EFT approach for gravitational binaries

Saturday, February 1, 2025 8:56 PM

References:

- Porto 1601.04914 (Review)
- Levi 1807.01699 (Review, focusing on spin)
- Goldberger, Rothstein, hep-th/0409156 (NRGR)

Basic idea of EFT:

If there exist a hierachy of scales.

↓

Separate UV from IR.

$$= \frac{1}{p^2 - M^2} = -\frac{1}{M^2}\left(1 + \frac{p^2}{M^2} + \cdots\right)$$

↓

In IR, Simpler dynamical Dof.

Universal description of interactions

Standard Model ⟷ GUT / Quantum gravity.

Newtonian mechanics ⟷ GR.

UV physics ⟶ $\sum_i C_i(\Lambda)\, O_i$

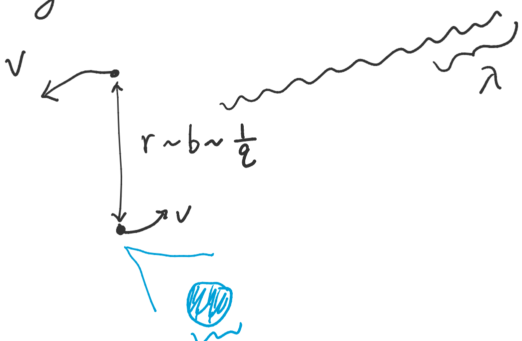
↑ Wilson coefficients; encode UV physics

↑ Higher dimensional operators, universal

In the context of gravitational binaries

- What are the relevant scales?
- What are the relevant Dof?
- How to integrate out certain Dof?

Gravitational binary



$r_s = GM$.

Bound system: $v^2 \sim \frac{GM}{r}$

Unbound scattering: $v^2 < 1$

$$\underbrace{\lambda_{\text{Compton}} \ll r_s}_{\text{classical}} \ll b \qquad \overbrace{r_s \ll b}^{\text{Point particle approximation is valid}}$$

$\underbrace{\qquad\qquad}_{\text{Classical}}$

$r_s \ll b \rightarrow \frac{Gm}{b} \sim Gmq \ll 1 \rightarrow$ Gmq is the small parameter in the classical perturbative expansion.

$\lambda_{\text{Compton}} \ll r_s \rightarrow Gm^2 \gg 1 \rightarrow$ usual loop expansion in QFT assumes $Gm^2 \ll 1$

* Classical physics is encoded in loop amplitudes
* Re-summation.

$\lambda_{\text{Compton}} \ll b \rightarrow \frac{q}{m} \ll 1 \rightarrow$ Bohr's correspondence principle. ($J \gg \hbar \rightarrow$ classical)

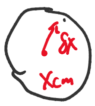
(or $J = mvr \gg \hbar$) "classical region" for gravitons.

Relevant DOF

Gravitons: potential $q, \ell \sim (v|\vec{q}|, \vec{q})$ } soft region $(\epsilon|\vec{q}|, \vec{q})$

radiation $q, \ell \sim (v|\vec{q}|, v\vec{q})$

hard $q, \ell \sim (m, \vec{p})$

BH / NS: 

Integrate out hard graviton, internal dynamics $\rightarrow$ point particle description

$$g_{\mu\nu} = g^L_{\mu\nu} + g^S_{\mu\nu}$$
$$x^\mu = x^\mu_{cm} + \delta x^\mu.$$

$S_{EH} = -\frac{1}{16\pi G}\int d^4x \sqrt{g}\, R.$

$$\int Dg^S_{\mu\nu}\, D\delta x^\mu \exp\left[i S_{EH}(g^L_{\mu\nu} + g^S_{\mu\nu}) + i S_{\text{matter}}(x^\mu_{cm} + \delta x^\mu, g^L_{\mu\nu} + g^S_{\mu\nu})\right] = \exp\left[i S_{EH}[g^L_{\mu\nu}] + i S_{pp}[x^\mu_{cm}, g^L_{\mu\nu}]\right]$$

* world-line EFT: $S_{pp} = -m\int d\tau + \underbrace{\sum_i C_i(r_s)\int d\tau\, O_i(\tau)}_{\text{Finite size effect}}$

$-m\int d\tau \rightarrow -m\int dt \sqrt{g_{\mu\nu}(x(t))\,\dot{x}^\mu(t)\,\dot{x}^\nu(t)}$

Example, $C_E\int d\tau\, E_{\mu\nu}E^{\mu\nu} + C_B\int d\tau\, B_{\mu\nu}B^{\mu\nu}$, $E_{\mu\nu} \rightarrow R_{\mu\alpha\nu\beta}u^\alpha u^\beta$, $B_{\mu\nu} \rightarrow {}^*R_{\mu\alpha\nu\beta}u^\alpha u^\beta$

* Field theory: $S_{pp} = \int d^4x \left(\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 + C_E R_{\mu\alpha\nu\beta}R^{\mu\rho\nu\delta}\partial^\alpha\partial_\rho\phi\, \partial^\beta\partial_\delta\phi + \cdots\right)$

Bottom-up construction of one-particle EFT

symmetries: general coordinate transformation

world-line reparameterization

symmetries : general coordinate transformation
world-line reparameterization
Lorentz invariance of the local frame

Two-body effective interaction.

We may view the point particles as the source of the massless field DOF.

$$Z[J] = \int D\phi\, e^{iS} = e^{iW[J]},$$

$$W[J] = -\int dt\, V[J] \longrightarrow$$ identified as the two-body potential.

For the problem of gravitational binaries

Integrating out soft region gravitons (hard region has been integrated out)

↑ use method of regions

gives the two-body effective interaction.

Consider a toy model:

$$S = -m_1 \int dt \left(1 - \frac{1}{M}\phi(z_1(t))\right) - m_2 \int dt \left(1 - \frac{1}{M}\phi(z_2(t))\right) + \int d^4x \left(\frac{1}{2}(\partial_\mu\phi)^2 - \lambda\phi^3\right)$$

$$= -m_1 \int dt - m_2 \int dt + \int d^4x \left[\frac{1}{2}(\partial_\mu\phi)^2 - \lambda\phi^3 + J(x)\phi(x)\right]$$

$$J(x) = \frac{m_1}{M}\int dt\, \delta^{(4)}(x - z_1(t)) + \frac{m_2}{M}\int dt\, \delta^{(4)}(x - z_2(t)).$$

$$z_1^\mu(t) = (t, \vec{z}_1(t)),\quad z_2^\mu(t) = (t, \vec{z}_2(t))$$

$$= \frac{m_1}{M}\delta^{(3)}(\vec{x} - \vec{z}_1(t)) + \frac{m_2}{M}\delta^{(3)}(\vec{x} - \vec{z}_2(t)).$$

$$Z[J,\lambda] = \sum_{n,l} \frac{(-i\lambda)^n}{n!}\frac{i^l}{l!}\int d^4x_1 \cdots d^4x_l\, J(x_1)\cdots J(x_l)$$

(this is a full quantum calculation)

$$\times \left\langle \phi(x_1)\cdots\phi(x_l) \int d^4y_1\, \phi^3(y_1) \cdots \int d^4y_n\, \phi^3(y_n)\right\rangle_{\text{free}}.$$

where $\langle \phi(x_1)\cdots\phi(x_n)\rangle_{\text{free}} = \int D\phi\, \phi(x_1)\cdots\phi(x_n) \exp\left[i\int d^4x \frac{1}{2}(\partial_\mu\phi)^2\right]$

(computed by Wick contractions)

Saddle point $Z = e^{\frac{i}{\hbar}S} \longrightarrow$ classical limit $\longrightarrow$ only connected tree-level Feynman diagrams

Mind symmetry factors!

Mind symmetry factors!

NRGR

Lagrangian: $\mathcal{L}_{pp} = -m\int d\tau = -m\int d\sigma \sqrt{g_{\mu\nu}(x(\sigma))\, u^\mu u^\nu}$

$$= \int d^4x \sqrt{-g(x)}\; \underbrace{g_{\mu\nu}(x)}_{}\; \underbrace{T^{\mu\nu}_{pp}}_{}$$

$$T^{\mu\nu}_{pp} = \frac{-m}{\sqrt{-g(x)}} \int d\sigma \frac{u^\mu u^\nu}{\sqrt{u^2}} \delta^4(x - x(\sigma))$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_{Pl}} h_{\mu\nu}$$

$u^\mu(\sigma) = \frac{dx^\mu(\sigma)}{d\sigma}$

choose $x^\mu(\sigma) = (\sigma, x^i(\sigma))$

then $u^\mu(\sigma) = (1, v^i(\sigma))$

perturbative expansion:

			0PN	1PN	2PN	3PN	4PN	5PN
(tree)	1PM	G	1	v^2	v^4	v^6	v^8	v^{10}
(1-Loop)	2PM	G^2		1	v^2	v^4	v^6	v^8
(2-Loop)	3PM	G^3			1	v^2	v^4	v^6
(3-Loop)	4PM	G^4				1	v^2	v^4
(4-loop)	5PM.	G^5					1	v^2

Consider PN expansion.

$$\mathcal{L}_{pp} = \int d\sigma \Big[-m\sqrt{1-v^2} - \frac{m h_{00}}{2M_{Pl}} - \frac{m h_{0i} v^i}{M_{Pl}} - \frac{m h_{ij} v^i v^j}{2M_{Pl}} - \frac{m h_{00} v^2}{4M_{Pl}} - \frac{m h_{0i} v^i v^2}{2M_{Pl}} + \frac{m (h_{00})^2}{8M_{Pl}^2} + \frac{m h_{00} h_{0i} v^i}{2M_{Pl}^2} + \cdots \Big]$$

Graviton propagator.

$$\langle h_{\mu\nu}(t_1, \vec{x}_1) h_{\alpha\beta}(t_2, \vec{x}_2) \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{i P_{\mu\nu;\alpha\beta}}{p^2 + i\epsilon} e^{-ip(x_1 - x_2)}$$

potential graviton $\quad \frac{1}{p^2} = \frac{1}{(p^0)^2 - (\vec{p})^2} = -\frac{1}{(\vec{p})^2}\left(1 + \frac{(p^0)^2}{(\vec{p})^2} + \cdots\right)$

$$= -iP \int \frac{d^3\vec{p}}{} \Big[\delta(t_1 - t_2) + \frac{d^2}{dt_1 dt_2} \delta(t_1 - t_2) \quad \Big] e^{i\vec{p}\cdot(\vec{x}_1 - \vec{x}_2)}$$

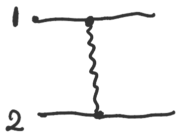
$$= -i P_{\mu\nu\alpha\beta}\int \frac{d^3\vec p}{(2\pi)^3}\left[\frac{\delta(t_1-t_2)}{\vec p^{\,2}} + \frac{\frac{d^2}{dt_1 dt_2}\delta(t_1-t_2)}{\vec p^{\,4}}+\cdots\right]e^{i\vec p\cdot(\vec x_1-\vec x_2)}$$

Graviton vertex :

At 1PN, only one vertex is relevant.

$$\langle h_{00}(t_1,\vec p_1)h_{00}(t_2,\vec p_2)h_{00}(t_3,\vec p_3)\rangle = \frac{i}{4M_{Pl}}\delta(t_1-t_2)\delta(t_1-t_3)\left(\frac{-i}{\vec p_1^{\,2}}\right)\left(\frac{-i}{\vec p_2^{\,2}}\right)\left(\frac{-i}{\vec p_3^{\,2}}\right)(\vec p_1^{\,2}+\vec p_2^{\,2}+\vec p_3^{\,2})(2\pi)^3\delta^{(3)}(\vec p_1+\vec p_2+\vec p_3)$$

First, let's consider the Newtonian potential



$$= \int d\sigma_1 d\sigma_2 \left(-\frac{im_1}{2M_{Pl}}\right)\langle h_{00}(\sigma_1, z_1(\sigma_1))\, h_{00}(\sigma_2, z_2(\sigma_2))\rangle\left(\frac{-im_2}{2M_{Pl}}\right)$$

$$= \left(-\frac{m_1m_2}{4M_{Pl}^2}\right)\int d\sigma_1 d\sigma_2\, (-iP_{0000})\int\frac{d^3p}{(2\pi)^3}\frac{\delta(\sigma_1-\sigma_2)}{\vec p^{\,2}}e^{i\vec p\cdot(\vec z_1(\sigma_1)-\vec z_2(\sigma_2))} \qquad (P_{0000}=\tfrac12)$$

$$= \frac{im_1m_2}{8M_{Pl}^2}\int d\sigma \int\frac{d^3p}{(2\pi)^3}\frac{1}{\vec p^{\,2}}e^{i\vec p\cdot(\vec z_1(\sigma)-\vec z_2(\sigma))}$$

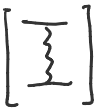
$$\hookrightarrow \int \frac{d^Dp}{(2\pi)^D}\frac{1}{(p^2)^\alpha}e^{ip\cdot x} = \frac{1}{(4\pi)^{D/2}}\frac{\Gamma(\frac D2-\alpha)}{\Gamma(\alpha)}\left(\frac{x^2}{4}\right)^{\alpha-\frac D2}$$

$D=3 \quad \alpha=1$

$$= \frac{1}{4\pi|x|}$$

$$=\int d\sigma \frac{i\, m_1m_2}{32\pi M_{Pl}^2|\vec z_1(\sigma)-\vec z_2(\sigma)|} \qquad \sim \frac{1}{M_{Pl}^2} = \kappa^2 = 32\pi G$$

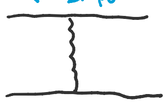
Is this correct ?

$W = (-i)\log Z = (-i)\Big[$  $\Big] = \int d\sigma \frac{Gm_1m_2}{|\vec z_1(\sigma)-\vec z_2(\sigma)|}$

$W = -\int d\sigma\, V \rightarrow V = -\frac{Gm_1m_2}{r}$ Newtonian potential

$O(v^1)$ is absent :

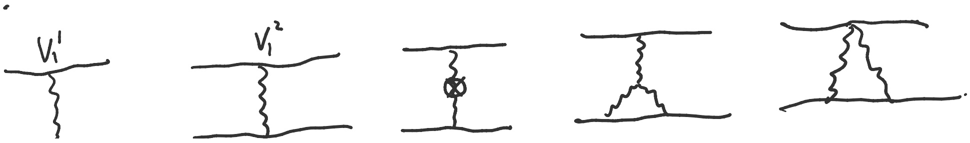
$\left(-\frac{im_1}{2M_{Pl}}h_{00}\right)$



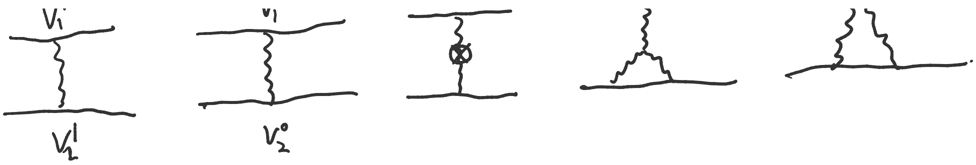
$\left(-\frac{im_2 h_{0i}v^i}{M_{Pl}}\right)$

$\sim \langle h_{00}h_{0i}\rangle \sim P_{000i}=0$

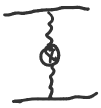
1PN :



$+$ flip.



+ flip.



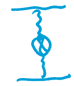
$$= \int d\sigma_1 d\sigma_2 \left(-\frac{i m_1}{2M_{Pl}}\right)(-iP_{0000}) \int \frac{d^3p}{(2\pi)^3} \frac{\frac{d^2}{d\sigma_1 d\sigma_2}\delta(\sigma_1-\sigma_2)}{(\vec{p}^{\,2})^2} e^{i\vec{p}\cdot(\vec{z}_1(\sigma_1)-\vec{z}_2(\sigma_2))} \left(-\frac{i m_2}{2M_{Pl}}\right)$$

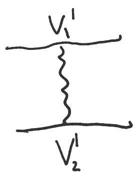
$$= \frac{i m_1 m_2}{8 M_{Pl}^2} \int d\sigma_1 d\sigma_2 \frac{d^2}{d\sigma_1 d\sigma_2}\delta(\sigma_1-\sigma_2)\left(-\frac{|z_1(\sigma_1)-z_2(\sigma_2)|}{8\pi}\right)$$

$$= -\frac{i m_1 m_2}{64\pi M_{Pl}^2} \int d\sigma_1 d\sigma_2\, \delta(\sigma_1-\sigma_2) \frac{d^2}{d\sigma_1 d\sigma_2} |z_1(\sigma_1)-z_2(\sigma_2)|$$

$$\simeq -\frac{i m_1 m_2}{64\pi M_{Pl}^2} \int d\sigma_1 d\sigma_2\, \delta(\sigma_1-\sigma_2) \frac{d}{d\sigma_2} \frac{\vec{v}_1\cdot(\vec{z}_1(\sigma_1)-\vec{z}_2(\sigma_2))}{|z_1(\sigma_1)-z_2(\sigma_2)|}$$

$$= \int d\sigma \frac{i m_1 m_2}{64\pi M_{Pl}^2}\left[\frac{\vec{v}_1\cdot\vec{v}_2}{r} - \frac{\vec{v}_1\cdot\vec{r}\;\vec{v}_2\cdot\vec{r}}{r^3}\right] \qquad r = |\vec{z}_1(\sigma)-\vec{z}_2(\sigma)|$$

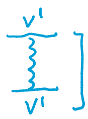
$V\Big[$  $\Big] = -\dfrac{G m_1 m_2}{2r}\left[\vec{v}_1\cdot\vec{v}_2 - \dfrac{\vec{v}_1\cdot\vec{r}\;\vec{v}_2\cdot\vec{r}}{r^2}\right]$

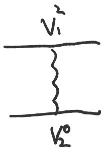


$$= \int d\sigma_1 d\sigma_2 \left(-\frac{i m_1 v_1^i}{M_{Pl}}\right)\underbrace{(-iP_{0i0j})}_{P_{0i0j} = -\frac{1}{2}\delta_{ij}} \int \frac{d^3p}{(2\pi)^3} \frac{\delta(\sigma_1-\sigma_2)\, e^{i\vec{p}\cdot(\vec{z}_1(\sigma)-\vec{z}_2(\sigma))}}{\vec{p}^{\,2}} \left(\frac{-i m_2 v_2^j}{M_{Pl}}\right)$$

$$= -\frac{i m_1 m_2}{2M_{Pl}^2}\frac{\vec{v}_1\cdot\vec{v}_2}{4\pi r}$$

$$= (-i)\frac{4G m_1 m_2}{r}\vec{v}_1\cdot\vec{v}_2$$

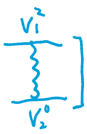
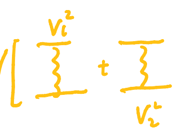
$V\Big[$  $\Big] = \dfrac{4G m_1 m_2}{r}\vec{v}_1\cdot\vec{v}_2$



$$= \int d\sigma_1 d\sigma_2 \left(-\frac{i m_1 v_1^i v_1^j}{2M_{Pl}}\right)(-iP_{ij00})\frac{\delta(\sigma_1-\sigma_2)}{4\pi r}\left(-\frac{i m_2}{2M_{Pl}}\right) \qquad P_{ij00} = \frac{1}{2}\delta_{ij}$$

$$+ \left(-\frac{i m_1 v_1^2}{4M_{Pl}}\right)(-iP_{0000})\frac{\delta(\sigma_1-\sigma_2)}{4\pi r}\left(-\frac{i m_2}{2M_{Pl}}\right) \qquad P_{0000} = \frac{1}{2}$$

$$= \int d\sigma_1\, (i)\frac{3G m_1 m_2 v_1^2}{2r}$$

$V\Big[$  $\Big] = -\dfrac{3G m_1 m_2 v_1^2}{2r}$ $\qquad V\Big[$  $\Big] = -\dfrac{3G m_1 m_2 (v_1^2+v_2^2)}{2r}$

$$= \int d\sigma_1 d\sigma_2 d\sigma_2' \left(\frac{i m_1}{8 M_{PL}^2}\right) \langle h_{00}(\sigma_1, \vec{z}_1(\sigma_1))\, h_{00}(\sigma_2, \vec{z}_2(\sigma_2)) \rangle \langle h_{00}(\sigma_1, \vec{z}_1(\sigma_1))\, h_{00}(\sigma_2', \vec{z}_2(\sigma_2')) \rangle \left(\frac{-i m_2}{2 M_{PL}}\right)^2$$

$$= -\frac{i m_1 m_2^2}{32 M_{PL}^4} \int d\sigma \underbrace{(-i P_{0000})^2}_{(-i/2)^2 = -1/4} \underbrace{\left[\int \frac{d^3p}{(2\pi)^3} \frac{1}{\vec{p}^2} e^{i\vec{p}\cdot\vec{r}}\right]^2}_{\frac{1}{(4\pi r)^2}}$$

$$= \int d\sigma\, (i)\, \frac{G^2 m_1 m_2^2}{2r^2}$$

V[] $= -\frac{G^2 m_1 m_2^2}{2r^2}$, V[ + ] $= -\frac{G^2 m_1 m_2 (m_1+m_2)}{2r^2}$

P

symmetry factor

$$= \frac{1}{2} \int d\sigma_1 d\sigma_2 d\sigma_2' \left(-\frac{i m_1}{2 M_{PL}}\right) \langle h_{00}(\sigma_1, \vec{z}_1(\sigma_1))\, h_{00}(\sigma_2, \vec{z}_2(\sigma_2))\, h_{00}(\sigma_2, \vec{z}_2(\sigma_2')) \rangle \left(-\frac{i m_2}{2 M_{PL}}\right)^2$$

$$= \frac{i m_1 m_2^2}{16 M_{PL}^3} \times (\cdots\cdots)$$

$$\int d\sigma_1 d\sigma_2 d\sigma_2' \langle h_{00}(\sigma_1, \vec{z}_1(\sigma_1))\, h_{00}(\sigma_2, \vec{z}_2(\sigma_2))\, h_{00}(\sigma_2, \vec{z}_2(\sigma_2')) \rangle$$

$$= \int d\sigma_1 d\sigma_2 d\sigma_2' \int \frac{d^D p_1}{(2\pi)^D} \frac{d^D p_2}{(2\pi)^D} \frac{d^D p_3}{(2\pi)^D} \frac{i}{4 M_{PL}} \delta(\sigma_1 - \sigma_2)\delta(\sigma_1 - \sigma_2') \left(\frac{-i}{\vec{p}_1^2}\right)\left(\frac{-i}{\vec{p}_2^2}\right)\left(\frac{-i}{\vec{p}_3^2}\right)(\vec{p}_1^2 + \vec{p}_2^2 + \vec{p}_3^2)(2\pi)^D \delta(\vec{p}_1 + \vec{p}_2 + \vec{p}_3)$$

$$\times e^{i\vec{p}_1\cdot\vec{z}_1(\sigma_1)} e^{i\vec{p}_2\cdot\vec{z}_2(\sigma_2)} e^{i\vec{p}_3\cdot\vec{z}_2(\sigma_2')}$$

solve $\vec{p}_3$

$$= \frac{-1}{4 M_{PL}} \int d\sigma \int \frac{d^D p_1}{(2\pi)^D} \frac{d^D p_2}{(2\pi)^D} \left[\frac{1}{\vec{p}_2^2 (\vec{p}_1 + \vec{p}_2)^2} + \underbrace{\frac{1}{\vec{p}_1^2 (\vec{p}_1 + \vec{p}_2)^2} + \frac{1}{\vec{p}_1^2 \vec{p}_2^2}}\right] e^{i\vec{p}_1\cdot(z_1(\sigma) - z_2(\sigma))}$$

p_2 integral is scaleless; set to zero in method of regions

$$= \frac{-1}{4 M_{PL}} \int d\sigma \int \frac{d^D p_1}{(2\pi)^D} \frac{1}{8(\vec{p}_1^2)^{1/2}} e^{i\vec{p}_1\cdot\vec{r}}$$

$$= -\frac{1}{64\pi^2 M_{PL}} \frac{1}{r^2}$$

$$\boxed{\int \frac{d^D k}{(2\pi)^D} \frac{1}{(\vec{k}^2)^\alpha ((\vec{k}+\vec{p})^2)^\beta} = \frac{1}{(4\pi)^{D/2}} \frac{\Gamma(\alpha+\beta-D/2)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(D/2-\alpha)\Gamma(D/2-\beta)}{\Gamma(D-\alpha-\beta)} (\vec{p}^2)^{\frac{D}{2}-\alpha-\beta}}$$

Final result:

$$= \frac{i m_1 m_2^2}{16 M_{PL}^3} \times \left(-\frac{1}{64\pi^2 M_{PL} r^2}\right) = (-i) \frac{G^2 m_1 m_2^2}{r^2}$$

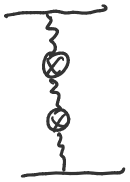
V[] $= \frac{G^2 m_1 m_2^2}{r^2}$, V[ + ] $= \frac{G^2 m_1 m_2 (m_1 + m_2)}{r^2}$

Adding everything together:

Adding everything together.

$$V_{1PN} = -\frac{Gm_1m_2}{2r}\left(3(\vec{v}_1^{\,2}+\vec{v}_2^{\,2}) - 7\,\vec{v}_1\cdot\vec{v}_2 - \frac{\vec{v}_1\cdot\vec{r}\;\vec{v}_2\cdot\vec{r}}{r^2}\right) + \frac{G^2 m_1 m_2 (m_1+m_2)}{2r^2}$$

2PN and beyond

*  $\longrightarrow V(x, \dot{x}, \ddot{x}, \ldots) \longrightarrow$ Ostrogradsky instability

$H \sim p$ instead of p^2

Higher order derivatives are spurious. They are due to our arbitrary choice of world-line time $z^\mu(\sigma) = (\sigma, \vec{z}(\sigma))$

They can be removed by a re-definition of position variables.

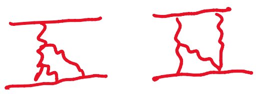
$$q = x + O(G, v^2) \longrightarrow V(q, \dot{q})$$

See Damour, Schafer, Journal of Mathematical Physics 32, 127. (1991)

* $W[J]$ in general is complex.

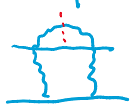
$$W[J] = \mathrm{Re}\,W[J] + i\,\mathrm{Im}\,W[J]$$

potential gravitons



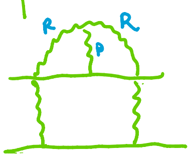
Conservative potential V.

radiation gravitons

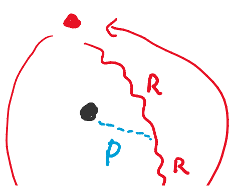


gives energy dissipated through GW.

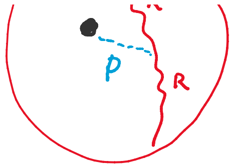
Tail effect.



non-local-in-time interactions in the conservative sector



Obstruct the analytic continuation between bound and scattering processes



between bound and scattering processes