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宇宙线加速的基本物理

The basic physics of cosmic-ray acceleration

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Major theoretical questions in CR astrophysics

- 宇宙线在哪里以及是如何加速到如此高能量的? (起源问题)

  How are CRs accelerated to such high energies? (Origin)

- 宇宙线如何在星系中传播和逃逸? (输运问题)

  How do CRs propagate/escape from galaxies? (Transport)

- 宇宙线如何影响和塑造星系生态系统? (反馈作用)

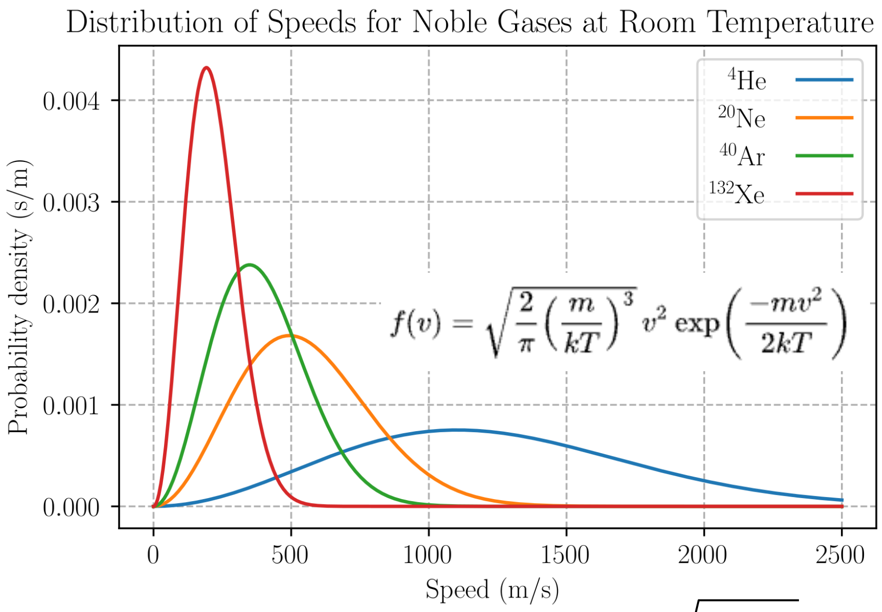
  How do CRs shape the galactic ecosystem? (Feedback)

Outline

- Overview
- Stochastic Fermi acceleration (2nd order)
- Diffusive shock acceleration (1st order)
- Case study: Ion-acceleration in non-relativistic shocks
- Particle acceleration in magnetic reconnection (brief)
- Computational methods: brief introduction

Origin of cosmic rays

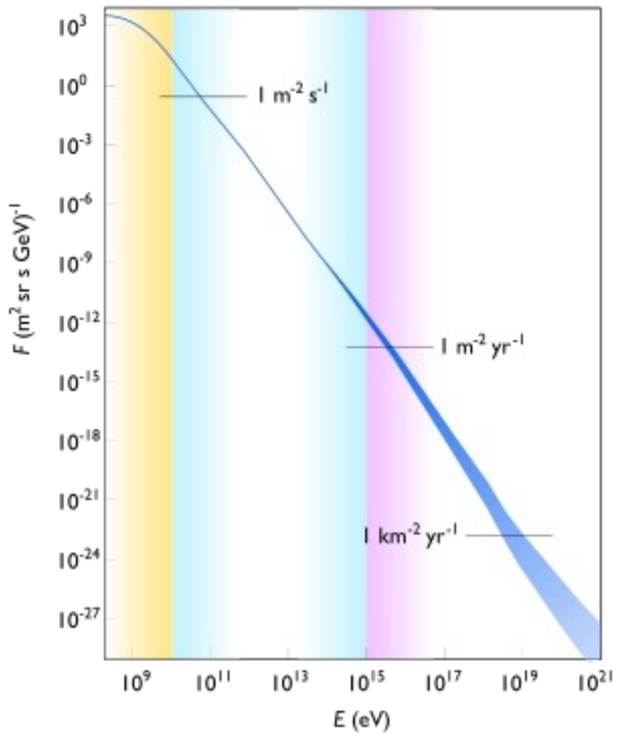
Velocity distribution in thermal equilibrium:



Typical velocity: $v_T = \sqrt{\frac{3kT}{m}}$

Need frequent collisions!

However, energy distribution of CRs:

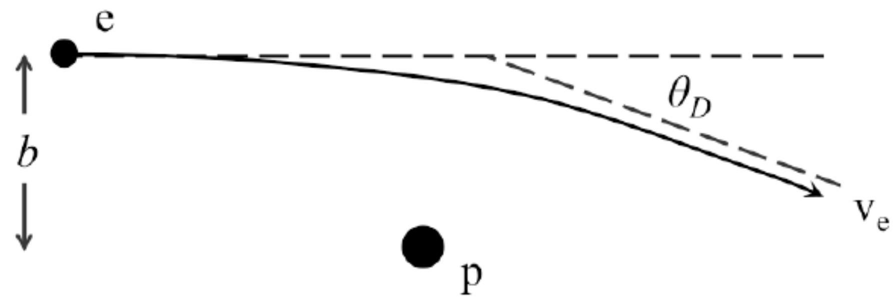


Need collisionless processes!

Coulumb cross section in ionized plasmas

Dominated by small-angle scattering:

$$\theta_D = \frac{b_o}{b} \text{ for } b \gg b_o \qquad \text{where} \quad b_o \equiv \frac{2e^2}{mv_e^2}$$



Deflection following random-walk process, mean deflection angle after time t:

$$\langle \Theta^2 \rangle = \int_{b_{\min}}^{b_{\max}} \left(\frac{b_o}{b}\right)^2 n v_e t \cdot 2\pi b db = 2\pi n b_o^2 v_e t \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

~ λ_D (Debye length)

~ b_o

Characteristic scattering rate:

$$\nu_D = \frac{1}{t_D} = 2\pi n b_o^2 v_e \ln\Lambda = \frac{8\pi n e^4}{m^2 v_e^3} \ln\Lambda$$

Coulumb logarithm (usually 10-30).

Coulumb collision timescale in plasmas

For thermal plasmas, collision timescale:

$$\tau_{ei} = \frac{3}{4\sqrt{2\pi}\ln\Lambda}\frac{(kT)^{3/2}m_e^{1/2}}{e^4 n_0} \approx 2.8\times 10^4 \left(\frac{10}{\ln\Lambda}\right)\left(\frac{T}{10^4\ \mathrm{K}}\right)^{3/2}\left(\frac{n_0}{\mathrm{cm}^{-3}}\right)^{-1}\ \mathrm{s}\ ,$$

$$\tau_{ee} = \frac{3}{4\sqrt{\pi}\ln\Lambda}\frac{(kT)^{3/2}m_e^{1/2}}{e^4 n_0} \approx 3.9\times 10^4 \left(\frac{10}{\ln\Lambda}\right)\left(\frac{T}{10^4\ \mathrm{K}}\right)^{3/2}\left(\frac{n_0}{\mathrm{cm}^{-3}}\right)^{-1}\ \mathrm{s}\ ,$$

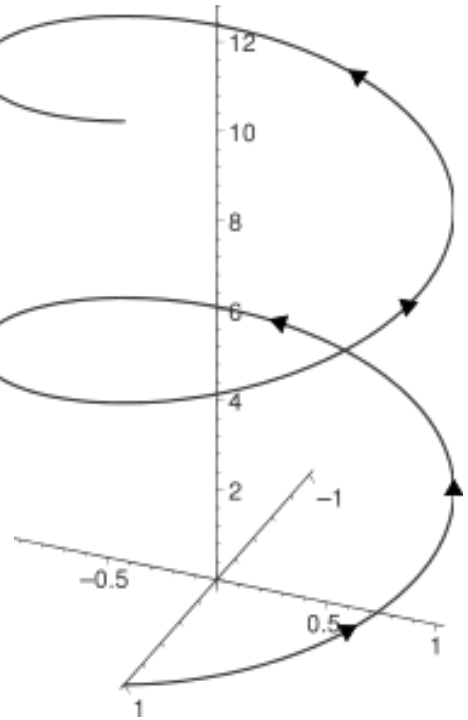
$$\tau_{ie} = \frac{3}{4\sqrt{2\pi}\ln\Lambda}\frac{(kT)^{3/2}m_i}{e^4 n_0 m_e^{1/2}} \approx 5.0\times 10^7 \left(\frac{10}{\ln\Lambda}\right)\left(\frac{T}{10^4\ \mathrm{K}}\right)^{3/2}\left(\frac{n_0}{\mathrm{cm}^{-3}}\right)^{-1}\ \mathrm{s}$$

Further compute (ion) mean free path:

$$\tau_{ii} = \frac{3}{4\sqrt{\pi}\ln\Lambda}\frac{(kT)^{3/2}m_i^{1/2}}{e^4 n_0} \approx 1.7\times 10^6 \left(\frac{10}{\ln\Lambda}\right)\left(\frac{T}{10^4\ \mathrm{K}}\right)^{3/2}\left(\frac{n_0}{\mathrm{cm}^{-3}}\right)^{-1}\ \mathrm{s}\ .$$

System	n (cm^{-3})	T (K)	B (G)	L (cm)	λ_{mfp} (cm)
Solar wind (1 AU)	1	10^5	10^{-5}	10^{13}	$6.4\cdot 10^{13}$
Galactic center ($20R_g$)	10^7	10^{11}	10	10^{13}	$6.4\cdot 10^{18}$
Galactic center ($10^4 R_g$)	10^4	10^8	0.01(?)	10^{15}	$6.4\cdot 10^{15}$
Hot ionized medium	3×10^{-3}	10^6	3×10^{-6}	10^{20}	$2.1\cdot 10^{18}$
Intracluster medium	10^{-2}	10^8	10^{-6}	10^{24}	$6.1\cdot 10^{21}$

Cosmic-rays in electromagnetic fields



The ISM is magnetized， typically B=1-10μG~10$^{-(10-9)}$T。

CRs are largely collisionless, subject to the Lorentz force：

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

B field => gyro-motion:

Gyro-frequency： $\Omega_e = \frac{17.6}{\gamma_e} \mathrm{s}^{-1} \left(\frac{B}{\mu \mathrm{G}} \right), \quad \Omega_i = \frac{9.58 \times 10^{-3}}{\gamma_i} \mathrm{s}^{-1} \left(\frac{B}{\mu \mathrm{G}} \right)$

Gyro-radius：

$$r_L = 0.223 \mathrm{AU} \left(\frac{E}{\mathrm{GeV}} \right) \left(\frac{B}{\mu \mathrm{G}} \right)^{-1} = 1.08 \times 10^{-6} \mathrm{pc} \left(\frac{E}{\mathrm{GeV}} \right) \left(\frac{B}{\mu \mathrm{G}} \right)^{-1}$$

Cosmic-rays in electromagnetic fields

The ISM is magnetized，typically B=1-10μG~$10^{-(10\text{-}9)}$T。

CRs are largely collisionless, subject to the Lorentz force：

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

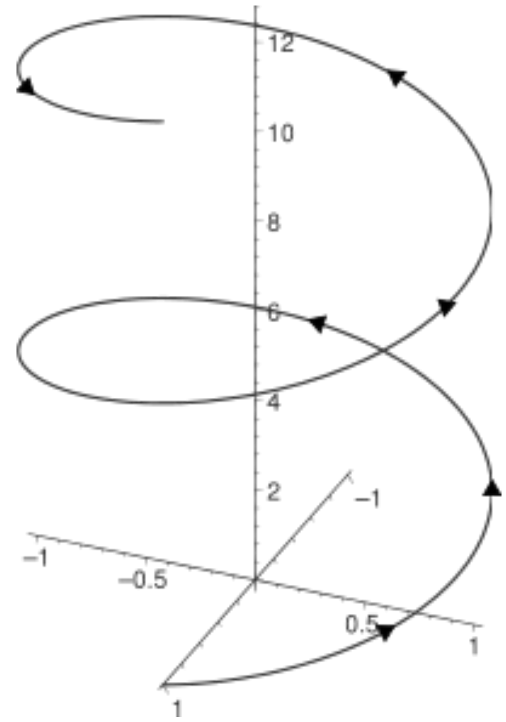
E field: primarily from background gas motion.

Being plasmas, expect E~0 in the ISM in comoving frame.

In the lab frame, we have $\boldsymbol{E}$=-$\boldsymbol{V}\times\boldsymbol{B}$

local gas velocity

Certain region may have $E_{\parallel}$ (e.g., reconnection) to direct accelerate particles.



CR scattering: analogy with gravity assist

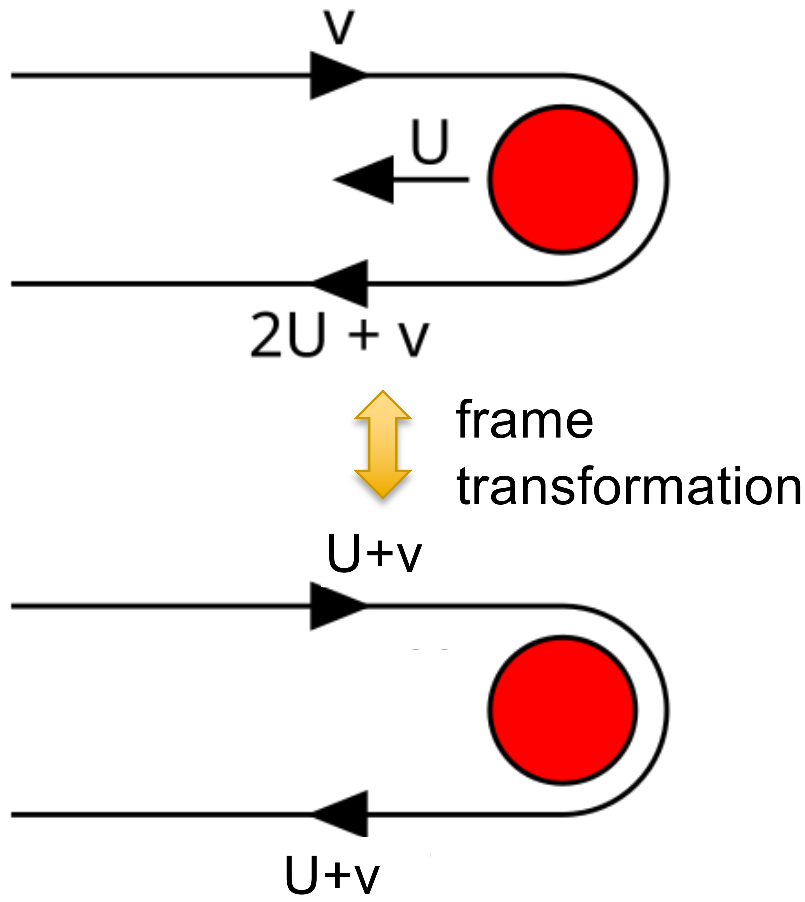
By making use of the relative motion and gravity of a planet/other bodies to alter the path and speed of a spacecraft.

Do we get acceleration/deceleration "for free"?

For CR scattering:

In scatterer's frame, energy does not change.

In observer's frame, work done by E field, energy from the scatterer.



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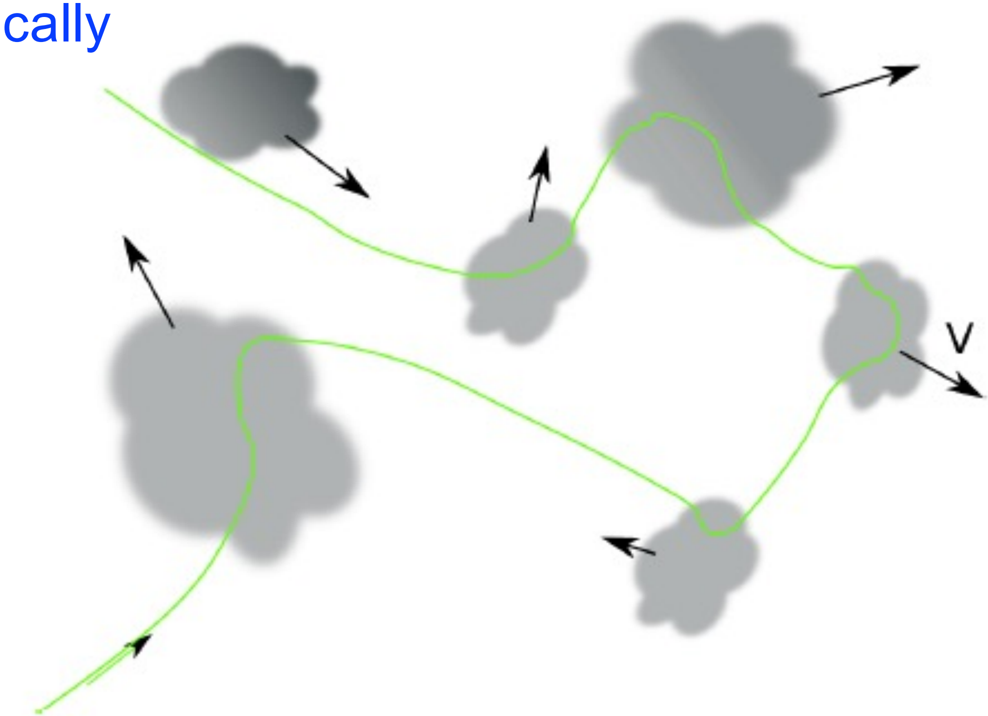
Stochastic Fermi acceleration: basic picture

Fermi (1949)

Fast particles gain/lose energy stochastically via reflections in "moving B fields".

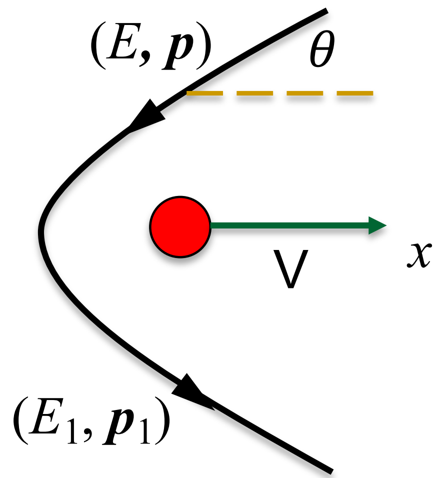
On average, particles gain energy from each reflection:

$$\frac{\Delta E}{E} \propto \left(\frac{V}{c}\right)^2$$



Usually operates in the presence of magnetohydrodynamic turbulence.

Stochastic Fermi acceleration



Let the scatterer's frame denoted by ‘, with velocity V along the x-direction:

$$E' = \gamma_V (E - V p_x) \ , \quad p'_x = \gamma_V \left(p_x - \frac{VE}{c^2} \right)$$

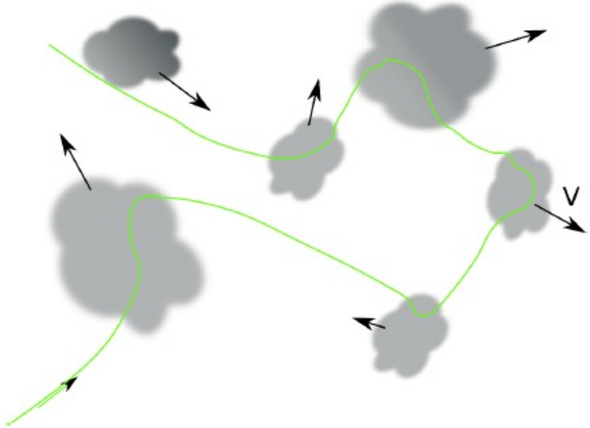
In scatter's frame, let the incident angle to x-axis to be θ. Upon exiting the scatterer, transform back to the lab frame:

$$E_1 = \gamma_V (E' + V p'_{x1}) = \gamma_V (E' - V p'_x) = \gamma_V^2 E \left(1 + \frac{2 V v \cos\theta}{c^2} + \frac{V^2}{c^2} \right)$$

Net energy gain：

$$\frac{E_1 - E}{E} = \frac{\Delta E}{E} \approx \frac{2 V v \cos\theta}{c^2} + 2\frac{V^2}{c^2}$$

Stochastic Fermi acceleration



Net energy gain:

$$\frac{E_1 - E}{E} = \frac{\Delta E}{E} \approx \frac{2V v \cos\theta}{c^2} + 2\frac{V^2}{c^2}$$

Angular distribution of scatterers seen by the particle:

$$f(\theta)d\theta = \frac{1}{2}\left(1 + \frac{V\cos\theta}{c}\right)\sin\theta d\theta$$

(higher chance of "head-on" than "tail-on" scattering)

Angular averaging:

$$\left\langle \frac{2V\cos\theta}{c} \right\rangle = \frac{2V}{c}\int_0^{\pi} f(\theta)\cos\theta d\theta = \frac{2}{3}\frac{V^2}{c^2}$$

Average energy gain per scattering:

$$\left\langle \frac{\Delta E}{E} \right\rangle \approx \frac{8}{3}\frac{V^2}{c^2}$$

Timescale and energy spectra

Let scttering mean free path be $L_{\rm scat}$, with scattering timescale $L_{\rm scat}/c$, we have:

$$\frac{dE}{dt} \approx \frac{8}{3}\frac{V^2}{c^2}E\left(\frac{c}{L_{\rm scat}}\right) \equiv \frac{E}{t_a}$$

=> acceleration timescale: $t_a \approx \frac{3}{8}\frac{cL_{\rm scat}}{V^2}$

Particles may escape acceleration site, over an escape timescale $t_{\rm esc}$.

Assuming we start with N_0 particles with initial energy E_0. After time Dt, there are $N_0\exp(-Dt/t_{esc})$ particles reach energy $E=E_0\exp(Dt/t_a)$. Therefore,

$$N(>E) = N_0\left(\frac{E}{E_0}\right)^{-t_a/t_{\rm esc}}$$

=> energy distribution $f(E) \propto E^{-(t_a/t_{\rm esc}+1)}$

We arrive at a power-law distribution, with slope set by $t_a/t_{\rm esc}$.

Mathematical description

Particle distribution function satisfies the Fokker-Planck Eq. In most simplified form:

$$\frac{\partial f(p,t)}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D_{pp}(p)\frac{\partial f}{\partial p}\right] + Q(p)$$

(Assuming particles are isotropic)

momentum diffusion

source and sink

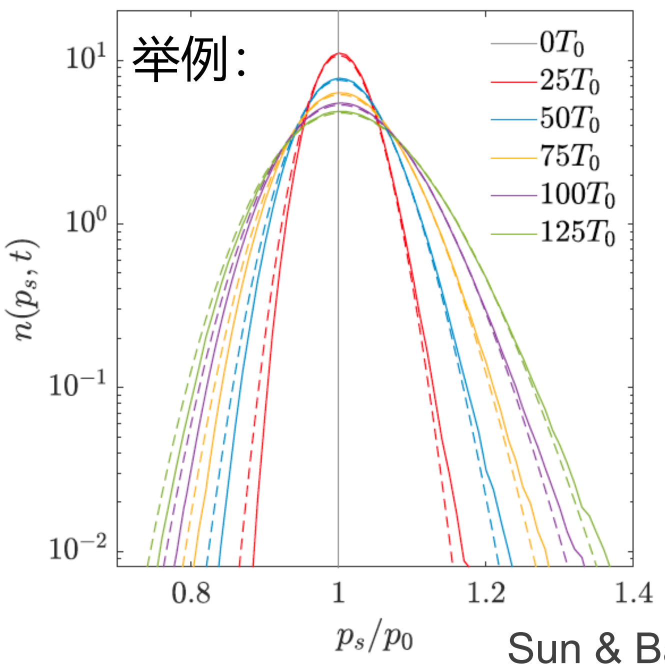
(Missing: spatial dependence, cooling, etc.)

"Diffusion" of momentum, being more likely to reach higher energy:

$$t_a \sim \frac{p^2}{D_{pp}(p)}$$

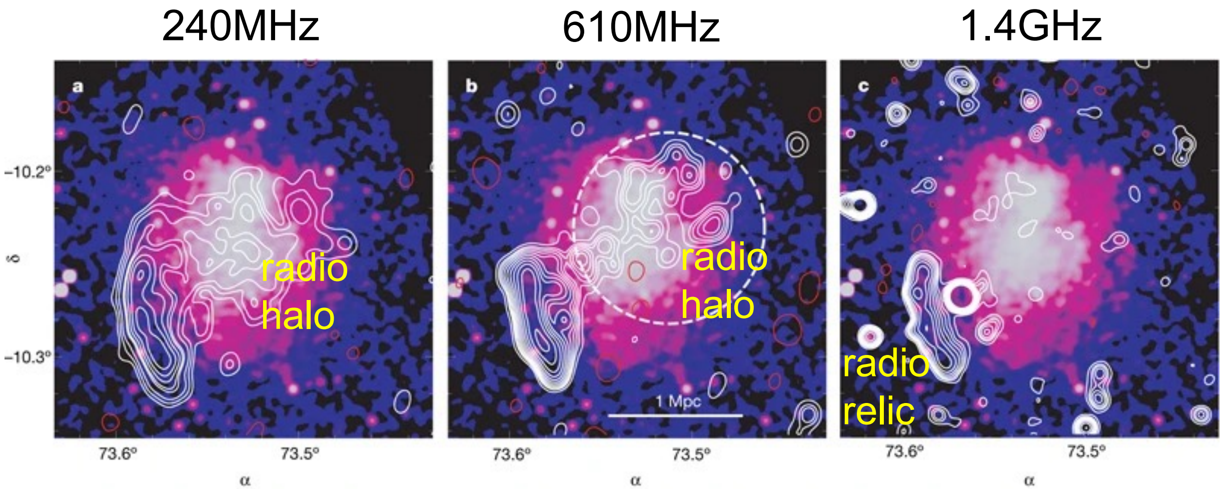
By comparison:

$$D_{pp}(p) \sim \frac{p^2 V^2}{c L_{\rm scat}}$$

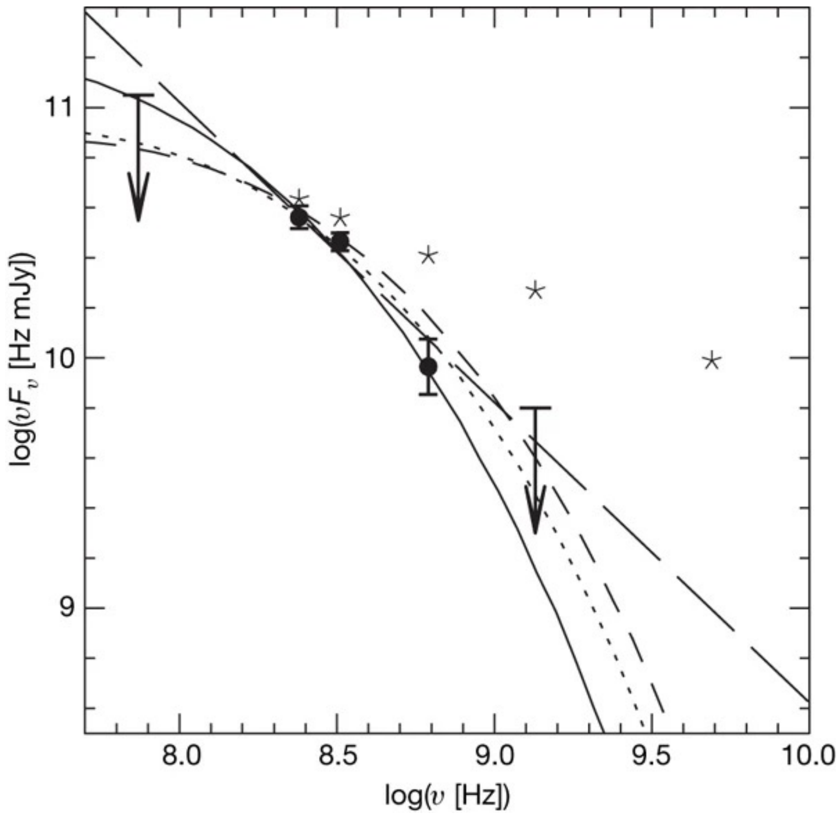


Sun & Bai 21

Astrophysical applications



Galaxy cluster Abell 521. Contours: radio; Color: X-ray.



Brunetti et al. 2008

Hot gas in galaxy clusters are weakly turbulent, (re-)accelerating electrons to produce radio halos.

Also find applications in solar flares and accretion disk corona.

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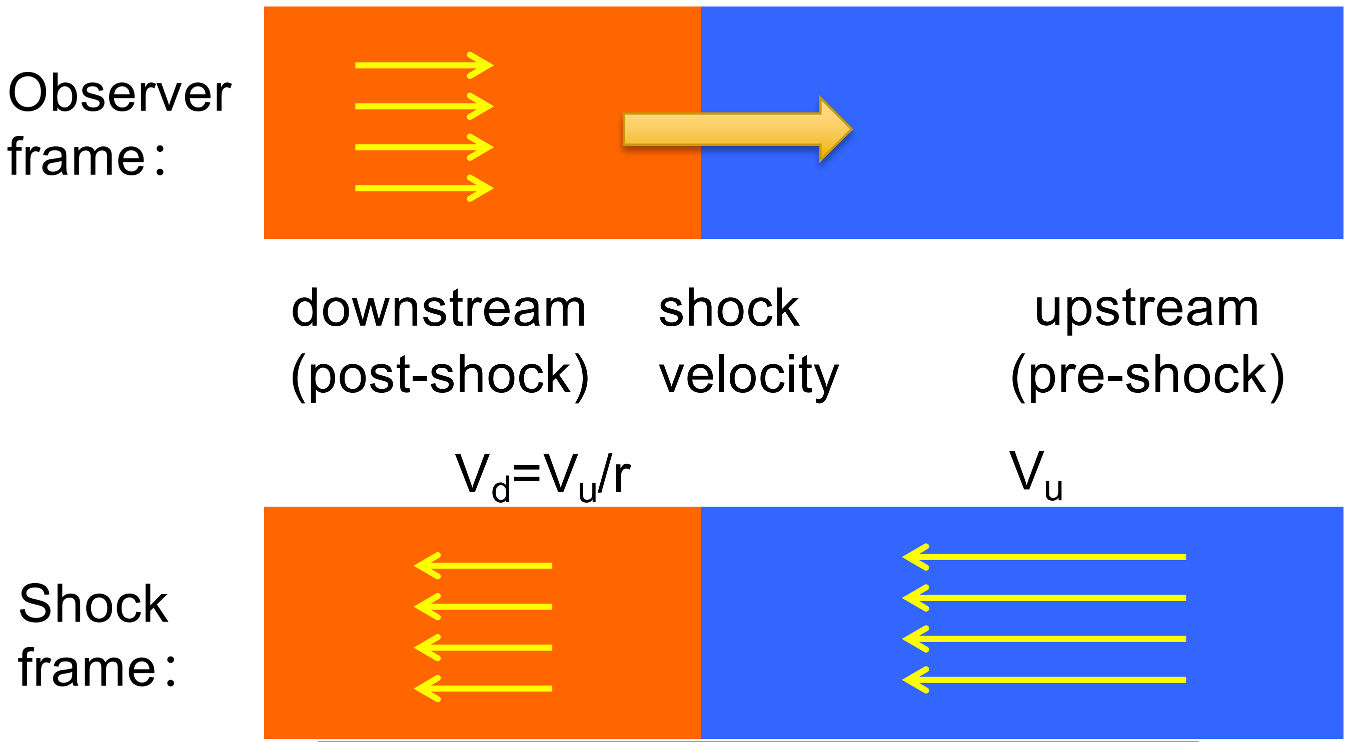
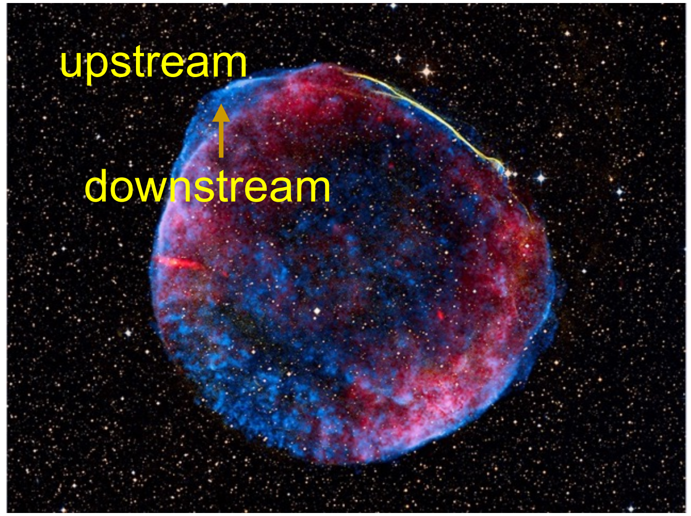
From 2nd order to 1st order: shocks





Converging flow across the shock

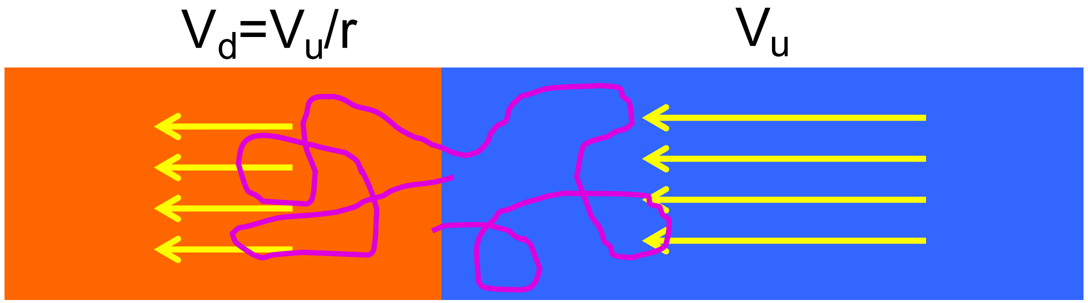
$$V \equiv V_u - V_d = (r-1)V_d = \frac{r-1}{r} V_u$$

Bell 1978
Blandford & Ostriker 1978

Diffusive Shock Acceleration

$$V \equiv V_u - V_d$$

Shock frame:



Assume rel. CRs in non-rel. shock (V<<c), with turbulence to scatter CRs.

Crossing the shock from downstream, CR energy in the upstream is (to 1st order):

$$E' = E + Vp\cos\theta\ , \qquad \text{or} \quad \frac{\Delta E}{E} \approx \frac{V\cos\theta}{c}$$

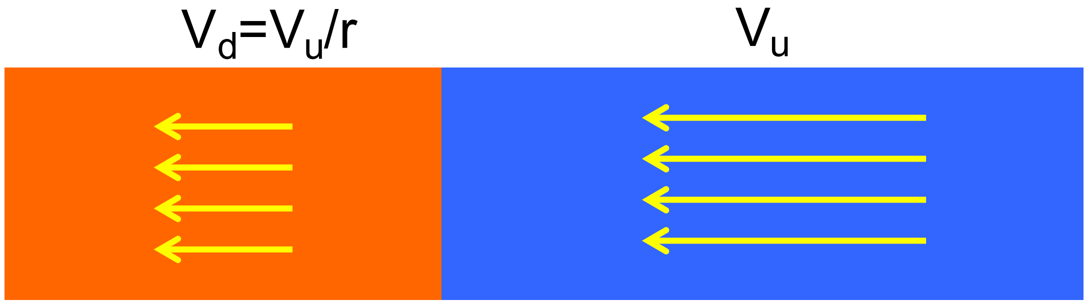
θ: angle to shock normal during crossing

Assuming CRs are originally isotropic, only those with $\theta \in (0, \pi/2)$ can cross.

Useful reference on DSA: Drury 1983, Rep. Prog. Phys.

Diffusive Shock Acceleration

Shock frame:



$$V \equiv V_u - V_d$$

Assume rel. CRs in non-rel. shock (V<<c), with turbulence to scatter CRs.

After angular averaging, mean energy gain is:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{V}{c} \int_0^{\pi/2} f(\theta) \cos\theta d\theta = \frac{2}{3}\frac{V}{c}$$

After crossing the shock back and forth for a full cycle:

$$\left\langle \frac{\Delta E}{E} \right\rangle_{\text{cycle}} = \frac{4}{3}\frac{V}{c}$$

Energy spectrum from DSA

Starting from N_0 particles with initial energy E_0.

After one cycle, a fraction P of all particles can enter the next cycle (others leave to shock downstream), and their energy increase by a factor A.

After k cycles, there will be $N=N_0P^k$ particles with energy E_0A^k.

Now, the cumulative distribution function (CDF) of particle energy becomes:

$$N(\geq E) = N_0 P^{\ln(E/E_0)/\ln A} = N_0 \exp\left[\ln\frac{E}{E_0}\cdot\frac{\ln P}{\ln A}\right] = N_0\left(\frac{E}{E_0}\right)^{\ln P/\ln A}$$

The energy distribution reads: $f(E) = \frac{dN}{dE} \propto E^{-1+\ln P/\ln A}$ $(-t_a/t_{esc})$

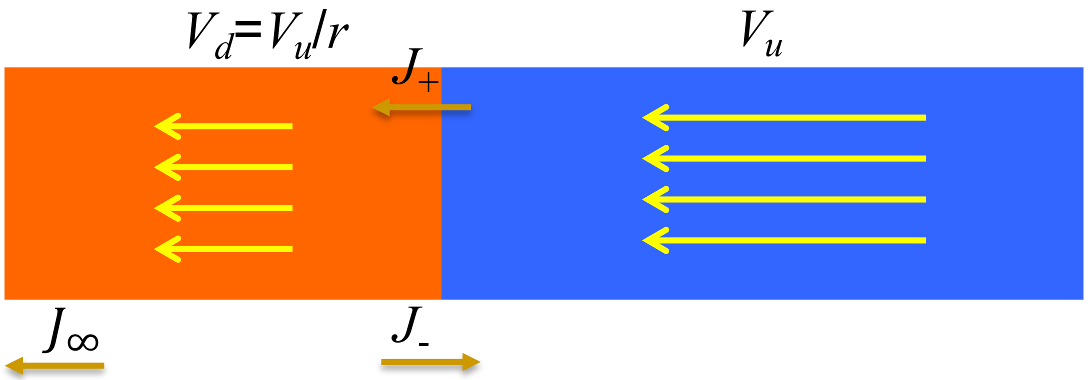
Power-law index: $s = 1 - \frac{\ln P}{\ln A}$

We already learned $A = 1 + (4/3)V/c$. Next we look at P.

Energy spectrum from DSA

$V \equiv V_u - V_d$

Shock frame:



CR flux conservation: $J_+ = J_- + J_\infty$ => $P = \frac{J_-}{J_+} = \frac{J_-}{J_- + J_\infty}$

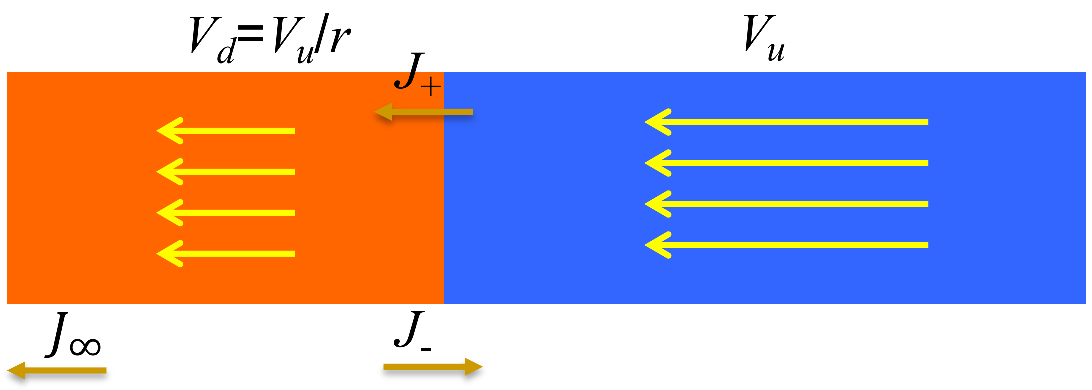
Asymptotic flux downstream: $J_\infty = n_0 V_d$

Flux returning upstream: $J_- = \int_{\cos\theta > 0} \frac{d\Omega}{4\pi} nc \cos\theta = \frac{nc}{4}$ => $P = \frac{c}{c + 4V_d} \approx 1 - \frac{4V_d}{c}$

Energy spectrum from DSA

$V \equiv V_u - V_d$

Shock frame:



Fractional energy gain per cycle: $A = 1 + (4/3)V/c.$

Probability of continued acceleration: $P = \dfrac{c}{c+4V_d} \approx 1 - \dfrac{4V_d}{c}$

Power-law index: $s = 1 - \dfrac{\ln P}{\ln A} \approx 1 + \dfrac{3V_d}{V} = 1 + \dfrac{3}{r-1}$ Compression ratio: $r = V_u/V_d$.

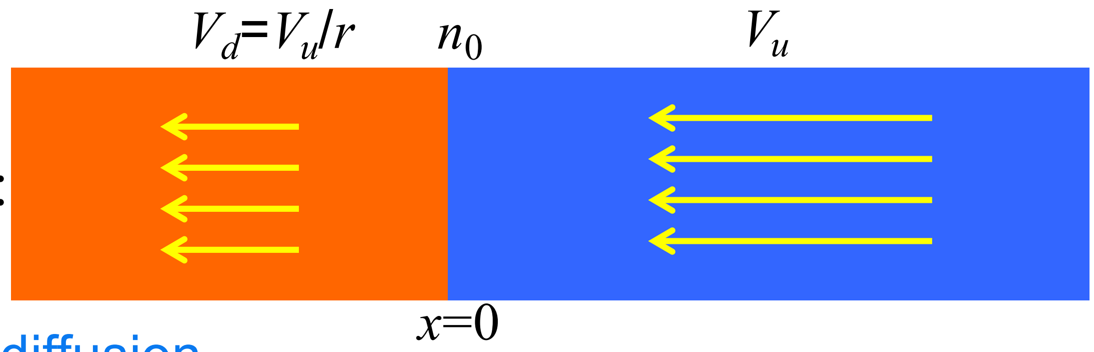
For strong shocks (M>>1), r=4 => s=2.

Independent of microphysics!

Timescale of DSA

$$V \equiv V_u - V_d$$

Shock frame:



Spatial diffusion coefficient: D_d D_u

CR density profile upstream satisfies: $nV_u - D_u \frac{dn}{dx} = 0$

Assuming constant D: $n(x) = n_0 e^{-V_u x / D_u}$ => $N_u = \int_{x=0}^{\infty} n(x) dx = \frac{n_0 D_u}{V_u}$

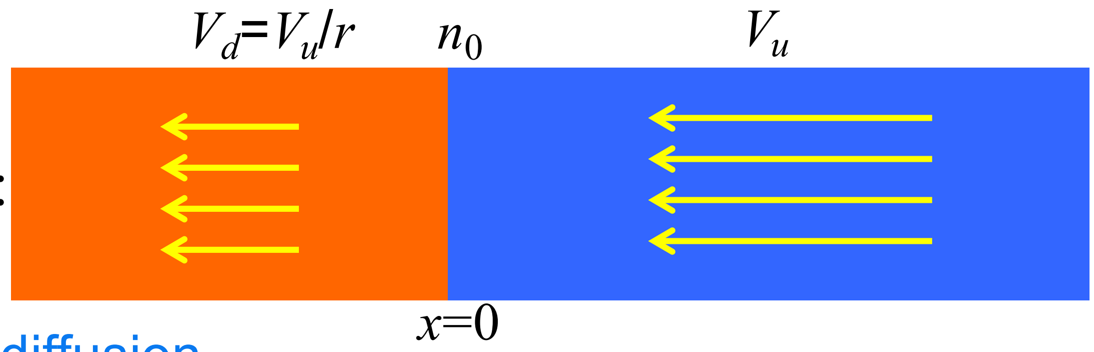
Time spent at upstream: $t_u = \frac{N_u}{J_-} = \frac{n_0 D_u}{V_u} \Big/ \frac{n_0 c}{4} = \frac{4 D_u}{V_u c}$

Similarly, time spent at downstream: $t_d = \frac{4 D_d}{V_d c}$

Timescale of DSA

$V \equiv V_u - V_d$

Shock frame:



Spatial diffusion coefficient: D_d D_u

Timescale of one acceleration cycle:

$$t_{\rm cycle} = t_u + t_d = \frac{4D_u}{V_u c} + \frac{4D_d}{V_d c}$$

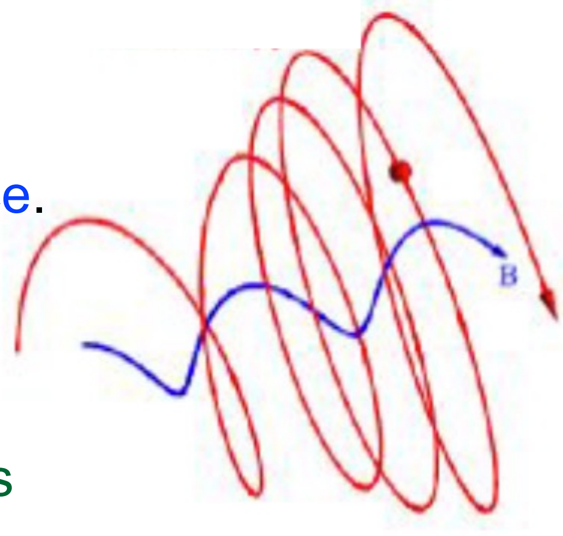
Mean rate of acceleration:

$$\frac{dE}{dt} = \frac{4V}{3c}\frac{E}{t_{\rm cycle}} \equiv \frac{E}{t_a}$$

=> $$t_a = \frac{3ct_{\rm cycle}}{4V} = \frac{3}{V}\left(\frac{D_u}{V_u} + \frac{D_d}{V_d}\right)$$

For e^-, synchrotron loss rate$\propto E^2$, maximum energy set by balancing the loss.

On diffusion coefficients



From detailed microphysics, mainly through gyro-resonance.

Rate of pitch angle scattering:

$$\nu_s \sim \Omega \left(\frac{\delta B}{B} \right)^2$$

At resonant wavelength

For small δB, known as *quasi-linear diffusion*

CR mean free path: $l_{\rm mfp} \sim c/\nu_{\rm s}$, diffusion coefficient: $D \sim c^2/\nu_{\rm s}$

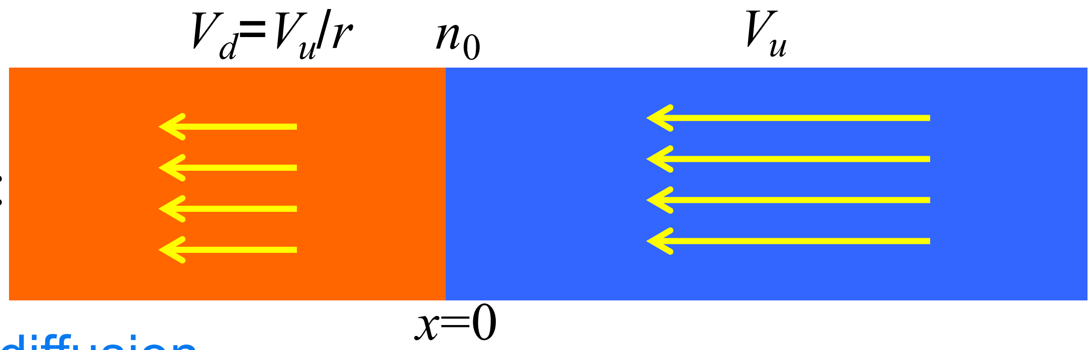
Bohm diffusion: $\delta B \sim B$, $l_{\rm mfp} \sim r_L$, $$D_{\rm Bohm} = \frac{1}{3} r_L c = \frac{Ec}{3ZeB}$$

Bohm diffusion gives the lower limit in diffusion coefficient.

Timescale of DSA

$V \equiv V_u - V_d$

Shock frame:



Spatial diffusion coefficient: D_d D_u

Recall: $D_{\rm Bohm} = \frac{1}{3} r_L c = \frac{Ec}{3ZeB}$

Acceleration timescale: $\frac{dE}{dt} = \frac{E}{t_a}$, $t_A = \frac{3}{V}\left(\frac{D_u}{V_u} + \frac{D_d}{V_d}\right)$

For strong shocks (V_d=V_u/4) with Bohm diffusion, B_d=4B_u => $E_{\rm max} \approx \frac{3}{8}\frac{ZeB_u}{c} V_u^2 T$ (Time)

(most optimistic)

For typical SNR shock, V_u~5000 km/s, T~2000yr, B_u~10μG => $E_{\rm max}$~ PeV.

(will decelerate afterwards)

Key questions in studying shock acceleration

- 粒子注入问题 (Injection problem)

  激波上游：粒子生来平等，如何选出“幸运儿”进入加速流程？来自微观物理。

- 粒子的扩散系数 (Diffusion coefficients)

  激波上/下游的湍流强度直接决定了激波加速的效率，来自微观物理。

- 粒子的反馈作用 (Feedback)：Non-linear DSA

  相当一部分激波能量用于粒子加速，激波已不再是纯磁流体激波。

Results sensitively depend on shock parameters, and are different between e/p.

Mathematical description

From the Boltzmann Eq., assuming particles are isotropic with local fluid, to obtain:

(Skilling 1975, Drury 1983)

$$\frac{\partial f}{\partial t} + u(x)\frac{\partial f(x,p)}{\partial x} = \frac{\partial}{\partial x}\left[D(x,p)\frac{\partial f(x,p)}{\partial x}\right] + \frac{p}{3}\frac{\mathrm{d}u(x)}{\mathrm{d}x}\frac{\partial f(x,p)}{\partial p} + Q(x,p)$$

Advection

Spatial diffusion

Acceleration (by compression)

Injection, energy loss

Microphysics

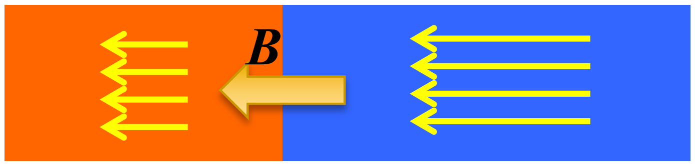
Feedback

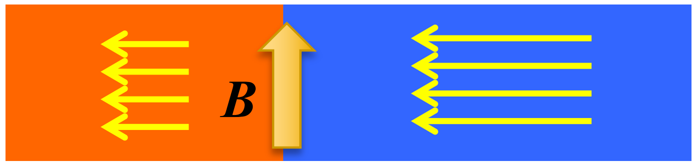
Classification of shocks

- **Relativistic:** Yes (e.g., GRB/Blazar) / No (e.g., SNR, stellar wind collision)
- **Composition:** electron-ion vs electron-positron
- **Magnetization:** Alfvenic Mach # (non-rel) or magnetization (relativistic)

$M_A = v_{\rm sh}/v_A$, $\sigma = (B^2/8\pi)/(nmc^2)$

- **Magnetic obiquity:** quasi-parallel vs quasi-perpendicular





Current understandings

- Outcome of relativistic shocks are reasonably understood through kinetic simulations.

  Review: Sironi, Keshet & Lemoine, 2015, Space Sci. Rev.

- For non-relativistic shocks, outcome of ion acceleration is reasonably understood, but electron acceleration is difficult and under debate.

  e.g., Guo et al. 2014a,b, Park et al. 2015, Matsumoto et al. 2017, Xu et al. 2020, Bohdan et al. 2017, 2020, Kumar & Reville 2021

  Review by Amano, Matsumoto et al. 2022, Reviews of Modern Plasma Physics.

- Kinetic simulations largely limited to microscopic scales and initial stage of acceleration. Need to access larger scales.

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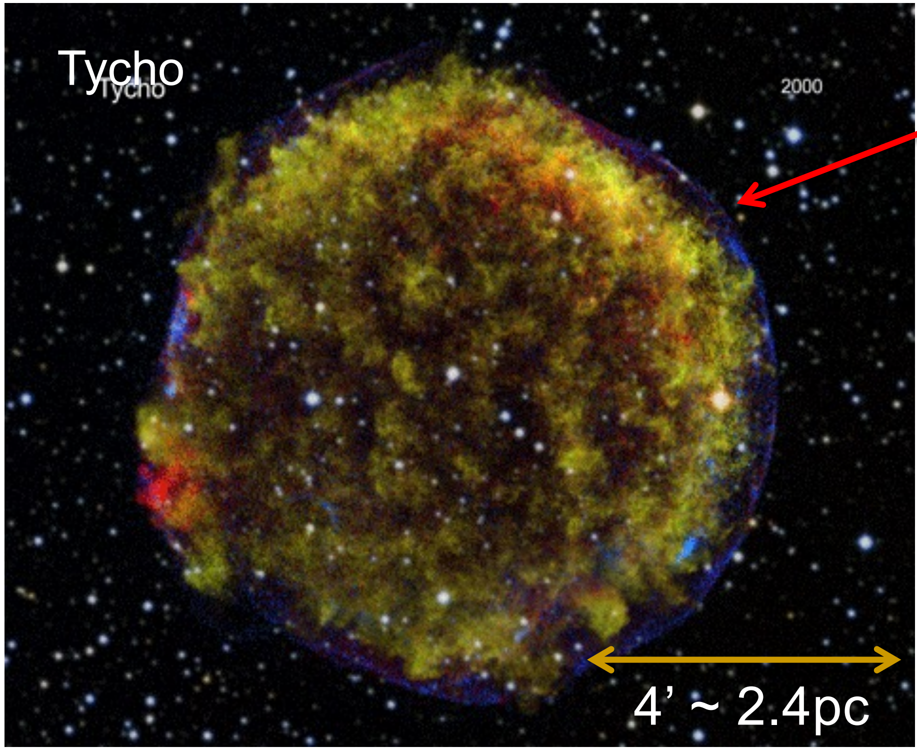
Cosmic accelerator: supernova remnant (SNR)



From Chandra X-ray telescope

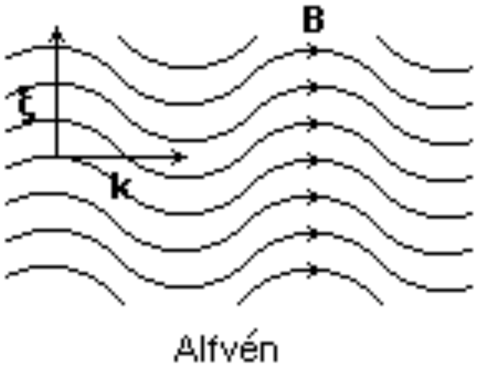
forward shock (primary site for particle acceleration)

X-ray rims due to synchrotron radiation from accelerated *electrons*
(ion acceleration should be more efficient but not directly visible)

Shock velocity: ~$10^{3\text{-}4}$ km/s

Free expansion for ~$10^{2\text{-}3}$ years where CR acceleration is most efficient

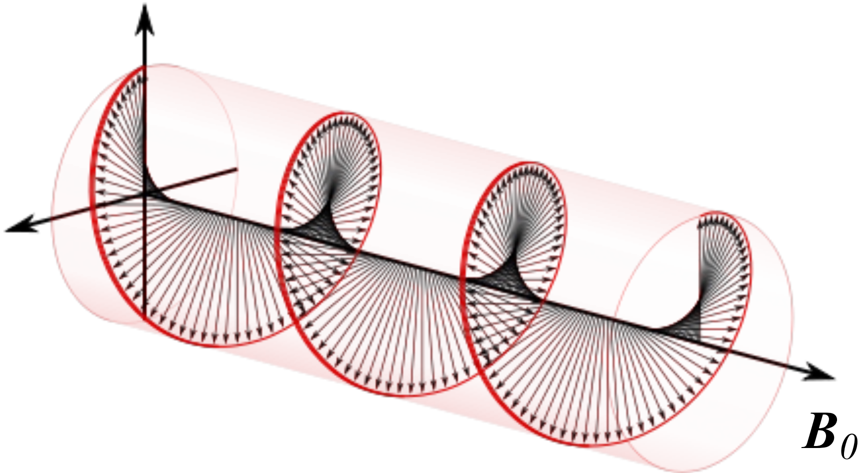
Alfven waves in magnetohydrodynamics



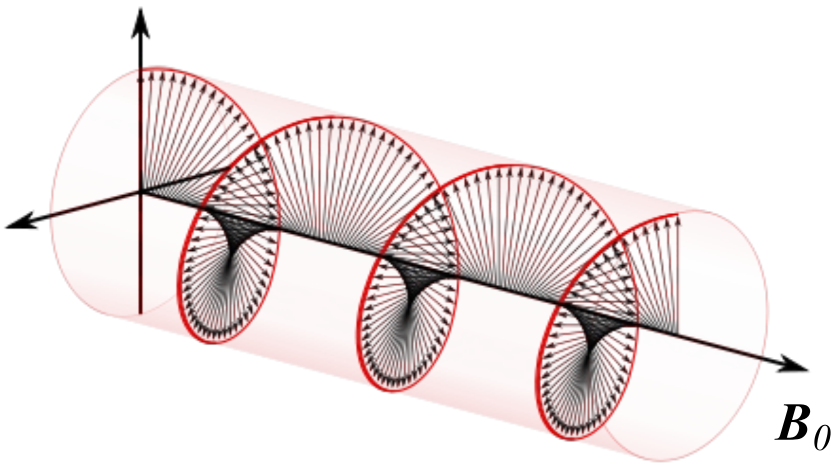
Incompressible, transverse wave; restoring force is magnetic tension.

$$v_A = \frac{B}{\sqrt{4\pi\rho}}$$

Left polarization:

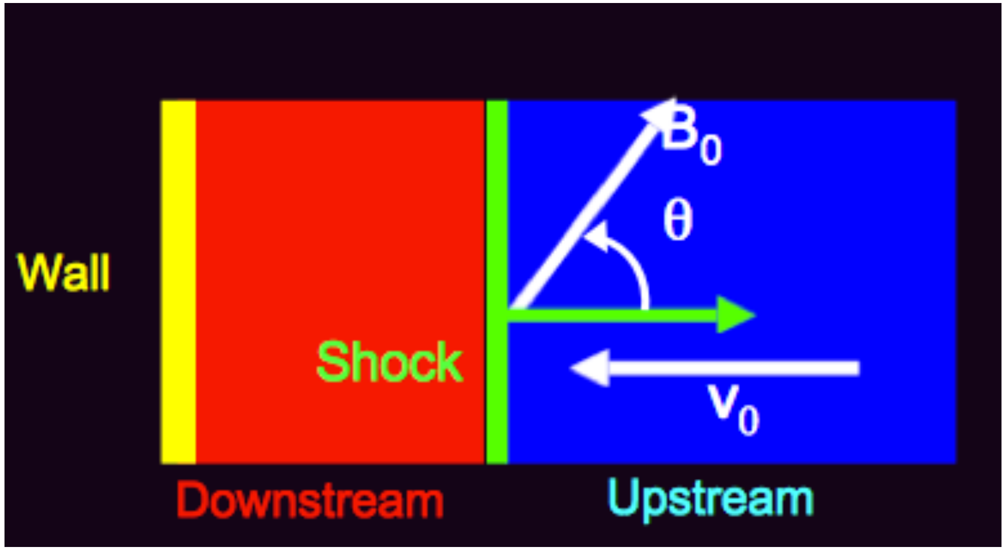


Right polarization:



Simulations of collisionless shocks: setup

- Colliding fast moving (cold) gas into a reflecting wall: simulation in the downstream frame.



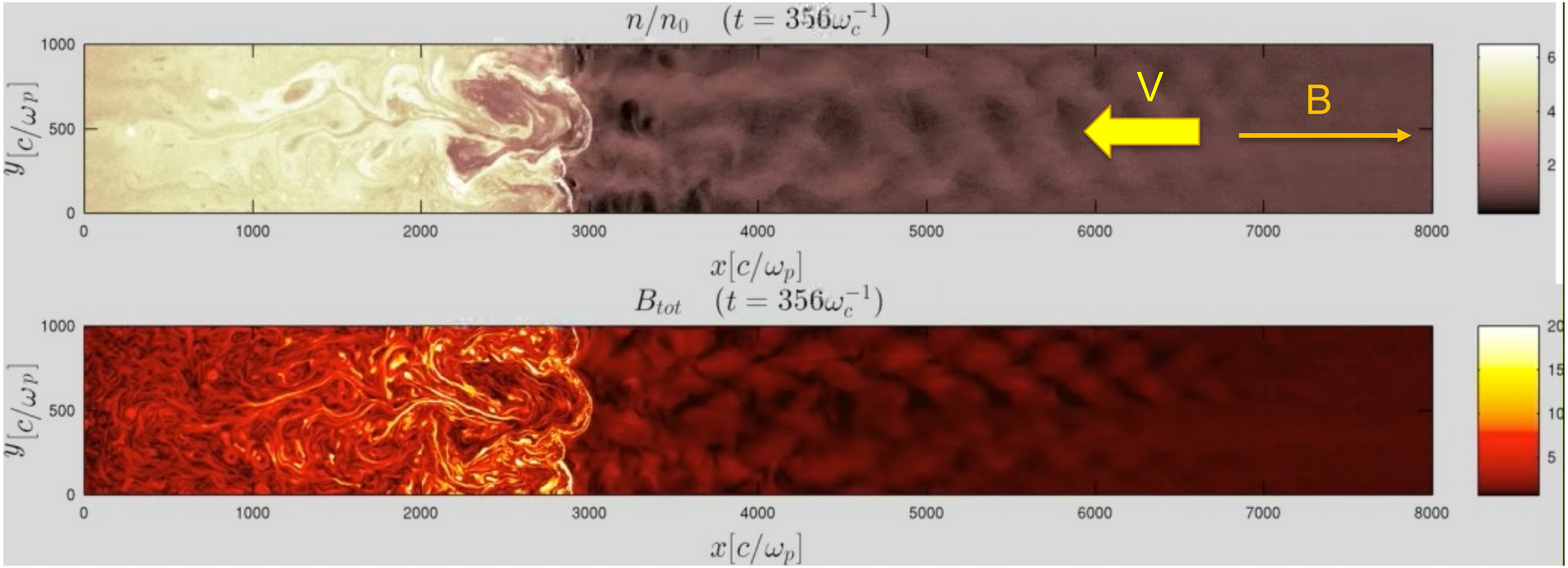
Pre-shock parameters:

- Alfvenic Mach number $M_A = v_0/v_A$
- Magnetic obliquity θ

SNR shocks: typical Mach number M_A~300-1000

Simulations typically use M_A~30-100.

Ion acceleration in non-relativistic shocks



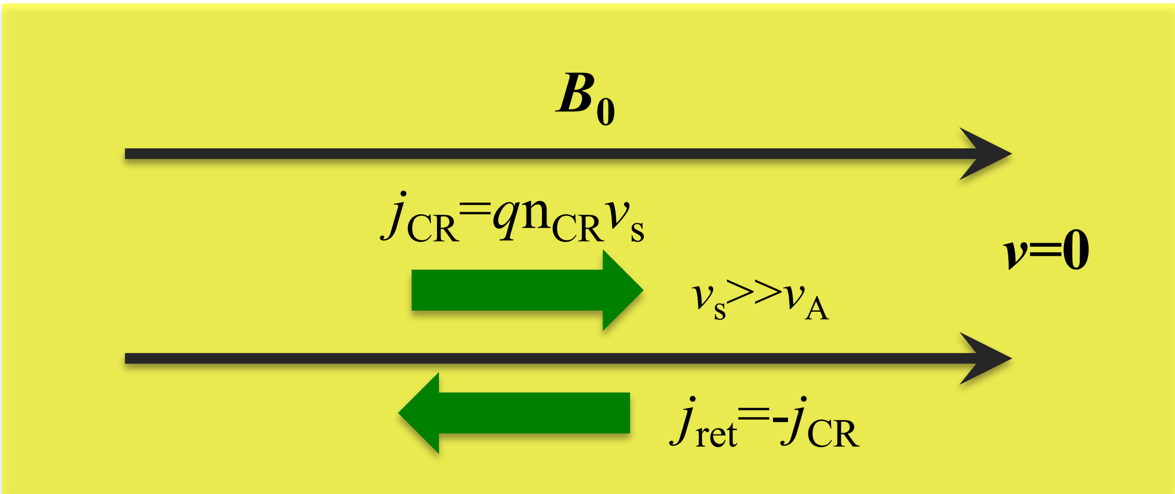
Caprioli & Spitkovsky 2013

- Upstream: onset of instabilities and turbulence
- Downstream: strong magnetic field amplification.

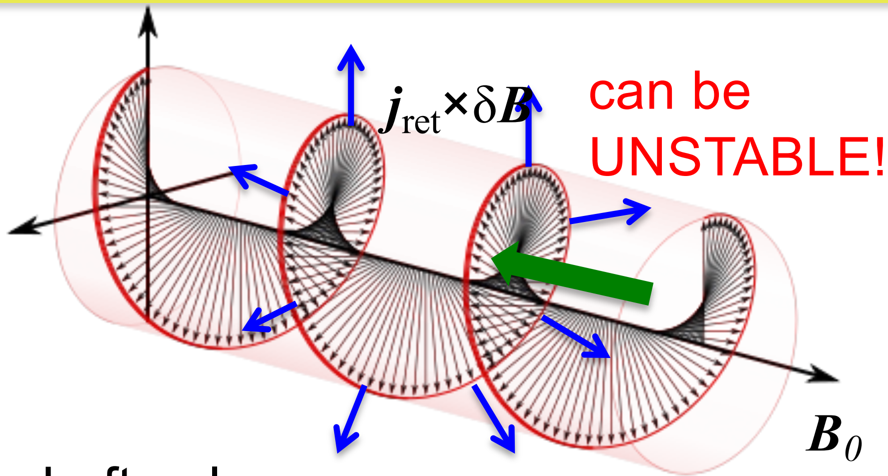
M_A=30, θ=0.

The Bell instability (non-resonant)

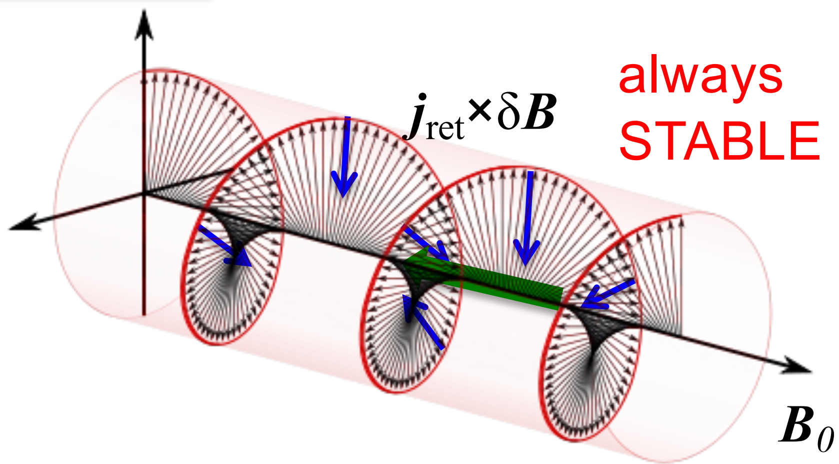
(Bell, 2004)



Freshly accelerated CRs into the upstream to trigger the Bell instability, leading to turbulence which scatter the CRs back.

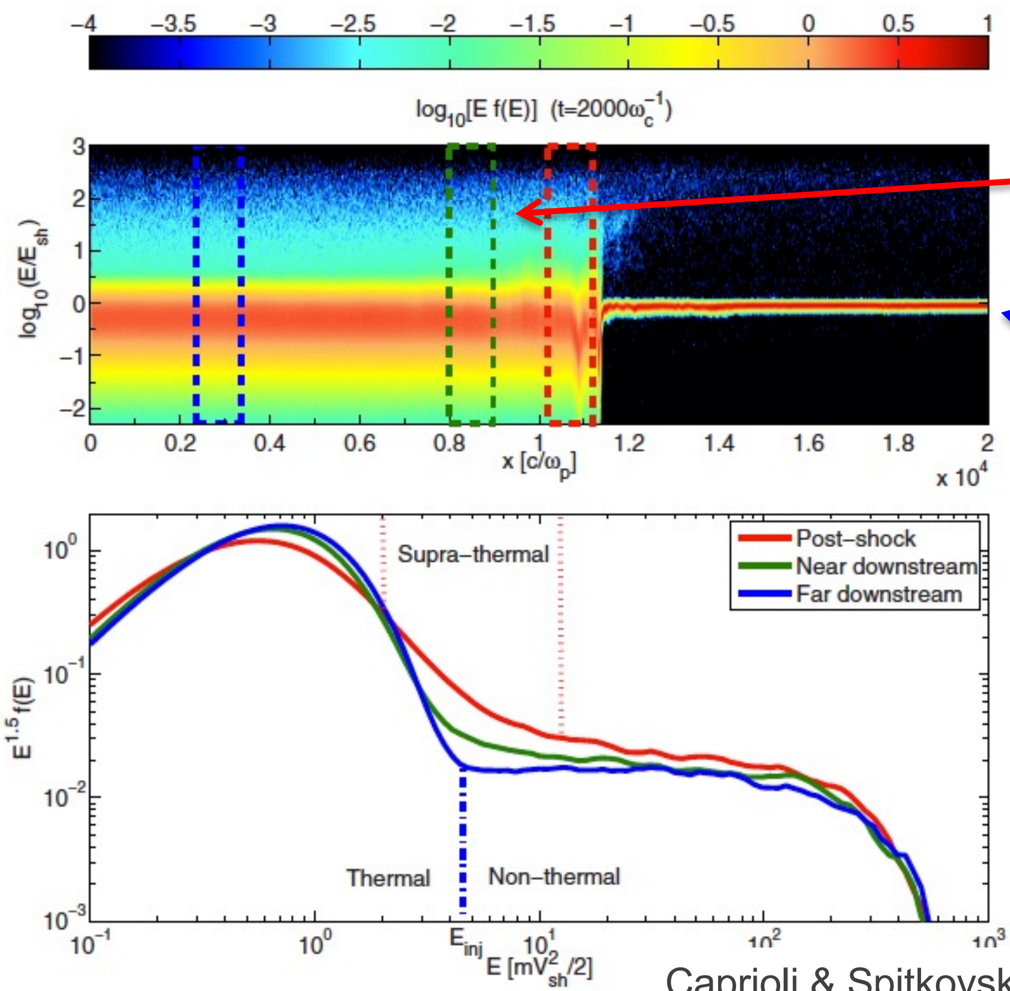


Left pol:



Right pol:

Particle energy spectra



Caprioli & Spitkovsky 2014

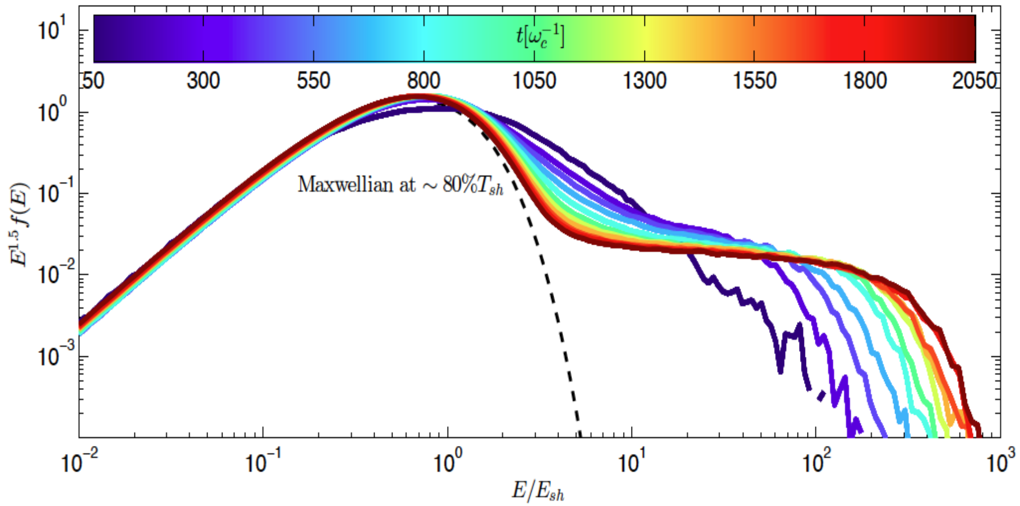
accelerated particles streaming into upstream

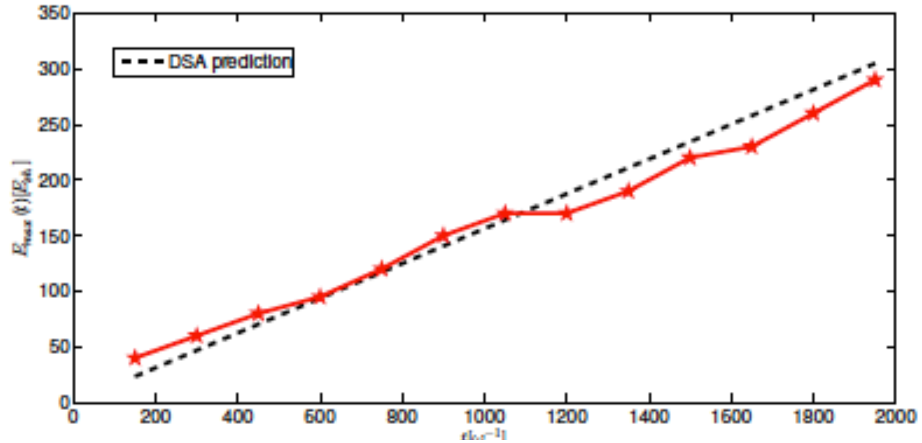
incoming cold upstream ion beam

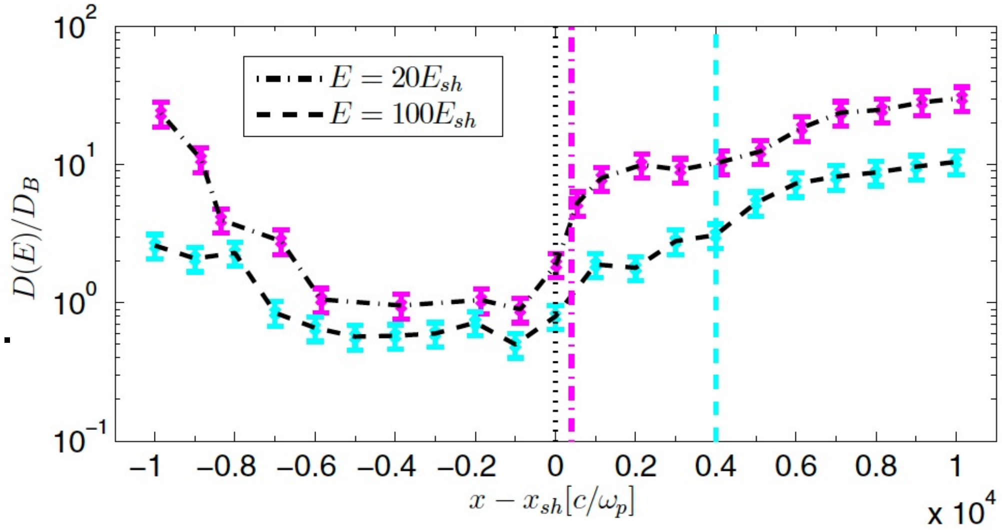
Non-rel particles, expect $f(E) \propto E^{-3/2}$.

~10-20% of shock kinetic energy converted to CRs.

Particle acceleration: time progression





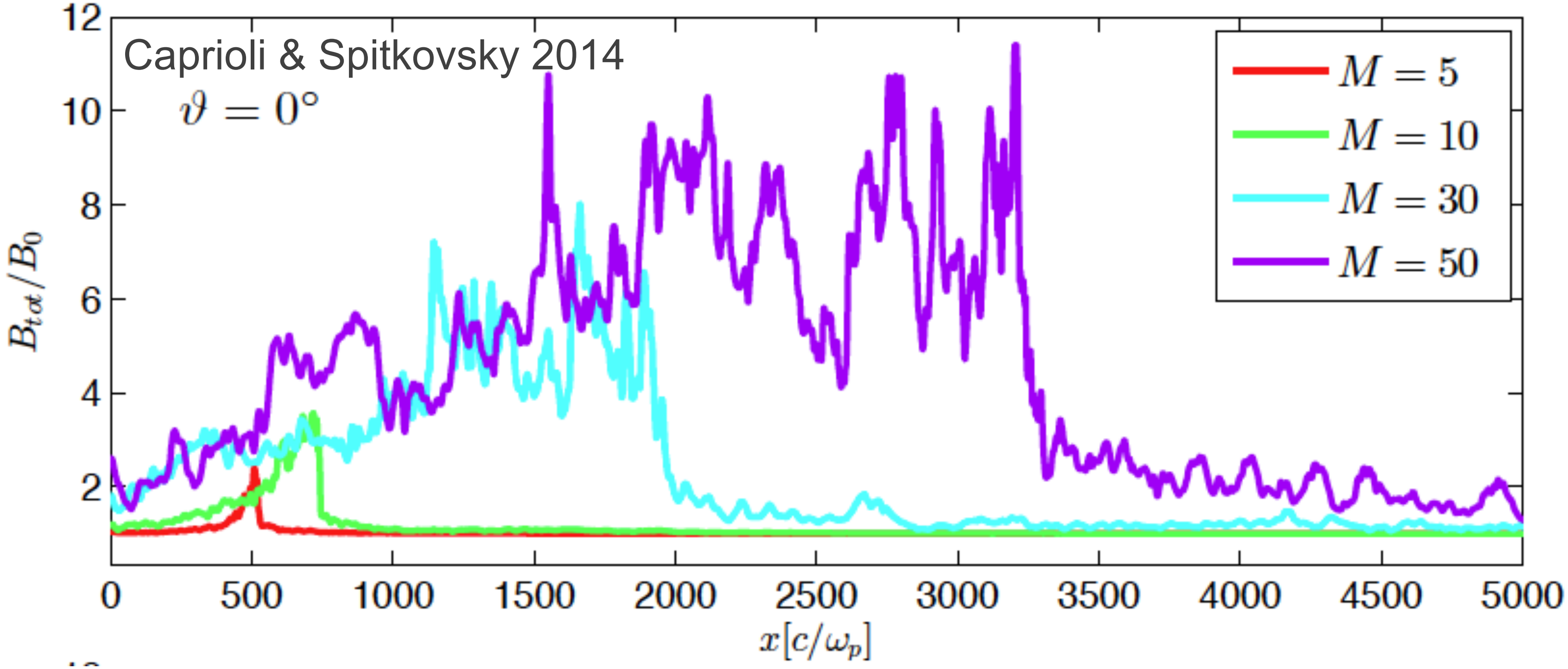


Highest particle energy increases linearly in time.

Diffusion is Bohm-like (higher in upstream).

Caprioli & Spitkovsky 2014

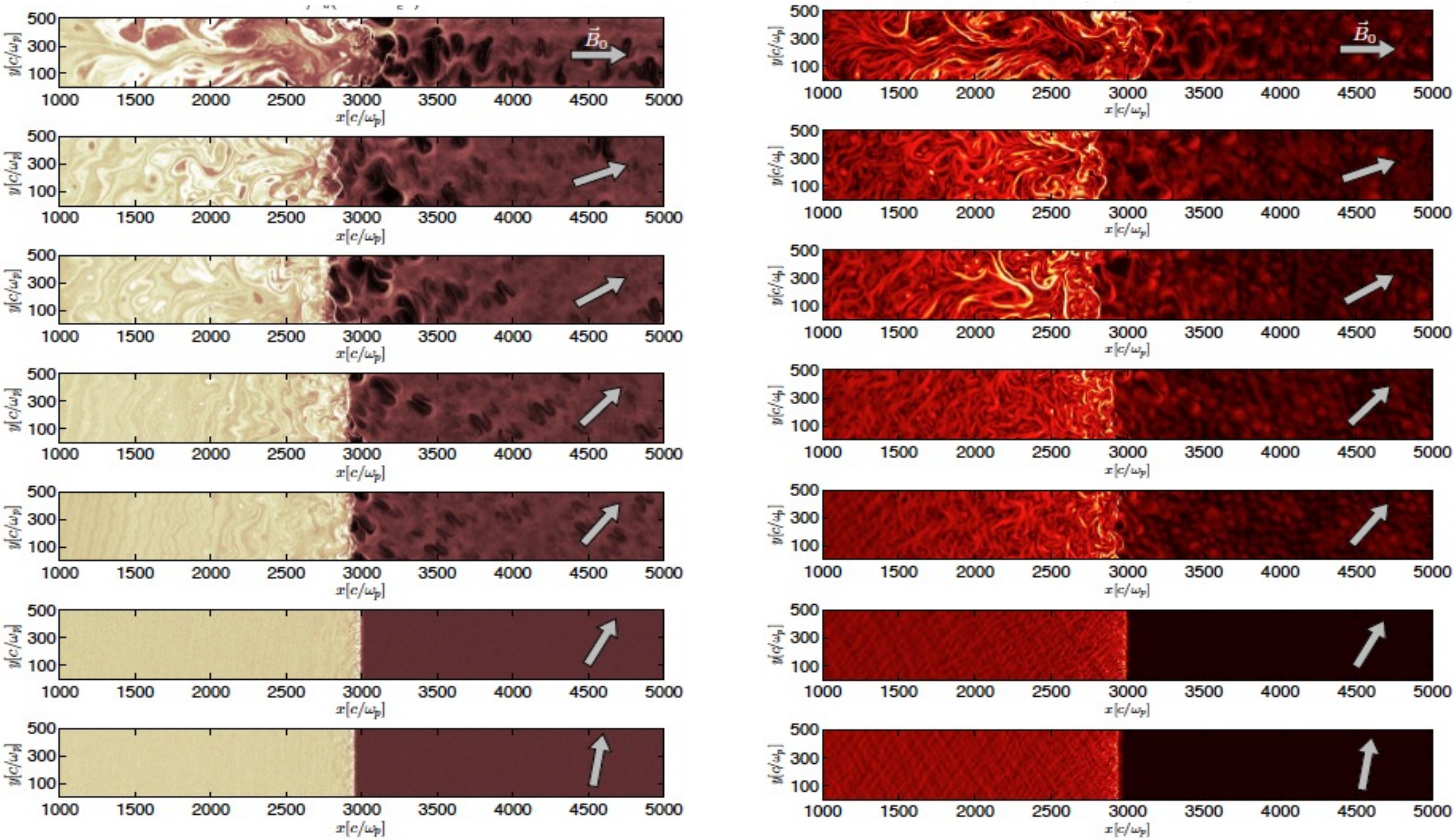
Magnetic field amplification



Pre-amplified in the shock upstream, and strongly amplified in the downstream due to compression and instabilities => reduce the acceleration timescale!

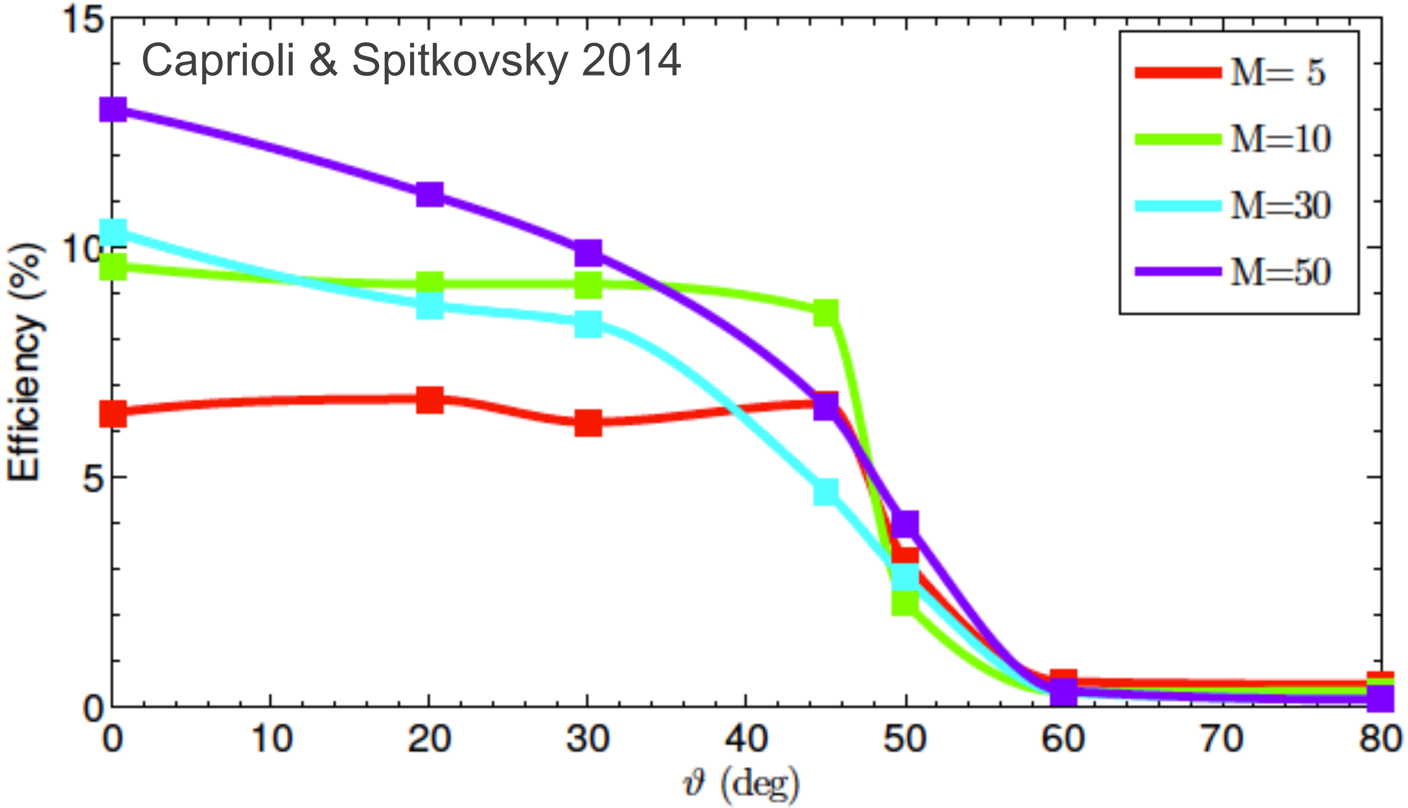
Higher Mach number -> stronger amplification

Dependence on magnetic obliquity



Caprioli & Spitkovsky 2014

Dependence on magnetic obliquity



Only quasi-parallel shocks can efficiently accelerate ions (?)

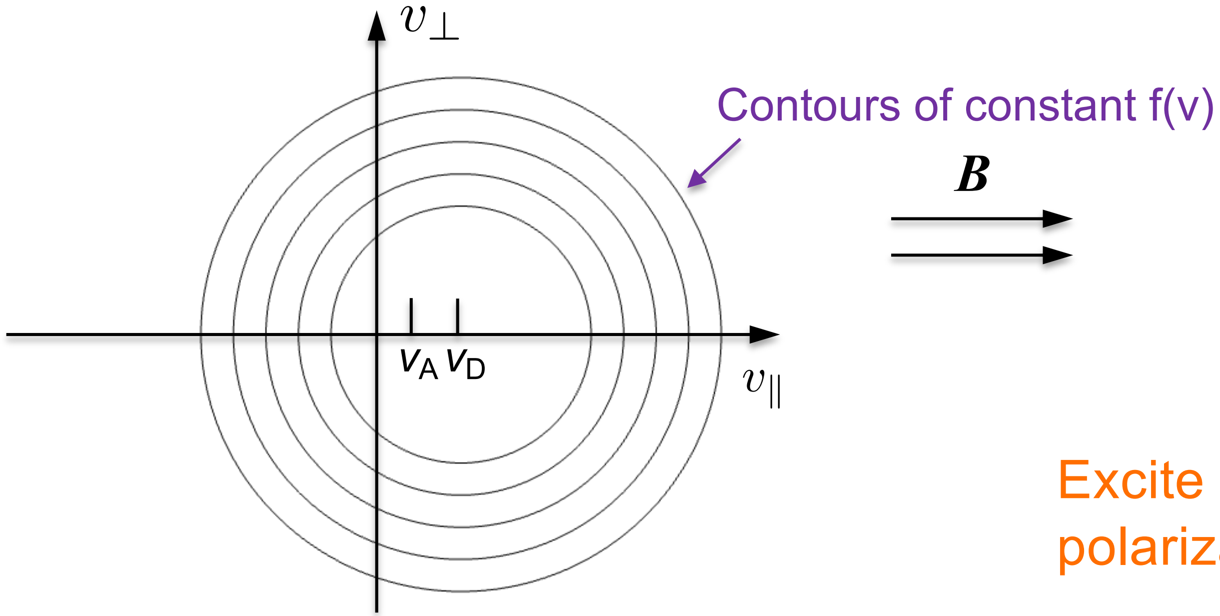
But see recent results by Xu, Spitkovsky & Caprioli (2020).

Note: the CR streaming instability

When the level of CR anisotropy exceeds $\sim v_A/c$, CRs resonantly excite Alfvén waves. (see book by Kulsrud 2005)

Classical case: CR streaming instability (Kulsrud & Pearce 1969)

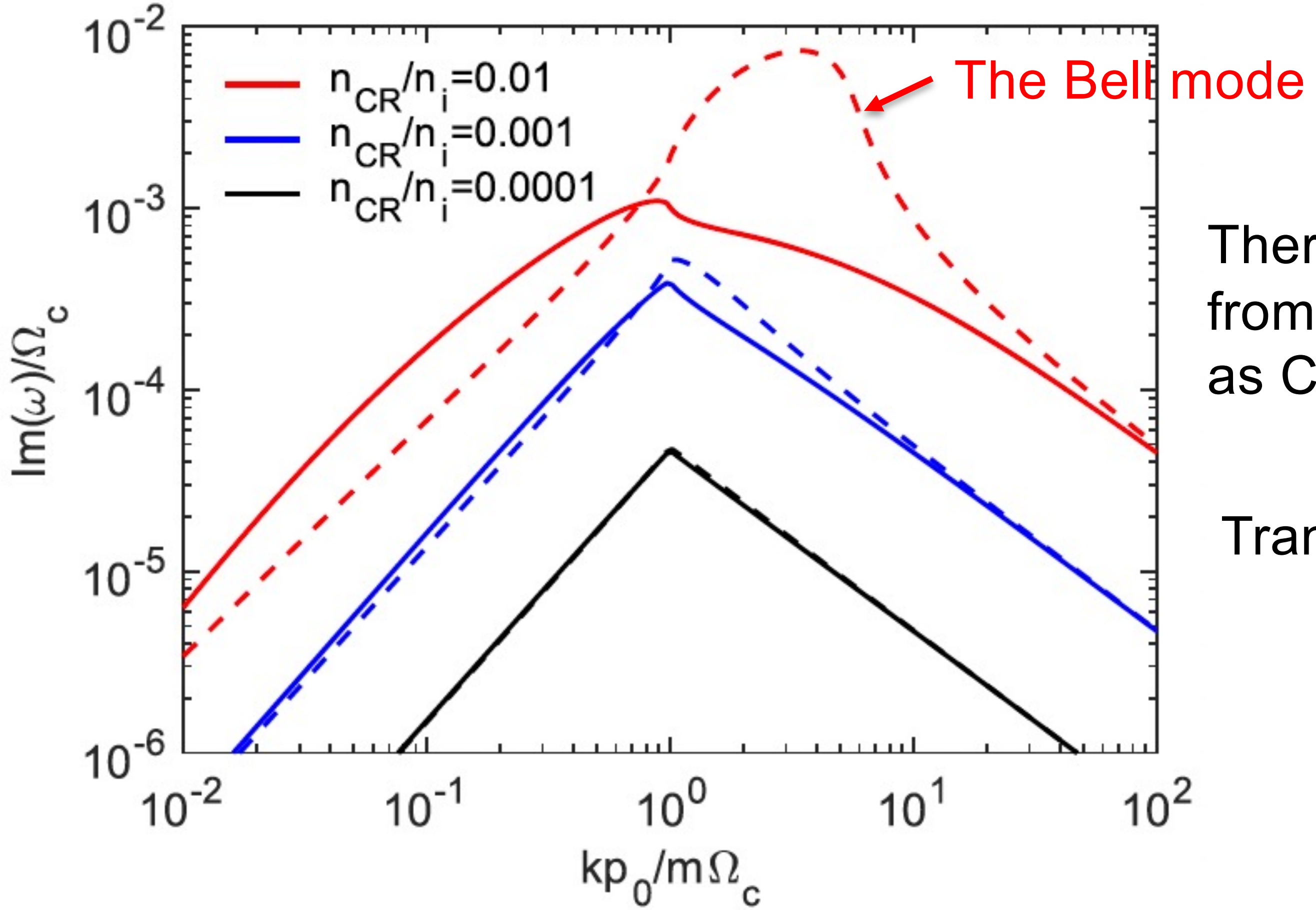
Triggered when the bulk CRs drift speed relative to gas is faster than v_A



CR streaming naturally arises when CRs escape from the source.

Excite forward-traveling waves (two polarizations) down the CR gradient.

Note: Streaming instability vs. Bell instability



Bai et al. 2019

There is a continuous transition from streaming to Bell instabilities as CR current strengthens.

Transition occurs at:

$$J_{\rm CR} \gtrsim \frac{c}{4\pi}\frac{B_0}{r_{L,0}}$$

Amato & Blasi (2009)

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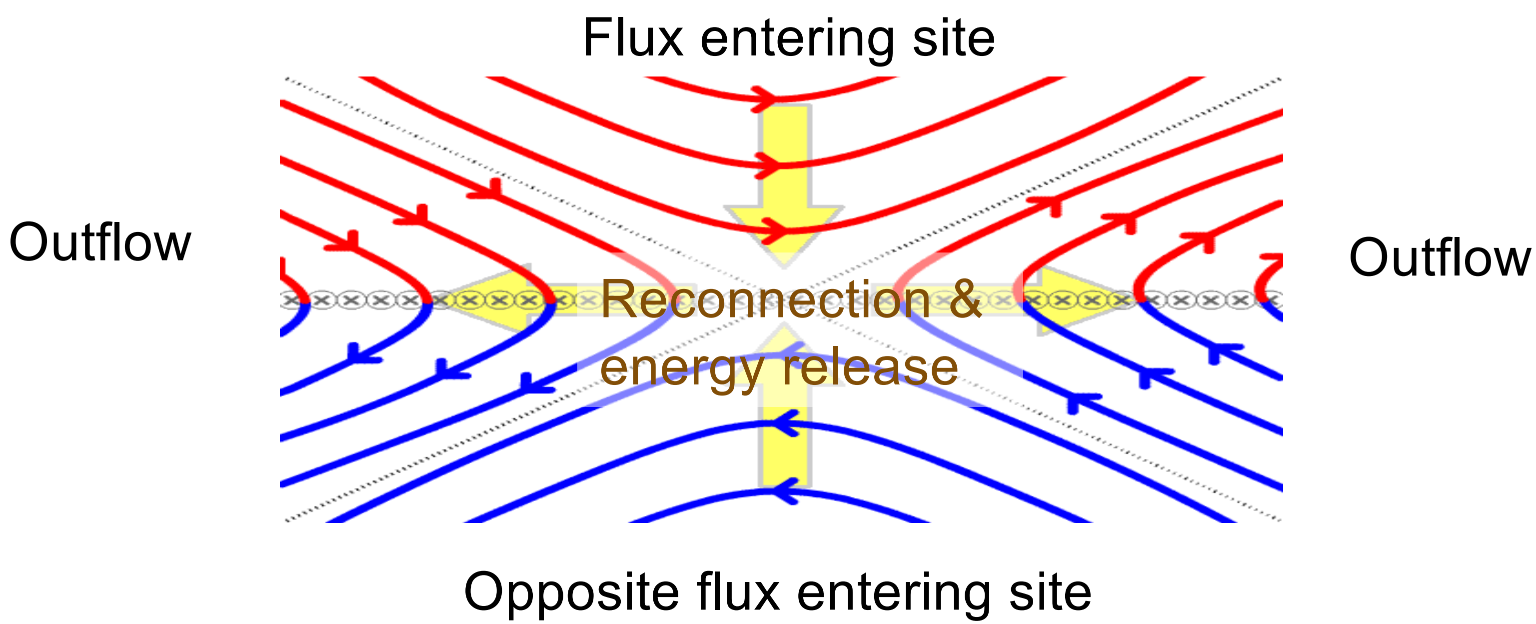
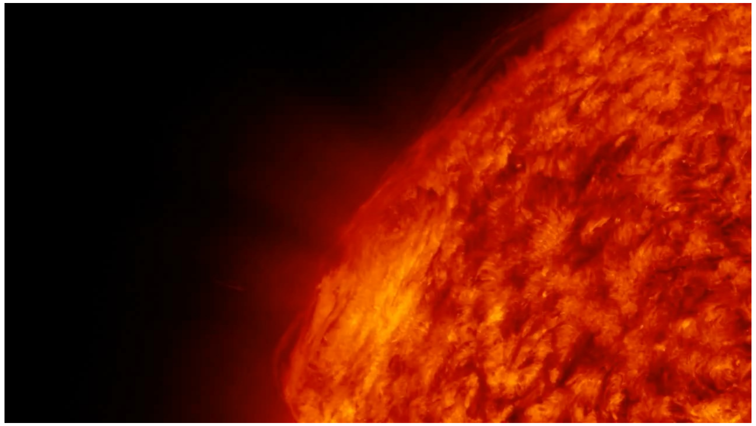
Magnetic reconnection

Astrophysical plasmas often contain stressed regions including current sheets with accumulation of magnetic energy. This energy is often released explosively via magnetic reconnection, which leads to reconfiguration of the magnetic field, along with high-speed flows, thermal heating and nonthermal particle acceleration.

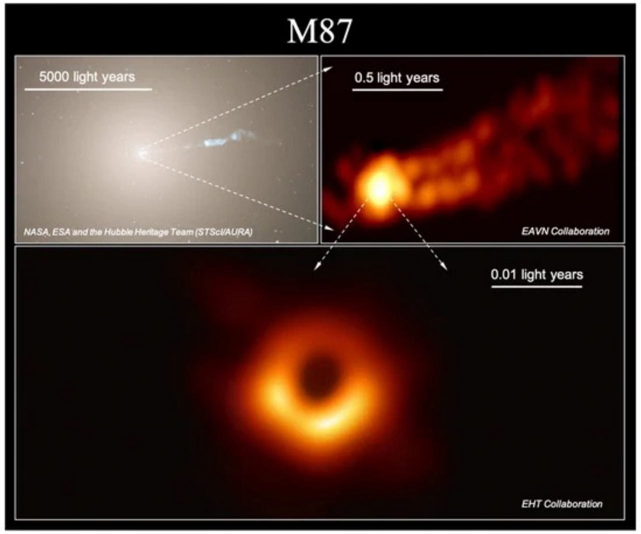
Ji et al. 2022



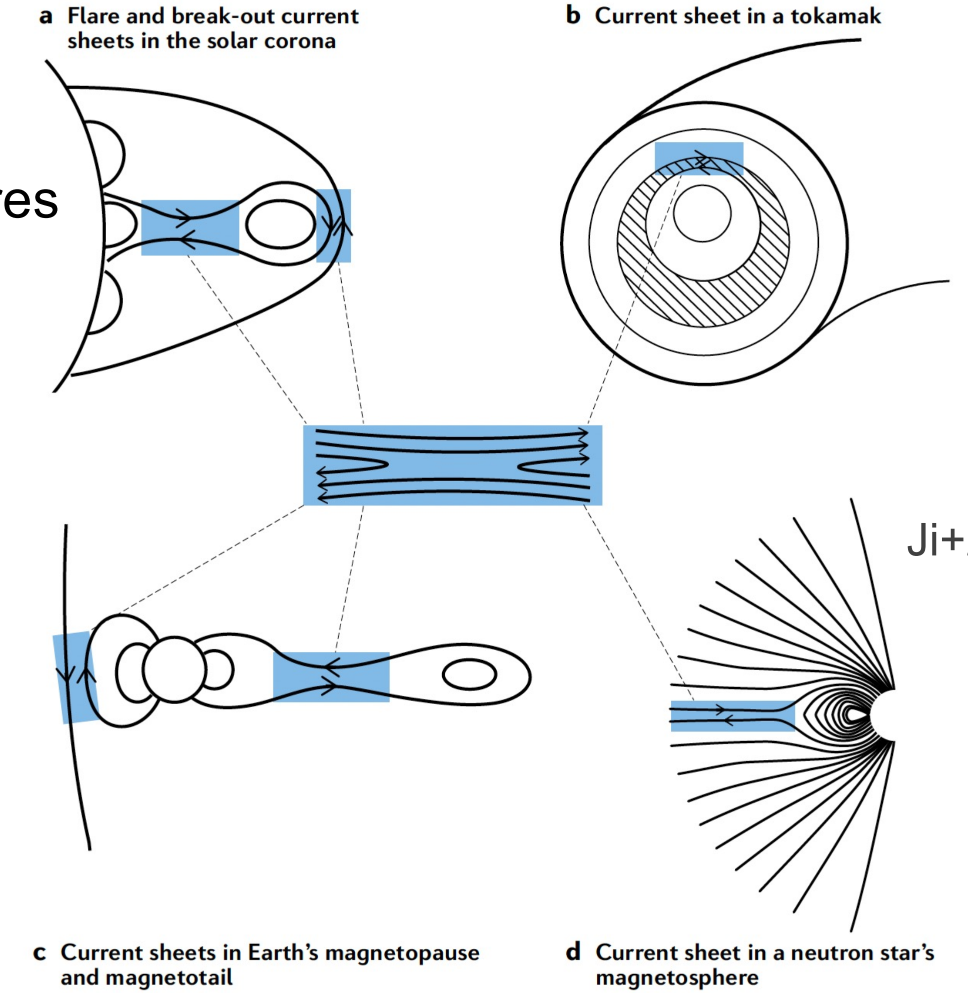
Astrophysical, space and laboratory applications



Solar flares
/CME



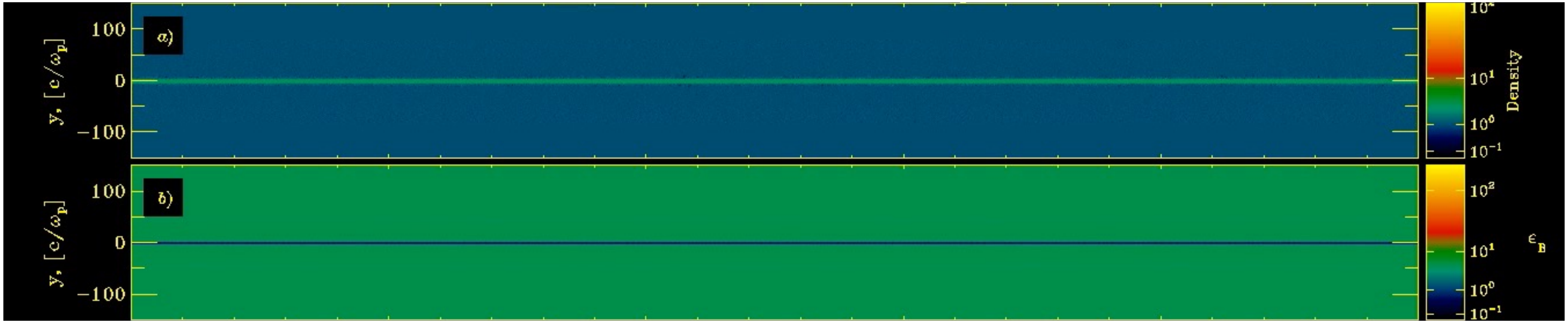
AGN jet
base?



Ji+22

Magnetic reconnection: kinetic simulations

Kinetic simulations of magnetic reconnection of an $e^{\pm}$ plasma.



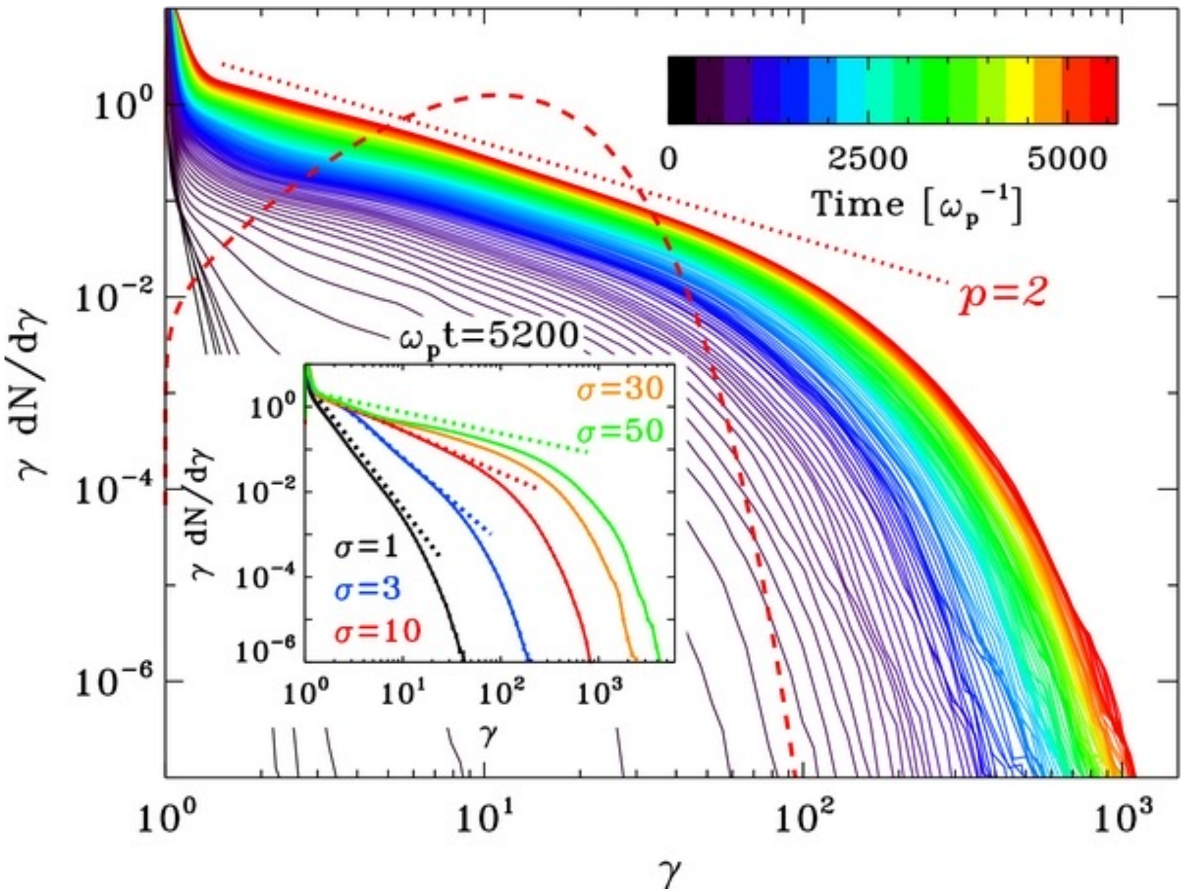
Sironi & Spitkovsky 2014, see also F. Guo et al. 2014

Magnetic islands formation by “tearing mode instability”, particle injection at X-point.

[There is also the “plasmoid instability” in MHD, Loureiro et al. 2007]

Islands merger towards bigger islands, accompanied by strong particle acceleration.

Particle acceleration in magnetic reconnection

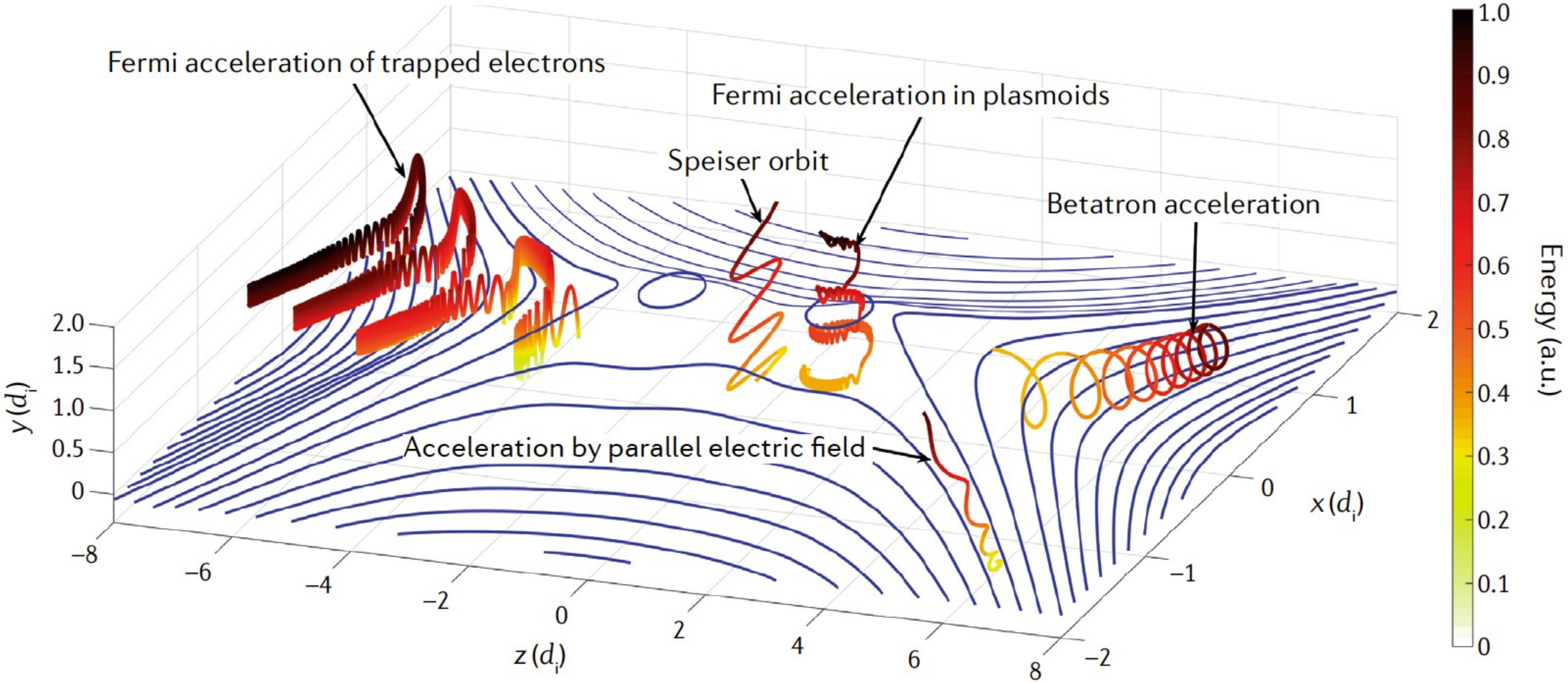


Sironi & Spitkovsky 2014

Formation of power law spectrum

Spectral index depends on magnetization / guide field.

Acceleration mechanisms



Ji et al. 2022, Nature Review Physics

Multi-scale phemonena call for multi-scale simulations in the era of exa-scale computing.

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The Vlasov-Maxwell equations

Plasma particles: $$\frac{\partial f_s}{\partial t} + \boldsymbol{v} \cdot \nabla f_s + \frac{q_s}{m_s}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right) \cdot \nabla_{\boldsymbol{v}} f_s = 0$$

EM fields: $$\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} \ ,$$
$$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \frac{4\pi}{c}\boldsymbol{j}$$

$$\boldsymbol{j} = \sum_s q_s \int \boldsymbol{v} f_s d^3 v$$

The Vlasov equation is in 6D, extremely expensive to solve.

Instead, use a population of particles as coarse-grain representation of the plasma.

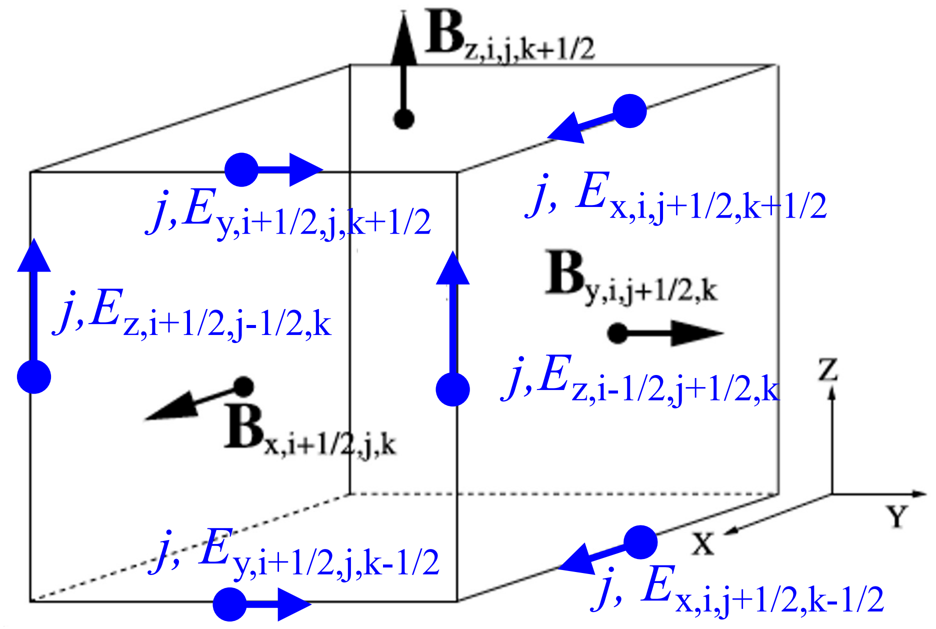
Solving Maxwell equations on the "Yee-mesh"

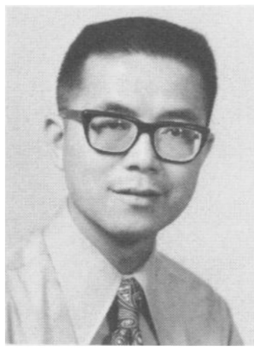
EM fields solved in staggerd mesh to preserve divergence-free of B field.

$$\nabla \cdot \boldsymbol{B} = \frac{B_{x,i+1/2,j,k} - B_{x,i-1/2,j,k}}{\Delta x} + \frac{B_{y,i,j+1/2,k} - B_{x,i,j-1/2,k}}{\Delta y} + \frac{B_{z,i,j,k+1/2} - B_{z,i,j,k-1/2}}{\Delta z}$$

Updates in Div(B) corresponds to differences in E that cancel exactly.

(a.k.a. constrained transport in MHD)



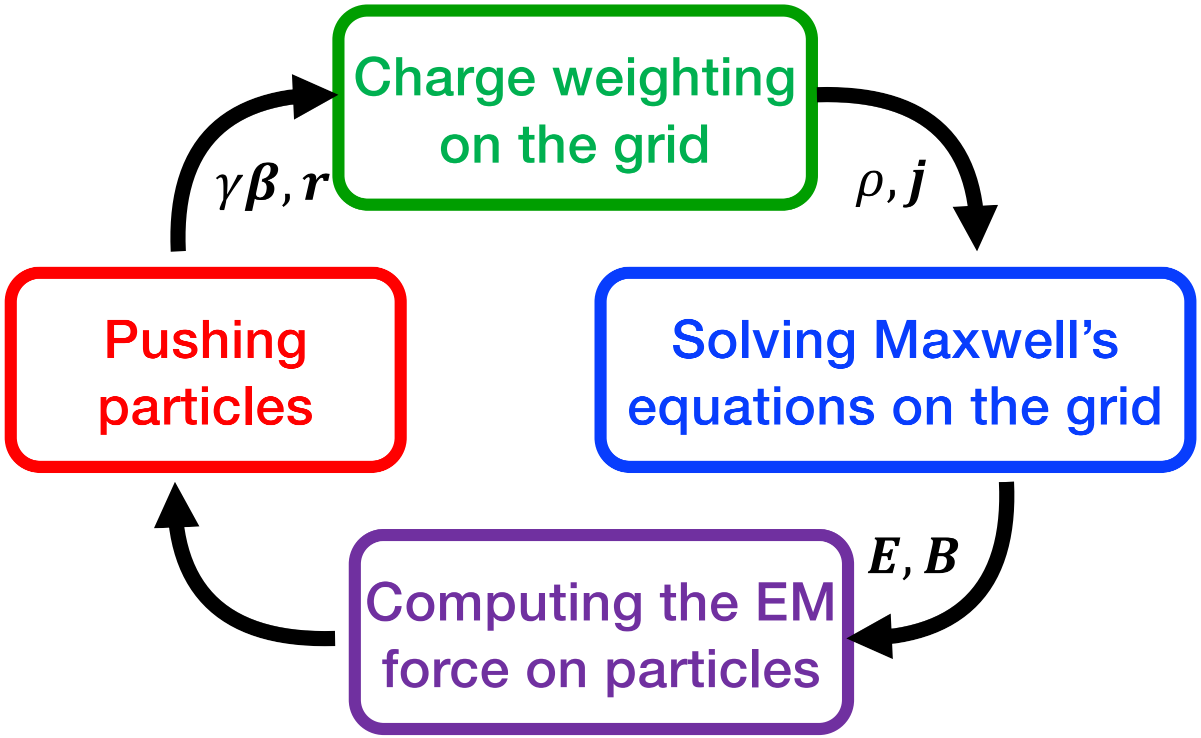


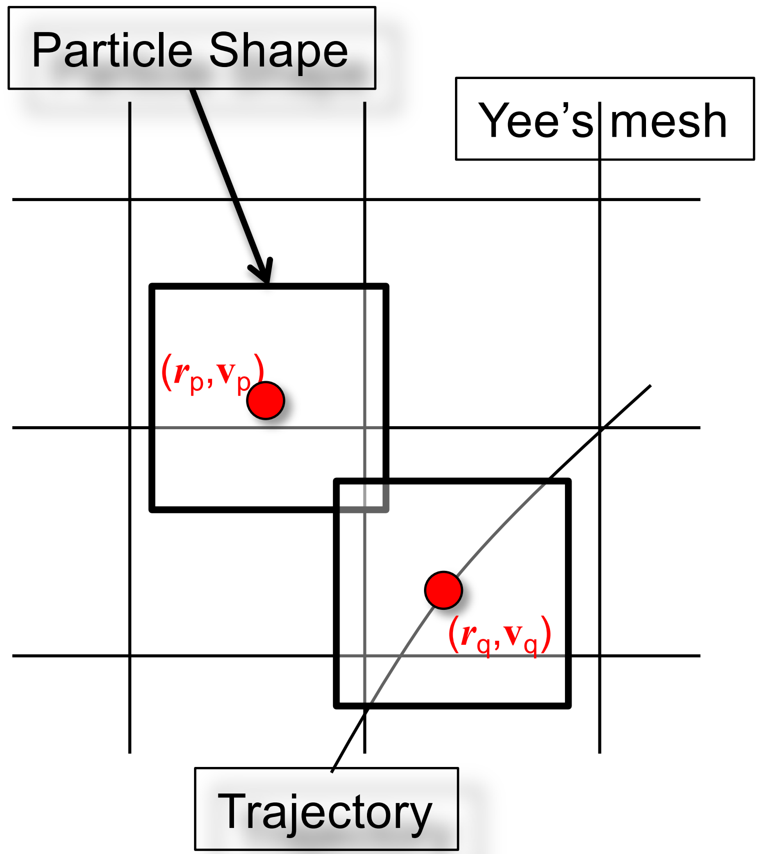
Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media

KANE S. YEE

Particle-in-cell method for plasma simulations

Particle-in-cell method:





Self-consistently solve the Vlasov-Maxwell equations using (super-)particles on staggered mesh.

Current deposition must ensure charge conservation.

Variants of PIC methods

- Full PIC: all particles are kinetic, no compromise
- Hybrid PIC: electrons as massless and conducting fluid, ions are kinetic

  Compromise the electron-scale physics (c/ω_{pe}) of background gas

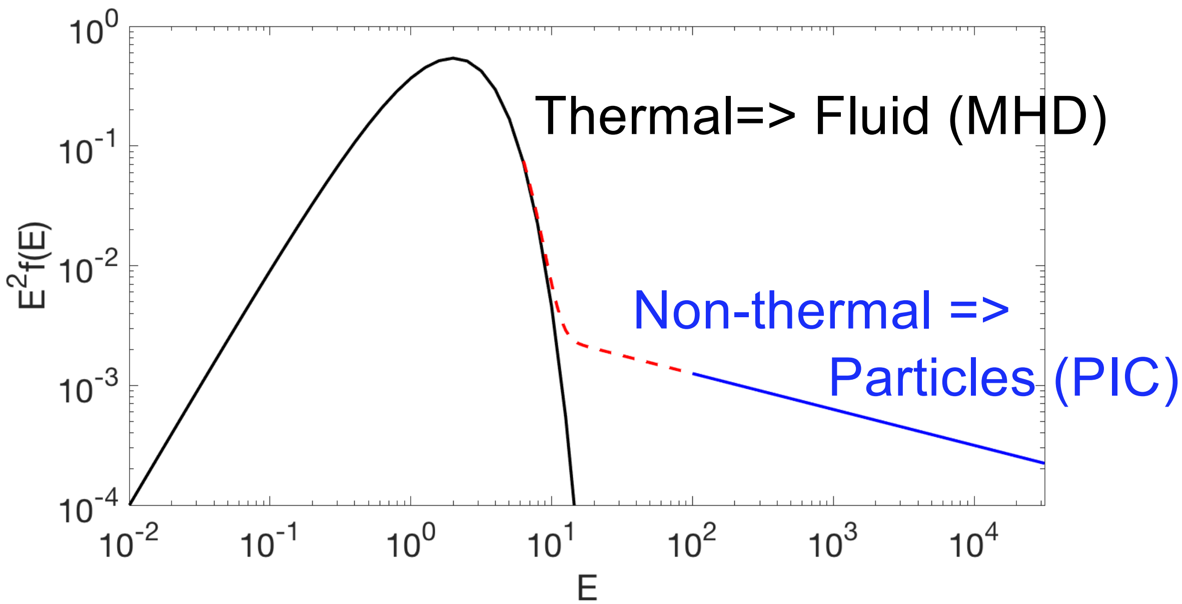
- MHD-PIC: background gas as fluid, non-thermal particles are kinetic

  Compromise the ion-scale physics (c/ω_{pi}) of background gas

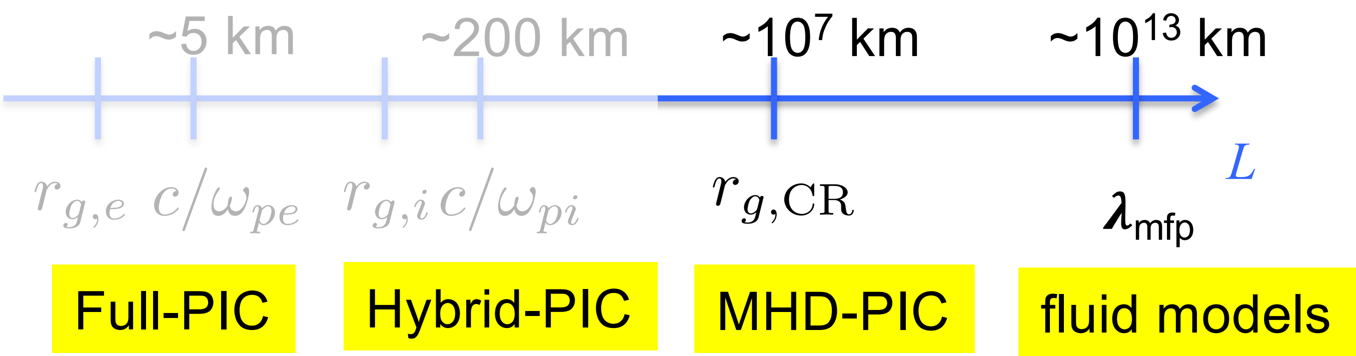
- MHD-g(uiding-center-)PIC: non-thermal particles in guiding-center orbit

  Compromise the gyro-scale physics of the non-thermal particles

Scale separation

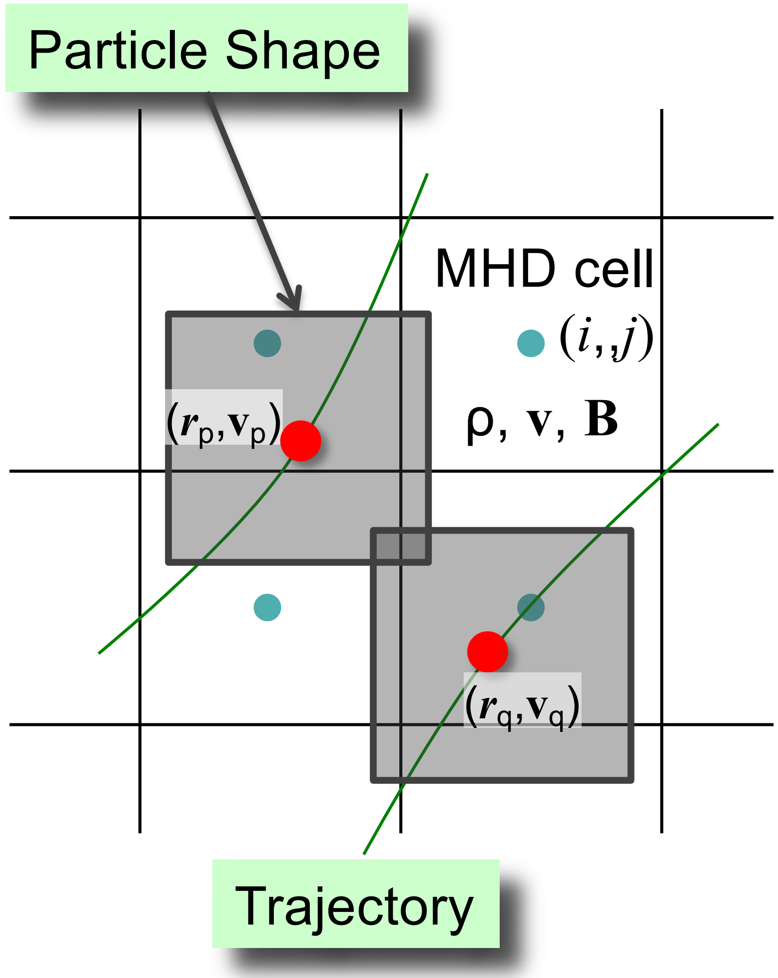


Generally need to resolve R_g.



MHD-PIC approach

- Each computational particle (i.e., super-particle) represents a large collection of real CR particles.
- Each super-particle carries an effective shape, designed to facilitate interpolation from the grid.
- Individual CR particles move under the electro-magnetic field from MHD.
- Total momentum and energy must conserve: particles feedback to MHD by depositing changes in momentum and energy locally.



The MHD-PIC approach

Equations for the (relativistic) CR particles:

$$\frac{d(\gamma_j \boldsymbol{u}_j)}{dt} = \frac{q_j}{m_j}\left(\boldsymbol{E} + \frac{\boldsymbol{u}_j}{c} \times \boldsymbol{B}\right)$$

Specify numerical speed of light c >> any MHD velocity.

Full equations for the gas:

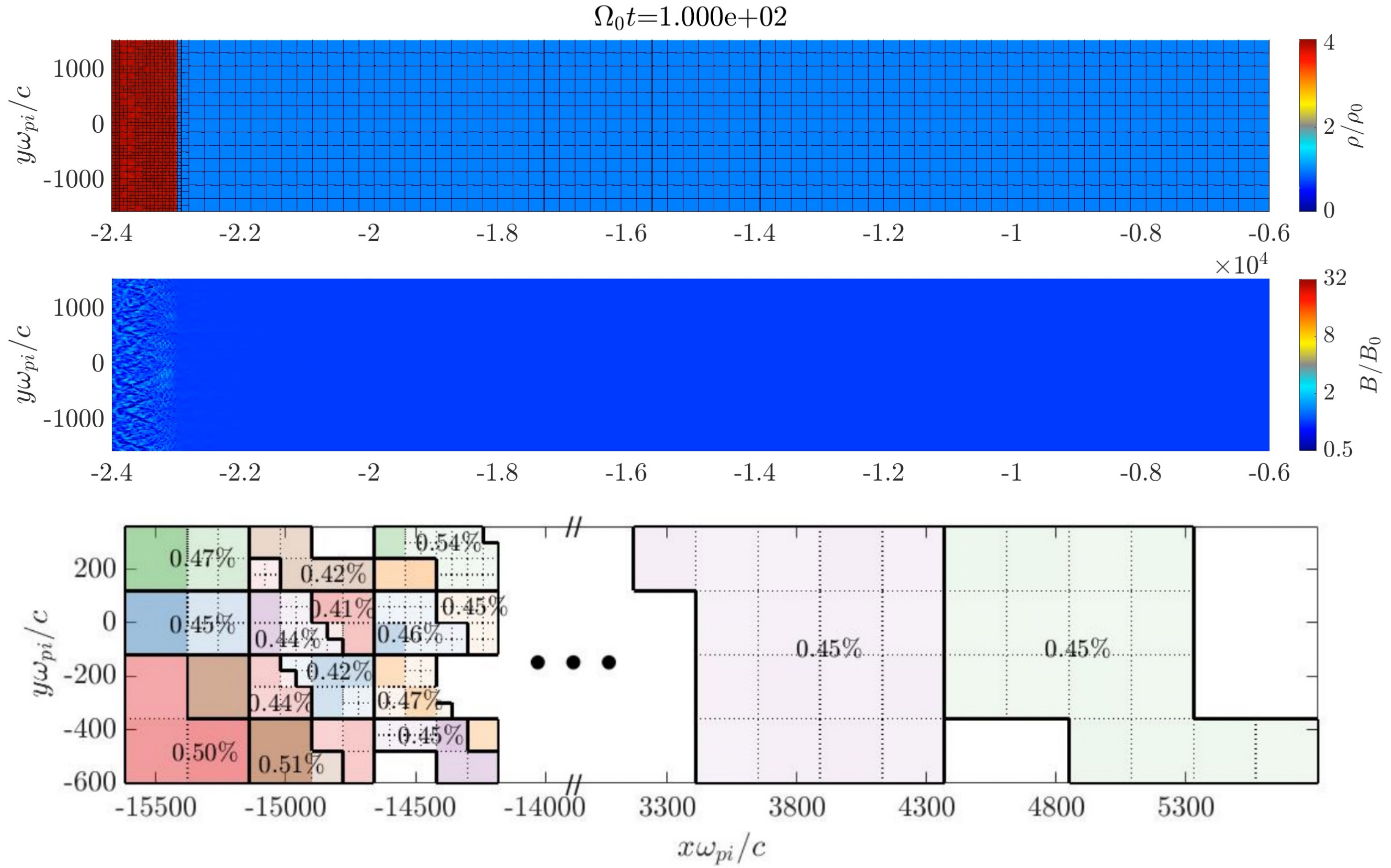
$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{B}\boldsymbol{B} + \mathbf{P}^*) = \text{- Lorentz force on the CRs}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + P^*)\boldsymbol{v} - \boldsymbol{B}(\boldsymbol{B} \cdot \boldsymbol{v})] = \text{- energy change rate on the CRs}$$

Known implementations in Athena (Bai, Caprioli, Sironi & Spitkovsky 2015), Pluto (Mignone et al. 2018), MPI-AMRVAC (van Marle et al. 2018), Athena++ (Sun & Bai 2023).

The MHD-PIC module in Athena++

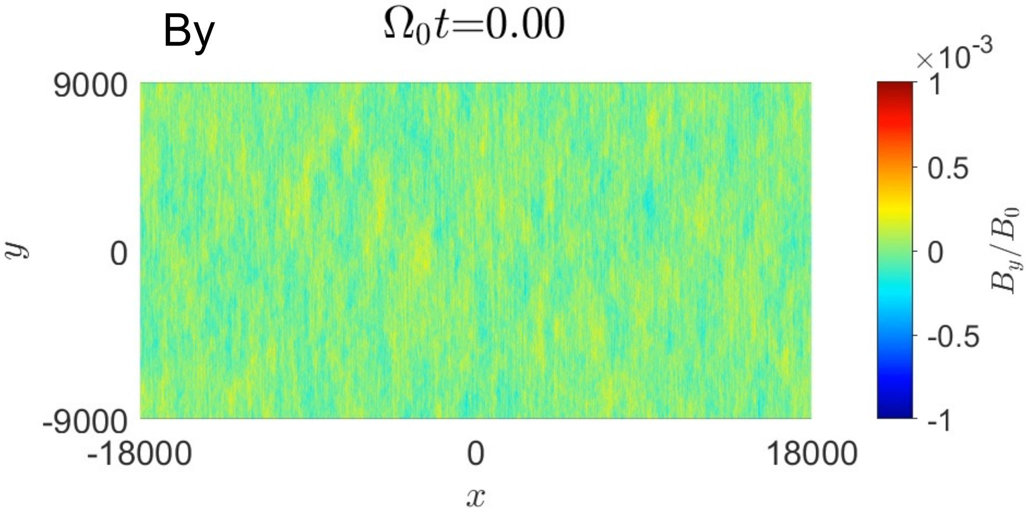
2D simulation of a parallel non-relativistic parallel shock:

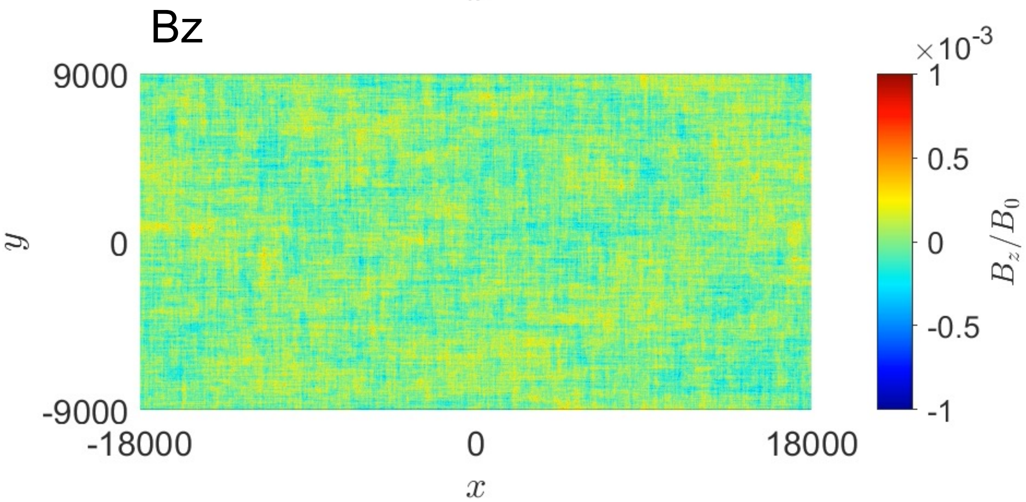


Flexible and efficient with adaptive mesh refinement.

Sun & Bai (2023)

Simulations of the CR streaming instability in 2D





Dispersion relation reproduced exactly.

$$\frac{\Gamma_i}{\Omega_c} = \left(\frac{k_\parallel v_d}{\omega} - 1\right)\frac{n_{\rm CR}}{n_i}\frac{2\sqrt{\pi}}{\kappa^{3/2}}\frac{\kappa+1}{\kappa}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})}\cdot$$

$$\sum_{n=-\infty}^{+\infty}\frac{1}{|\tilde{k}_\parallel|}\int_{|n|\tilde{p}_{res}}^{+\infty} \mathrm{d}\tilde{p}\,\tilde{p}\left[1+\frac{\tilde{p}^2}{\kappa}\right]^{-(\kappa+2)}\cdot$$

$$\begin{Bmatrix} \frac{n^2}{\tilde{k}_\perp^2} J_n^2\left(\tilde{k}_\perp\sqrt{\tilde{p}^2 - n^2\tilde{p}_{res}^2}\right) & \text{Alfvén} \\ (\tilde{p}^2 - n^2\tilde{p}_{res}^2)\left[J_n'\left(\tilde{k}_\perp\sqrt{\tilde{p}^2 - n^2\tilde{p}_{res}^2}\right)\right]^2 \cos^2\alpha & \text{Fast} \\ (\tilde{p}^2 - n^2\tilde{p}_{res}^2)\left[J_n'\left(\tilde{k}_\perp\sqrt{\tilde{p}^2 - n^2\tilde{p}_{res}^2}\right)\right]^2 \sin^2\alpha & \text{Slow} \end{Bmatrix}$$

Zeng, Bai & Sun, to be submitted

MHD-guiding-center PIC in Athena++

- For problems with excessively small gyro-radii, we can adopt the guiding-center approximation (Drake et al. 2019), for electrons (for now).

$$\vec{v}_p = \vec{v}_{drift} + v_{||}\vec{b}$$

including ExB drift velocity v_E.+ grad B, curvature drift velocities.

In the parallel direction: $$\frac{\mathrm{d}}{\mathrm{d}t} p_{k||} = (p_{k||} - \gamma_k v_{g||}) \boldsymbol{v}_g \cdot \boldsymbol{\kappa} - \frac{\mu_k}{\gamma_k} \boldsymbol{b} \cdot \boldsymbol{\nabla} B - eE_{||}$$ (Northrop 1963)

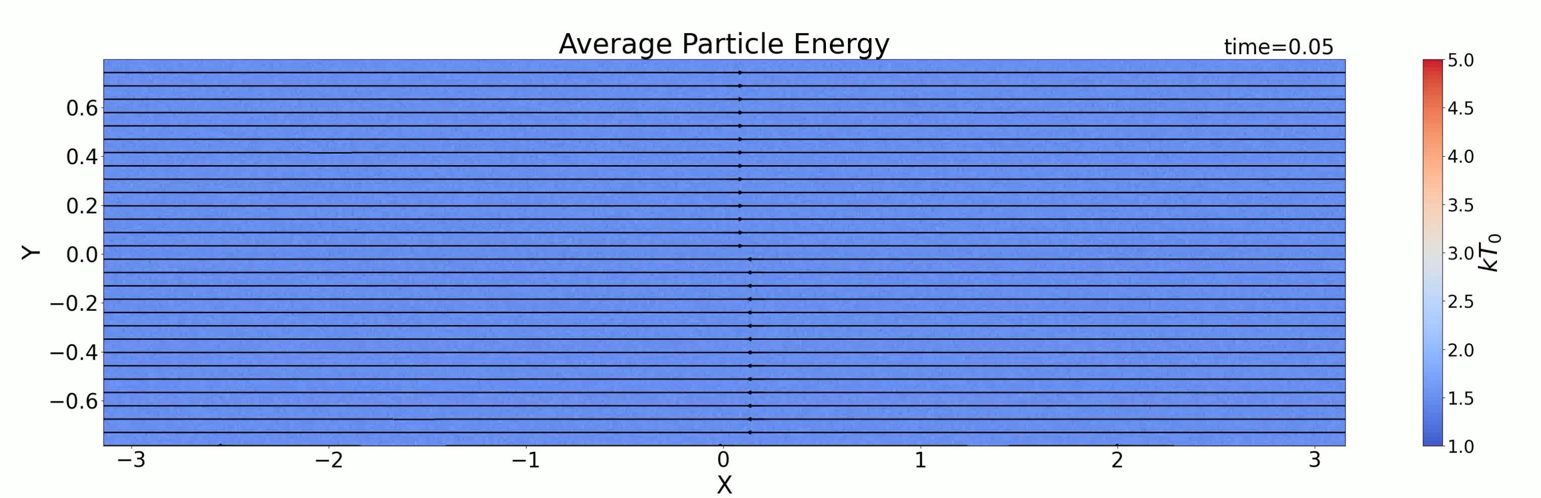
In the perpendicular direction: $\mu_{\mathrm{e}} = p_{\perp}^2 / 2B$ conservation.

Parallel E field: $$E_{||} = -\frac{1}{n_e e} \boldsymbol{b} \cdot \nabla \cdot \boldsymbol{P}_e$$

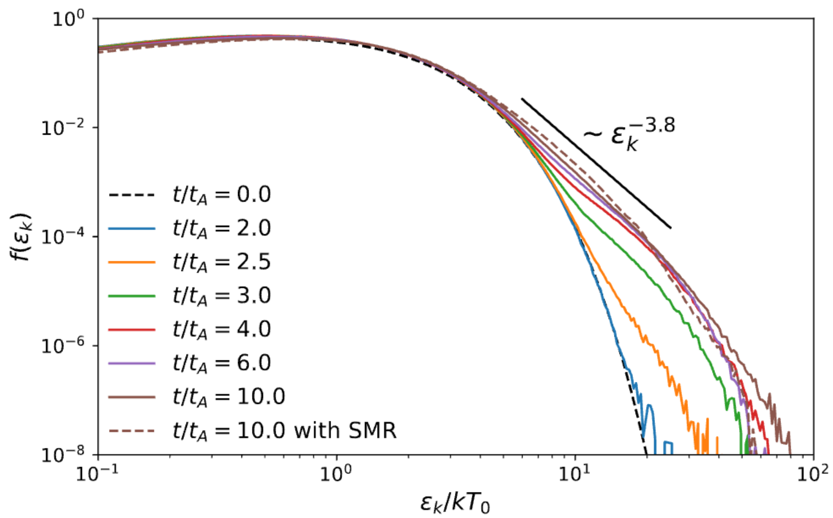
Backreaction from PIC to MHD included to ensure conservation laws.

Hu, Bai & Sun, to be submitted

MHD-PIC with guiding center CRs in Athena++



With weak guide field, particles fill the volume.



- Particle acceleration during islands merger, forming a (narrow) power-law spctrum.
- Due to the confinement, high-energy particles cannot access the regions where the acceleration is most active.

Hu, Bai & Sun, to be submitted

Summary

- Stochastic Fermi acceleration: 2nd order, $s=(t_a/t_{esc}+1)$.
- Diffusive shock acceleration: 1st order, $s=1+3/(r-1)$.
- Case study: Ion-acceleration in non-relativistic shocks

  Microphysics matters! Sets injection, turbulent diffusion, and feedback.

- Particle acceleration in magnetic reconnection

  Explosive, power-law spectra depending on magnetization.

- Computational methods: brief introduction

  Extending PIC towards multi-scales, i.e., MHD-PIC and MHD-gPIC methods.