Holographic imaginary potential of a quark antiquark pair in the presence of gluon condensation

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1. Introducing the AdS/QCD duality

- Holographic principle says that a theory of quantum gravity in a region of space should be described by a non-gravitational theory living at the boundary of that region. In particular, one may think of the quantum field theory as living on the $z = 0$ slice, the boundary of the entire space.

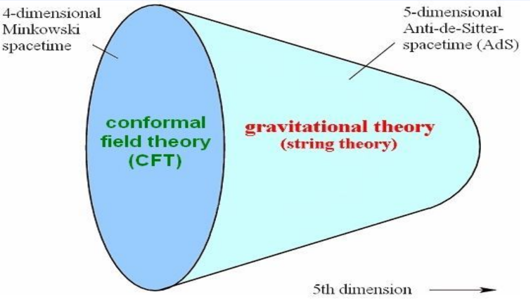
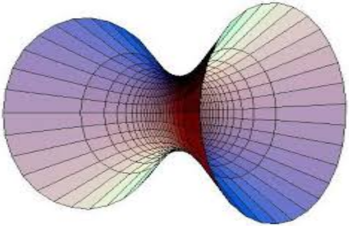
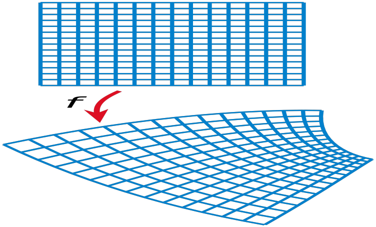


Figure: 1. AdS_5/CFT_4 conjecture

a) What is AdS space?
b) What is a conformal theory?


(a)



(b)

Figure: 2. a) Anti de Sitter spacetime has a negative radial coordinate, b) Conformal theories are invariant under conformal transformations, means they preserve angles.

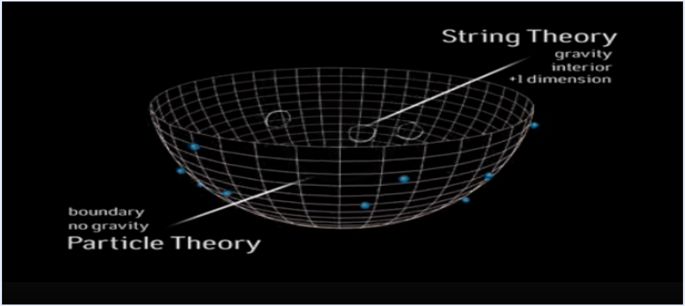


Figure: 3. The relation between string theory and particle physics description from gauge/gravity point of view.

- Original example: the conjectured equivalence between a certain conformal gauge theory and a certain gravitational theory in Anti de Sitter spacetime.
- We know string theory as a well-known quantum gravitational theory.

2. General aspects of the duality

Does it work for QCD?

- String theory was first invented to describe strong interactions.
- Different vibration modes of a string provided an economical way to explain many resonances discovered in the sixties which obey the so-called Regge behavior, between the mass and the angular momentum of a particle.
- Also confinement provided a physical picture for possible stringy degrees of freedom in QCD.
  Due to confinement, gluons at low energies behave to some extent like flux tubes which can close on themselves or connect a quark-antiquark pair, which naturally suggests a possible string formulation.

- The open strings describe particles: "quarks", "antiquarks", on their endpoints.

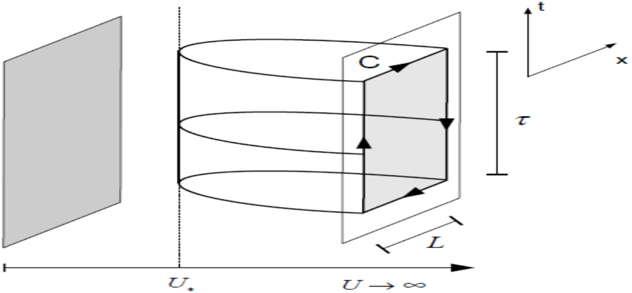


Figure: 4. Quark- anti quark pair structure from AdS/QCD point of view.

3. Holographic imaginary potential of a quark antiquark pair in the presence of gluon condensation

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- For a moving heavy quark antiquark in a QGP, we use gauge/gravity duality to study both real and imaginary parts of the potential in a gluon condensate theory.

- The complex potential is derived from the Wilson loop by considering the thermal fluctuations of the worldsheet of the Nambu-Goto holographic string.

- The potential of the pair describes the interaction energy between quark and anti-quark and the thermal width of the $Q\bar{Q}$ is estimated by the imaginary part of the interaction energy at finite temperature.

1. S. I. Finazzo and J. Noronha, JHEP 11 (2013) 042 and JHEP 01 (2015) 051.

3.1 Heavy quark potential and its relation with Wilson loop

The heavy quark potential (the vacuum interaction energy) is related to the vacuum expectation value of the Wilson loop as,

$$\lim_{\mathcal{T} \longrightarrow 0} \langle W(\mathcal{C}) \rangle_0 \sim e^{i\mathcal{T} V_{Q\bar{Q}}(L)}, \tag{1}$$

where $\mathcal{C}$ is a rectangular loop of spatial length L and extended over $\mathcal{T}$ in the time direction.

The expectation value of the Wilson loop can be evaluated in a thermal state of the gauge theory with the temperature T. From this point of view $V_{Q\bar{Q}}(L)$ is the heavy quark potential at finite temperature and its imaginary part defines a thermal decay width.

To estimate the thermal width mentioned, one can use worldsheet fluctuations of the Nambu-Goto action.

The gluon condensate

$$G_2 = \langle 0| \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a,\mu\nu} |0\rangle \tag{2}$$

with $G^a_{\mu\nu}$ the gluon field strength tensor, was introduced in QCD in the framework of the short distance operator product expansion applied to the two-point correlation function of heavy and light quark current operators.

A non-zero trace of the energy-momentum tensor appears in a full quantum theory of QCD. The anomaly implies a non-zero GC which can be calculated as

$$\triangle G_2(T) = G_2(T) - G_2(0) = -(\varepsilon(T) - 3P(T)) \tag{3}$$

where $G_2(T)$ denotes the thermal GC, $G_2(0)$, being equal to the condensate value at the deconfinement transition temperature, is the zero temperature condensate value, $\varepsilon(T)$ is the energy density, P(T) is the pressure of the QGP system.

M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl.Phys.B 147 (1979) 385.

The well-known modified holographic model introducing the gluon condensation in the boundary theory is given by the following background,

$$ds^2 = \frac{R^2}{z^2}(A(z)dx_i^2 - B(z)dt^2 + dz^2), \tag{4}$$

where in this dilaton black hole background , $A(z), B(z), f$ are defined as,

$$\begin{aligned} A(z) &= (1+fz^4)^{\frac{f+a}{2f}}(1-fz^4)^{\frac{f-a}{2f}}, \\ B(z) &= (1+fz^4)^{\frac{f-3a}{2f}}(1-fz^4)^{\frac{f+3a}{2f}}, \\ f^2 &= a^2 + c^2, \end{aligned} \tag{5}$$

a is related to the temperature by $a = \frac{(\pi T)^4}{4}$ and the dilaton field is given by,

$$\phi(z) = \frac{c}{f}\sqrt{\frac{3}{2}}\ln\frac{1+fz^4}{1-fz^4} + \phi_0. \tag{6}$$

Note that the dilaton black hole solution is well defined only in the range $0 < z < f^{-\frac{1}{4}}$, where f determines the position of the singularity and z_f behaves as an IR cutoff. For $a = 0$, (4) reduces to the dilaton-wall solution. Meanwhile, for $c = 0$, it becomes the Schwarzschild black hole solution.

Before getting through the GC model and applying it, let's have an exact look at (5), there are considerations need to be taken into account.

- In $f^2 = a^2 + c^2$, c and a are independent parameters, so GC is constant implying temperature dependence is absent.
- An analogous situation happens in a work by *Y. Kim et al.* They discussed that the gluon condensate is very sensitive to the QCD deconfinement transition since its value changes drastically with the deconfinement transition.
- They calculate the gluon condensate dependence of the heavy quark potential in AdS/CFT to study how the property of the heavy quarkonium is affected by a relic of the deconfinement transition.
- They observe that the heavy quark potential becomes deeper as the value of the gluon condensate decreases.
- They finally argue that dropping gluon condensate and pure thermal effect are competing each other in the physics of heavy quarkonium at high temperature.
- Although this is a novel work, the contribution of the coupled dilaton field with the background is missed there.

Y. Kim, B.-H. Lee, C. Park and S.-J. Sin, **Phys. Rev. D 80 (2009) 105016** [arXiv:0808.1143 [hep-th]]

- In a work by *P. Colangelo et al* it shows that, when temperature is not very high, GC strongly depends on T and μ (chemical potential).
- Also they discussed that at high temperature the gluon condensate is independent of temperature and chemical potential.
- Generally, they have found that the T dependence of the gluon condensate coincides with the one obtained in lattice QCD at $\mu = 0$. At large temperature and density, the condensate does not depend on μ.

P. Colangelo, F. Giannuzzi, S. Nicotri and F. Zuo, **Phys. Rev. D 88 (2013) 115011** [arXiv:1308.0489 [hep-ph]]

- An analysis of the temperature dependence of the leading contributions to the gluon condensate for SU(N) lattice gauge theory is presented by **G.Boyd et al** indicated that the GC appears a drastic change near T_c.

- The gluon condensate is calculated directly from the new lattice calculations of the interaction measure. It is shown how these computations provide a simple picture for the melting of the condensate around the deconfinement temperature.

G. Boyd and D.E. Miller, [**BI-TP-96-28**] hep-ph/9608482.

To avoid any distortion of the actual physics of gluon condensation, we restrict our framework to high temperatures only. Where the GC parameter is completely independent of the temperature.

considering all the mentioned points, now we are ready to go ahead. Let's get started our study specifically...
To remind, we will calculate the imaginary potential of a heavy quark-antiquark in the presence of GC, when temperature is high enough and when the dilaton field is coupled the the background.

S_{str} is the classical Nambu-Goto action of a string in the bulk,

$$S_{str} = \frac{1}{2\pi\alpha'} \int d\sigma d\tau e^{\frac{\phi(z)}{2}} \sqrt{-det(G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu)}. \tag{7}$$

To account for the effect of speed, we can consider a reference frame in which the plasma is at rest and the dipole moves with a constant speed $-\eta$ in the x_3 direction. Boosting the metric (4) we obtain,

$$\begin{aligned} ds^2 &= \frac{1}{z^2}\Big(A(z)\, dx_i^2 + [\cosh^2\eta\, A(z) - \sinh^2\eta\, B(z)]\, dx_3^2 \\ &- [\cosh^2\eta\, B(z) - \sinh^2\eta\, A(z)]\, dt^2 - 2[A(z) - B(z)]\, \sinh\eta \cosh\eta\, dx_3\, dt \\ &+ dz^2\Big), \end{aligned} \tag{8}$$

from now on, we can consider the dipole in the gauge theory, which has a gravitational dual with metric (8).

- The quarks are located at $x_3 = \frac{L}{2}$ and $x_3 = -\frac{L}{2}$,

  to write the action we consider two different positions for the pair,

- Pair alignment transverse to the axis of the quarks:

  The spacetime target functions are $X^\mu = (\tau = t, \sigma = x_1, cte, cte, z(x))$

- Pair alignment parallel to the axis of the quarks:

  The spacetime target functions are $X^\mu = (\tau = t, \sigma = cte, cte, x_3, z(x))$

For transverse case,

$$S_{str} = \frac{\mathcal{T}}{2\pi\alpha'}\int_{-L/2}^{L/2} d\sigma\sqrt{f_1(z)\cosh^2\eta - f_2(z)\sinh^2\eta + (f_3(z)\cosh^2\eta - f_4(z)\sinh^2\eta)z'^2(\sigma)}. \tag{9}$$

For parallel case,

$$S_{str} = \frac{\mathcal{T}}{2\pi\alpha'}\int_{-L/2}^{L/2} d\sigma\sqrt{f_1(z) + G(z)z'^2(\sigma)}, \tag{10}$$

where, $z' = \frac{dz}{d\sigma}$ and we defined,

$$\begin{aligned}
f_1(z) &= \frac{\omega^2(z)}{z^4}A(z)\,B(z), \qquad f_2(z) = \frac{\omega^2(z)}{z^4}A^2(z) \\
f_3(z) &= \frac{\omega^2(z)}{z^4}B(z), \qquad f_4(z) = \frac{\omega^2(z)}{z^4}A(z), \\
\omega(z) &= e^{\frac{\phi(z)}{2}} = (\frac{1+fz^4}{1-fz^4})^{\frac{c}{f}\sqrt{\frac{3}{8}}}.
\end{aligned} \tag{11}$$

$$\begin{aligned}
F(z) &= f_1(z)\cosh^2\eta - f_2(z)\sinh^2\eta, \\
G(z) &= f_3(z)\cosh^2\eta - f_4(z)\sinh^2\eta.
\end{aligned} \tag{12}$$

Finally, we proceed from $\mathrm{Re}\,V_{Q\bar{Q}} = S_{str}^{reg}/\mathcal{T}$.

The action depends only on $\sigma = x$ and the associated Hamiltonian is a constant of the motion. With the corresponding position of the deepest position in the bulk being z_*, Hamiltonian is,

$$H = \frac{F(z)}{\sqrt{F(z) + G(z) z'^2(\sigma)}} = cte = \sqrt{F(z_*)}. \tag{13}$$

From the Hamiltonian (13), we can write the equation of motion for $z(x)$ as,

$$\frac{dz}{dx} = \left[\frac{F(z)}{G(z)}\left(\frac{F(z)}{F(z_*)} - 1\right)\right]^{\frac{1}{2}}. \tag{14}$$

and we can relate L to z_* as follows,

$$\frac{L}{2} = \int_0^{z_*} \left[\frac{F(z)}{G(z)}\left(\frac{F(z)}{F(z_*)} - 1\right)\right]^{-\frac{1}{2}} dz. \tag{15}$$

Now we are ready to study ReV.

(a) (b)

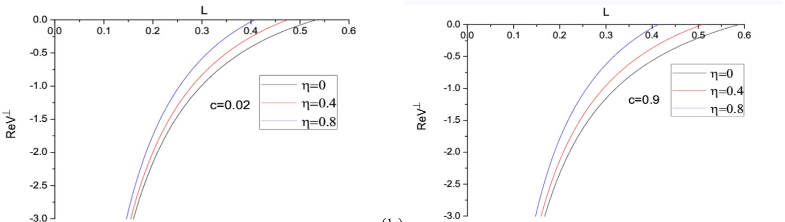


Figure: 1.Re $V_{Q\bar{Q}}$ as a function of L for a $Q\bar{Q}$ pair oriented transverse to the axis of the quarks.

This plots show the ReV as a function of L with the pair oriented transverse to the axis of the quarks, in the presence of GC. The results show that increasing speed leads to a decrease in dissociation length while c has the opposite effect, means increasing c increases dissociation length.

(In this figure and all other plots from now on, we consider $T = 200MeV$).

3.4 Real potential of heavy quark antiquark by holographic approach

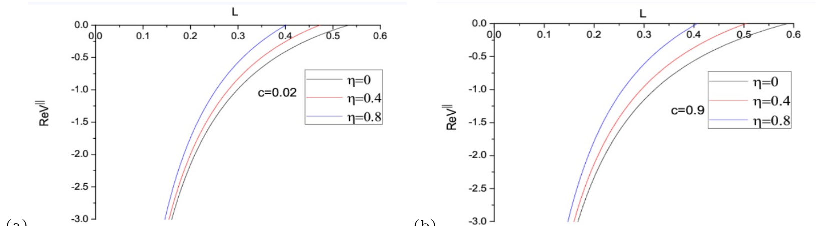


Figure: 2. $\mathrm{Re}\, V_{Q\bar{Q}}$ as a function of L for a $Q\bar{Q}$ pair oriented parallel to the axis of the quarks.

Similar to previous case, increasing speed leads to decreasing the dissociation length while c has the opposite effect.

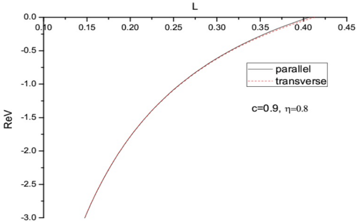


Figure: 5. $\mathrm{Re}\, V_{Q\bar{Q}}$ as a function of L, as a comparison between the parallel and the transverse cases.

The plot shows a comparison between the ReV for the parallel and the transverse cases. Although the difference is not significant, the plots show that the effect of the GC is slightly stronger for the parallel case. In other words, increasing c increases the dissociation length in both the transverse and the parallel cases (previous figures), this effect appears stronger when the dipole moves parallel to the axis of the quarks.

3.5 Imaginary potential of heavy quark antiquark by holographic approach

Consider the effect of worldsheet fluctuations around the classical configuration $r = \frac{1}{z}$,

$$r(x) = r_*(x) \to r(x) = r_*(x) + \delta r(x), \tag{16}$$

Considering the fluctuations in the partition function and dividing the interval of x into $2N$ points (where $N \longrightarrow \infty$) we obtain,

$$Z_{str} \sim \lim_{N\to\infty} \int d[\delta r(x_{-N})] \dots d[\delta r(x_N)] e^{iS_{NG}(r_*(x)+\delta r(x))}, \tag{17}$$

- We expand $r_*(x_j)$ around $x = 0$ and keep only terms up to second order of it because thermal fluctuations are important around r_* which means $x = 0$,
- Considering small fluctuations and insert them in the action, the action is written as,

  $$S_j^{NG} = \frac{\mathcal{T}\Delta x}{2\pi\alpha'}\sqrt{C_1 x_j^2 + C_2}, \tag{18}$$

  where C_1 and C_2 are functions of f_1, f_2, f_3, f_4.
- To have $ImV_{Q\bar{Q}} \neq 0$, the function in the square root (18) should be negative. Then, we consider the j-th contribution to Z_{str} and apply it in the action. $D(\delta r_j) < 0 \Longrightarrow -x_* < x_j < x_*$ leads to an imaginary part in the square root.
- The total contribution of the $D(\delta r)$ to the imaginary part, will be available with a continuum limit.

3.5 Imaginary potential of heavy quark antiquark by holographic approach

Applying all these conditions in the NG action, we find the imaginary potential as follows,

- for transverse case

$$\mathrm{Im}\, V_{Q\bar{Q}} = -\frac{1}{2\sqrt{2}\alpha'}\sqrt{G_*}z_*^2\left[\frac{F_*}{z_*^2 F_*'} - \frac{z_*^2 F_*'}{4z_*^3 F_*' + 2z_*^4\, F_*''}\right], \tag{19}$$

  and,

- for parallel case,

$$\mathrm{Im}\, V_{Q\bar{Q}} = -\frac{1}{2\sqrt{2}\alpha'}\sqrt{G_*}z_*^2\left[\frac{f_{1*}}{z_*^2 f_{1*}'} - \frac{z_*^2 f_{1*}'}{4z_*^3 f_{1*}' + 2z_*^4\, f_{1*}''}\right]. \tag{20}$$

where, we have defined $F(z), G(z)$, $f_{1,2,3,4}(z)$ before, and $*$ simply describes the turning point of the string.

3.5 Imaginary potential of heavy quark antiquark by holographic approach

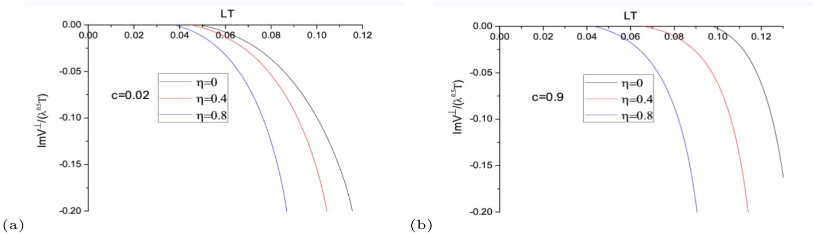
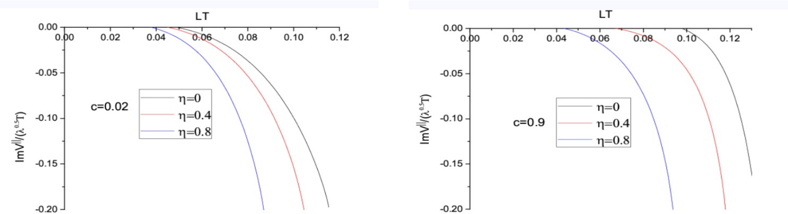


Figure: 3. $ImV_{Q\bar{Q}}$ as a function of LT for a $Q\bar{Q}$ pair oriented transverse to the axis of the quarks.

With increasing speed the ImV begins to become non-zero for smaller values of LT. Also, the onset of the ImV occurs for smaller LT and the absolute value of the ImV decreases, implying that quarkonium melts more easily which is consistent with the results of the previous works. Thus our results show that the pair's thermal width decreases with increasing its speed relative to the plasma, while c has the opposite effects.



(a) (b)

Figure: 4. $ImV_{Q\bar{Q}}$ as a function of LT for a $Q\bar{Q}$ pair oriented parallel to the axis of the quarks.

In parallel case, results are similar to the transverse case, means the thermal width of the pair increases with increasing GC and these effects are the opposite of the speed effects.

3.5 Imaginary potential of heavy quark antiquark by holographic approach

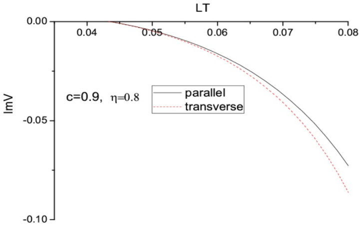


Figure: 6. $ImV_{Q\bar{Q}}$ as a function of LT, as a comparison between the parallel and the transverse cases.

This is a comparison between the ImV for the parallel and the transverse cases. Similar to ReV, the plots show that the effect of the GC is stronger for the parallel case. As shown previously, increasing c increases the thermal width in both the transverse and the parallel cases. This effect appears stronger when the dipole moves parallel to the axis of the quarks.

4. Results and discussions

4. Results and discussions

- By using holography approach, one can study some aspects of QCD, since quarks and gluons could be formulated as strings.
- An important phenomenon is **gluon condensation** in QCD which explain **(de)confinement**, hence could be a condition for the **phase transition** and it is useful to study the nonperturbative nature of the QGP.
- By using the holographic GC model, we investigated potential of a heavy moving quarkonia in a QGP.
- According to Wilson loop relation with the potential, it develops an imaginary part. To find it, one could consider the thermal worldsheet fluctuations in string action, known as Noronha's method.
- **Increasing speed** leads to a **decrease in dissociation length** while **c has the opposite effect** for both transverse and parallel cases.
- The effect of c appears **slightly stronger when the dipole moves parallel** to the axis of the quarks.
- For both transverse and parallel case, with increasing gluon condensation the imaginary part of the potential starts to become nonzero for larger values of LT and the onset of the imaginary potential happens for larger LT and ImV . Our results thus indicate that the **thermal width of the $Q\bar{Q}$ pair increases with increasing gluon condensation**. These effects are the opposite of the speed effects.

5. Future plans

- a $5d$ TSB model, (JHEP 05 (2014) 101) introducing a disordering parameter α in the medium, could be used to study a variety of physical observables. The question is how they depend on disorder.

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Thanks for your attention