非平衡量子色动力学
Non-equilibrium QCD matter

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Outline

- **The QCD matter**
  - From the Big Bang to the Little Bang
- **Far-from-equilibrium QCD matter**
  - The turbulent nature of quark-gluon plasma (QGP)
- **Early stage of heavy-ion collisions (HICs)**
  - The pre-hydrodynamic QGP in HICs
  - Probing the pre-hydrodynamic QGP in HICs
- **Quantum speedup for the QCD matter**
  - An exciting new avenue for computing
- **Conclusions**

The QCD matter

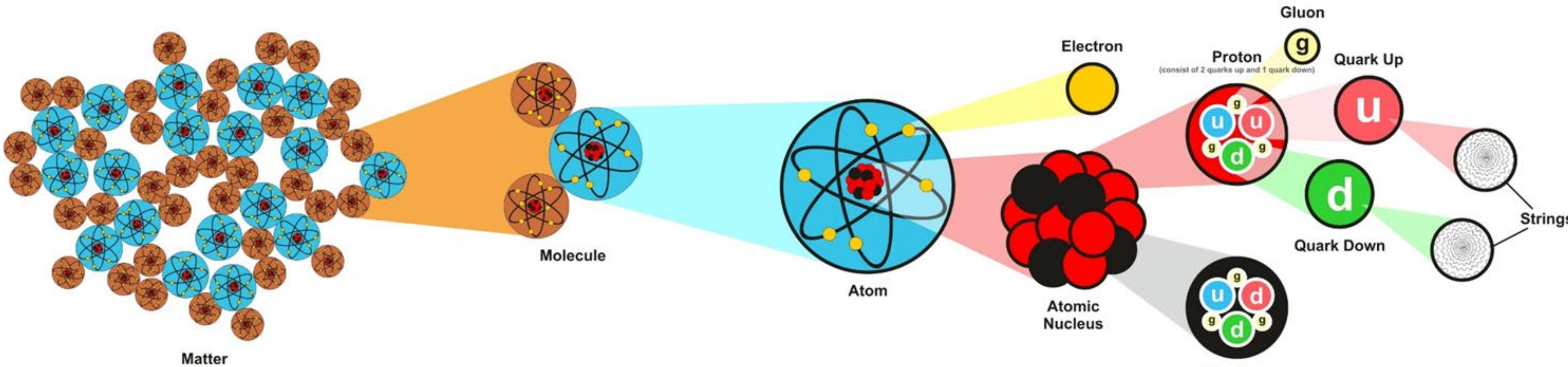
From the Big Bang to the Little Bang

Matter

Physics is the natural science of matter

Reductionism:

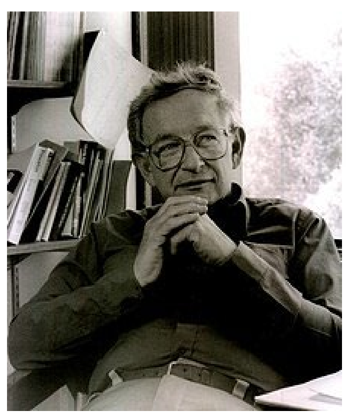
- Constitutes and finer structure of matter

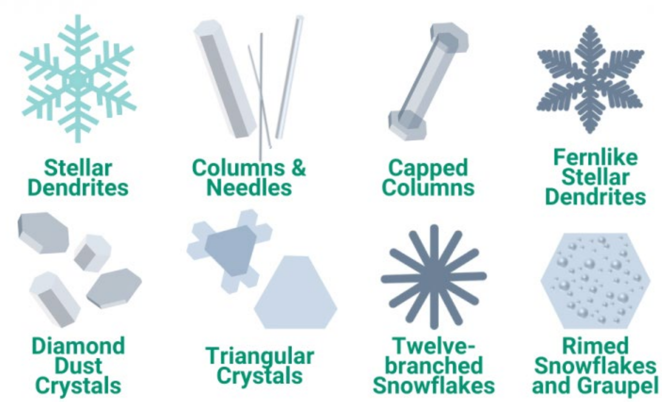


Lower energy → Higher energy

Emergence:

- More is different
  多者异也





Quarks and gluon

Reductionism: Quantum Chromodynamics (QCD)

- One of the theory with finest structure experimentally verified

Hagedorn temperature (1960s):

- The number of hadronic (e.g. proton, neutron, etc.) states diverges when approaching T_H:

$$\lim_{T \to T_H} Tr\left[e^{-\beta H}\right] = \infty$$

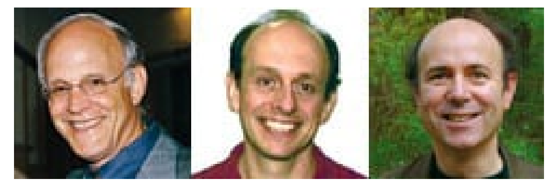
- Absolute hot? Indicating new degrees of freedom beyond T_H. All hadrons are expected to be made of these new degrees of freedom

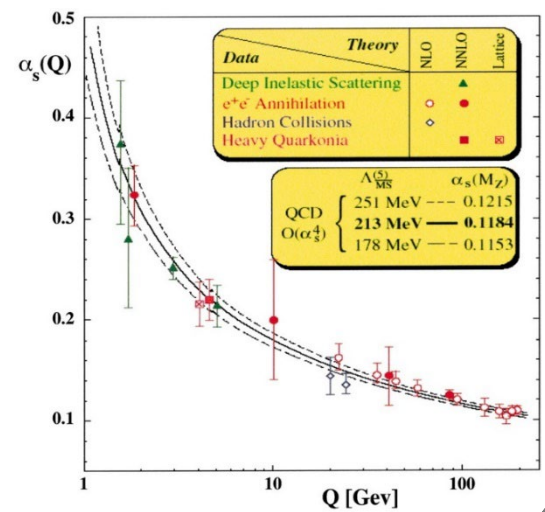
Asymptotic freedom (1970s):
(Gross, Wilczek, Politzer, 2004 Nobel Prize)

- Quantum Chromodynamics (QCD)

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f}^{N_f} \bar{\psi}_f \,(i\gamma^\mu D_\mu - m)\psi_f \; - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}$$

- Running coupling becomes weaker at larger exchange momentum)
- Deconfinement of quark/gluon from hadron (new degree of freedom).





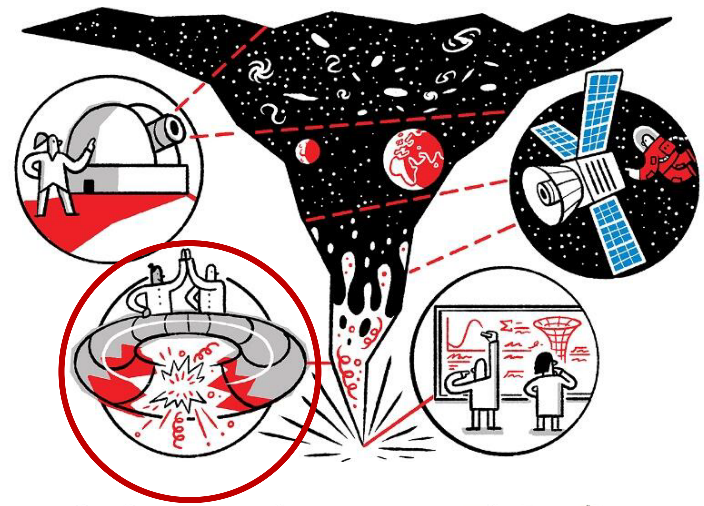
The QCD Plasma

Emergence: Quark-gluon plasma (QGP)

- A new phase of and the hottest matter in the Universe

Where to find it:

- A few microseconds after the Big Bang in nature



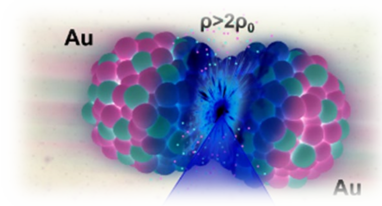
Heavy-ion collisions as the Little Bang:

- Smash nucleus to produce a bulk medium of free quarks and gluon

"More is different" in high-energy nuclear physics:

核子重如牛，对撞生新态

Probing the QCD plasma

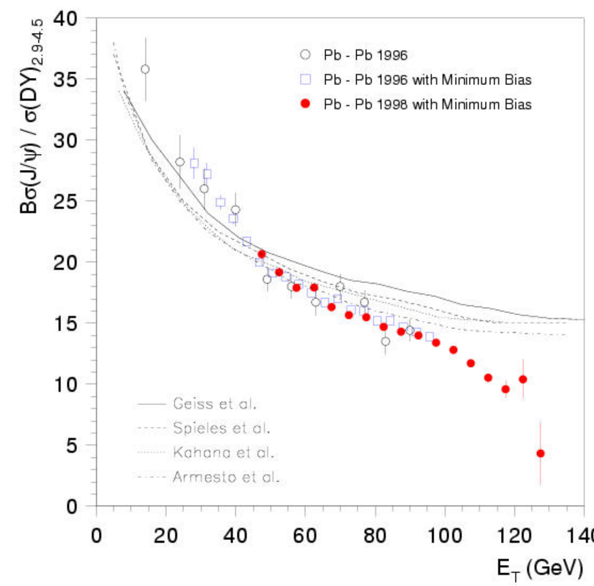


Heavy-ion collisions (HICs)

- Largest experiment in human history

High energy heavy-ion collisions (1980s - current):

- Super Proton Synchrotron (SPS) at CERN (1980s, 1990s, 2000s)
- Then Relativistic Heavy-Ion Collider (RHIC) at Brookhaven (2000s, …)
- Then Large Hardon Collider (LHC) at CERN (2000s, …)



Earliest signal of the quark-gluon plasma (2000s):

- J/ψ abnormal suppression at SPS@CERN
- Theoretically predicted by Matsui & Satz (1986)

Fruitful physics in heavy-ion collisions:

- A complex multi-stage experiment, including:
  - Initial production of quarks and gluon,
  - Thermalization of the non-equilibrium QGP,
  - Dynamic production of hard and electromagnetic probes in the QGP,
  - Hadronization,
  - …

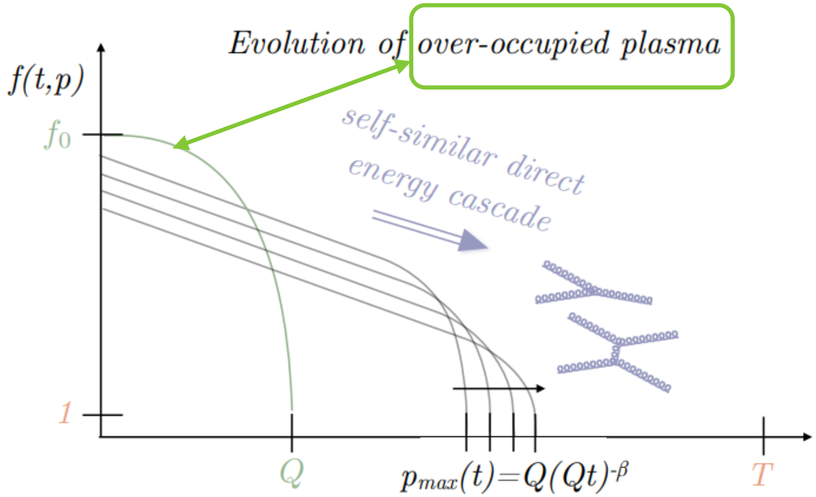
Far-from-equilibrium QCD matter

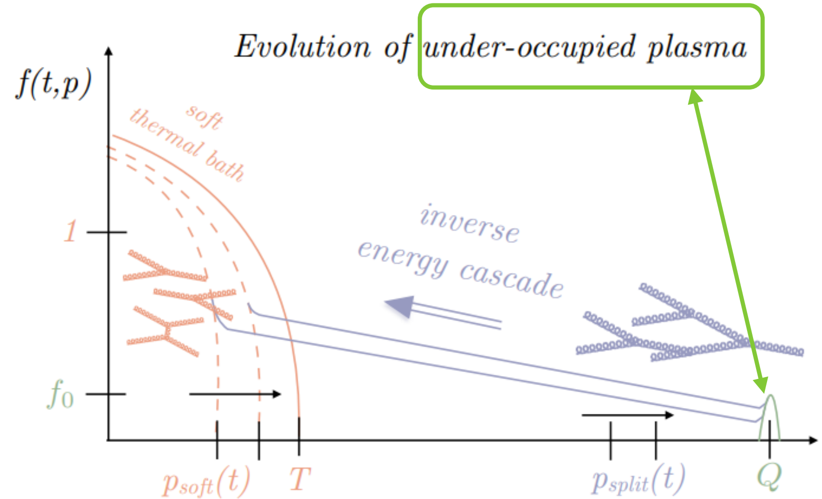
The turbulent nature of quark-gluon plasma

Thermalization of the QCD plasma

Two typical far-from-equilibrium systems

- Over-occupied and under-occupied plasmas





Over-occupied plasma:

- Separation of scale

  $\langle p \rangle_0 \ll T$
- Direct energy cascade

  Low → High momentum
- Initial state in HICs

Under-occupied plasma:

- Separation of scale

  $\langle p \rangle_0 \gg T$
- Inverse energy cascade

  High → Low momentum
- Jets in HICs

Non-equilibrium QCD plasma

QCD effective kinetic theory (QCD EKT)

- The state-of-the-art tool to study non-equilibrium QCD plasma

2-point correlations from the QCD

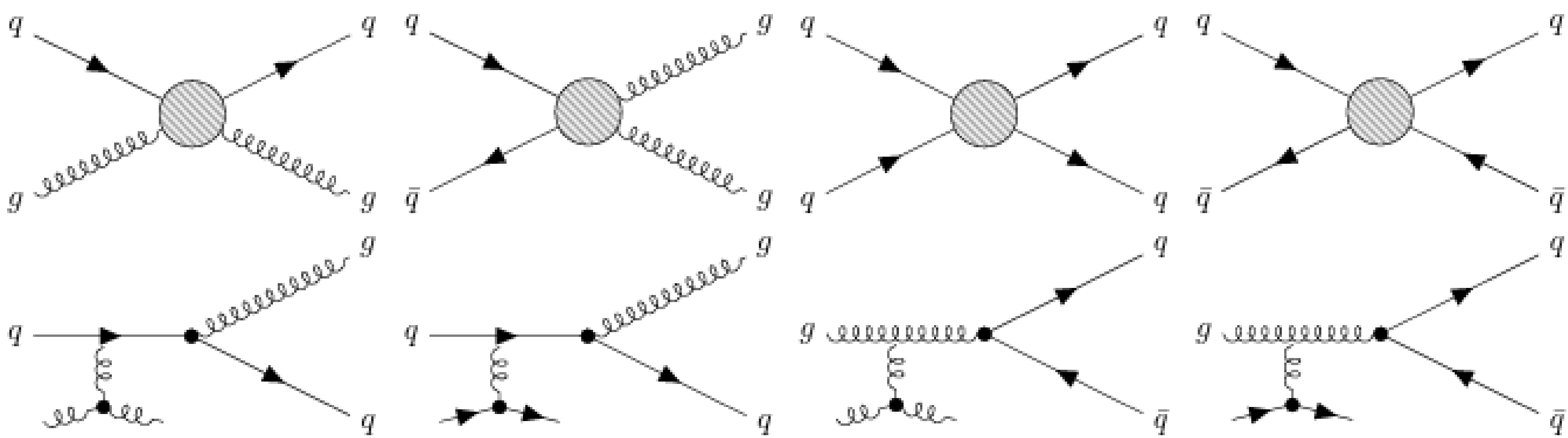
$$\mathcal{L}_{\rm QCD} = \sum_{f}^{N_f} \bar{\psi}_f \, (i\gamma^\mu D_\mu - m)\psi_f \; - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}$$

Set of coupled Boltzmann equations for quarks and gluon distribution:

$$\left(\frac{\partial}{\partial \tau} - \frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right) f_a(\tau, p_T, p_\parallel) = -C_a^{2\leftrightarrow 2}[f](\tau, p_T, p_\parallel) - C_a^{1\leftrightarrow 2}[f](\tau, p_T, p_\parallel)$$

$$a = g, u, \bar{u}, d, \bar{d}, s, \bar{s}$$

Including both elastic and inelastic scatterings in the QCD:



Turbulence of the QCD plasma

Self-similar energy cascade

- Turbulence in over-occupied QCD plasma

Self-similar scaling spectra:

$$f_g(p,t) = (t/t_0)^{\alpha} f_0 f_S\left((t/t_0)^{\beta}\frac{p}{\langle p\rangle_0}\right)$$

Universal Scaling Function

$$f_S\left((t/t_0)^{\beta}\frac{p}{\langle p\rangle_0}\right)$$

Scaling Exponents from Yang-Mills plasma

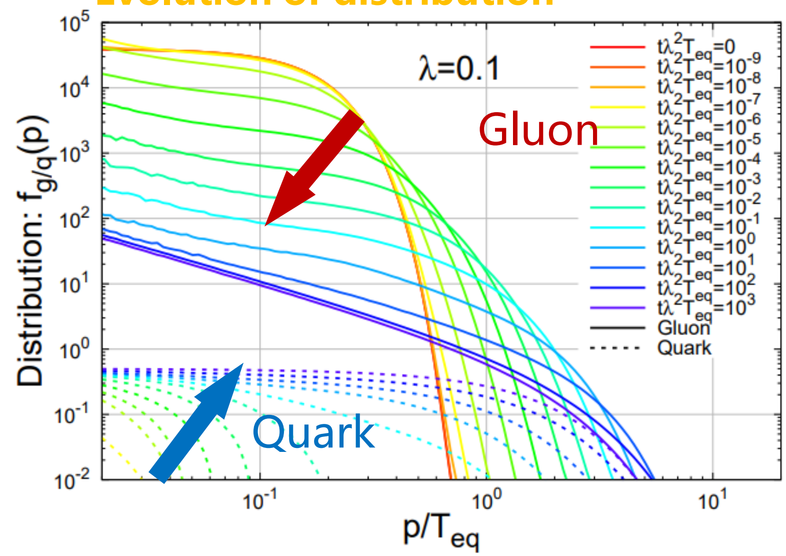
$$\alpha = -\frac{4}{7}, \beta = -\frac{1}{7}$$

Scaling works for the QCD plasma:
gluon dominated

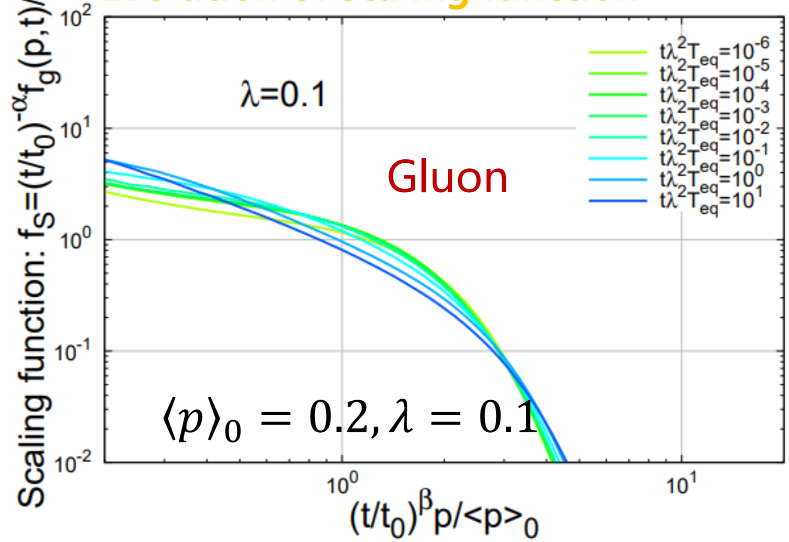
Quark spectra following gluon spectrum

X Du, S Schlichting, Phys. Rev. D 104 (2021) 054011

Evolution of distribution



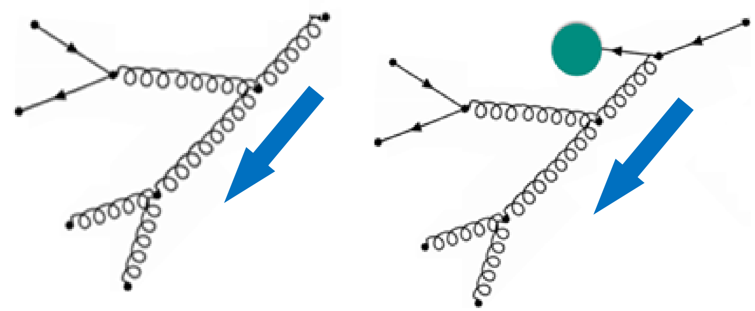
Evolution of scaling function



Turbulence of the QCD plasma

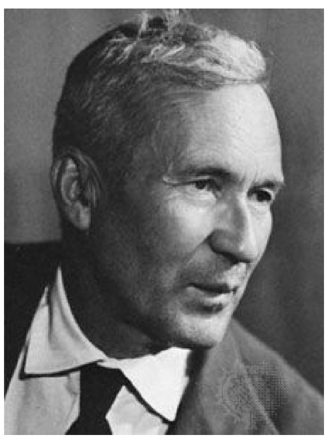
Kolmogorov-Zakharov spectra

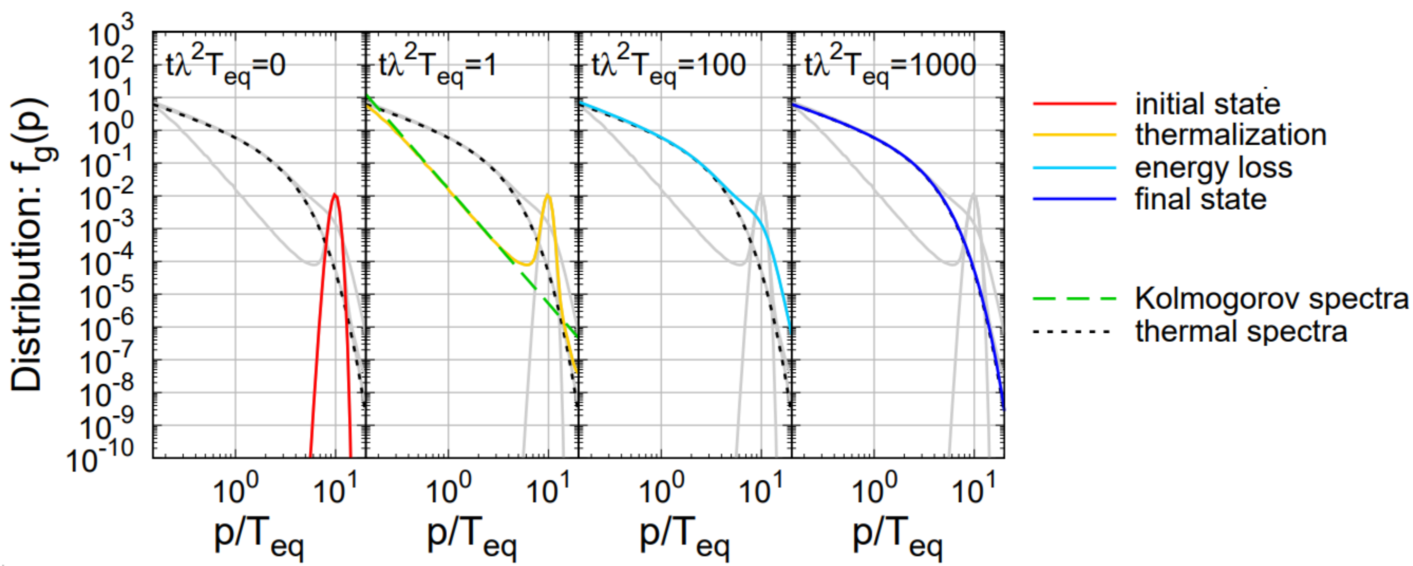
- Turbulence in under-occupied QCD plasma



Turbulence:

$$f_{KZ}(p,t) = \eta(t)\left(\frac{\langle p\rangle_0}{p}\right)^{\kappa}$$





Andrey Kolmogorov:
-5/3 power law in classical turbulence

QCD EKT simulation:
power law in the QCD turbulence

X Du, S Schlichting, Phys. Rev. D 104 (2021) 054011

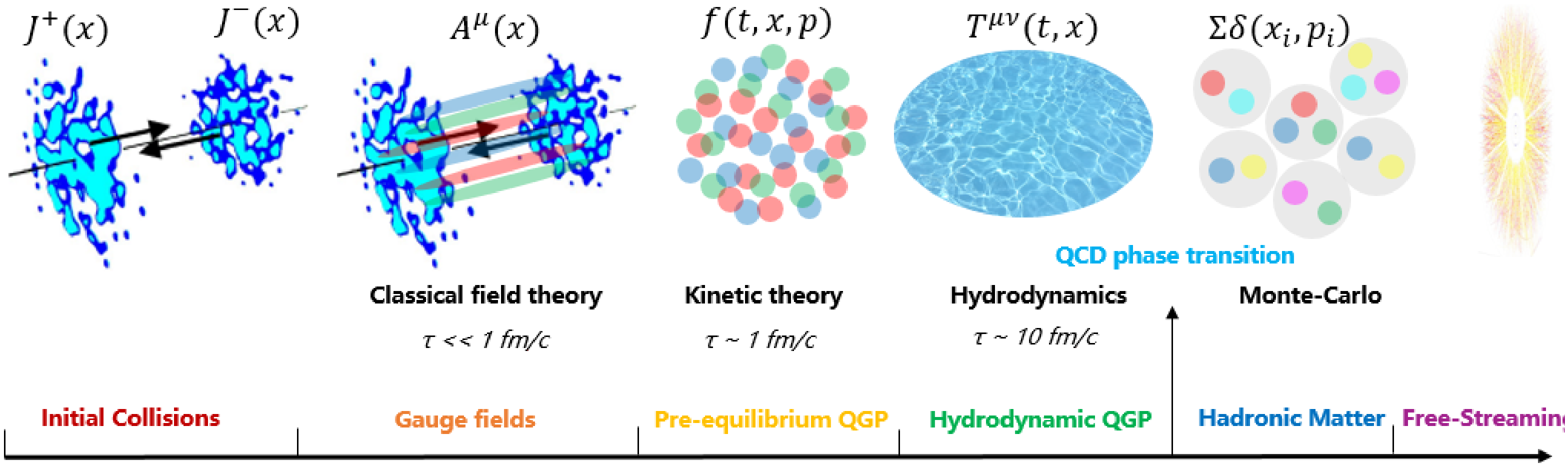
Early stage of heavy-ion collisions I

The pre-hydrodynamic QGP in HICs

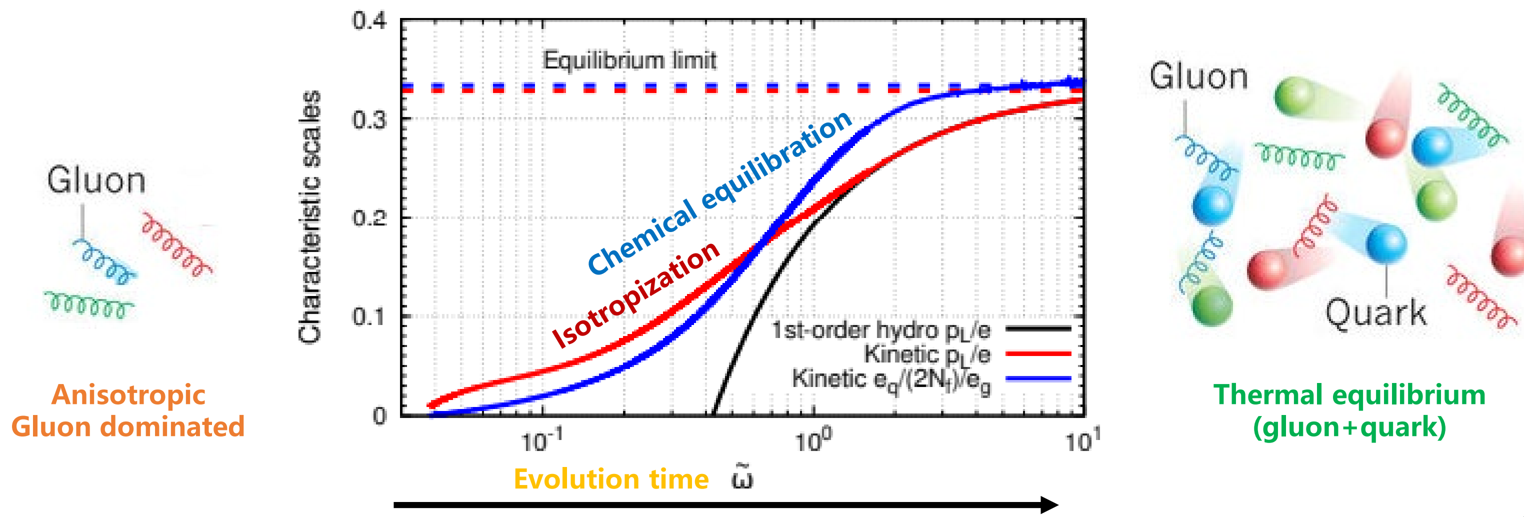
Non-equilibrium QCD plasma in HICs

Heavy-ion collision: A multi-stage experiment

- Where does the QGP thermalization occur in HICs? Early stage



Equilibration/thermalization of the QGP:



Kinetic equilibration

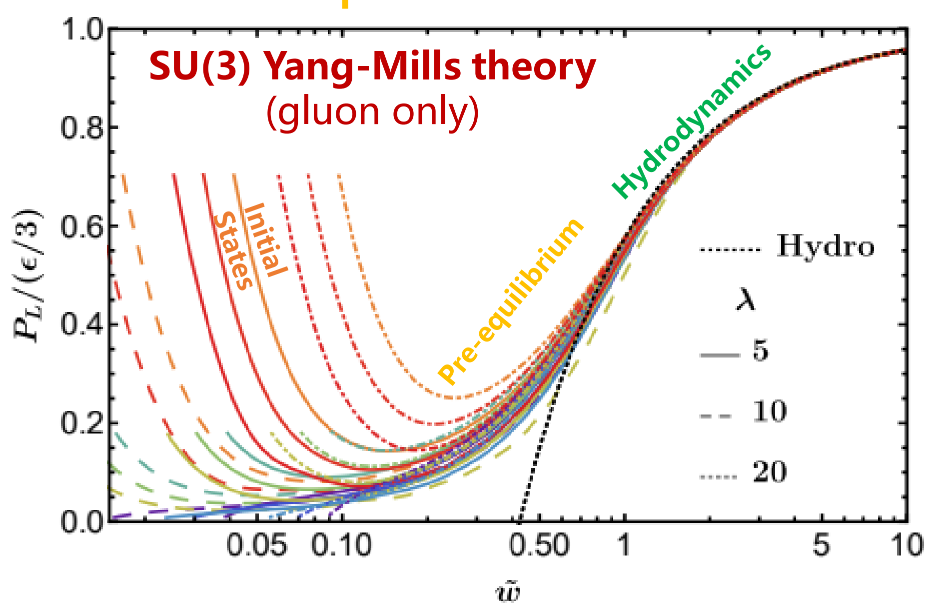
Universal attractor solution in HICs

- The second law of thermodynamics

Anisotropization and isotropization:

- Longitudial expansion in the early stage of HICs (anisotropization)
- Hydrodynamization (isotropization)

Evolution of pressure



Memory loss

- Different initial state tends to reach a unique point

Universality

- The unique point can occur even before the hydrodynamics become valid

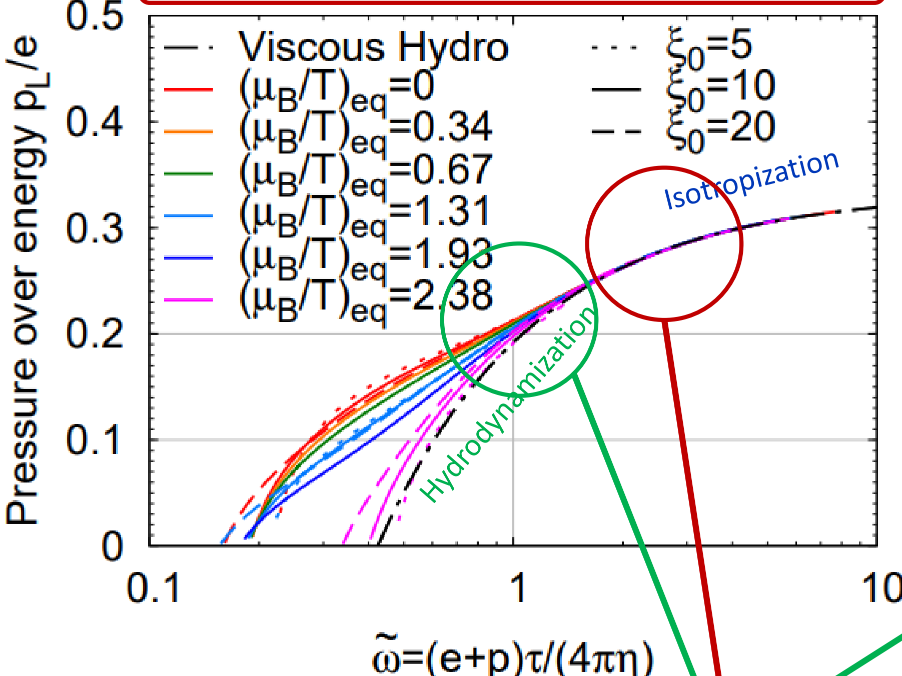
X Du, M Heller, S Schlichting, V Svensson, Phys. Rev. D 106 (2022) 014016

Chemical equilibration

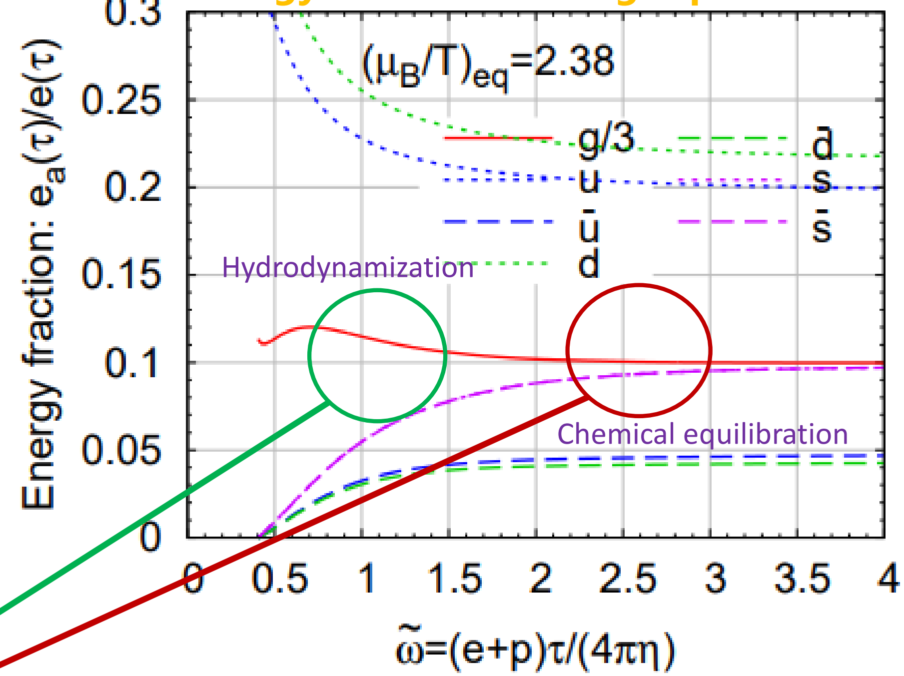
Quarks slow down the equilibration

Evolution of pressure

QCD theory (gluon + quark/antiquark)



Energy fraction of all light partons



Quarks slow down the equilibration

(Chemical equilibration persists after hydrodynamization)

X Du, Schlichting, Phys. Rev. D 104 (2021) 054011

X Du, Schlichting, Phys. Rev. Lett. 127 (2021) 122301

Attractor solution

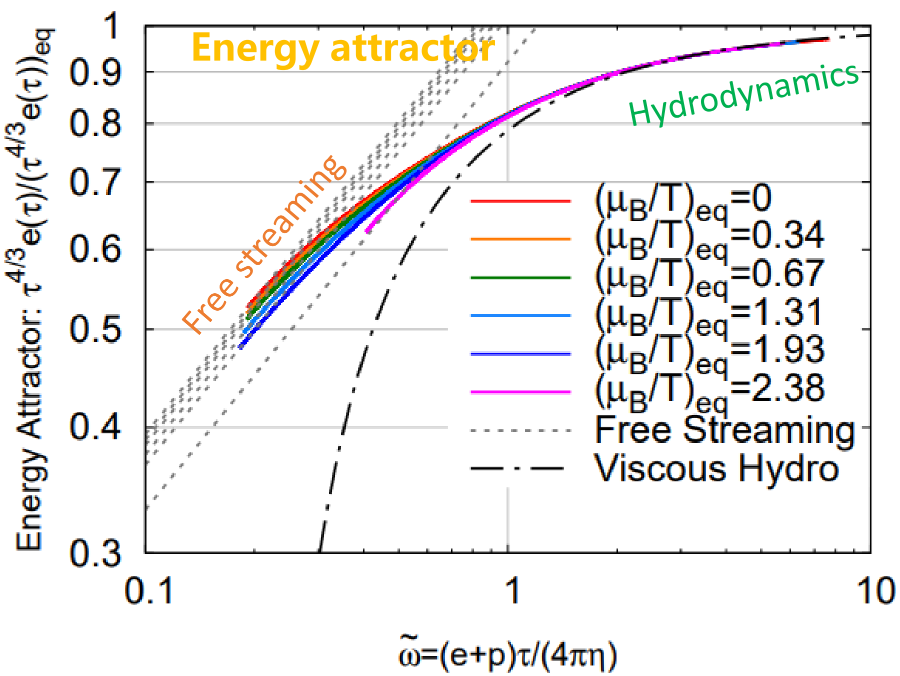
Conservation in equilibration

- Thermalization is about change, what is unchanged during thermalization?

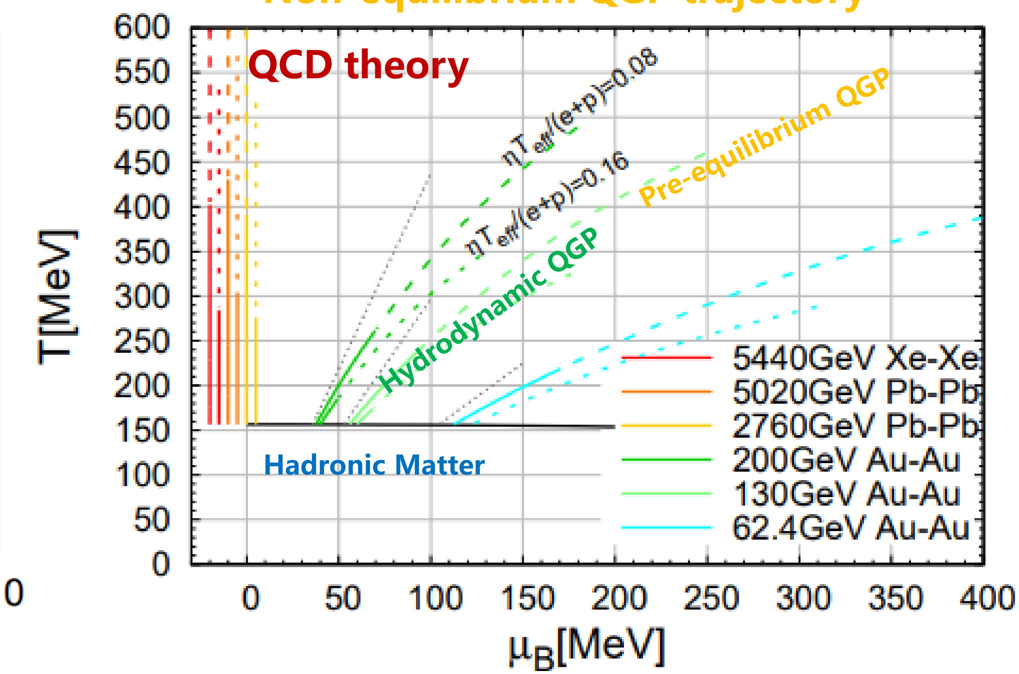
Energy and charge conservation:

$$\left(\tau^{4/3}e\right)_{\tilde{\omega}} = \left(4\pi\frac{\eta T_{\rm eff}}{e+p}\right)^{\frac{4}{9}}\left(\frac{\pi^2}{30}\nu_{\rm eff}\right)^{\frac{1}{9}}(\tau e)_0^{\frac{8}{9}}C_\infty\mathcal{E}(\tilde{\omega})$$

$$\left(\tau\Delta n_f\right)_{\tilde{\omega}} = \left(\tau\Delta n_f\right)_0$$



Non-equilibrium QGP trajectory



X Du, S Schlichting, Phys. Rev. Lett. 127 (2021) 122301

Fluctuation on top of the attractor

Fluctuation propagation in equilibration

- Provide a complete picture of the pre-hydrodynamic plasma in HICs and initial condition for hydrodynamic simulations

$$\left(\frac{\partial}{\partial\tau} - \frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right) f_a(\tau,p) = -C_a\,[f](\tau,p)$$

Bulk medium in average:

- Attractor from conservation

$$\left(\frac{\partial}{\partial\tau} + v\cdot\frac{\partial}{\partial x} - \frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right) \delta f_a(\tau,x,p) = -\delta C_a\,[f,\delta f](\tau,x,p)$$

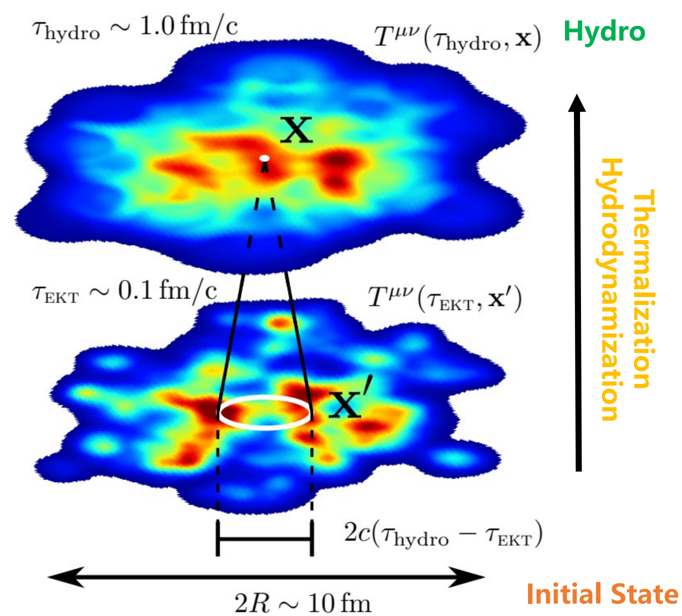
Hot spots as fluctuation:

- Linear response theory: Energy-momentum tensor /charge-current vector responses to perturbations/fluctuation (hot spots)

$$\delta T^{\mu\nu}_x(\tau_{\rm hydro},x) = \int d^2x'\, G^{\mu\nu}_{\alpha\beta}(x,x',\tau_{\rm hydro},\tau_{\rm EKT})\,\delta T^{\alpha\beta}_x(\tau_{\rm EKT},x')$$

$$\delta J^{\mu}_x(\tau_{\rm hydro},x) = \int d^2x'\, F^{\mu}_{\alpha}(x,x',\tau_{\rm hydro},\tau_{\rm EKT})\,\delta J^{\alpha}_x(\tau_{\rm EKT},x')$$

Thermalization
Hydrodynamization



T Dore, **X Du**, S Schlichting, will appear on arXiv soon…

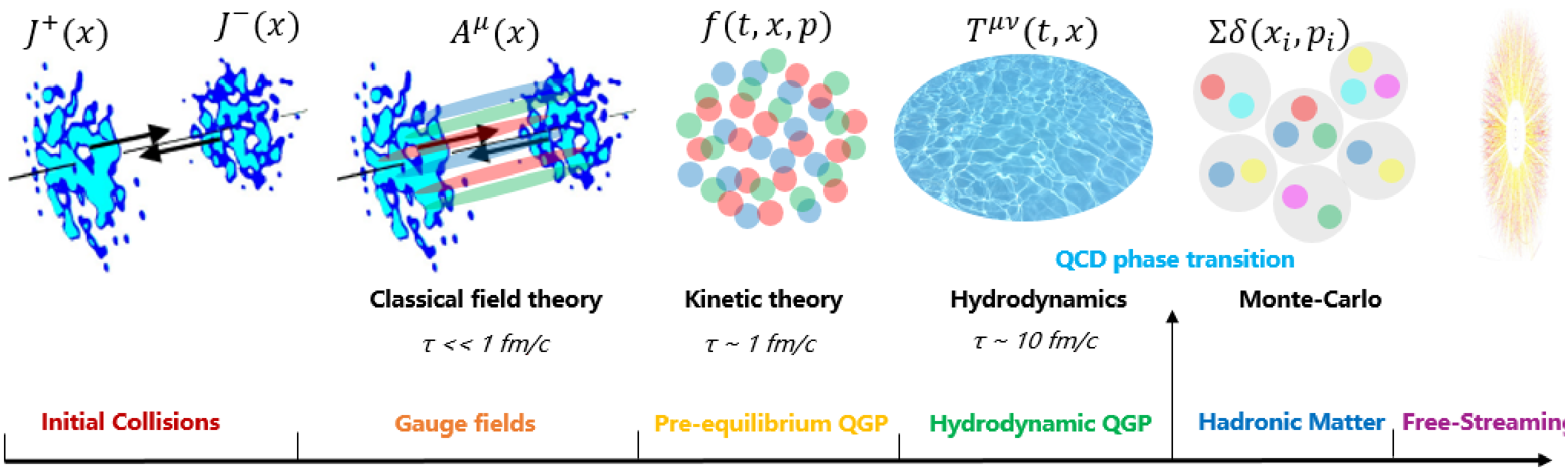
Early stage of heavy-ion collisions II

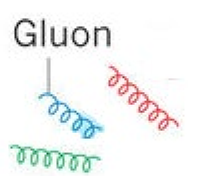
Probing the pre-hydrodynamic QGP in HICs

Non-equilibrium QCD plasma in HICs

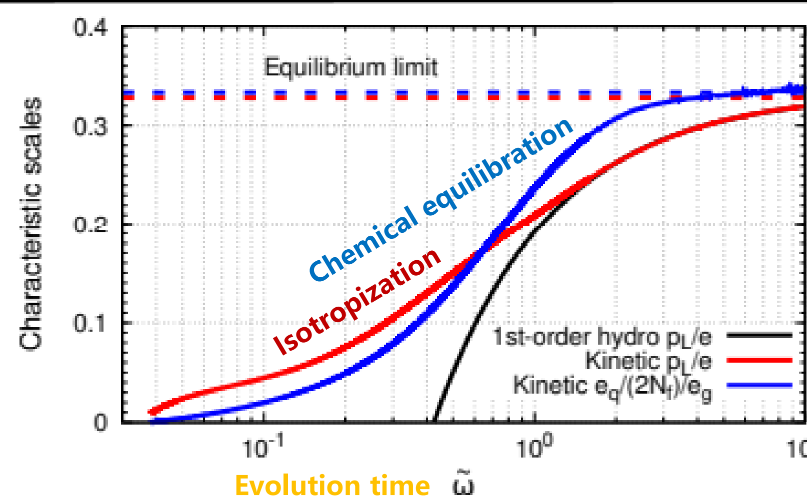
Phenomenology of the pre-equilibrium stage

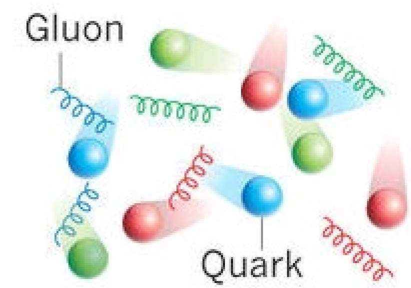
- How to probe/measure the pre-equilibrium stage?





Anisotropic
Gluon dominated





Thermal equilibrium
(gluon+quark)

Possible probe:

- Electromagnetic probe, such as di-leptons: no further interaction with the QGP

Di-lepton as a probe

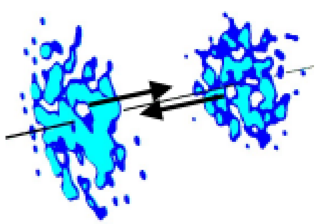
Electromagnetic probes in heavy-ion collisions

- Di-lepton calculations in HICs were focusing on thermal production

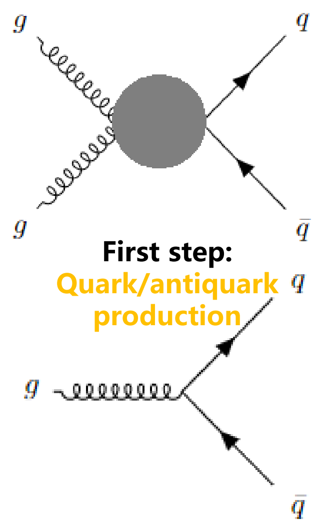
Di-lepton production in the pre-equilibrium QGP in HICs:

- Speed of Isotropization/Chemical equilibration of quark/anti-quark

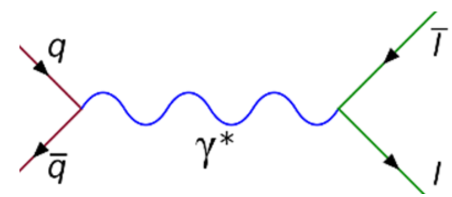
Initial State



Anisotropic
Gluon dominated



First step:
Quark/antiquark production



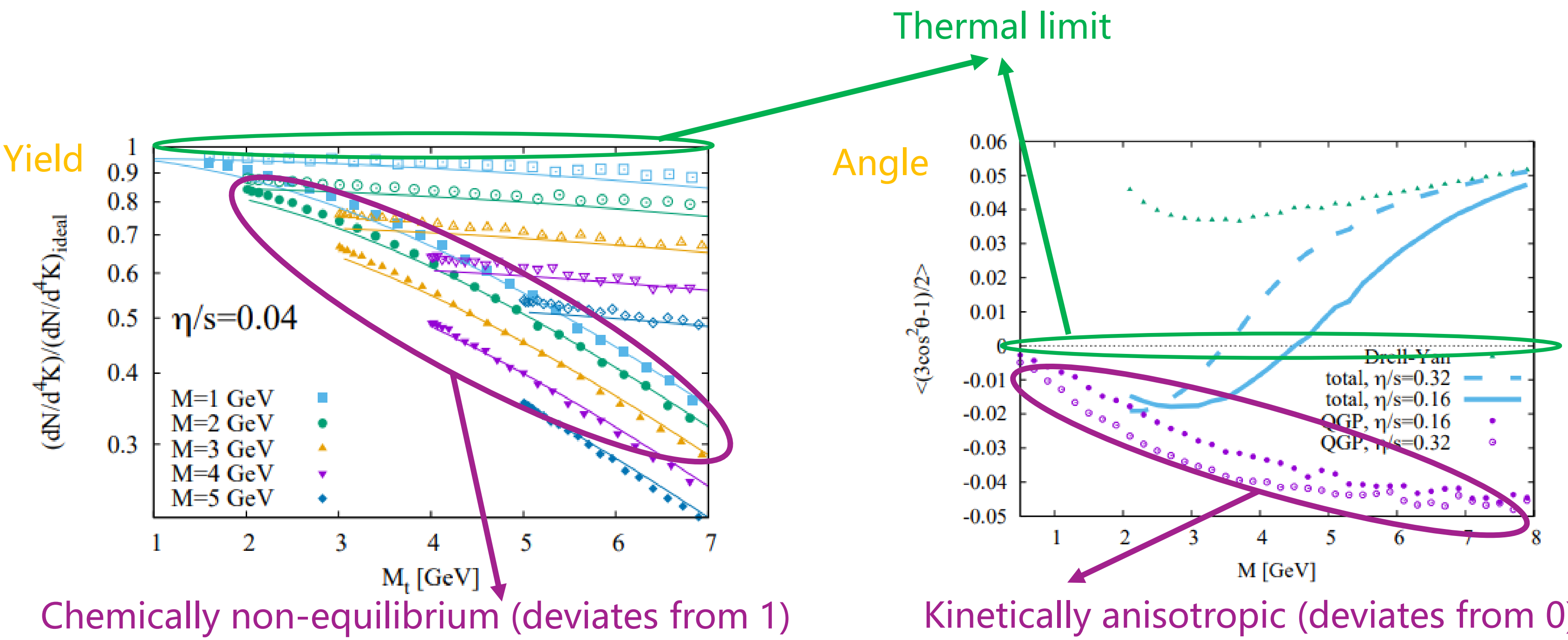
Second step:
Di-lepton production
Catches the chemical equilibration

$$\frac{dN^{l+l-}}{d^4x d^4K} = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} 4N_c \sum_f f_q(x,p_1) f_{\bar{q}}(x,p_1) v_{q\bar{q}} \sigma_{q\bar{q}}^{l+l-} \delta^{(4)}(K - P_1 - P_2)$$

Di-lepton as a probe

Electromagnetic probes in heavy-ion collisions

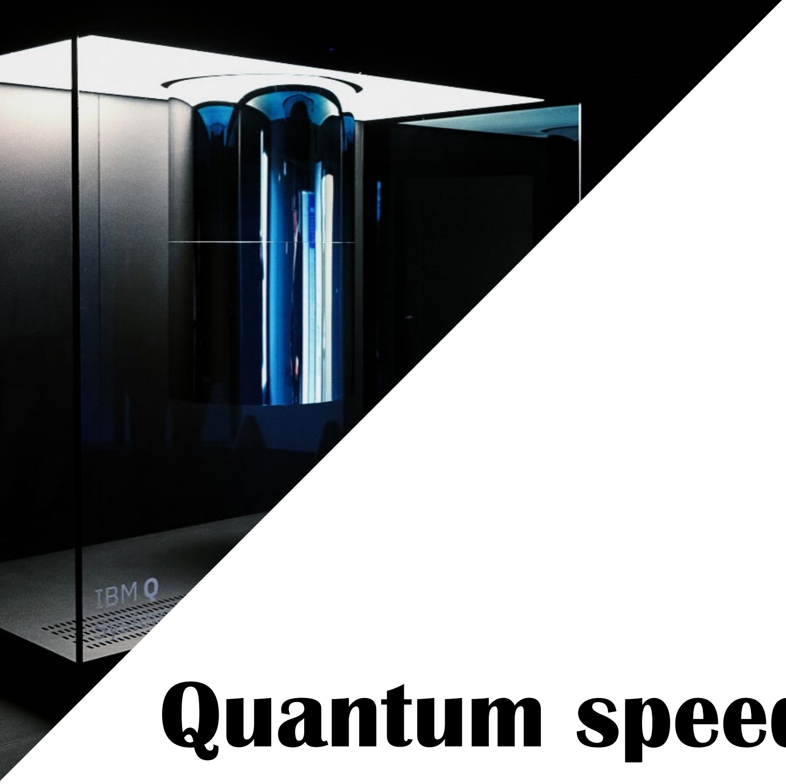
- Di-lepton may serve as a speedometer of equilibration of the QGP



M Coquet, **X Du**, JY Ollitrault, S Schlichting, M. Winn, Phys. Lett. B821 (2021) 136626

M Coquet, **X Du**, JY Ollitrault, S Schlichting, M. Winn, Nucl. Phys. A. 1030 (2023) 122579

M Coquet, **X Du**, JY Ollitrault, S Schlichting, M. Winn, Phys. Rev. Lett. 132 (2024) 232301



Quantum speedup for the QCD matter

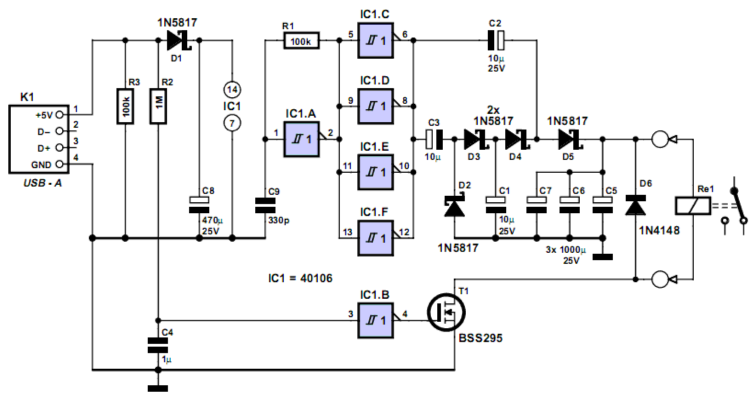
An exciting new avenue for computing

Quantum computing

Gate-based digital quantum computing

- Quantum computing is parallel computing in nature
- Quantum computing can potentially speed up calculation

Circuit of a digital computer:

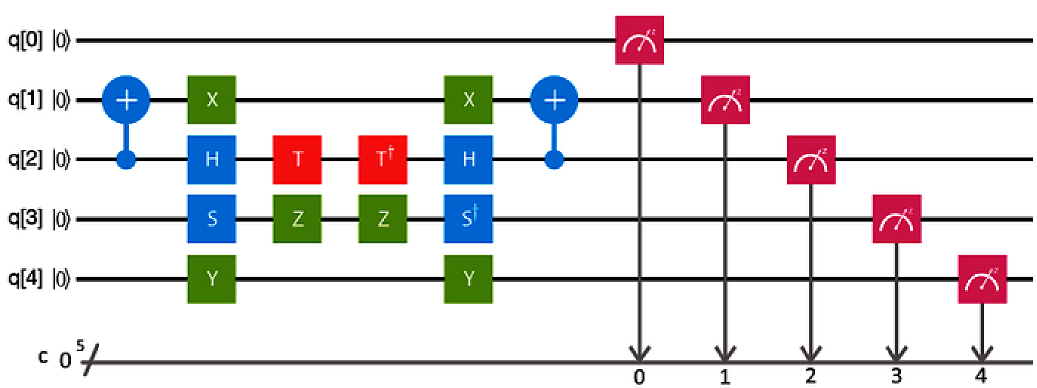


Classical bits:
0 and 1

Typical classical gates:
AND, NOT, OR, etc…

Typical classical circuits:
Adder, Multiplication, etc…

Quantum circuit of a digital quantum computer:



Quantum bits (qubit):
$|0\rangle$ and $|1\rangle$ and superposition of them with quantum phase

$$\frac{|0\rangle + e^{i\varphi}|1\rangle}{\sqrt{2}}$$

Typical quantum gates:
X(not), Y(rotation), Z(phase flip), Hadamard(superposition), etc…

Typical quantum circuits:
Adder, Fourier Transform, etc…

Heavy quark thermalization

Hard probes in heavy-ion collisions

- Distinguished scale compared to the thermal QCD plasma

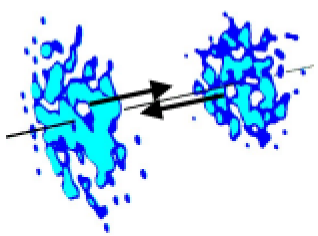
Hard probe energy (jet energy/heavy quark mass) $E \gg T$ Medium temperature (Light parton energy in medium)

Time scales in thermalization:

Heavy quark production $\tau_O \sim 1/M$

QGP thermalization $\tau_H \sim 1/T$

Heavy quark thermalization $\tau_R \sim M/T^2$



$\tau_O \ll \tau_H \ll \tau_R$

- The QCD plasma thermalizes much faster than the heavy quarks
- Heavy quark thermalizes mostly in the thermal QCD plasma (also in most of simulations)

Heavy quark thermalization

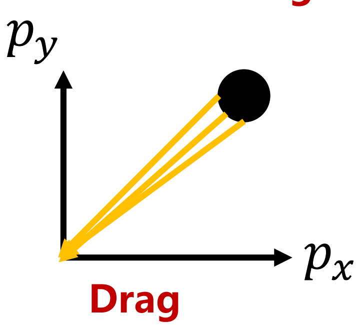
Heavy quark dynamics

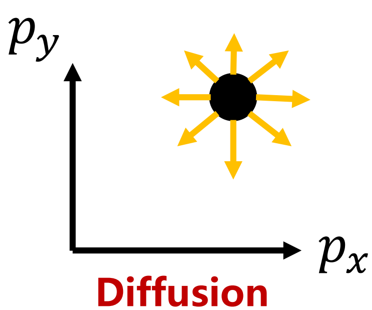
- Large mass, low velocity, elastic kicks from the medium dominate

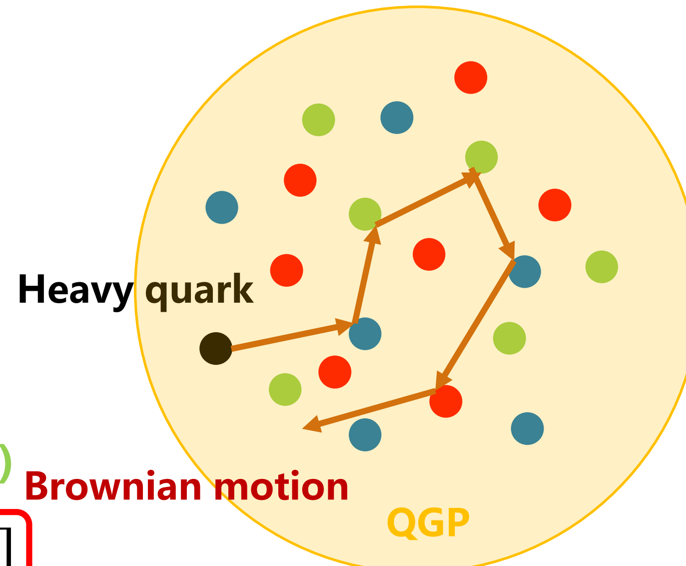
Stochastic differential equation (SDE) for heavy quark dynamics:

$$dp_i = \boxed{-Ap_i dt} + \boxed{\sigma_{ij} dW_j}$$

Drag — Diffusion — Stochastic term (dW_j)







From the SDE to partial differential equation (PDE)

$$\partial_t f(p) = \boxed{\partial_{p_i}[Ap_i f(p,t)]} + \boxed{\partial_{p_i}\partial_{p_j}[B_{ij} f(p,t)]}$$

Drag — Diffusion

$$B_{ij} = \sigma_{ik}\sigma_{kj}/2$$

Drag: Dissipation/Energy loss

Diffusion: Momentum broadening

Thermalization
(Fluctuation-dissipation theorem)

Heavy quark on a quantum circuit

Stochastic process on quantum circuit

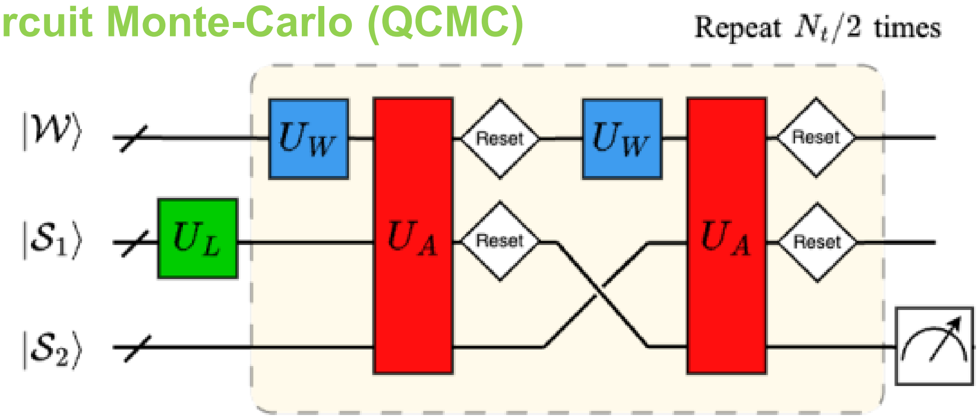
- Similar to classical circuit

$$p_i(t+dt) = p_i(t) - Ap_i(t)dt + \sigma_{ij} dW_j$$

Multiplication   Random number generator

Adder

Quantum circuit Monte-Carlo (QCMC)



(a) The depth-oriented QCMC with resets

Depth-oriented QCMC

We have to implement **reset** gates to implement U_W and addition to recycle quantum register

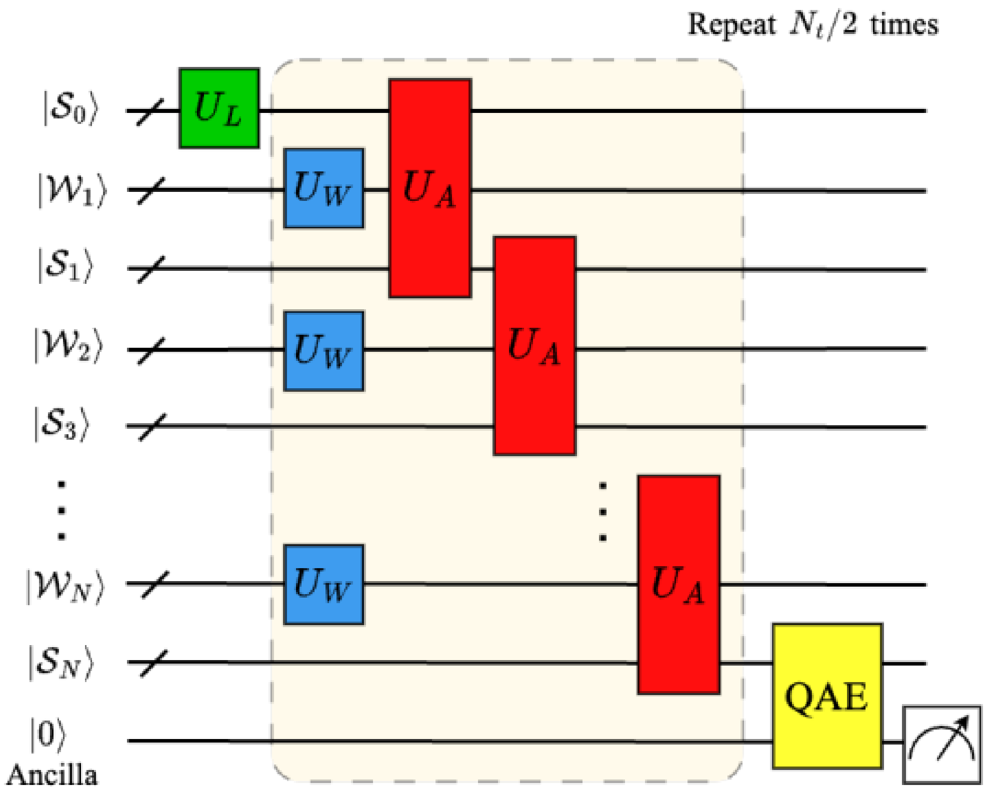
Heavy quark on a quantum circuit

Stochastic process on quantum circuit

- Quantum speedup

Accelerated Quantum circuit Monte-Carlo (aQCMC)

Breadth-oriented aQCMC



(b) The breadth-oriented aQCMC with the QAE

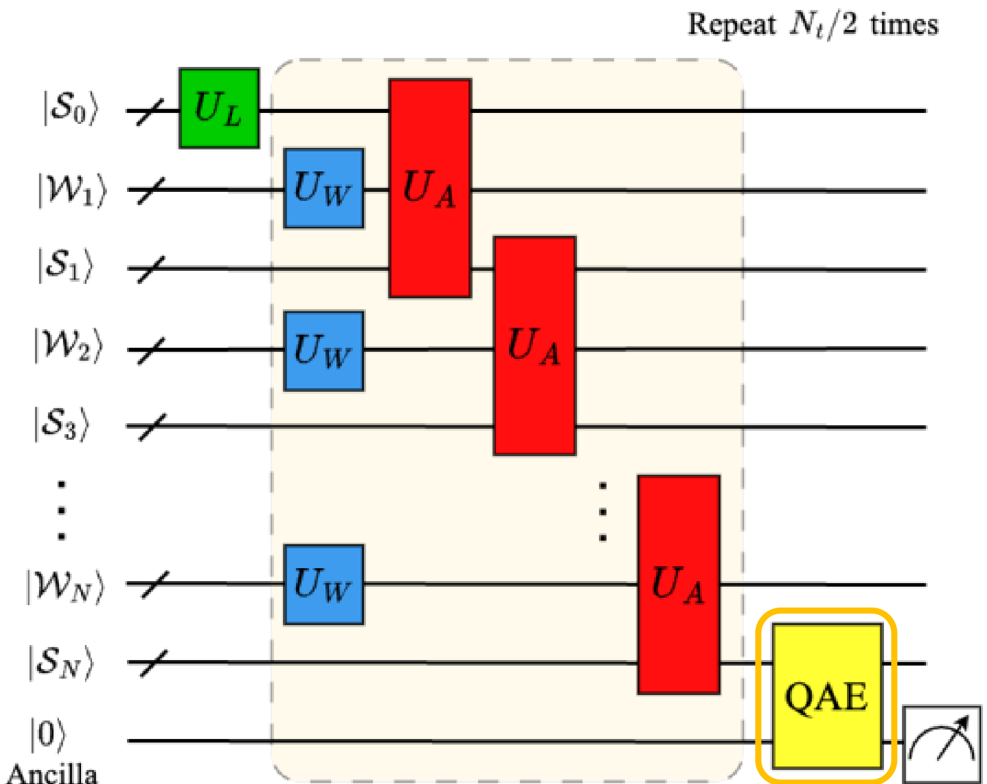
No **reset** gates, no recycle of quantum registers, the whole circuit is **unitary**

Heavy quark on a quantum circuit

Stochastic process on quantum circuit

- Quantum speedup

Accelerated Quantum circuit Monte-Carlo (aQCMC)



(b) The breadth-oriented aQCMC with the QAE

Breadth-oriented aQCMC

The quantum speed up algorithm **Quantum Amplitude Estimation (QAE)** requires a **Grover's operator** that can be constructed with a unitary circuit

Heavy quark on a quantum circuit

Quantum Amplitude Estimation (QAE)

- Quantum speedup

Oracle

$$A_F|\psi\rangle_n|0\rangle = \cos(\theta)|\psi_0^*\rangle_n|0\rangle + \sin(\theta)|\psi_1^*\rangle_n|1\rangle$$

$$a = \sin^2(\theta)$$

Momentum Distribution

$$|\psi\rangle_n = \sum_{i=0}^{2^n-1} \sqrt{P(i)}|i\rangle_n$$

Expectation value

$$a = \sum_{i=0}^{2^n-1} F(i)P(i)$$

Iteration of Grover's operator

$$Q^k A_F|\psi\rangle_n|0\rangle = \cos((2k+1)\theta)|\psi_0^*\rangle_n|0\rangle + \sin((2k+1)\theta)|\psi_1^*\rangle_n|1\rangle$$

Bad state — Good state

Likelihood

$$L_k(h,N) = [\sin^2((2k+1)\theta)]^h[\cos^2((2k+1)\theta)]^{N-h}$$

Combined Likelihood

$$L(h,N) = \prod_{k=0}^{M} L_k(h,N)$$

Quantum speedup

Quantum Amplitude Estimation (QAE)

- Quantum speedup

Lower bound of error

Quantum

$$\epsilon > \frac{\sqrt{a(1-a)}}{N_q}$$

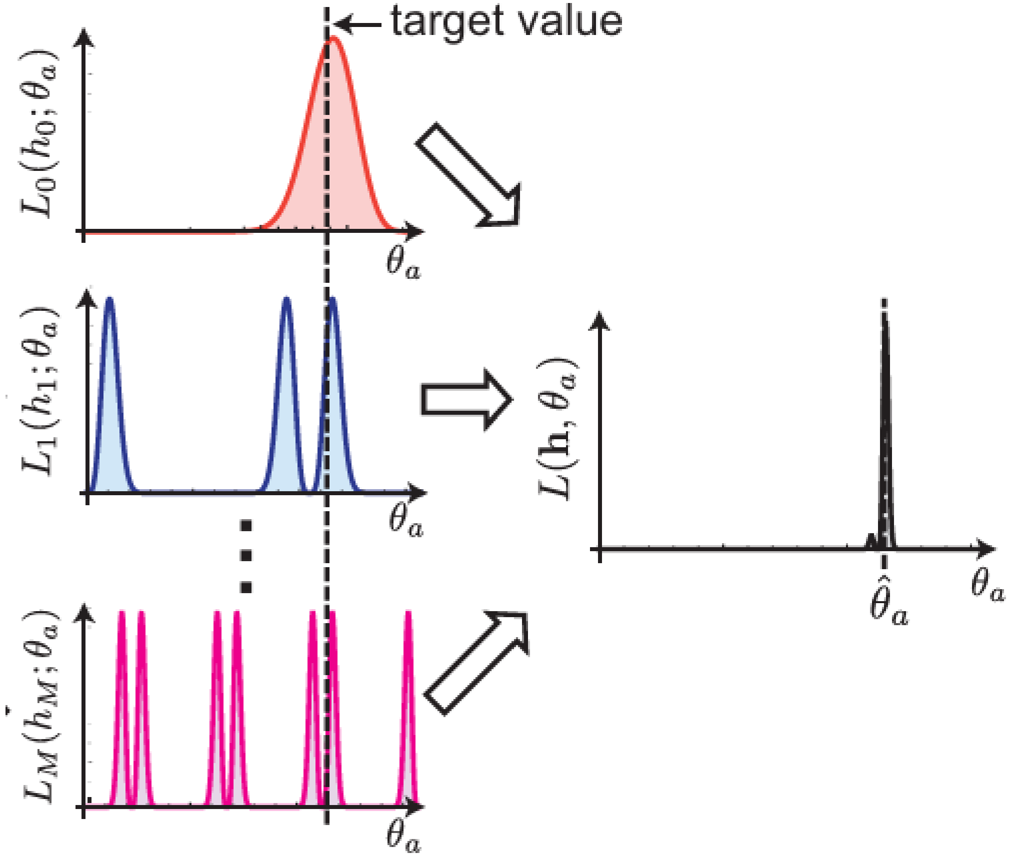
Classical

$$\epsilon > \frac{\sqrt{a(1-a)}}{\sqrt{N_q}}$$

Combined Likelihood

$$L(h,N) = \prod_{k=0}^{M} L_k(h,N)$$

$$a = \sin^2(\theta)$$



Y Suzuki et al., Quantum Information Processing, 19, 75, 2020

Quantum speedup

Quantum Amplitude Estimation (QAE)

- Quantum speedup

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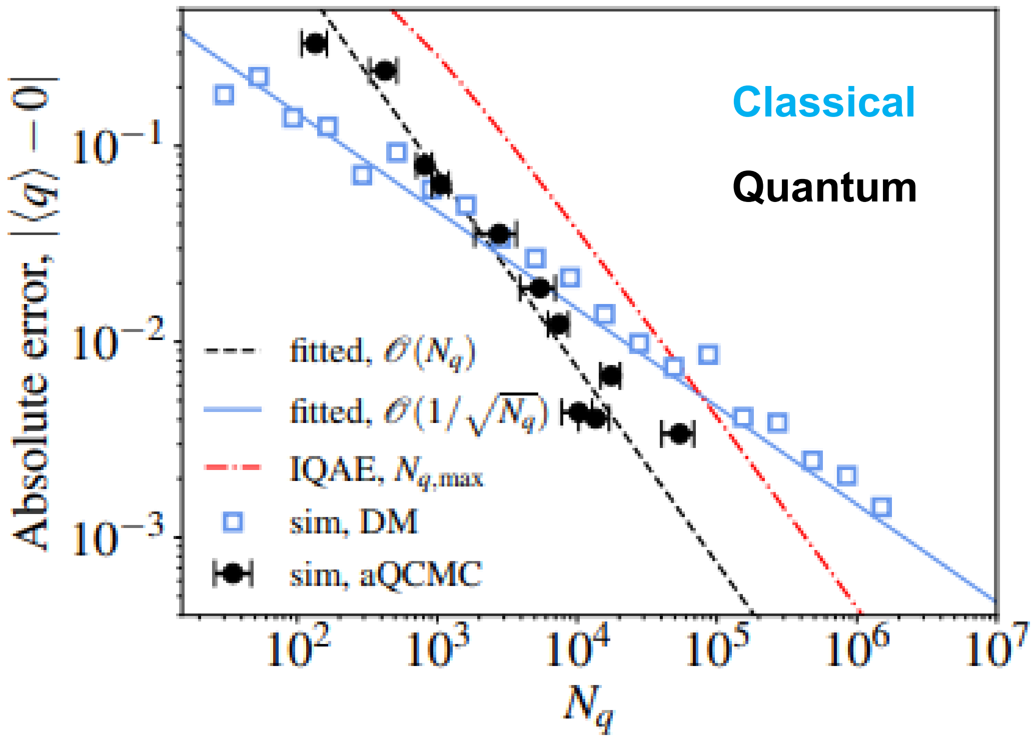


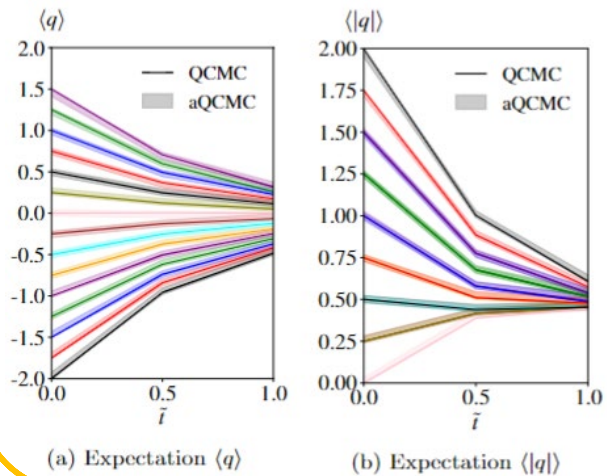
X Du, W Qian, Phys. Rev. D 109 (2024) 076025

Y Suzuki et al., Quantum Information Processing, 19, 75, 2020

Heavy quark on a quantum circuit

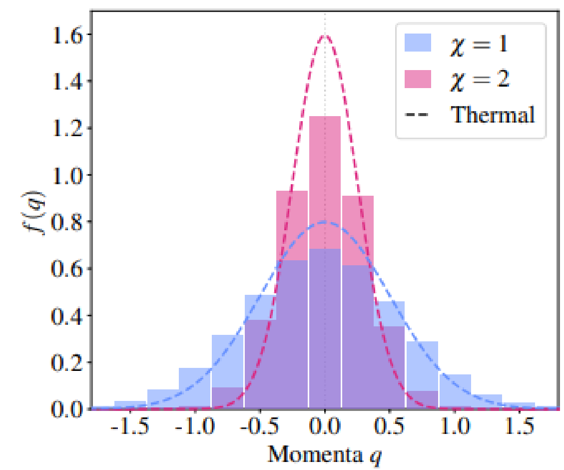
Simulation results on heavy quark thermalization

Direct measurement vs QAE



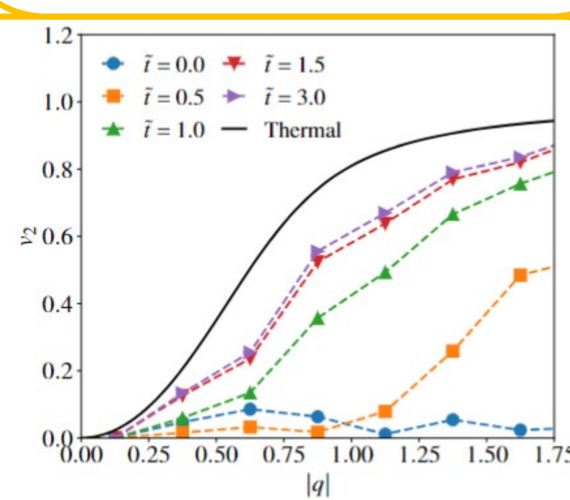
(a) Expectation ⟨q⟩ (b) Expectation ⟨|q|⟩

Thermalization



Elliptic flow in anisotropic medium

$$v_2 = \frac{\int f(q, \cos(\phi), t) \cos(2\phi)\, d\phi}{\int f(q, \cos(\phi), t) d\phi}$$



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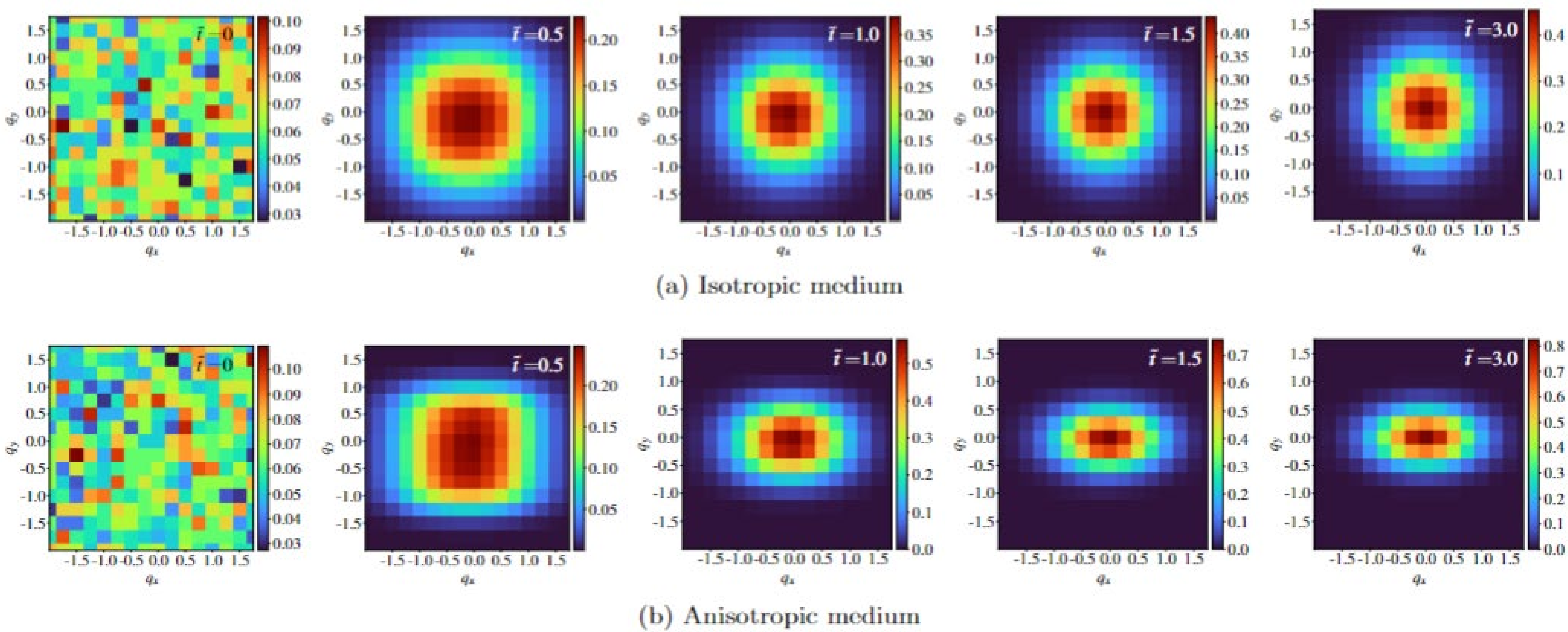
Heavy quark on a quantum circuit

Simulation results on heavy quark thermalization

Time evolution of density

Isotropically, anisotropically towards thermal equilibrium



(a) Isotropic medium

(b) Anisotropic medium

X Du, W Qian, Phys. Rev. D 109 (2024) 076025

Conclusions

Summary

- **The QCD matter**
  - Philosophy of reductionism and emergence

- **Far-from-equilibrium QCD matter**
  - Self-similarity and Kolmogorov spectra as signatures of turbulence

- **Early stage of heavy-ion collisions (HICs)**
  - Kinetic and chemical equilibrations, attractor, etc…
  - Di-lepton as a probe for the pre-hydrodynamic QGP in HICs

- **Quantum speedup for the QCD matter**
  - Heavy quark thermalization on quantum computer and quantum speedup

谢谢大家 Thanks!