Black Hole Ringdown

杨桓
清华大学天文系

Collaborators: Beatrice Bonga, Neev Khera, Sizheng Ma

高能理论论坛， 中科院高能物理研究所，2024年5月29日

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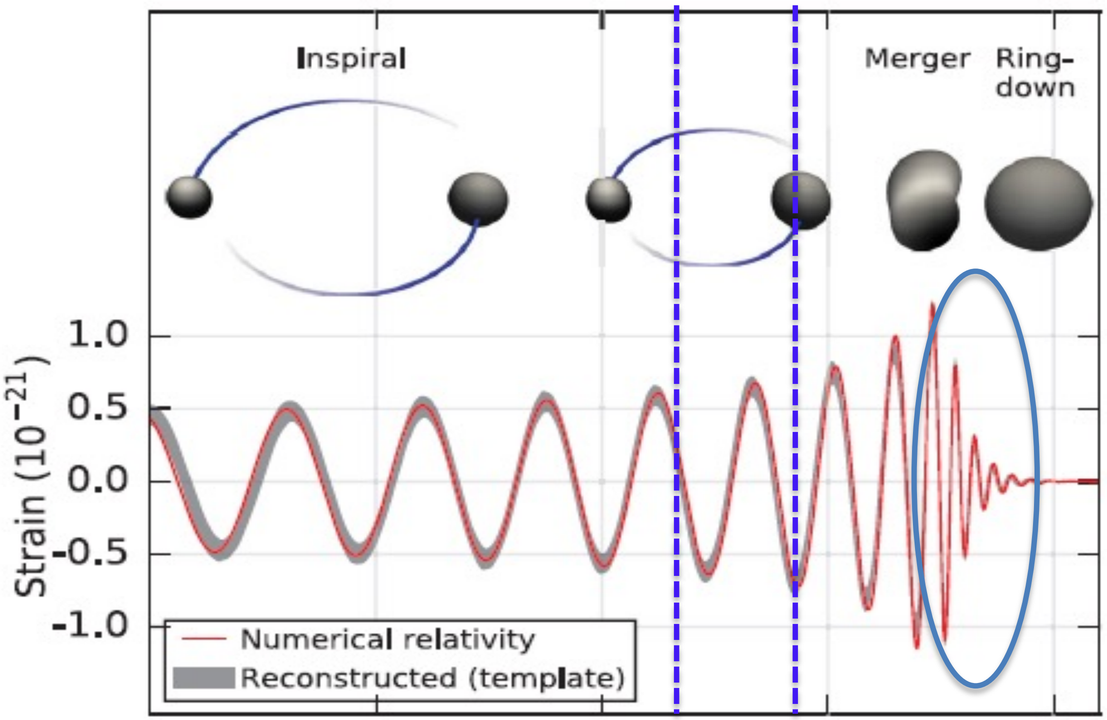

Post-Newtonian Supercomputer BH perturbation

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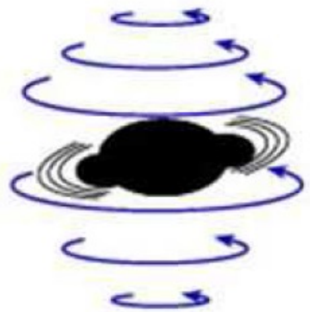
- A brief review of the history of black hole perturbation.
- Development of ringdown theory and data analysis in the past decade.
- Ringdown frontier: nonlinear modes.
- Beyond black hole spectroscopy and future prospects.



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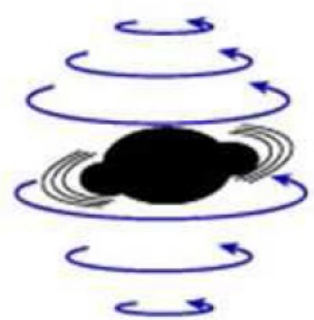


How to describe the ringing of a black hole

- The ringdown of a bell (perturbation of the bell couples to the sound wave in the air)

- Black hole ringdown requires the understanding of dynamics of spacetime perturbations near a black hole.

The perturbations of a Schwarzschild black hole

PHYSICAL REVIEW VOLUME 108, NUMBER 4 NOVEMBER 15, 1957

Stability of a Schwarzschild Singularity

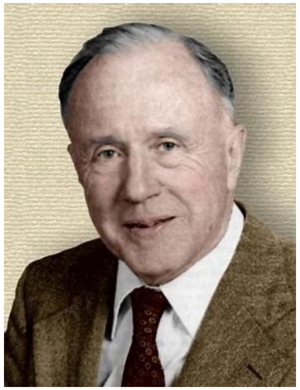
TULLIO REGGE, *Istituto di Fisica della Università di Torino, Torino, Italy*

AND

JOHN A. WHEELER, *Palmer Physical Laboratory, Princeton University, Princeton, New Jersey*

(Received July 15, 1957)

It is shown that a Schwarzschild singularity, spherically symmetrical and endowed with mass, will undergo small vibrations about the spherical form and will therefore remain stable if subjected to a small nonspherical perturbation.



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VOLUME 24, NUMBER 13 PHYSICAL REVIEW LETTERS 30 MARCH 1970

EFFECTIVE POTENTIAL FOR EVEN-PARITY REGGE-WHEELER GRAVITATIONAL PERTURBATION EQUATIONS*

Frank J. Zerilli

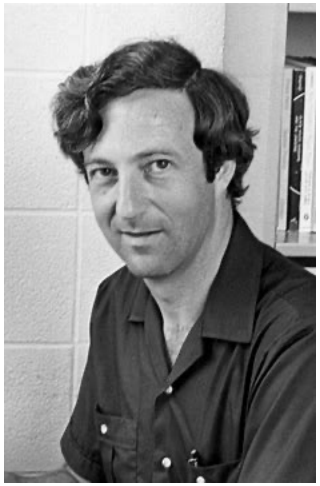
Physics Department, University of North Carolina, Chapel Hill, North Carolina 27514

(Received 29 January 1970)

The Schrödinger-type equation for odd-parity perturbations on a background geometry has been extended to the even-parity perturbations. This should greatly simplify the analysis for calculations of gravitational radiation from stars and from objects falling into black holes.

The perturbations of a rotating black hole

THE ASTROPHYSICAL JOURNAL, **185**: 635–647, 1973 October 15



PERTURBATIONS OF A ROTATING BLACK HOLE. I. FUNDAMENTAL EQUATIONS FOR GRAVITATIONAL, ELECTROMAGNETIC, AND NEUTRINO-FIELD PERTURBATIONS*

SAUL A. TEUKOLSKY†
California Institute of Technology, Pasadena
Received 1973 April 12

ABSTRACT

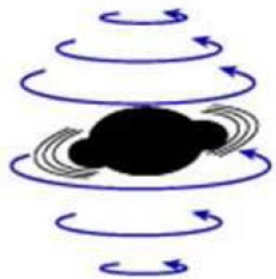
This paper derives linear equations that describe dynamical gravitational, electromagnetic, and neutrino-field perturbations of a rotating black hole. The equations decouple into a single gravitational equation, a single electromagnetic equation, and a single neutrino equation. Each of these equations is completely separable into ordinary differential equations. The paper lays the mathematical groundwork for later papers in this series, which will deal with astrophysical applications: stability of the hole, tidal friction effects, superradiant scattering of electromagnetic waves, and gravitational-wave processes.
Subject headings: black holes — gravitation — neutrinos — relativity — rotation

$$\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\varphi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\psi}{\partial\varphi^2}$$
$$- \Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{\partial\psi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) - 2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\psi}{\partial\varphi}$$
$$- 2s\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\psi}{\partial t} + (s^2\cot^2\theta - s)\psi = 4\pi\Sigma T\,.$$

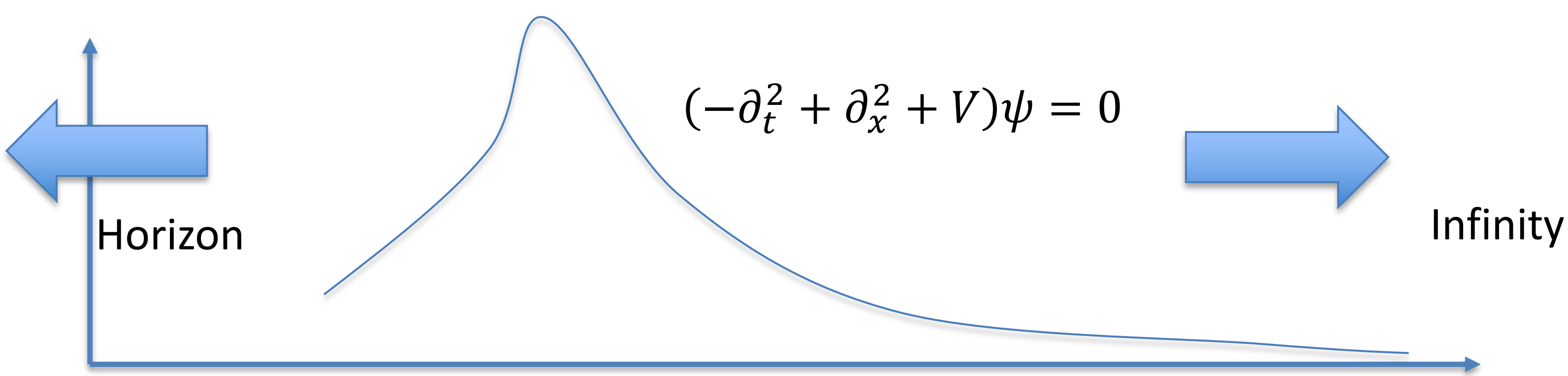
Solving for the modes

- Black holes are like imperfect cavities.

$$h(t) \propto e^{-i\omega t}, \quad \omega = \omega_R - i\omega_I$$

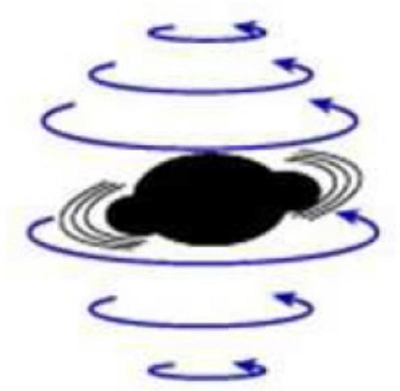


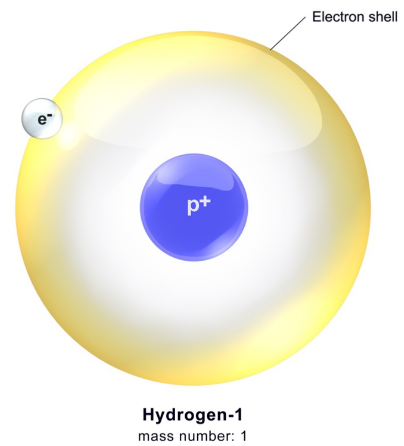
- These decaying oscillations can be decomposed into eigenmodes: black hole quasinormal modes. Chandrasekhar and Detweiler [1975] first solved for a few modes of Schwarzschild black hole.



Solving for the modes (rotating black holes)

- For rotating black holes, the BH oscillation modes (quasinormal modes) are characterized by (l,m,n): angular, azimuthal and radial node number.





- A powerful numerical scheme invented by E. W. Leaver in 1980s.

Proc. R. Soc. Lond. A **402**, 285-298 (1985)
Printed in Great Britain

An analytic representation for the quasi-normal modes of Kerr black holes

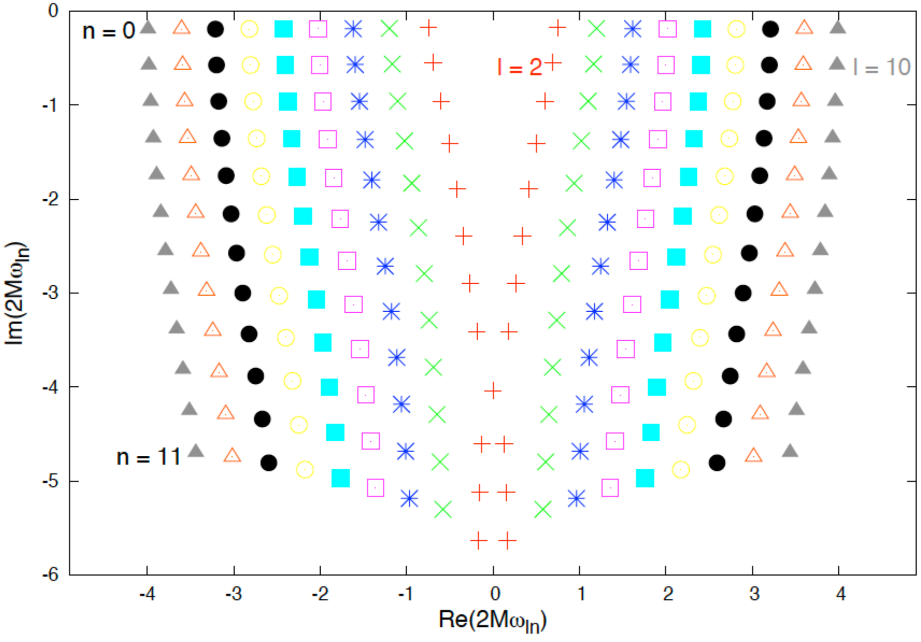
BY E. W. LEAVER
Department of Physics, University of Utah, Salt Lake City, Utah 84112, *U.S.A*

Black hole spectroscopy

- The oscillation mode spectrum represents a type of fundamental properties of black holes within GR or modified gravity theories.
- Measuring the BH spectrum -> BH spectroscopy: a way to probe BH spacetime and test GR.

Schwarzschild QNM

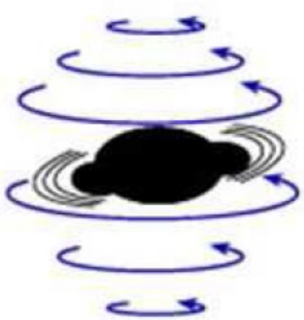
Dolan & Ottewill 2011



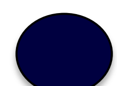
No-hair theorem and ringdown modes

- No Hair Theorem : astrophysical black holes are characterized by their mass M and spin J , so should their (linear) oscillation modes.

40 years



$\sim 0.1\mathrm{ms} \times \frac{M}{M_\odot}$



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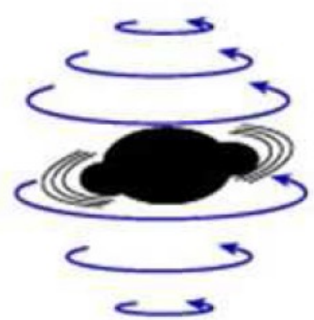
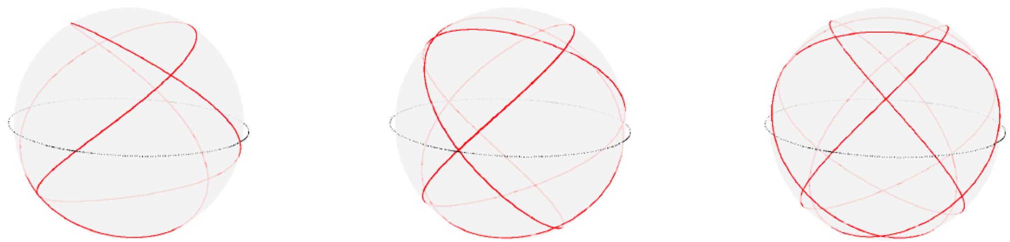


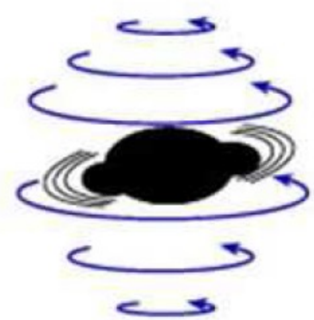
The development of WKB theory

- Intuitively, the modes can be viewed as waves propagating around the BH. In the short-wavelength limit, they have one-to-one correspondence to the geometric rays propagating in the photon ring/spherical photon orbit. Mode frequency is related to the orbital frequencies.



H. Yang *et al.*, PRD 2012

- It also implies a relation between the photon ring shape that can be measured from EHT and modes that can be measured with LISA.



H. Yang, PRD 2022

Near-extremal black holes

- Astrophysical black holes can spin up by accreting materials. According to the thin-disk model, $a \leq 0.998$ [Novikov & Thorne 1973]

Gargantua: a = $1\text{-}10^{-14}$

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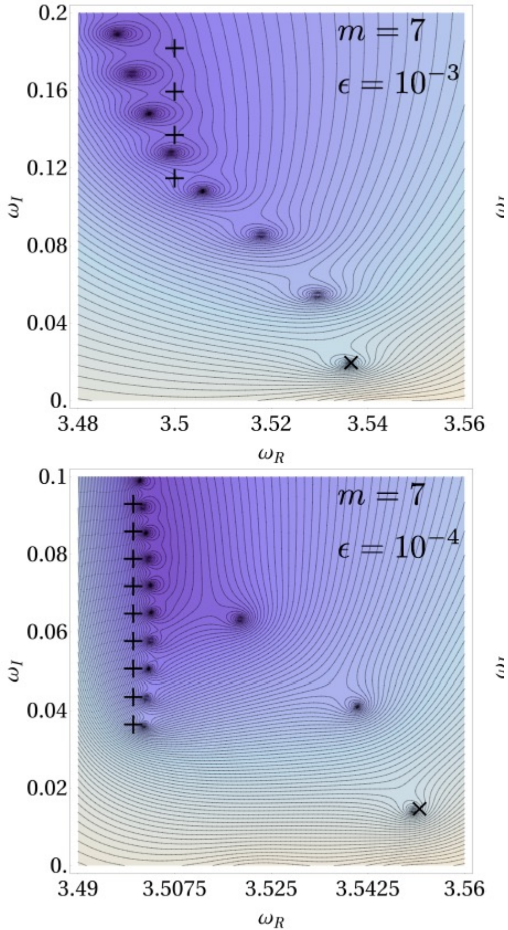
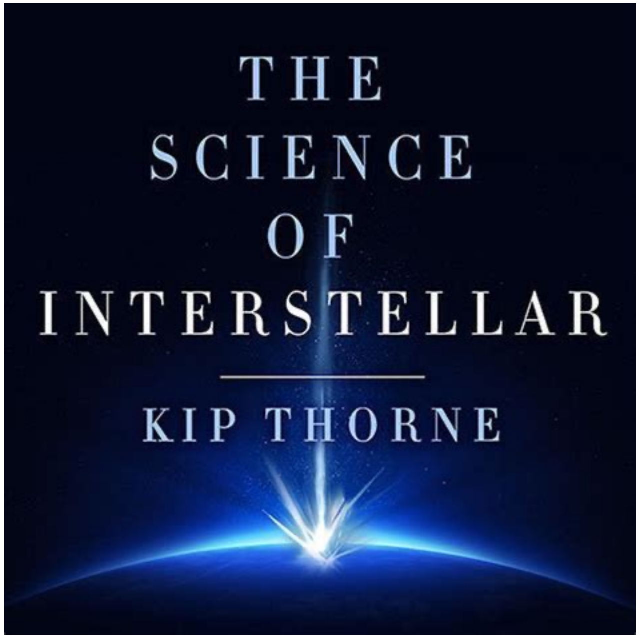
- Mode frequency is given by [S. Hod, PRD 2008] [H. Yang *et al.*, PRD 2012,2013]



$$\omega_R \approx \frac{m}{2}, \quad \omega_I \propto [n + O(1)]\sqrt{1-a}$$

$$\epsilon = 1 - a$$



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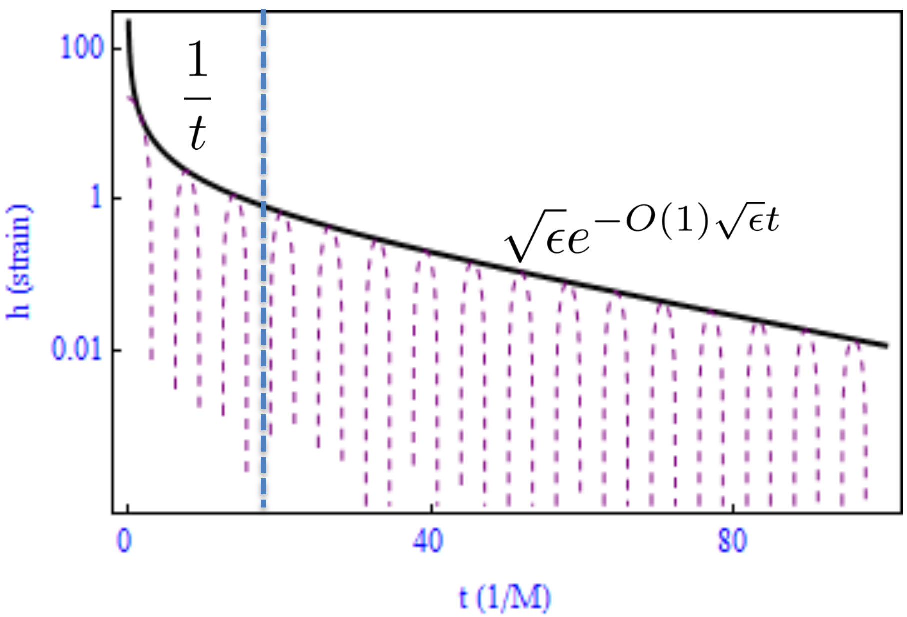
- Collective excitation of modes: power-law decay at intermediate times.

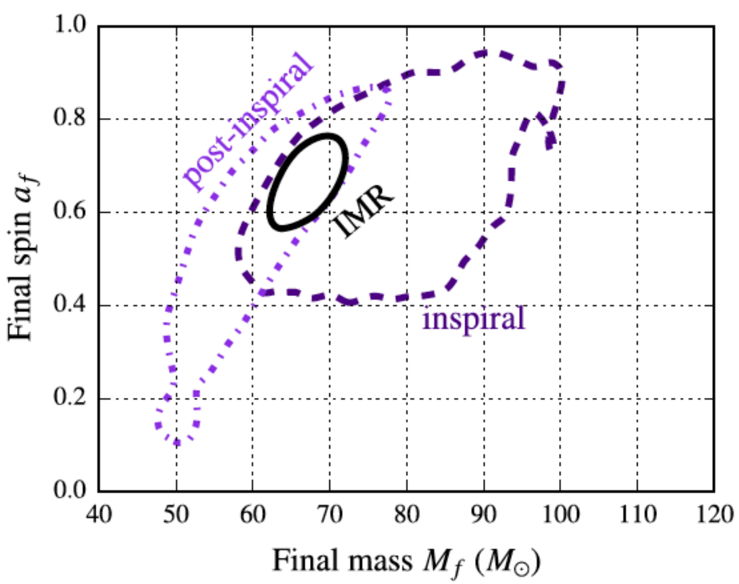


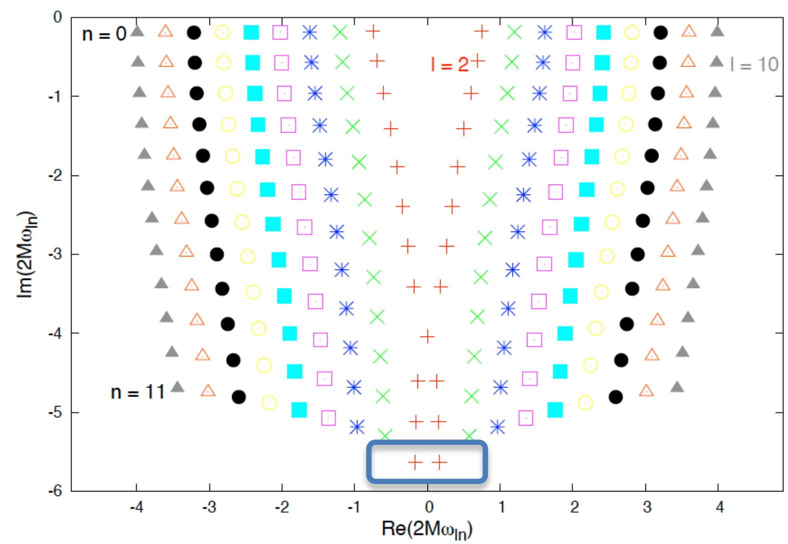
$$h \sim \sqrt{\epsilon} \sum_{n=0}^{\infty} e^{-[n+O(1)]\sqrt{\epsilon} t} \sim \sqrt{\epsilon} \frac{e^{-O(1)\sqrt{\epsilon} t}}{1 - e^{-\sqrt{\epsilon} t}}$$

H. Yang *et al.*, PRD 2013

The analysis of GW150914

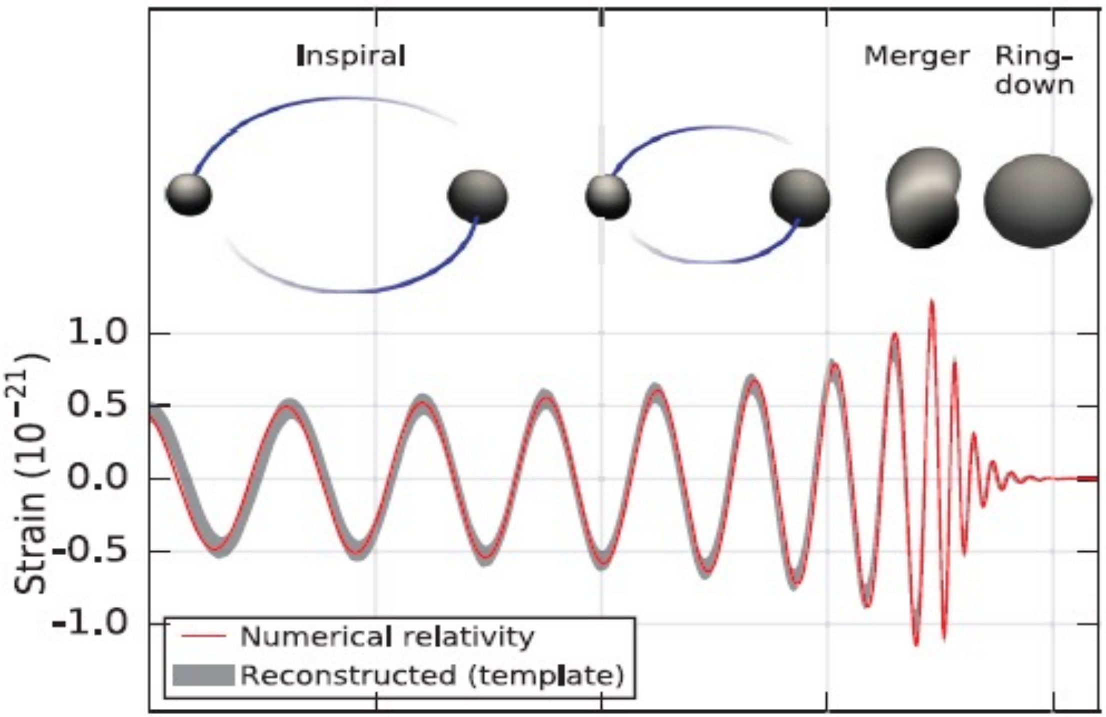
- The ringdown SNR for GW 150914 is about 8, the dominant mode (22) with lowest damping rate is detected.
- Inspiral-merger-ringdown test is performed using the inferred final black hole mass and spin [LVC PRL 2016]





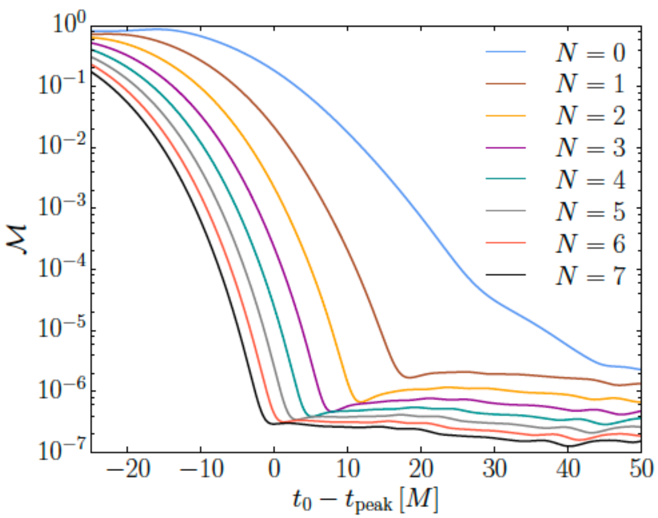
The search for sub-dominant modes

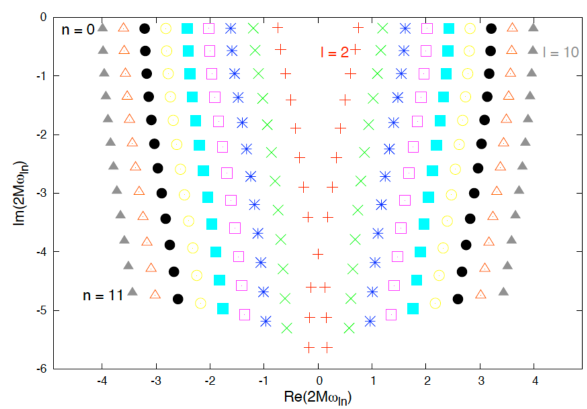
- Other sub-dominant QNMs usually have much lower SNR.
- The ringdown SNR depends on the starting time of the fitting: earlier starting time means larger SNR. However, choosing earlier starting time may face contamination from nonlinear effects, etc.

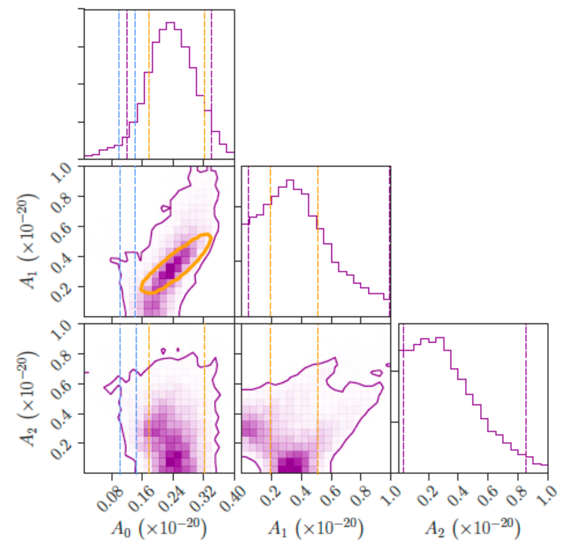


The debate on overtone detection

- There are claims about higher overtone might be important in early ringdown, e.g. [Geisler et al. 2019]. Later studies claimed to find the first overtone in GW150914 by fitting from the peak [e.g. Isi et al. 2019].



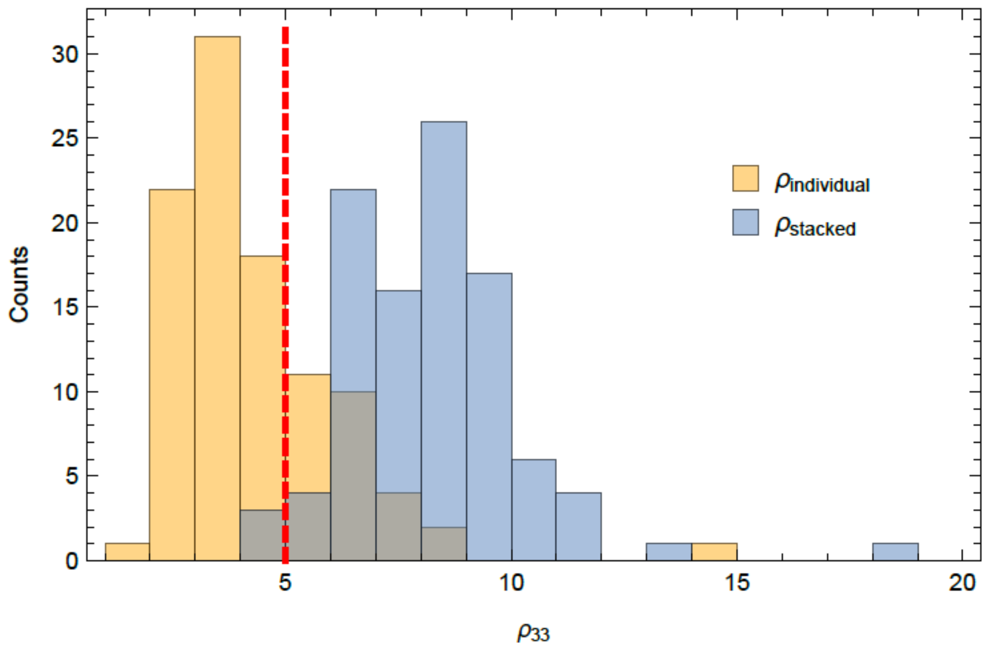




- Other studies questioned the data analysis method, i.e. the sensitive dependence on the starting time [Cotesta et al. 2022]. It is inevitable that other effects are fitted into QNMs for early starting time.

Stacking Method for Black hole Spectroscopy

- The statistical significance of subdominant mode may be amplified by combining information from a set of events.
- How to stack events?: Use the information from inspiral waveform measurement and estimate the QNM phase and frequency. Then stack them coherently.
- A factor of 2-3 amplification for the SNR of 33 mode!



One–year observation with full Advanced LIGO sensitivity

H. Yang *et al.*, PRL 2017

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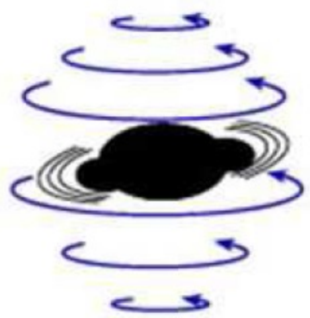
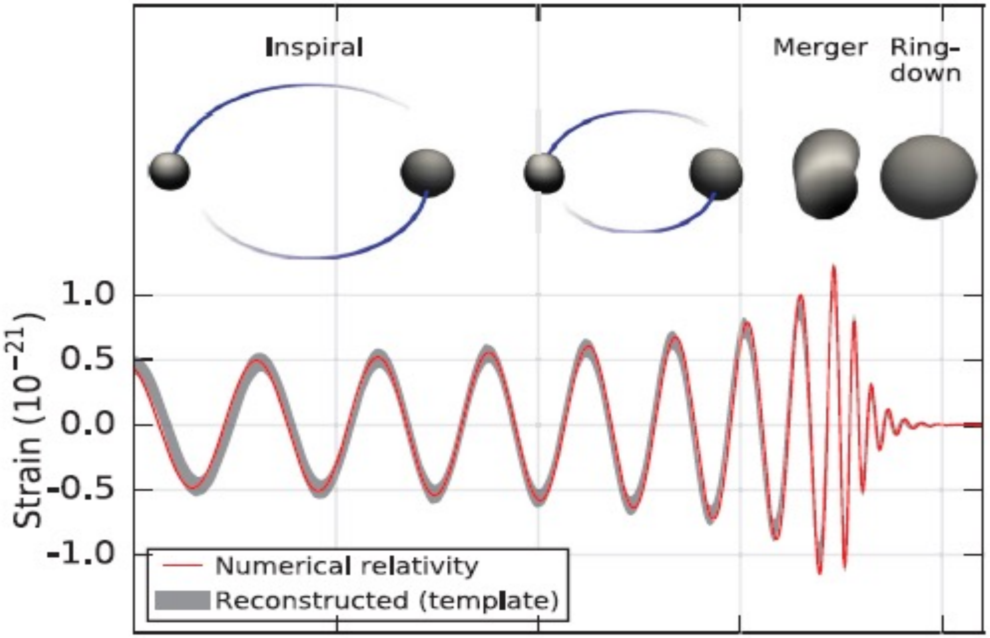
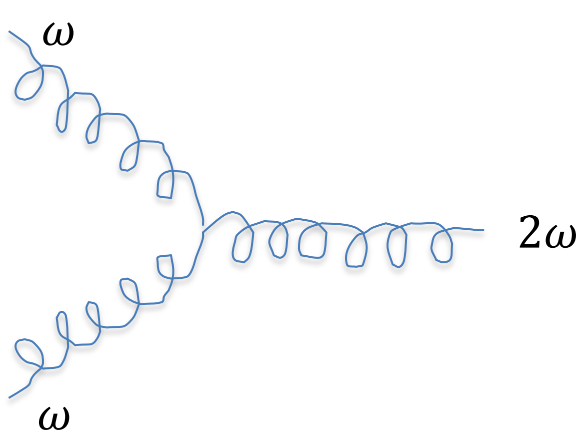


Nonlinearities in the ringdown

- Nonlinearities are naturally expected in the early part of the ringdown signal.



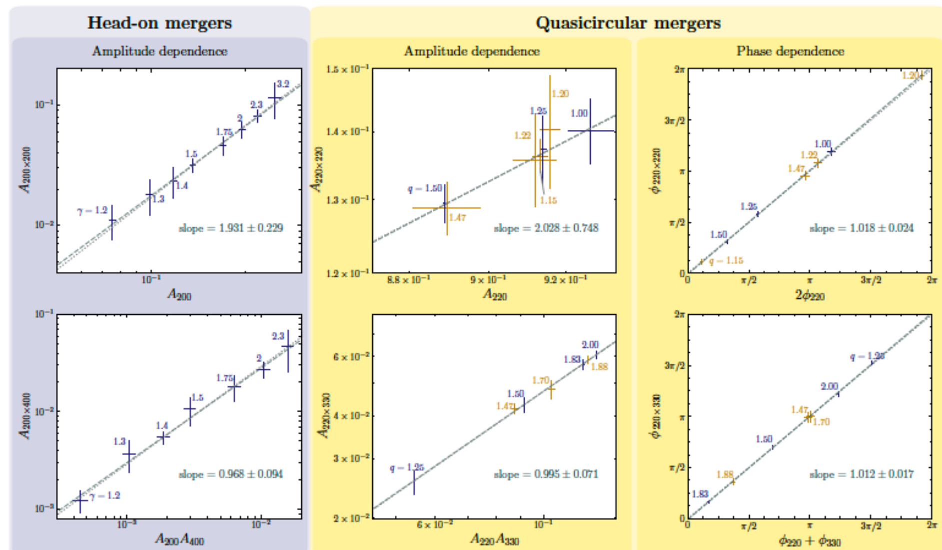


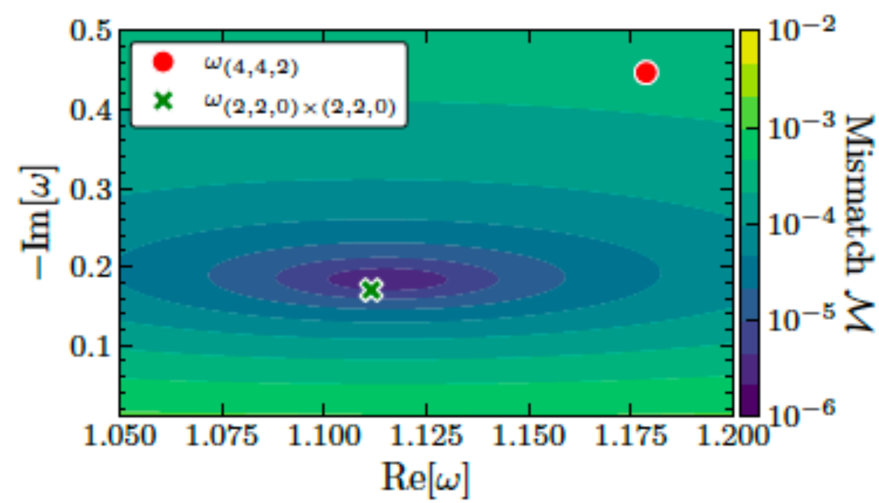
- One interesting form of nonlinearity shows up as quadratic quasinormal mode: mode (ω) $\times$ mode (ω) $\rightarrow$ mode (2ω). The amplitude and phase of the quadratic mode is uniquely determined by the underlying linear mode.

Quadratic mode in the GW

- Quadratic quasinormal modes are indeed found in numerical simulations [Mitman et al., Cheung et al. PRL 2022]:

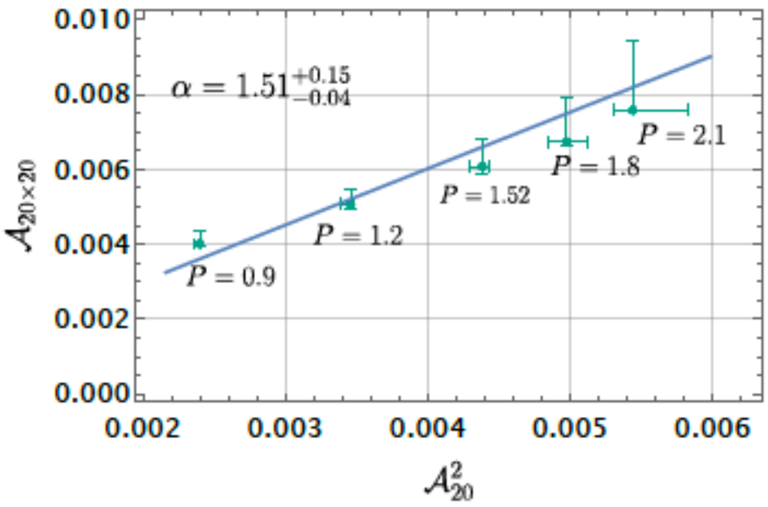
- Using linear modes plus additional modes for fitting. Checking the quadratic dependence in amplitudes and check the frequency of the additional mode.

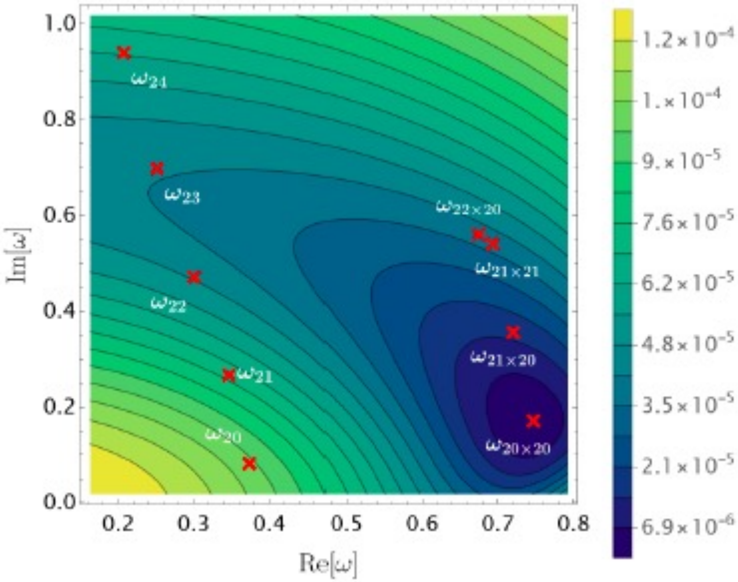




Quadratic mode on the horizon

- Black hole horizon geometry may be described by the shear and other quantities.
- We have searched for quadratic modes in the horizon shear and indeed we found them [Khera,.. HY, PRL 2023]:

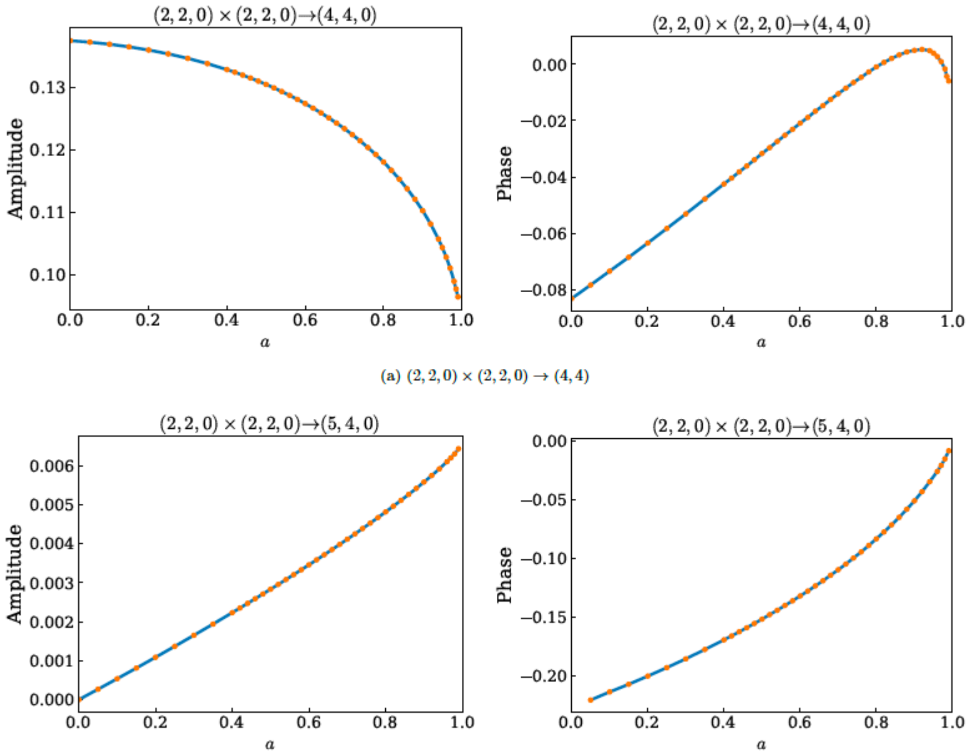




- First time nonlinear deformation of BH horizon is quantitatively described.

The quadratic mode amplitude

- The quadratic mode amplitude and phase can be computed using second-order Teukolsky equation [S. Ma, HY, 2024]:



- This result will be useful for future nonlinear mode detections.

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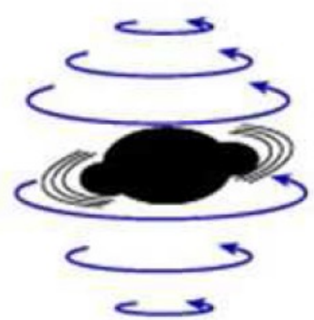

Beyond black hole spectroscopy

- Black hole spectroscopy targets the QNM spectrum of the final black hole, where the ringdown signal contains more information: the amplitude and phase of modes, non-mode content, various nonlinearities, tail, etc.

- The nonlinearities tend to matter in the early inspiral, where tail matter more in the late inspiral.

- Do we want to give up all these additional info?

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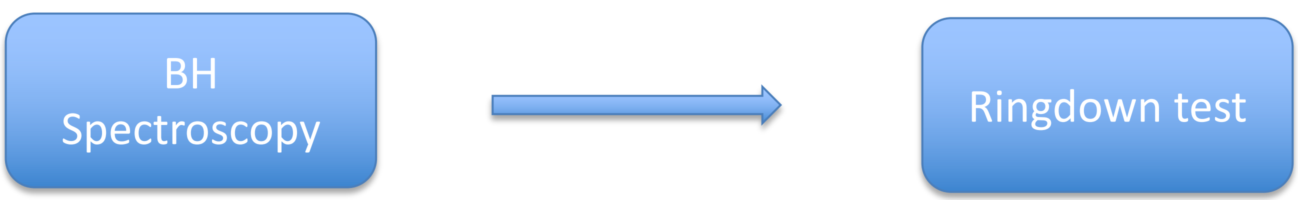
- Do we want to give up all these additional info?

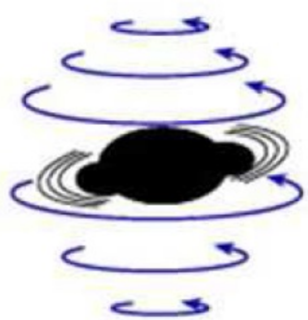
- We may want to obtain a complete ringdown description instead!



Checklist

- Linear effects: quasinormal modes (spectrum, amplitude and phase), source terms ("direct part" of the signal, horizon modes), tails.

- Nonlinear effects: nonlinear quasinormal modes (amplitude and phase), drifts of background in the early inspiral, new mode excitations because of nonlinear couplings, gravitational wave memory, etc.



Conclusion

- We have come a long way in understanding the black hole ringdown. Now we have data to play with!

- The dominant mode is detected, but other modes are much weaker.

- To boost the SNR, we may choose earlier starting time of fitting, but that comes with various systematics, including nonlinear effects. We may also stack events.

- In the future, we want to go beyond black hole spectroscopy and try to understand the full ringdown signal.