Radiation mechanism

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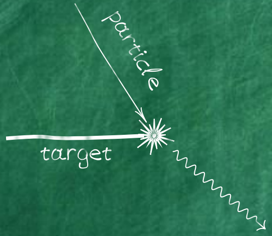
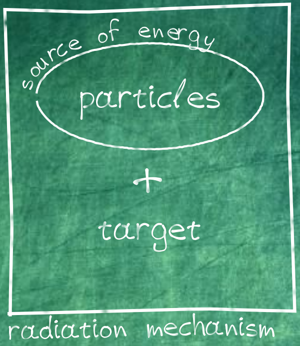
Outline

- Introduction
- Fokker-Planck Equation
- Lorentz transformations
- Anatomy of Inverse Compton Scattering

Radiation

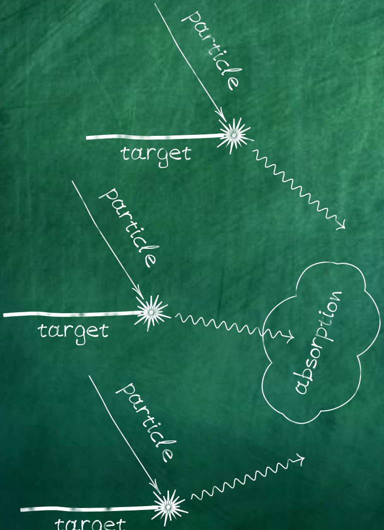

particle
target


source of energy
particles
+
target
radiation mechanism

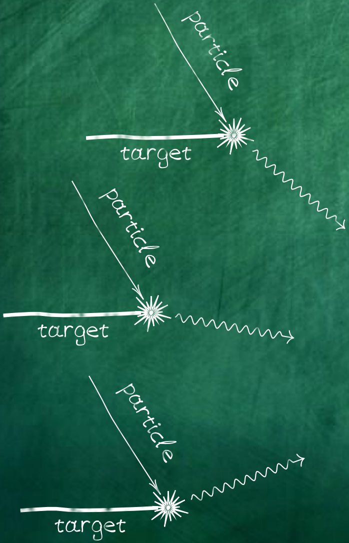
Radiation

radiation mechanism
+
particle distribution
+
absorption

factors to account for



Radiation

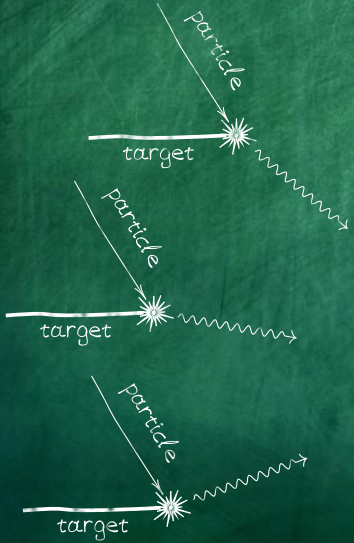


radiation mechanism
+
particle distribution
+
absorption

factors to account for

Why do we need this?!!!

Radiation



radiation mechanism
+
particle distribution
+
absorption

factors to account for

particle distribution
-or-
target
-or-
absorption

Why do we need this?

Radiation

Feature	particle distribution	radiation process	absorption
Spectrum	!	!	!
Morphology	!	✔	✔
Time Variability	!	✔	!
Polarization	✔	!	✔

where ! means a "strong dependence"

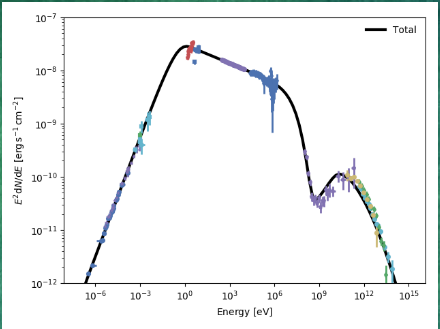
For interpretation of non-thermal emission an accurate description of the particle spectrum is as important as accurate calculation of the emission. When particles lose energy by radiating this influences the particle distribution thus the description of the particle spectrum should include the impact of radiation. However, non-radiative processes may also have a substantial influence on the particle spectrum and spatial distribution.

Description of non-thermal particles

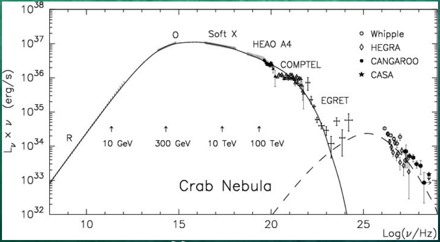
Distribution of high energy particles depends on some parameter(s):

- Energy: $dN = f dE$
- Momentum: $dN = f d^3 p$
- Coordinate: $dN = f dE dx$
- Coordinates: $dN = f dE d^3 r$
- Phase-space coordinates:
  $dN = f d^3 r d^3 p$

Here f is distribution function, and its definition may vary depending on the context. For each problem one needs to select an adequate distribution function that allows accounting for all relevant processes.



NAIMA, Zabalza 2015



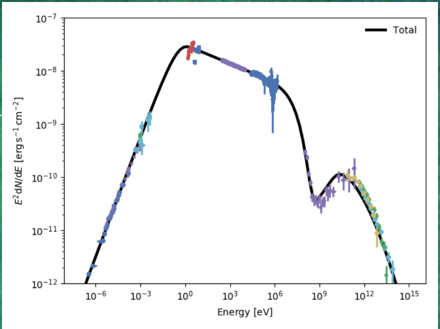
Aharonian&Atoyan 1998

Description of non-thermal particles

Distribution of high energy particles depends on some parameter(s):

- Energy: $dN = f dE$ — One-zone model
- Momentum: $dN = f d^3p$
- Coordinate: $dN = f dE dx$
- Coordinates: $dN = f dE d^3r$
- Phase-space coordinates: $dN = f d^3r d^3p$ — MHD for transport

Here f is distribution function, and its definition may vary depending on the context. For each problem one needs to select an adequate distribution function that allows accounting for all relevant processes.



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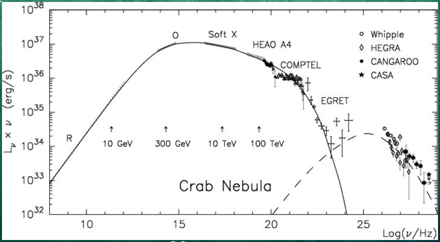


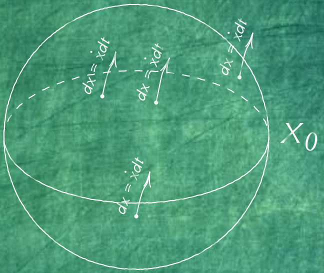
Aharonian&Atoyan 1998

Description of Non-Thermal Particles

If one ignores the particle spin –which still might be important in some astrophysical conditions, e.g., in pulsar magnetosphere– the phase-space distribution function provides the most complete description: $dN = f d^3r d^3p$

There is a quite simple equation for the distribution function, Boltzmann Equation:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f}{\partial \mathbf{p}} = 0$$



$$\frac{\partial}{\partial t}\int_{X_0} f\,dx = -\int_{\partial X_0} F(f)\,d\mathbf{S}_x$$

$$\int_{X_0}\left(\frac{\partial f}{\partial t} + \mathrm{div}F(f)\right)dx = 0$$

$$\frac{\partial f_a}{\partial t} + \frac{\partial(\dot{\mathbf{r}} f_a)}{\partial \mathbf{r}} + \frac{\partial(\dot{\mathbf{p}} f_a)}{\partial \mathbf{p}} = 0$$

Description of Non-Thermal Particles

If one ignores the particle spin –which still might be important in some astrophysical conditions, e.g., in pulsar magnetosphere– the phase-space distribution function provides the most complete description: $dN = f d^3r d^3p$

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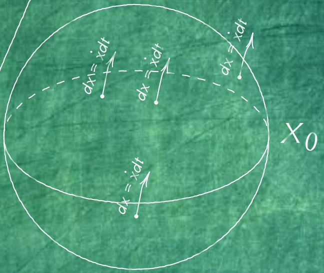
$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f}{\partial \mathbf{p}} = \left[\frac{\partial f}{\partial t}\right]_{\rm col}$$

What about particle collisions?!!!



$$\frac{\partial}{\partial t}\int\limits_{X_0} f\,dx = -\int\limits_{\partial X_0} F(f)\,d\mathbf{S}_x$$

$$\int\limits_{X_0}\left(\frac{\partial f}{\partial t} + {\rm div}F(f)\right)dx = 0$$

$$\frac{\partial f_a}{\partial t} + \frac{\partial(\dot{\mathbf{r}}f_a)}{\partial \mathbf{r}} + \frac{\partial(\dot{\mathbf{p}}f_a)}{\partial \mathbf{p}} = 0$$

Boltzmann Equations

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f}{\partial \mathbf{p}} = \left[\frac{\partial f}{\partial t}\right]_{\text{col}}$$

- The collision integral $\left[\frac{\partial f}{\partial t}\right]_{\text{col}}$ accounts for many processes:
  - particle injection
  - acceleration
  - scattering
  - energy losses

  etc – i.e., for *ALL* plasma and radiation physics
- In the simplest case, the Boltzmann collision integral is

$$\left[\frac{\partial f_a}{\partial t}\right]_{\text{st}} = \sum_b \int d^3 p_1 v_{\text{rel}} d\sigma \left(f_a(x') f_b(x_1') - f_a(x) f_b(x_1) \right)$$

Boltzmann Collision Integral in Astrophysics

$$\left[\frac{\partial f_a}{\partial t}\right]_{\rm st} = \sum_b \int d^3 p_1 v_{\rm rel} d\sigma \left(f_a(x') f_b(x_1') - f_a(x) f_b(x_1) \right)$$

- Boltzmann collision integral is widely used in kinetics of neutral gases, e.g., to describe an admixture propagation
- In astrophysics the collision integral in this form is used to describe, e.g., the electromagnetic cascading:

$$\frac{\partial f_e}{\partial t} + \mathbf{v}\frac{\partial f_e}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f_e}{\partial \mathbf{p}} = \int d_\gamma c\, d\bar{\sigma}_{\gamma\gamma} f_\gamma(p_\gamma) - c\bar{\sigma}_{ic} f_e$$

$$\frac{\partial f_\gamma}{\partial t} + \mathbf{c}\frac{\partial f_\gamma}{\partial \mathbf{r}} = \int d_e c\, d\bar{\sigma}_{ic} f_e(p_e) - c\bar{\sigma}_{\gamma\gamma} f_\gamma$$

Boltzmann Equation

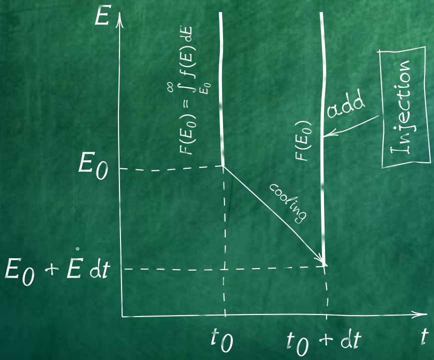
$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f}{\partial \mathbf{p}} = \left[\frac{\partial f}{\partial t}\right]_{\text{col}}$$

- Equation with the collision integral $\left[\frac{\partial f}{\partial t}\right]_{\text{col}}$ cannot be solved for astrophysical applications
- It is possible to divide the physics included in the collision integral in two parts: complex (e.g., acceleration) and simple (cooling, which can be treated under the continuous-loss approximation)
- Also in the most cases particles are isotropic in some system, thus particle energy is a good parameter

Significant simplification in the case of energy losses

Distribution function & Injection

$$dN = f(E,t)\,dE \qquad dN = q(E,t)\,dE\,dt$$



$$F(E,t) = \int_E^{\infty} f(E',t)\,dE'$$

$$F(E + \dot{E}dt, t + dt) =$$

$$F(E,t) + dt \int_E^{\infty} q(E',t)\,dE'$$

Fokker–Planck Equation

$$\frac{\partial f}{\partial t} + \frac{\partial(\dot{E} f)}{\partial E} = q(E,t)$$

Significant simplification in the case of energy losses

$$F(E+\dot{E}dt, t+dt) = F(E,t) + dt\int_E^\infty q(E',t)dE'$$

$$F(E,t) + \frac{\partial F}{\partial E}\dot{E}dt + \frac{\partial F}{\partial t}dt = F(E,t) + dt\int_E^\infty q(E',t)dE'$$

$$\frac{\partial}{\partial E} \Longrightarrow$$

accounting for $\frac{\partial F}{\partial E} = -f$ $\qquad \frac{\partial}{\partial E}\int_E^\infty q(E',t)dE' = -q$

Fokker-Planck Equation

$$\boxed{\frac{\partial f}{\partial t} + \frac{\partial(\dot{E}f)}{\partial E} = q(E,t)}$$

Fokker–Planck Equation Solution

$$f(E,t) = \frac{1}{\dot{E}} \int_E^{E_{\mathrm{eff}}} q(E')\, dE', \quad \text{where } t = \int_E^{E_{\mathrm{eff}}} \frac{dE'}{\dot{E}(E')}$$

$$\dot{E} = \dot{E}_{syn} + \dot{E}_{ic} + \dot{E}_{ad} + etc / \dot{E}_{syn} + \dot{E}_{pp} + \dot{E}_{p\gamma} + etc$$

Fast Cooling (Saturation)

$$E_{\mathrm{eff}} \to \infty$$

$$f(E) = \frac{1}{\dot{E}} \int_E^{\infty} dE'\, q(E')$$

Slow Cooling

$$E \approx E_{eff} \quad i.e. \quad t \ll \frac{E}{\dot{E}}$$

$$f(E,t) = q(E) \cdot t$$

Spectral Breaks: Particle Distribution

Solution of the Fokker-Planck Equation:

$$f(E,t) = \frac{1}{\dot{E}} \int_{E}^{E_{\rm eff}} q(E')\, dE', \text{ where } t = \int_{E}^{E_{\rm eff}} \frac{dE'}{\dot{E}(E')}$$

Let us consider the simplest case:

$$q(E,t) = \theta(E - E_{min})\theta(E_{max} - E)E^{-\alpha}, \text{ where } \dot{E} \propto E^{\beta}$$

Cooling energy is $E_c = \dot{E}(E_c)t$, where t is the source age

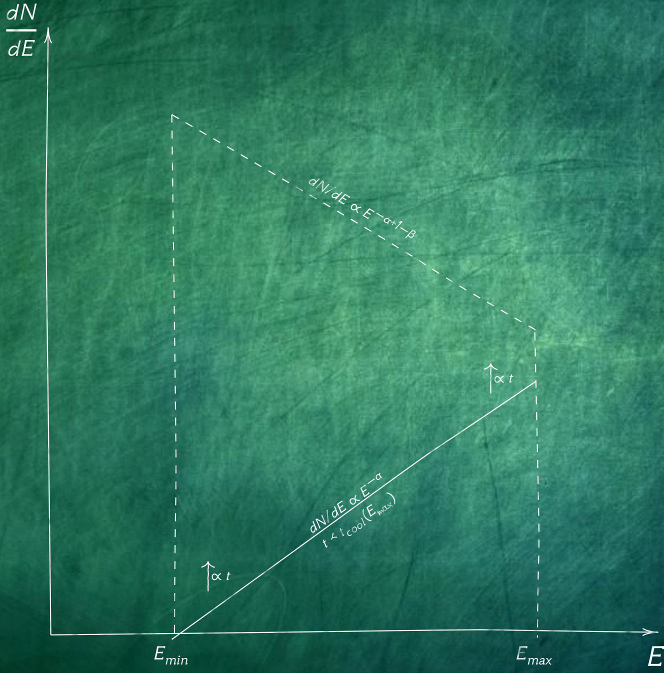
$E_c > E_{min}$ then break at E_c and range of energy is from E_{min} to E_{max}:

$$f(E) \propto \begin{cases} (E^{-\alpha+1} - E_{max}^{-\alpha+1})E^{-\beta} \\ E^{-\alpha} \end{cases}$$

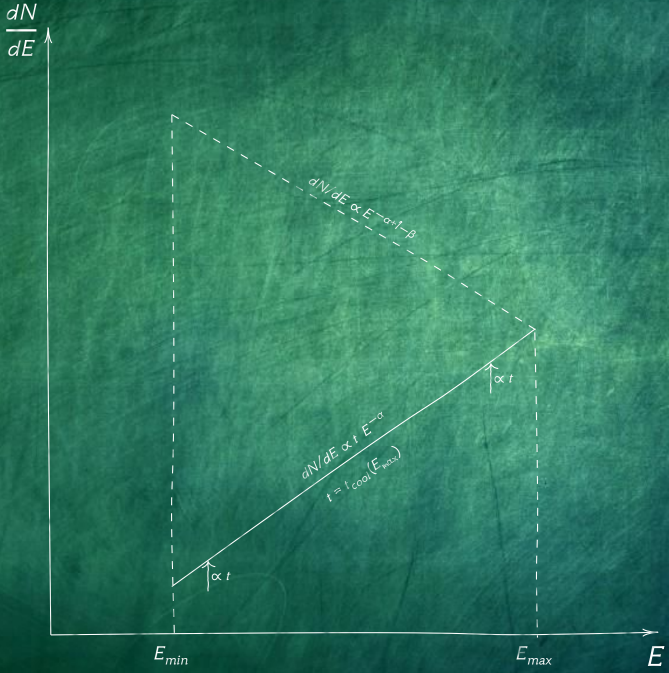
$E_c < E_{min}$ then break at E_{min} and range of energy is from E_c to E_{max}

$$f(E) \propto \begin{cases} (E^{-\alpha+1} - E_{max}^{-\alpha+1})E^{-\beta} \\ E^{-\beta} \end{cases}$$

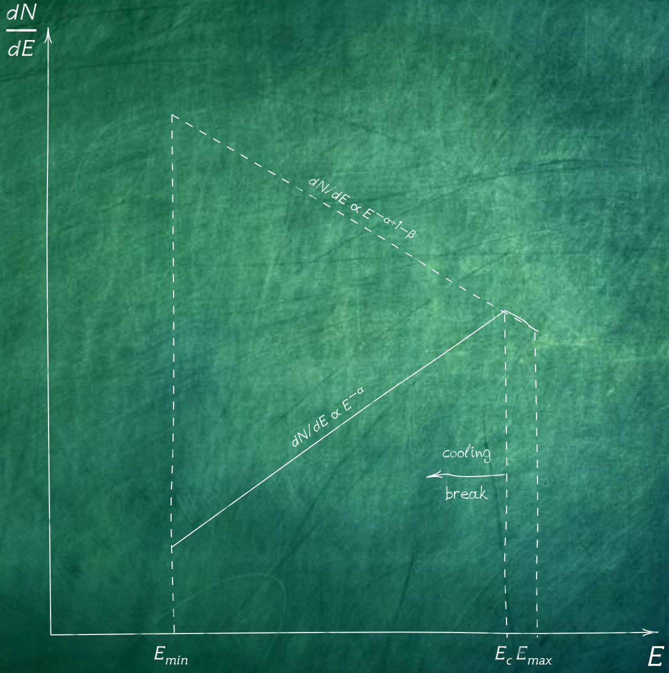
Cooling break



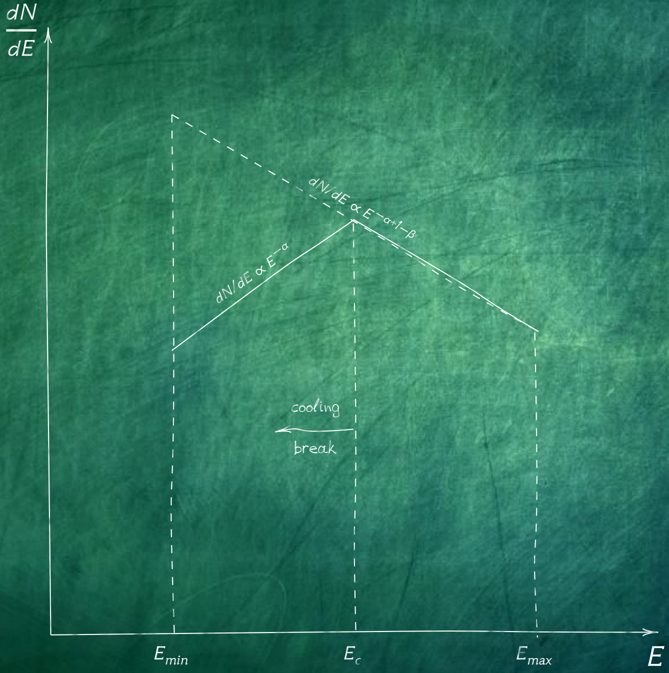
Cooling break



Cooling break



Cooling break

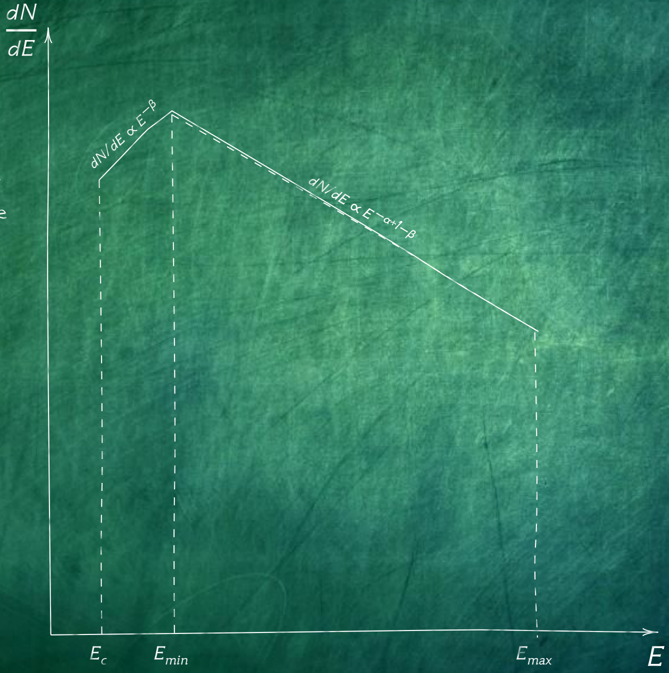


Cooling break

In case with pure power-law injection and cooling the particle distribution may have one break and three different slops:

- $E^{-\alpha-\beta+1}$
- $E^{-\alpha}$
- $E^{-\beta}$

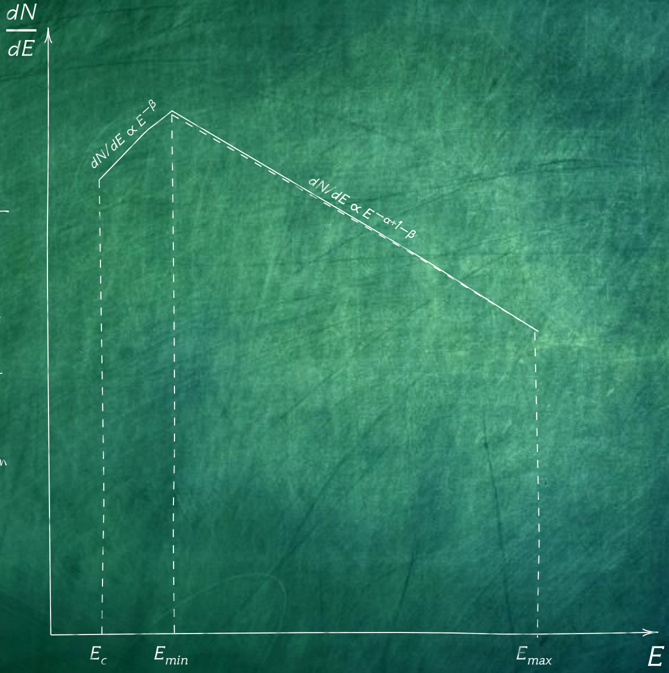
here α and β are the power-law indexes of the acceleration spectrum and the cooling rate.



Cooling break

In particular, particle distributions cannot have two breaks. More complicated particle distributions are allowed if

- the injection spectrum is not a power-law (e.g., each acceleration spectrum has a high-energy cutoff);
- (ii) the loss rate has a non-power-law dependence on energy



Used simplifications

- Phenomenological treatment of the acceleration process
  - High energy cut-off may depend on the loss rate and the source age
  - Some acceleration processes cannot be treated as a power-law injection, e.g. converter mechanism by Derishev
- Energy losses might be non-power law
- Particles lose energy by small fractions, which is not true for some processes, e.g. IC in the Klein-Nishina regime
- The Fokker-Planck equation describes a one-zone model

Spectral Breaks: Particle Distribution

Solution of the Fokker–Planck Equation:

$$f(E,t) = \frac{1}{\dot{E}} \int_{E}^{E_{\rm eff}} q(E')\, dE', \quad \text{where } t = \int_{E}^{E_{\rm eff}} \frac{dE'}{\dot{E}(E')}$$

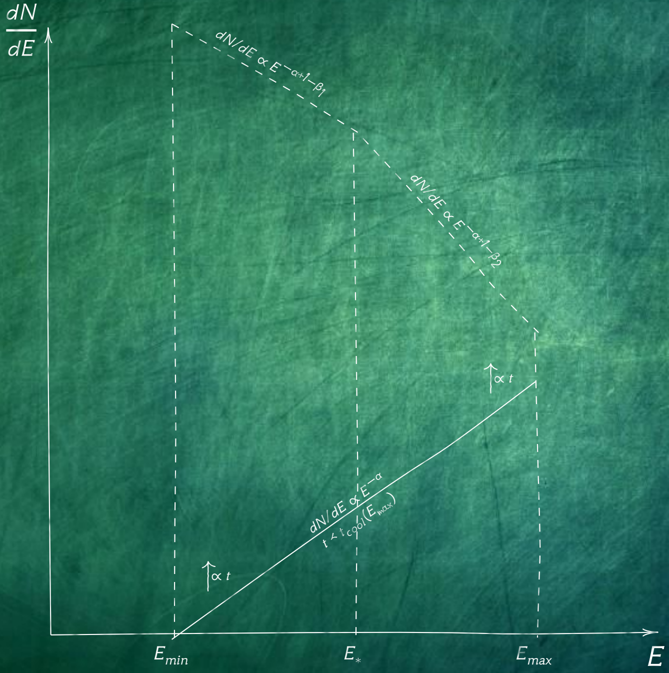
Let us consider ~~the simplest~~ case:

$$q(E,t) = \theta(E - E_{min})\theta(E_{max} - E)E^{-\alpha}, \text{ and } \dot{E} \propto \begin{cases} E^{\beta_1} & \text{for } E < E_* \\ E^{\beta_2} & \text{for } E > E_* \end{cases}$$

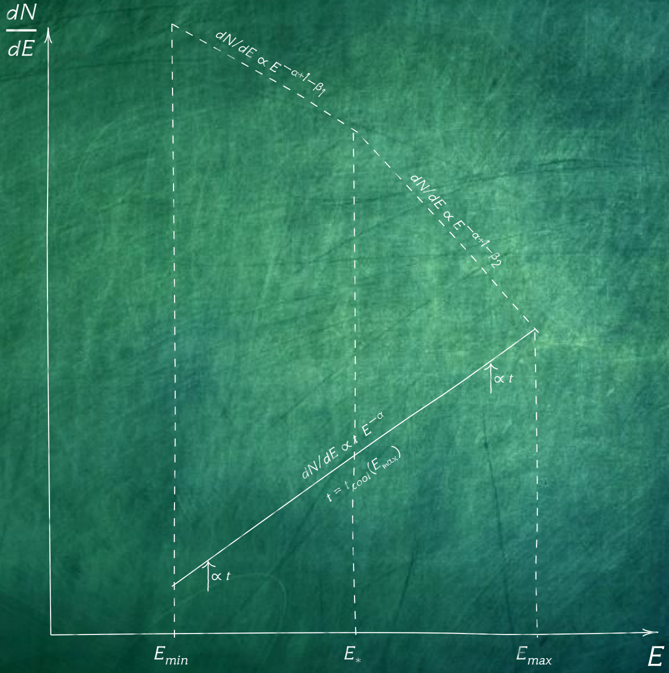
Cooling energy is $E_c = \dot{E}(E_c)t$, where t is the source age. There should be four different cases:

- $E_{max} < E_c$
- $E_* < E_c < E_{max}$
- $E_{min} < E_c < E_*$
- $E_c < E_{min}$

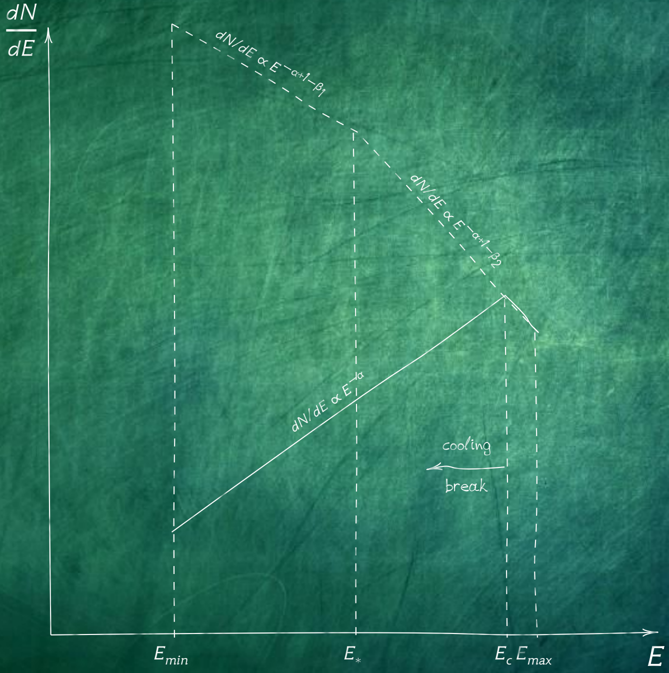
Cooling break



Cooling break



Cooling break



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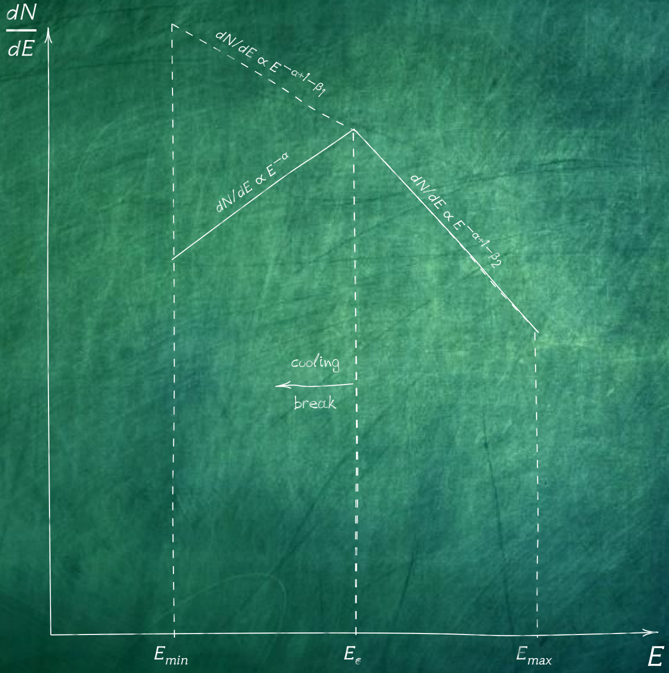

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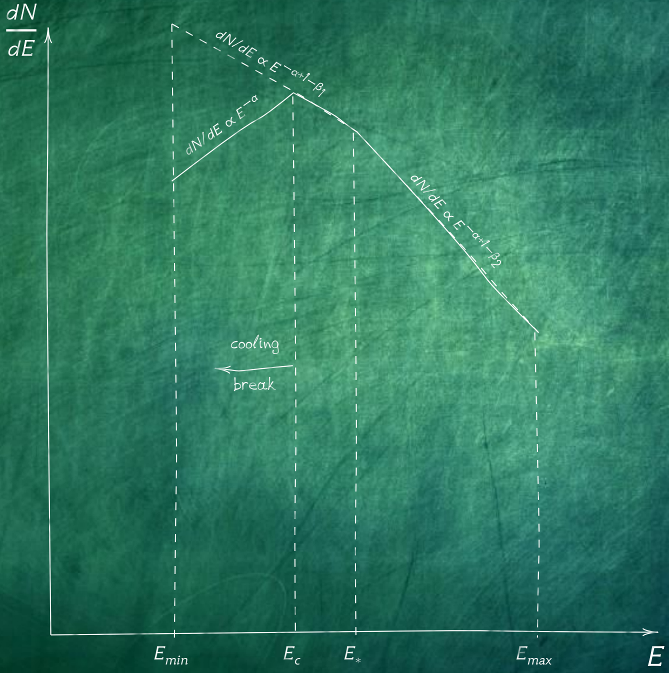
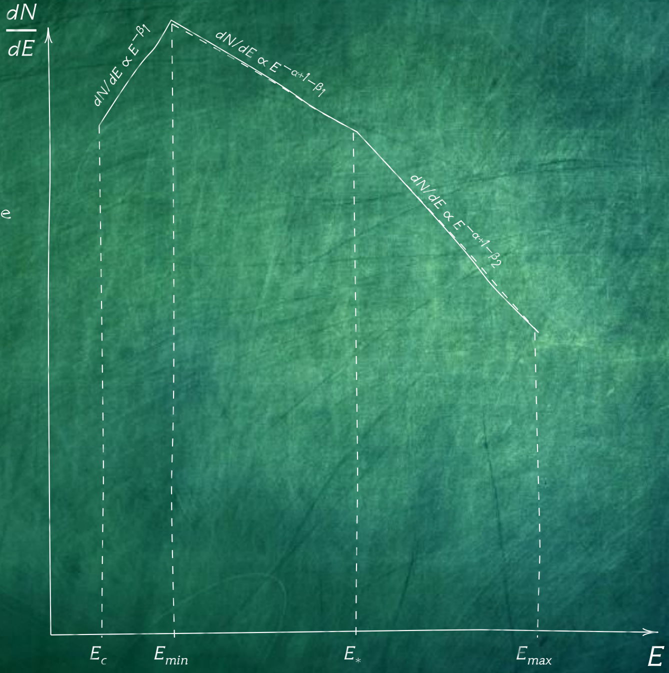


Cooling break

Two break particle spectra can be realized if one break is cooling break and the second one is caused by the change of the cooling regime. The breaks are then

- $\beta_2 - \beta_1$
- $\beta_1 - 1$ or $\alpha - 1$

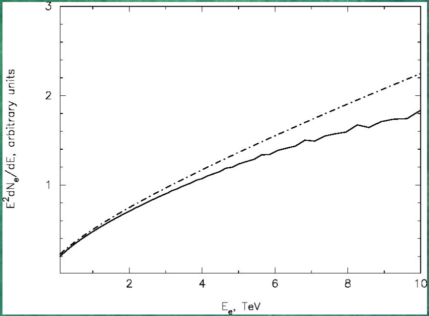
here α is the injection spectrum and β_1/β_2 is the power-law dependence of the cooling rate.



Continuous Loss Approximation for Klein-Nishina regime

Klein-Nishina and Continues Loss approximation

- Particles lose energy by small fractions, which is not true for some processes, e.g. IC in the Klein-Nishina regime



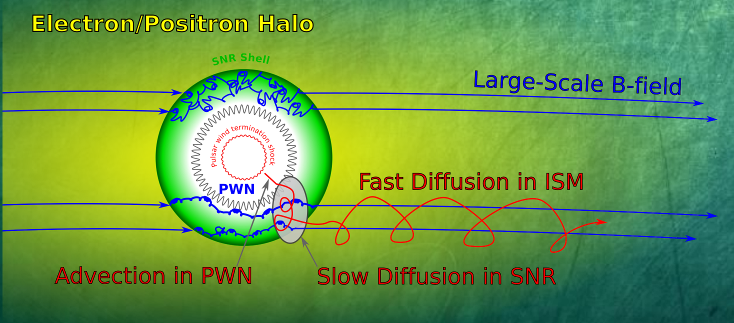
Solid line: $c\sigma_{ic} f(\gamma) = q(\gamma) + c \int_{\gamma}^{\infty} d\gamma' f(\gamma') \frac{d\sigma}{dE_\gamma}(\gamma', \gamma' - \gamma)$

Dash-dotted line: $f(\gamma) = \frac{1}{\dot{E}_{ic}} \int_{\gamma}^{\infty} d\gamma' q(\gamma')$

Transport Equation with Diffusion and Escape

$$\frac{\partial f}{\partial t} + \frac{\partial(\dot{E} f)}{\partial E} + \nabla\left(D\nabla f\right) + \frac{f}{\tau} = q(E,t)$$

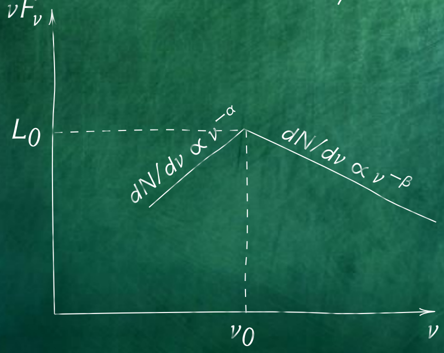
also can be solved analytically for homogeneous diffusion (see e.g. Ginzburg's "Astrophysics of Cosmic Rays") or numerically (e.g., lecture by Gwenael Giacinti). The latter case is most likely relevant for interpreting gamma-ray observations



νF_ν peak gives the luminosity

νF_ν peaking distribution

$$\nu F_\nu = E_\gamma^2 \frac{dN_\gamma}{dE_\gamma}$$



$$L_\gamma = \int d\nu\, \nu \frac{dn}{d\nu} =$$

$$\int_{\nu_{min}}^{\nu_0} d\nu\, \nu^{1-\alpha} + \int_{\nu_0}^{\nu_{max}} d\nu\, \nu^{1-\beta}$$

$$\nu_{min} \ll \nu_0 \ll \nu_{max}$$

$$L_\gamma = L_0 \left(\frac{1}{2-\alpha} + \frac{1}{\beta - 2} \right)$$

For $\alpha = 1.5$ and $\beta = 2.5$

$$\boxed{L_\gamma = 4 L_0}$$

Radiation Production

Emission of a Particle (two channels)

Single particle spectra:

$$\frac{dN_i}{d\nu} = K_i\,(\nu, E_0)$$

Total luminosity (per particle):

$$L = \dot{E}_1(E_0) + \dot{E}_2(E_0)$$

Luminosity per channel:

$$L_i = \frac{\dot{E}_i}{\dot{E}_1 + \dot{E}_2} L$$

Ratio of the humps:

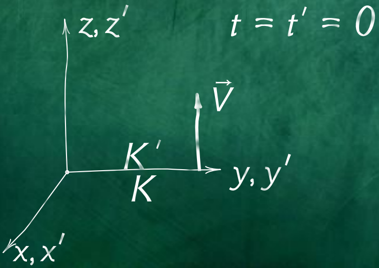
$$\frac{L_1}{L_2} = \frac{\dot{E}_1}{\dot{E}_2} = \frac{w_B}{w_{ph}}$$

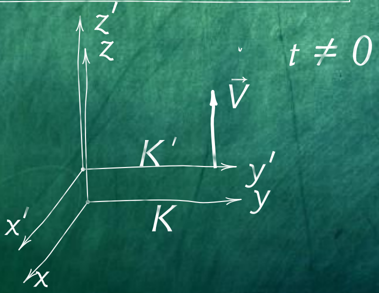
Lorentz Transformation

Lorentz transformations relate physical quantities in different inertia reference frames. Lorentz transformations are essential for radiation process in two important ways:

- Emitting particles move with relativistic speed, thus *all* the processes occurring in the co-moving frame or the center-of-mass frame are a subject for Lorentz transformation
- If the emission is produced in relativistically moving media, then one needs to transform it to the observer frame

$$t' = \Gamma\left(t - \frac{V}{c^2} z\right), \quad z' = \Gamma(z - Vt), \quad x', y' = x, y$$

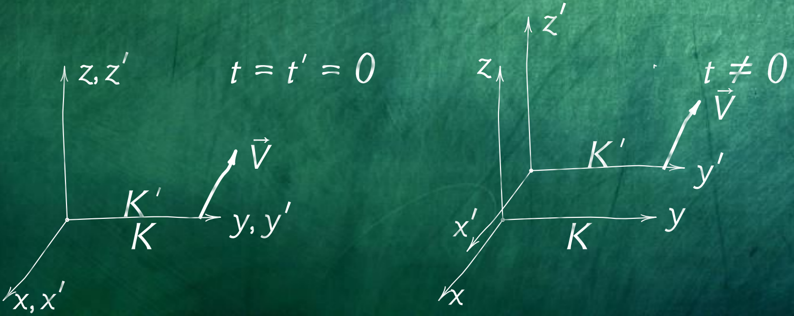




Lorentz transformation: Vector Form

$$t' = \Gamma\left(t - \frac{V}{c^2} z\right), \quad z' = \Gamma(z - Vt), \quad x', y' = x, y$$

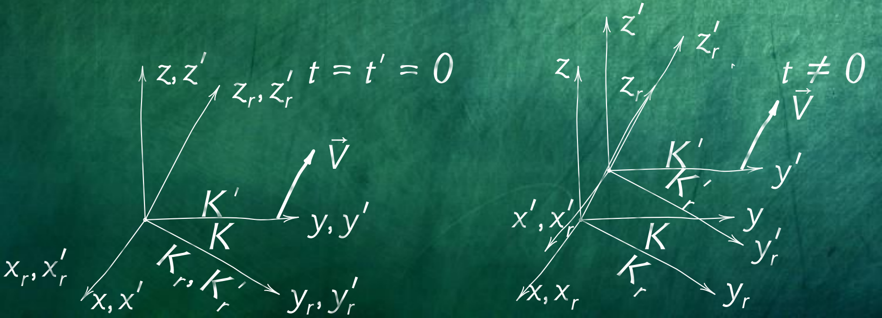
but what to do if the relative speed is not aligned with a coordinate axis?!!!



Lorentz transformation: Vector Form

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = R^{-1} L R \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

Superposition of three linear transformations: R is rotation and L is Lorentz transformation. Trivial or even boring...



Lorentz transformation: Vector Form

$$t' = \Gamma\left(t - \frac{V}{c^2}z\right), \quad z' = \Gamma(z - Vt), \quad x', y' = x, y$$

Let us look at this problem from somewhat different point of view: $z = \vec{V}\vec{r}/V$ and $\vec{e}_z = \vec{V}/V$. Then

$$t' = \Gamma\left(t - \frac{\vec{V}\vec{r}}{c^2}\right)$$

$$\vec{r}' = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z + (\Gamma(z - Vt) - 1)\vec{e}_z$$

$$\vec{r}' = \vec{r} + \Gamma\vec{V}t + (\Gamma - 1)\frac{\vec{V}\vec{r}}{V}\frac{\vec{V}}{V} = \vec{r} + \Gamma\vec{V}\left(t + \frac{\vec{V}\vec{r}}{c^2}\frac{\Gamma}{\Gamma+1}\right)$$

Lorentz transformation: Vector Form

$$t' = \Gamma\left(t - \frac{V}{c^2} z\right), \quad z' = \Gamma(z - Vt), \quad x', y' = x, y$$

Let us look at this problem from somewhat different point of view: $z = \vec{V}\vec{r}/V$ and $\vec{e}_z = \vec{V}/V$. Then

This is a very useful method, which is substituting quantities in "invariant form". Here we are dealing with the rotation transformations, so the form is "rotation invariant", but it is also an extremely useful tool to deal with Lorentz transformations.

$$t' = \Gamma\left(t - \frac{\vec{V}\vec{r}}{c^2}\right)$$

$$\vec{r}' = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z + (\Gamma(z - Vt) - 1)\vec{e}_z$$

$$\vec{r}' = \vec{r} + \Gamma\vec{V}t + (\Gamma - 1)\frac{\vec{V}\vec{r}}{V}\frac{\vec{V}}{V} = \vec{r} + \Gamma\vec{V}\left(t + \frac{\vec{V}\vec{r}}{c^2}\frac{\Gamma}{\Gamma+1}\right)$$

used here

$$\frac{\Gamma - 1}{V^2} = \frac{1}{c^2}\frac{\Gamma^2}{\Gamma+1}$$

Mastering Lorentz Transformations (LTs)

LTs is an essential element of high-energy astrophysics. Even they are fundamental and basic, it is still a source of often confusion and mistakes. There are two essential methods to master LT, which are worthy of learning and practicing:

- Using Lorentz invariant quantities

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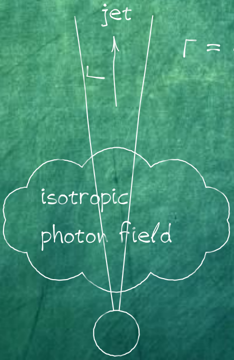

$$\Gamma = \frac{1}{\sqrt{1-\beta^2}}$$

If a photon momentum makes an angle θ to the jet bulk speed, what is its energy in the co-moving frame?

That is a trivial question, but illustrates the method:

- The photon's energy is ν' in the co-moving frame, then it equals to the following Lorentz invariant expression: $\boxed{\nu' = k'u'}$, where k' is 4-momentum of the photon and u' is 4-speed of the jet.
- In the lab frame we obtain $\nu' = ku = \nu(1 - \beta\cos\theta)$

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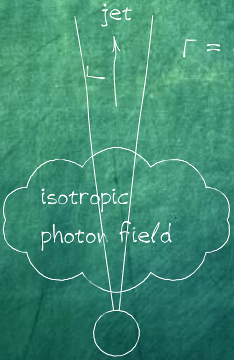

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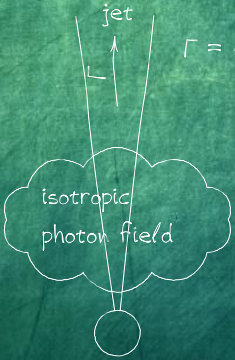
Energy density of an external photon field in relativistic jet

Why is this complicated?

- One needs to account for the transformation of energy and volume
- Energy of different photons transformed differently to the co-moving frame



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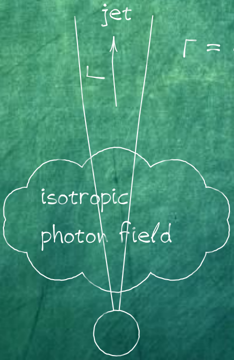

$$\Gamma = \frac{1}{\sqrt{1-\beta^2}}$$

If T^{ij} is energy-momentum tensor, then T^{00} is energy density. In the photon gas "rest frame" the tensor has a simple form:

$$T^{ij} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon/3 & 0 & 0 \\ 0 & 0 & \varepsilon/3 & 0 \\ 0 & 0 & 0 & \varepsilon/3 \end{pmatrix}$$

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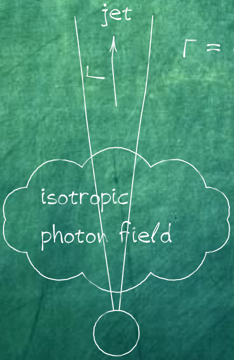

$$\Gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Tensors have clear transformation properties

$$T'^{ik} = L^i_m L^k_l T^{ml}$$

thus we immediately obtain

$$\varepsilon' = L^0_0 L^0_0 \varepsilon + L^0_3 L^0_3 \frac{\varepsilon}{3} = \Gamma^2 \varepsilon \left(1 + \frac{\beta^2}{3}\right)$$

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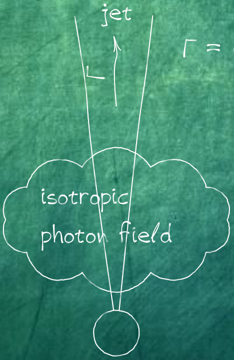

$$\Gamma = \frac{1}{\sqrt{1-\beta^2}}$$

We can also use (kind of) the first approach and add the bulk speed, $u^i = (1, 0, 0, 0)$ to the expression of the tensor:

$$T^{ik} = w u^i u^k - \eta^{ik} p$$

Here $w = \varepsilon + p$, $p = \varepsilon/3$, and η^{ij} is metric tensor

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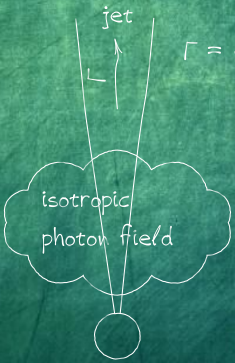


$$\Gamma = \frac{1}{\sqrt{1-\beta^2}}$$

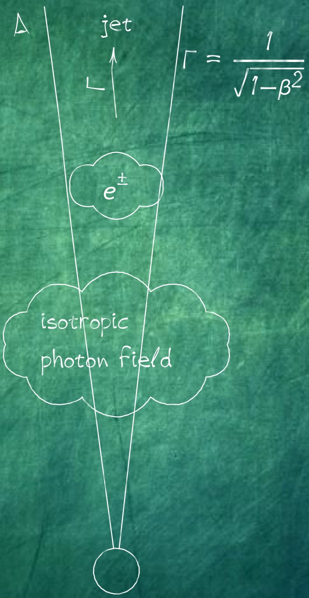
if $u = (\Gamma, 0, 0, \beta\Gamma)$ the component is

$$T^{00} = w\Gamma^2 - p = \Gamma^2\varepsilon\left(1 + \frac{1}{3}\frac{\Gamma^2 - 1}{\Gamma^2}\right)$$

$$\varepsilon' = \Gamma^2\varepsilon\left(1 + \frac{\beta^2}{3}\right)$$

Inverse Compton Emission from Jets

Relativistic electron(-positron) gas confined in a jet, which moves with bulk Lorentz factor Γ. Photon gas has a know energy-momentum distribution in the Lab frame (e.g, thermal isotropic). How to compute IC emission in the direction of the observer?



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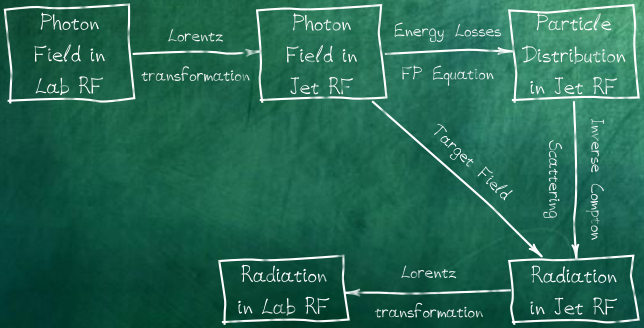

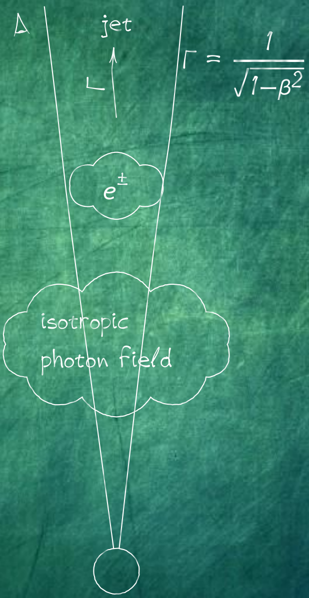

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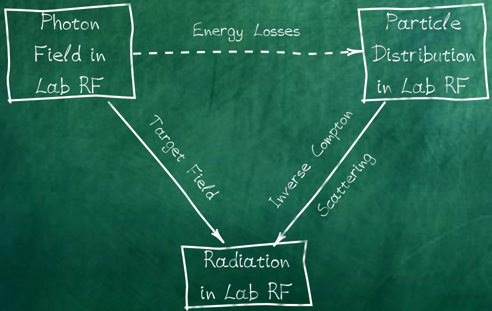

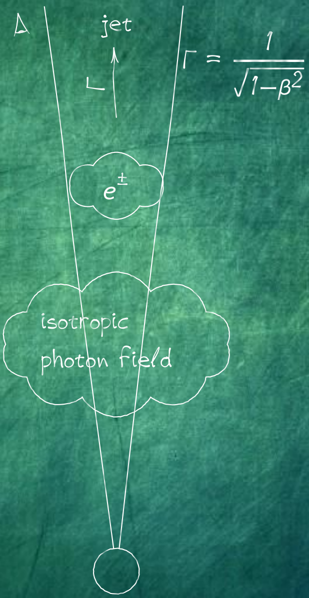

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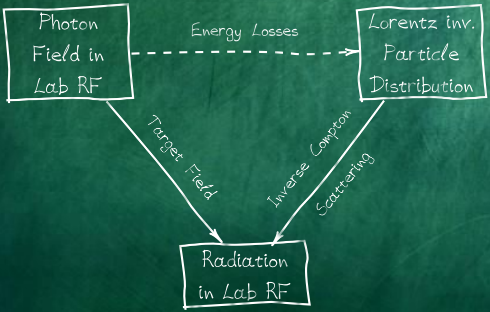
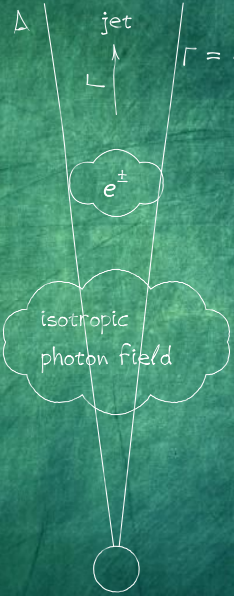




$$\Gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Lorentz Invariant distribution function

Distribution function in the phase space is a Lorentz invariant:

$$dN = f(\vec{r},\vec{p})\, d^3r\, d^3p$$

$$f(\vec{r},\vec{p}) = f'(\vec{r}',\vec{p}')$$

here $(\vec{r},\vec{p})$ and $(\vec{r}',\vec{p}')$ are related by the LT

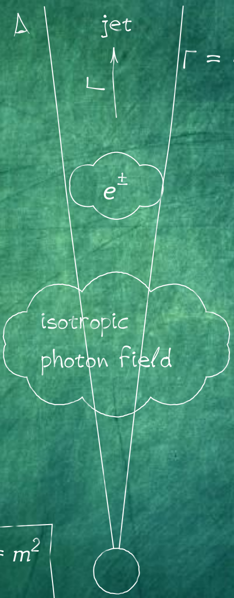
$$dN = n\, d^3r\, dE = \int d\Omega_p\, f\, d^3r\, d^3p$$

$$dN = n\, d^3r\, dE = 4\pi f\, d^3r\, p^2\, dp$$

$$f = \left[\frac{c^2}{4\pi p E}\right] n$$

$$E^2 - m^2 p^2 = m^2$$

$$E\, dE = m^2\, p\, dp$$



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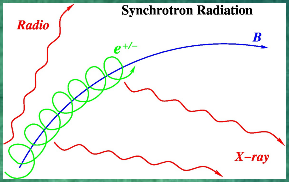
Synchrotron Radiation

- Single Particle Spectrum:

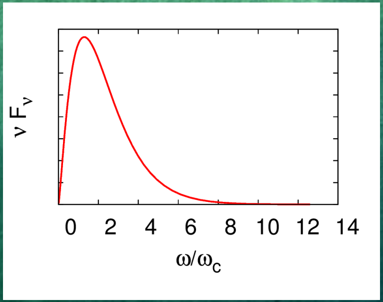
$$\frac{dI_{syn}}{d\omega} = \frac{\sqrt{3}}{2\pi}\frac{e^3 B}{mc^2} F\left(\frac{\omega}{\omega_c}\right)$$



where $\omega_c = \frac{3eB\gamma^2}{2mc}$ and $F(x) = x\int_x^{\infty} K_{5/3}(x')dx'$

- Energy Losses: $\dot{E}_{syn} = -\frac{4}{3}U_B c\gamma^2$

- Spectrum transformation: $\alpha \Rightarrow \Gamma = \frac{\alpha+1}{2}$



Acceleration of non-thermal particle proceeds in magnetized media therefore accelerated particles unavoidable interact with magnetic field generating non-therm emission – synchrotron radiation

Inverse Compton Scattering

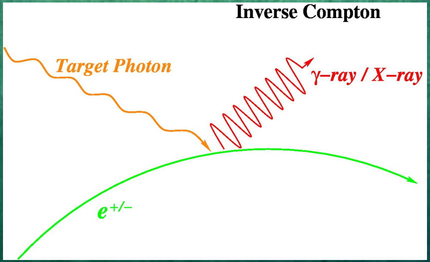
Single Particle Spectrum:

$$\frac{dI_{ic}}{d\omega} = \frac{r_o^2 \pi m_e^3 c^4 \kappa T^2}{3\hbar^3 E}\left[\frac{(\omega/E)^2}{2(1-\omega/E)} + 1\right]$$

Energy Losses: $\dot{E}_{syn} = -\frac{4}{3}U_{ph}c\gamma^2$

Spectrum transformation: $\alpha \Rightarrow \Gamma = \frac{\alpha+1}{2}$

Background photons should present in any source, in many cases IC scattering appears to be comparable to the synchrotron radiation.



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The slope transformation is the same as for synchrotron radiation

$$\Gamma = \frac{\alpha + 1}{2}$$

Energy Losses: $\dot{E}_{syn} = -\frac{4}{3} U_{ph} c \gamma^2$

Does this implies a very simple range of synchrotron–IC spectra?

Spectrum transformation: $\alpha \Rightarrow \Gamma = \frac{\alpha+1}{2}$

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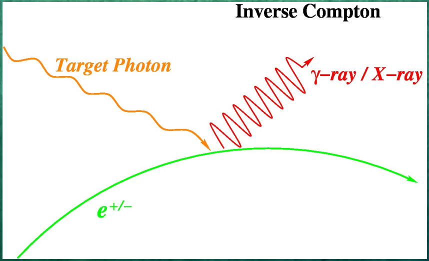
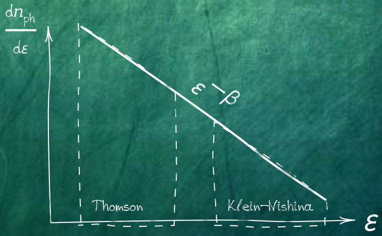


Spectral slope of IC component

- Let's assume that there is a power-law distribution of relativistic electrons, $dn_e / dE_e \propto E_e^{-\alpha}$. What is the slope of the IC component?
- There is a standard answer from the textbook: it depends on the scattering regime

$$\frac{dn_\gamma}{d\omega} \propto \begin{cases} \omega^{-(\alpha+1)/2} & \text{Thomson regime} \\ \omega^{-(\alpha+1)} & \text{Klein-Nishina regime} \end{cases}$$

- However, this rule of thumb doesn't work if the target photon field is a broadband power-law, $dn_{ph} / d\varepsilon \propto \varepsilon^{-\beta}$
- What is the spectral slope in this case?



Anatomy of IC scattering

☞ Using δ-functional approximation one can study the properties of IC scattering on a power-law target analytically:

$$\frac{d\dot{n}_\gamma}{d\omega} = |\frac{\dot{E}}{\omega}|\delta(\omega - \bar{\omega})$$

- This approach allows obtaining the position of spectral breaks and expected slopes
- IC component can feature up to three physically motivated breaks!

Under the δ-function approximation, we can (approximately) compute the spectrum of gamma-ray emission:

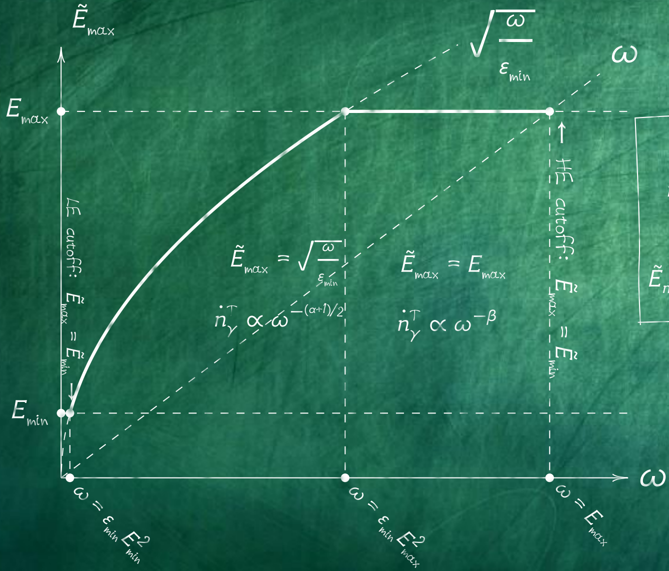
$$\dot{n}_\gamma \approx \frac{\omega^{-\beta}}{(2\beta - \alpha - 1)} \left(\tilde{E}_{max}^{(2\beta-\alpha-1)} - \tilde{E}_{min}^{(2\beta-\alpha-1)} \right)$$

where

$$\tilde{E}_{max} = \min \left(E_{max}, \sqrt{\frac{\omega}{\varepsilon_{min}}} \right)$$

$$\tilde{E}_{min} = \max \left(\omega, E_{min}, \sqrt{\frac{\omega}{\varepsilon_{max}}} \right)$$

Anatomy of IC Scattering



$$\dot{n}_{\gamma} \approx \frac{\omega^{-\beta} \tilde{E}_{max}^{(2\beta-\alpha-1)}}{(2\beta-\alpha-1)}$$

$$\tilde{E}_{max} = \min\left(E_{max}, \sqrt{\frac{\omega}{\varepsilon_{min}}}\right)$$

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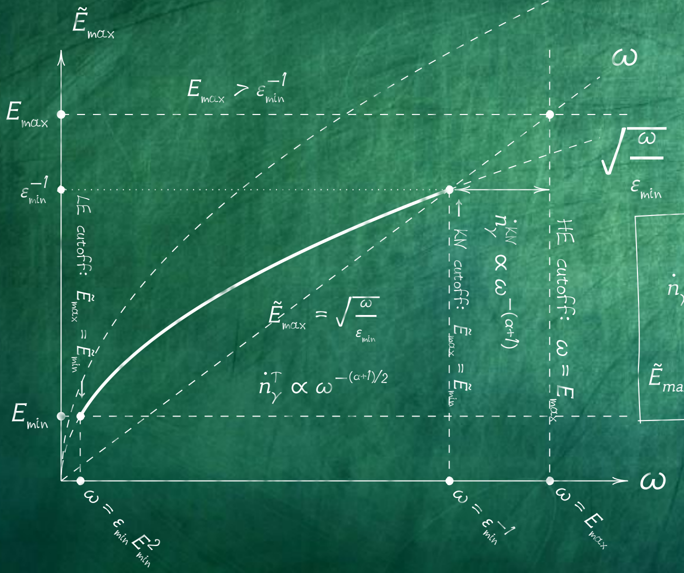

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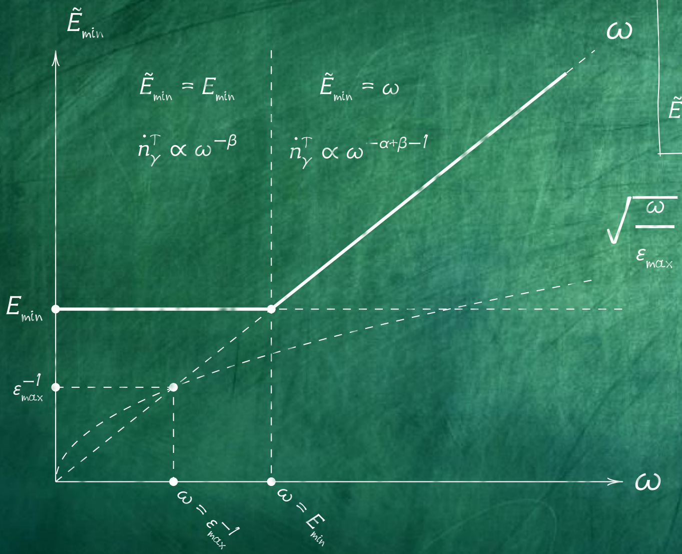

$$\dot{n}_\gamma \approx \frac{\omega^{-\beta}\tilde{E}_{min}^{(2\beta-\alpha-1)}}{(\alpha+1-2\beta)}$$

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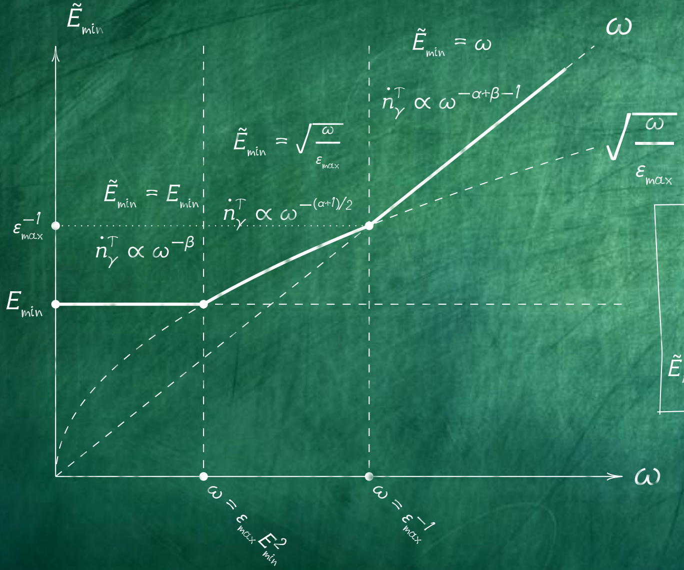

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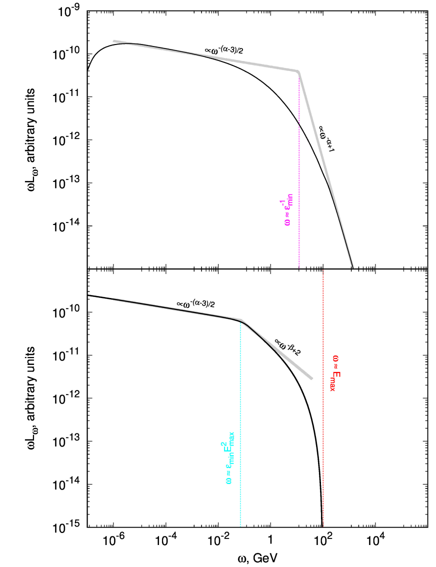
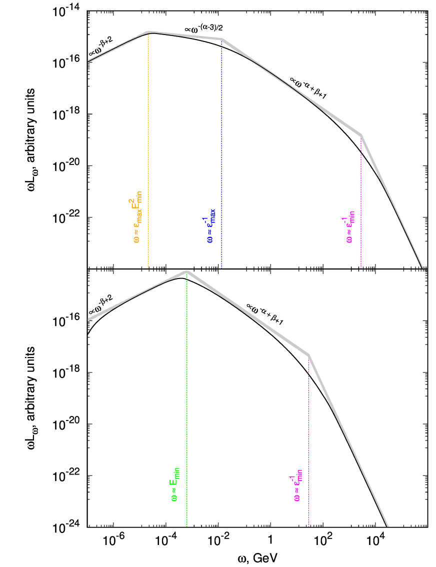
Anatomy of IC scattering: Example





Summary

- When one models non-thermal emission, it is often more important to implement a physically justified description for particle distribution than include an accurate treatment of the emission processes
- Of course, there are significant uncertainties in particle acceleration processes, and we cannot implement a self-consistent model for particles. However, there are certain spectral features, which properties are firmly determining by the basic theory (such as cooling breaks), and they should not be ignored in modeling
- If you plan to model emission from relativistic sources, don't neglect mastering Lorentz transformations: better understanding of relativistic physics will help you to avoid mistakes and find more efficient way for computing radiation
- Even the most simple radiation process, such as IC scattering, may appear complex enough if one start looking into details