A Novel Quantum Realization of Jet Clustering

arXiv:2407.09056

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Motivation:

1, Quarks and gluons carry color charge and can not travel freely. Once generated in high-energy collisions, quarks, and gluon would fragment into numerous particles called jets.

2, Exploring the application of quantum technologies to jet clustering is key to fostering innovation for both sides.

Contents:

1, MaxCut problem and mapping jet clustering to maxCut

2, Adiabatic Quantum Computing (AQC) and from AQC to Quantum Approximate Optimization Algorithm (QAOA)

3, the performance of QAOA on jet clustering

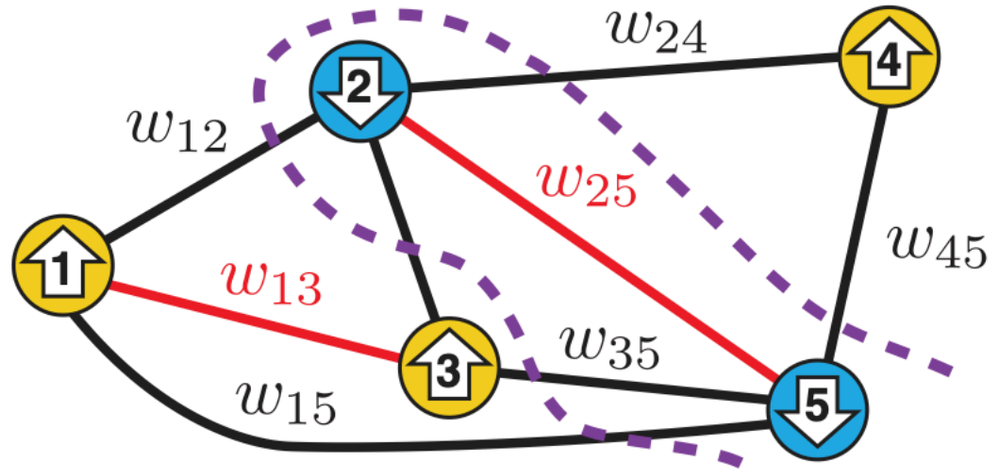
MaxCut problem

undirected graph $G = (V, E)$

V: set of vertices

E: set of edges

$w_{ij} > 0$: the weight of the edge $(ij) \in E$



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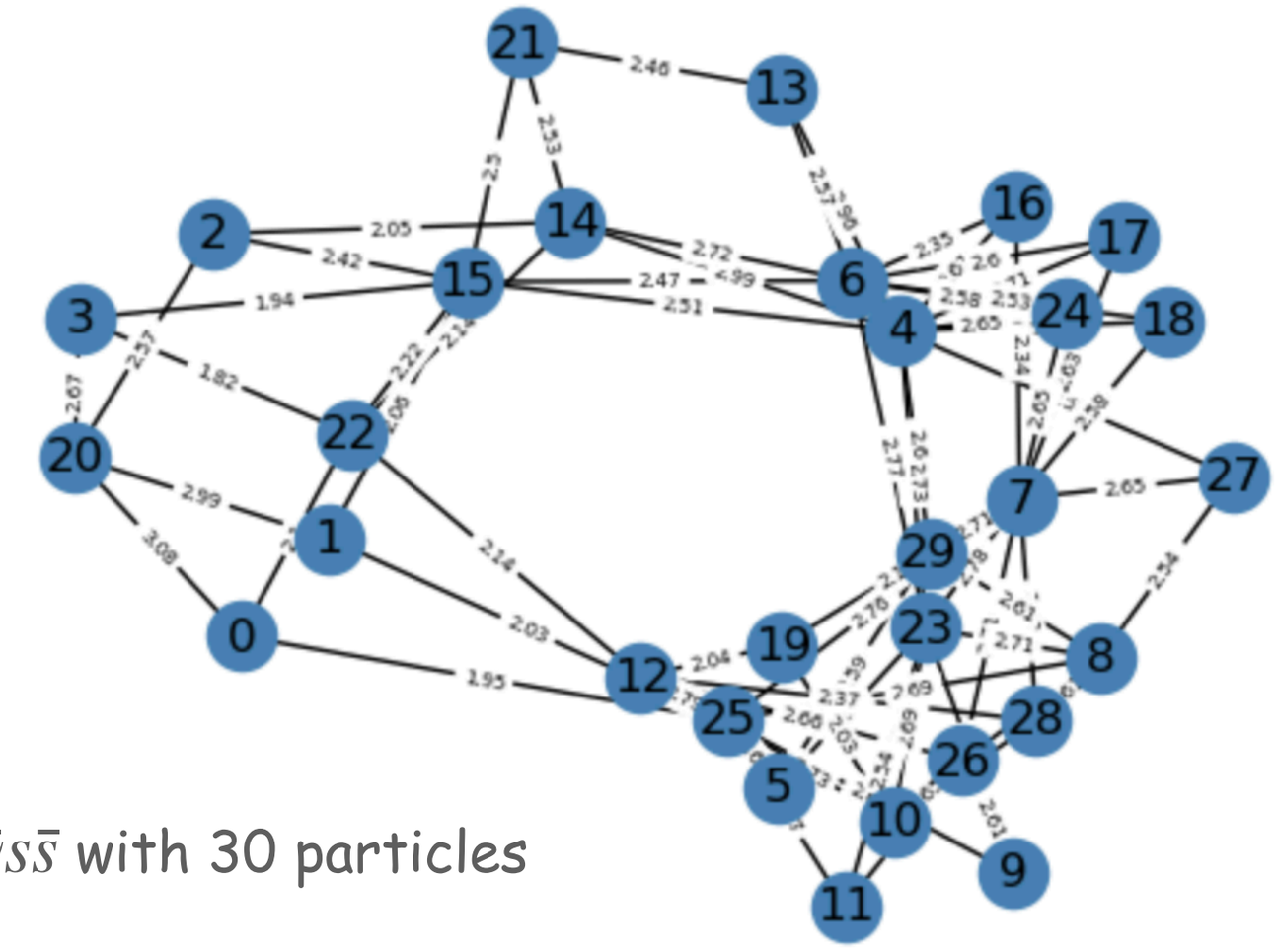
goal: partition the graph vertices into two complementary subsets to maximize the total weight of edges with two vertices belong to two subsets $C(x) = \sum_{i,j=1}^{|V|} w_{ij} x_i (1 - x_j)$

A collision event can be represented as a graph

particles as vertices
the angle of two particles as the edge weight
only the k leading large edges are retained (k-regular graph)



$e^+e^- \to ZH \to \nu\bar{\nu}s\bar{s}$ with 30 particles

Adiabatic Quantum Computing (AQC)

- the driver Hamiltonian (H_D) encodes some quantum state that is easy to prepare its ground state
- the problem Hamiltonian (H_C) encodes a quantum state we are interested in as its ground state
- the idea underlines the AQC: start with a ground state that is easy to prepare and wish to end up with the quantum state we are interested in.
- This transition is accomplished via the adiabatic theorem, which states that a system in the ground state of some Hamiltonian will remain in the ground state if the Hamiltonian is changed slowly enough.

the process of AQC :

1, define the Hamiltonian: $H(t) = (1 - s(t))H_D + s(t)H_C$ and let our quantum system evolve under it, $U(t) = \tau e^{\frac{-i}{h}\int_o^t H(T)dT}$.

2, We discretize $U(T)$ into intervals Δt small enough that the Hamiltonian is approximately constant over each interval.

3, Let $U(b, a)$ represent time evolution from time a to time b

$$U(T,0) = U(T, T - \Delta t)U(T - \Delta t, T - 2\Delta t) \dots U(\Delta t,0) = \prod_{j=1}^{P} U(j\Delta t, (j-1)\Delta t) \approx \prod_{j=1}^{P} e^{-iH(j\Delta t)\Delta t}$$

$$U(T,0) \approx \prod_{j=1}^{P} e^{-i(1-s(j\Delta t))H_D\Delta t} e^{-is(j\Delta t)H_C\Delta t} = \prod_{j=1}^{P} e^{-i\beta_P H_D} e^{-i\gamma_P H_C} = \prod_{j=1}^{P} \hat{U}_D(\beta_j)\hat{U}_C(\gamma_j)$$

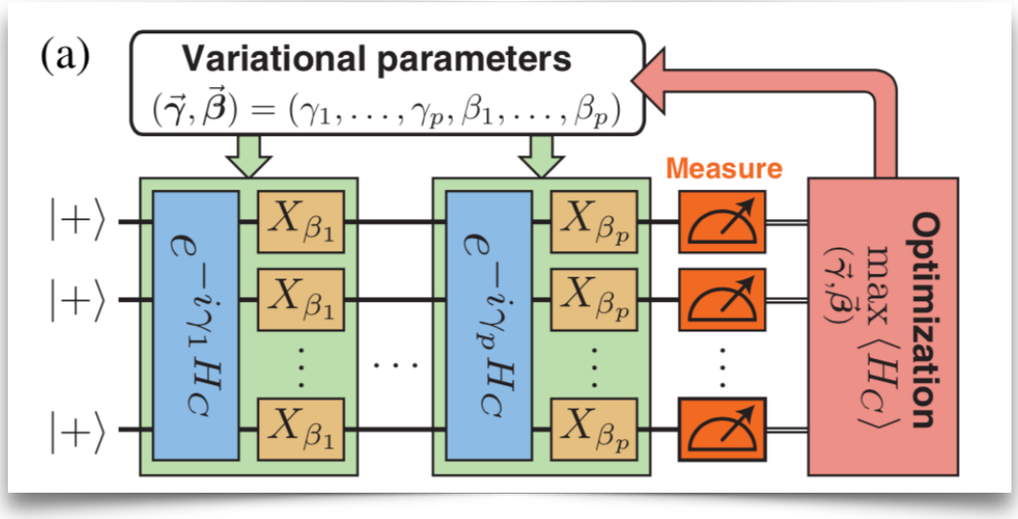
Thus we can approximate AQC by repeatedly letting the system evolve under H_C for some time γ_j and then H_D for some time β_j.

QAOA for MaxCut problem

1. $U(T,0) \approx \prod_{j=1}^{P} e^{-i\beta_P H_D} e^{-i\gamma_P H_C} = \prod_{j=1}^{P} \hat{U}_D(\beta_j)\hat{U}_C(\gamma_j)$

2. $H_D : B = \sum_{j=1}^{n} \sigma_j^x$

3. $H_C : C = \frac{1}{2} \sum_{(i,j)\in E} W_{ij}(I - \sigma_i^z \sigma_j^z)$



4. Initialize the system in the state $|s\rangle = |+\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}} \sum_{x\in(0,1)^n} |x\rangle$

5. Construct the circuit (ansatz) by applying the unitaries $\hat{U}_C(\gamma_j)$ and $\hat{U}_D(\beta_j)$ repeatedly P times

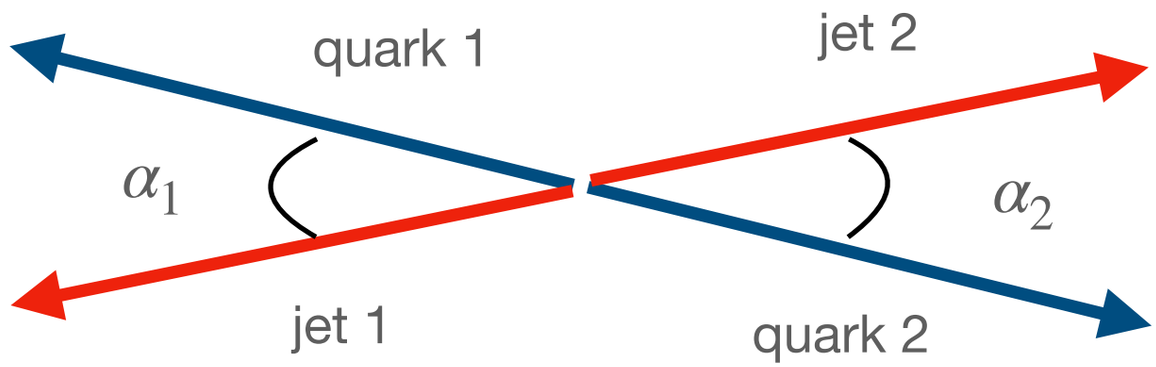
6. The final state output by the circuit is $|\psi_P(\gamma,\beta)\rangle = \hat{U}_D(\beta_P)\hat{U}_C(\gamma_P)\ldots\hat{U}_D(\beta_1)\hat{U}_C(\gamma_1)|s\rangle$

7. The expectation value $\hat{H}_C$ with respect to the state $|\psi_P(\gamma,\beta)\rangle$ is calculated through repeated measurements (1024 times in this analysis) of the final state on the computational basis, $F_P(\gamma,\beta) = \langle\psi_P(\gamma,\beta)|\hat{H}_C|\psi_P(\gamma,\beta)\rangle$

8. A classical optimization algorithm is employed to iteratively update the parameters γ and β to find the optimal set of parameters (γ^*, β^*) such that the expectation value $F_P(\gamma,\beta)$ is maximized.

samples and the criteria of jet clustering performance

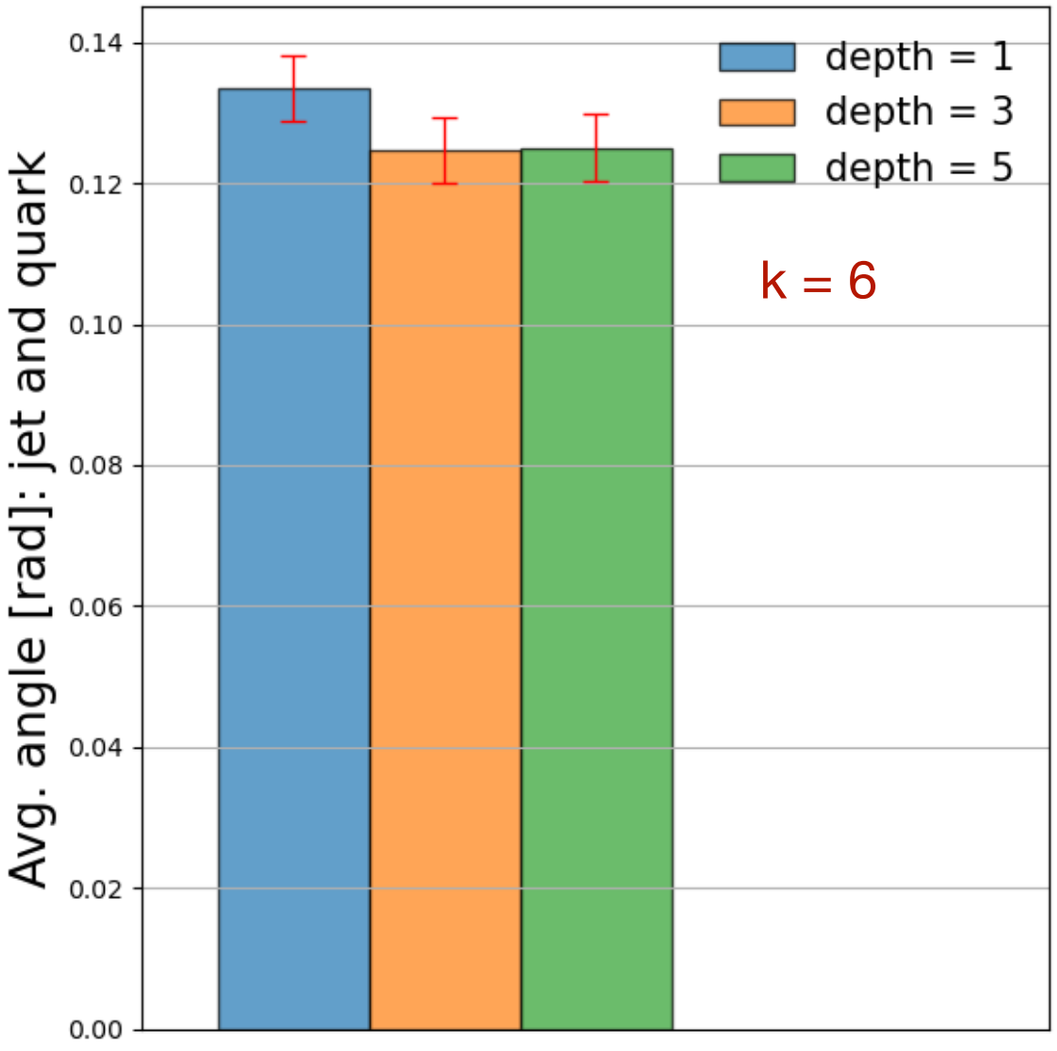
4000 $e^+e^- \to ZH \to \nu\bar{\nu}s\bar{s}$ with 30 particles



criterion is $\alpha = \alpha_1 + \alpha_2$

jet clustering performace v.s. QAOA depth (P)

With more layers, QAOA better approximates the continuous adiabatic process, where the system evolves slowly enough to stay in the ground state of the Hamiltonian, leading to higher probabilities of finding the optimal solution.



approximation ratio (r) : the probability of finding the ground state

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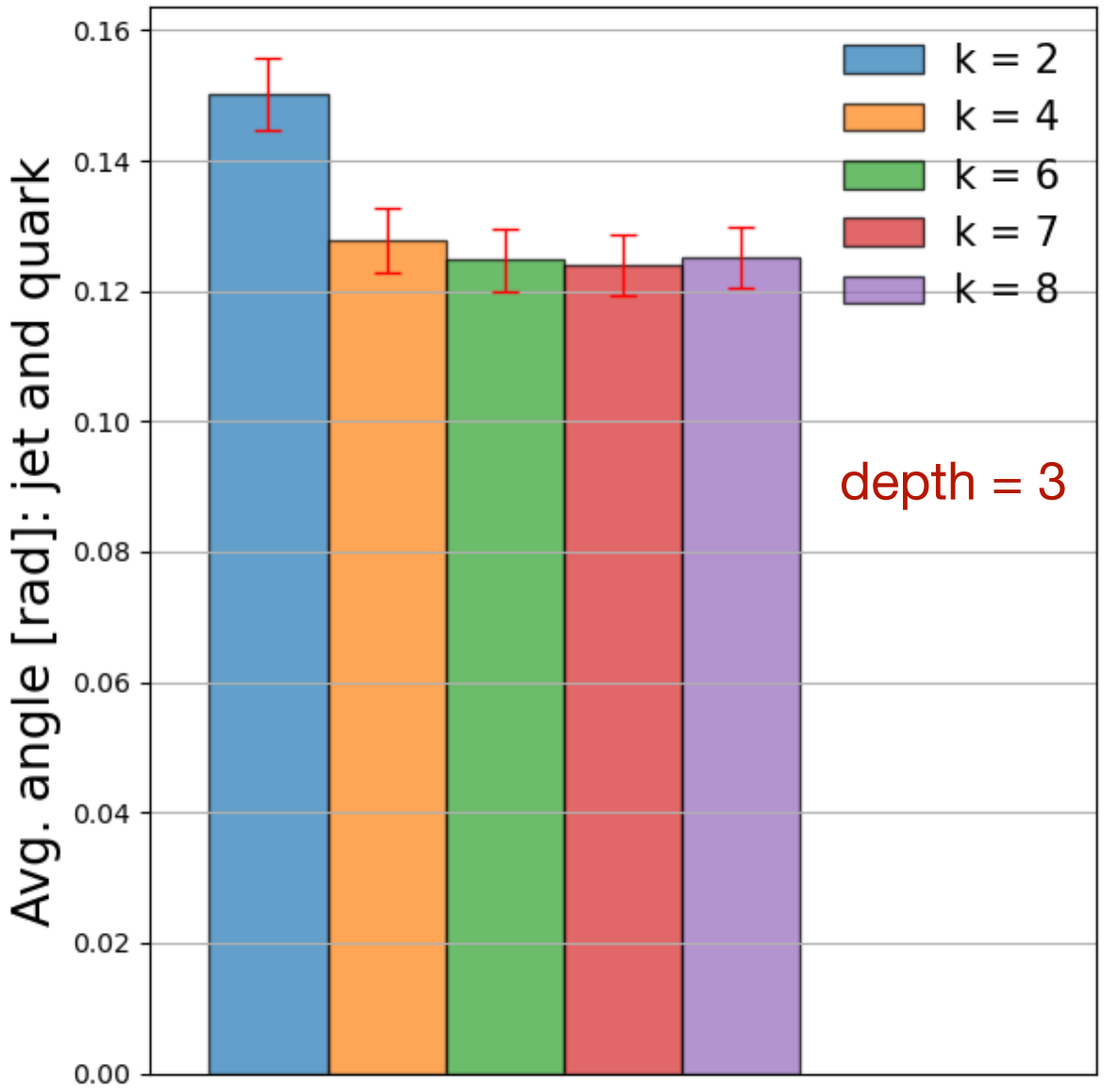
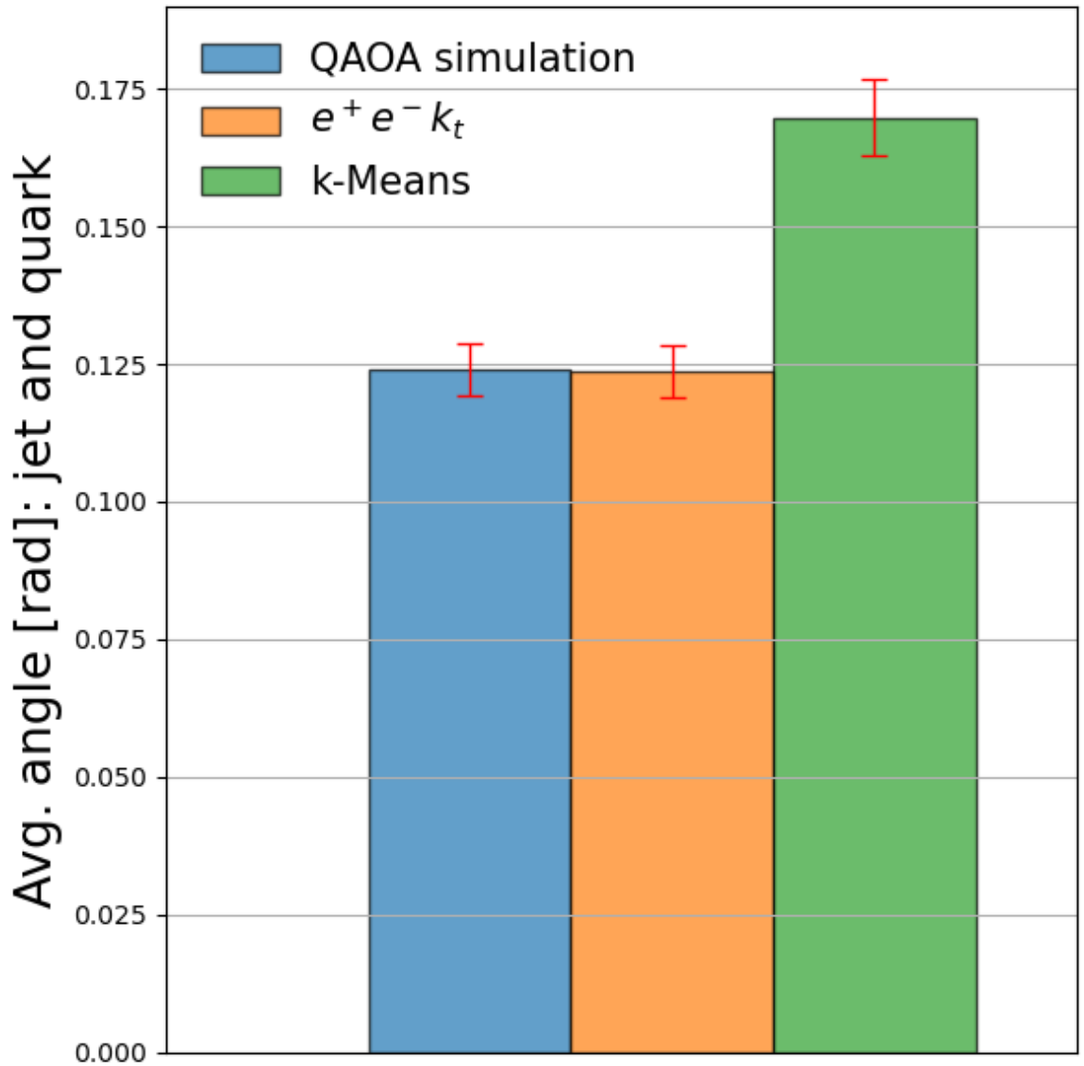
jet clustering performance v.s. k (k-regular graph)



A higher k-value signifies that a node is linked to more nodes within the graph, indicating a more complex complied quantum circuit.

Conversely, a smaller k-value may result in suboptimal performance due to inadequate linkage between particles belonging to distinct jets, thus failing to accurately cluster particles from separate jet.

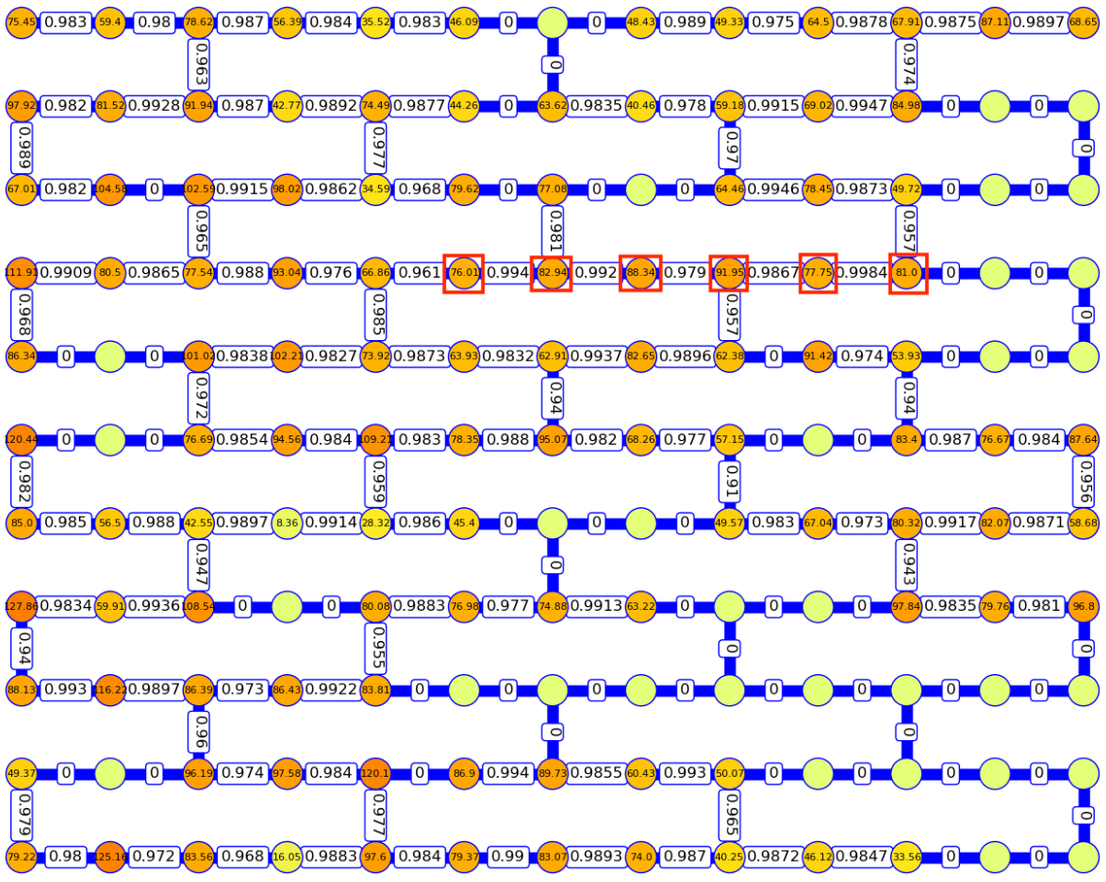
compare jet clustering performance obtained by QAOA, $e^+e^-k_t$ and k-Means



depth=3
k=7

This comparison highlights the potential of the QAOA in the jet clustering problem.

the Baihua quantum processor in BAQIS

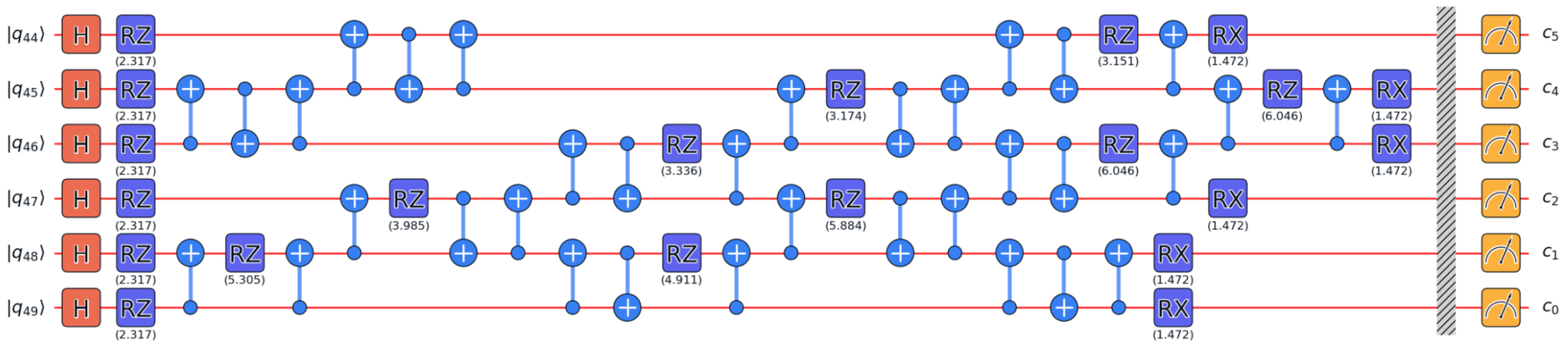


- 123 operational qubits
- relaxation time T_1 of 73.994 us
- dephasing time T_2^* of 29.02 us
- fidelity of single-qubit gate 99.9%
- fidelity of two-qubit gates (CZ) 98.9%

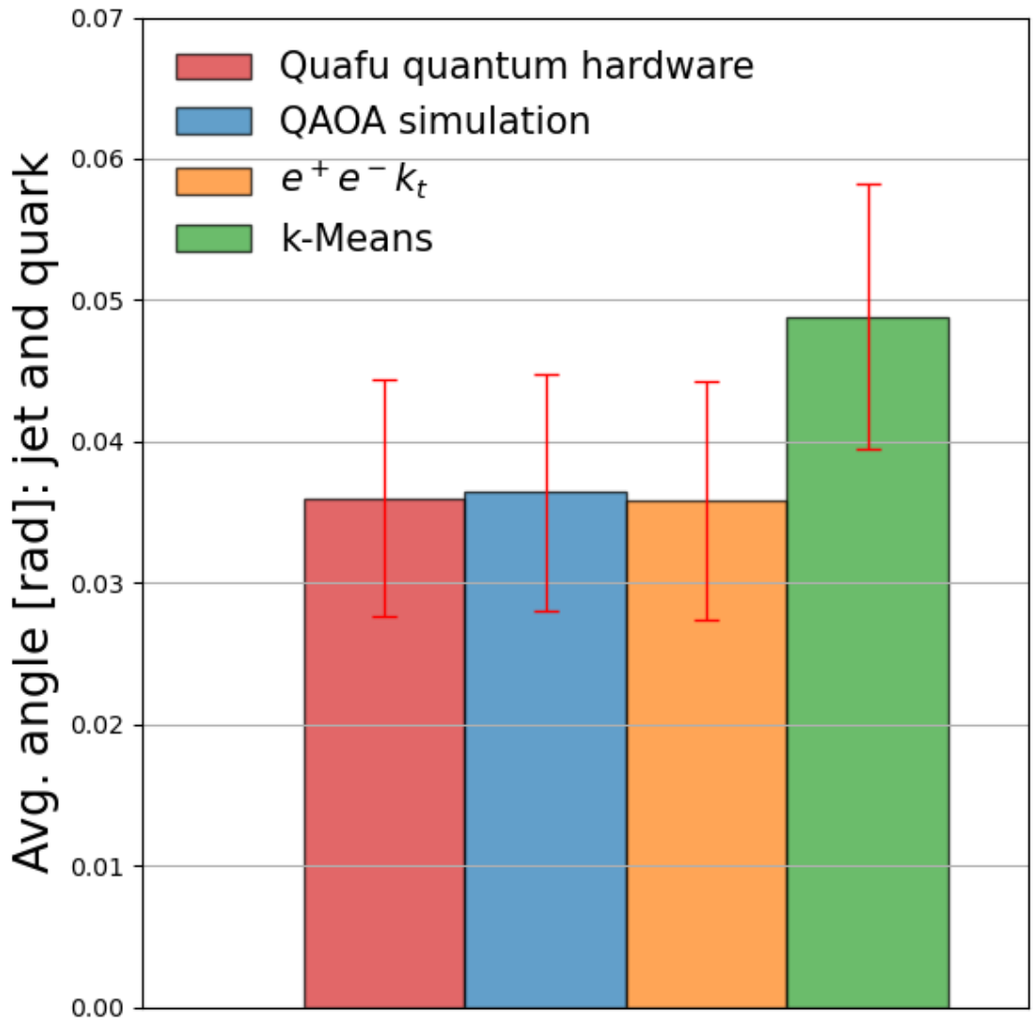
Conduct on the Baihua processor

$e^+e^- \to ZH \to \nu\bar{\nu}s\bar{s}$ with 6 particles

compiled QAOA circuit on Baihua processor reaches a depth of 26 with 34 CNOT gates and 27 single-qubit gates.



180 $e^+e^- \to ZH \to \nu\bar{\nu}s\bar{s}$ with 6 particles



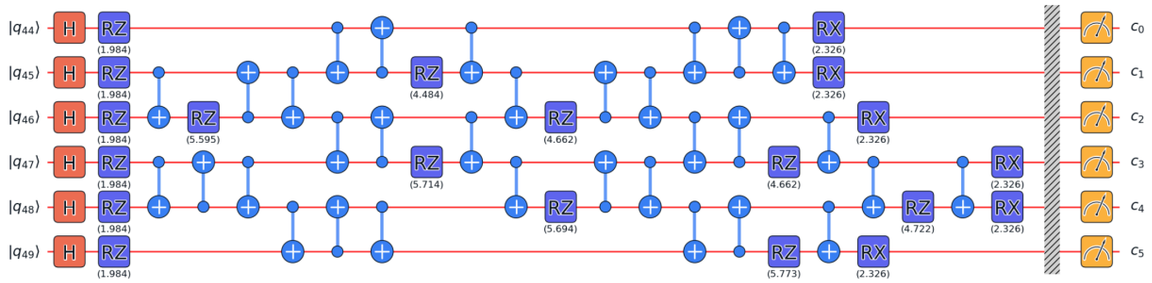
For this small-sized problem, the quantum hardware can achieve similar performance to a noiseless quantum computer simulator.

Summary

- The rapid development of quantum algorithms and hardware devices enables the execution of small-scale but representative applications on quantum computers.
- We apply a quantum combinatorial optimization algorithm, QAOA, on jet clustering. With small-sized Higgs->ss samples, QAOA running on quantum simulator and quantum hardware can reach the similar performance to classical jet clustering algorithm, ee_kt.
- The current generation of quantum processors faces several bottlenecks, including qubit coherence times, error rates, and connectivity, that must be overcome to realize their full potential.

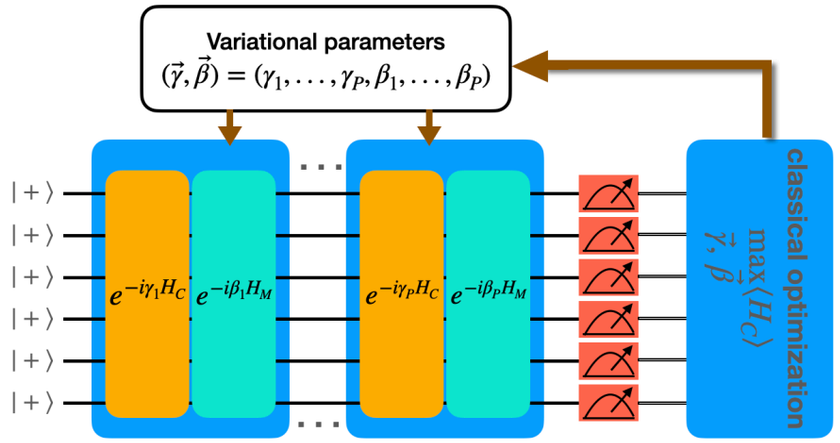
Many thanks !

Backup



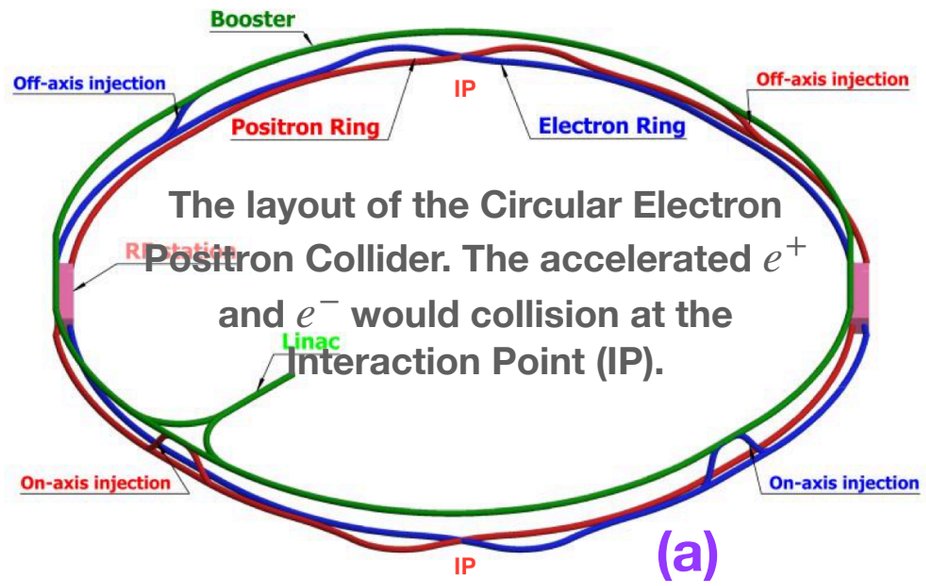
(f)

compiled quantum circuit



jet clustering with QAOA

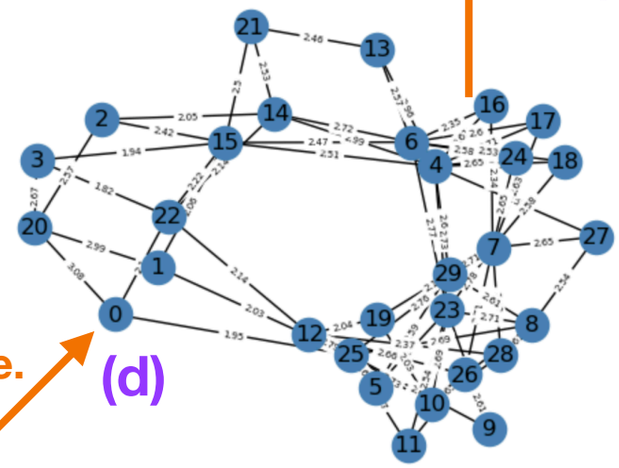
(e)



The layout of the Circular Electron Positron Collider. The accelerated e^+ and e^- would collision at the Interaction Point (IP).

(a)

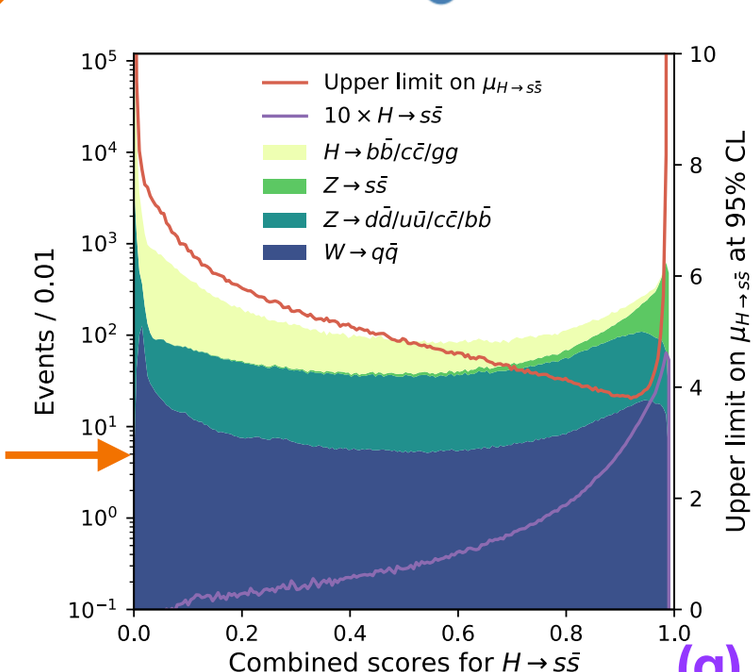
See the event as a graph, where particles as nodes and angle of two particles as edge.



(d)

(b)

(c)



(g)

The collision of e^+ and e^- can generate quarks, gluons, and leptons.

The quarks and gluons would immediately transform into collimated particle sprays known as jets.

With jet clustering and other techniques, the related physics analyses can be performed.

Introduction of adiabatic quantum computation and the evolution from AQC to QAOA

AQC : a theoretical framework

components : 1, the driver Hamiltonian (H_D) encodes some quantum state that is easy to prepares its ground state 2, the problem Hamiltonian (H_P) encodes a quantum state we are interested in as its ground state

the idea underline the AQC : we start with a ground state that is easy to prepare and wish to end up with the quantum state we are interested in. This transition is accomplished via the adiabatic theorem, which states that a system in the ground state of some Hamiltonian will remain in the ground state if the Hamiltonian is changed slowly enough.

the process of AQC : 1, define the Hamiltonian: $H(t) = (1 - s(t))H_D + s(t)H_P$ and let our quantum system evolve under it. Unfortunately, time evolution under this time-dependent Hamiltonian involves very messy integral that is hard to evaluate : $U(t) = \tau e^{\frac{-i}{h}\int_o^t H(T)dT}$

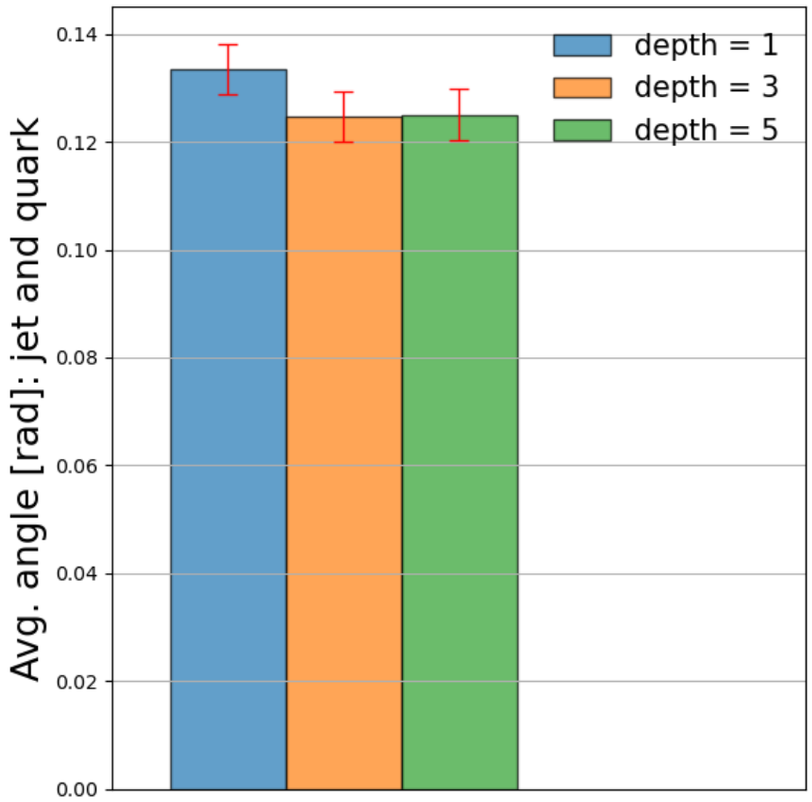
2, **We discretize $U(T)$ into intervals of Δt small enough that the Hamiltonian is approximately constant over each interval.**

3, Let $U(b, a)$ represent time evolution from time a to time b

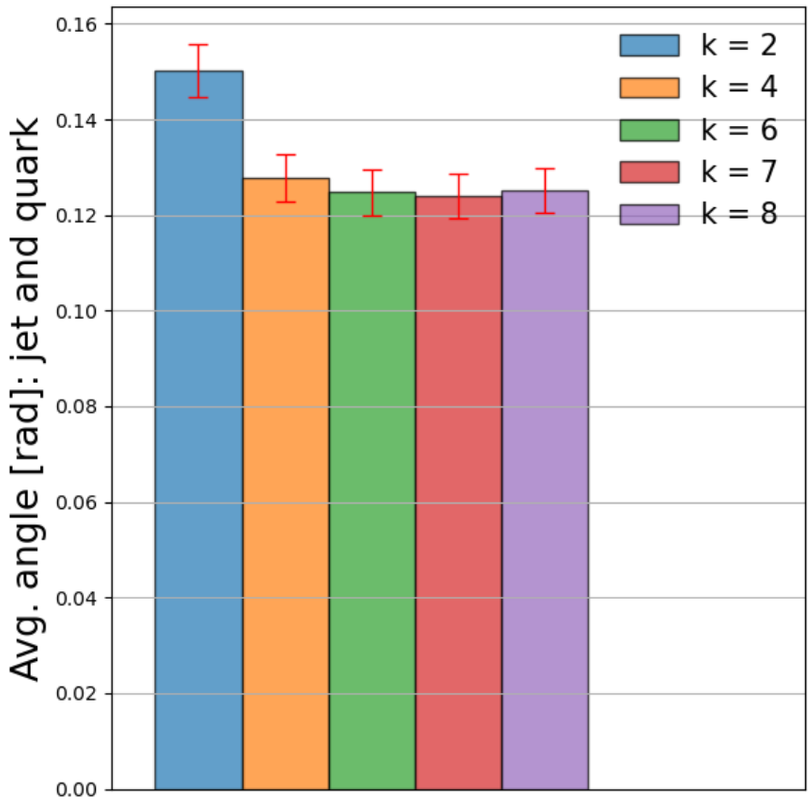
$$U(T,0) = U(T, T - \Delta t)U(T - \Delta t, T - 2\Delta t) \dots U(\Delta t,0) = \prod_{j=1}^{P} U(j\Delta t, (j-1)\Delta t) \approx \prod_{j=1}^{P} e^{-iH(j\Delta t)\Delta t}$$

since $H(j\Delta t) = (1 - s(j\Delta t))H_D + s(j\Delta t)H_P$ **we get** $U(T,0) \approx \prod_{j=1}^{P} e^{-i(1-s(j\Delta t))H_D\Delta t} e^{-is(j\Delta t)H_P\Delta t}$

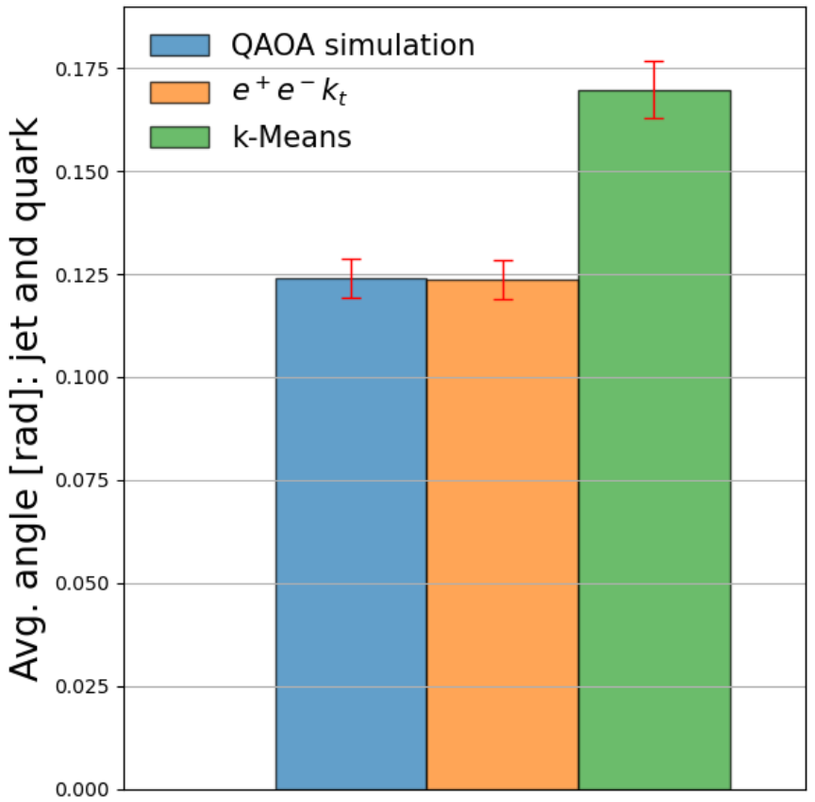
Thus we can approximate AQC by repeatedly letting the system evolve under H_P for some $s(j\Delta t)\Delta t$ and then H_D for some small $(1 - s(j\Delta t))\Delta t$


depth = 1
depth = 3
depth = 5
Avg. angle [rad]: jet and quark
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14

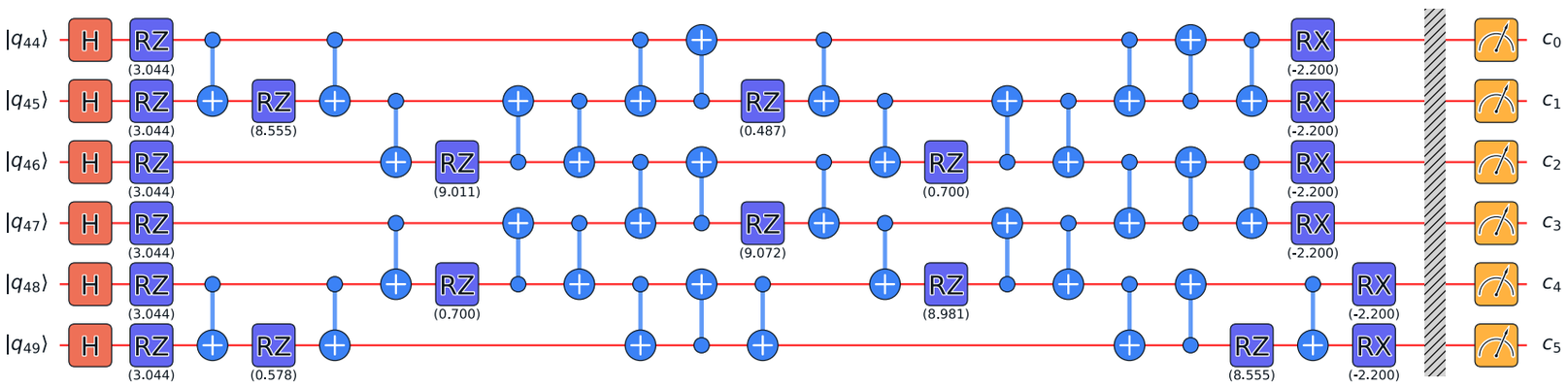
(a)


k = 2
k = 4
k = 6
k = 7
k = 8
Avg. angle [rad]: jet and quark
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16

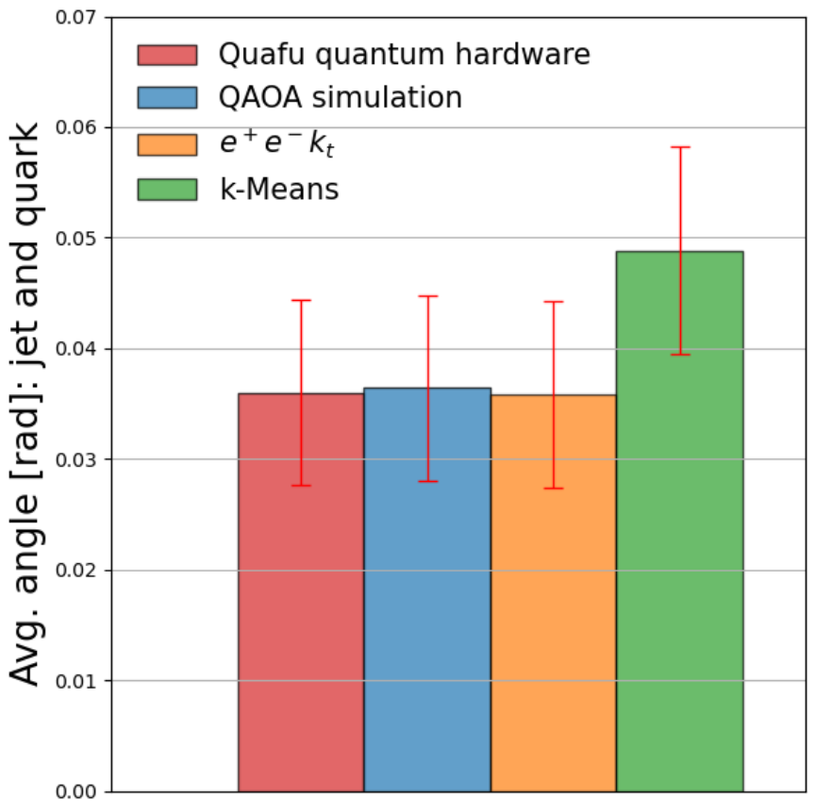
(b)


QAOA simulation
$e^+e^-k_t$
k-Means
Avg. angle [rad]: jet and quark
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175

(c)


$|q_{44}\rangle$
$|q_{45}\rangle$
$|q_{46}\rangle$
$|q_{47}\rangle$
$|q_{48}\rangle$
$|q_{49}\rangle$
H
RZ
RX
(3.044)
(8.555)
(0.487)
(9.011)
(0.700)
(9.072)
(8.981)
(0.578)
(-2.200)
c_0
c_1
c_2
c_3
c_4
c_5

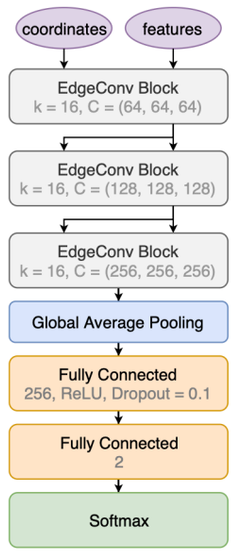
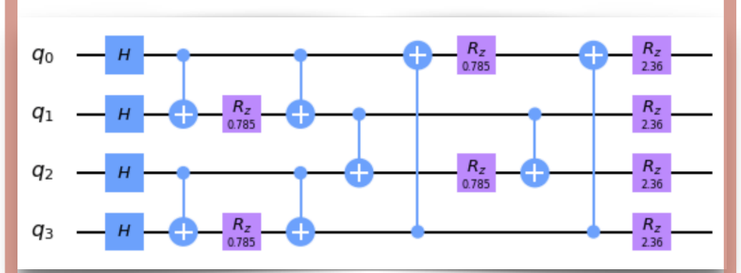
(d)


Quafu quantum hardware
QAOA simulation
$e^+e^-k_t$
k-Means
Avg. angle [rad]: jet and quark
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07

(e)

VQA v.s. ML

parameters update

	input	construct	output	cost function
ML	**features of samples** - [part_isElectron, null] - [part_isMuon, null] - [part_isNeutralHadron, null] - [part_isPhoton, null] - [part_d0, null] - [part_d0err, 0, 1, 0, 1] - [part_dz, null] - [part_dzerr, 0, 1, 0, 1] - [part_deta, null] - [part_dphi, null] - [part_pt_log, -1.5, 1.0] - [part_e_log, -0.687, 1.0] - [part_logptrel, -4.7, 1.0] - [part_logerel, -4.473, 1.0] - [part_deltaR, 2.1, 2.3] - [part_charge, null] - [part_isChargedHadron, null]	**model** 	**prediction** **classification or regression** **y’**	**f(y, y’)**
VQA	**initial state** $\|\psi(0)\rangle$	**quantum circuit** 	**final state** $\|\psi(\theta)\rangle$	$E(\theta) = \langle\psi(\theta)\|H\|\psi(\theta)\rangle$