重味重子物理研究
的机遇与挑战

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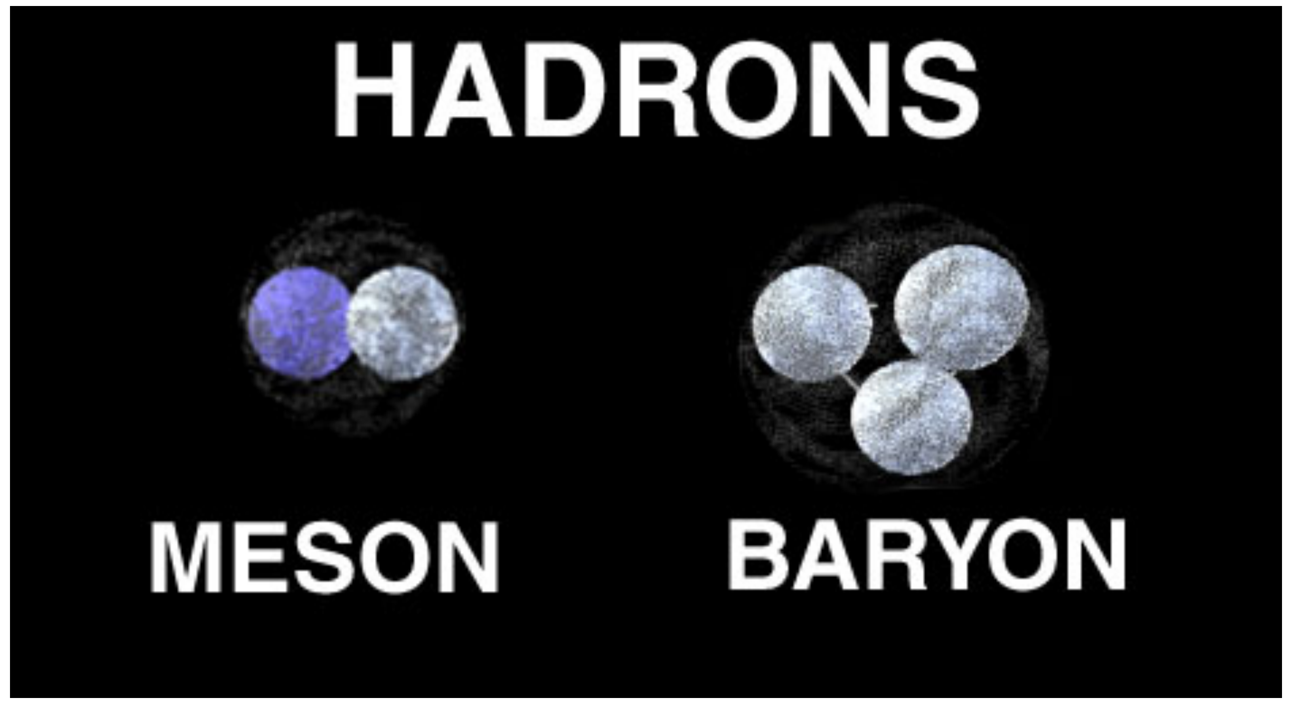
Outline

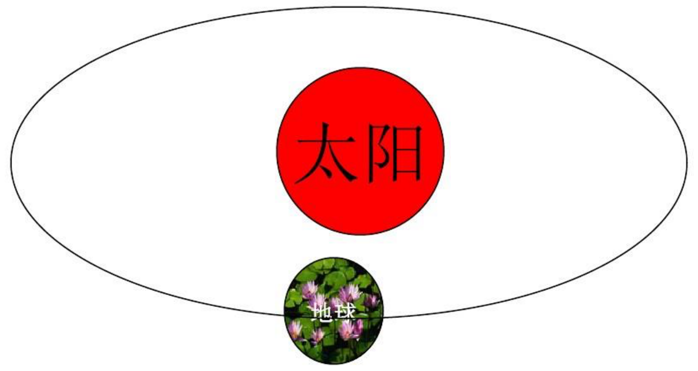
- Why baryon physics
- Bottom-baryon decays
- Charm-baryon decays
- Summary

Heavy flavor physics

- Heavy flavor physics has achieved a great progress in the heavy meson systems during the past two decades.
- It established the KM mechanism for the CP violation in B meson decays.
- However, the studies on heavy-flavor baryons are limited.



2-body



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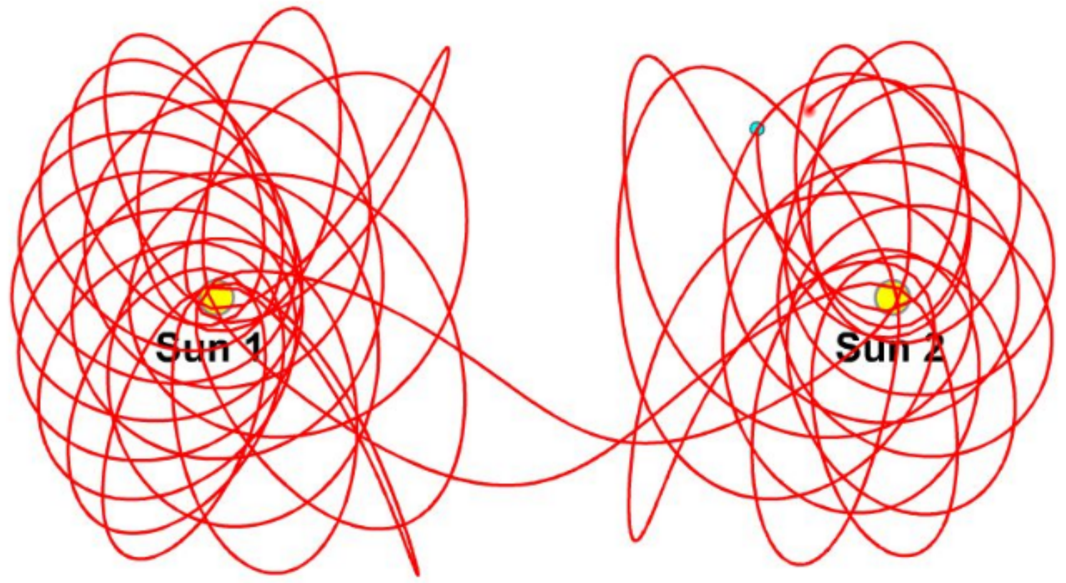
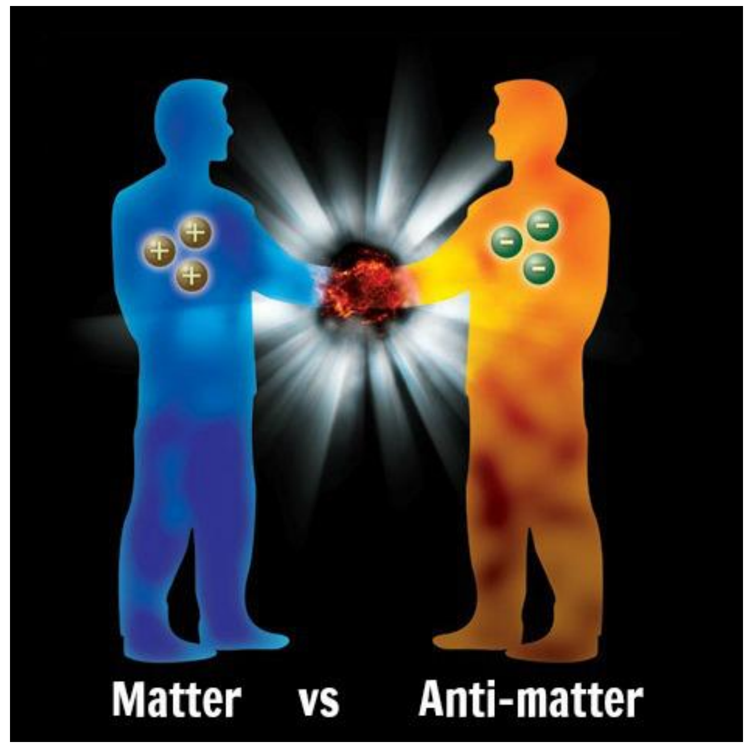


It is a non-trivial extension.

More is different.

CP violation in baryons



- Sakharov conditions for **Baryogenesis:**
  1) **baryon** number violation
  2) C and **CP violation**
  3) out of thermal equilibrium
- **CPV: SM < BAU. => new source of CPV, NP**
- CPV well established in K, B and D mesons,
  but CPV never established in any baryon
- **The visible matter in the Universe is mainly made of baryons**

CP violation in baryons

CP violation in baryons

- In 2017, LHCb reported 3σ evidence of CPV in $\Lambda_b \to p\pi\pi\pi$ [Nature Physics, 2017]
- In 2019 and 2022, BESIII reported the measurement of CPV in $\Lambda^0 \to p\pi^-$ [Nature Physics, 2019; PRL 2022]
- In 2022, BESIII reported the measurement of CPV in $\Xi^- \to \Lambda^0\pi^-$ [Nature 2022]
- So far, no CPV in the baryon sector has been observed yet.

Opportunities

- **LHCb is a baryon factory !!** **Large Production:** $\frac{f_{\Lambda_b}}{f_{u,d}} \sim 0.5 \longrightarrow \frac{N_{\Lambda_b}}{N_{B^{0(-)}}} \sim 0.5$

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- Precision of baryon CPV measurements has reached to the order of **1%** [LHCb, PLB2018]

$$A_{CP}(\Lambda_b^0 \to p\pi^-) = (-3.5 \pm 1.7 \pm 2.0)\,\%\,, \quad A_{CP}(\Lambda_b^0 \to pK^-) = (-2.0 \pm 1.3 \pm 1.0)\,\%$$

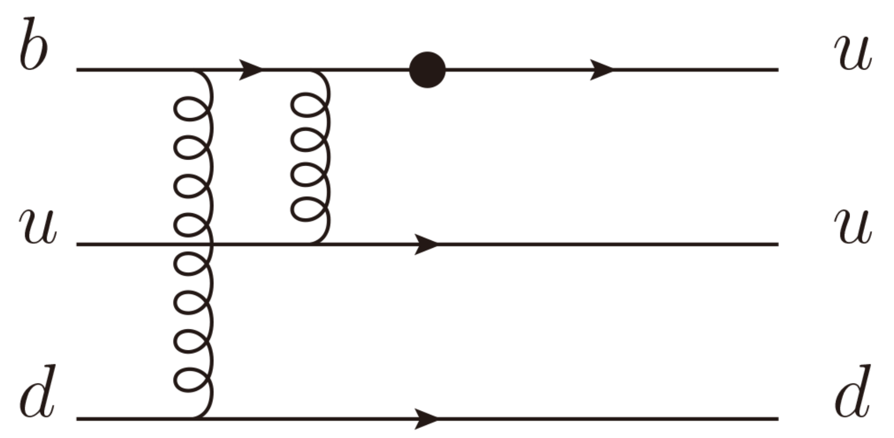
- CPV in some B-meson decays are as large as **10%**:

$$A_{CP}(\overline{B}^0 \to K^+\pi^-) = -\,(8.34 \pm 0.32)\,\%\,, \quad A_{CP}(\overline{B}_s^0 \to K^+\pi^-) = +\,(21.3 \pm 1.7)\,\%$$

- **It can be expected that CPV in b-baryons might be observed soon !!**

Theoretical Challenges

1. QCD dynamics for non-leptonic decays



- One more energetic quark, one more hard gluon.
  Counting rule of power expansion is violated by α_s .

2. Non-perturbative inputs

- Theoretical uncertainties are dominated by the non-perturbative input parameters, such as the light-cone distribution amplitudes (LCDA).

3. Observables

- T-odd triple products $(\vec{p}_1 \times \vec{p}_2) \cdot \vec{p}_3$, 3σ signal in $\Lambda_b \to p\pi\pi\pi$[LHCb2017].
  Defined by kinematics, but unclear relation to the decay amplitudes.
  No way for theoretical explanations and predictions.

Dynamics of Non-leptonic Decays

Dynamics of Non-leptonic Decays

- **QCD studies on baryons are limited**

- Generalized factorization [Hsiao, Geng, 2015; Liu, Geng, 2021]:
  lost of non-factorizable contributions, such as W-exchange diagrams.
- QCDF [Zhu, Ke, Wei, 2016, 2018]: based on diquark picture, No W-exchange diagrams.
- PQCD [Lu, Wang, Zou, Ali, Kramer, 2009]: only considering the leading twists of LCDAs.

- **Currently, no complete QCD-inspired method for non-leptonic b-baryon decays**

	EXP	GF	PQCD	QCDF
$Br(\Lambda_b \to p\pi)[\times 10^{-6}]$	4.3 ± 0.8	4.2+-0.7	$4.66^{+2.22}_{-1.81}$	4.11~4.57
$Br(\Lambda_b \to pK)[\times 10^{-6}]$	5.1 ± 0.9	4.8+-0.7	$1.82^{+0.97}_{-1.07}$	1.70~3.15
$A_{CP}(\Lambda_b \to p\pi)[\%]$	-2.5 ± 2.9	-3.9+-0.2	-32^{+49}_{-1}	-3.74~-3.08
$A_{CP}(\Lambda_b \to pK)[\%]$	-2.5 ± 2.2	5.8+-0.2	-3^{+25}_{-4}	8.1~11.4

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- **Why CPV in $\Lambda_b \to p\pi,\ pK$ is so small?**

Baryons are Different

Baryons are Different

- **Baryons are very different from mesons!!**

- **Factorization:** Heavy-to-light form factor is factorizable at leading power in SCET.
  No end-point singularity! [Wei Wang, 1112.0237] Taking $\Lambda_b \to \Lambda$ as an example,

$$\xi_\Lambda = f_{\Lambda_b} \Phi_{\Lambda_b}(x_i) \otimes J(x_i, y_i) \otimes f_\Lambda \Phi_\Lambda(y_i)$$

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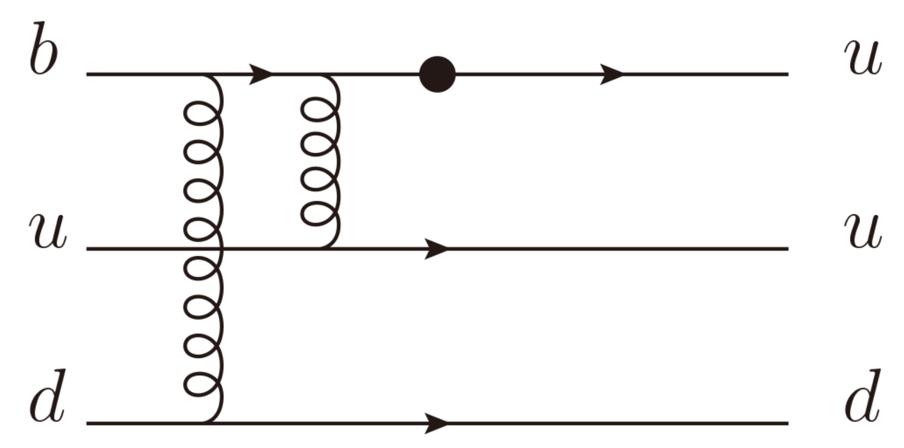
- However, the **leading-power result is one order of magnitude smaller** than the total one
  - Leading power: $\xi_\Lambda(0) = -0.012$ [W.Wang, 2011]
  - Total form factor: $\xi_\Lambda(0) = 0.18$ [Y.L.Shen, Y.M.Wang, 2016]
- Two hard gluons suppressed by α_s^2 at the leading power. Compared to the soft contributions in the power corrections.



PQCD approach

- PQCD successfully predicted CPV in B meson decays [Keum, H.n.Li, Sanda, 2000; C.D.Lu, Ukai, M.Z.Yang, 2000].

直接CP破坏(%)	GFA	QCDF	2000 PQCD	2004 exp.
$B \to \pi^+\pi^-$	-5 ± 3	-6 ± 12	$+30 \pm 20$	$+32 \pm 4$
$B \to K^+\pi^-$	$+10 \pm 3$	$+5 \pm 9$	-17 ± 5	-8.3 ± 0.4

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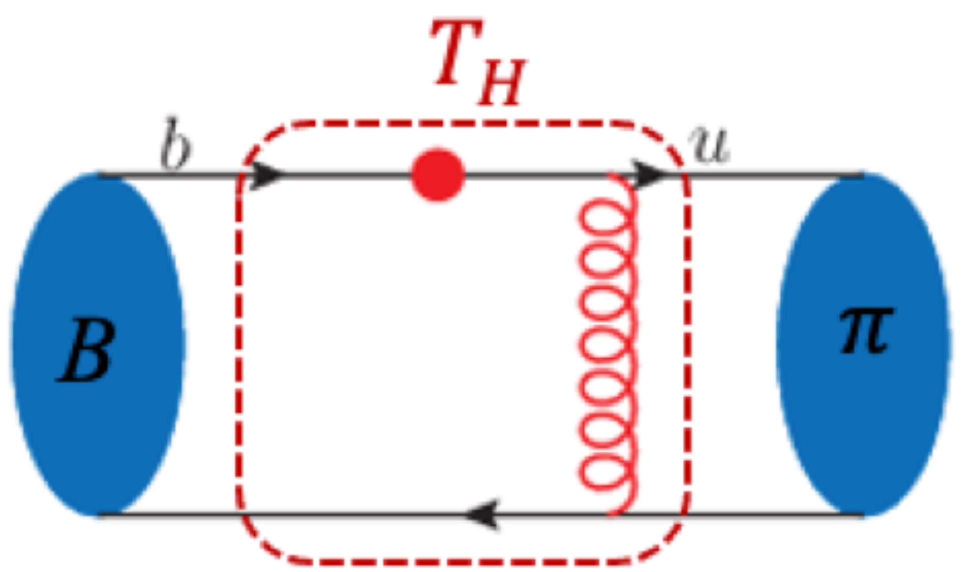
- under collinear factorization:
  - Endpoint singularity: propagator $\sim 1/x_1x_2Q^2 \to \infty$ when $x_{1,2} \to 0,1$



$$M(Q^2) = \int_0^1 dx_1 dx_2\, \phi_B(x_2,\mu^2) * T_H\left(x_1, x_2, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) * \phi_\pi(x_1,\mu^2)$$

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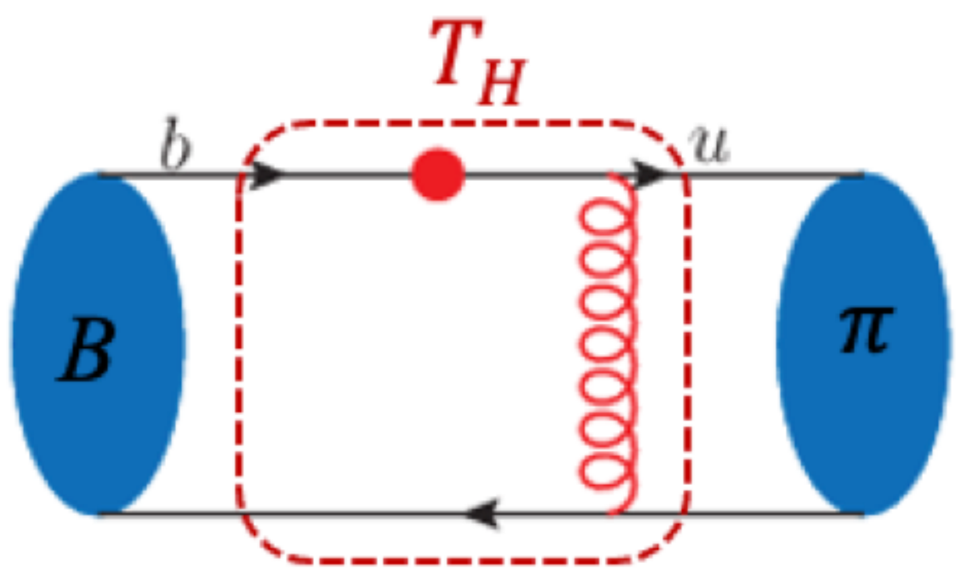

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- PQCD approach (based on k_T factorization): retain transverse momentum of parton k_T,
  - propagator $\sim 1/(x_1x_2Q^2 + k_T^2)$

$$M(Q^2) = \int_0^1 dx_1 dx_2 \int \boldsymbol{dk_{1T} dk_{2T}} \phi_B(x_2, \boldsymbol{k_{2T}}, \mu^2) * T_H\left(x_1, x_2, \boldsymbol{k_{2T}}, \boldsymbol{k_{1T}}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) * \phi_\pi(x_1, \boldsymbol{k_{1T}}, \mu^2)$$

$\Lambda_b \to p$ form factors in PQCD

- In 2009, the form factors are two orders of magnitude smaller than LatticeQCD/experiments, considering only the leading twist of LCDAs of baryons. [C.D.Lu, Y.M.Wang, et al, 2009]
- In 2022, when consider contributions of high-twist LCDAs, they are consistent with LatticeQCD. [J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, 2022]

	Lattice/exp	PQCD(2009)	PQCD(2022)
$f_1^{\Lambda_b \to p}(0)$	0.22 ± 0.08	0.002 ± 0.001	0.27 ± 0.12

	twist-3	twist-4	twist-5	twist-6	total
exponential					
twist-2	0.0007	-0.00007	-0.0005	-0.000003	0.0001
twist-3^{+-}	-0.0001	0.002	0.0004	-0.000004	0.002
twist-3^{-+}	-0.0002	0.0060	0.000004	0.00007	0.006
twist-4	0.01	0.00009	$\boxed{0.25}$	0.0000007	0.26
total	0.01	0.008	0.25	0.00007	$0.27 \pm 0.09 \pm 0.07$

Theoretical progresses: PQCD

proton

Λ_b

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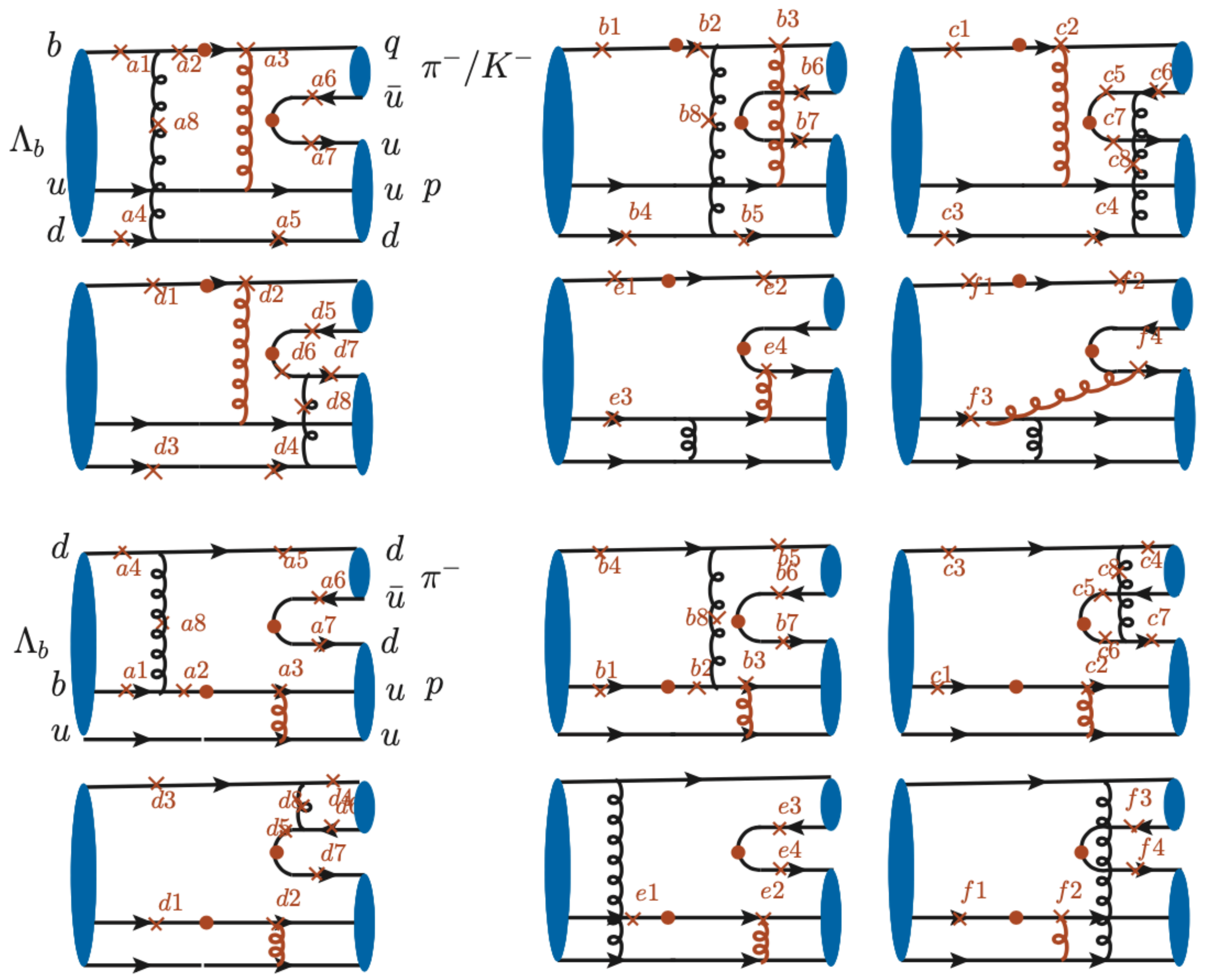
- Safely twist expansion. Twist-6 of proton is highly suppressed.
- Perturbation protected. Results are given with $\mu \geq$ 1GeV.

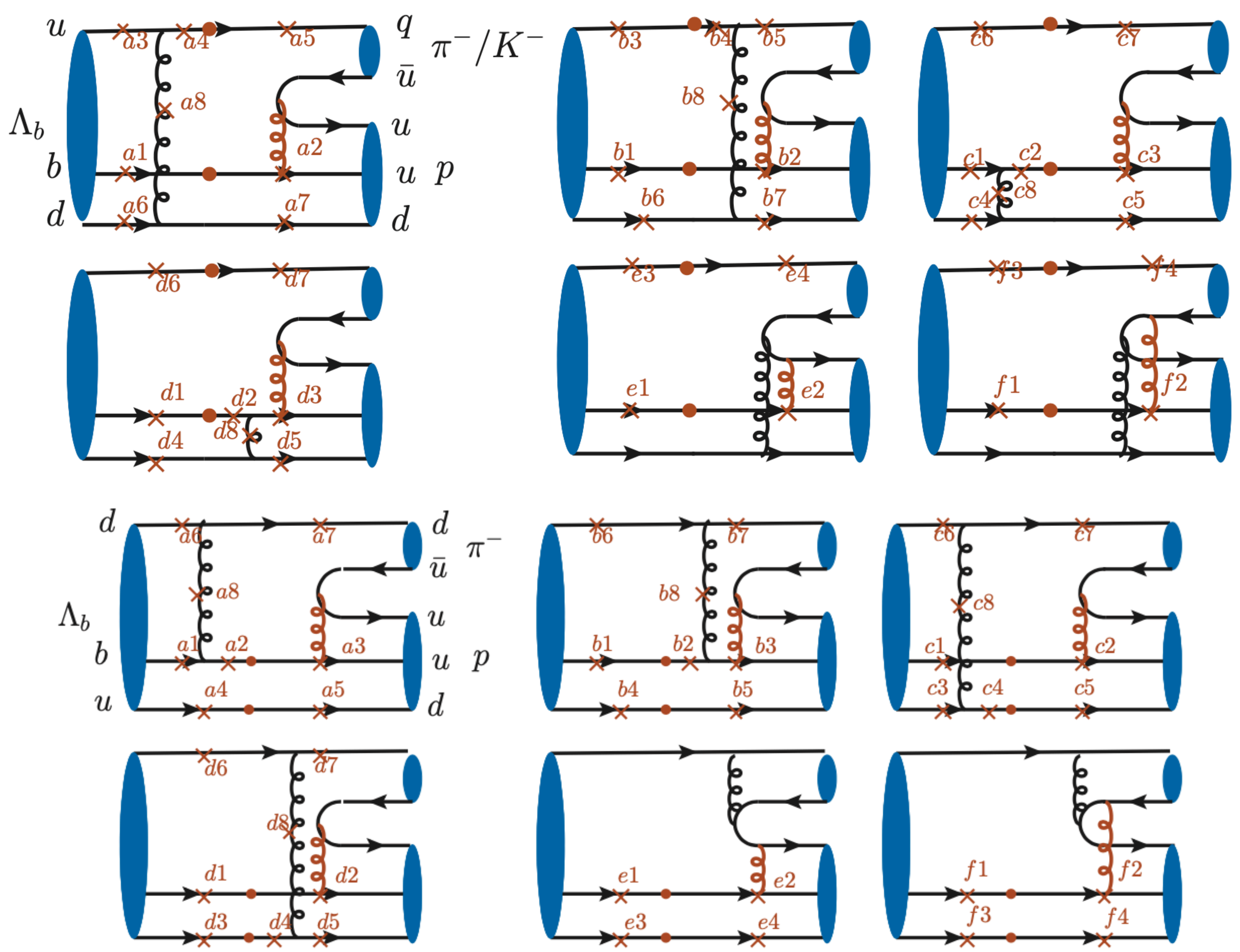
J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, 2202.04804

Non-leptonic decays

- $\Lambda_b \to \Lambda_c \pi, \Lambda_c K, \Lambda J/\Psi, \Lambda\phi$ are recently studied by [C.Q.Zhang, J.M.Li, M.K.Jia, Zhou Rui, 2022]
- **It can be expected that PQCD can predict CPV of b-baryons**

J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, in preparation





There are 200 Feynman diagrams for $\Lambda_b \to p\pi$, and 120 diagrams for $\Lambda_b \to pK$.

Observables

$$\mathscr{M} = i\bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$

$$A_{CP}^{\text{dir}} = \frac{-2A\,|S^T|^2 r_1 \sin\Delta\phi_1 \sin\Delta\delta_1 - 2B\,|P^T|^2 r_2 \sin\Delta\phi_2 \sin\Delta\delta_2}{A\,|S^T|^2(1 + r_1^2 + 2r_1\cos\Delta\phi_1\cos\Delta\delta_1) + B\,|P^T|^2(1 + r_2^2 + 2r_2\cos\Delta\phi_2\cos\Delta\delta_2)}$$

$$A_{CP}^{S} = \frac{-2r_1\sin\Delta\phi_1\sin\Delta\delta_1}{(1 + r_1^2 + 2r_1\cos\Delta\phi_1\cos\Delta\delta_1)}$$

$$A_{CP}^{P} = \frac{-2r_2\sin\Delta\phi_2\sin\Delta\delta_2}{(1 + r_2^2 + 2r_2\cos\Delta\phi_2\cos\Delta\delta_2)}$$

$$A = \frac{(M_{\Lambda_b} + M_p)^2 - M_M^2}{M_{\Lambda_b}^2}$$

$$B = \frac{(M_{\Lambda_b} - M_p)^2 - M_M^2}{M_{\Lambda_b}^2}$$

Numerical Results

$$Br(\Lambda_b \to p\pi^-) = 4.65 \times 10^{-6}$$
$$A_{CP}^{dir}(\Lambda_b \to p\pi^-) = 4.29\%$$
$$A_{CP}^{S}(\Lambda_b \to p\pi^-) = 15\%$$
$$\Delta\delta_S = -19.31^\circ$$
$$r_S = 0.24$$
$$A_{CP}^{P}(\Lambda_b \to p\pi^-) = -6\%$$
$$\Delta\delta_P = -277.18^\circ$$
$$r_P = 0.03$$

$$Br(\Lambda_b \to pK^-) = 3.89 \times 10^{-6}$$
$$A_{CP}^{dir}(\Lambda_b \to pK^-) = -5.3\%$$
$$A_{CP}^{S}(\Lambda_b \to pK^-) = -4\%$$
$$\Delta\delta_S = -4.69^\circ$$
$$r_S = 4.83$$
$$A_{CP}^{P}(\Lambda_b \to pK^-) = -32\%$$
$$\Delta\delta_P = -135.31^\circ$$
$$r_P = 0.30$$

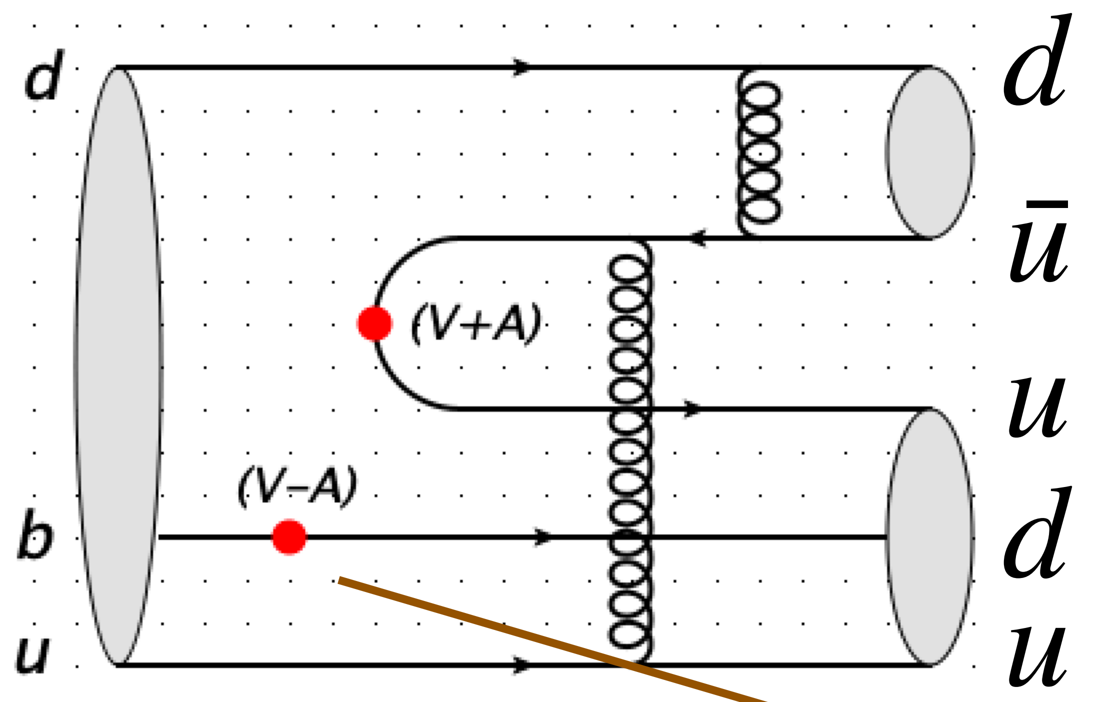
余纪新、韩佳杰、李亚、李湘楠、肖振军、于福升，2024

$\Lambda_b \to p\pi^-$	$\lvert S\rvert$	$\phi(S)^\circ$	Real(S)	Imag(S)	$\lvert P\rvert$	$\phi(P)^\circ$	Real(P)	Imag(P)
T_f	832.17	180.00	-832.17	0.00	1172.14	180.00	-1172.14	0.00
T_{nf}	87.54	81.77	12.53	86.64	352.14	81.92	49.49	348.65
C'	29.00	-21.42	27.00	-10.59	46.29	4.80	46.13	3.87
E_2	79.80	42.40	58.93	53.81	67.41	-59.80	33.90	-58.26
B	15.95	-62.67	7.32	-14.17	20.90	33.07	17.51	11.40
Tree	735.54	170.95	-726.39	115.69	1069.70	163.40	-1025.10	305.66
$P_f^{C_1}$	68.67	180.00	-68.67	-0.00	3.42	180.00	-3.42	-0.00
$P_{nf}^{C_1}$	2.48	74.02	0.68	2.38	13.33	84.06	1.38	13.25
P^{C_2}	16.52	68.47	6.06	15.36	18.25	-104.55	-4.59	-17.66
$P^{E_1^u}$	10.56	88.27	0.32	10.55	8.75	-69.74	3.03	-8.21
P^B	1.60	113.43	-0.64	1.47	1.48	-9.91	1.46	-0.25
$P^{E_1^d} + P^{E_2}$	3.44	73.24	0.99	3.29	3.25	162.19	-3.10	1.00
Penguin	69.60	151.64	-61.25	33.06	12.98	-113.79	-5.23	-11.88

$\Lambda_b \to pK^-$	$\|S\|$	$\phi(S)^\circ$	Real(S)	Imag(S)	$\|P\|$	$\phi(P)^\circ$	Real(P)	Imag(P)
T_f	1018.63	180.00	-1018.63	0.00	1439.70	180.00	-1439.70	0.00
T_{nf}	94.76	82.95	11.63	94.04	442.59	82.49	57.81	438.80
E_2	109.88	45.33	77.25	78.13	92.24	-60.79	45.02	-80.51
Tree	945.56	169.51	-929.75	172.17	1384.04	165.00	-1336.87	358.28
$P_f^{C_1}$	90.09	180.00	-90.09	-0.00	3.85	0.00	3.85	-0.00
$P_{nf}^{C_1}$	2.37	68.85	0.86	2.21	17.06	85.45	1.35	17.01
$P^{E_1^u}$	13.41	86.40	0.84	13.39	11.13	-70.96	3.63	-10.52
$P^{E_1^d}$	8.11	78.15	1.67	7.94	2.88	-124.36	-1.62	-2.38
Penguin	89.87	164.82	-86.73	23.54	8.30	29.69	7.21	4.11

CPV are cancelled by S- and P-wave amplitudes

$$\mathcal{M} = i\bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$

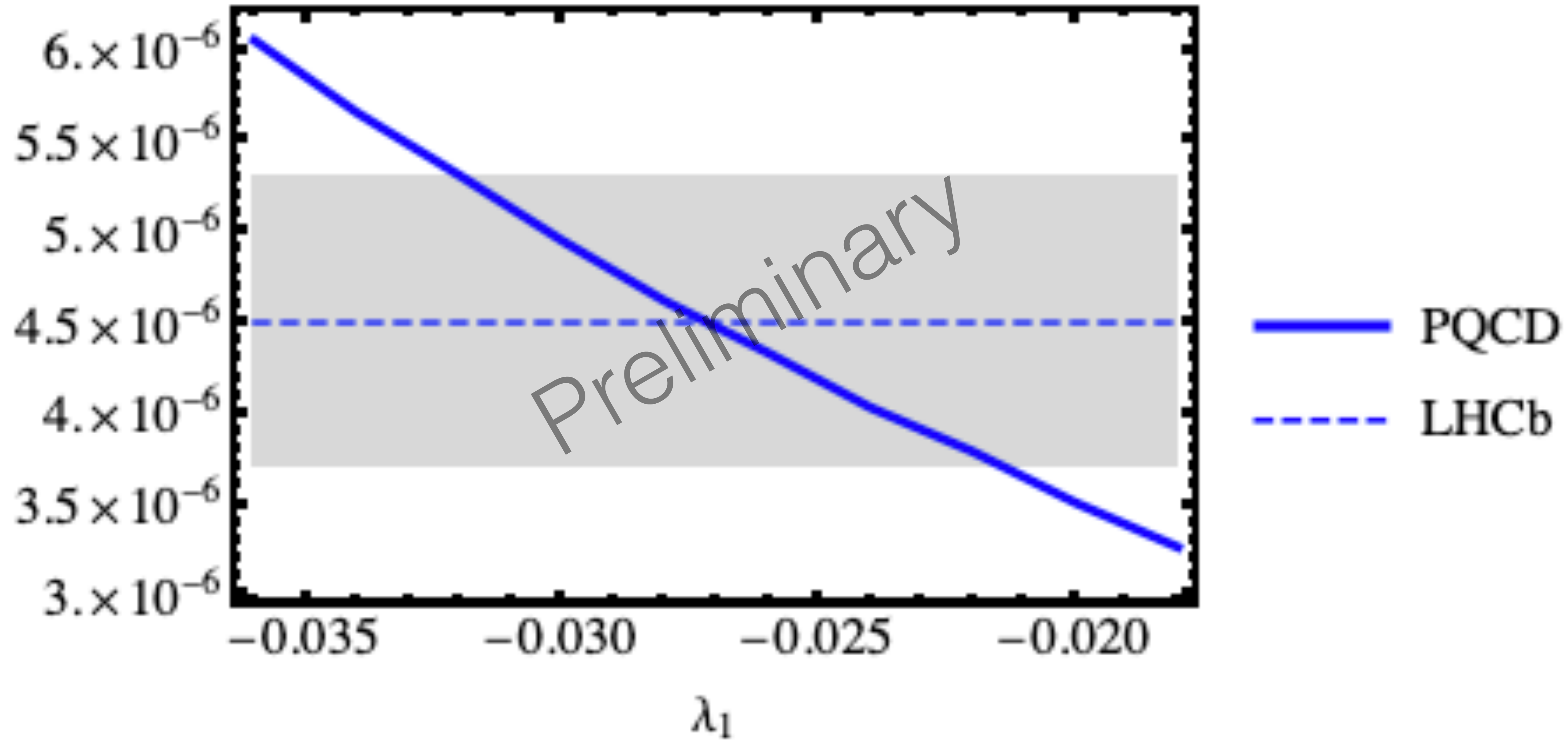


$$\gamma_5\gamma^\mu(1+\gamma_5)(\not{p} - \not{p}' + \not{k}_2')\gamma^\nu(\not{p} - \not{p}' - \not{k}_3')\gamma^\rho\gamma_5\not{q}\gamma_\rho\gamma_5(-\not{p} + m_b)\gamma_\nu\not{p}'\gamma_\mu(1-\gamma_5)$$

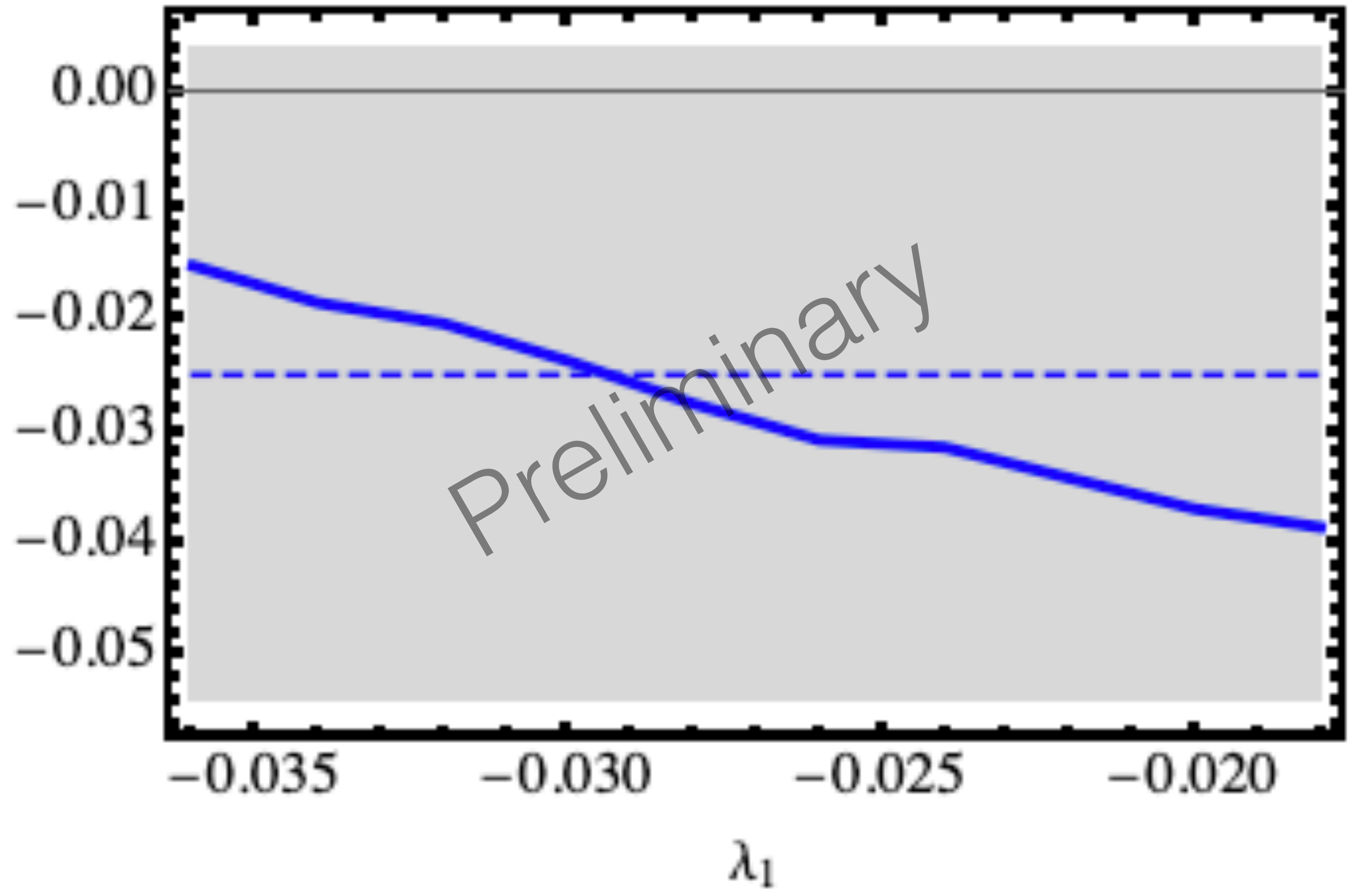
- Minus sign comes from V-A current in penguin diagram.
- Non-factorizable contributions, benefitted by PQCD.

Dependence on the input parameter

$BR(\Lambda_b^0 \to p\pi^-)$



$A_{CP}(\Lambda_b^0 \to p\pi^-)$



- λ_1 is one parameter in the proton LCDA. Within the allowed region of λ_1
- CPV is less sensitive to the input parameters of LCDAs.

Prospects: LCDAs

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    - Higher twists.

How to predict CPV in multi-body decays?

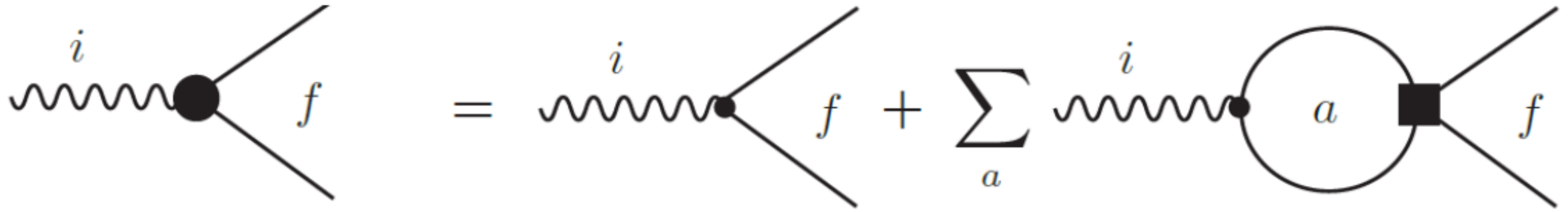
$$\Lambda_b^0 \to p\pi^-,\ pK^-$$

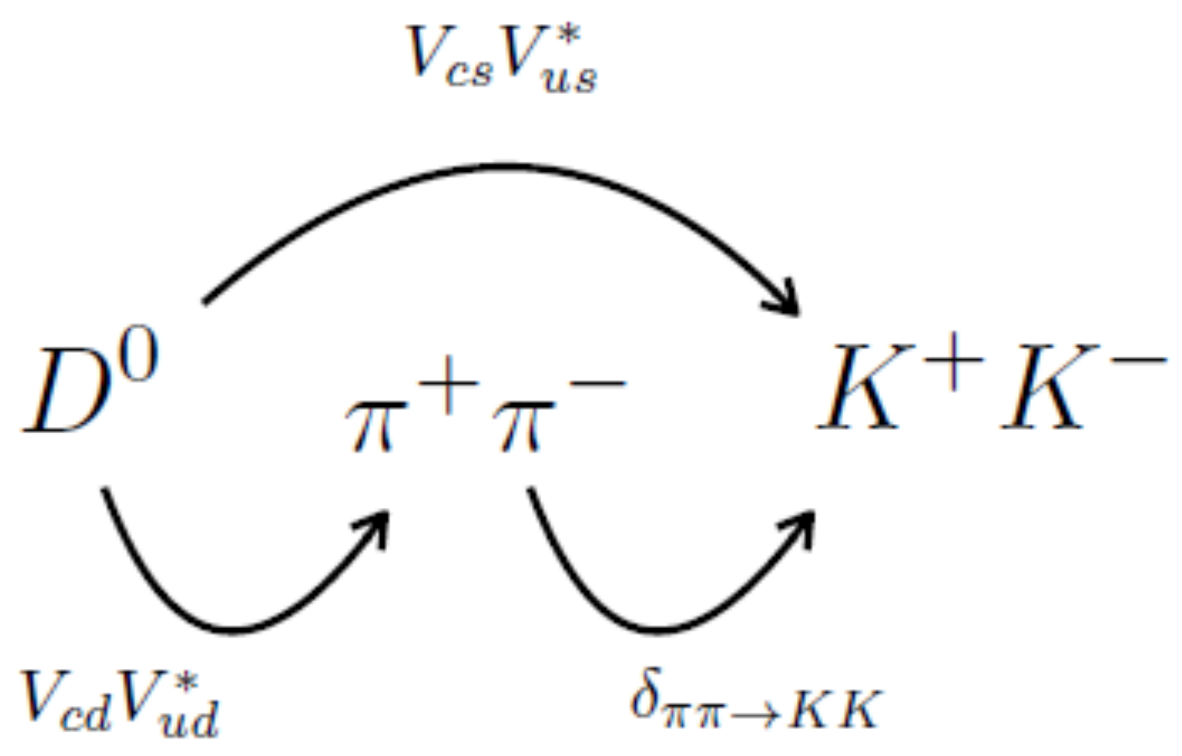
$$\Lambda_b^0 \to pK_S^0\pi^-,\ pK_S^0K^-,\ p\pi^0\pi^-,\ p\pi^0K^-$$

$$\Lambda_b^0 \to \Lambda^0K^+K^-,\ \Lambda^0K^+\pi^-,\ \Lambda^0\pi^+\pi^-$$

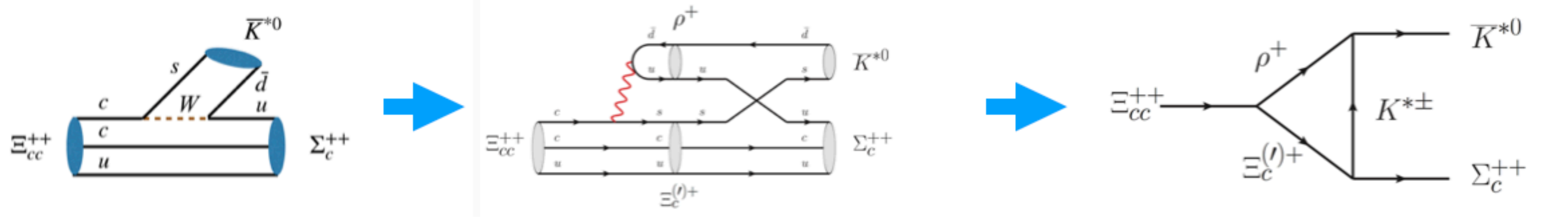
$$\Lambda_b^0 \to p\pi^+\pi^-\pi^-,\ pK^-\pi^+\pi^-,\ pK^-K^+\pi^-,\ pK^-K^+K^-$$

Final-state interactions





- Rescattering mechanism for charm CPV, Data-driven extraction of the $\pi\pi \to KK$ scatterings [Bediaga, Frederico, Magalhaes, PRL2023; Pich, Solomonidi, Silva, PRD2023].
- Rescattering mechanism have been successfully used to predict the discovery channel of $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ [FSY, Jiang, Li, Lu, Wang, Zhao, '17]

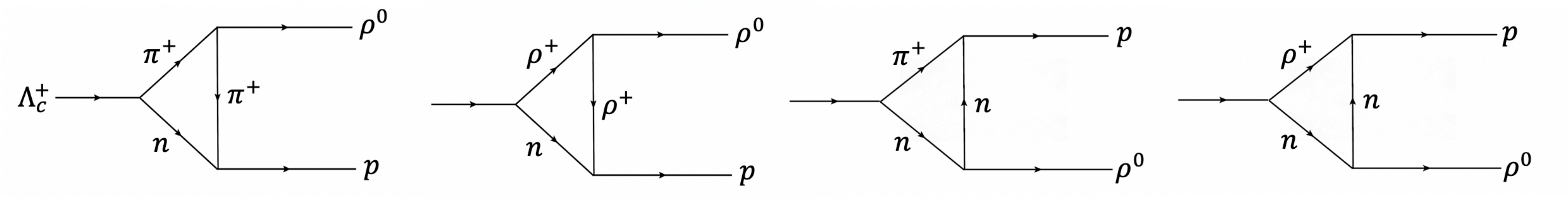


Topology: C — quark exchange — hadronic triangle

Triangle diagrams

- Much more channels are included in the rescattering mechanism

$V_{ud}V_{cd}^{*}$



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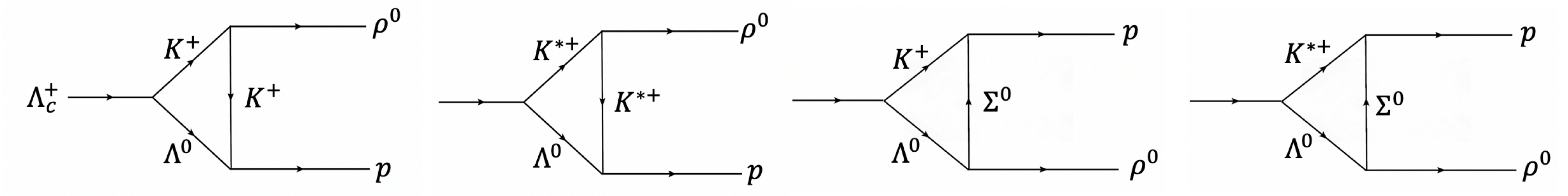

CPV can be easily obtained within the rescattering mechanism

$$\lambda_d\, A_d + \lambda_s\, A_s$$

Branching Ratios

Only one parameter explains all the 8 experimental data!

表 I: The branching ratio of $\Lambda_c^+ \to \mathcal{B}_8 V$ processes with $\eta = 0.6 \pm 0.1$.

Decay modes	Topology	$\mathcal{B}R_{\rm SD}(\%)$	$\mathcal{B}R_{\rm LD}(\%)$	$\mathcal{B}R_{\rm tot}(\%)$	$\mathcal{B}R_{\rm exp}(\%)$
$\Lambda_c^+ \to \Lambda^0 \rho^+$	T, C', E_2, B	6.12	$2.30^{+1.18}_{-1.94}$	$6.26^{+2.44}_{-1.39}$	4.06 ± 0.52
$\Lambda_c^+ \to \Sigma^+ \rho^0$	C', E_2, B	–	–	$0.77^{+1.38}_{-0.53}$	< 1.7
$\Lambda_c^+ \to \Sigma^+ \omega$	C', E_2, B	–	–	$2.06^{+0.40}_{-1.78}$	1.7 ± 0.21
$\Lambda_c^+ \to \Sigma^+ \phi$	E_1	–	–	$0.33^{+0.08}_{-0.29}$	0.39 ± 0.06
$\Lambda_c^+ \to p \bar{K}^{*0}$	C, E_1	3.26×10^{-3}	$3.76^{+1.37}_{-3.43}$	$3.70^{+1.29}_{-3.39}$	1.96 ± 0.27
$\Lambda_c^+ \to \Xi^0 K^{*+}$	E_2, B	–	–	$1.94^{+0.40}_{-1.68}$	–
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$\Lambda_c^+ \to \Lambda^0 K^{*+}$	T, C', E_2, B	2.92	$2.78^{+1.28}_{-1.02}$	$4.71^{+0.48}_{-0.20}$	–
$\Lambda_c^+ \to \Sigma^0 K^{*+}$	C', E_2, B	–	–	$1.60^{+0.89}_{-0.62}$	–
$\Lambda_c^+ \to \Sigma^+ K^{*0}$	C', E_1	–	–	$2.10^{+1.37}_{-0.86}$	3.5 ± 1.0
$\Lambda_c^+ \to p \phi$	C	1.78×10^{-3}	$1.44^{+1.14}_{-0.66}$	$1.37^{+1.13}_{-0.65}$	1.06 ± 0.14
$\Lambda_c^+ \to p \omega$	C, C', E_1, E_2, B	1.48×10^{-3}	$1.28^{+0.46}_{-0.37}$	$1.26^{+0.45}_{-0.37}$	0.83 ± 0.11
$\Lambda_c^+ \to p \rho^0$	C, C', E_1, E_2, B	1.81×10^{-3}	$2.79^{+1.89}_{-1.29}$	$2.72^{+1.87}_{-1.27}$	–
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$\Lambda_c^+ \to p K^{*0}$	C, C'	9.28×10^{-4}	$0.53^{+3.67}_{-0.38}$	$0.55^{+3.71}_{-0.39}$	–
$\Lambda_c^+ \to n K^{*+}$	T, C'	3.66	$0.44^{+1.64}_{-0.30}$	$5.08^{+1.95}_{-0.66}$	–

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$$BR(\Lambda_c^+ \to p\pi^+\pi^-) = (4.60 \pm 0.26) \times 10^{-3}$$

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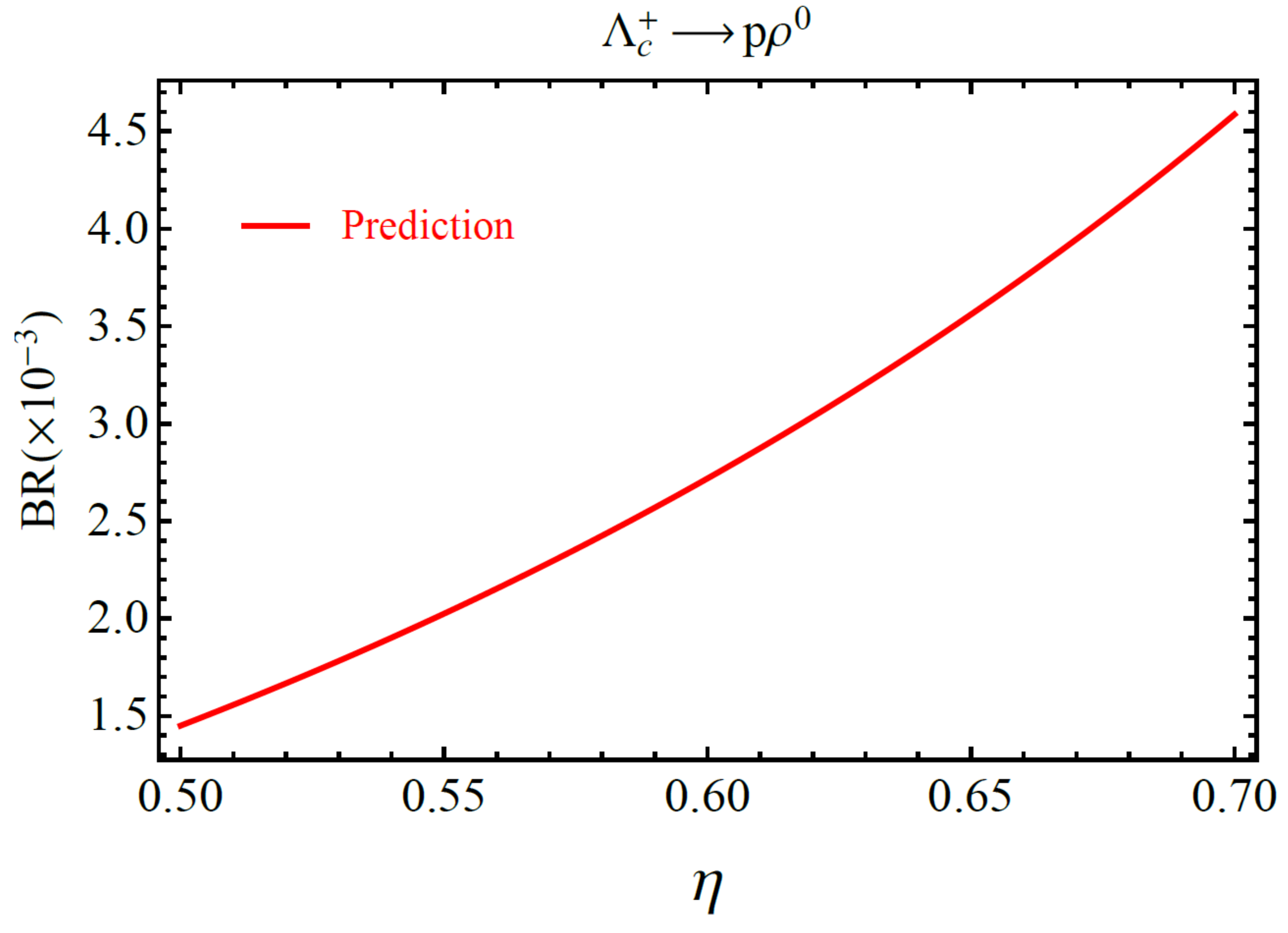
Dependence on η

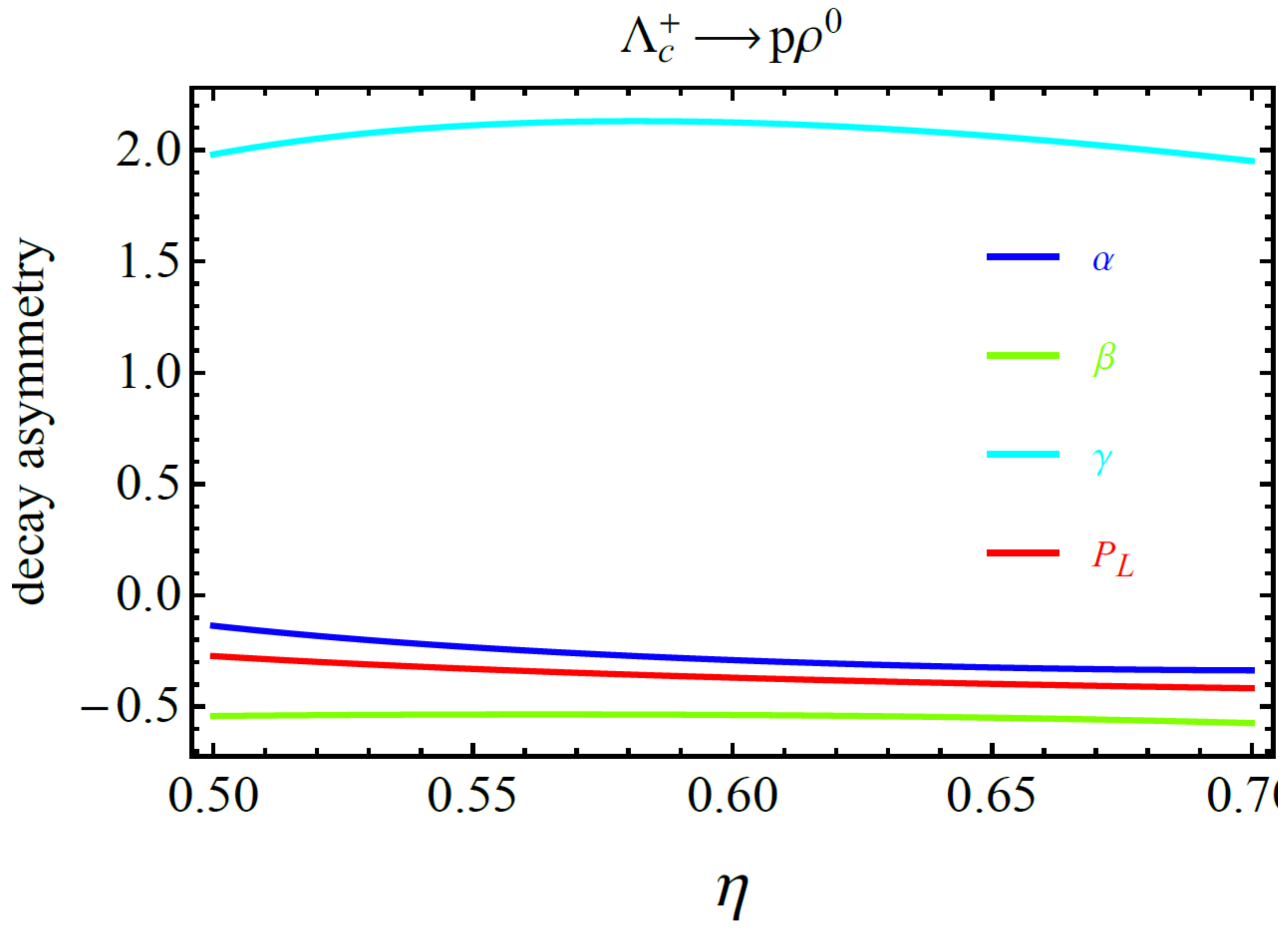
Branching fractions



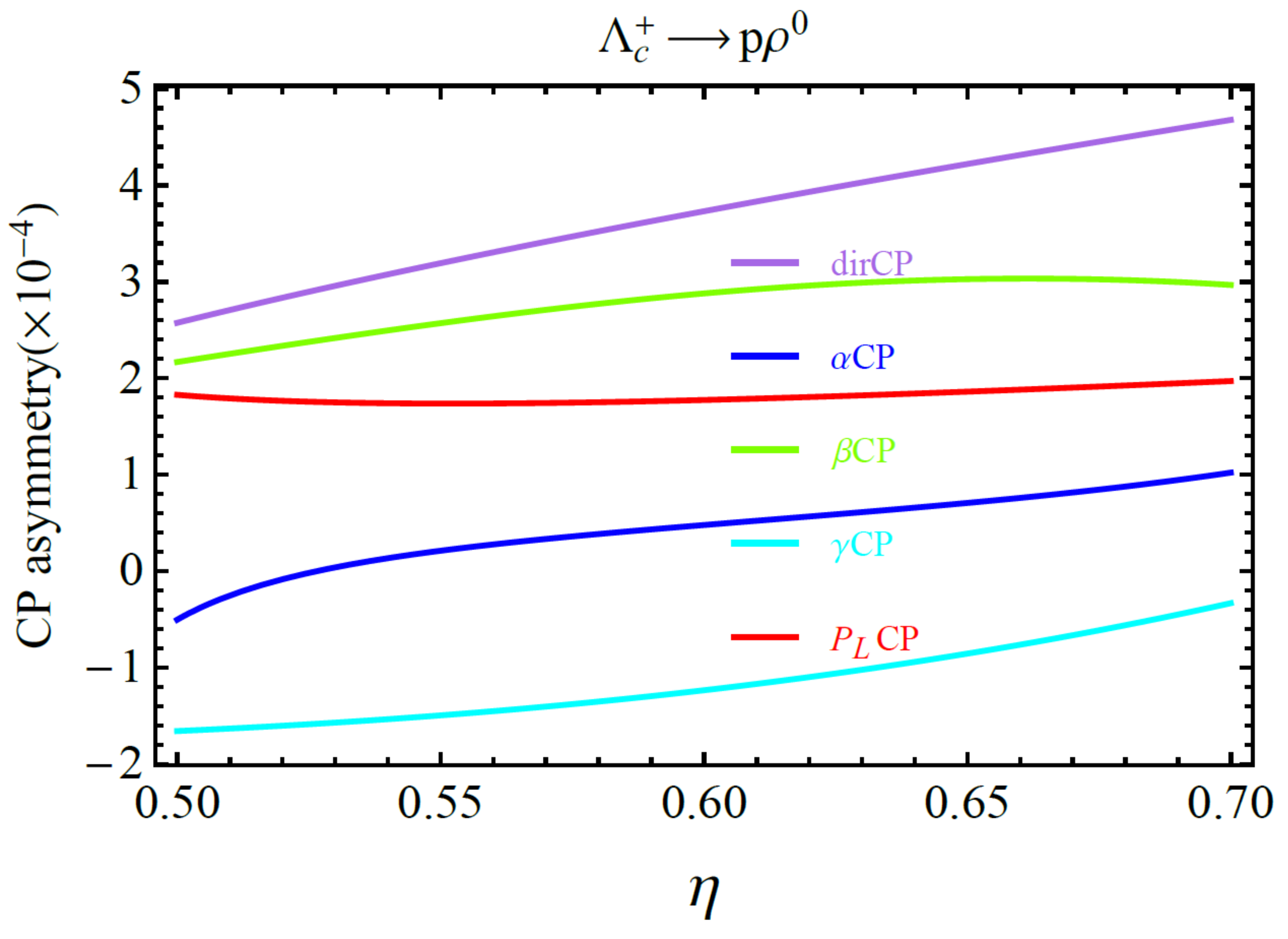
- Branching fractions are very sensitive to η

$$\Gamma \propto \quad 2(|S|^2 + |P_2^2|) + \frac{E_V^2}{m_V^2}(|S+D|^2 + |P_1|^2)$$

Decay asymmetries



CP violations



- The decay asymmetries and CPV are insensitive to η, whose dependences are mostly cancelled by the ratios

$$\alpha = \frac{\left|H_{1,\frac{1}{2}}\right|^2 - \left|H_{-1,-\frac{1}{2}}\right|^2}{\left|H_{1,\frac{1}{2}}\right|^2 + \left|H_{-1,-\frac{1}{2}}\right|^2} \qquad A_{CP} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

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CPV in Λ_b decay by FSI

- Reasonable strong phases
- Can predict more processes:

$$\Lambda_b \to p\pi^-, pK^-, \Lambda\phi, \qquad \Lambda_b \to p\rho^-, pK^{*-}, pa_1(1260)^-, pK_1^-(1270,1400), \qquad \Xi_b^0 \to pK^-$$

$$\Lambda_b \to \Lambda^0 + \phi/f_0(980),\ \Lambda^0 + \rho^0/f_0(500,980),\ p + K^{*-}/\kappa(700),\ p + K_{0,2}^{*-}(1430)$$

$$\Lambda_b \to N^*(1520,1535) + K_S^0,\ N^* + \overline{K}^{*0}/\kappa(700),\ N^* + \rho^0/f_0(500),\ N^* + \phi/f_0(980)$$

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- Prospects: Strong couplings with $N^*(1520)$, $N^*(1535)$, $\Lambda^*(1520)$

Observables

- Baryons have nonzero spins which can construct more observables and thus are helpful to find large CPV for measurements.
- Direct CPV in the decays: $a_{CP}^{\mathrm{dir}} \propto \sin\delta_s \sin\phi_w$. Sensitive to the strong phases.

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- Momentum $\vec{p}$ and spin $\vec{s}$ are odd under T operation. T-odd triple product: $(\vec{s}_1 \times \vec{s}_2) \cdot \vec{p}$
- Example (1): $\vec{s}_i \times \vec{s}_f \cdot \vec{p}$ measures the β parameter in $\Lambda \to p\pi$ [Lee, Yang, 1957]

  It was found that $a_{CP}^{\beta} \propto \beta + \bar{\beta} \propto \cos\delta_s \sin\phi_w$ [Donoghue, Pakvasa, 1985]

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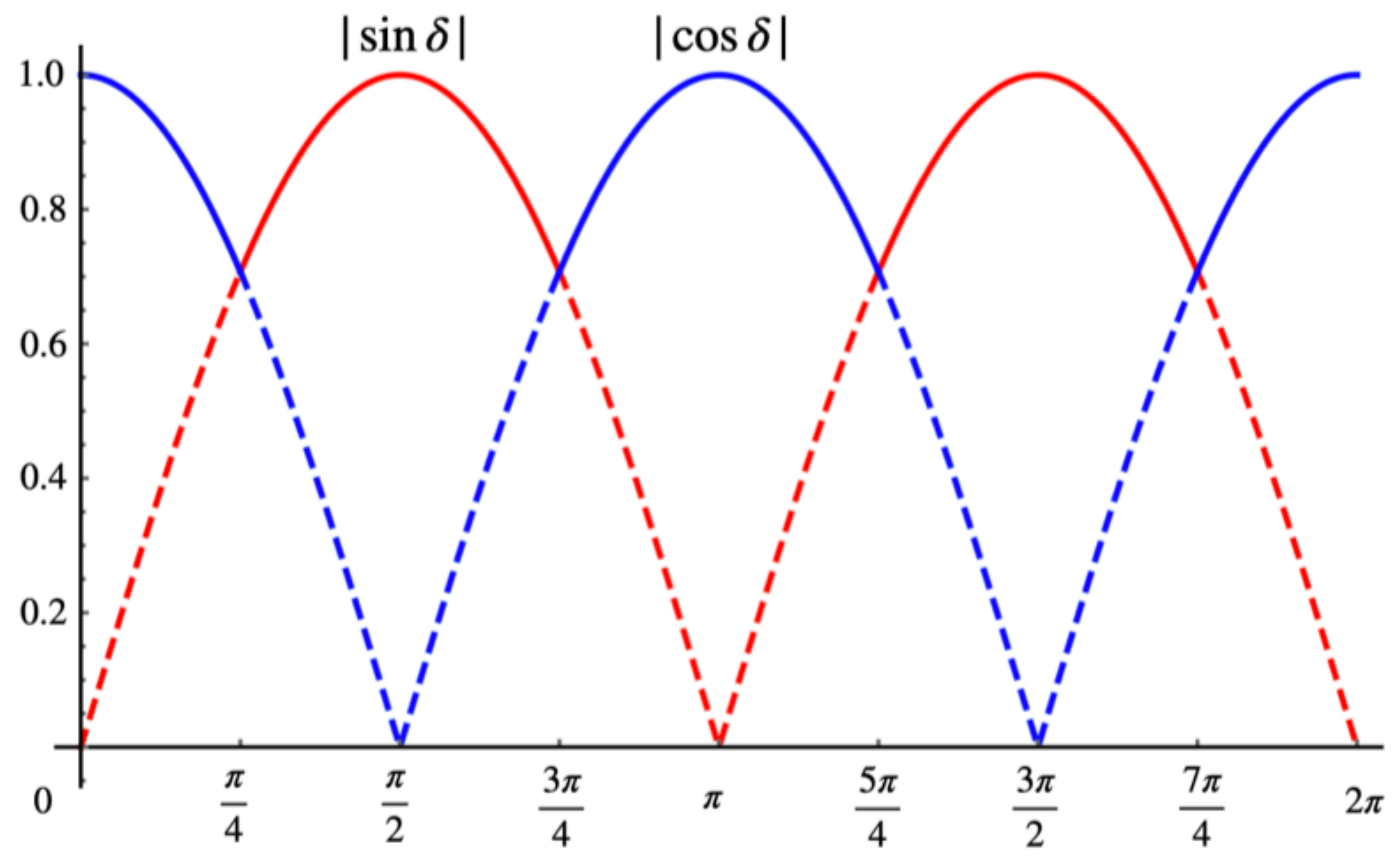
- Example (2): It was proposed to measure $A_B \propto N(\vec{p}\cdot\vec{\epsilon}_1 \times \vec{\epsilon}_2 > 0) - N(\vec{p}\cdot\vec{\epsilon}_1 \times \vec{\epsilon}_2 < 0)$

  in $B \to VV$, whose CPV is $A_B + A_{\overline{B}} \propto \cos\delta_s \sin\phi_w$ [Valencia, 1989]

Complementarity: $\cos\delta_s$ vs $\sin\delta_s$

- Precise prediction on strong phases is far beyond control currently
- Complimentary CPV observables proportional to $\sin\delta$ or $\cos\delta$ cover all the $(0,\ 2\pi)$ region
- Whatever the strong phase is, either $|\sin\delta|$ or $|\cos\delta|$ would be larger than 0.7 which is large enough for measurements

$a_{CP}^{(2)}$ $\qquad$ $a_{CP}^{(1)}$



- But keep in mind that not all the CPV observables of $\cos\delta$ and $\sin\delta$ are exactly complementary, since they might have different strong phases.

$$a_{CP}^{(1)} \propto \cos\delta_s \sin\phi_w$$

$$a_{CP}^{(2)} \propto \sin\delta_s \sin\phi_w$$

Why $\cos\delta_s$? What conditions?

[J.P.Wang, Q.Qin, FSY, 2211.07332]

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- To find the exactly complementary observables, we should know
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- **Two conditions:**

(1) For a basis of final states and a unitary transformation so that $UT|\psi_n\rangle = e^{i\alpha}|\psi_n\rangle$

(2) Q_- is invariant under this unitary transformation, $UQ_-U^\dagger = Q_-$

[J.P.Wang, Q.Qin, FSY, 2211.07332]

Why $\cos\delta_s$? What conditions?

- **Proof:**

$$\begin{aligned}\langle f|Q_-|f\rangle &= \langle i|S^\dagger Q_- S|i\rangle \\ &= \sum_{m,n}\langle\psi_i|S^\dagger|\psi_m\rangle\langle\psi_m|Q_-|\psi_n\rangle\langle\psi_n|S|\psi_i\rangle \\ &= \sum_{m,n} A_m^* A_n \langle\psi_m|Q_-|\psi_n\rangle \ .\end{aligned}$$

pure imaginary

$$\begin{aligned}\langle\psi_m|Q_-|\psi_n\rangle &= \langle\psi_m|\mathcal{T}^\dagger\mathcal{T}\, Q_-|\psi_n\rangle^* \\ &= -\langle\psi_m|\mathcal{T}^\dagger Q_- \mathcal{T}|\psi_n\rangle^* \\ &= -\langle\psi_m|\mathcal{T}^\dagger\, \mathcal{U}^\dagger\mathcal{U}\, Q_-\, \mathcal{U}^\dagger\mathcal{U}\, \mathcal{T}|\psi_n\rangle^* \\ &= -\langle\psi_m|\mathcal{T}^\dagger\mathcal{U}^\dagger\, Q_-\, \mathcal{U}\mathcal{T}|\psi_n\rangle^* \\ &= -\langle\psi_m|Q_-|\psi_n\rangle^* \ ,\end{aligned}$$

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\end{aligned}
$$

$$\langle f|Q_-|f\rangle \propto \sum_{m,n} Im(A_m^* A_n)$$

[J.P.Wang, Q.Qin, FSY, 2211.07332]

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$$\begin{aligned}\langle f|Q_-|f\rangle &= \langle i|S^\dagger Q_- S|i\rangle \\ &= \sum_{m,n}\langle\psi_i|S^\dagger|\psi_m\rangle\langle\psi_m|Q_-|\psi_n\rangle\langle\psi_n|S|\psi_i\rangle \\ &= \sum_{m,n} A_m^* A_n \langle\psi_m|Q_-|\psi_n\rangle \ .\end{aligned}$$

pure imaginary

$$\begin{aligned}\langle\psi_m|Q_-|\psi_n\rangle &= \langle\psi_m|\mathcal{T}^\dagger\mathcal{T}\,Q_-|\psi_n\rangle^* \\ &= -\langle\psi_m|\mathcal{T}^\dagger Q_-\mathcal{T}|\psi_n\rangle^* \\ &= -\langle\psi_m|\mathcal{T}^\dagger\,\mathcal{U}^\dagger\mathcal{U}\,Q_-\,\mathcal{U}^\dagger\mathcal{U}\,\mathcal{T}|\psi_n\rangle^* \\ &= -\langle\psi_m|\mathcal{T}^\dagger\mathcal{U}^\dagger\,Q_-\,\mathcal{U}\mathcal{T}|\psi_n\rangle^* \\ &= -\langle\psi_m|Q_-|\psi_n\rangle^* \ ,\end{aligned}$$

$$\langle f|Q_-|f\rangle \propto \sum_{m,n} Im(A_m^* A_n)$$

$$A_{\rm CP}^{Q_-} \equiv \frac{\langle f|Q_-|f\rangle - \langle\bar f|\bar Q_-|\bar f\rangle}{\langle f|Q_-|f\rangle + \langle\bar f|\bar Q_-|\bar f\rangle} \quad \propto \sum_{m,n} Im(A_m^* A_n - \bar A_m^* \bar A_n) \propto \cos\delta_s \sin\phi_w$$

Quod erat demonstrandum.

[J.P.Wang, Q.Qin, FSY, 2211.07332]

CPV induced by T-odd and T-even

$$a_{CP}^{\mathrm{T-odd}} \propto \sum_{m,n} Im(A_m^* A_n - \bar{A}_m^* \bar{A}_n) \propto \cos\delta_s \sin\phi_w$$

$$a_{CP}^{\mathrm{T-even}} \propto \sum_{m,n} Re(A_m^* A_n - \bar{A}_m^* \bar{A}_n) \propto \sin\delta_s \sin\phi_w$$

- Example: $\Lambda_c^+ \to \Lambda^0 K^+$, Lee-Yang decay-asymmetry parameter

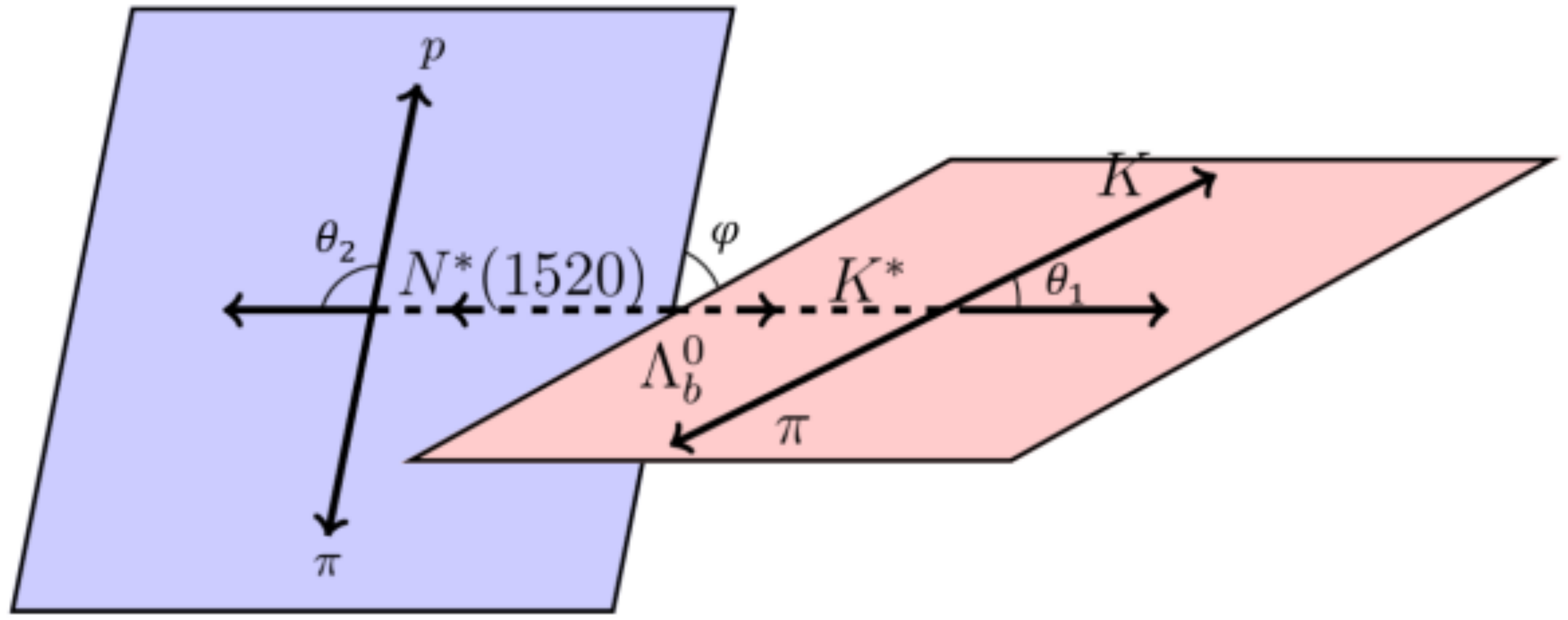
T-even: $\vec{s}_i \cdot \vec{p}$ $\qquad \alpha \propto Re[S^*P]$ $\qquad a_{CP}^{\alpha} = \dfrac{\alpha + \overline{\alpha}}{\alpha - \overline{\alpha}} \propto \sin\delta$

T-odd: $(\vec{s}_i \times \vec{s}_f) \cdot \vec{p}$ $\qquad \beta \propto Im[S^*P]$ $\qquad a_{CP}^{\beta} = \dfrac{\beta + \overline{\beta}}{\beta - \overline{\beta}} \propto \cos\delta$

complimentary

Angular distributions

$$
\begin{aligned}
\frac{d\Gamma}{dc_1\,dc_2\,d\varphi} \propto\; & -\frac{s_1^2 s_2^2}{\sqrt{3}} \mathrm{Im}\left(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{-1,-\frac{1}{2}} + \mathcal{H}_{+1,+\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}\right)\sin 2\varphi \\
& + \frac{s_1^2 s_2^2}{\sqrt{3}} \mathrm{Re}\left(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{-1,-\frac{1}{2}} + \mathcal{H}_{+1,+\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}\right)\cos 2\varphi \\
& - \frac{4 s_1 c_1 s_2 c_2}{\sqrt{6}} \mathrm{Im}\left(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{0,+\frac{1}{2}} + \mathcal{H}_{0,-\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}\right)\sin \varphi \\
& + \frac{4 s_1 c_1 s_2 c_2}{\sqrt{6}} \mathrm{Re}\left(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{0,+\frac{1}{2}} + \mathcal{H}_{0,-\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}\right)\cos \varphi
\end{aligned}
$$

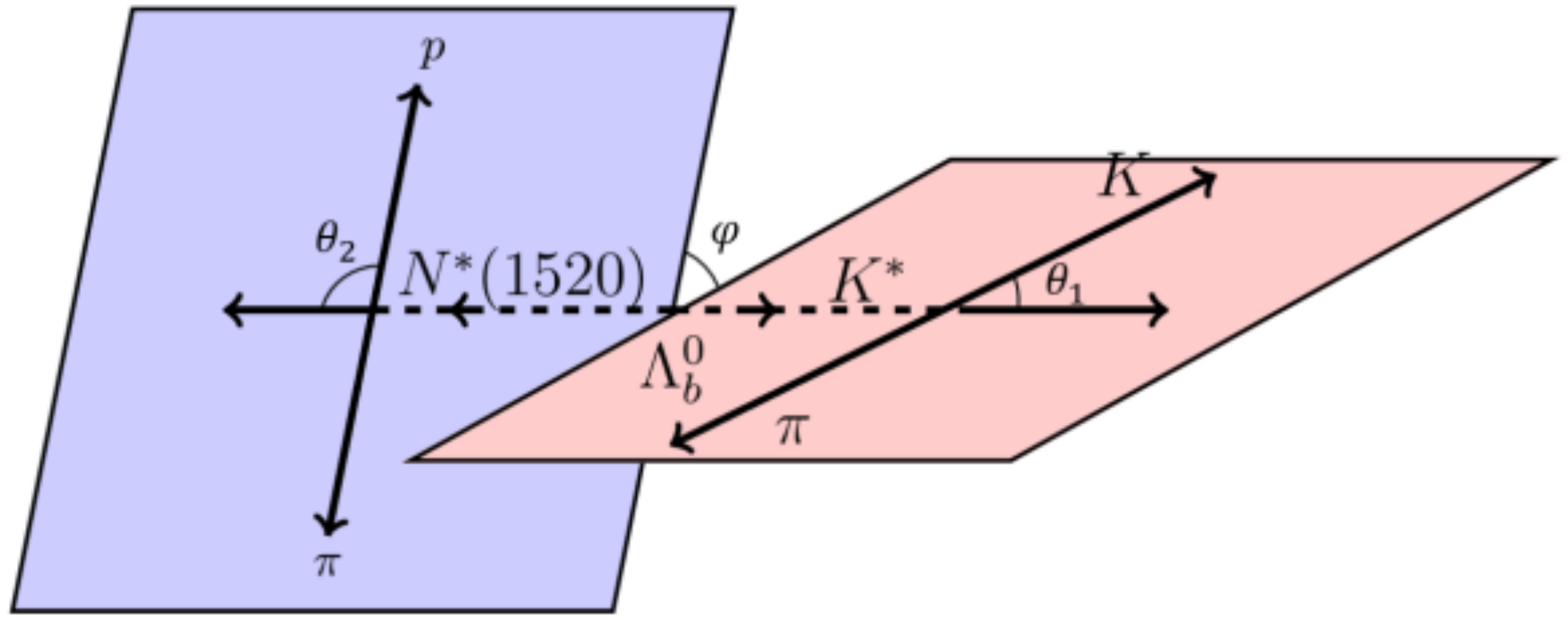


$$\sin\varphi = (\vec{n}_a \times \vec{n}_b)\cdot\hat{p}_b = \vec{n}_a\cdot(\vec{n}_b\times\hat{p}_b) \propto (\vec{p}_1\times\vec{p}_2)\cdot\vec{p}_4$$

$$\sin 2\varphi = 2\sin\varphi\cos\varphi \propto [(\vec{p}_1\times\vec{p}_2)\cdot(\vec{p}_3\times\vec{p}_4)][(\vec{p}_1\times\vec{p}_2)\cdot\vec{p}_4].$$

Angular distributions

$$\frac{d\Gamma}{dc_1\,dc_2\,d\varphi} \propto -\frac{s_1^2 s_2^2}{\sqrt{3}}\mathrm{Im}\left(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{-1,-\frac{1}{2}}+\mathcal{H}_{+1,+\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}\right)\sin 2\varphi$$
$$+\frac{s_1^2 s_2^2}{\sqrt{3}}\mathrm{Re}\left(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{-1,-\frac{1}{2}}+\mathcal{H}_{+1,+\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}\right)\cos 2\varphi$$
$$-\frac{4s_1c_1s_2c_2}{\sqrt{6}}\mathrm{Im}\left(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{0,+\frac{1}{2}}+\mathcal{H}_{0,-\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}\right)\sin\varphi$$
$$+\frac{4s_1c_1s_2c_2}{\sqrt{6}}\mathrm{Re}\left(\mathcal{H}_{+1,+\frac{3}{2}}\mathcal{H}^*_{0,+\frac{1}{2}}+\mathcal{H}_{0,-\frac{1}{2}}\mathcal{H}^*_{-1,-\frac{3}{2}}\right)\cos\varphi$$



$$\sin\varphi = (\vec{n}_a \times \vec{n}_b)\cdot\hat{p}_b = \vec{n}_a\cdot(\vec{n}_b\times\hat{p}_b) \propto (\vec{p}_1\times\vec{p}_2)\cdot\vec{p}_4$$

$$\sin 2\varphi = 2\sin\varphi\cos\varphi \propto [(\vec{p}_1\times\vec{p}_2)\cdot(\vec{p}_3\times\vec{p}_4)][(\vec{p}_1\times\vec{p}_2)\cdot\vec{p}_4].$$

- Triple-product of momentum, $(\vec{p}_1\times\vec{p}_2)\cdot\vec{p}_3$, is not good. $\sin\varphi$ with $\sin\theta_1\cos\theta_1\sin\theta_2\cos\theta_2$
- Angular distributions of resonant contributions are necessary. It is more clear in theory.

Summary

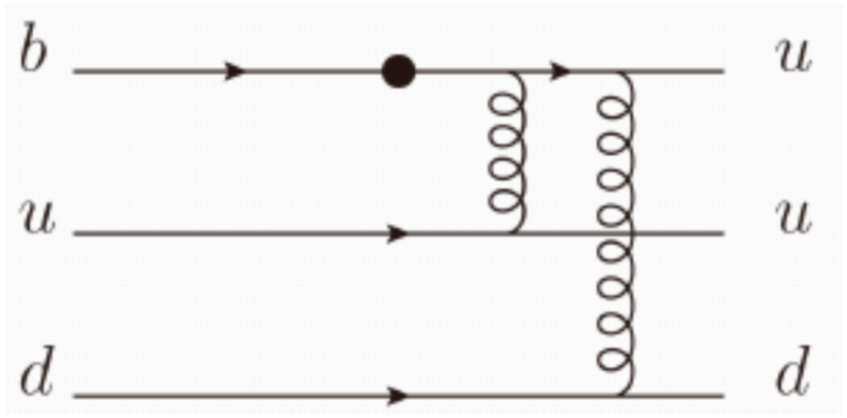
- Baryon physics is an opportunity of heavy flavor physics at the current stage.
- LHCb Run3 begins collecting more data.
- We are ready to predict CPV of heavy-flavor baryon decays.

Thank you very much!

Backups

Theoretical progresses: PQCD

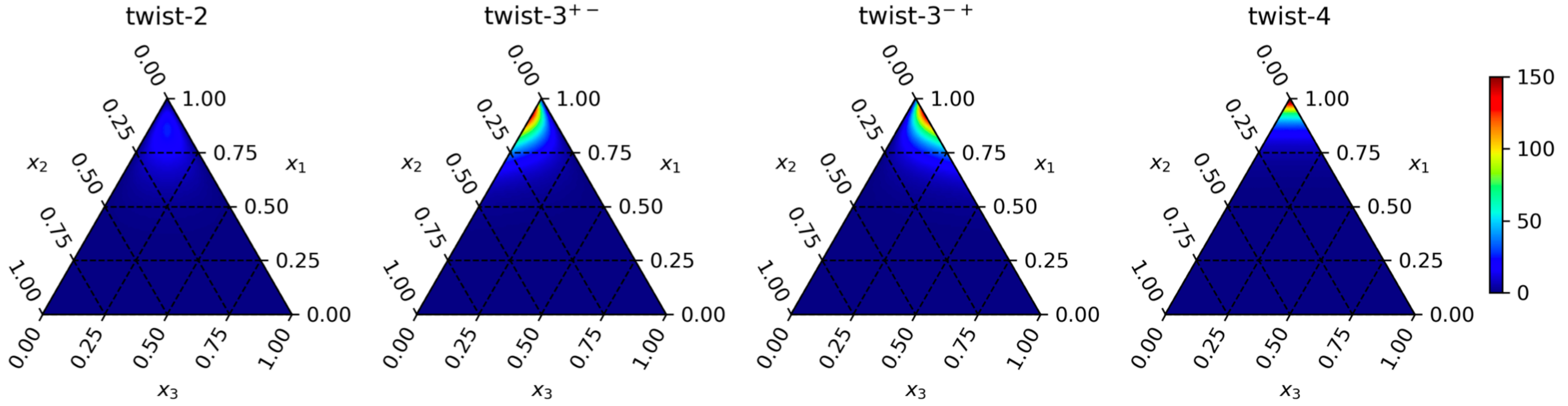
- Power-suppressed contribution incredibly surpasses the leading-power one



$$= \int [dx][db] \cdot \phi_{\Lambda_b} \cdot T_H \cdot \phi_p$$

$$r = \frac{m_p}{M_{\Lambda_b}}$$

D_7	twist-3	twist-4	twist-5	twist-6
twist-2	~ 0	$r \cdot 2\sqrt{2}(1-x_1)x_3$	$r^2 \cdot 2\sqrt{2}x_3$	$r^3 \cdot 4\sqrt{2}(1-x_1)(1-x_2')$
twist-3^{+-}	$x_3(1-x_1)$	$r \cdot x_3$	$r^2 \cdot (1-x_1)(1-x_2')$	~ 0
twist-3^{-+}	~ 0	$r \cdot x_3$	$r^2 \cdot (1-x_1)(1-x_2')$	$r^3 \cdot (1-x_2')$
twist-4	$4\sqrt{2}x_3$	$\boxed{r \cdot 2\sqrt{2}(1-x_1)(1-x_2')}$	$\boxed{r^2 \cdot 2\sqrt{2}(1-x_2')}$	~ 0



$$x_1 \to 1,\ x_{2,3} \to 0$$

J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, 2202.04804

Light-Cone Distribution Amplitudes: Λ_b

$$(Y_{\Lambda_b})_{\alpha\beta\gamma}(x_i,\mu) = \frac{1}{8\sqrt{2}N_c}\Big\{f^{(1)}_{\Lambda_b}(\mu)[M_1(x_2,x_3)\gamma_5 C^T]_{\gamma\beta}+f^{(2)}_{\Lambda_b}(\mu)[M_2(x_2,x_3)\gamma_5 C^T]_{\gamma\beta}\Big\}[\Lambda_b(p)]_\alpha$$

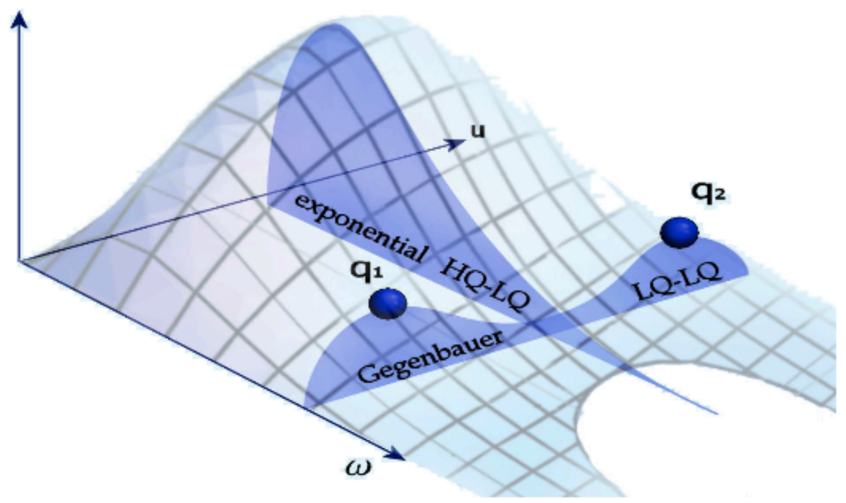
$$M_1(x_2,x_3) = \frac{\not{\bar{n}}\not{n}}{4}\psi_3^{+-}(x_2,x_3) + \frac{\not{n}\not{\bar{n}}}{4}\psi_3^{-+}(x_2,x_3),$$

$$M_2(x_2,x_3) = \frac{\not{n}}{\sqrt{2}}\psi_2(x_2,x_3) + \frac{\not{\bar{n}}}{\sqrt{2}}\psi_4(x_2,x_3),$$

$$(Y_{\Lambda_b})_{\alpha\beta\gamma}(x_i,\mu) = \frac{f'_{\Lambda_b}}{8\sqrt{2}N_c}[(\not{p}+m_{\Lambda_b})\gamma_5 C]_{\beta\gamma}[\Lambda_b(p)]_\alpha \psi(x_i,\mu),$$

$$\psi(x_i) = N x_1 x_2 x_3 \; exp\left(-\frac{m^2_{\Lambda_b}}{2\beta^2 x_1} - \frac{m_l^2}{2\beta^2 x_2} - \frac{m_l^2}{2\beta^2 x_3}\right),$$

Light-Cone Distribution Amplitudes: Λ_b



Model-I: Gegenbauer-1

$$\psi_2(x_2,x_3) = m_{\Lambda_b}^4 x_2 x_3 \left[\frac{1}{\epsilon_0^4} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_0} + a_2 C_2^{3/2}\left(\frac{x_2-x_3}{x_2+x_3}\right)\frac{1}{\epsilon_1^4} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_1}\right]$$
$$\psi_3^{+-}(x_2,x_3) = \frac{2m_{\Lambda_b}^3 x_2}{\epsilon_3^3} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_3},$$
$$\psi_3^{-+}(x_2,x_3) = \frac{2m_{\Lambda_b}^3 x_3}{\epsilon_3^3} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_3},$$
$$\psi_4(x_2,x_3) = \frac{5}{\mathcal{N}} m_{\Lambda_b}^2 \int_{m_{\Lambda_b}(x_2+x_3)/2}^{s_0} ds e^{-s/\tau}(s - m_{\Lambda_b}(x_2+x_3)/2)^3,$$

with the Gegenbauer moment $a_2 = 0.333^{0.250}_{-0.333}$, the Gegenbauer polynomial $C_2^{3/2}(x) = 3(5x^2-1)/2$, the parameters $\epsilon_0 = 200^{+130}_{-60}$ MeV, $\epsilon_1 = 650^{+650}_{-300}$ MeV and $\epsilon_3 = 230 \pm 60$

Ball, Braun, Gardi, 0804.2424, PLB 2008

Model-II: Gegenbauer-2

$$\psi_2(x_2,x_3) = m_{\Lambda_b}^4 x_2 x_3 \frac{a_2^{(2)}}{{\epsilon_2^{(2)}}^4} C_2^{3/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}/(x_2+x_3)/\epsilon_2^{(2)}},$$
$$\psi_3^{+-}(x_2,x_3) = m_{\Lambda_b}^3 (x_2+x_3)\left[\frac{a_2^{(3)}}{{\epsilon_2^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_2^{(3)}} + \frac{b_3^{(3)}}{{\eta_3^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\eta_3^{(3)}}\right]$$
$$\psi_3^{-+}(x_2,x_3) = m_{\Lambda_b}^3 (x_2+x_3)\left[\frac{a_2^{(3)}}{{\epsilon_2^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_2^{(3)}} - \frac{b_3^{(3)}}{{\eta_3^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\eta_3^{(3)}}\right]$$
$$\psi_4(x2,x3) = m_{\Lambda_b}^2 \frac{a_2^{(4)}}{{\epsilon_2^{(4)}}^2} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_2^{(4)}},$$

$a_2^{(2)} = 0.391 \pm 0.279$, $a_2^{(3)} = -0.161^{+0.108}_{-0.207}$, $a_2^{(4)} = -0.541^{+0.173}_{-0.09}$, $b_3^{(3)} = -0.24^{+0.24}_{-0.147}$, $\epsilon_2^{(2)} = 0.551^{+\infty}_{-0.356}$ GeV, $\epsilon_2^{(3)} = 0.055^{+0.01}_{-0.02}$ GeV, $\epsilon_2^{(4)} = 0.262^{+0.116}_{-0.132}$ GeV and $\eta_3^{(3)} = 0.633 \pm 0.099$ GeV.

Ali, Hambrock, Parkhomenko, W.Wang, 2012

Light-Cone Distribution Amplitudes: Λ_b

Model-III: Exponential

$$\begin{aligned}
\psi_2(x_2,x_3) &= \frac{x_2 x_3}{\omega_0^4} m_{\Lambda_b}^4 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0}, \\
\psi_3^{+-}(x_2,x_3) &= \frac{2x_2}{\omega_0^3} m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0}, \\
\psi_3^{-+}(x_2,x_3) &= \frac{2x_3}{\omega_0^3} m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0}, \\
\psi_4(x_2,x_3) &= \frac{1}{\omega_0^2} m_{\Lambda_b}^2 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},
\end{aligned}$$

$\omega_0 = 0.4$ GeV

Model-IV: Free Parton

$$\begin{aligned}
\psi_2(x_2,x_3) &= \frac{15 x_2 x_3 m_{\Lambda_b}^4 (2\bar\Lambda - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})}{4\bar\Lambda^5} \Theta(2\bar\Lambda - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b}) \\
\psi_3^{+-}(x_2,x_3) &= \frac{15 x_2 m_{\Lambda_b}^3 (2\bar\Lambda - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})^2}{4\bar\Lambda^5} \Theta(2\bar\Lambda - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b}), \\
\psi_3^{-+}(x_2,x_3) &= \frac{15 x_3 m_{\Lambda_b}^3 (2\bar\Lambda - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})^2}{4\bar\Lambda^5} \Theta(2\bar\Lambda - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b}), \\
\psi_4(x_2,x_3) &= \frac{5 m_{\Lambda_b}^2 (2\bar\Lambda - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})^3}{8\bar\Lambda^5} \Theta(2\bar\Lambda - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b}),
\end{aligned}$$

$\bar\Lambda \equiv (m_{\Lambda_b} - m_b)/2 \approx 0.8$ GeV

Bell, Feldmann, Y.M.Wang, Yip, 1308.6114, JHEP2013

Light-Cone Distribution Amplitudes: proton

$$\langle 0|\, \varepsilon^{ijk} u_\alpha^{i'}(a_1 z)\, [a_1 z, a_0 z]_{i',i}\, u_\beta^{j'}(a_2 z)\, [a_2 z, a_0 z]_{j',j}\, d_\gamma^{k'}(a_3 z)\, [a_3 z, a_0 z]_{k',k}\, |P(P,\lambda)\rangle$$

$$\begin{aligned}
&4\,\langle 0|\, \varepsilon^{ijk} u_\alpha^i(a_1 z) u_\beta^j(a_2 z) d_\gamma^k(a_3 z)\,|P\rangle = \\
&= S_1 M C_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + S_2 M C_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma + P_1 M\,(\gamma_5 C)_{\alpha\beta}\, N_\gamma^+ + P_2 M\,(\gamma_5 C)_{\alpha\beta}\, N_\gamma^- \\
&\quad + V_1\,(\not{p}C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + V_2\,(\not{p}C)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma + \frac{V_3}{2} M\,(\gamma_\perp C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^+\right)_\gamma \\
&\quad + \frac{V_4}{2} M\,(\gamma_\perp C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^-\right)_\gamma + V_5 \frac{M^2}{2pz}\,(\not{z}C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + \frac{M^2}{2pz} V_6\,(\not{z}C)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma \\
&\quad + A_1\,(\not{p}\gamma_5 C)_{\alpha\beta}\, N_\gamma^+ + A_2\,(\not{p}\gamma_5 C)_{\alpha\beta}\, N_\gamma^- + \frac{A_3}{2} M\,(\gamma_\perp \gamma_5 C)_{\alpha\beta}\left(\gamma^\perp N^+\right)_\gamma \\
&\quad + \frac{A_4}{2} M\,(\gamma_\perp \gamma_5 C)_{\alpha\beta}\left(\gamma^\perp N^-\right)_\gamma + A_5 \frac{M^2}{2pz}\,(\not{z}\gamma_5 C)_{\alpha\beta}\, N_\gamma^+ + \frac{M^2}{2pz} A_6\,(\not{z}\gamma_5 C)_{\alpha\beta}\, N_\gamma^- \\
&\quad + T_1\,(i\sigma_{\perp p} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^+\right)_\gamma + T_2\,(i\sigma_{\perp p} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^-\right)_\gamma + T_3 \frac{M}{pz}\,(i\sigma_{pz} C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma \\
&\quad + T_4 \frac{M}{pz}\,(i\sigma_{zp} C)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma + T_5 \frac{M^2}{2pz}\,(i\sigma_{\perp z} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^+\right)_\gamma + \frac{M^2}{2pz} T_6\,(i\sigma_{\perp z} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^-\right)_\gamma \\
&\quad + M \frac{T_7}{2}\,(\sigma_{\perp\perp'} C)_{\alpha\beta}\left(\sigma^{\perp\perp'} \gamma_5 N^+\right)_\gamma + M \frac{T_8}{2}\,(\sigma_{\perp\perp'} C)_{\alpha\beta}\left(\sigma^{\perp\perp'} \gamma_5 N^-\right)_\gamma\,, \qquad (2.9)
\end{aligned}$$

Braun, Fries, Mahnke, Stein,
hep-ph/0007279, NPB 2000

Light-Cone Distribution Amplitudes: proton

- Twist-3 LCDAs

$$V_1(x_i) = 120x_1x_2x_3[\phi_3^0 + \phi_3^+(1-3x_3)],$$
$$A_1(x_i) = 120x_1x_2x_3(x_2-x_1)\phi_3^-,$$
$$T_1(x_i) = 120x_1x_2x_3[\phi_3^0 + \frac{1}{2}(\phi_3^- - \phi_3^+)(1-3x_3)].$$

- Twist-4 LCDAs

$$V_2(x_i) = 24x_1x_2[\phi_4^0 + \phi_4^+(1-5x_3)],$$
$$V_3(x_i) = 12x_3[\psi_4^0(1-x_3) + \psi_4^-(x_1^2+x_2^2-x_3(1-x_3)) + \psi_4^+(1-x_3-10x_1x_2)],$$
$$A_2(x_i) = 24x_1x_2(x_2-x_1)\phi_4^-,$$
$$A_3(x_i) = 12x_3(x_2-x_1)[(psi_4^0 + \psi_4^+) + \psi_4^-(1-2x_3)],$$
$$T_2(x_i) = 24x_1x_2[\xi_4^0 + \xi_4^+(1-5x_3)],$$
$$T_3(x_i) = 6x_3[(\xi_4^0+\phi_4^0+\psi_4^0)(1-x_3) + (\xi_4^- + \phi_4^- - \psi_4^-)(x_1^2+x_2^2-x_3(1-x_3)) + (\xi_4^+ + \phi_4^+ + \psi_4^+)(1-x_3-10x_1x_2)],$$
$$T_7(x_i) = 6x_3[(-\xi_4^0+\phi_4^0+\psi_4^0)(1-x_3) + (-\xi_4^- + \phi_4^- - \psi_4^-)(x_1^2+x_2^2-x_3(1-x_3)) + (-\xi_4^+ + \phi_4^+ + \psi_4^+)(1-x_3-10x_1x_2)],$$
$$S_1(x_i) = 6x_3(x_2-x_1)[(\xi_4^0+\phi_4^0+\psi_4^0+\xi_4^++\phi_4^++\psi_4^+) + (\xi_4^-+\phi_4^--\psi_4^-)(1-2x_3)],$$
$$P_1(x_i) = 6x_3(x_2-x_1)[(\xi_4^0-\phi_4^0-\psi_4^0+\xi_4^+-\phi_4^+-\psi_4^+) + (\xi_4^--\phi_4^-+\psi_4^-)(1-2x_3)].$$

- Twist-5 LCDAs

$$V_4(x_i) = 3[\psi_5^0(1-x_3) + \psi_5^-(2x_1x_2-x_3(1-x_3)) + \psi_5^+(1-x_3-2(x_1^2+x_2^2))],$$
$$V_5(x_i) = 6x_3[\phi_5^0 + \phi_5^+(1-2x_3)],$$
$$A_4(x_i) = 3(x_2-x_1)[-\psi_5^0 + \psi_5^- x_3 + \psi_5^+(1-2x_3)],$$
$$A_5(x_i) = 6x_3(x_2-x_1)\phi_5^-,$$
$$T_4(x_i) = \frac{3}{2}[(\xi_5^0+\psi_5^0+\phi_5^0)(1-x_3) + (\xi_5^-+\phi_5^--\psi_5^-)(2x_1x_2-x_3(1-x_3)) + (\xi_5^++\phi_5^++\psi_5^+)(1-x_3-2(x_1^2+x_2^2))],$$
$$T_5(x_i) = 6x_3[\xi_5^0 + \xi_5^+(1-2x_3)],$$
$$T_8(x_i) = \frac{3}{2}[(\psi_5^0+\phi_5^0-\xi_5^0)(1-x_3) + (\phi_5^- - \phi_5^- - \xi_5^-)(2x_1x_2-x_3(1-x_3)) + (\phi_5^++\phi_5^+-\xi_5^+)(\mu)(1-x_3-2(x_1^2+x_2^2))],$$
$$S_2(x_i) = \frac{3}{2}(x_2-x_1)[-(\psi_5^0+\phi_5^0+\xi_5^0) + (\xi_5^-+\phi_5^--\psi_5^0)x_3 + (\xi_5^++\phi_5^++\psi_5^0)(1-2x_3)],$$
$$P_2(x_i) = \frac{3}{2}(x_2-x_1)[(\psi_5^0+\phi_5^0-\xi_5^0) + (\xi_5^--\phi_5^-+\psi_5^0)x_3 + (\xi_5^+-\phi_5^+-\psi_5^0)(1-2x_3)].$$

- Twist-6 LCDAs

$$V_6(x_i) = 2[\phi_6^0 + \phi_6^+(1-3x_3)],$$
$$A_6(x_i) = 2(x_2-x_1)\phi_6^-,$$
$$T_6(x_i) = 2[\phi_6^0 + \frac{1}{2}(\phi_6^- - \phi_6^+)(1-3x_3)],$$

- LCDAs V_i, A_i, T_i, S_i, P_i are functions of parameters $\phi_i^{\pm,0}, \psi_i^{\pm,0}, \xi_i^{\pm,0}$

Braun, 2001

$$V_1(x_i) = 120x_1x_2x_3[\phi_3^0 + \phi_3^+(1-3x_3)],$$
$$A_1(x_i) = 120x_1x_2x_3(x_2-x_1)\phi_3^-,$$
$$T_1(x_i) = 120x_1x_2x_3[\phi_3^0 + \frac{1}{2}(\phi_3^- - \phi_3^+)(1-3x_3)].$$

- The parameters $\phi_i^{\pm,0}, \psi_i^{\pm,0}, \xi_i^{\pm,0}$ depend on 8 parameters

$$\phi_3^0 = \phi_6^0 = f_N, \qquad \phi_4^0 = \phi_5^0 = \frac{1}{2}(\lambda_1 + f_N),$$
$$\xi_4^0 = \xi_5^0 = \frac{1}{6}\lambda_2, \qquad \psi_4^0 = \psi_5^0 = \frac{1}{2}(f_N - \lambda_1).$$

$$\psi_4^+ = -\frac{1}{4}\Big(\lambda_1(-2+5f_1^d+5f_1^u) + f_N(2+5A_1^u-5V_1^d)\Big),$$
$$\xi_4^- = \frac{5}{16}\lambda_2(4-15f_2^d),$$
$$\xi_4^+ = \frac{1}{16}\lambda_2(4-15f_2^d),$$

$$\phi_4^- = \frac{5}{4}\Big(\lambda_1(1-2f_1^d-4f_1^u) + f_N(2A_1^u-1)\Big),$$
$$\phi_4^+ = \frac{1}{4}\Big(\lambda_1(3-10f_1^d) - f_N(10V_1^d-3)\Big),$$
$$\psi_4^- = -\frac{5}{4}\Big(\lambda_1(2-7f_1^d+f_1^u) + f_N(A_1^u+3V_1^d-2)\Big),$$

$$\phi_5^- = \frac{5}{3}\Big(\lambda_1(f_1^d-f_1^u) + f_N(2A_1^u-1)\Big),$$
$$\phi_5^+ = -\frac{5}{6}\Big(\lambda_1(4f_1^d-1) + f_N(3+4V_1^d)\Big),$$
$$\psi_5^- = \frac{5}{3}\Big(\lambda_1(f_1^d-f_1^u) + f_N(2-A_1^u-3V_1^d)\Big),$$
$$\psi_5^+ = \frac{5}{3}\Big(\lambda_1(-1+f_1^u) + f_N(1+A_1^u+V_1^d)\Big),$$
$$\xi_5^- = -\frac{5}{4}\lambda_2 f_2^d,$$
$$\xi_5^+ = \frac{5}{12}\lambda_2(2-3f_2^d),$$

$$\phi_6^- = \frac{1}{2}\Big(\lambda_1(1-4f_1^d-2f_1^u) + f_N(1+4A_1^u)\Big),$$
$$\phi_6^+ = \Big(\lambda_1(1-2f_1^d) + f_N(4V_1^d-1)\Big).$$

	$f_N(GeV^2)$	$\lambda_1(GeV^2)$	$\lambda_2(GeV^2)$	V_1^d	A_1^u	f_1^d	f_2^d	f_1^u
QCDSR(2001) [8]	$(5.3 \pm 0.5) \times 10^{-3}$	$-(2.7 \pm 0.9) \times 10^{-2}$	$(5.1 \pm 1.9) \times 10^{-2}$	0.23 ± 0.03	0.38 ± 0.15	0.6 ± 0.2	0.15 ± 0.06	0.22 ± 0.15
QCDSR(2006) [9]	$(5.0 \pm 0.5) \times 10^{-3}$	$-(2.7 \pm 0.9) \times 10^{-2}$	$(5.4 \pm 1.9) \times 10^{-2}$	0.23 ± 0.03	0.38 ± 0.15	0.4 ± 0.05	0.22 ± 0.05	0.07 ± 0.05
LCSR(2006) [9]	$(5.0 \pm 0.5) \times 10^{-3}$	$-(2.7 \pm 0.9) \times 10^{-2}$	$(5.4 \pm 1.9) \times 10^{-2}$	0.3	0.13	0.33	0.25	0.09

Light-Cone Distribution Amplitudes: proton

Table 2: Parameters in the proton LCDAs in units of 10^{-2} GeV^2 [73]. The accuracy of those parameters without uncertainties is of order of 50%.

	ϕ_i^0	ϕ_i^-	ϕ_i^+	ψ_i^0	ψ_i^-	ψ_i^+	ξ_i^0	ξ_i^-	ξ_i^+
twist-3 $(i=3)$	0.53 ± 0.05	2.11	0.57						
twist-4 $(i=4)$	-1.08 ± 0.47	3.22	2.12	1.61 ± 0.47	-6.13	0.99	0.85 ± 0.31	2.79	0.56
twist-5 $(i=5)$	-1.08 ± 0.47	-2.01	1.42	$1.61 \pm .047$	-0.98	-0.99	0.85 ± 0.31	-0.95	0.46
twist-6 $(i=6)$	0.53 ± 0.05	3.09	-0.25						

Parameters of LCDAs of proton

Model	Method	$f_N \cdot 10^3$ Gev^2	$\lambda_1 \cdot 10^3$ Gev^2	$\lambda_2 \cdot 10^3$ Gev^2	A_1^u	V_1^d	f_1^u	f_1^d	f_2^d	Ref.
	QCDSR	5.0(5)	-27(9)	54(19)						
ASY		-	-	-	0	1/3	1/10	3/10	4/15	
CZ	QCDSR	5.3(5)	-	-	0.47	0.22	-	-	-	[1]
KS	QCDSR	5.1(3)	-	-	0.34	0.24	-	-	-	[2]
COZ	QCDSR	5.0(3)	-	-	0.39	0.23	-	-	-	[3]
SB	QCDSR	-	-	-	0.38	0.24	-	-	-	[4]
BK	PQCD	6.64	-	-	0.08	0.31	-	-	-	[5]
BLW	QCDSR	-	-	-	0.38(15)	0.23(3)	0.07(5)	0.40(20)	0.22(5)	[6]
BLW	LCSR（LO）	-	-	-	0.13	0.30	0.09	0.33	0.25	[6]
ABO1	LCSR（NLO）	-	-	-	0.11	0.30	0.11	0.27	-	[7]
ABO2	LCSR（NLO）				0.11	0.30	0.11	0.29	-	[7]
LAT09	LATTICE	3.23 (63)	-35.57 (65)	70.02 (13)	0.19 (2)	0.20 (1)	-	-	-	[8]
LAT14	LATTICE	3.07 (36)	-38.77 (18)	77.64 (37)	0.07 (4)	0.31 (2)	-	-	-	[9]
LAT19	LATTICE	3.54（6）	-44.9 (42)	93.4 (48)	0.30 (32)	0.192 (22)	-	-	-	[10]

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