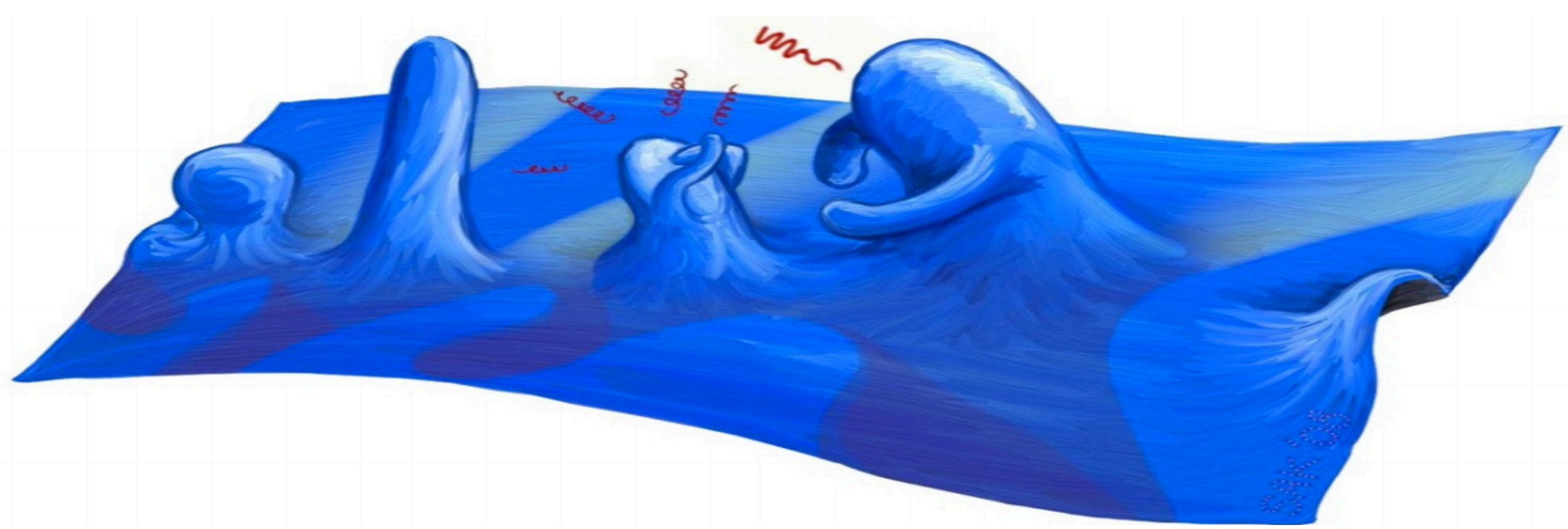


mapping

DOF to qubits

time evolution
to quantum gates





non-trivial vacuum,
composite initial state,
bosonic and fermionic DOF,
symmetries, …

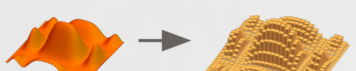
Where are we?

General Framework

(2010s) galactic algorithms

(2020s) pocket of methods for every steps,
continuum limits,
error corrections

(2030s) ?



$|q\rangle^N \rightarrow |G\rangle$

↓

$\mathcal{U}\,|G\rangle^L \rightarrow |\psi_0\rangle$

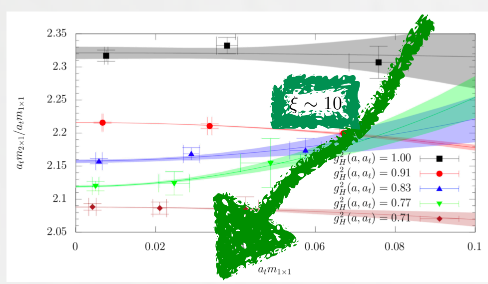
↓

$\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$

↓

$\langle \mathcal{O} \rangle$

various methods



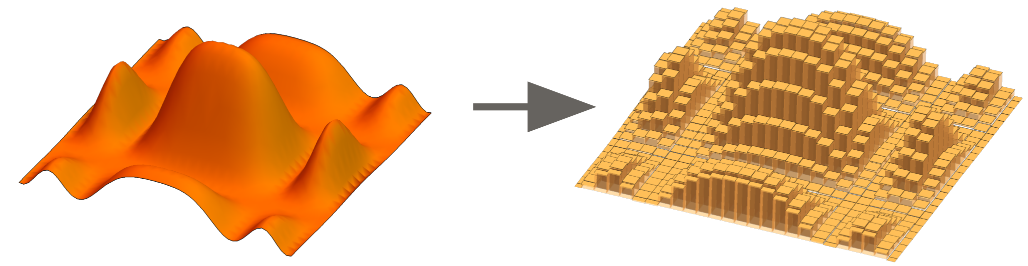
S. P. Jordan,
K. S. M. Lee,
J. Preskill

2011-

2020 -

2030s -

Quantum Simulation for Quantum Field Theory



- Discretization of space: KS Hamiltonian
  空间离散化
- Digitization of field degree of freedom:
  场的数字化
  truncation, discrete subgroup

$$|q\rangle^N \rightarrow |G\rangle$$

- Initialization of registers as a state: stochastically

$$\mathcal{U}\,|G\rangle^L \rightarrow |\psi_0\rangle$$

- Propagation of state-discrete time: trotterization, etc
  时间演化算符

$$\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$$

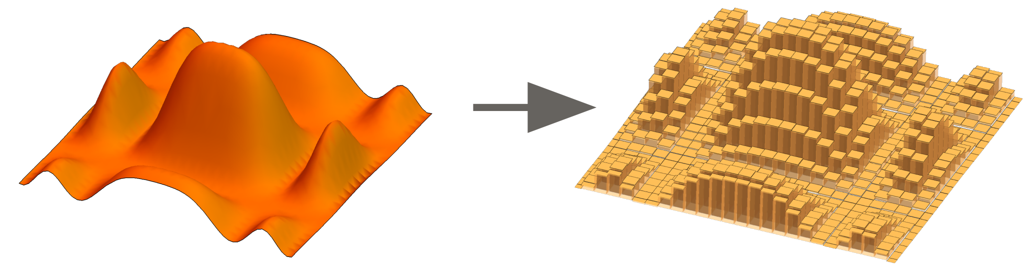
- Evaluation of observables

$$\langle\mathcal{O}\rangle$$

- Error Mitigation

See references in [M. Carena, H. Lamm, **YYL**, W. Liu, PRD. **104**, 094519]

Quantum Simulation for Quantum Field Theory



- ***Discretization of space:*** KS Hamiltonian
  空间离散化
- ***Digitization of field degree of freedom***:
  场的数字化
  truncation, discrete subgroup
- Initialization of registers as a state: stochastically
- ***Propagation of state-discrete time:*** trotterization, etc
  时间演化算符
- Evaluation of observables
- Error Mitigation

$$|q\rangle^N \rightarrow |G\rangle$$

$$\downarrow$$

$$\mathcal{U}\,|G\rangle^L \rightarrow |\psi_0\rangle$$

$$\downarrow$$

$$\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$$

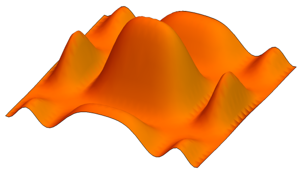
$$\downarrow$$

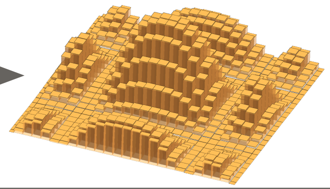
$$\langle\mathcal{O}\rangle$$

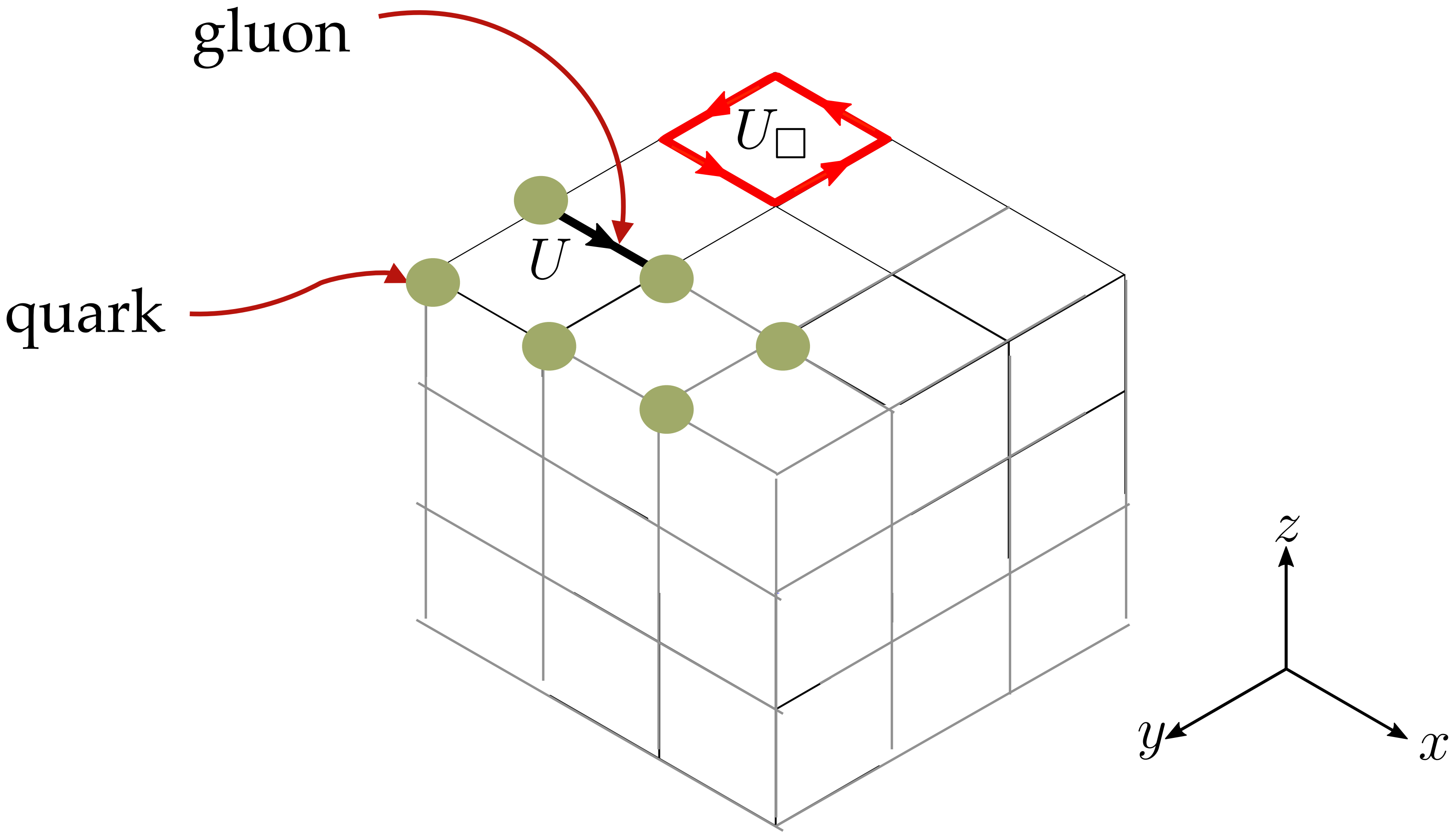
See references in [M. Carena, H. Lamm, **YYL**, W. Liu, PRD. **104**, 094519]

- 规范场的哈密顿量—KS哈密顿量，改进哈密顿量
- 规范场的量子模拟—普遍方法，简化体系举例

Discretization

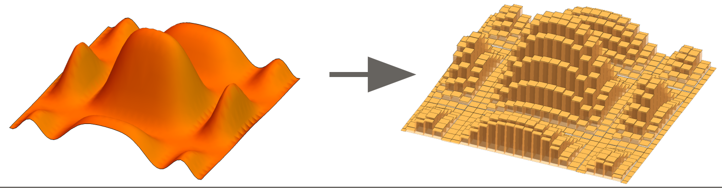




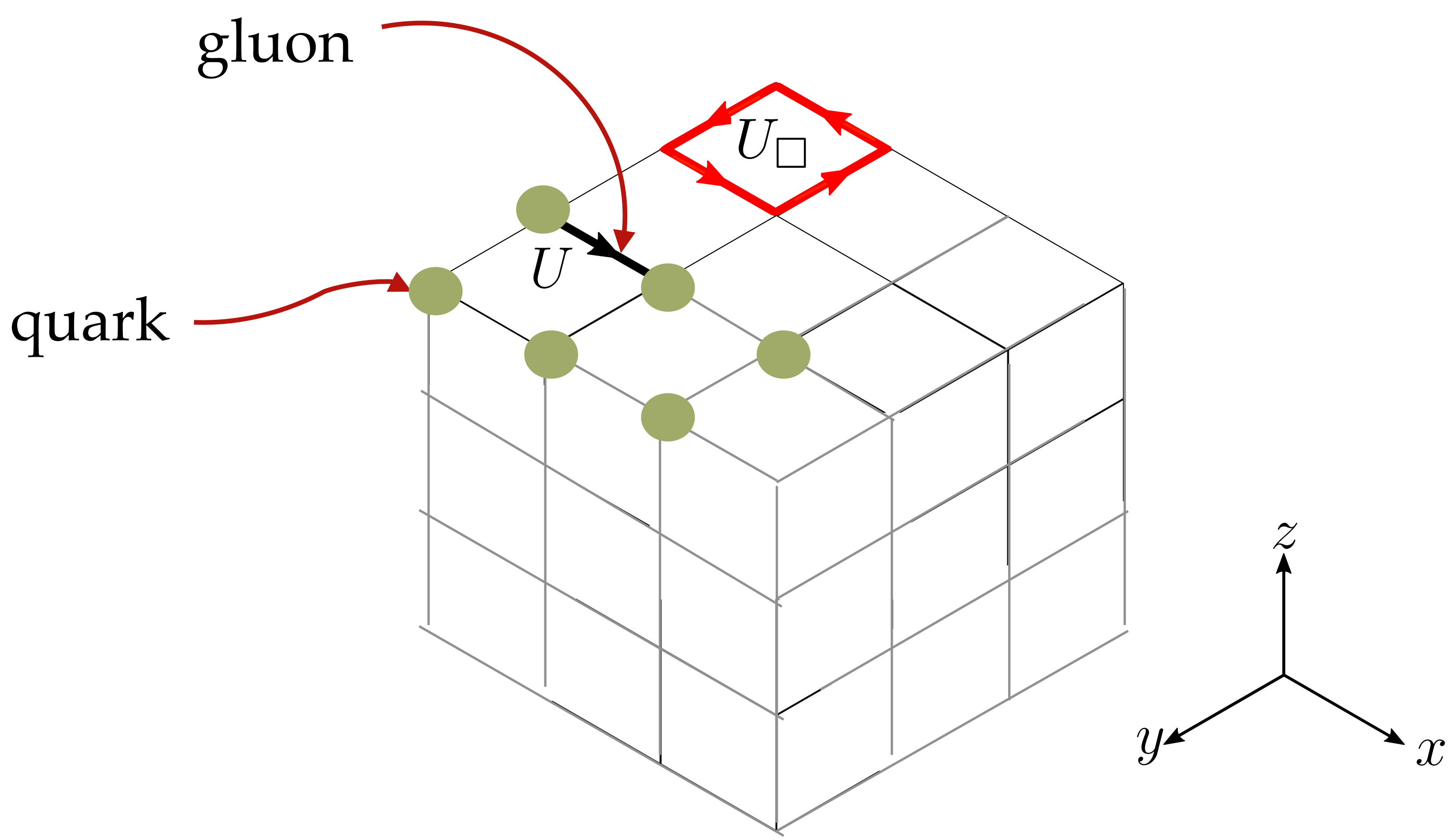


$$U_i(x) = e^{ig \int_a^0 dt A_i(x+t\hat{i})}$$

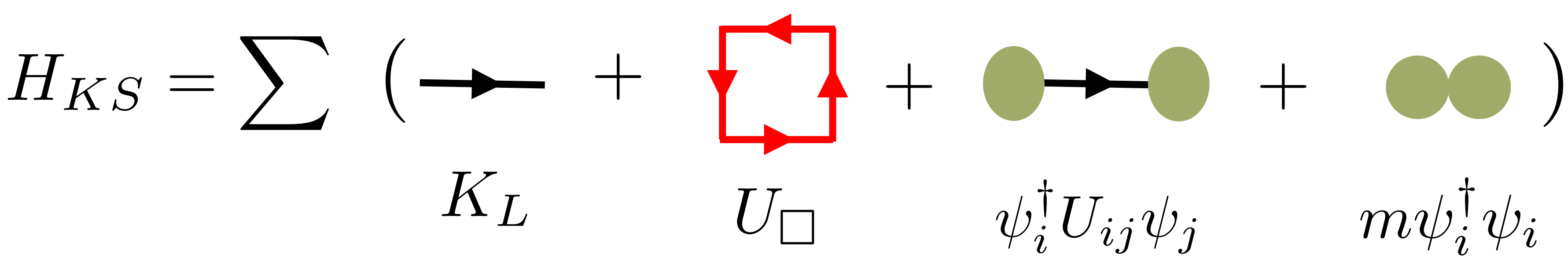
Discretization

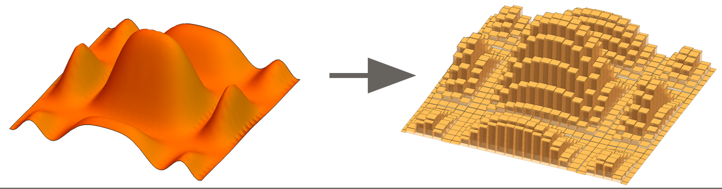


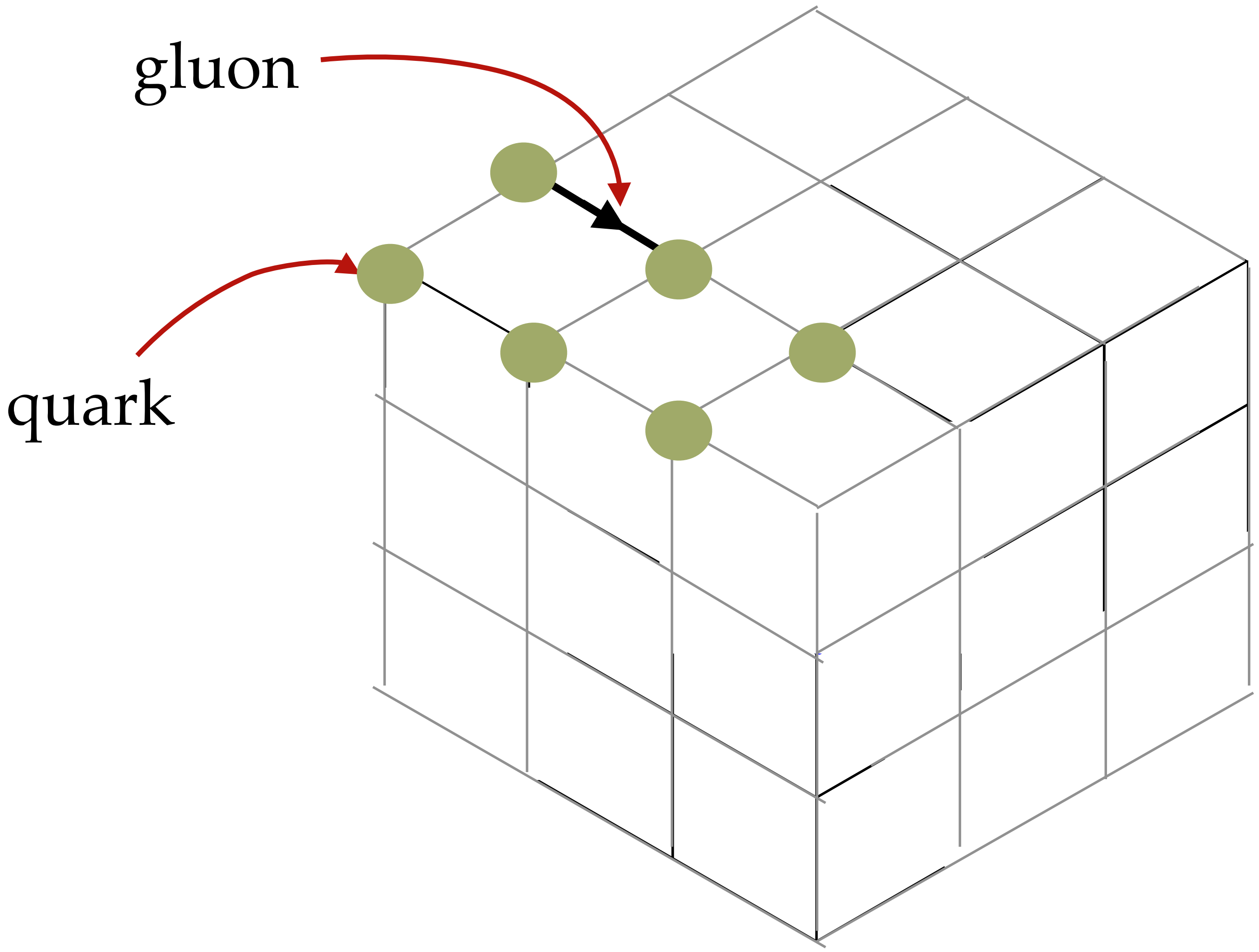
infinities in QFT



KS Hamiltonian Phys. Rev. D 11, 395 (1975)



Discretization  infinities in QFT



spatial dimension d

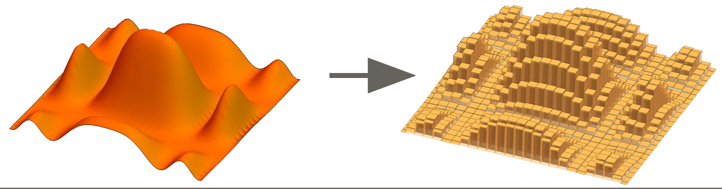
lattice spacing a

$$(na)^{-1} \lesssim E \lesssim a^{-1}$$

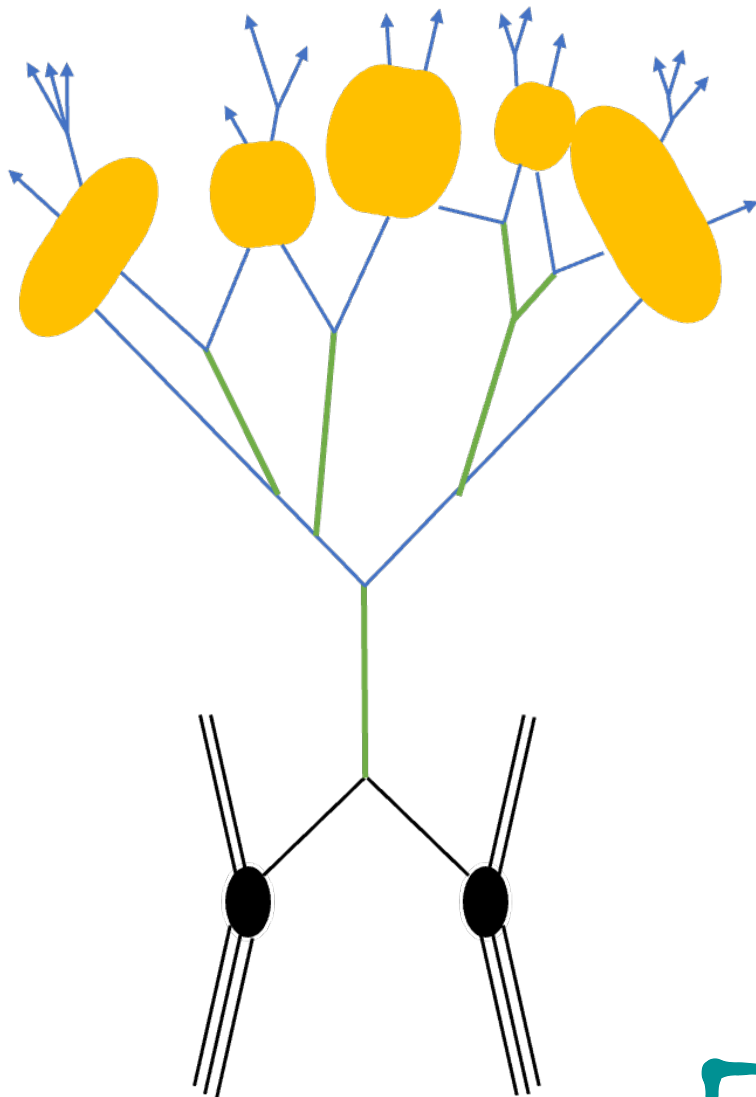
$$n_{\text{qubits}} \sim (n_{\text{gluon}} + n_{\text{quark}}) \times n^d$$

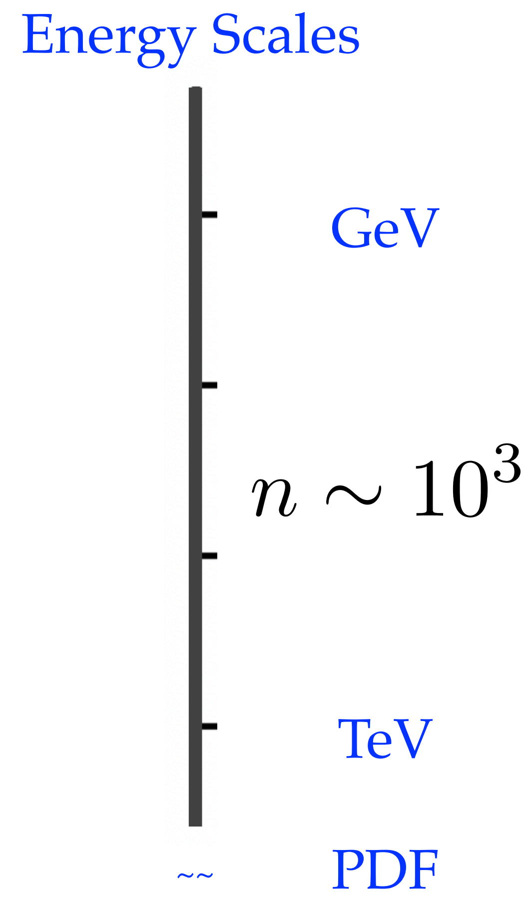
$$\sim f \times n^d$$

Discretization



infinities in QFT



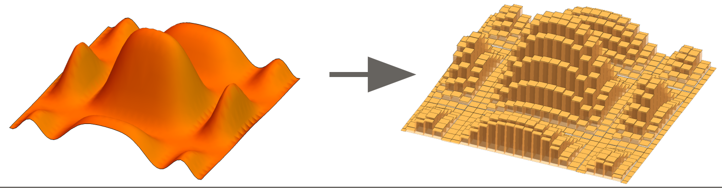


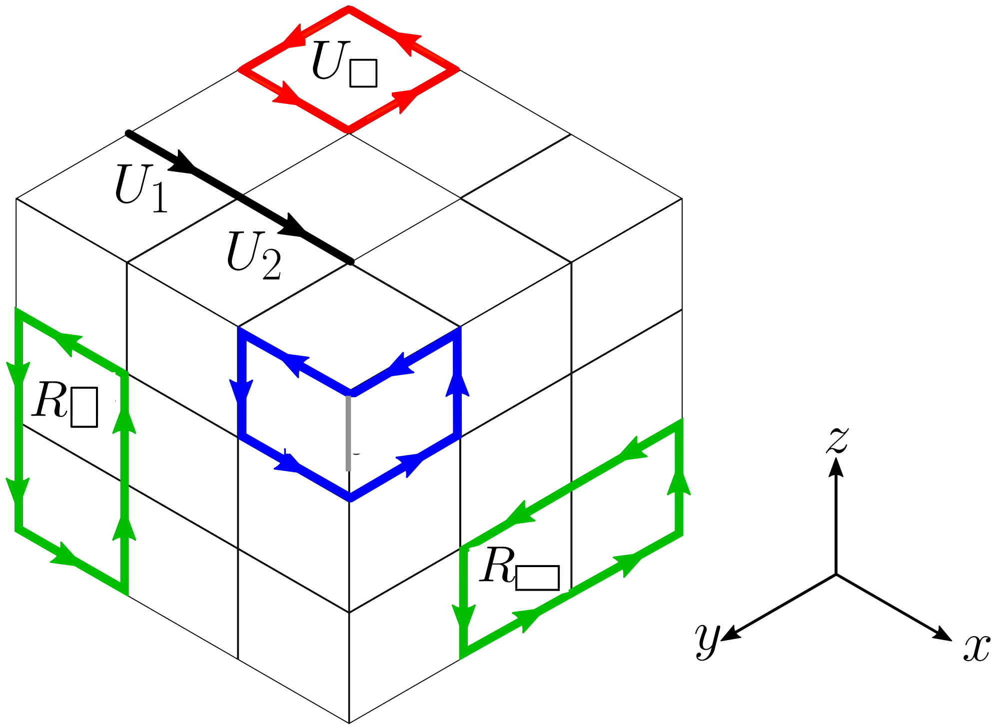
$$n_{\text{qubits}} \sim f \times n^d$$

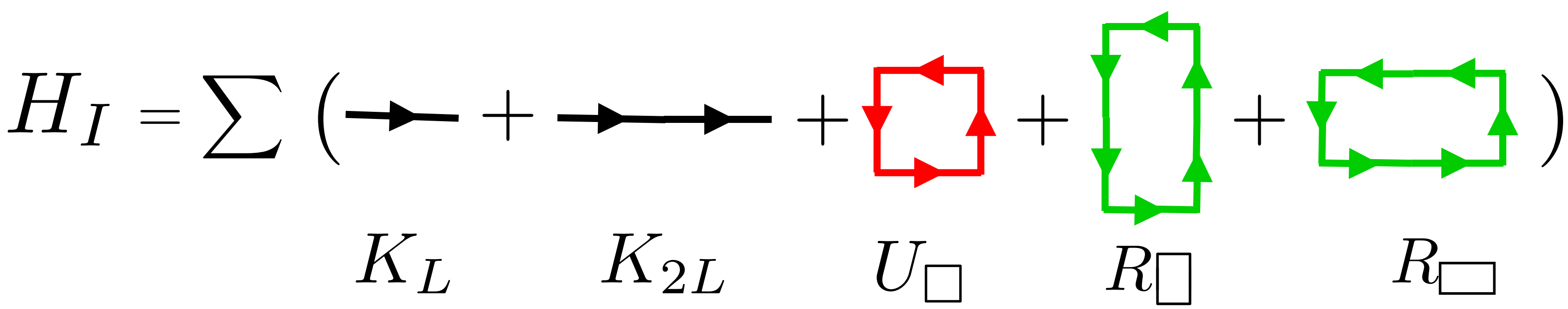
non-perturbative regime

$$100\text{MeV} \lesssim E \lesssim \text{GeV}$$

$$n_q \sim f \times 1000$$

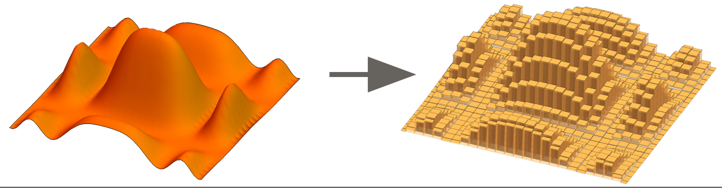




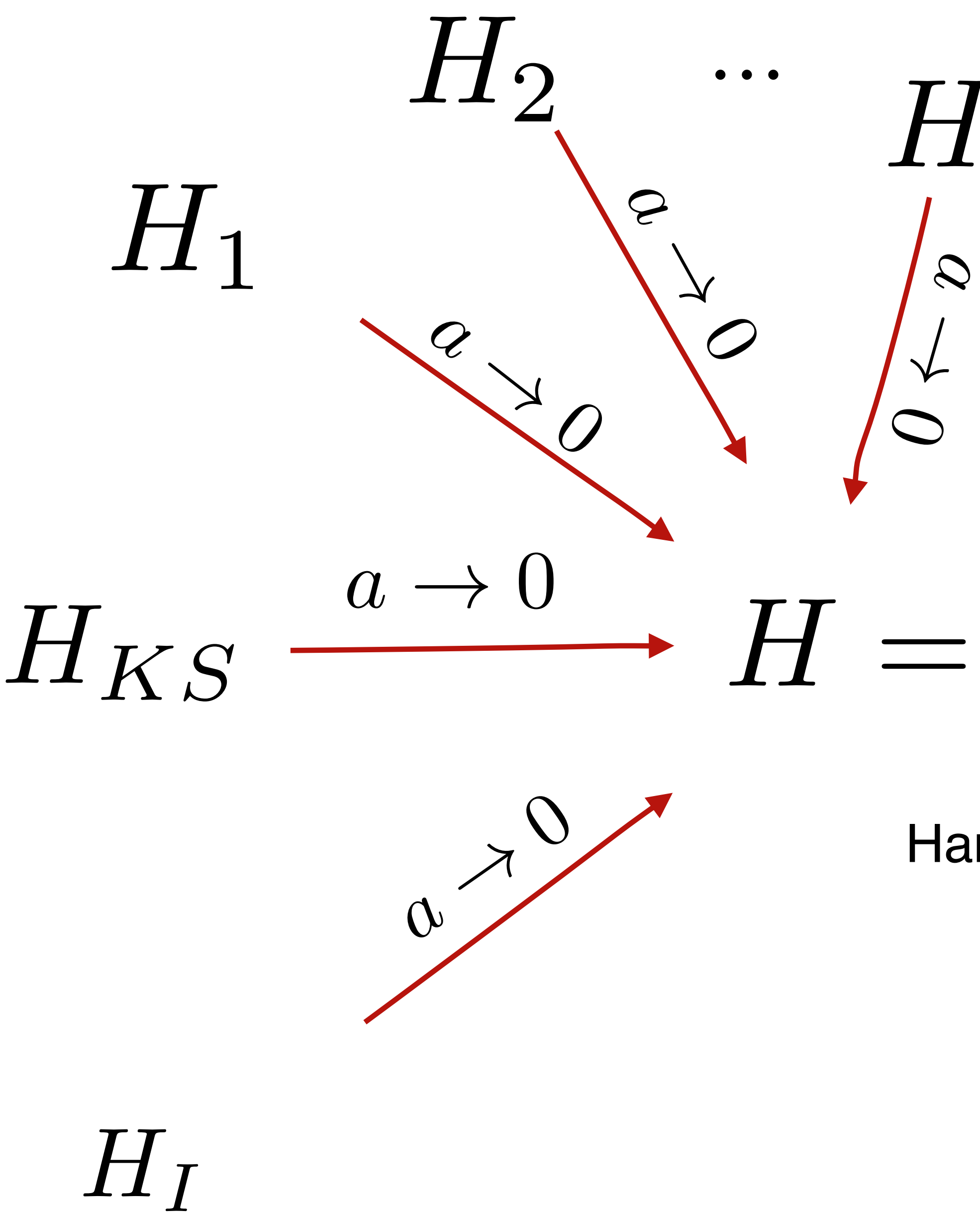


improved Hamiltonian

Discretization



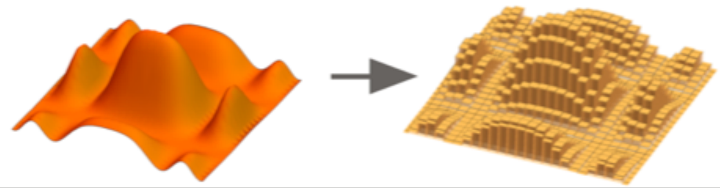
infinities in QFT



$$H = \int dx (E^2 + B^2)$$

Hamiltonian in continuous space

• 规范场的哈密顿量 Discretization



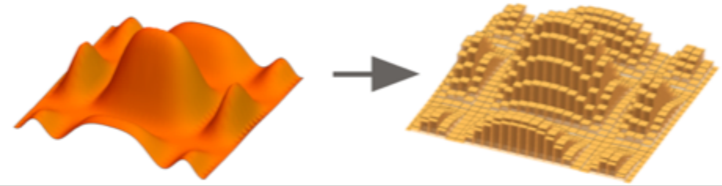
Symanzik improved Hamiltonian, correcting classical a^2 errors

$$V_{KS} = -\sum_{\mathbf{x},i<j} \frac{2}{g_s^2 a} \mathrm{Re\,Tr}\, P_{ij}(\mathbf{x}) \qquad P_{ij}(x) = 1 - \frac{1}{N}\mathrm{ReTr}\left\{ \square_{ij} \right\}$$

$$P_{ij}(x) = 1 - \frac{1}{N}\mathrm{ReTr}\exp\left\{ ig\oint_{\square} A\cdot dx \right\} \approx \frac{g^2 a^4}{2N}\mathrm{Tr}\left\{F_{ij}(x)F_{ij}(x)\right\} + \frac{g^2 a^6}{12N}\mathrm{Tr}\left\{F_{ij}(x)(D_i^2 + D_j^2)F_{ij}(x)\right\} + \dots$$

[J. Carlsson, et al, hep-lat/0105018]

• 规范场的哈密顿量 Discretization



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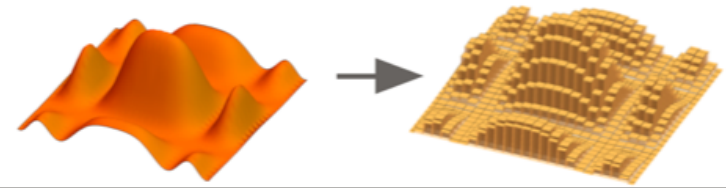
$$P_{ij}(x) \;=\; 1 - \frac{1}{N}\mathrm{ReTr}\exp\left\{ ig\oint_{\square} A\cdot dx \right\} \;\approx\; \frac{g^2a^4}{2N}\mathrm{Tr}\left\{F_{ij}(x)F_{ij}(x)\right\} + \frac{g^2a^6}{12N}\mathrm{Tr}\left\{F_{ij}(x)(D_i^2 + D_j^2)F_{ij}(x)\right\} + \ldots$$

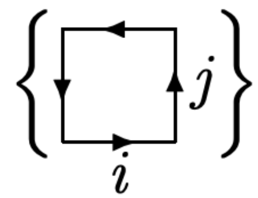
$$R_{ij}(x) = 1 - \frac{1}{N}\mathrm{ReTr}\left\{ \square\!\square_{i,j} \right\} \;=\; \frac{4g^2a^4}{2N}\mathrm{Tr}\left\{F_{ij}(x)F_{ij}(x)\right\} + \frac{4g^2a^6}{24N}\mathrm{Tr}\left\{F_{ij}(x)(4D_i^2 + D_j^2)F_{ij}(x)\right\} + \ldots$$

[J. Carlsson, et al, hep-lat/0105018]

• 规范场的哈密顿量 Discretization 

Symanzik improved Hamiltonian, correcting classical a^2 errors

$$V_{KS} = -\sum_{\mathbf{x},i<j} \frac{2}{g_s^2 a} \mathrm{Re\,Tr}\, P_{ij}(\mathbf{x}) \qquad P_{ij}(x) = 1 - \frac{1}{N}\mathrm{ReTr}\left\{ \square \right\}$$



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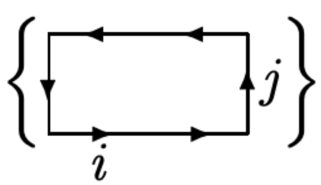
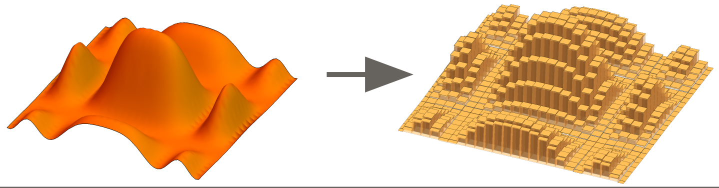


$$V_I = \frac{2N}{ag^2}\sum_{x,i<j}\left[\frac{5}{3}P_{ij}(x) - \frac{1}{12}\left(R_{ij}(x)+R_{ji}(x)\right)\right] = \beta_{V0}V_{KS} + \beta_{V1}V_{\mathrm{rect}}$$

deviations from the continuum
starts from a^2g^2 at quantum level

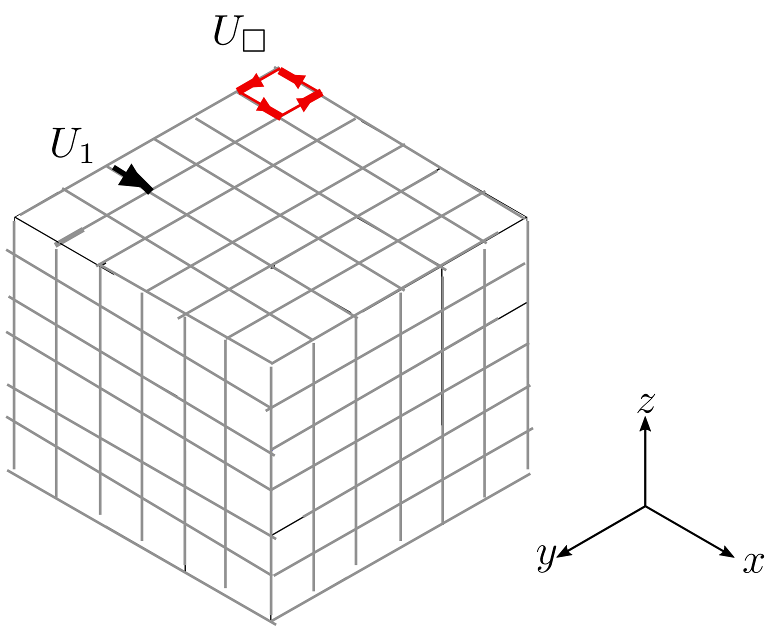
[J. Carlsson, et al, hep-lat/0105018]

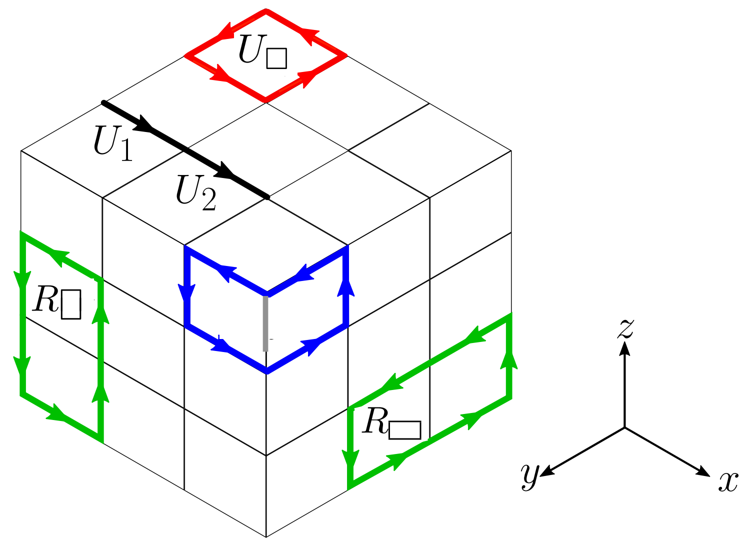
Discretization



infinities in QFT

$$|\langle H_{KS}(a) - H \rangle| \sim |\langle H_I(2a) - H \rangle|?$$





improved Hamiltonian

non-perturbative regime

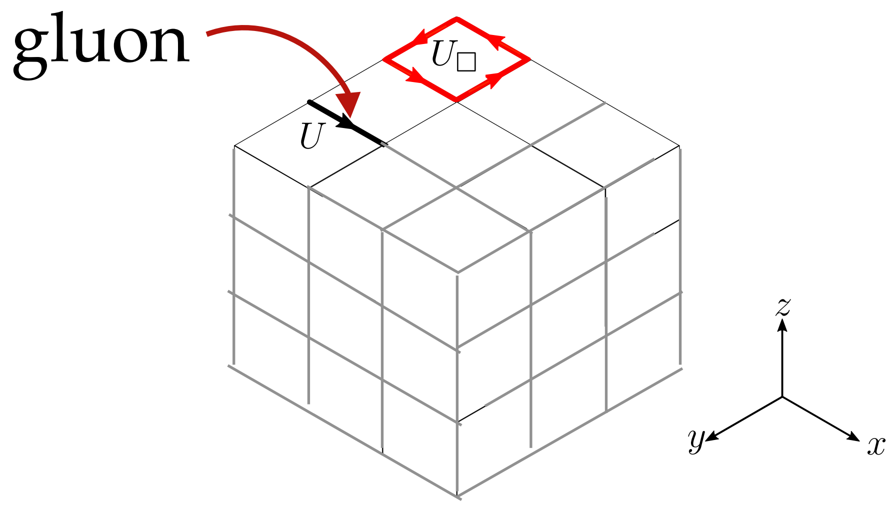
$$100\text{MeV} \lesssim E \lesssim \text{GeV}$$

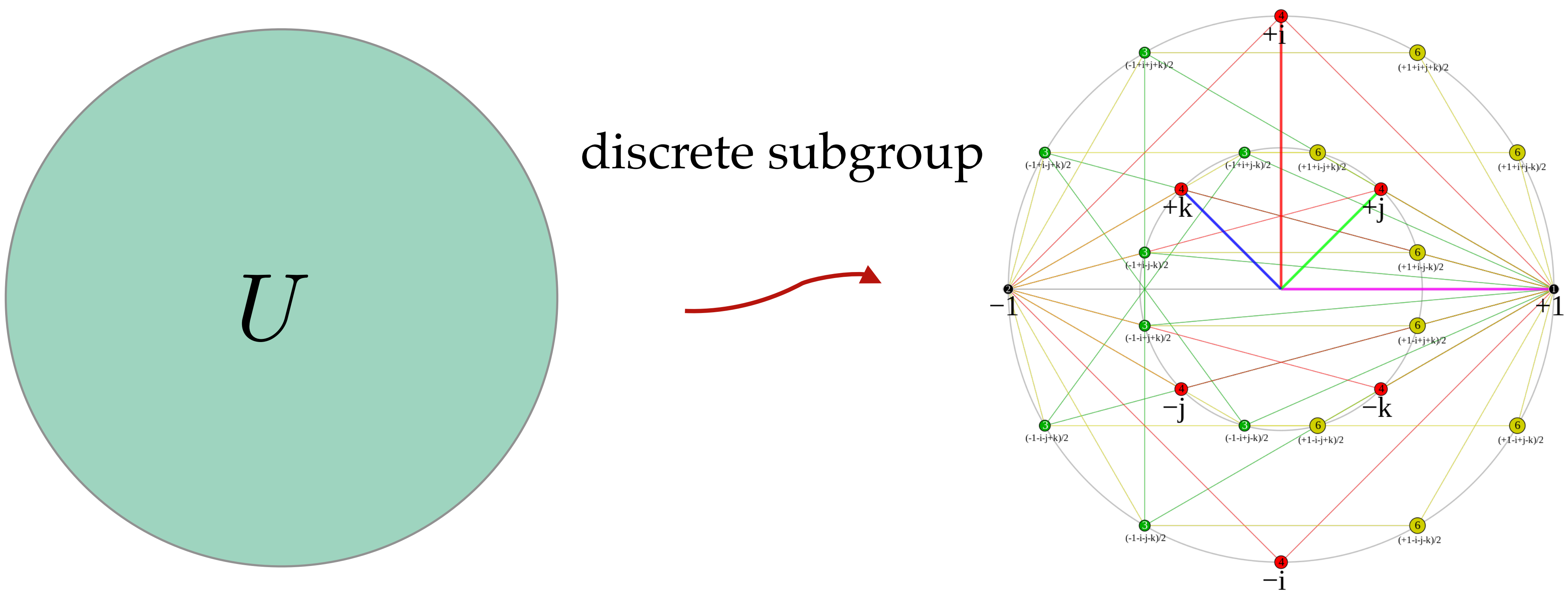
$$n_q \sim f \times 125$$

Digitization

infinities in field variables

Digitization $|q\rangle^N \rightarrow |G\rangle$ infinities in QFT



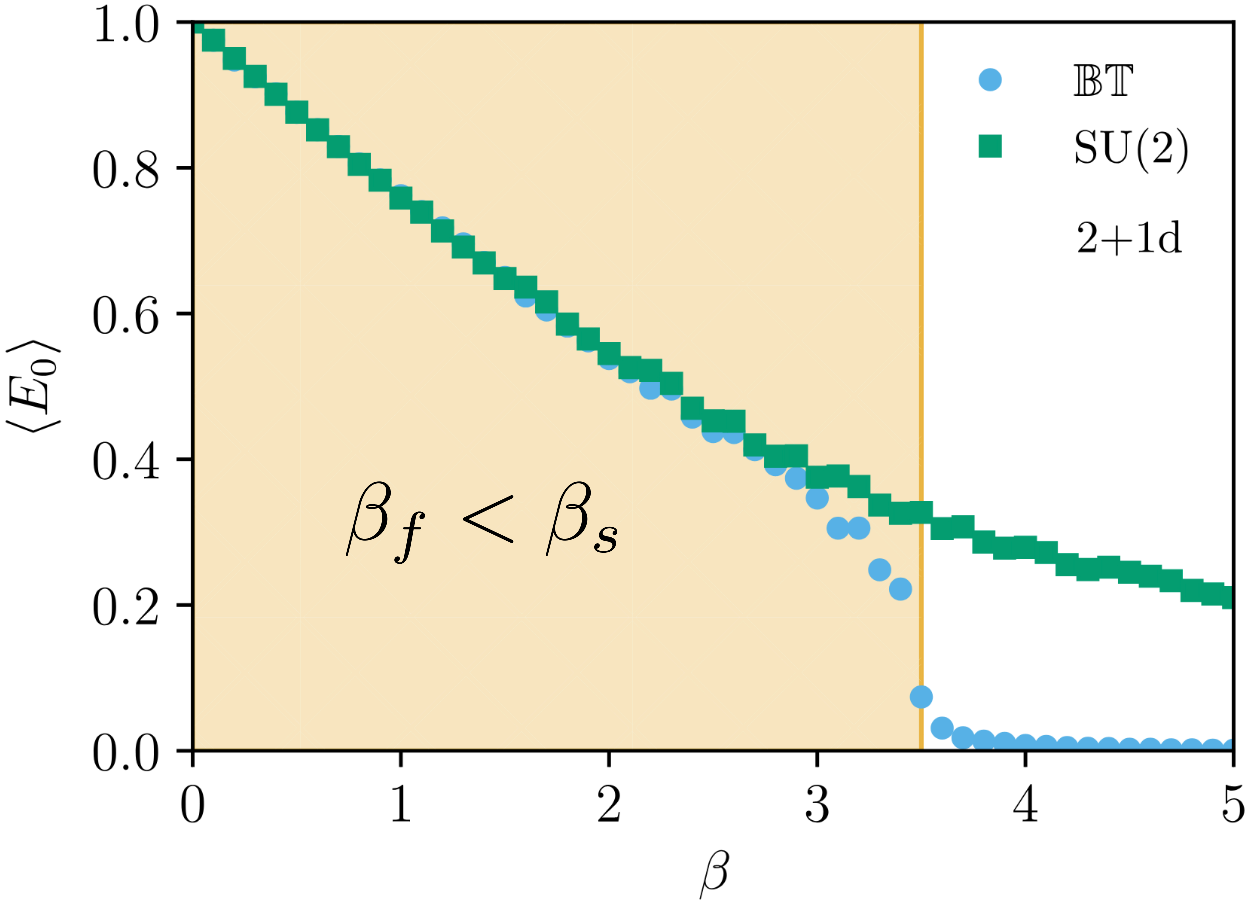


continuous field variables

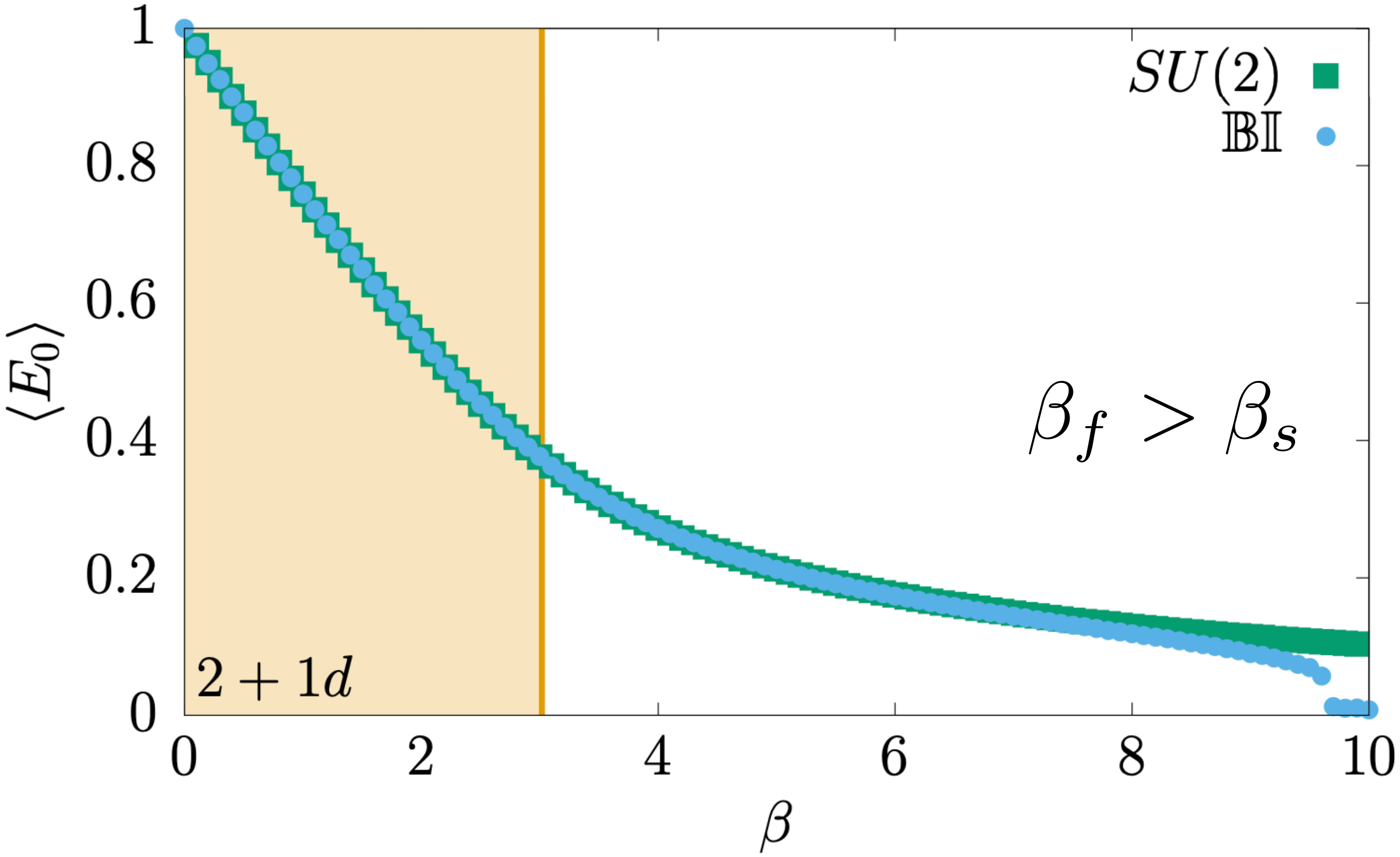
$G-$register : $|U\rangle$

Digitization $|q\rangle^N \rightarrow |G\rangle$ infinities in QFT

[Gustafson, Lamm, Lovelace, Mush, PRD **106**, 114501]

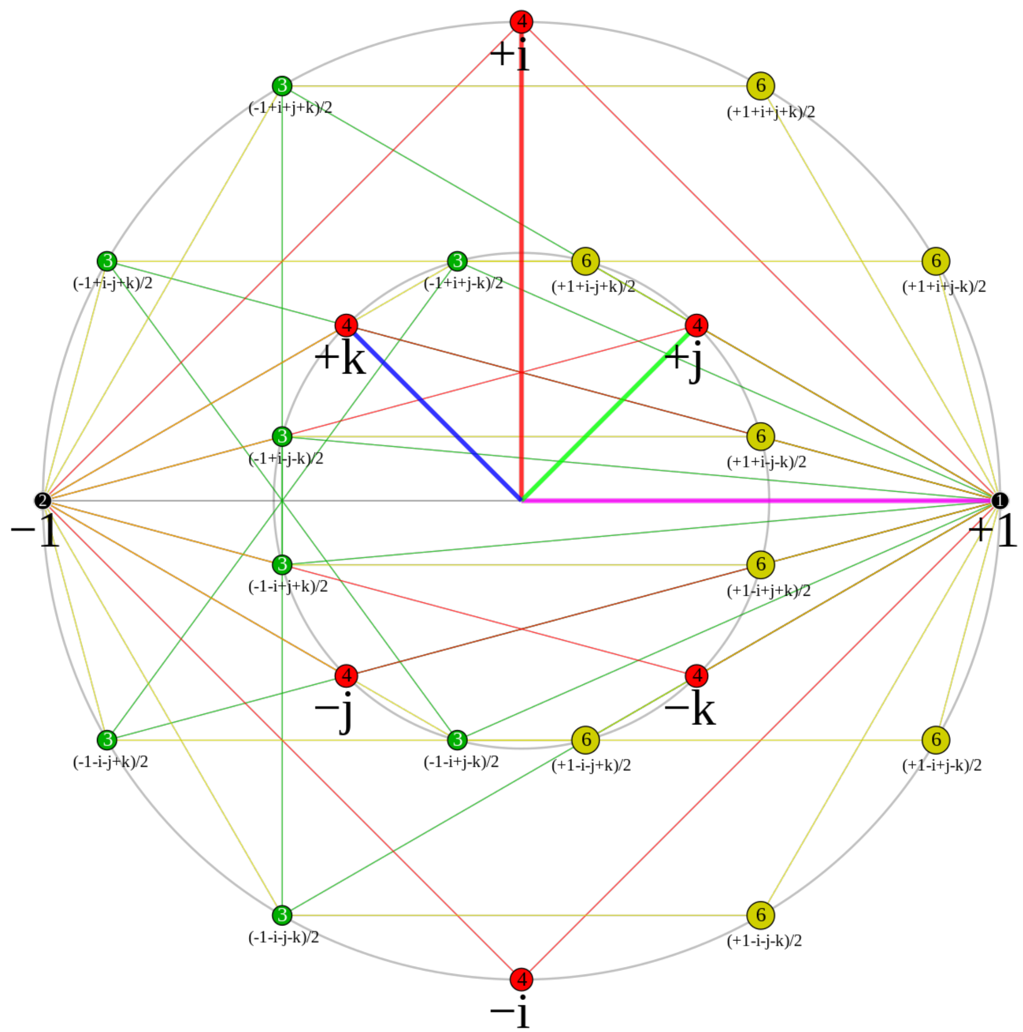


[Lamm,**YYL**, Shu, Wang, Bin, arXiv:2405.12890]



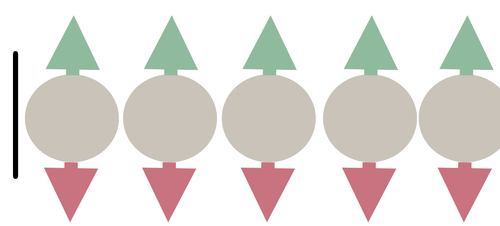
In the Scaling Regime: significantly reduces the errors in simulating SU(2) physics

Digitization $\quad |q\rangle^N \rightarrow |G\rangle$

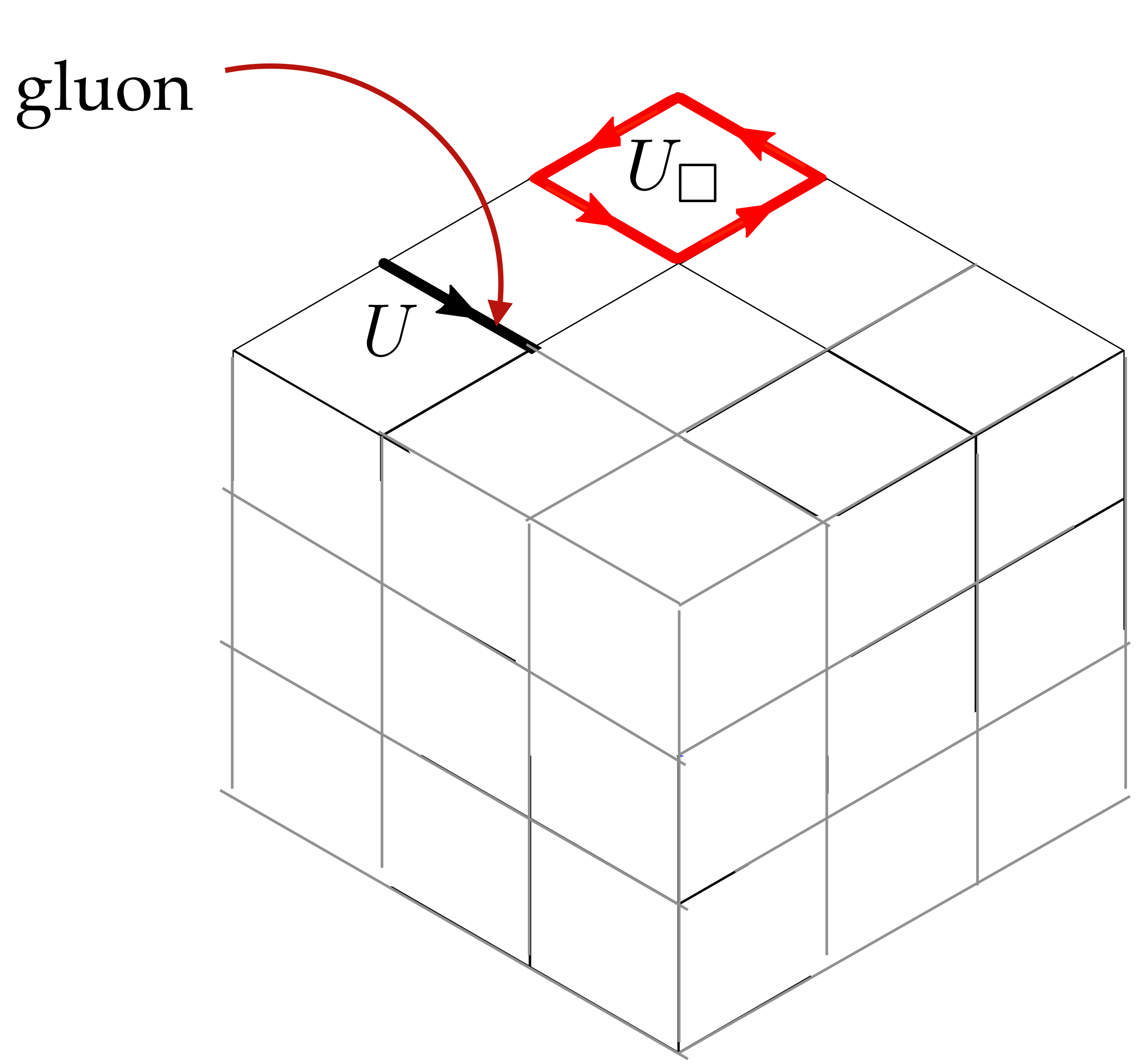


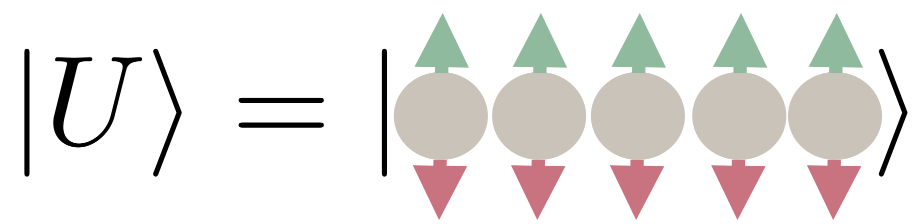
$$U = (-1)^m \mathbf{i}^n \mathbf{j}^o \mathbf{l}^{p+2q}$$

binary variables : m, n, o, p, q

$$|U\rangle = |$$  $$\rangle$$

Digitization $\quad |q\rangle^N \to |G\rangle$



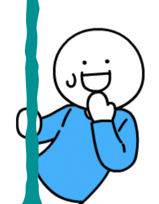


$$n_q \sim 5 \times n^d$$

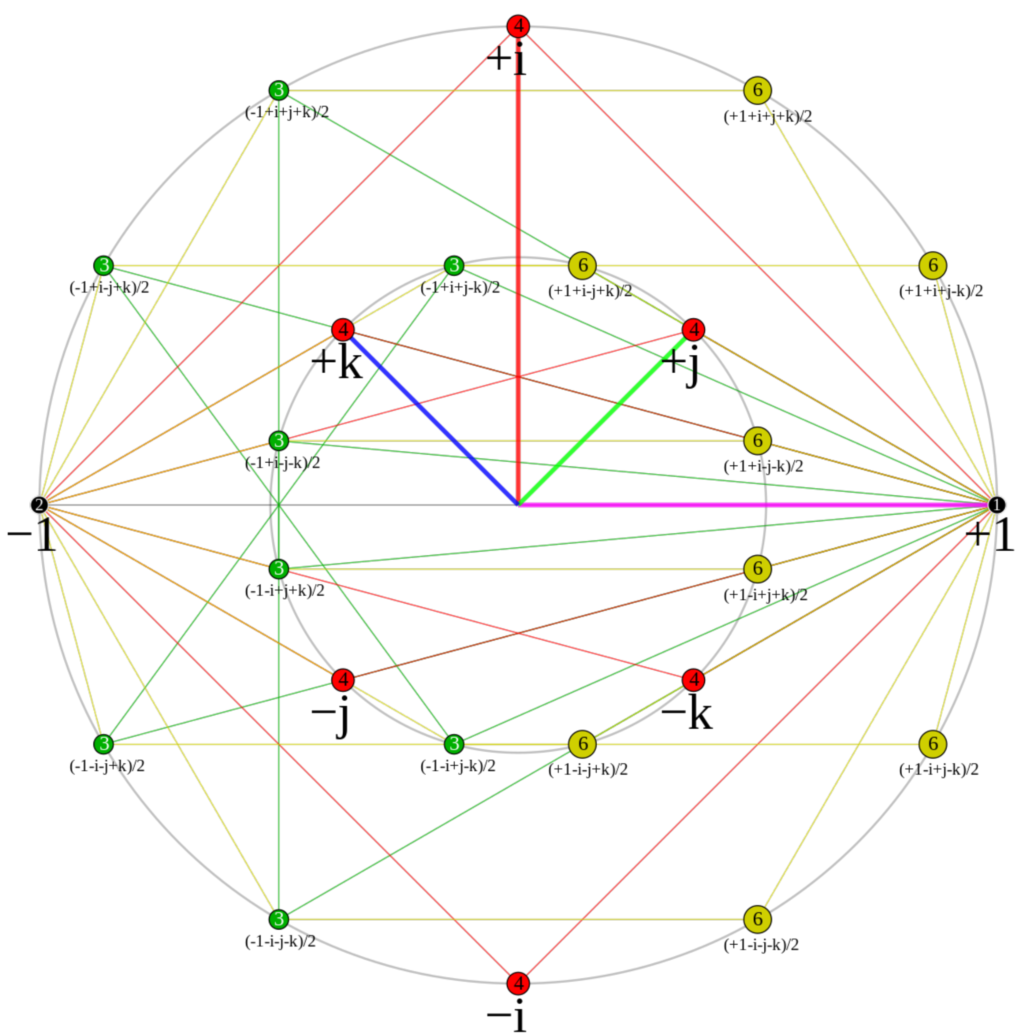
non-perturbative regime

$100\text{MeV} \lesssim E \lesssim \text{GeV}$

$H_I \qquad n_q \sim 625$

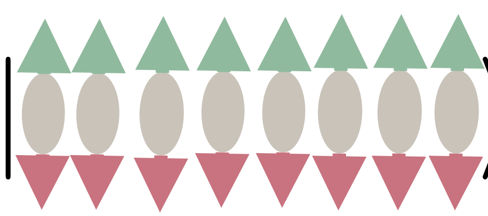


M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601



block product encoding: BT, BI

$$U = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbf{F}_3, ad - bc \equiv 1 \pmod 3 \right\}$$

$$|U\rangle = |$$  $$\rangle$$

[Lamm, **YYL**, Shu, Wang, Bin, arXiv:2405.12890]

qudit system?

- 规范场的量子模拟

Propagation $\mathcal{U}|\psi_0\rangle \rightarrow |\psi(t)\rangle$

Digital quantum computers

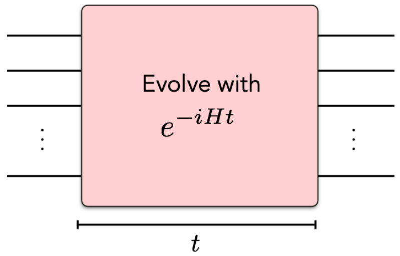
building blocks:
One-qubit/two-qubit gate set

$$||\mathcal{U} - e^{-iHt}|| < \epsilon$$

Propagation

$$\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$$

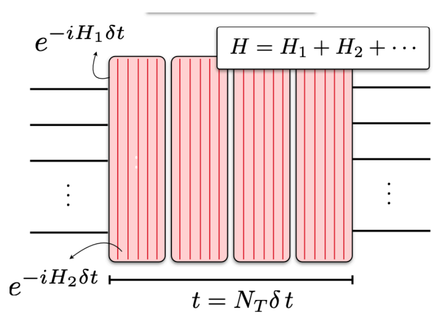
ANALOG



Cold neutral atoms, Trapped ions, Cavity quantum electrodynamics
Superconducting circuits
…

DIGITAL

superconducting qubit / trapped-ion system



building blocks:
one-qubit / two-qubit gate set

$$||\mathcal{U} - e^{-iHt}|| < \epsilon$$

optimal asymptotically?
overload of resources?
easy implementation?

Trotter-Suzuki decomposition

$$H = \sum_{l=1}^{\Gamma} H^{(l)}$$

$$\mathcal{U} = \left[\prod_{l=1}^{\Gamma} e^{-itH^{(l)}/r}\right]^r$$

p-th order trotterization: $\mathcal{O}\left((\frac{t}{r})^p\right)$

Errors depends on t and r
No ancillary overhead
Simpler implementation

Taylor series expansion (LCU)

$$e^{-iHt} = (e^{-iHt/r})^r \equiv V^r$$

$$V \approx \tilde{V} = \sum_{k=0}^{K} \frac{1}{k!}\left(\frac{-iHt}{r}\right)^k$$

$$\mathcal{U} = \tilde{V}^r$$

$$||\tilde{V} - V|| < \epsilon/r$$

K values depends on the aimed errors
Ancillary qubits are needed
Complex circuits implementation

Quantum singular value transformation

$$e^{-iHt} = \cos(Ht) - i\sin(Ht)$$

$$e^{i\phi_0\sigma_z}\prod_{j=1}^{k}\left(W(x)e^{i\phi_j\sigma_z}\right) = \begin{bmatrix} P(x) & iQ(x)\sqrt{1-x^2} \\ iQ^*(x)\sqrt{1-x^2} & P^*(x) \end{bmatrix}$$

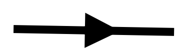
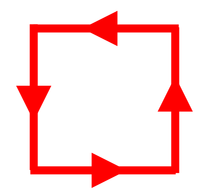
$$W(x) := \begin{bmatrix} x & i\sqrt{1-x^2} \\ i\sqrt{1-x^2} & x \end{bmatrix}$$

Jacobi-Anger expansion for cos and sin

error: truncation order of the expansion
Ancillary qubits are needed
Complex circuits implementation

Quantum signal processing, blocking encoding, off-diagonal Hamiltonian expansion, etc… [Bauer et al, arXiv:2204.03381]

Propagation with gauge redundant encodings

$H_{KS} = \sum \Big($  $+$  $\Big)$

$K_L \qquad U_\square$

$G-$register : $|U\rangle$

$G-$register : $|g\rangle$

$$\mathfrak{U}_\times |g\rangle |h\rangle = |g\rangle |gh\rangle$$

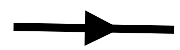
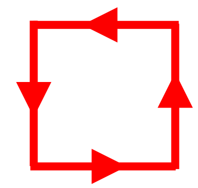
$$\mathfrak{U}_{-1} |g\rangle = |g^{-1}\rangle$$

$$\mathfrak{U}_{\mathrm{Tr}}(\theta) |g\rangle = e^{i\theta\, \mathrm{Re\,Tr}\, g} |g\rangle$$

$$\mathfrak{U}_F \sum_{g\in G} f(g) |g\rangle = \sum_{\rho\in\hat{G}} \hat{f}(\rho)_{ij} |\rho, i, j\rangle$$

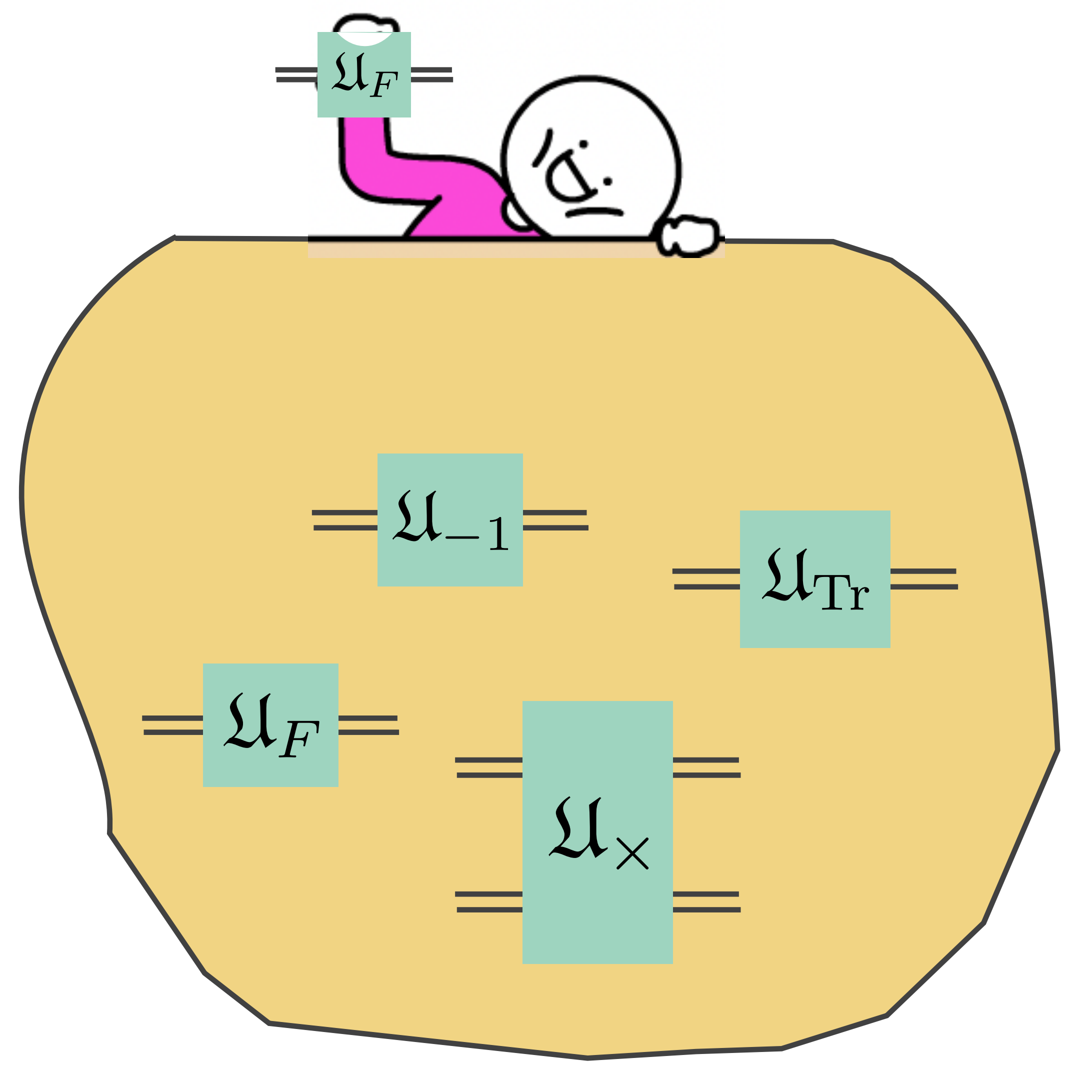
$$\mathcal{U}(t) = e^{-iH_{KS}t}$$
$$\approx \left[e^{-i\delta t K_L} e^{-i\delta t U_\square}\right]^{t/\delta t}$$

Propagation with gauge redundant encodings

$H_{KS} = \sum \Big($  $+$  $\Big)$

K_L $\qquad$ $U_\square$

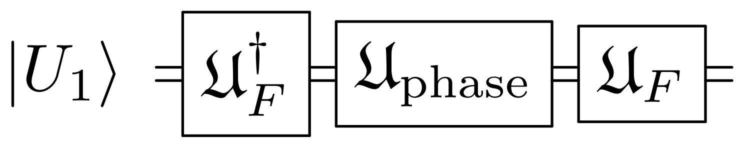
$G-$register : $|U\rangle$ ═

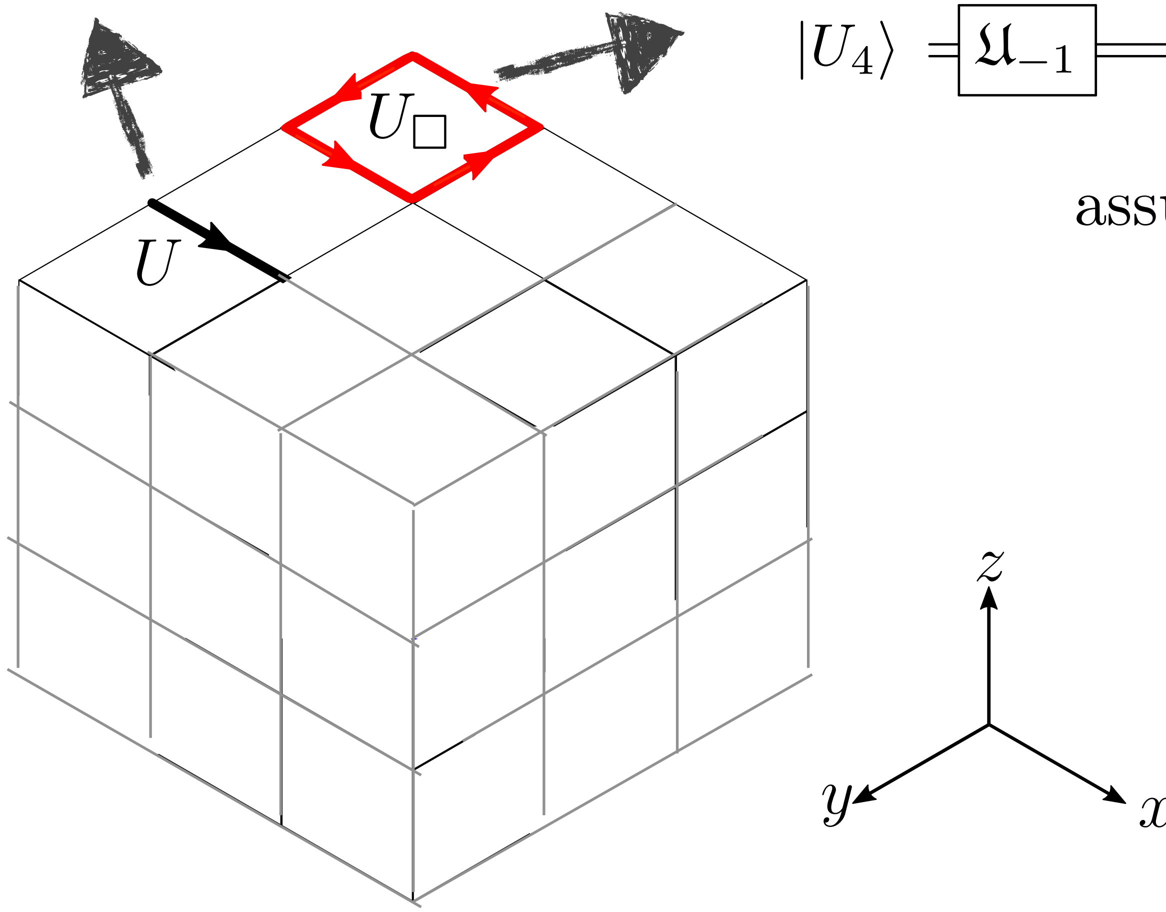


$$\mathcal{U}(t) = e^{-iH_{KS}t}$$
$$\approx \left[e^{-i\delta t K_L} e^{-i\delta t U_\square}\right]^{t/\delta t}$$

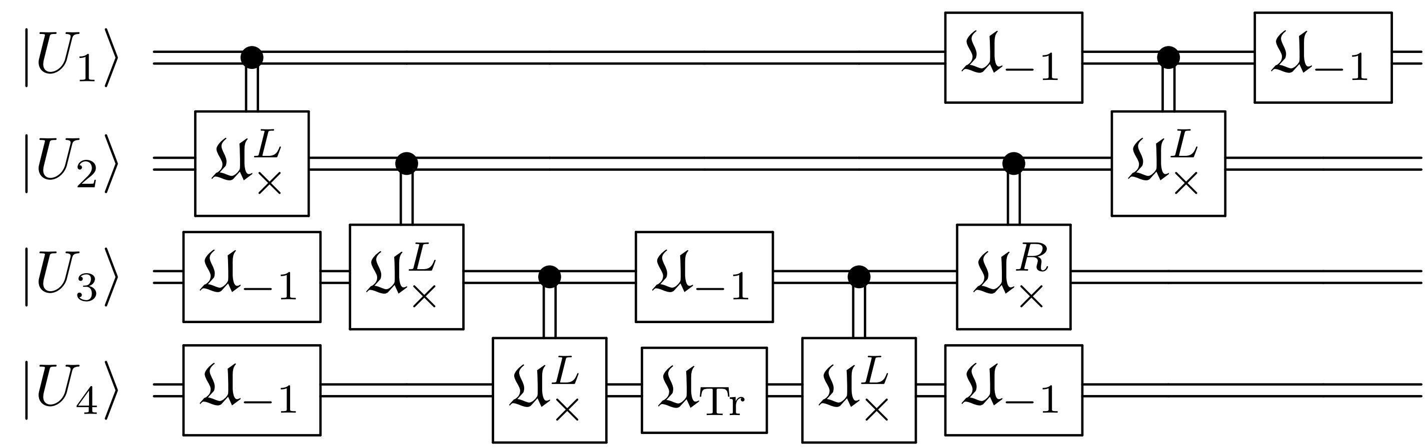
Propagation with gauge redundant encodings

$$e^{-i\delta t K_L}$$





$$e^{-i\delta t U_\Box}$$

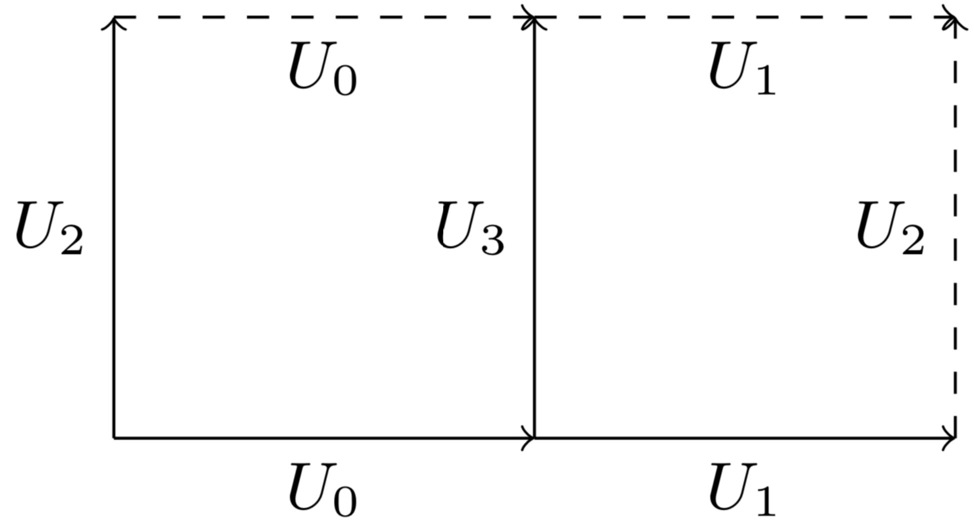


assuming linear register connectivity

M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601

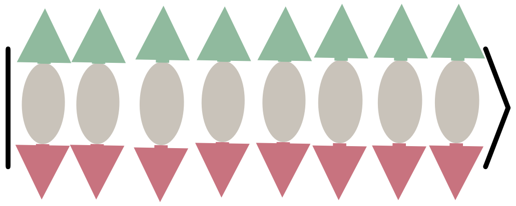
- 规范场的量子模拟—简化体系举例

Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$



block product encoding: BT

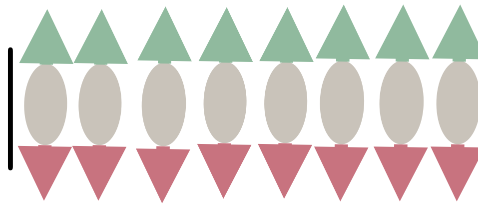
$$U = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbf{F}_3, ad - bc \equiv 1 \pmod 3 \right\}$$

$|U\rangle = |$$\rangle$

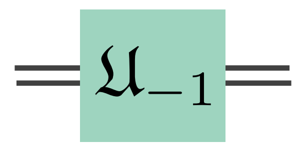
$$|a\rangle : \ |0\rangle = |00\rangle, \ |1\rangle = |01\rangle, \ |2\rangle = |10\rangle$$

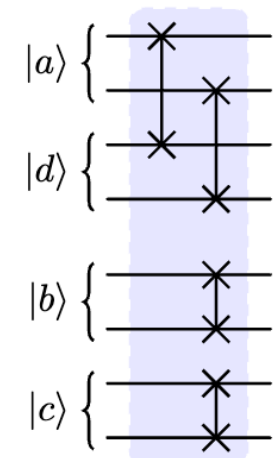
[Lamm, **YYL**, Shu, Wang, Bin, arXiv:2405.12890]

Propagation $\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$

$|U\rangle = |$$\rangle$

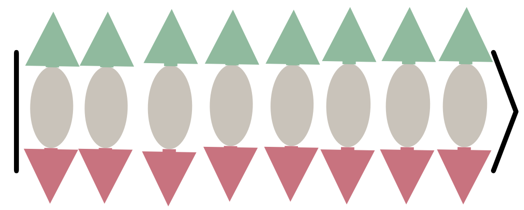
$$g^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$



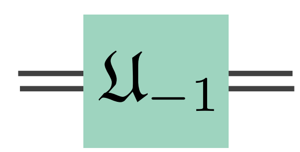


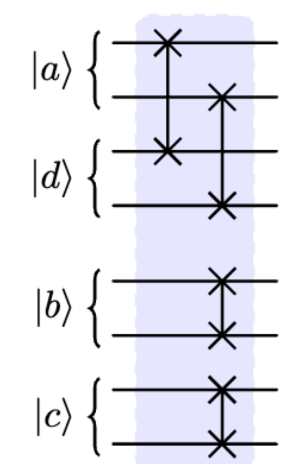
[Lamm,**YYL**, Shu, Wang, Bin, arXiv:2405.12890]

Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$

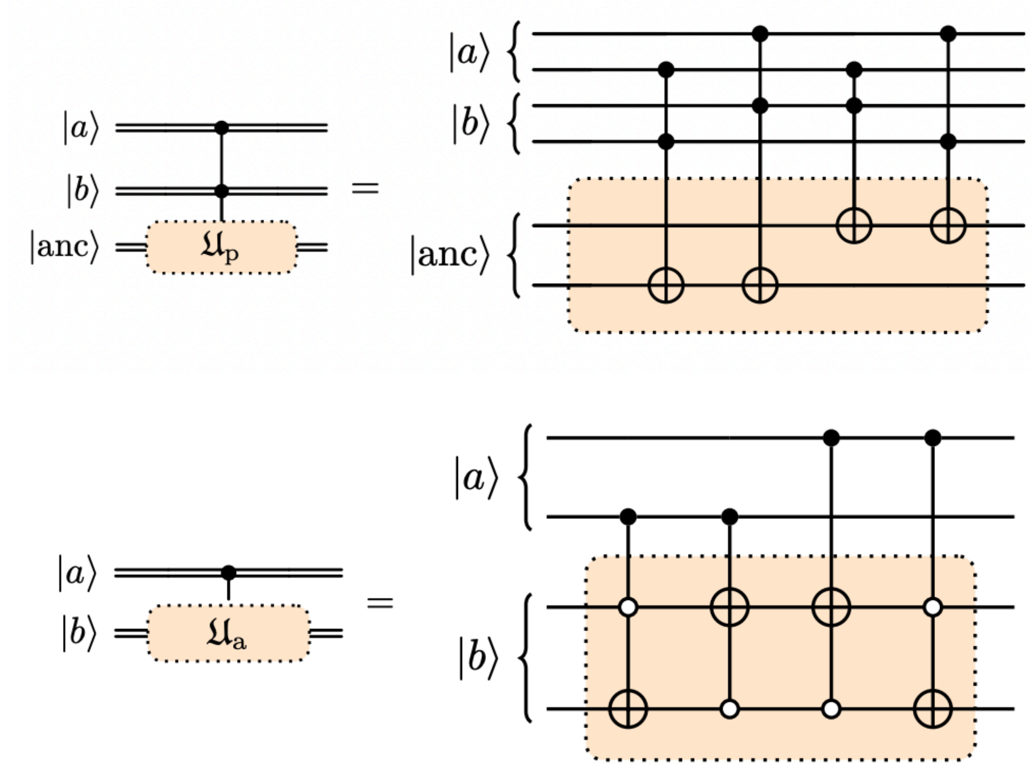
$|U\rangle =$ 

$$g^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$



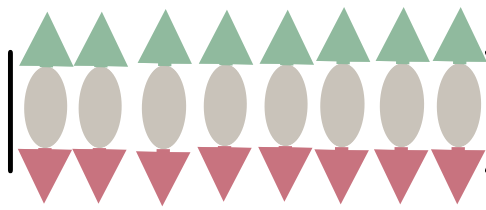


$$\begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix}\begin{pmatrix} a_j & b_j \\ c_j & d_j \end{pmatrix} = \begin{pmatrix} a_i a_j + b_i c_j & a_i b_j + b_i d_j \\ c_i a_j + d_i c_j & c_i b_j + d_i d_j \end{pmatrix}$$

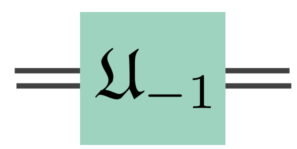


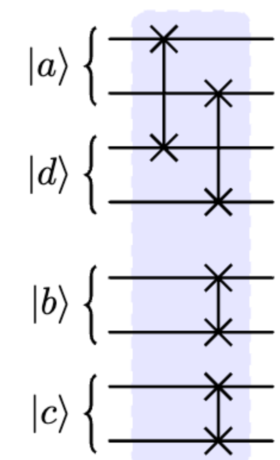
[Lamm, **YYL**, Shu, Wang, Bin, arXiv:2405.12890]

Propagation $\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$

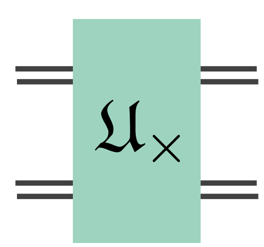
$$|U\rangle = |$$  $$\rangle$$

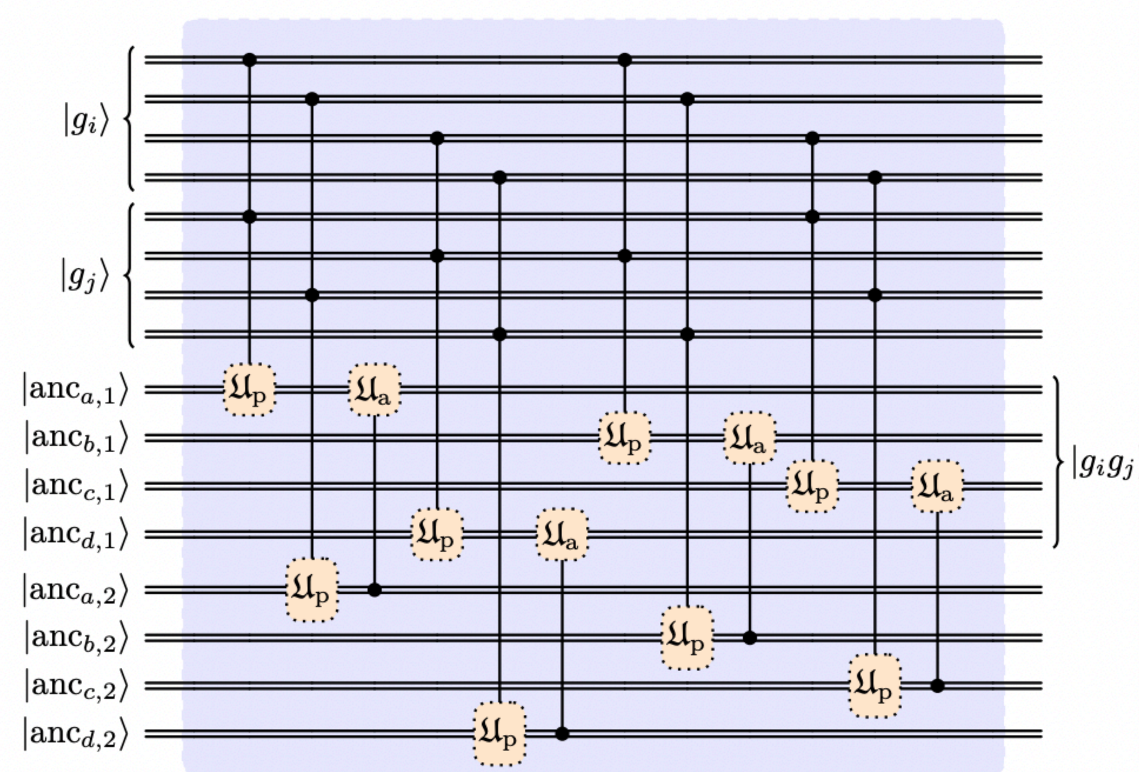
$$g^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$





$$\begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix} \begin{pmatrix} a_j & b_j \\ c_j & d_j \end{pmatrix} = \begin{pmatrix} a_i a_j + b_i c_j & a_i b_j + b_i d_j \\ c_i a_j + d_i c_j & c_i b_j + d_i d_j \end{pmatrix}$$

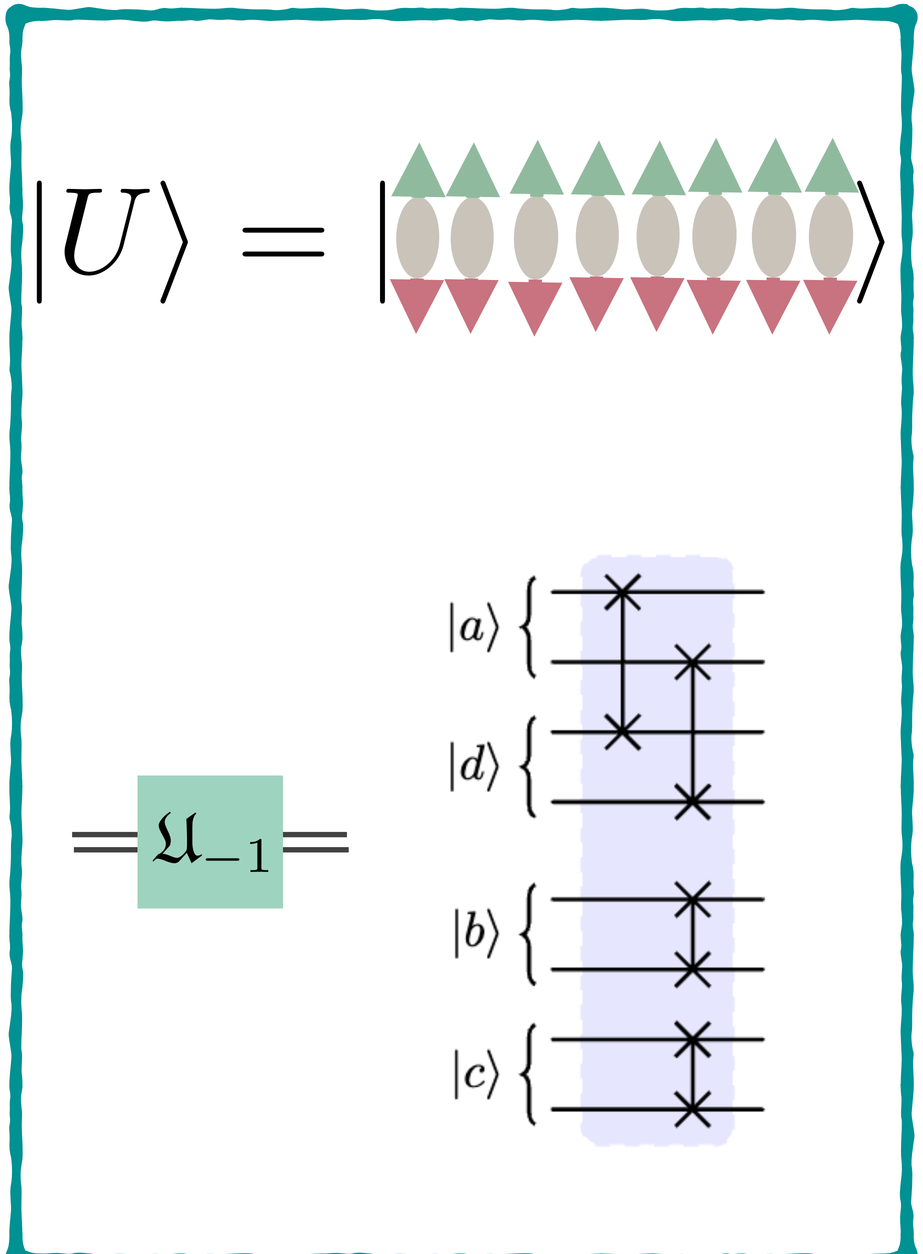


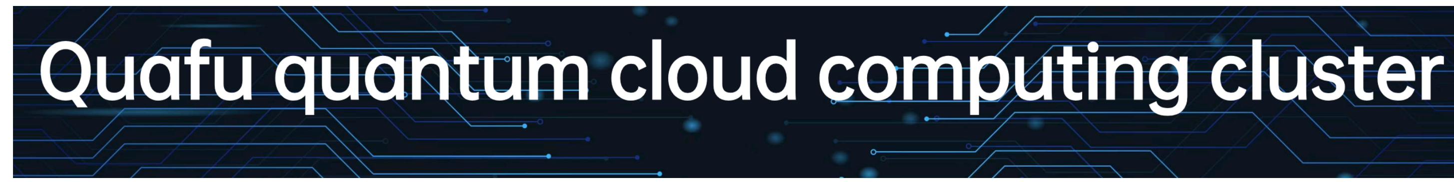


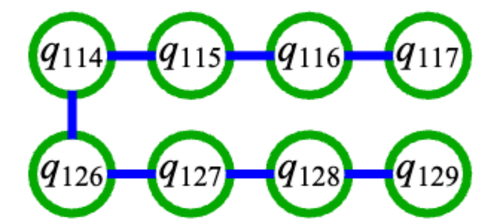
$\mathfrak{U}_\times$

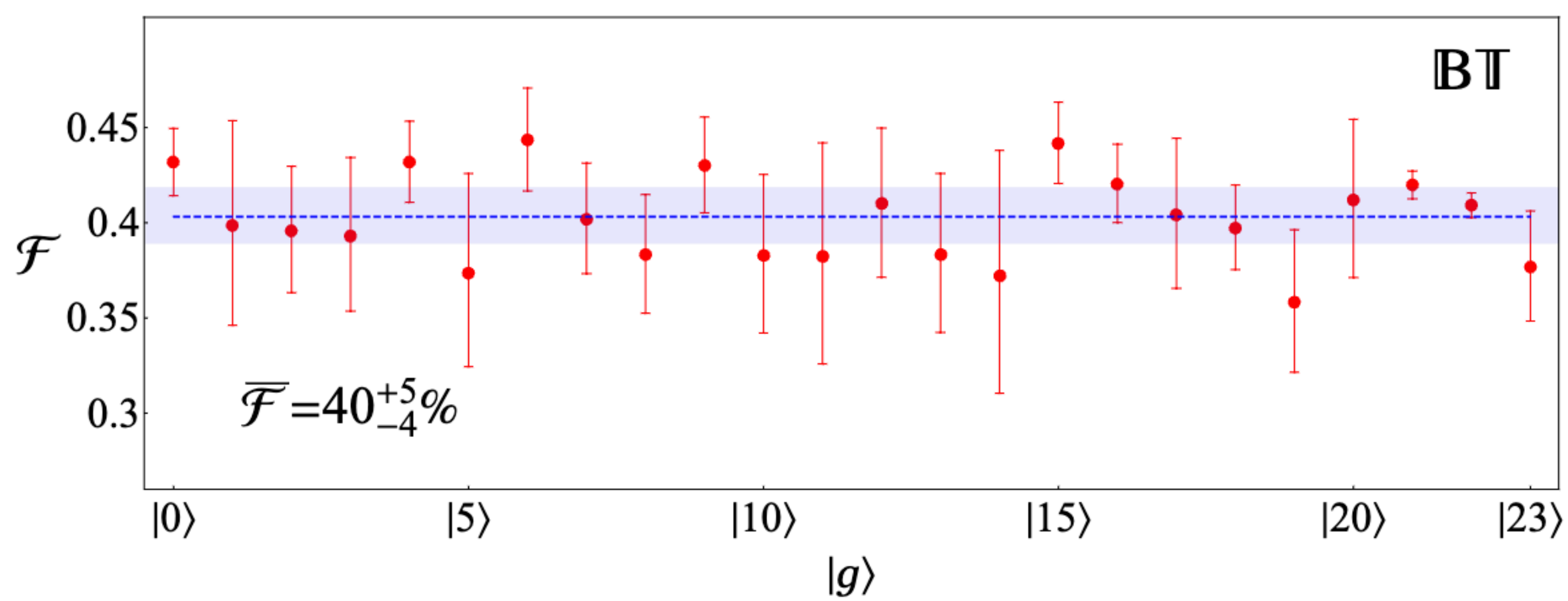
[Lamm, **YYL**, Shu, Wang, Bin, arXiv:2405.12890]

Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$









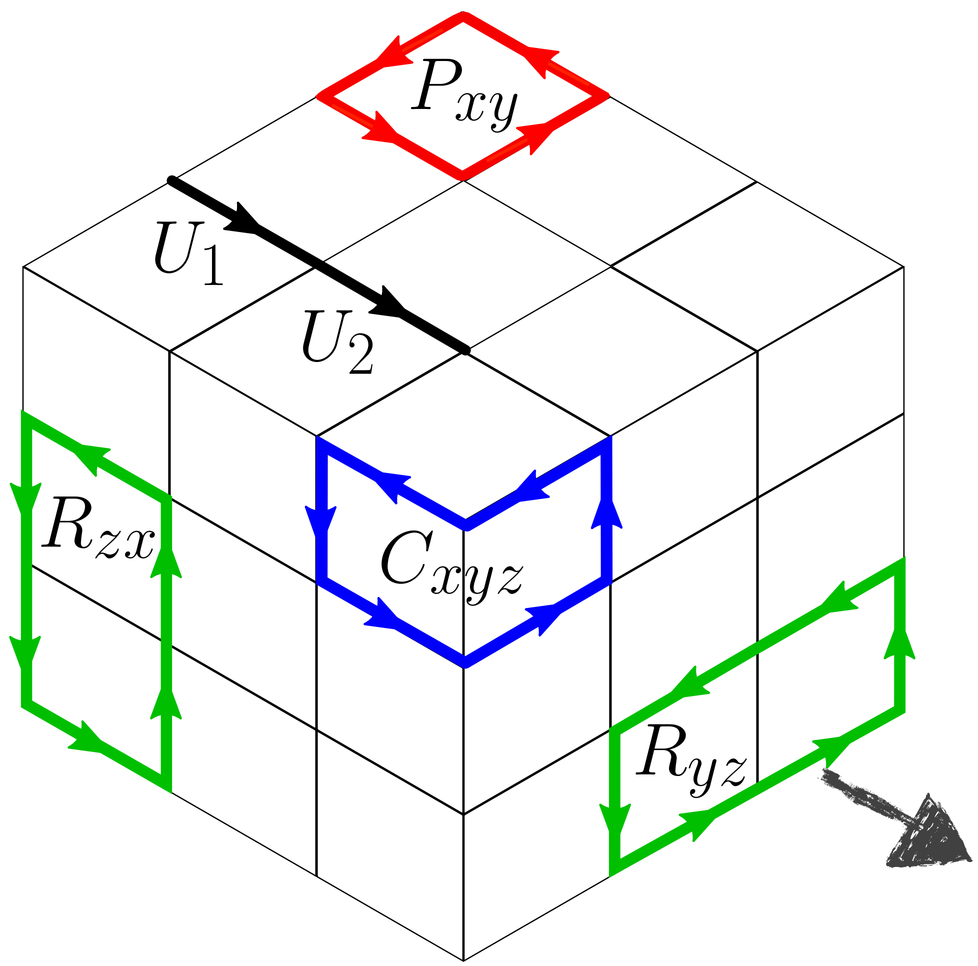
[Lamm, **YYL**, Shu, Wang, Bin, arXiv:2405.12890]

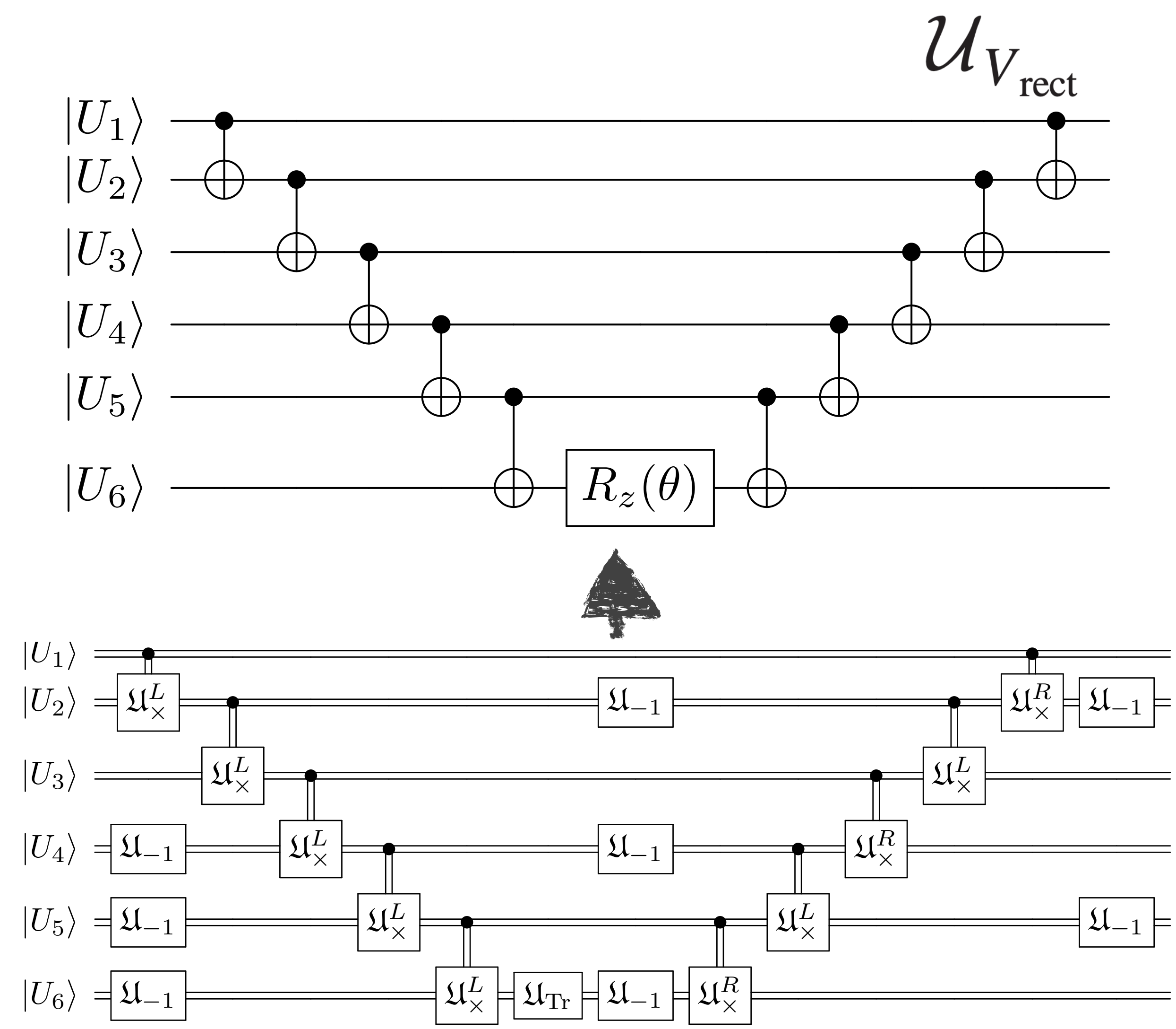
• 规范场的量子模拟—简化体系

$\mathbb{Z}_2$

$$1 \to |0\rangle$$
$$-1 \to |1\rangle$$

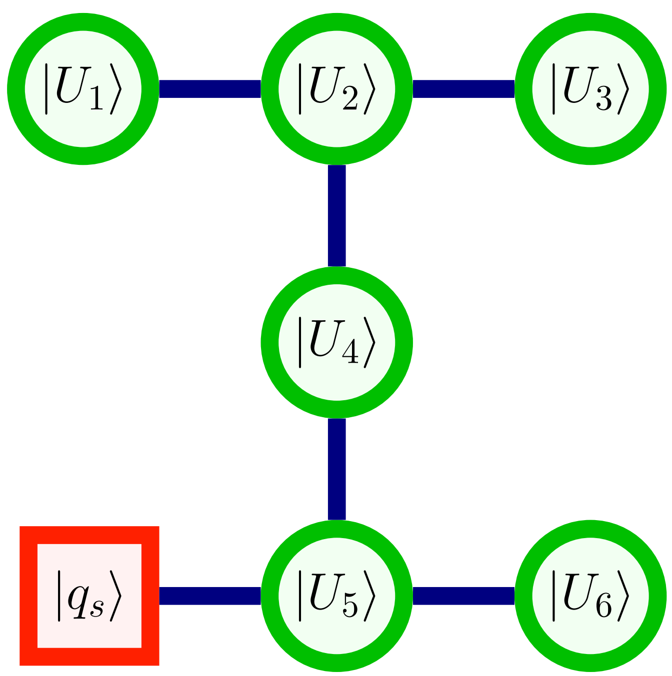
$\mathfrak{U}_F$	H
$\mathfrak{U}_{\text{phase}}$	$R_z(\theta)$
$\mathfrak{U}_{\text{Tr}}$	$R_z(\theta)$
$\mathfrak{U}_{-1}$	$\mathbb{1}$
$\mathfrak{U}_{\times}$	CNOT



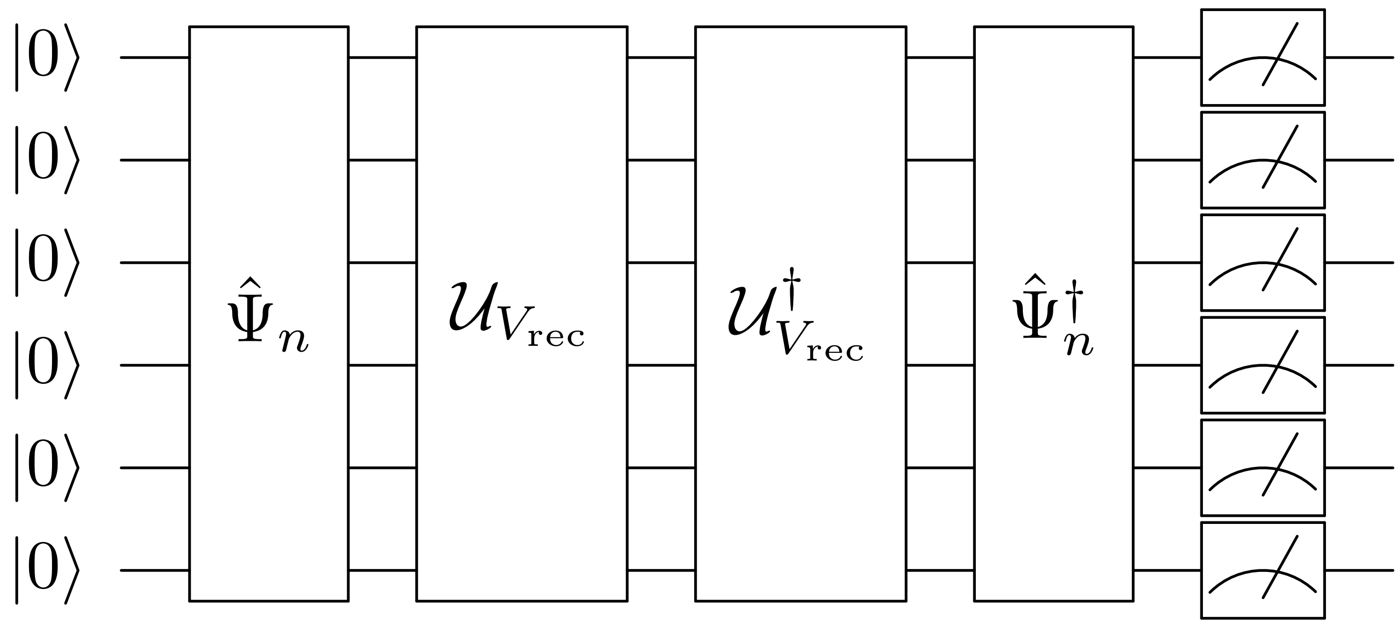


M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601

- 规范场的量子模拟—简化体系



ibm_perth device



$$\hat{\Psi}_n = \prod_{m\leq n} H_m^{\otimes}$$

$$\left[\prod_i (\sigma_i^{b_i})^{\otimes}\right] \mathrm{CNOT} \otimes \mathbb{1}_4 \left[\prod_i (\sigma_i^{a_i})^{\otimes}\right] = \mathrm{CNOT} \otimes \mathbb{1}_4$$

w_H :number of states measured in the 1 state

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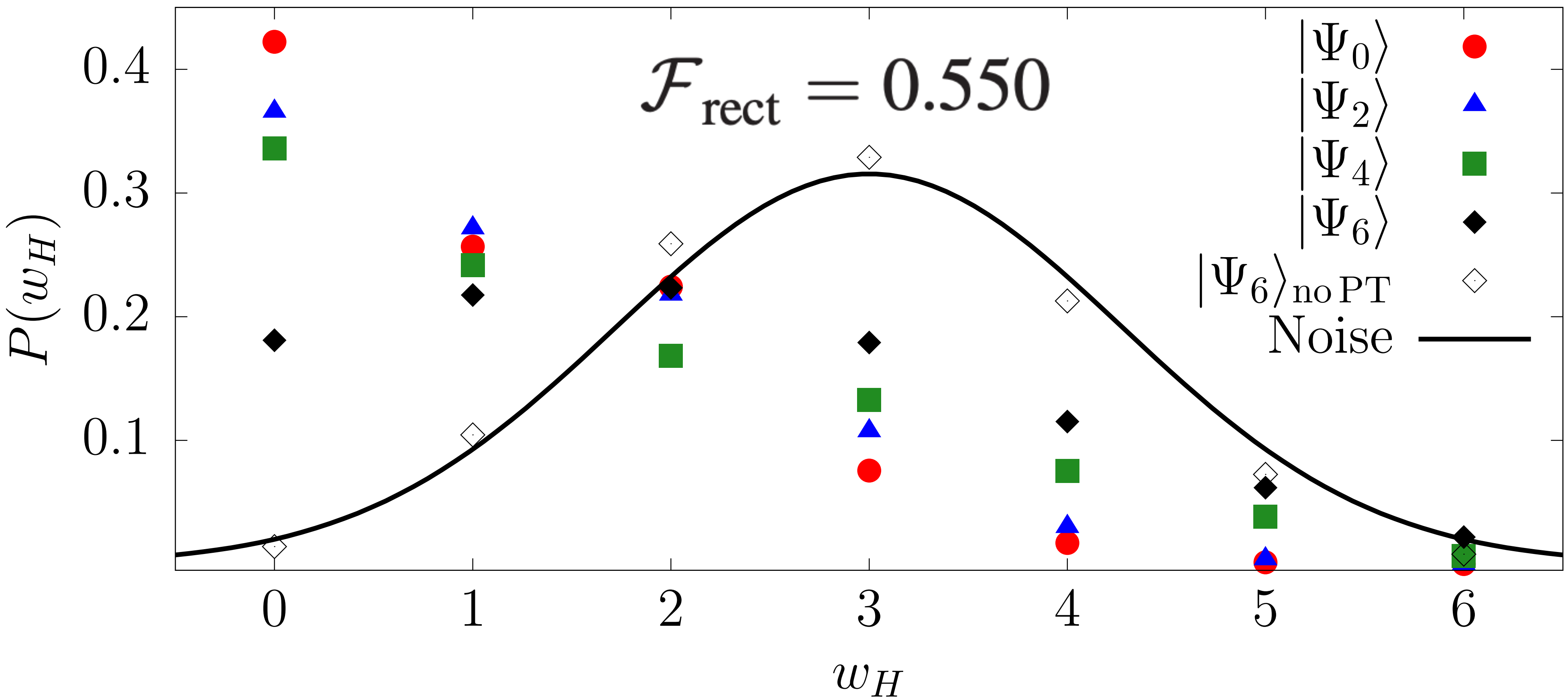
• 规范场的量子模拟—简化体系



$$\mathcal{F}_\delta \approx 0.25$$

demonstration of improved Hamiltonian is allowed in the near future

M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601

- 费米子场的量子模拟

- 费米子场的量子模拟

With **locality, hermiticity, and translational symmetry,** fermions have **doublers** (Nielsen-Ninomiya Theorem)

e.g. spin -1/2 fermion in a single spatial dimension

$$\psi : \text{two components in } 1+1\text{d}$$

$$i\gamma^0 \partial_t \psi(x,t) + i\gamma^1 \partial_x \psi(x,t) - m\psi(x,t) = 0$$

$$i\gamma^0 \partial_t \psi(x,t) + i\gamma^1 \frac{\psi(x+a,t) - \psi(x-a,t)}{2a} - m\psi(x,t) = 0$$

• 费米子场的量子模拟

With **locality, hermiticity, and translational symmetry,**
fermions have **doublers** (Nielsen-Ninomiya Theorem)

e.g. spin -1/2 fermion in a single spatial dimension

$$\text{plane wave ansatz} : \psi(x,t) = e^{i(Et-px)}u(p,E)$$

$$\text{Continuum} : -\infty < p < +\infty$$

$$(-\gamma^0 E + \gamma^1 p - m)u(p,E) = 0$$

$$\text{Lattice} : -\pi/a < p < +\pi/a$$

$$(-\gamma^0 E + i\gamma^1 \frac{e^{-ipa} - e^{+ipa}}{2a} - m)u(p,E) = 0$$

i.e.

$$(-\gamma^0 E + \gamma^1 \frac{\sin(pa)}{a} - m)u(p,E) = 0$$

• 费米子场的量子模拟

With **locality, hermiticity, and translational symmetry,**
fermions have **doublers** (Nielsen-Ninomiya Theorem)

e.g. spin -1/2 fermion in a single spatial dimension

Continuum :$u = (-\gamma^0 E + \gamma^1 p + m)v$

$$(E^2 - p^2 - m^2)v = 0$$

Lattice :$u = (-\gamma^0 E + \gamma^1 \sin(pa)/a + m)v$

$$(E^2 - (\frac{\sin pa}{a})^2 - m^2)v = 0$$

• 费米子场的量子模拟

With **locality, hermiticity, and translational symmetry**, fermions have **doublers** (Nielsen-Ninomiya Theorem)

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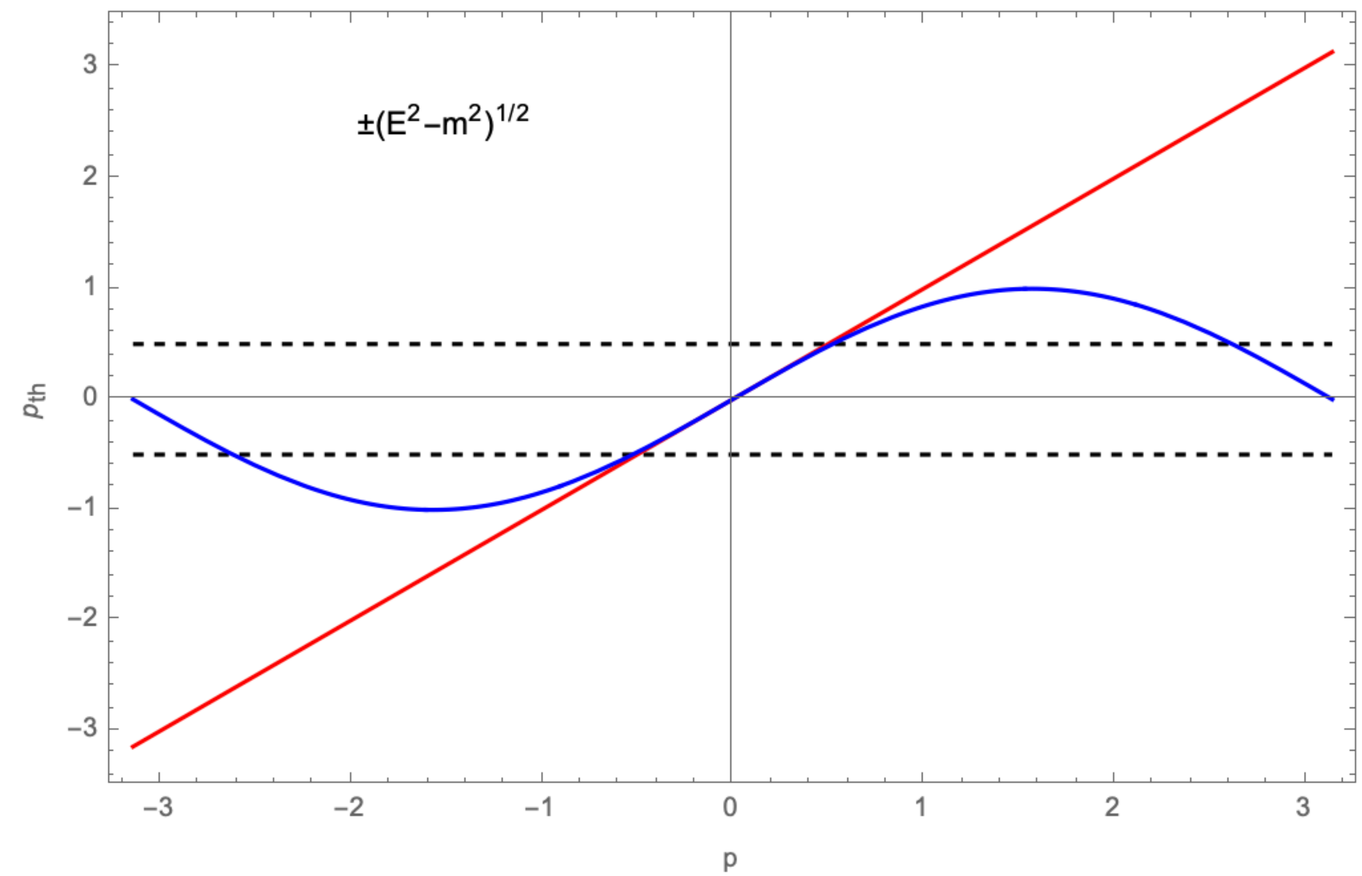


For every discretized spacetime dimension, states are "duplicated"

• 费米子场的量子模拟

Ways to discretize fermions without unphysical states:

— Staggered (KS) fermions: chirality components on different lattice sites

— Wilson Fermions: add new terms to give doublers heavy masses

— Domain wall Fermions: increase dimensionality

— Overlap Fermions: remove doubles with non-local operators

…

? Further improvements to remove lattice artifacts
Not all are formulated in Hamiltonian

• 费米子场的量子模拟

— Staggered fermions: chirality components on different lattice sites

sacrifice translational symmetry

$$\psi_1(n) = c_{2n}, \psi_2(n) = c_{2n+1}$$

Fermion anti-commutation relation, nontrivial maps to qubits

$$\{c_i, c_j^\dagger\} = \delta_{ij}, \qquad \{c_i, c_j\} = \{c_i^\dagger, c_j^\dagger\} = 0,$$

Create / annihilation operator from Pauli matrices satisfy commutation relation on different site

$$\{\hat{\sigma}_j^+, \hat{\sigma}_j^+\} = I, \qquad [\hat{\sigma}_i^+, \hat{\sigma}_j^-] = \delta_{ij}\hat{\sigma}_i^z, \qquad [\hat{\sigma}_i^z, \hat{\sigma}_j^\pm] = \pm 2\delta_{ij}\hat{\sigma}_i^\pm$$

• 费米子场的量子模拟

Jordan-Wigner transformation to preserve the anti commutation relations

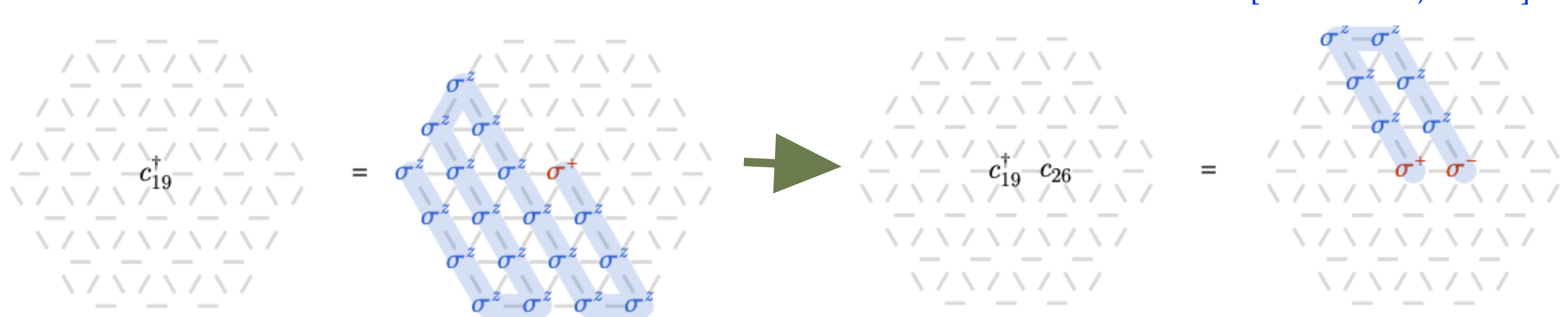
$$c_i = \left(\prod_{j<i} \hat{\sigma}_j^z\right) \hat{\sigma}_i^+$$

$$c_i^\dagger = \left(\prod_{j<i} \hat{\sigma}_j^z\right) \hat{\sigma}_i^-$$

leading to non-local interactions

Good in 1+1d, but expensive in higher dimension, non-local terms

[Everett You, UCSD]



- 费米子场的量子模拟—举例

• 费米子场的量子模拟

—1+1 D Schwinger Model

$$\gamma^0 = \hat{\sigma}^z \text{ and } \gamma^1 = i\hat{\sigma}^y$$

$$\hat{H}_{\text{cont}} = \int dx \left[-i\bar{\Psi}(x)\gamma^1 \left(\partial_1 + ig\hat{A}_1(x) \right) \Psi(x) + m\bar{\Psi}(x)\Psi(x) + \frac{1}{2}\hat{E}^2(x) \right]$$

Two-component spinor fields $\Psi(x) = \left(\hat{\Psi}_{e^-}(x), \hat{\Psi}^\dagger_{e^+}(x) \right)^T$

Canonical momentum $\hat{E}(x) = -\partial_0 \hat{A}_1(x)$

$$[\hat{A}_1(x), \hat{E}(x')] = -i\delta(x - x')$$

[arXiv:1612.08653]

• 费米子场的量子模拟

— on Lattice

$$\hat{\theta}_n = -ag\hat{A}_1(x_n) \qquad \hat{L}_n = \tfrac{1}{g}\hat{E}(x_n)$$

$$\hat{\Phi}_n = \sqrt{a}\hat{\Psi}_{e^-}(x_n) \text{ for even } n \text{ and } \hat{\Phi}_n = \sqrt{a}\hat{\Psi}^{\dagger}_{e^+}(x_n) \text{ for odd } n$$

$$\hat{H}_{\text{cont}} = \int dx \left[-i\bar{\Psi}(x)\gamma^1 \left(\partial_1 + ig\hat{A}_1(x)\right)\Psi(x) + m\bar{\Psi}(x)\Psi(x) + \frac{1}{2}\hat{E}^2(x) \right]$$

$$\hat{H}_{\text{lat}} = -iw\sum_{n=1}^{N-1}\left[\hat{\Phi}^{\dagger}_n e^{i\hat{\theta}_n}\hat{\Phi}_{n+1} - H.C.\right] + m\sum_{n=1}^{N}(-1)^n\hat{\Phi}^{\dagger}_n\hat{\Phi}_n + J\sum_{n=1}^{N-1}\hat{L}^2_n, (1)$$

Gauge invariance!

[arXiv:1612.08653]

• 费米子场的量子模拟

— on Lattice

$$\hat{H}_{\text{lat}} = -iw\sum_{n=1}^{N-1}\left[\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n}\hat{\Phi}_{n+1} - H.C.\right] + m\sum_{n=1}^{N}(-1)^n\hat{\Phi}_n^\dagger\hat{\Phi}_n + J\sum_{n=1}^{N-1}\hat{L}_n^2, (1)$$

Jordan-Wigner transformation $\hat{\Phi}_n = \prod_{l<n}\left[i\hat{\sigma}_l^z\right]\hat{\sigma}_n^-$

↓

Gauge transformation to remove the \theta operator

↓

$$\hat{\sigma}_n^- \rightarrow \left[\prod_{l<n} e^{-i\hat{\theta}_l}\right]\hat{\sigma}_n^-$$

$$\hat{H}'_{\text{lat}} = w\sum_{n=1}^{N-1}\left[\hat{\sigma}_n^+\hat{\sigma}_{n+1}^- + H.C.\right] + \frac{m}{2}\sum_{n=1}^{N}(-1)^n\hat{\sigma}_n^z + J\sum_{n=1}^{N-1}\hat{L}_n^2$$

[arXiv:1612.08653]

• 费米子场的量子模拟

— on Lattice

$$\hat{H}'_{\rm lat} = w \sum_{n=1}^{N-1} \left[\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + H.C.\right] + \frac{m}{2} \sum_{n=1}^{N} (-1)^n \hat{\sigma}_n^z + J \sum_{n=1}^{N-1} \hat{L}_n^2$$

Gauss's Law

$$\hat{G}_n = \hat{L}_n - \hat{L}_{n-1} - \hat{\Phi}^\dagger{}_n \hat{\Phi}_n + \tfrac{1}{2}\left[1 - (-1)^n\right]$$

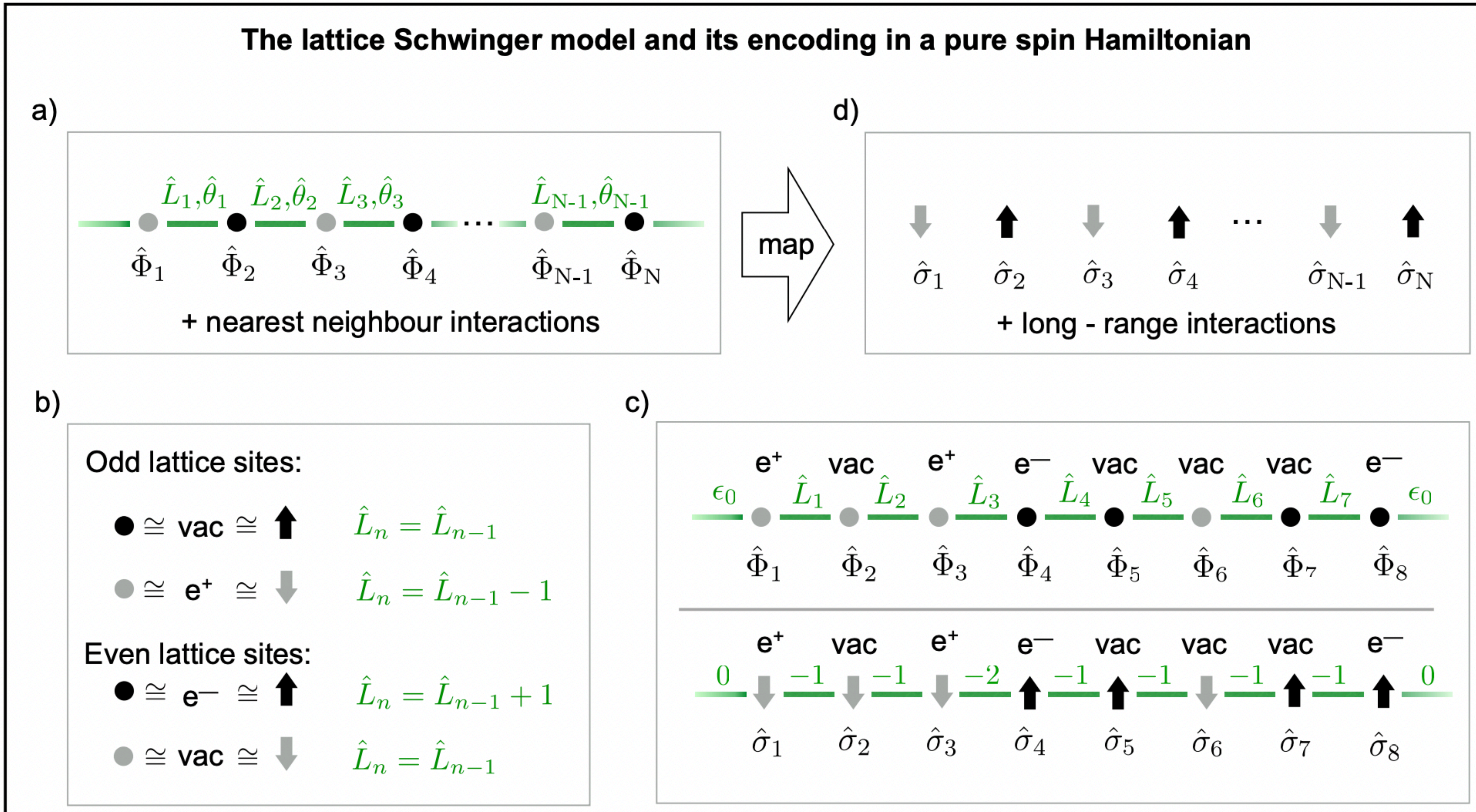
$$\hat{L}_n - \hat{L}_{n-1} = \tfrac{1}{2}\left[\hat{\sigma}_n^z + (-1)^n\right]$$

$$\hat{L}_n = \epsilon_0 + \tfrac{1}{2} \textstyle\sum_{l=1}^{n} \left(\hat{\sigma}_l^z + (-1)^l\right)$$

[arXiv:1612.08653]

• 费米子场的量子模拟

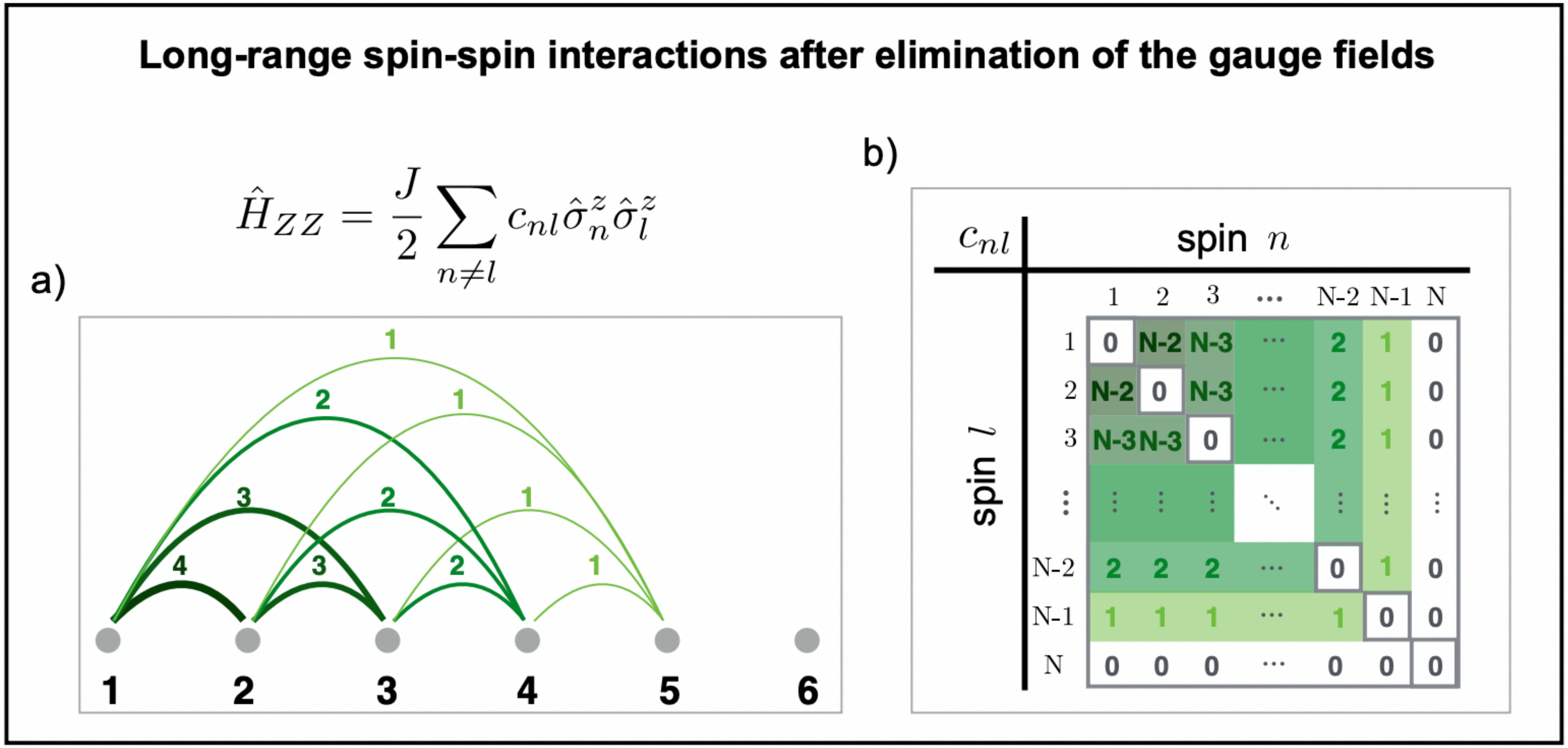
— on Lattice



[arXiv:1612.08653]

- 费米子场的量子模拟

— on Lattice



[arXiv:1612.08653]

• 费米子场的量子模拟

$$\hat{H}_{ZZ} = \frac{J}{2}\sum_{n=1}^{N-2}\sum_{l=n+1}^{N-1}(N-l)\hat{\sigma}_n^z\hat{\sigma}_l^z,$$

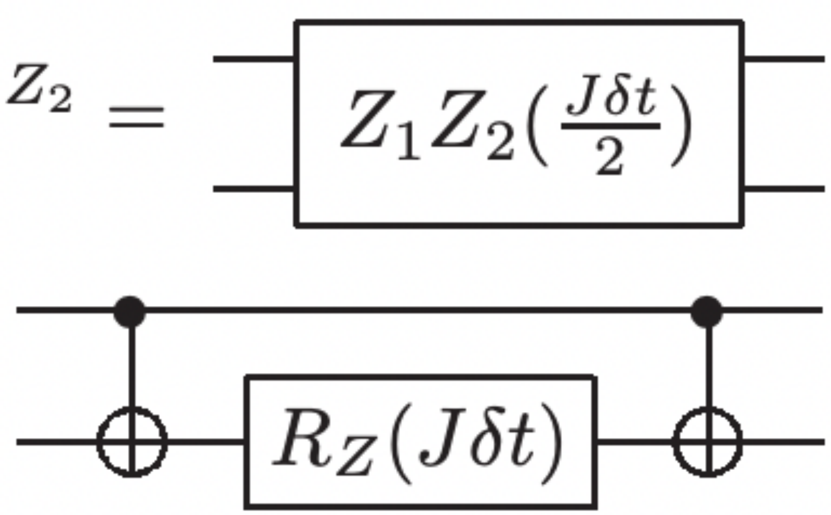
$$\hat{H}_{\pm} = w\sum_{n=1}^{N-1}\left[\hat{\sigma}_n^+\hat{\sigma}_{n+1}^- + H.C.\right],$$

$$\hat{H}_Z = \frac{m}{2}\sum_{n=1}^{N}(-1)^n\hat{\sigma}_n^z - \frac{J}{2}\sum_{n=1}^{N-1}(n\mathrm{mod}2)\sum_{l=1}^{n}\hat{\sigma}_l^z$$

Quantum circuits

$$e^{-\mathrm{i}\frac{J\delta t}{2}Z_1Z_2} =$$

$$:=$$



[arXiv:1612.08653]

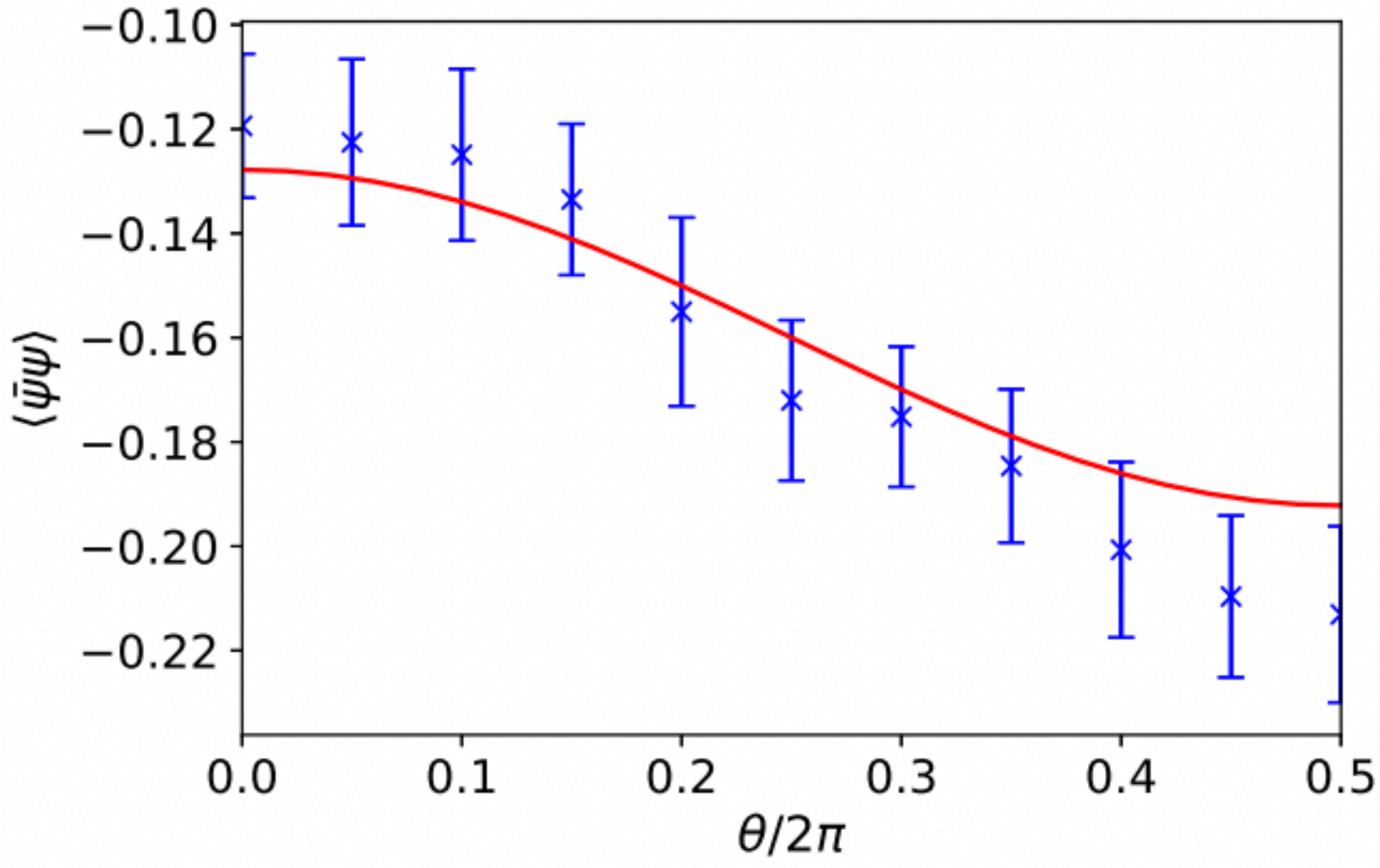
[M. Nielsen, I. Chuang, Quantum computation and quantum information]

- 费米子场的量子模拟

$$\langle\overline{\psi}(x)\psi(x)\rangle = \langle\text{vac}|\overline{\psi}(x)\psi(x)|\text{vac}\rangle$$

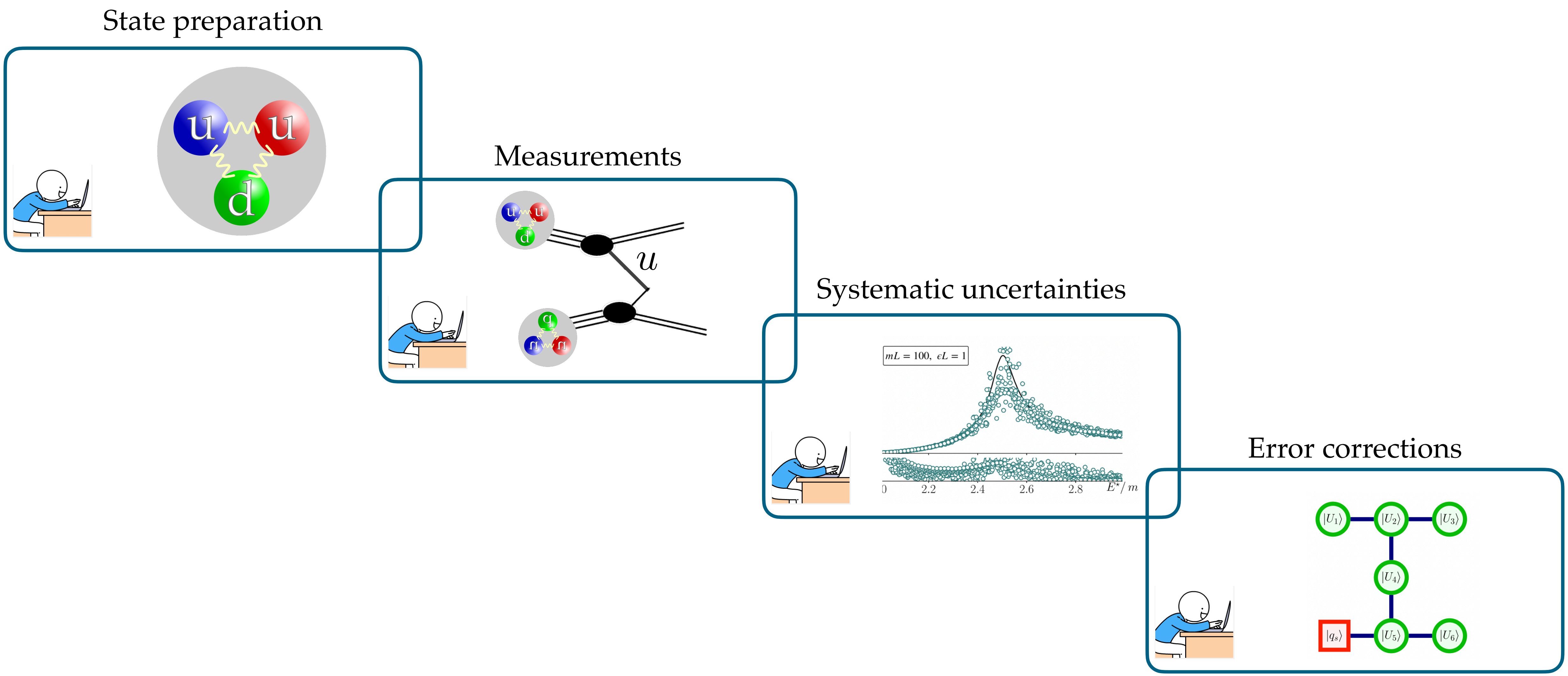
$$\frac{1}{2Na}\langle\text{vac}|\sum_{n=1}^{N}(-1)^n Z_n|\text{vac}\rangle$$

$$\langle\overline{\psi}(x)\psi(x)\rangle \approx -0.160g + 0.322m\cos\theta$$



[B. Chakraborty et al., arXiv:2001. 00485]

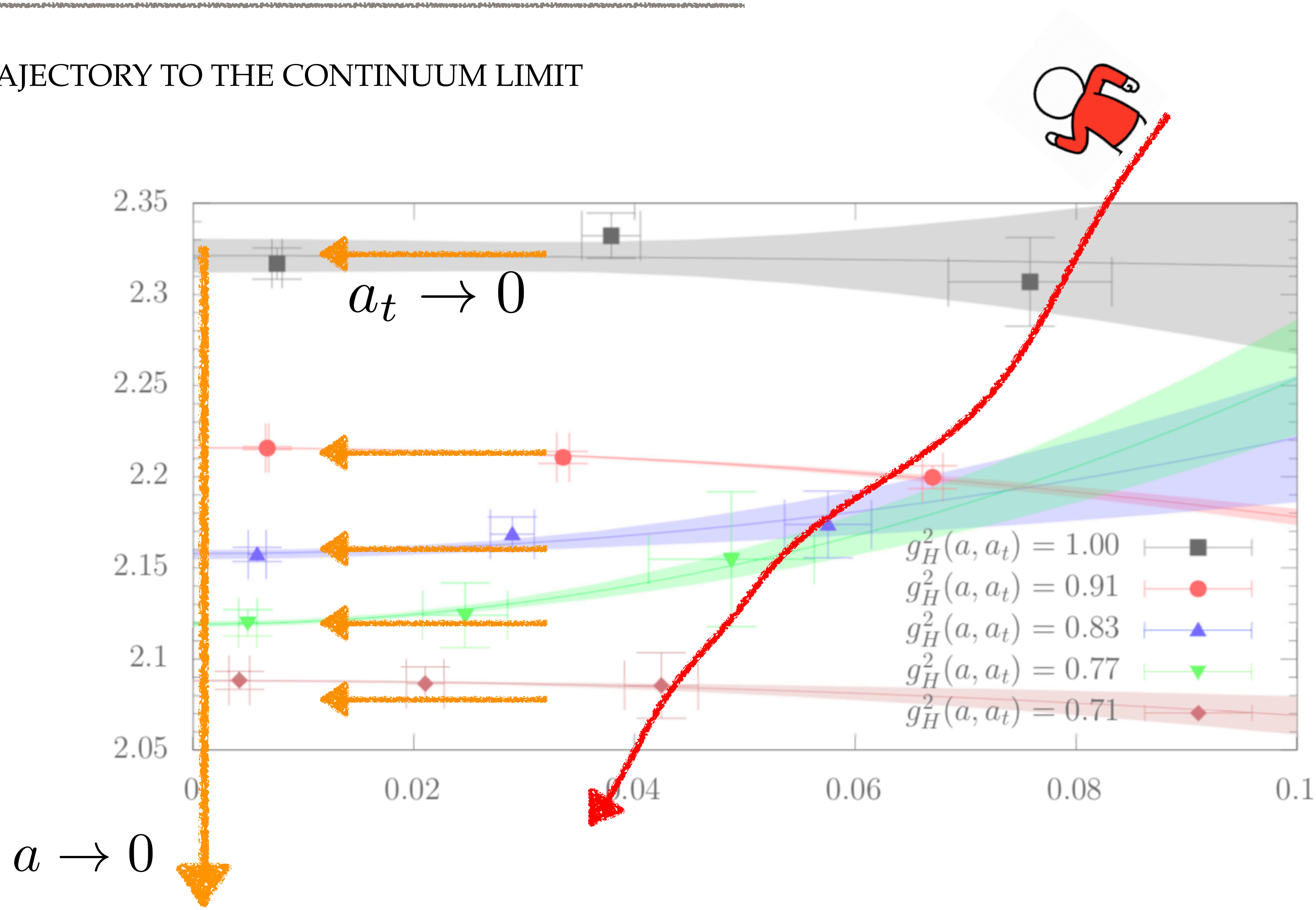
To reach the observables — How to do…



and reach the continuum limit

To reach observables in the continuum limit

TRAJECTORY TO THE CONTINUUM LIMIT



continuous space-time limit

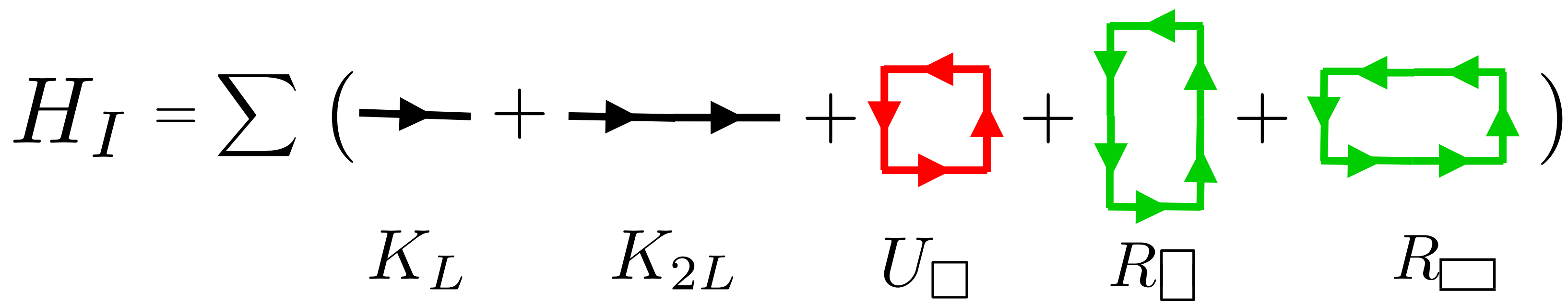
[Carena, Lamm, **YYL**, Liu, PRD. **104**, 094519]

Could we further reduce the resources needed?

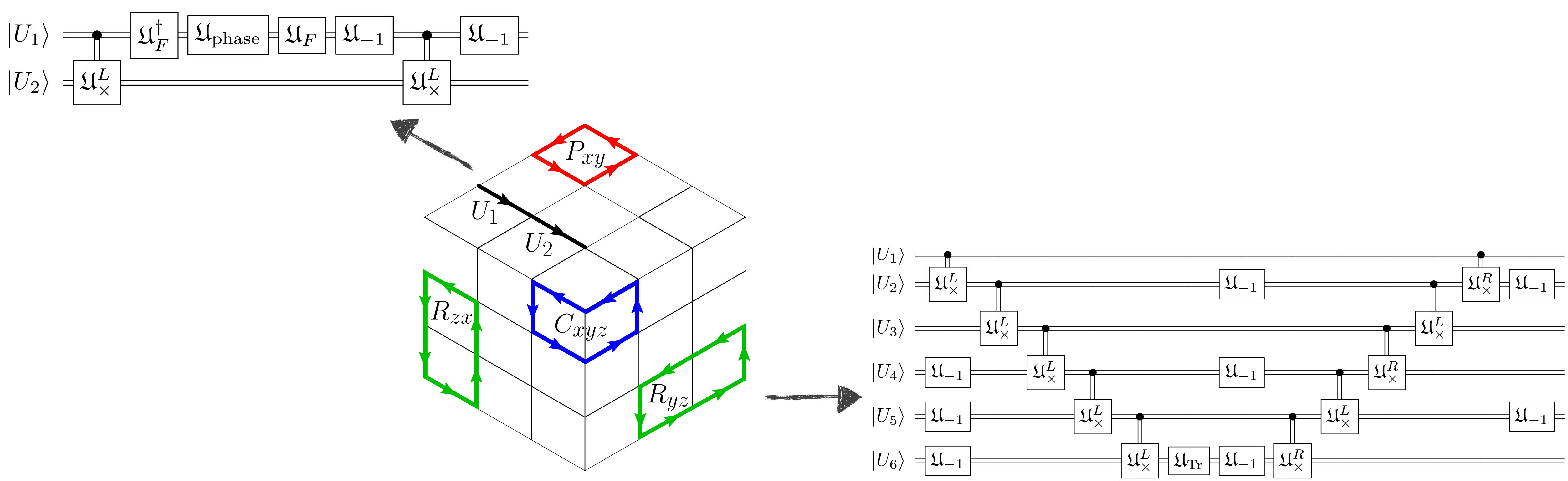
—— improved Hamiltonian

—— gauge invariant subspace encoding?

Improved Hamiltonian



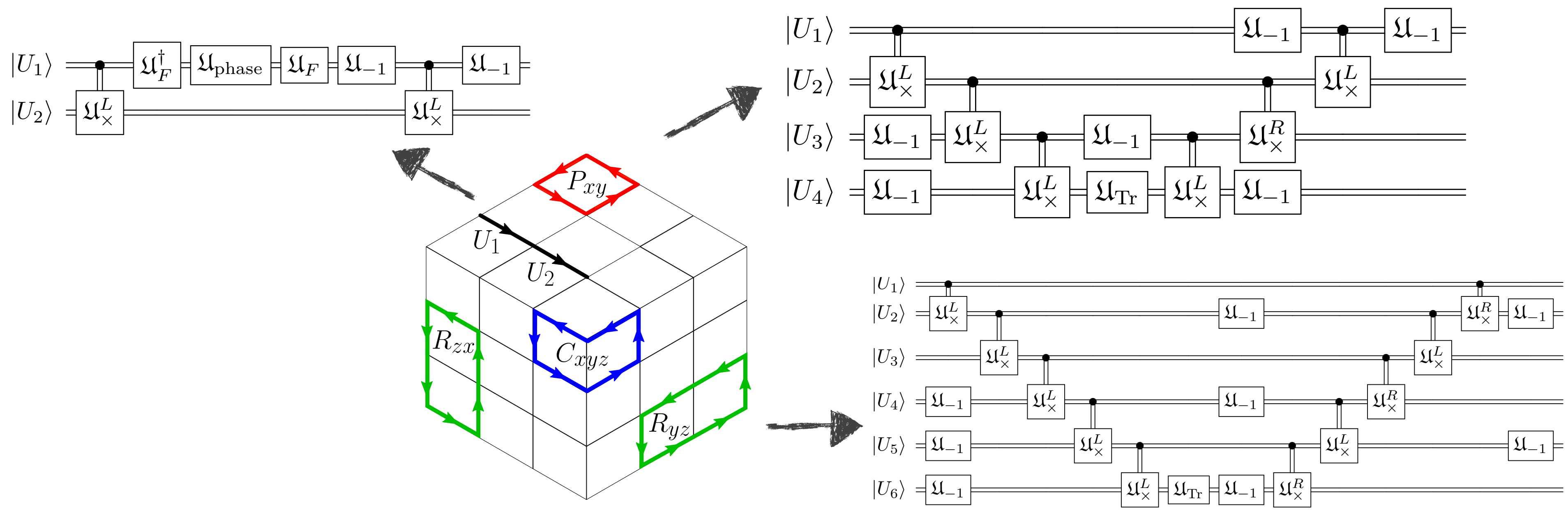
$$\langle U_1', U_2' | \mathcal{U}_{K_{2L}} | U_1, U_2 \rangle = \delta_{U_1' U_2', U_1 U_2} \langle U_1' | e^{i\theta K_{L_1}} | U_1 \rangle$$



Demonstration of improved Hamiltonian is allowed in the near future

M. Carena, H. Lamm, **YYL**, W. Liu, PRL. **129**, 051601

•规范场的量子模拟-Trotterization



Gate	$N[\hat{K}_{KS}+\hat{V}_{KS}]$	$N[\hat{K}_{2L}+\hat{V}_{\text{rect}}]$
$\mathfrak{U}_F$	2	2
$\mathfrak{U}_{\text{phase}}$	1	1
$\mathfrak{U}_{\text{Tr}}$	$\frac{d-1}{2}$	$d-1$
$\mathfrak{U}_{-1}$	$3(d-1)$	$2+8(d-1)$
$\mathfrak{U}_{\times}$	$6(d-1)$	$4+20(d-1)$

- # of Gates here for a single trotter is increasing only multiplicatively, could be compensated by the decreasing of links.
- Larger trotter steps, instead could be used for improved Hamiltonian.

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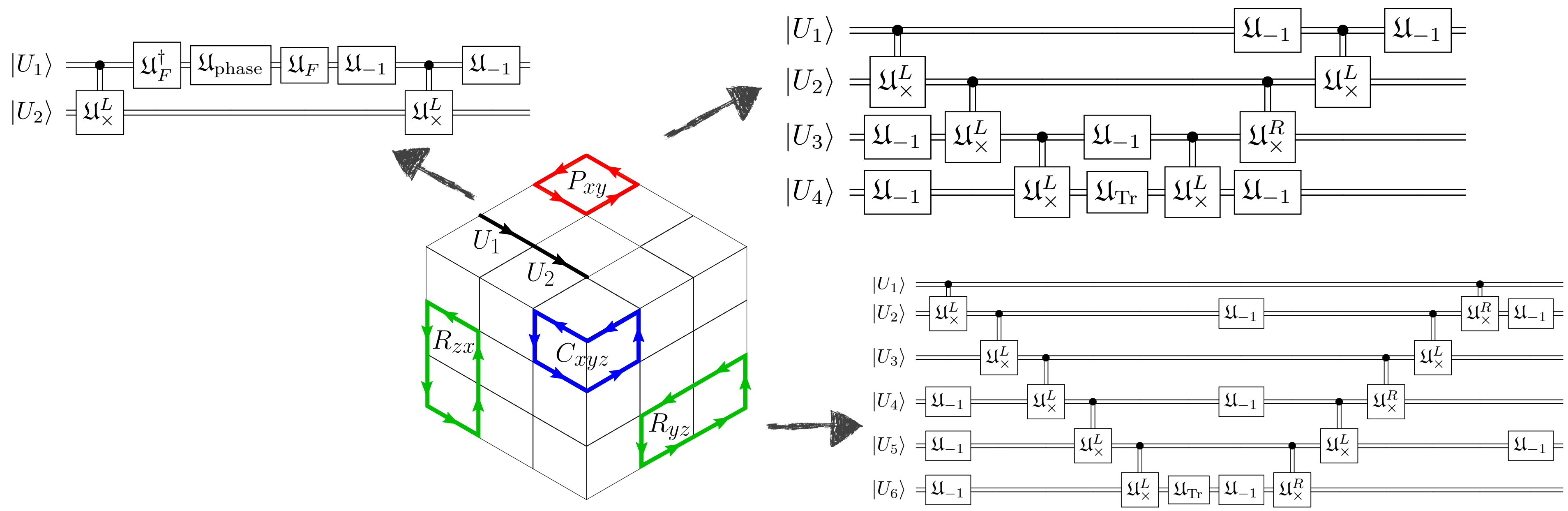
- 规范场的量子模拟-Trotterization



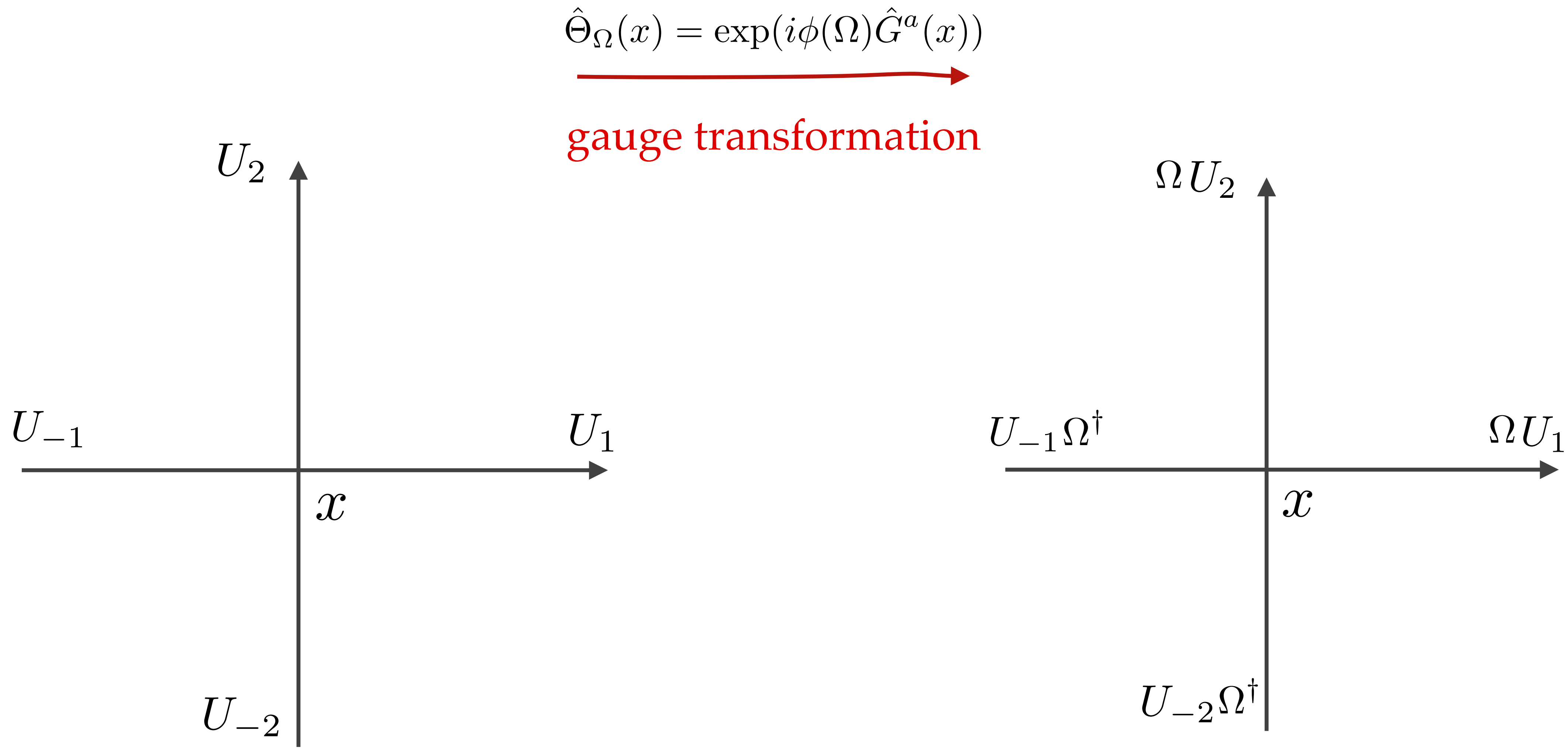
Gate	$N[\hat{K}_{KS}+\hat{V}_{KS}]$	$N[\hat{K}_{2L}+\hat{V}_{\rm rect}]$
$\mathfrak{U}_F$	2	2
$\mathfrak{U}_{\rm phase}$	1	1
$\mathfrak{U}_{\rm Tr}$	$\frac{d-1}{2}$	$d-1$
$\mathfrak{U}_{-1}$	$3(d-1)$	$2+8(d-1)$
$\mathfrak{U}_\times$	$6(d-1)$	$4+20(d-1)$

So far, circuits for improved Hamiltonian are designed, reducing the number of qubits required, with comparable or less quantum gates.

Demonstration

M. Carena, H. Lamm,**YYL**, W. Liu, PRL. **129**, 051601

Keeping Gauge Redundancy or Not?



gauge equivalent states

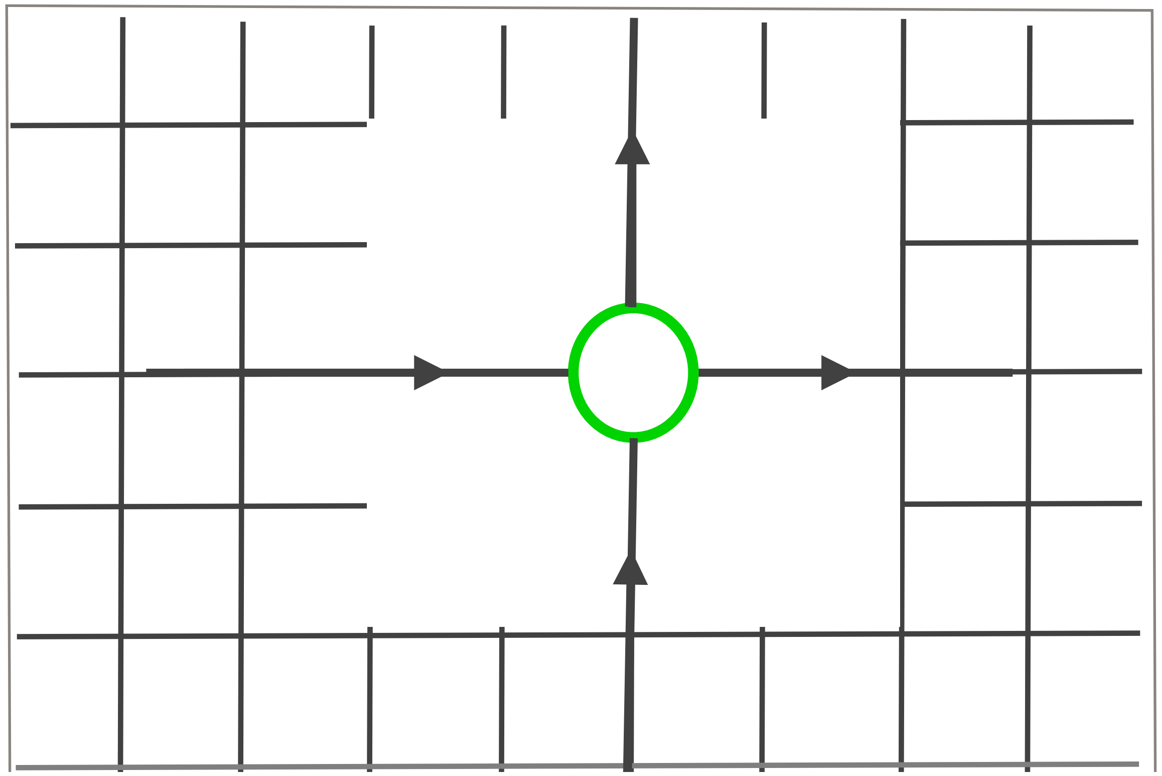
$$\hat{\Theta}_\Omega(x)\,|U_{-1}U_1U_{-2}U_2\rangle = |U'_{-1}U'_1U'_{-2}U'_2\rangle$$

Digitization $\quad |q\rangle^N \rightarrow |G\rangle$

gauge invariant states

$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

neutral charge



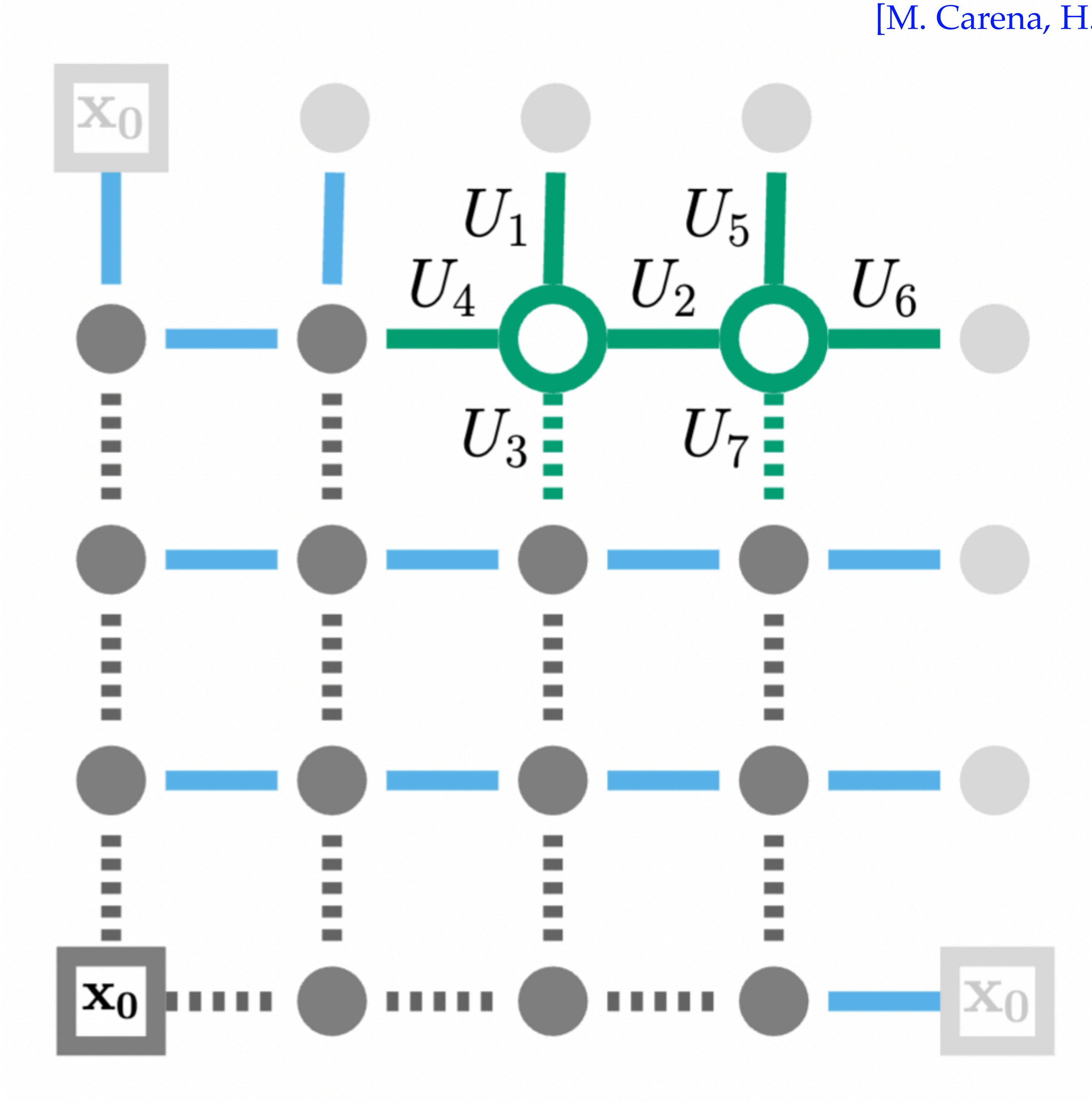
$$\hat{\Theta}_\Omega(x) = \exp(i\phi(\Omega)\hat{G}^a(x))$$

$$\hat{G}^a(x) = \sum_{i=1}^{d}\left[\hat{E}^a_R(x-e_i,e_i) - \hat{E}^a_L(x,e_i)\right]$$

$$\hat{G}^a(x)\,|\psi_{\text{phys}}\rangle = 0$$

Digitization $|q\rangle^N \to |G\rangle$

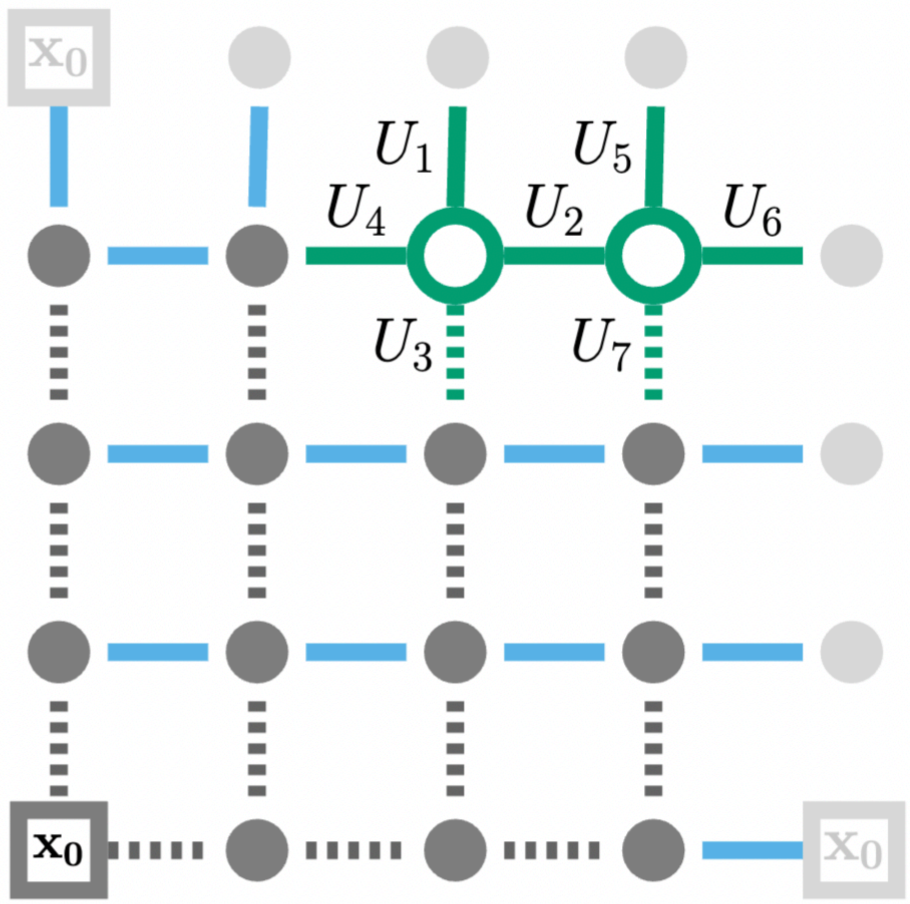
[M. Carena, H. Lamm, **YYL**, W. Liu, arXiv:2402.16780]



Gauss's Law

Digitization $\qquad |q\rangle^N \to |G\rangle$

[M. Carena, H. Lamm, **YYL**, W. Liu, arXiv:2402.16780]



$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L}$$

$$\hat{\Theta}_\Omega(x)\,|\psi\rangle = |\psi'\rangle$$

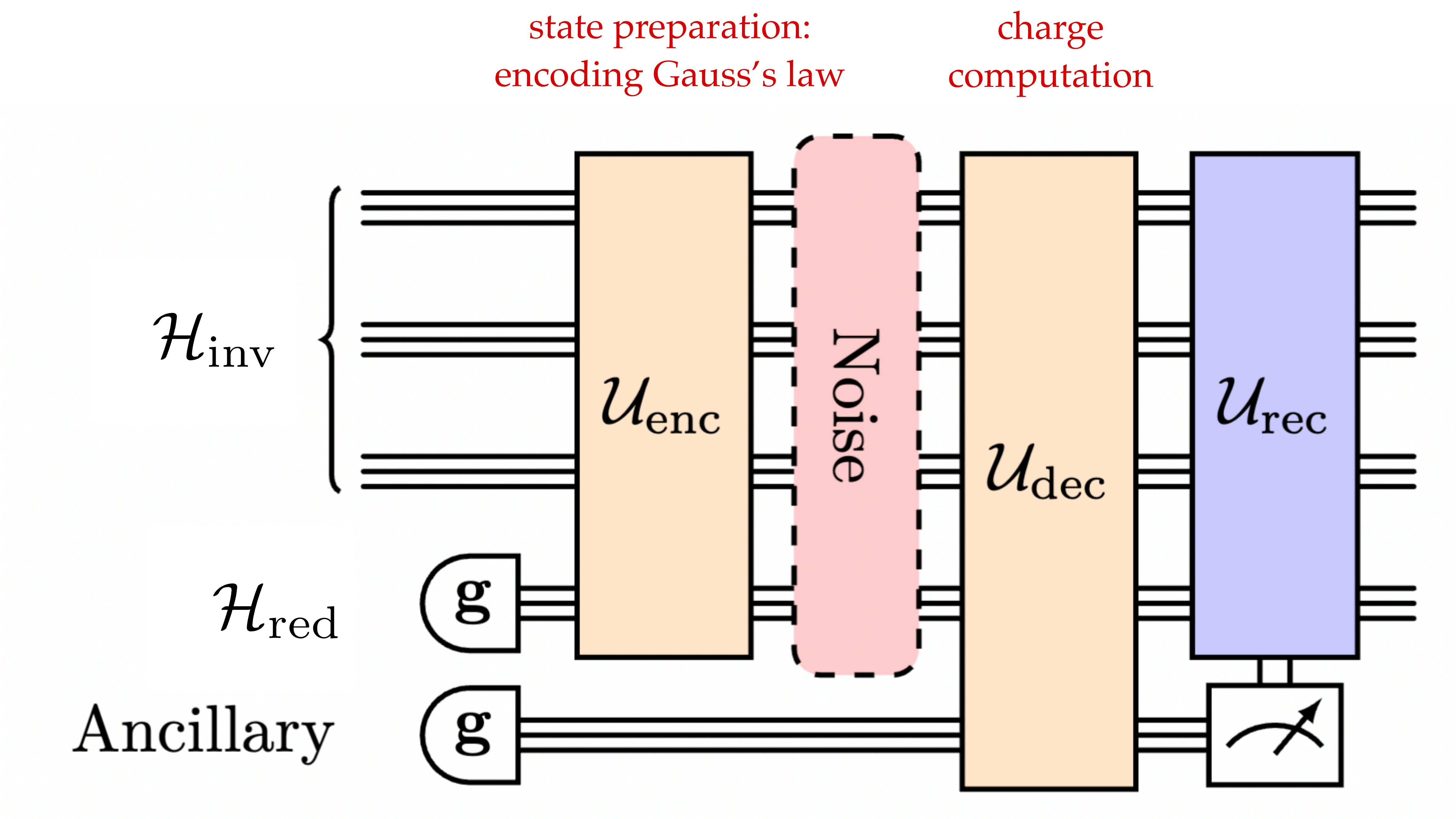
gauge redundant

$$\hat{G}^a(x)\,|\psi\rangle = 0$$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L - N_V + 1}$$

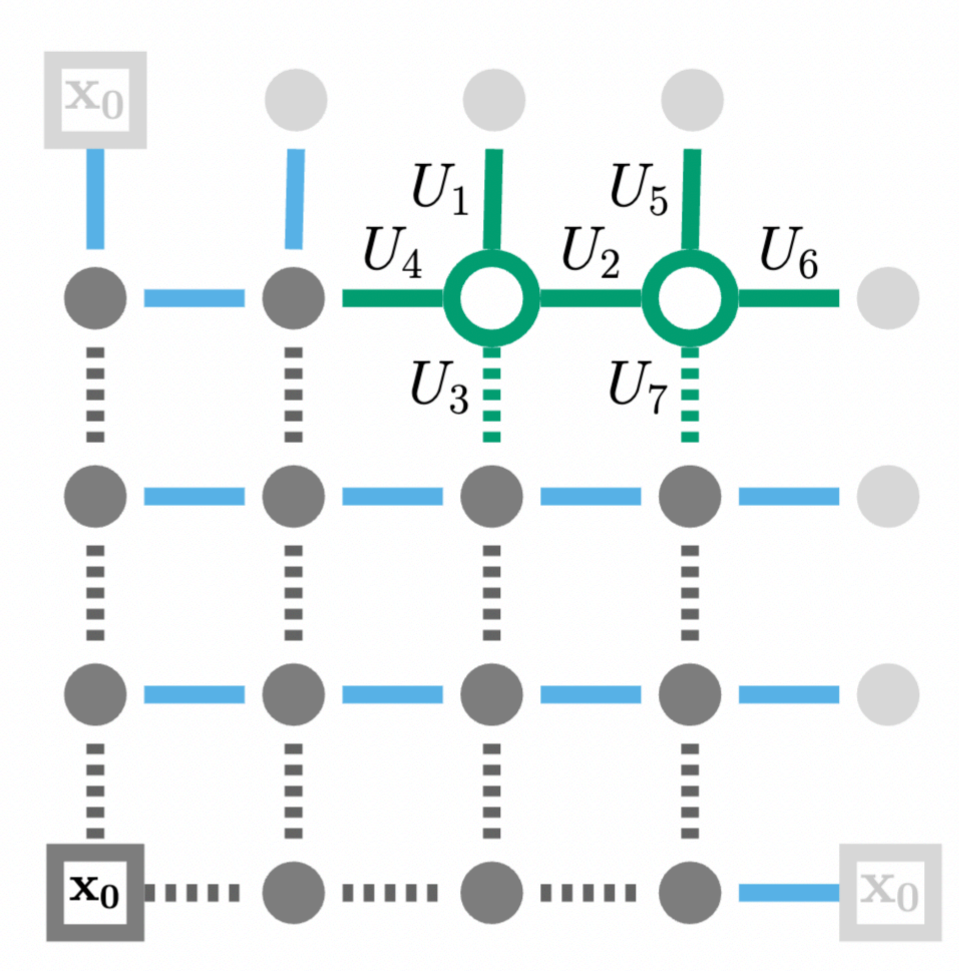
$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

Digitization $|q\rangle^N \rightarrow |G\rangle$

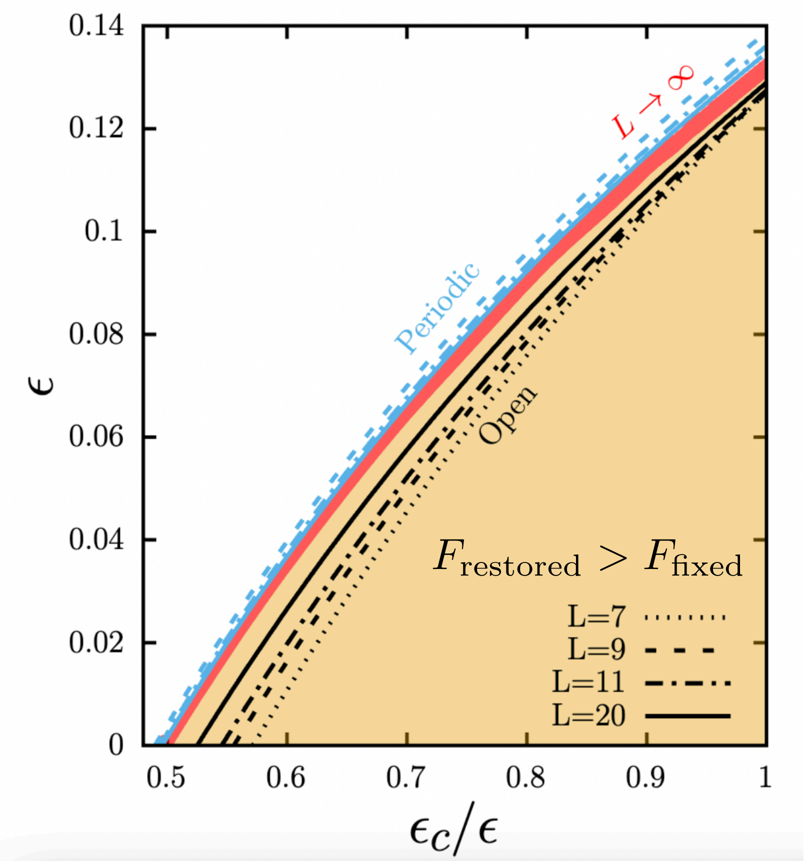


Gauge redundancy utilized for error corrections

Digitization $|q\rangle^N \to |G\rangle$



[M. Carena, H. Lamm, **YYL**, W. Liu, arXiv:2402.16780]



$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L}$$

$$\hat{\Theta}_\Omega(x)\,|\psi\rangle = |\psi'\rangle$$

gauge redundant

$$\hat{G}^a(x)\,|\psi\rangle = 0$$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L - N_V + 1}$$

$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

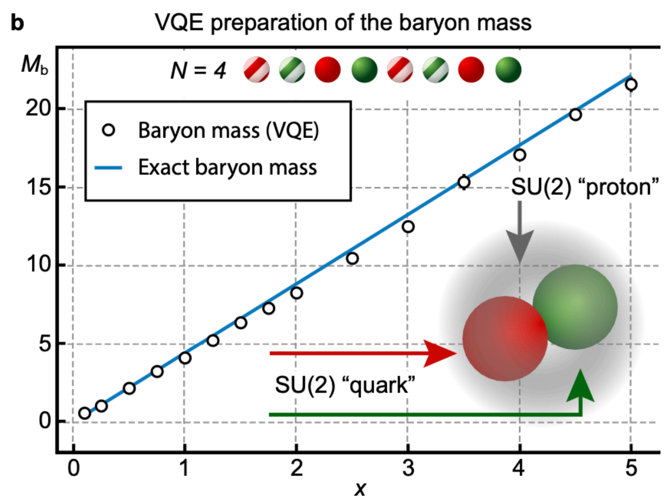
Now - Noisy Intermediate Scale Quantum (NISQ) era

more than 50 well controlled qubits, not error-corrected yet

Physics Benchmarks

Physics Benchmarks for Quantum Computing

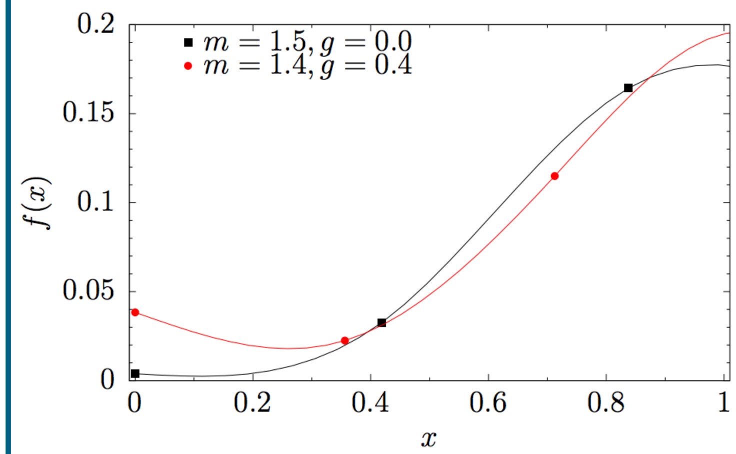
proton state preparation



Atas et al, Nat Commun 12, 6499 (2021)

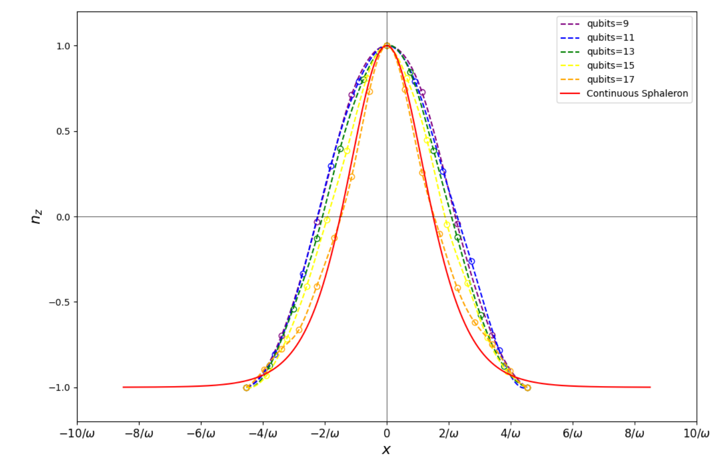
PDF

$$\langle\Psi|\mathcal{O}(t)\mathcal{O}(0)|\Psi\rangle = \frac{\partial}{\partial\epsilon_t}\frac{\partial}{\partial\epsilon_0}\langle\Psi|e^{-iHt}e^{-i\mathcal{O}\epsilon_t}e^{iHt}e^{i\mathcal{O}\epsilon_0}|\Psi\rangle$$



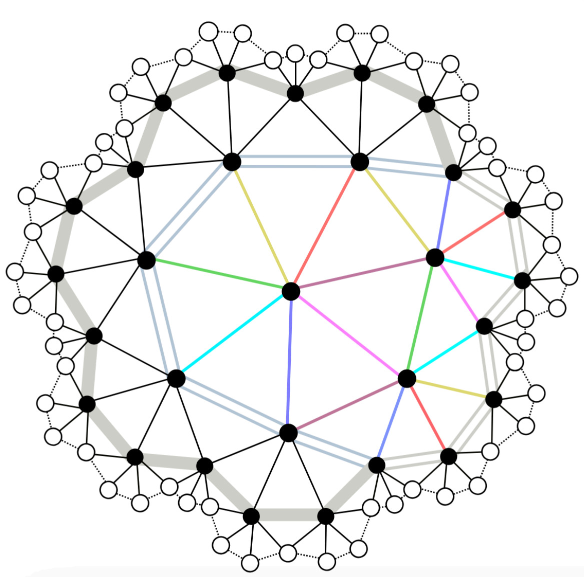
Lamm, et al., T. Li, et al,

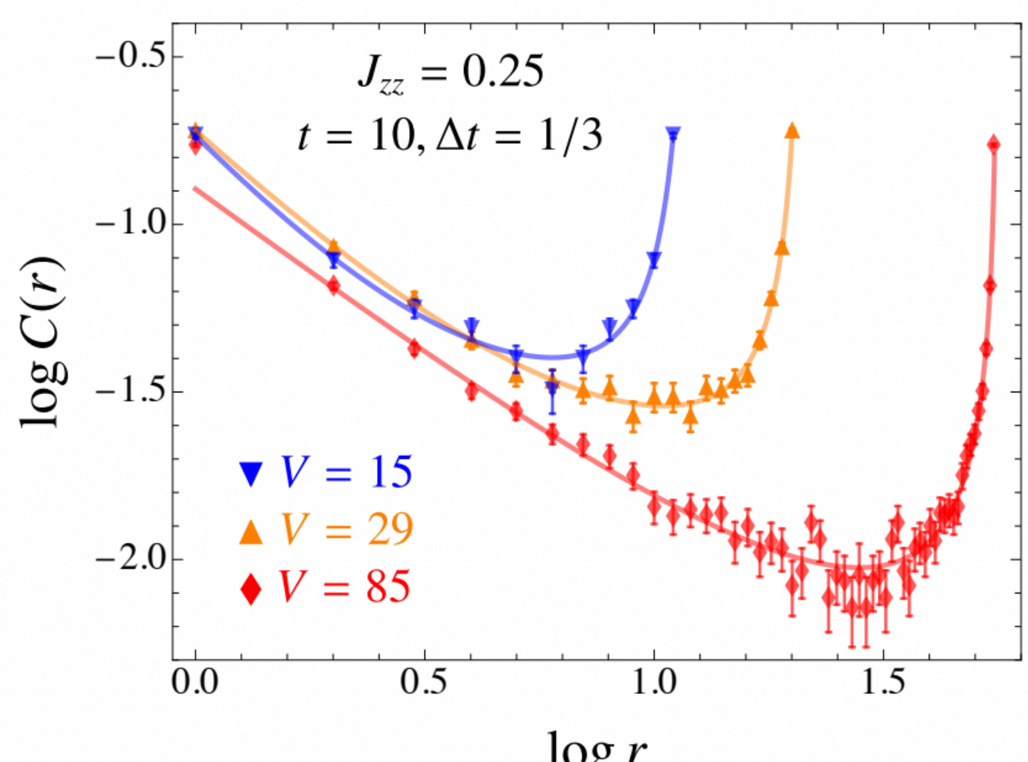
topological objects preparation



Huang, **YYL**, Liu, Wang, Zhang, in preparation

Holography





[**YYL,** Sajid, Unmuth-Yockey, arXiv:2312.10544]

entanglement entropy?

Quantum Machine Learning

computational complexity improvements, computational speed-ups

Supervised Learning—better separation power?

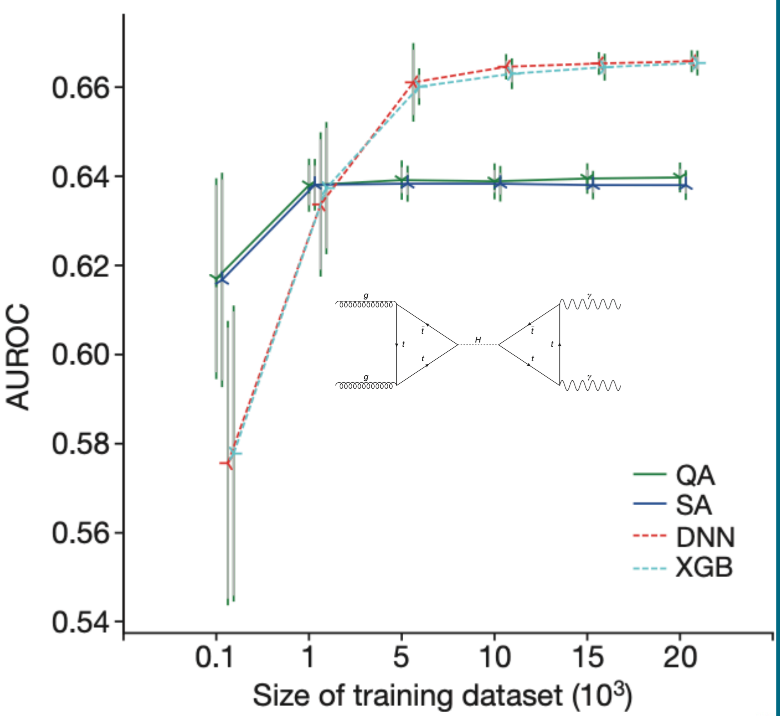
Quantum variational circuits, quantum annealing, QSVM, etc.

Quantum Annealing

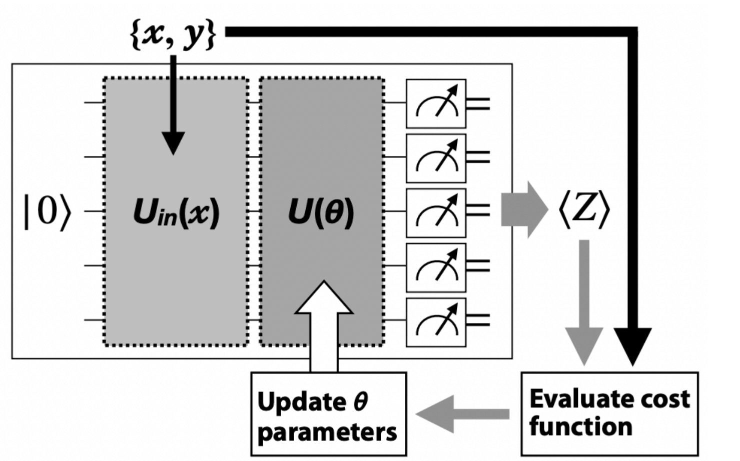
$$C_{ij} = \sum_{\tau} c_i(\boldsymbol{x}_\tau) c_j(\boldsymbol{x}_\tau), \quad C_i = \sum_{\tau} c_i(\boldsymbol{x}_\tau) y_\tau$$

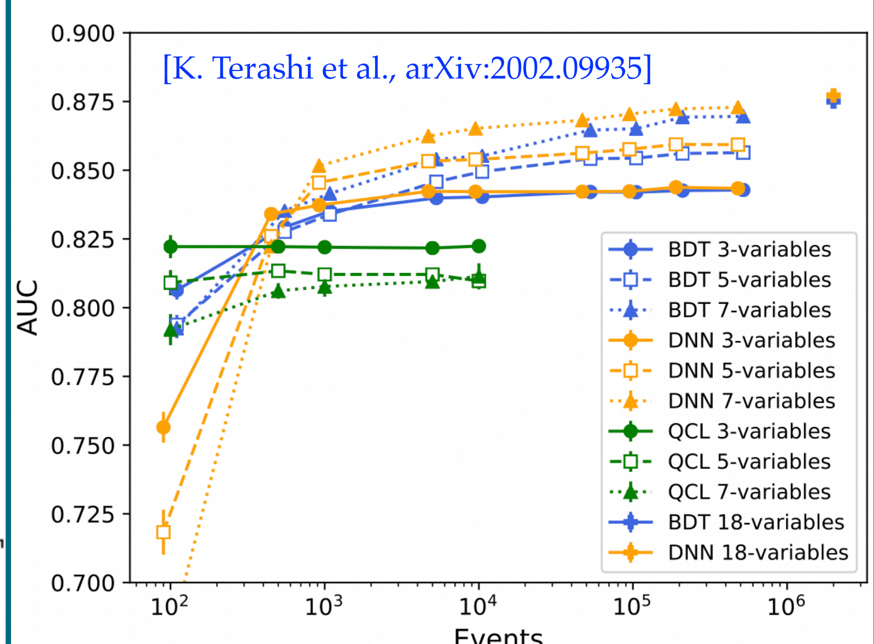
$$H = \sum_{i,j} J_{ij} s_i s_j + \sum_i h_i s_i$$

$$R(\boldsymbol{x}) = \sum_i s_i^g c_i(\boldsymbol{x}) \in [-1, 1]$$



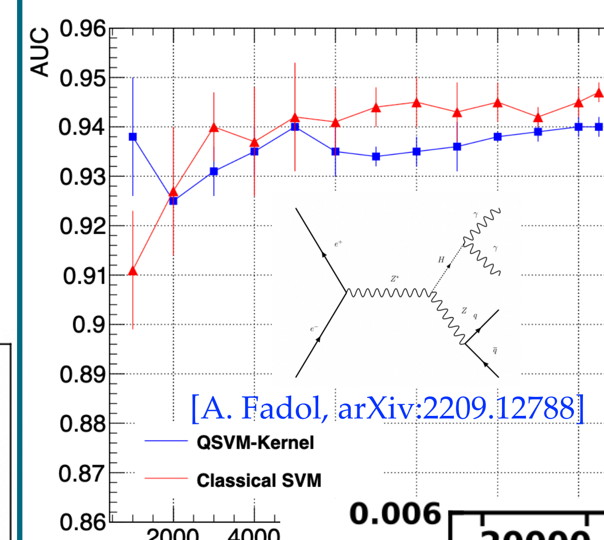
Variational Quantum Approach

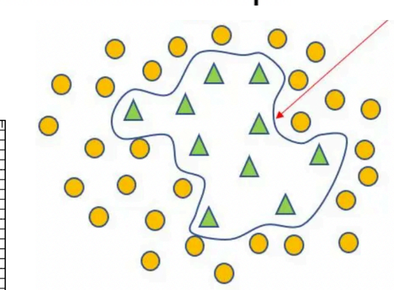




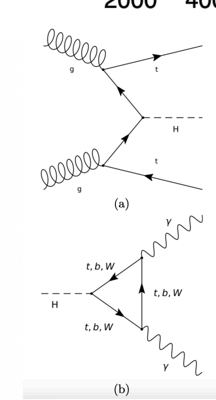
Quantum Support Vector Machine

$$k(\vec{x}_i, \vec{x}_j) = \left| \left\langle 0^{\otimes N} \middle| \mathcal{U}^\dagger_{\Phi(\vec{x}_i)} \mathcal{U}_{\Phi(\vec{x}_j)} \middle| 0^{\otimes N} \right\rangle \right|^2$$

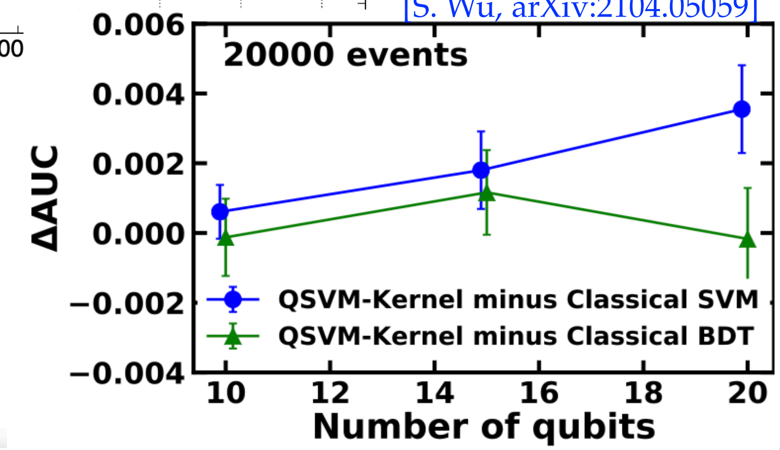




larger sample size?

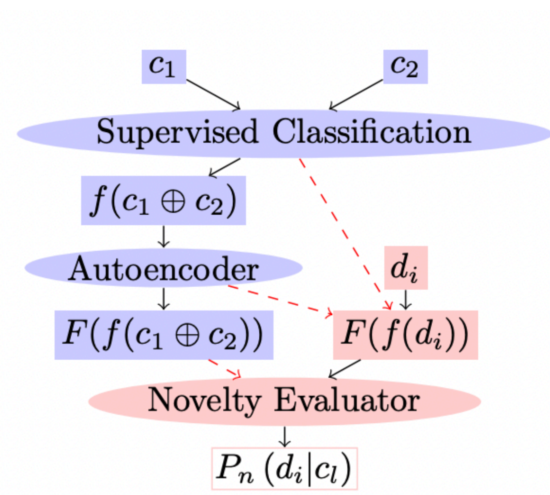


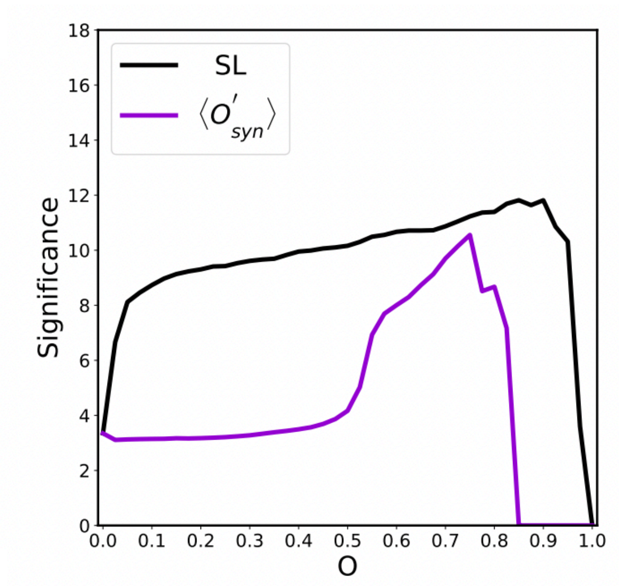
[S. Wu, arXiv:2104.05059]



Quantum Machine Learning - Anomaly detection

ANOMALY DETECTION





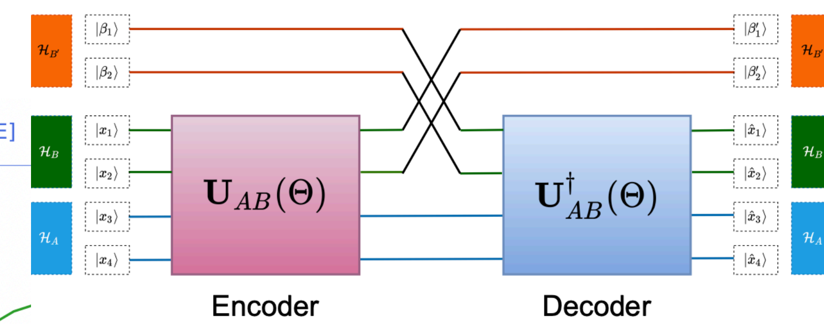
(a) $t\bar{t}h$

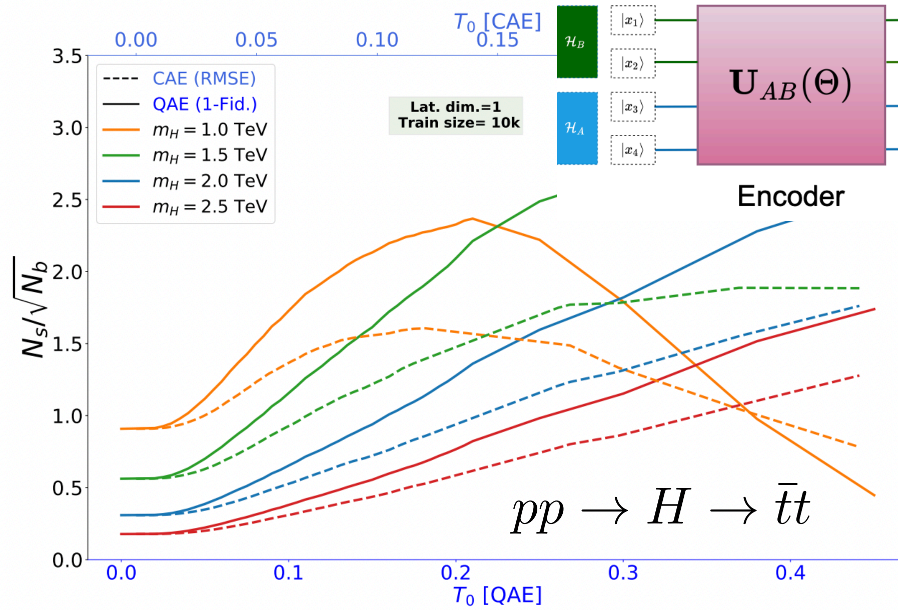
[J. Hajer, **YYL**, T. Liu, H. Wang, *PRD* 101 7, 076015]
[X.-H. Jiang, **YYL**, A. Juste, T. Liu, JHEP 10 (2022) 085]

- arXiv:1808.08992: ``Searching for New Physics with Deep Autoencoders'', Marco Farina, Yuichiro Nakai, and David Shih
- arXiv:1808.08992: ``QCD or What?'', Theo Heimel, Gregor Kasieczka, Tilman Plehn, and Jennifer M Thompson
- arXiv:1811.10276, ``Variational Autoencoders for New Physics Mining at the Large Hadron Collider'', Olmo Cerri, Thong Q. Nguyen, Maurizio Pierini, Maria Spiropulua and Jean-Roch Vlimant
- arXiv:1903.02032, ``A robust anomaly finder based on autoencoder'', Tuhin S. Roy and Aravind H. Vijay
- arXiv:1905.10384, ``Adversarially-trained autoencoders for robust unsupervised new physics searches'', Andrew Blance, Michael Spannowsky, and Philip Waite
- … … … …

QUANTUM ANOMALY DETECTION

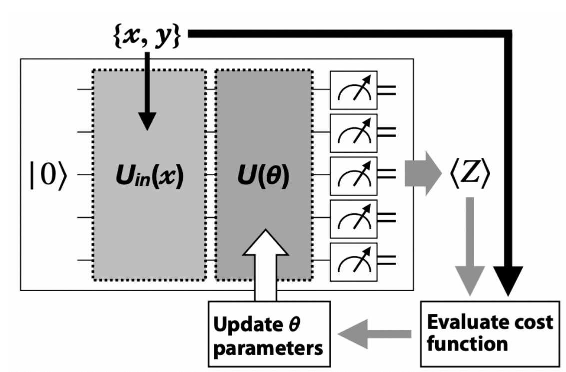
Quantum auto encoder





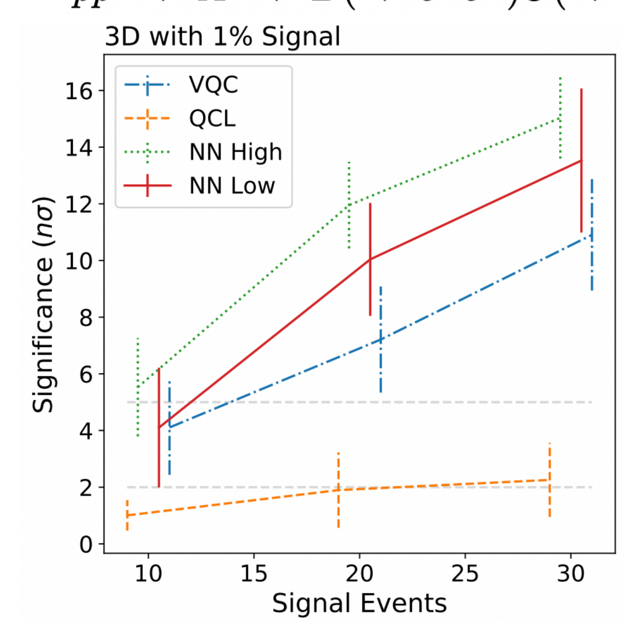
[Ngairangbam, Spannowsky, Takeuchi, PRD. **105**, 095004]

Quantum weakly-supervised learning (bg VS bg + ϵ signal)



[Terashi et al, arXiv:2002.09935]

$pp \to A \to B(\to e^+e^-)C(\to \mu^+\mu^-)$

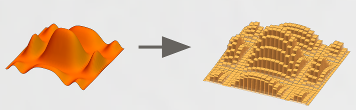


[Alvi, Bauer, Nachman, arXiv:2206.08391]

"Quantum potential for first-principle calculations!"

(2030s) narrow down the framework with
- improving algorithms —— efficient Fourier transformations
- theoretical studies of uncertainties —— phase diagrams for improved H
- hardware co-design —— qudits for blocking encodings
- benchmark studies
- …

HEP case calculations for experiments



$|q\rangle^N \to |G\rangle$

↓

$\mathcal{U}\,|G\rangle^L \to |\psi_0\rangle$

↓

$\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$

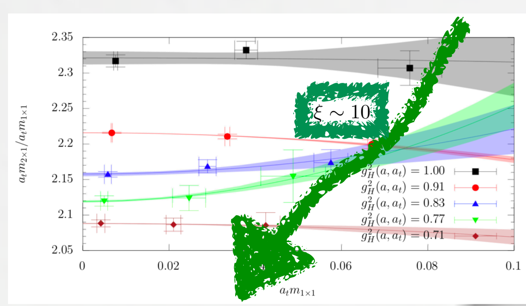
↓

$\langle \mathcal{O} \rangle$

S. P. Jordan,
K. S. M. Lee,
J. Preskill

2011-

various methods



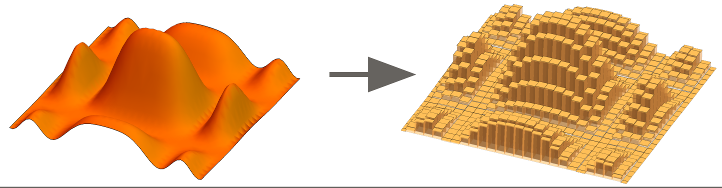
2020 -

2030s -

Thank you

BACK UP

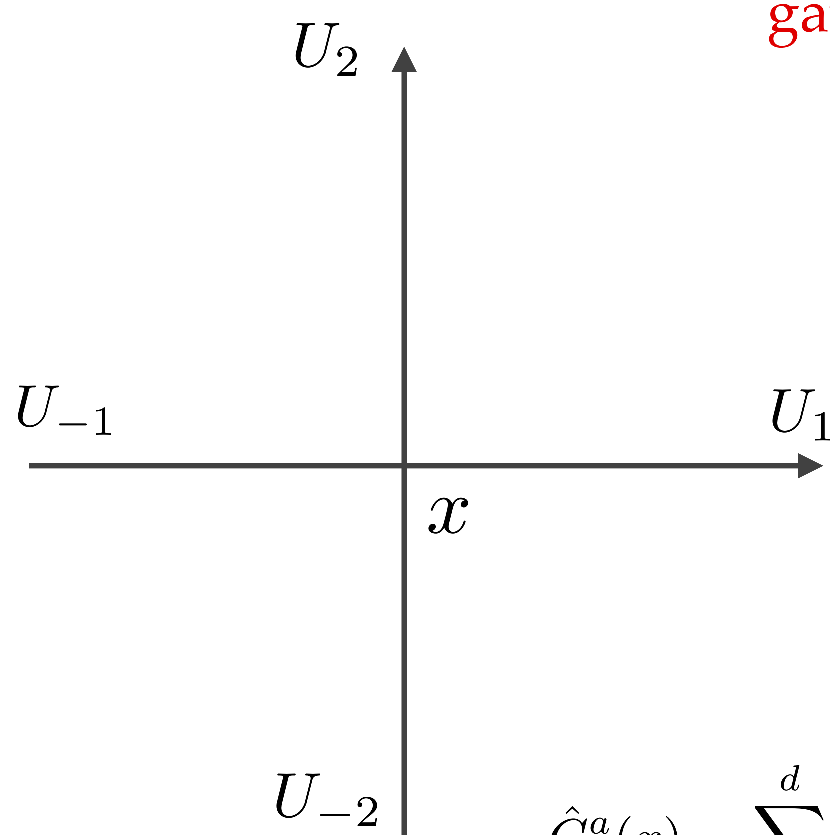
Discretization

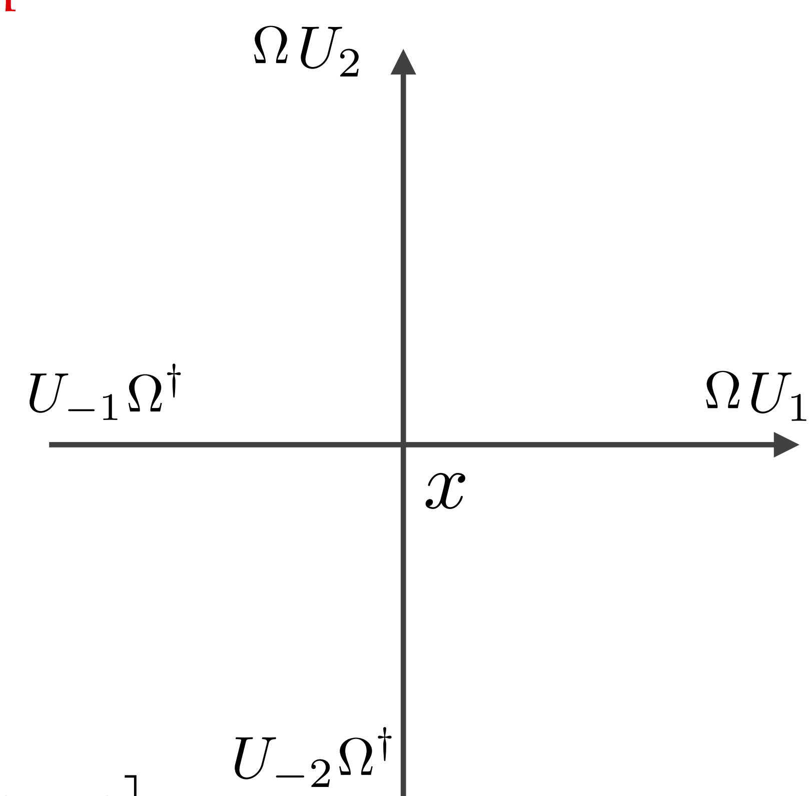


infinities in QFT

$$\hat{\Theta}_{\Omega}(x) = \exp(i\phi(\Omega)\hat{G}^a(x))$$

gauge transformation



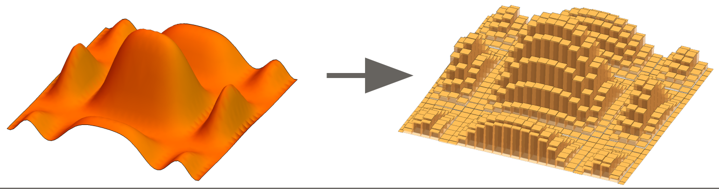


$$\hat{G}^a(x) = \sum_{i=1}^{d} \left[\hat{E}^a_R(x - e_i, e_i) - \hat{E}^a_L(x, e_i) \right]$$

lattice analog of covariant divergence of chromo-electric field

quadratic Casmir : $\hat{E}^2 \left| j m_L m_R \right\rangle = j(j+1) \left| j m_L m_R \right\rangle$ $\quad \left| j m_L m_R \right\rangle \overset{\text{FT}}{\leftrightarrow} \left| U \right\rangle$

Discretization



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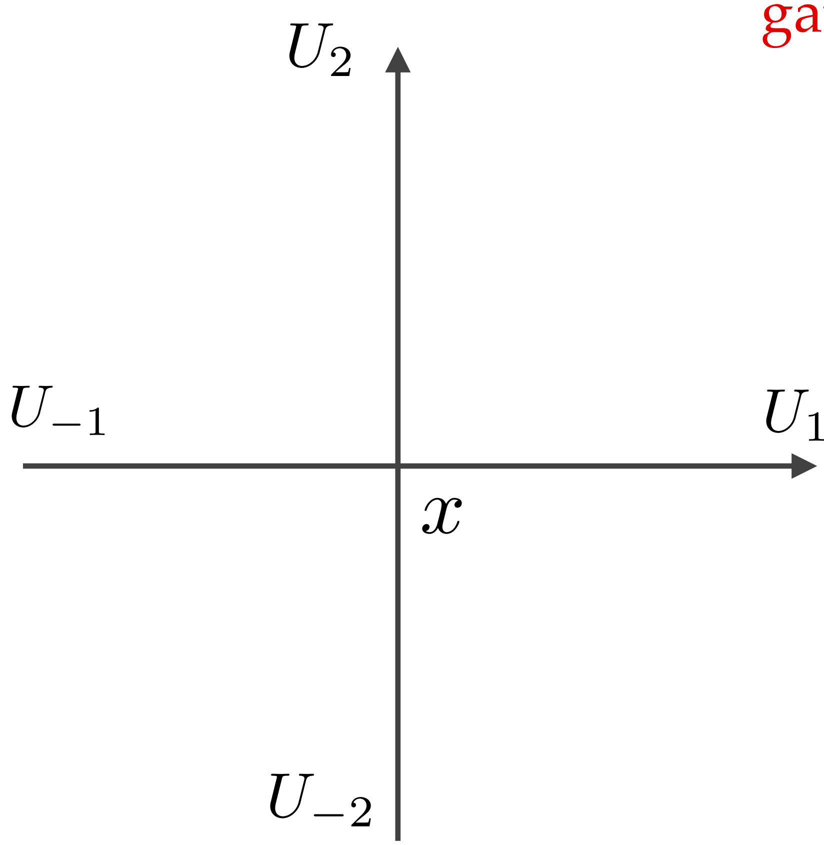
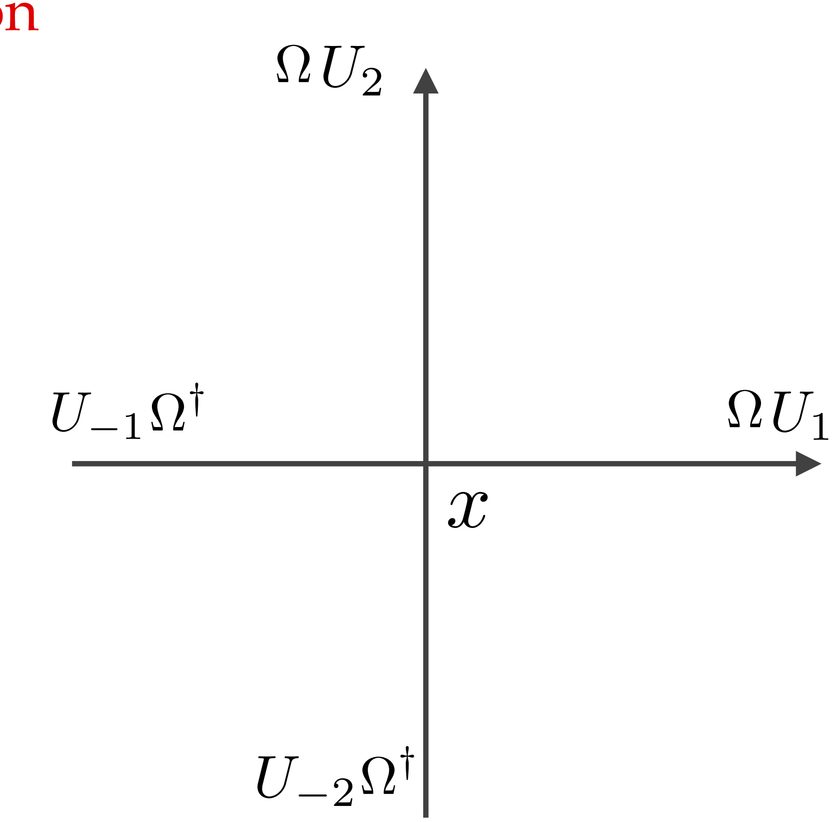
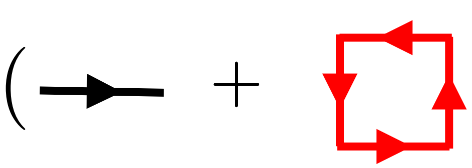




gauge invariant Hamiltonian

$H_{KS} = \sum$ (



)

K_L $U_\square$

quadratic Casimir