广西师范大学

Study of SU(3) Deconfinement Phase Transition under rotation with Matrix model

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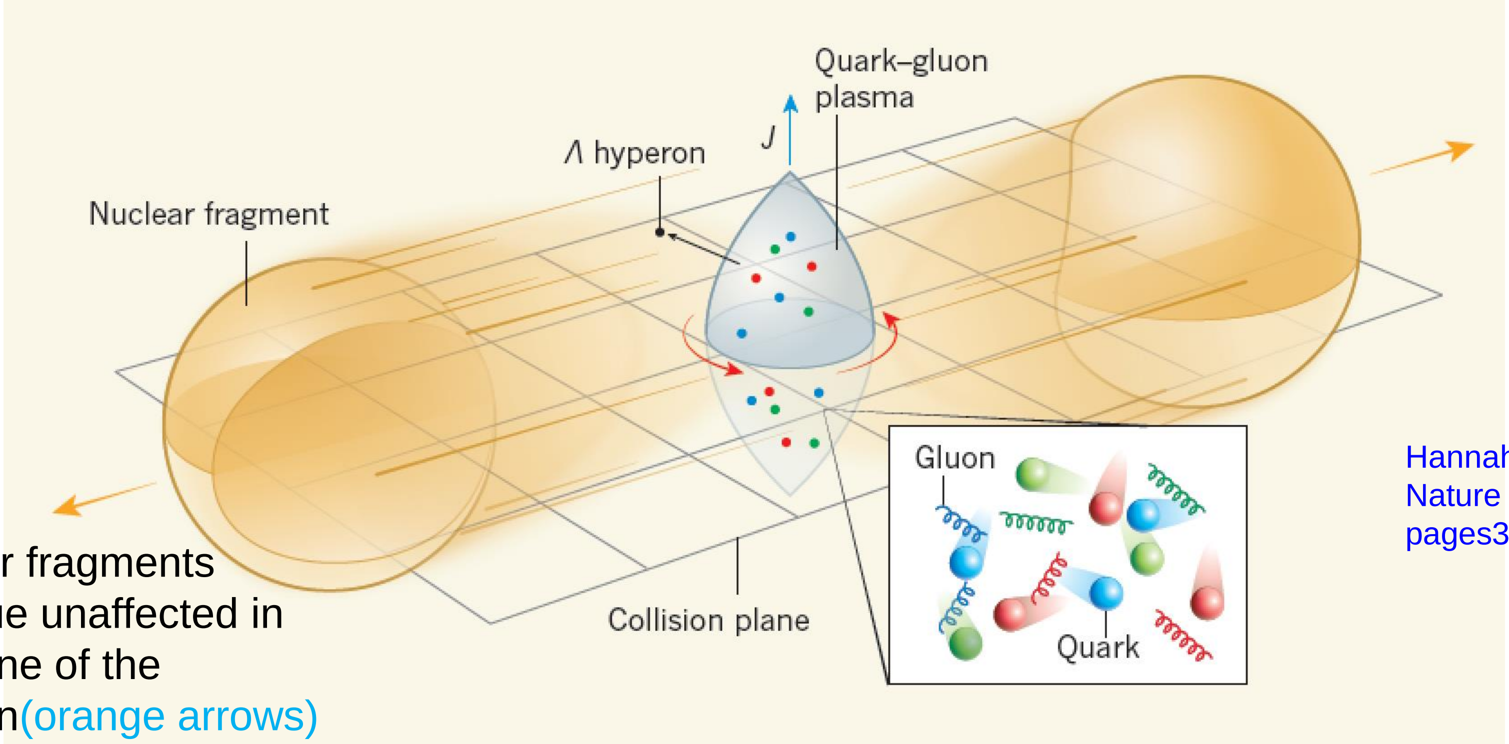
2024年6月21日

Outline:

- Background and Motivations
- Matrix model in rotation
- Numerical results
- Summary and outlook

Background and Motivations



Nuclear fragments continue unaffected in the plane of the collision(orange arrows)

Hannah Petersen, Nature volume 548, pages34–35 (2017).

Fig. 1 Measuring the rotation of the quark–gluon plasma

1) QGP rapidly rotates (red arrows), with an angular momentum (J, blue arrow), perpendicular to the plane of the collision.

2) The vorticity of QGP can be determined by using the polarization of Λ hyperons (black arrow), due to its self-analysis, on experiment.

Background and Motivations

1) In experiment RHIC-STAR

L. Adamczyk et al. (The STAR Collaboration). Nature 548, 62–65 (2017).

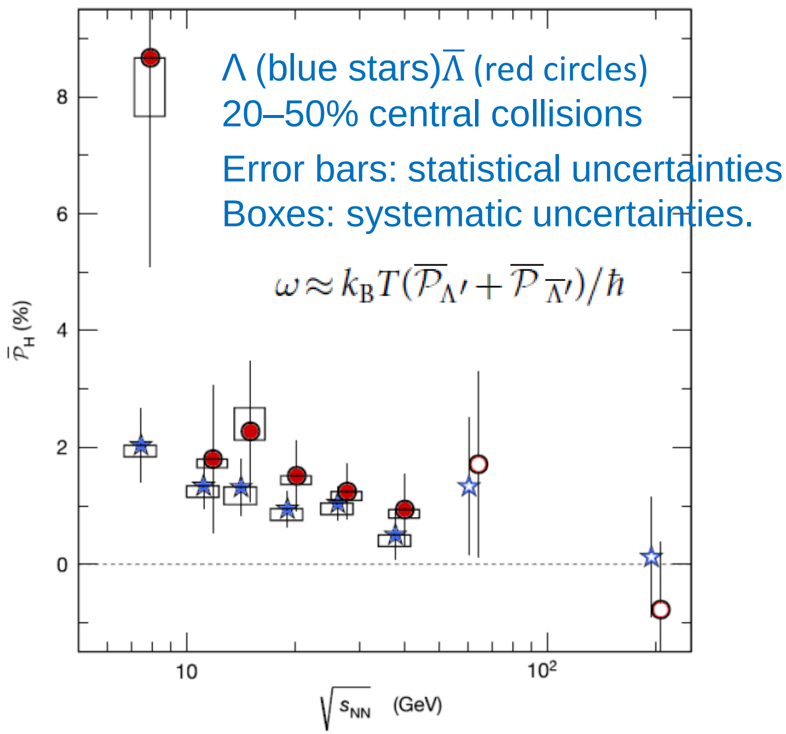


Fig. 2 The hyperon average polarization in Au + Au collisions.

$$\Omega \simeq (9 \pm 1) \times 10^{21} \mathrm{s}^{-1} \sim 6\mathrm{MeV}$$

2) Hydrodynamic simulations

Yin Jiang, Zi-Wei Lin, and Jinfeng Liao, Physical Review C 94, 044910 (2016), (AMPT model)

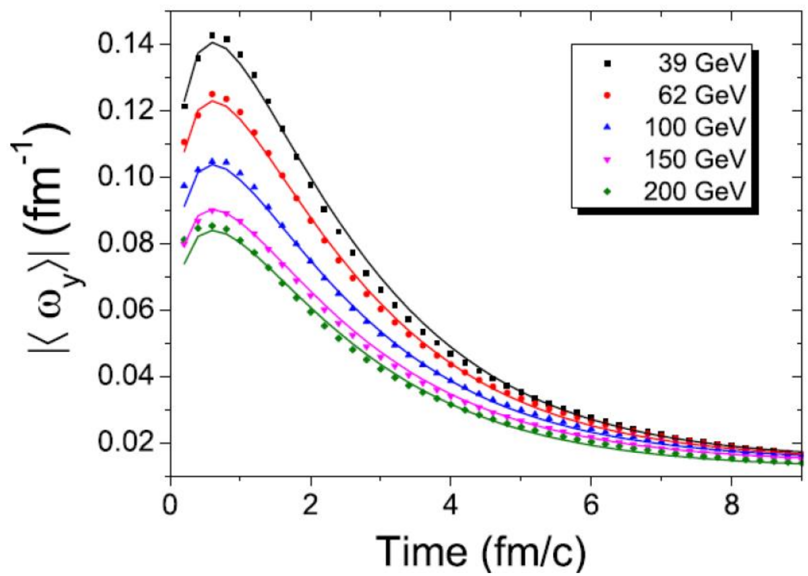


Fig. 3 Averaged vorticity $\langle \omega_y \rangle$ as a function of time at various $\sqrt{s_{NN}}$ for fixed $b = 7$fm in Au-Au collision.

$$\Omega \sim 0.1 - 0.2\mathrm{fm}^{-1} \sim 20 - 40\mathrm{MeV}$$

These values of angular velocity lead to relativistic rotation of QGP.

Background and Motivations

Rotation affect →

Chiral vortical effect

G. Y. Prokhorov, O. V. Teryaev, and V. I. Zakharov, J. High Energy Phys. 02 (2019) 146.

Polarization of particles

O. V. Teryaev and V. I. Zakharov, Phys. Rev. D 96, 096023 (2017).

QCD phase transition

1) Chiral phase transition

Rotation suppresses the chiral critical temperature ($\omega \uparrow, T_c \downarrow$)

Lattice QCD、NJL model、Holography、HRG model、Compact QED in 2+1…

Hao-Lei Chen, Kenji Fukushima, Xu-Guang Huang, and Kazuya Mameda, Phys. Rev. D 93, 104052 (2016);
Yin Jiang and Jinfeng Liao, Phys. Rev. Lett. 117, 192302 (2016);
M. N. Chernodub and Shinya Gongyo, J. High Energy Phys. 01 (2017) 136; Phys. Rev. D 95, 096006 (2017);
Xinyang Wang, Minghua Wei, Zhibin Li, and Mei Huang, Phys. Rev. D 99, 016018 (2019);
Zheng Zhang, Chao Shi, Xiao-Tao He, Xiaofeng Luo, and Hong-Shi Zong, Phys. Rev. D 102, 114023 (2020);
N. Sadooghi, S. M. A. Tabatabaee Mehr, and F. Taghinavaz, Phys. Rev. D 104, 116022 (2021).

Explanation: Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016).

The rotation tends to align the spins of quarks and antiquarks along the rotation axis, thus suppressing the scalar pairing and, therefore, lowering the scalar fermionic condensate.

Background and Motivations

2) The deconfinement phase transition

Lattice QCD   first-principles lattice calculation

a) Rotation increase the bulk critical temperature, and independence of spatial boundary conditions（Open BC (OBC)、Periodic (PBC)、Dirichlet BC (DBC)

$$\frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2, C_2 > 0$$

V. V. Braguta, A. Yu. Kotov, D. D. Kuznedelev, and A. A. Roenko, JETP Lett. 112, 6–12(2020);Phys. Rev. D 103, 094515 (2021);
Victor Braguta, A. Yu. Kotov, Denis Kuznedelev, and Artem Roenko, PoS LATTICE2021, 125 (2022).

b) The local critical temperature at the rotation axis does not depend on Ω;
There is no such notion as a single (global) critical temperature for vortical quark-gluon plasma;
The non-perturbative dynamics of the gluon plasma do not comply with the oversimplified picture of the Tolman-Ehrenfest (TE) law.

Victor V. Braguta, Maxim N. Chernodub, and Artem A. Roenko, e-Print: 2312.13994 [hep-lat].

$$T(r) = \frac{T_0}{\sqrt{1-\Omega^2 r^2}} = \frac{T_0}{\sqrt{1+\Omega_I^2 r^2}}$$

Background and Motivations

Holography QCD | hadron resonance gas

Xun Chen, Lin Zhang, Danning Li, Defu Hou, and Mei Huang, J. High Energy Phys. 07 (2021) 132;
Yuki Fujimoto, Kenji Fukushima, and Yoshimasa Hidaka, Phys. Lett. B 816, 136184 (2021).

Rotation decrease the critical temperature $(\omega \uparrow, T_c \downarrow)$

Compact QED in 2+1 $(\omega \uparrow, T_c \downarrow)$ M. N. Chernodub, Phys. Rev. D 103, 054027 (2021).

two transition temperatures: $T_{c1}(\Omega) < T < T_{c2}(\Omega)$ Mixed inhomogeneous phase

$T < T_{c1}(\Omega)$ confining phase $T > T_{c2}(\Omega)$ deconfining phase

Perturbative method Shi Chen, Kenji Fukushima, Yusuke Shimada, Phys. Rev. Lett. 129 (2022) 24, 242002.

At sufficiently large Ω_I , the system goes through a phase transition perturbatively. $(\omega \uparrow, T_c \downarrow)$

KvBLL caloron Yin Jiang, Phys. Lett. B 853 (2024) 138655.

For static coupling constant g, $\omega \uparrow, T_c \downarrow$; For $g(\omega)$, $\omega \uparrow, T_c \uparrow\downarrow$

Bag model Kazuya Mameda, Keiya Takizawa, Phys.Lett.B 847 (2023) 138317.

For static bag constant B_0, $\omega \uparrow, T_c \downarrow$; For revolving bag constant $B(\omega)$, $\omega \uparrow, T_c \uparrow$

Matrix model in rotation

The matrix model (MMO) currently

Matrix models for deconfinement phase transition

- Perturbation (T^4) ⟹ Based on the **1-loop effective potential**
- Non-Perturbation ($m_g^2 T^2$) ⟹ **Effective potential with the gluonic effective mass m_g**

Peter N. Meisinger, Travis R. Miller, Michael C. Ogilvie, Phys. Rev. D 65 (2002) 034009.

It can well describe the physical behavior of QGP near the phase transition point, but in order to compare with Lattice QCD quantitatively, it needs to be improved.

Improvement currently

- Deconfinement phase transition in pure gauge theory ⇨ By considering the dependent relationship of m on T

  Yun Guo, JHEP 11 (2014) 111.

- Chiral phase transition in QCD ⇨ By introducing the perturbative contribution of the 1-loop fermionic effective potential

Robert D. Pisarski, Vladimir V. Skokov, Phys. Rev. D 94 (2016) 3, 034015.

Matrix model in rotation

The background field

Definition of the background field

The classical (constant) **background field A_0^{cl}** is described by the diagonal matrix of the color space:

$$(A_0^{cl})_{ab} = \frac{2\pi T}{g} q_a \delta_{ab}$$

A. T. Bhattacharya, A. Gocksch, C. P. Korthals Altes and R. D. Pisarski, Phys. Rev. Lett. 66, 998 (1991).

where: $q_{ab} = q_a - q_b$ and $q_1 + q_2 + q_3 + ... + q_N = 0$, T is temperature.

A_0^{cl} takes the eigenvalue of the Wilson line as its variable, where the **Wilson line** is defined as:

$$\mathbf{L}(\vec{x}) = \mathcal{P}\exp[ig \int_0^{1/T} d\tau\, A_0^{cl}(\vec{x},\tau)]$$

The corresponding **Polyakov loop** is: $\ell = \frac{1}{N}\mathrm{Tr}\mathbf{L}$

phase	temperature	ℓ from LQCD	method
hadronic	$T < T_c$	$\ell = 0$	eff. theory (HRG)
QGP	$T >\sim 3T_c$	$\ell \approx 1$	(HTL) pert. theory
"semi"-QGP	$T_c < T <\sim 3T_c$	$0 < \ell < 1$	BF eff. theory

Matrix model in rotation

Perturbative calculation of the constrained effective potential Γ

Definition of Γ

$$\exp(-V\beta\Gamma(\ell_k)) = \int DA_\mu \prod_{k=1}^{N-1} \delta(\ell_k - \bar{\ell}_k) \exp\left(-\frac{S(A)}{g^2}\right)$$

C. P. Korthals Altes. Nucl. Phys. B 420 (1994) 637.

where: V is the volume of the system under consideration; $\beta = \frac{1}{T}$;

g is the coupling constant; N is the number of colors;

$$\bar{\ell}_k \equiv \frac{1}{N}\overline{\mathrm{Tr}\,\mathbf{L}^k} = \frac{1}{N}\frac{\int_V d^3\vec{x}\,\mathrm{Tr}\,\mathbf{L}^k(\vec{x})}{V}$$

$S(A)$ is the action for the gauge field;

It is Equivalent to: $\int DA_\mu \exp[-\frac{S(A)}{g^2} - j\int d\vec{x}\,\mathrm{Tr}\mathbf{L}(\vec{x})]$ Gauge independent

Fourier transforming the delta function constraint:

$$\exp(-V\beta\Gamma(\ell_k)) = \int DA_\mu\, d\epsilon\, \exp\left(-\frac{1}{g^2}S_{con}(A,\epsilon)\right)$$

The "constrained" is reflected in the effective action due to $\delta(\ell_k - \bar{\ell}_k)$:

$$S_{\rm con}(A,\epsilon) = i\sum_{k=1}^{N-1} \epsilon_k\left(\ell_k - \bar{\ell}_k\right) + S(A)$$

where: ϵ_k is the extra fields.

Expand A_μ and ϵ field : $A_\mu = A_\mu^{\rm cl} + B_\mu$ and $\epsilon = \epsilon_c + \epsilon_{\rm q}$

$A_\mu^{cl} = A_0^{cl}\delta_{\mu 0} = \frac{2\pi T q}{g}\delta_{\mu 0}$, B_μ and ϵ_q is the quantum fluctuation

Matrix model in rotation

Add the gauge fixing and ghost contributions into S_{con} ⇨ Expanding S_{con} in terms of B_μ and ϵ_q ⇨ to LO ⇨ 1-loop effective potential $\Gamma^{(1)} = F_b^{(1)}$

Yun Guo, Qianqian Du, JHEP 05 (2019) 042. ⇨ to order g^2 ⇨ 2-loop result

Compute the effective potential in a perturbative way.

Applied to compute

The free energies for SU(∞) gauge theory on a small sphere up to 3-loop order;

The quark/gluon self-energies for SU(N) gauge theory at leading order.

O. Aharony, J. Marsano, S. Minwalla, et., Phys. Rev. D 71, 125018 (2005);
Y. Hidaka and R. D. Pisarski, Phys. Rev. D 80, 036004 (2009).

The 1-loop free energy can be obtained by $F = \dfrac{\partial(T \ln Z)}{\partial V}$

The partition function ($\ln Z$) is calculated mainly by the technique of Feynman diagram expansion.

$F = -\frac{1}{\beta V}$  $+\frac{1}{\beta V}$ 

(a) (b)

Fig. 4 Curly lines gluons, and dashed lines ghosts.

Matrix model in rotation

- Lagrangian density in the background field (in Euclidean spacetime)

$$\mathcal{L} = \frac{1}{2}\,\mathrm{tr}\left(G_{\mu\nu}^2\right) + \frac{1}{\xi}\,\mathrm{tr}\left(D_\mu^{\mathrm{cl}} B_\mu\right)^2 - 2\,\mathrm{tr}\left(\bar{\eta} D_\mu^{\mathrm{cl}} D_\mu \eta\right)$$

the field strength tensor , $G_{\mu\nu} = [D_\mu, D_\nu]/(-ig) = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$

where: covariant derivative in the adjoint representation $D_\mu = \partial_\mu - ig[A_\mu,\ .\ ]$

η is the ghost field; ξ is the gauge fixing parameter,

The Feynman rules can be derived straight-forwardly in the *double line basis*.

Y. Hidaka and R. D. Pisarski. Phys.Rev. D80:036004,2009.

Double line basis

P. Cvitanovic, Phys. Rev. D 14, 1536 (1976).

Compared to the usual Cartan basis ⇨ It is believed to be more efficient when compute in the presence of a constant A_0^{cl}

Classical covariant derivative acts upon a bosonic field, $D_\mu^{cl} B_\nu^{ba} t^{ab} = -i\, P_\mu^{ab} B_\nu^{ba} t^{ab}$

where: $P_\mu^{ab} = \left(\omega_n + C^{ab}, \vec{p}\right) = (\omega_n^{ab}, \vec{p}\,)$ with ω_n=2nπT, $C^{ab} = 2\pi T q^{ab}$

The computation in $A_0^{cl} \neq 0$ is a trivial **generalization** of that in the $A_0^{cl} = 0$ case

Matrix model in rotation

- **without gluon mass**

$$\ln Z_b^{(1)} = -\sum_{ab}\sum_{n}\sum_{\mathbf{p}} \ln\{\beta^2[(\omega_n^{ab})^2+\mathbf{p}^2]\}\left(1-\frac{1}{N_c}\delta_{ab}\right)$$

$$\sum_{\mathbf{p}} = V\int\frac{d^3\mathbf{p}}{(2\pi)^3} \quad \beta=\frac{1}{T} \quad N_c=3$$

$$\Gamma_b^{(1)} = F_b^{(1)} = -\sum_{ab}\frac{1}{6\pi^2}\int_0^\infty q^3 dq\left(N_{ab}^+(q)+N_{ab}^-(q)\right)\left(1-\frac{1}{N}\delta_{ab}\right)$$

$$= \frac{2\pi^2T^4}{3}\sum_{ab}B_4(q^{ab})+\frac{\pi^2T^4}{45},$$

with $N_{ab}^{\pm}(q) = \frac{1}{e^{\beta q\mp i\beta C^{ab}}-1}$, $B_4(x)$ is Bernoulli polynomial

A_0^{cl} =0， $\ell=1$, the system is in a full deconfinement phase, with no phase transition

⇨

Transform to the confined phase by the matrix model which is constructed by adding nonperturbative terms

Matrix model in rotation

- **with gluon mass (MMO assume the ghost fields have the same mass with gluon) (The result is gauge independent)**

Peter N. Meisinger, Travis R. Miller, Michael C. Ogilvie, Phys. Rev. D 65 (2002) 034009.

$$\ln Z_b^{(1)} = -\sum_{ab}\sum_{n}\sum_{\mathbf{p}} \ln\{\beta^2[(\omega_n^{ab})^2 + \mathbf{p}^2 + m_g^2]\}\left(1 - \frac{1}{N_c}\delta_{ab}\right),$$

$$F_b^{(1)} = \sum_{ab} T \int_0^\infty \frac{p^2 dp}{2\pi^2}\left[\ln(1 - e^{-\beta\sqrt{p^2+m_g^2} - i\beta C^{ab}}) + \ln(1 - e^{-\beta\sqrt{p^2+m_g^2} + i\beta C^{ab}})\right]\left(1 - \frac{1}{N_c}\delta_{ab}\right)$$

Expanding m_g at high temperature, original Matrix model for SU(3)

$$\mathcal{V} = -\frac{8\pi^2 T^4}{45} + \frac{2\pi^2 T^4}{3}\sum_{ab}|q^{ab}|^2(1 - |q^{ab}|)^2 + \frac{2m_g^2 T^2}{3} + \frac{m_g^2 T^2}{2}\sum_{ab}|q^{ab}|(|q^{ab}| - 1)$$

LO, perturbative part — **NLO, non-perturbative part**

Matrix model in rotation

By setting the $T_d = 0.27\text{GeV}$, we fix the parameter gluon mass $m_{g,d} = 0.596\text{GeV}$ at the phase transition point.

The background field q can be obtained by the variational approach $\frac{\partial \mathcal{V}}{\partial q} = 0$

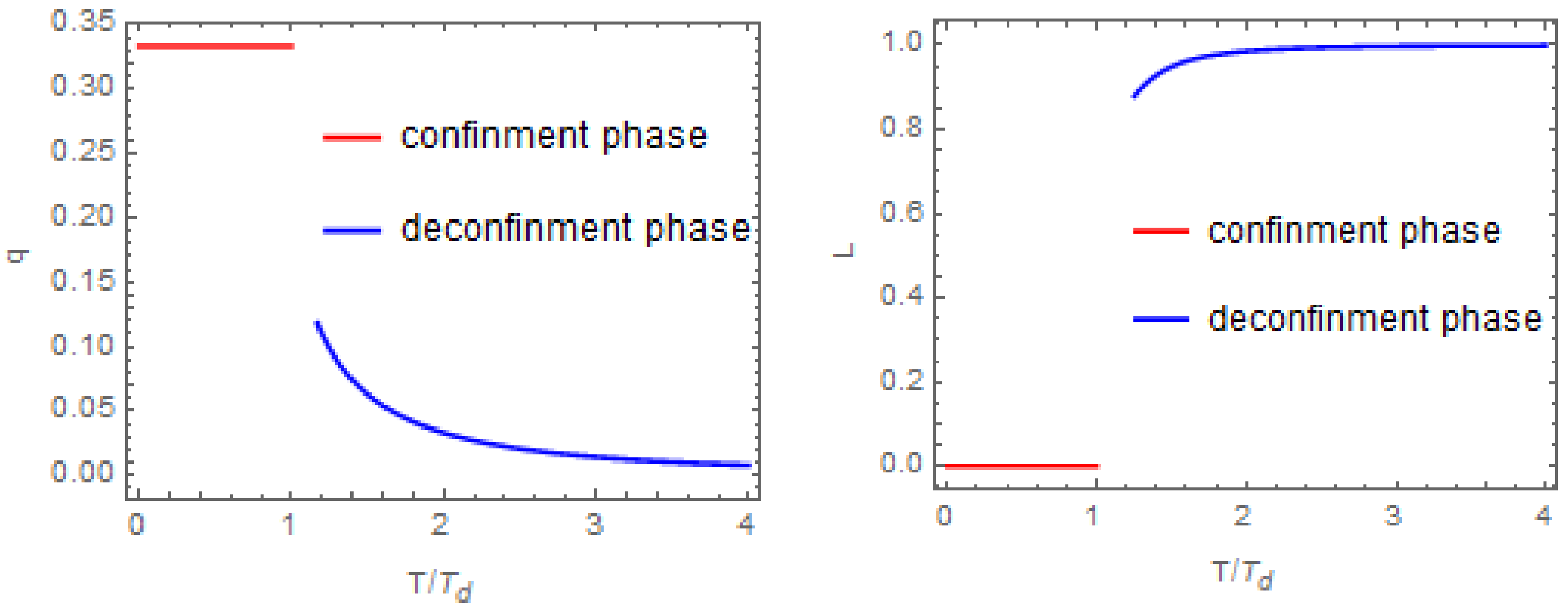


Fig. 4 The dependence of the background field and Polyakov loo on temperature.

1) First-order phase transition, $T < T_d$, $q = \frac{1}{3}$, $\ell = 0$, $T_d < T < 3T_d$, $0 < \ell < 1$

2) From LQCD, $0 < \ell < 1$, in the semi-QGP region ($T_d \sim 3T_d$) , and the background field can well describe this behavior.

Matrix model in rotation

The results of the improved Matrix model to compare with Lattice QCD quantitatively.

Yun Guo, JHEP 11 (2014) 111.

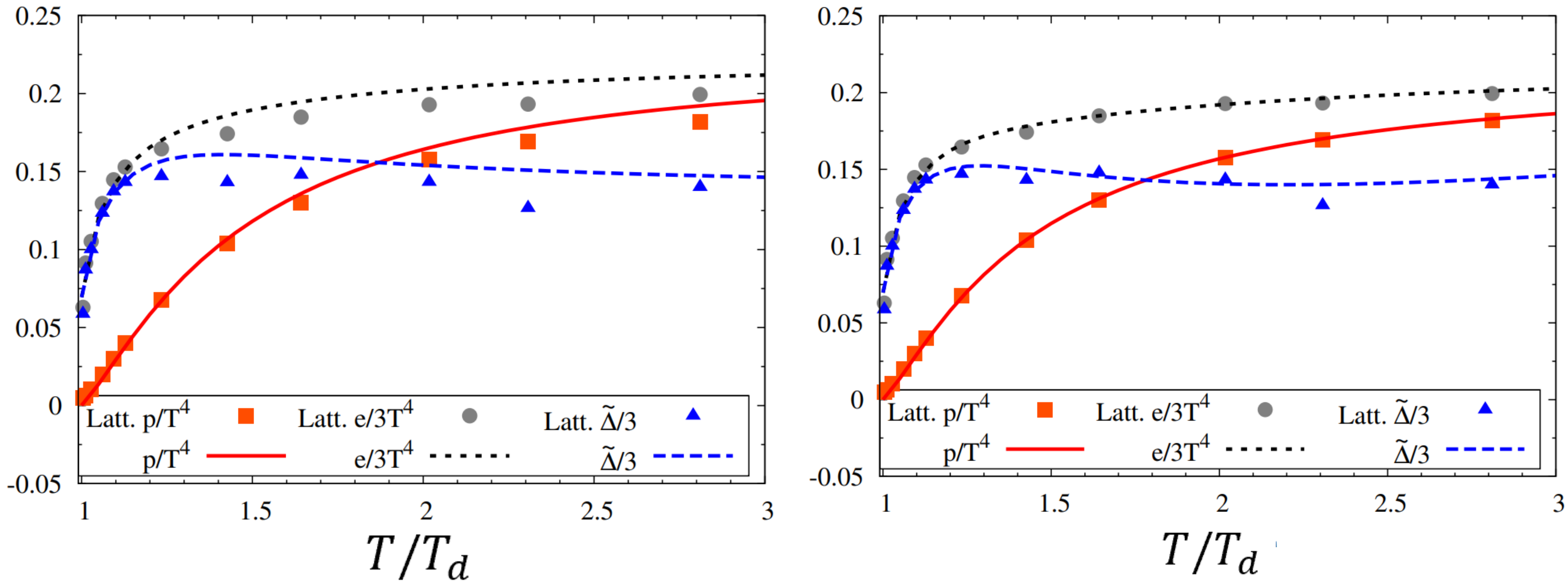
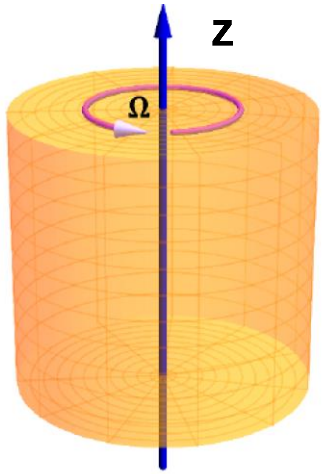


Fig. 4 Comparison of the SU(3) dimensionless pressure and energy density obtained from the lattice simulation, matrix models.

Matrix model in rotation



Consider a **bosonic system** in a **cylindrical volume**, which **rigidly rotates** about the **fixed z-axis** with a **constant angular velocity $\boldsymbol{\Omega}$**, with the **radius $\boldsymbol{R}$, $\boldsymbol{R\Omega} < \mathbf{1}$** to preserve the causality.

1) Choose the cylindrical coordinates in the **corotating reference frame** $\tilde{x}^{\mu} = (\tilde{t}, \tilde{\varphi}, \tilde{\rho}, \tilde{z})$

2) Apply the **Dirichlet condition** to the field at the cylindrical boundary

Alexander Vilenkin,, Phys. Rev. D 21 (1980) 2260-2269. M. N. Chernodub, Phys. Rev. D 103, 054027 (2021).

According to Vilenkin's and Chernodub's paper, for a scalar field, by solving the rotated EOM in the finite temperature field theory, the Matsubara frequency for bosons is:

$$\omega_{n,\Omega}^{(b)} = 2\pi T n - i\Omega m$$ with m is the angular quantum numbers, $m \in \mathbb{Z}$.

The integral for the three momentum p change to:

$$\int \frac{d^3p}{(2\pi)^3} \longrightarrow \frac{1}{2\pi^2 R^2} \sum_{m=-\infty}^{\infty} \sum_{\ell=1}^{\infty} \int_{-\infty}^{\infty} dp_z \frac{J_m^2(\frac{\tilde{\rho}}{R}\kappa_{m\ell})}{J_{m+1}^2(\kappa_{m\ell})} \qquad J_m(\kappa_{ml}) = 0, \quad l = 1, 2, \ldots.$$

where: l is the radial quantum numbers, p_z is the longitudinal quantum number. κ_{ml} is the ℓth positive root of the Bessel function J_m

Matrix model in rotation

- **The eigenmodes and eigenvalues for ghost (scalar field)**

EOM: $$D^{cl}_{\mu,s}D^{cl}_{\mu,s}\eta_{nJ}(\tilde{\tau},\tilde{\rho},\tilde{\phi},\tilde{z}) = -\tilde{E}^2_{nJ}\eta_{nJ}(\tilde{\tau},\tilde{\rho},\tilde{\phi},\tilde{z})$$

with boundary condition:

$$\eta_{nJ}(\tilde{\tau},\tilde{\rho},\tilde{\phi},\tilde{z})|_{\tilde{\rho}=R} = 0$$

where: $D^{cl}_{\mu,s}D^{cl}_{\mu,s} = (\partial_{\tilde{\tau}} - iC^{ab} - i\Omega\partial_{\tilde{\phi}})^2 + \tilde{\nabla}^2$

with: $\tilde{\nabla}^2 = \partial^2_{\tilde{\rho}} + \frac{1}{\tilde{\rho}}\partial_{\tilde{\rho}} + \frac{1}{\tilde{\rho}^2}\partial^2_{\tilde{\phi}} + \partial^2_{\tilde{z}}$

Eigenmodes: $$\eta_{nJ}(\tilde{\tau},\tilde{\rho},\tilde{\phi},\tilde{z}) = e^{-i\omega_n\tilde{\tau} - im\tilde{\phi} - ik_z\tilde{z}}\frac{J^2_m(\frac{\tilde{\rho}}{R}\kappa_{m,\ell})}{\sqrt{2}\pi R|J_{m+1}(\kappa_{m,\ell})|}$$

Eigenvalue: $$\tilde{E}^2_{nJ} = k^2_z + \left(\frac{\kappa_{m,\ell}}{R}\right)^2 + (\omega_n + C^{ab} - im\Omega)^2$$

$$P^{ab}_{\mu,\Omega} = (\omega_n - i\Omega m + C^{ab}, \vec{p}_J) = (\omega^{ab}_n - i\Omega m, \vec{p}_J) \qquad \vec{p}^{\,2}_J = p^2_z + p^2_T \qquad p_T = \frac{\kappa_{m,\ell}}{R}$$

Matrix model in rotation

Shi Chen, Kenji Fukushima, Yusuke Shimada, Phys. Rev. Lett. 129 (2022) 24, 242002.

- **The eigenmodes and eigenvalues for gluons (covariant vector field)**

EOM: $$D^{cl}_{\mu,v} D^{cl}_{\mu,v} B_{\mu,nJ}(\tilde{\tau},\tilde{\rho},\tilde{\phi},\tilde{z}) = -\tilde{E}^2_{nJ} B_{\mu,nJ}(\tilde{\tau},\tilde{\rho},\tilde{\phi},\tilde{z})$$

with boundary condition: $B_{\mu,nJ}(\tilde{\tau},\tilde{\rho},\tilde{\phi},\tilde{z})|_{\tilde{\rho}=R} = 0$

Four Eigenmodes:

Two unphysical (nontransverse) polarizations,

$$B^{(1,2)}_{nJ}(\tilde{\tau},\tilde{\rho},\tilde{\phi},\tilde{z}) = \eta_{nJ}(\tilde{\tau},\tilde{\rho},\tilde{\phi},\tilde{z})\xi^{(1,2)} \qquad \xi^{(1)} = (1,0,0,0)^T \qquad \xi^{(2)} = (0,0,0,1)^T$$

The two contributions to gluon loop are canceled by the ghost loop.

Two unphysical (transverse) polarizations,

$$B^{(-)}_{nJ}(\tilde{\tau},\tilde{\rho},\tilde{\phi},\tilde{z}) = e^{-i\omega_n\tilde{\tau}-im\tilde{\phi}-ik_z\tilde{z}} \frac{J^2_{m-1}(\frac{\tilde{\rho}}{R}\kappa_{m-1,\ell})}{2\pi R|J_m(\kappa_{m-1,\ell})|}\xi^- \qquad \xi^+ = (0,\tilde{\rho},i,0)^T$$

$$B^{(+)}_{nJ}(\tilde{\tau},\tilde{\rho},\tilde{\phi},\tilde{z}) = e^{-i\omega_n\tilde{\tau}-im\tilde{\phi}-ik_z\tilde{z}} \frac{J^2_{m+1}(\frac{\tilde{\rho}}{R}\kappa_{m+1,\ell})}{2\pi R|J_{m+2}(\kappa_{m+1,\ell})|}\xi^+ \qquad \xi^- = (0,\tilde{\rho},-i,0)^T$$

Eigenvalue: $$\tilde{E}^2_{nJ} = k_z^2 + \left(\frac{\kappa_{m\pm1,\ell}}{R}\right)^2 + (\omega_n + C^{ab} - im\Omega)^2$$

Matrix model in rotation

- **1-loop effective potential with gluon mass under rotation**

$$\ln Z_b^{(1)} = -\frac{1}{2}\sum_{ab}\sum_n V\sum_{\mathbf{J_1}}\ln\{\beta^2[(\omega_{n,\Omega}^{ab})^2+\mathbf{p}_{J1}^2+m_g^2]\}\left(1-\frac{1}{N_c}\delta_{ab}\right)$$
$$-\frac{1}{2}\sum_{ab}\sum_n V\sum_{\mathbf{J_2}}\ln\{\beta^2[(\omega_{n,\Omega}^{ab})^2+\mathbf{p}_{J2}^2+m_g^2]\}\left(1-\frac{1}{N_c}\delta_{ab}\right)$$

$$\sum_{\mathbf{J_1}} f(p_{T1}) = \frac{1}{2\pi^2R^2}\sum_{m=-\infty}^{\infty}\sum_{\ell=1}^{\infty}\int_{-\infty}^{\infty}dp_z\frac{J_{m-1}^2(\frac{\tilde\rho}{R}\kappa_{m-1,\ell})}{J_m^2(\kappa_{m-1,\ell})}f\left(\frac{\kappa_{m-1,\ell}}{R}\right)$$

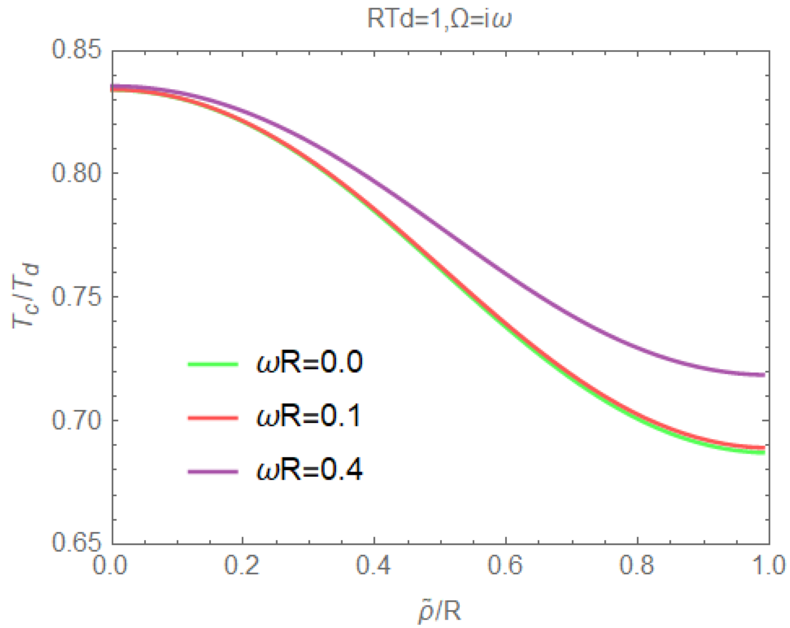
$$\sum_{\mathbf{J_2}} f(p_{T2}) = \frac{1}{2\pi^2R^2}\sum_{m=-\infty}^{\infty}\sum_{\ell=1}^{\infty}\int_{-\infty}^{\infty}dp_z\frac{J_{m+1}^2(\frac{\tilde\rho}{R}\kappa_{m+1,\ell})}{J_{m+2}^2(\kappa_{m+1,\ell})}f\left(\frac{\kappa_{m+1,\ell}}{R}\right)$$

Expanding m_g at high T, the **Matrix model under rotation is**

$$\mathcal{V}_\Omega = \frac{1}{2\pi^2R^2}\sum_{ab}\sum_{m=-\infty}^{\infty}\sum_{\ell=1}^{\infty}\sum_{n=1}^{\infty}\left\{\left[-\frac{T}{n}p_{T1}K_1(\beta n p_{T1})+\frac{m_g^2}{2}K_0(\beta n p_{T1})\right]\frac{J_{m-1}^2(\tilde\rho p_{T1})}{J_m^2(\kappa_{m-1,\ell})}\right.$$
$$\left.+\left[-\frac{T}{n}p_{T2}K_1(\beta n p_{T2})+\frac{m_g^2}{2}K_0(\beta n p_{T2})\right]\frac{J_{m+1}^2(\tilde\rho p_{T2})}{J_{m+2}^2(\kappa_{m+1,\ell})}\right\}$$
$$\times[e^{i\beta n(C^{ab}-i\Omega m)}+e^{-i\beta n(C^{ab}-i\Omega m)}]\left(1-\frac{1}{N_c}\delta_{ab}\right)$$

Numerical results

Setting the $T_d = 0.27\text{GeV}$, keep the parameter gluon mass $m_{g,d} = 0.596\text{GeV}$ at the phase transition point, $RT_d = 1$, $0 < \tilde{\rho} < R$, $R\omega < 1$.



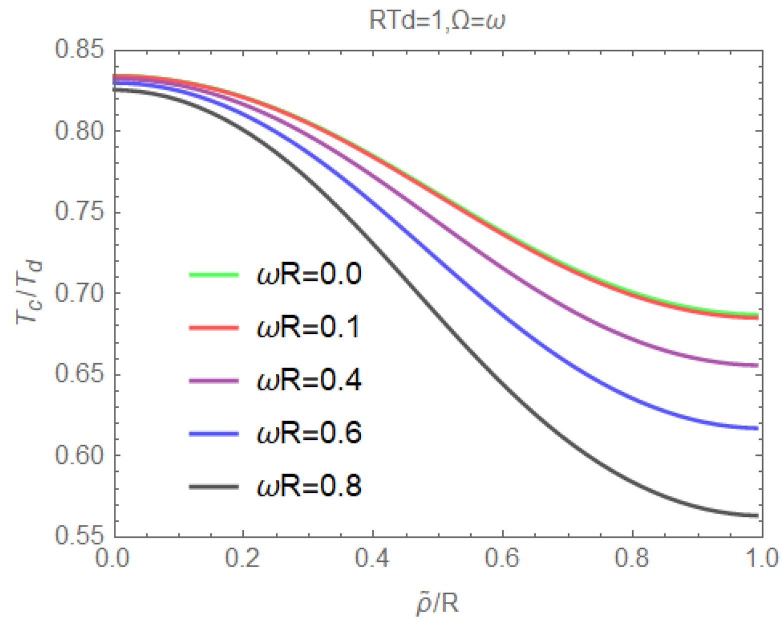


Fig. 5 Dimensionless phase transition temperature T_c/T_d as a function of $\tilde{\rho}/R$, with $\tilde{\rho}$ is the distance from the axis. The left side is the result with $\Omega = i\omega$, and the right side corresponds to $\Omega = \omega$.

1) First-order phase transition, $T < T_d$, $q = \frac{1}{3}$, $\ell = 0$;

2)Tolman-Ehrenfest (TE) law $T(r) = \dfrac{T_0}{\sqrt{1-\Omega^2 r^2}} = \dfrac{T_0}{\sqrt{1+\Omega_I^2 r^2}}$.

Numerical results

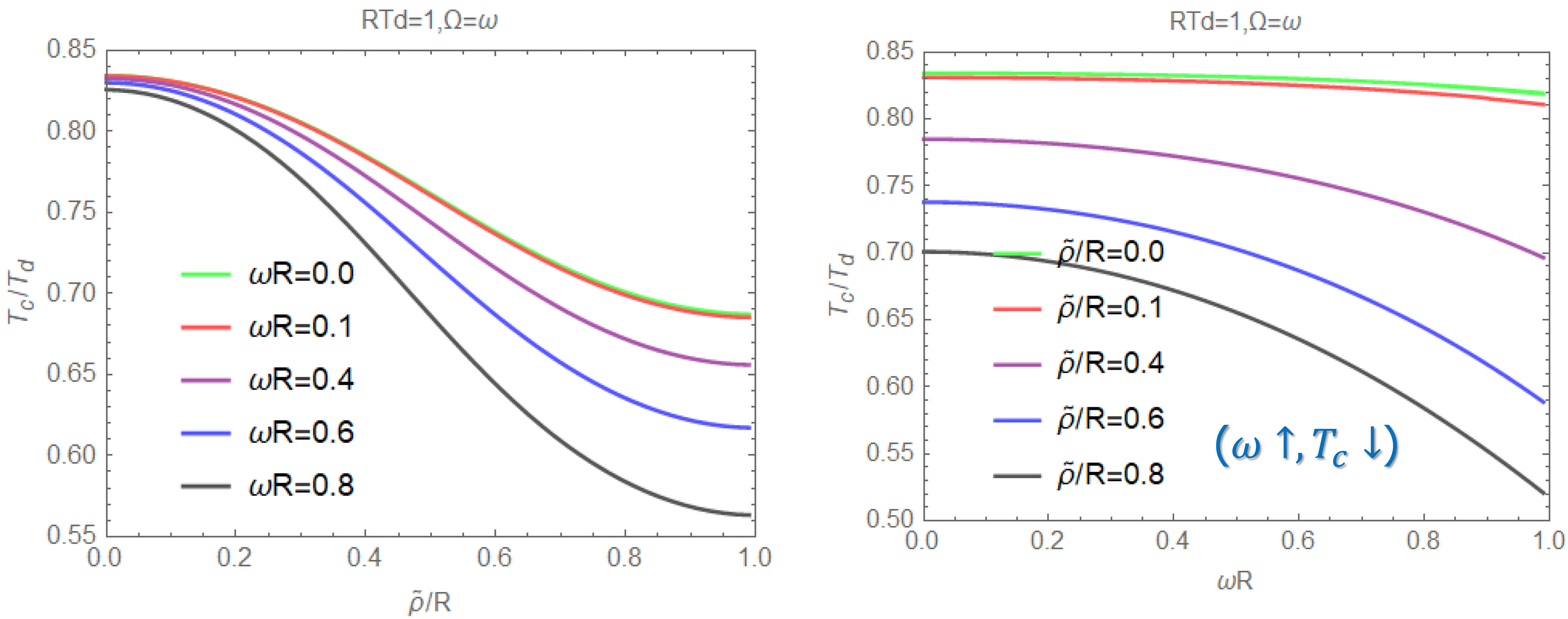


Fig. 6 Dimensionless phase transition temperature T_c/T_d as a function of $\tilde{\rho}/R$, and ωR.

1) On the axis ($\tilde{\rho}$=0), T_c remains nearly unchanged with ω, and the difference between $\omega = 0$, and $\omega R = 0.99$ is minimal. This discrepancy is likely due to numerical calculations and the selection of cutoffs for m, n, ℓ.

Numerical results

Setting the $T_d = 0.27\text{GeV}$, gluon mass $m_{g,d}(\omega) = (1 + 0.1\,|\omega|/0.28)0.596\text{GeV}$ at the phase transition point, $RT_d = 1,\ 0 < \tilde{\rho} < R,\ R\omega < 1$. Yin Jiang, Phys. Lett. B 853 (2024) 138655.

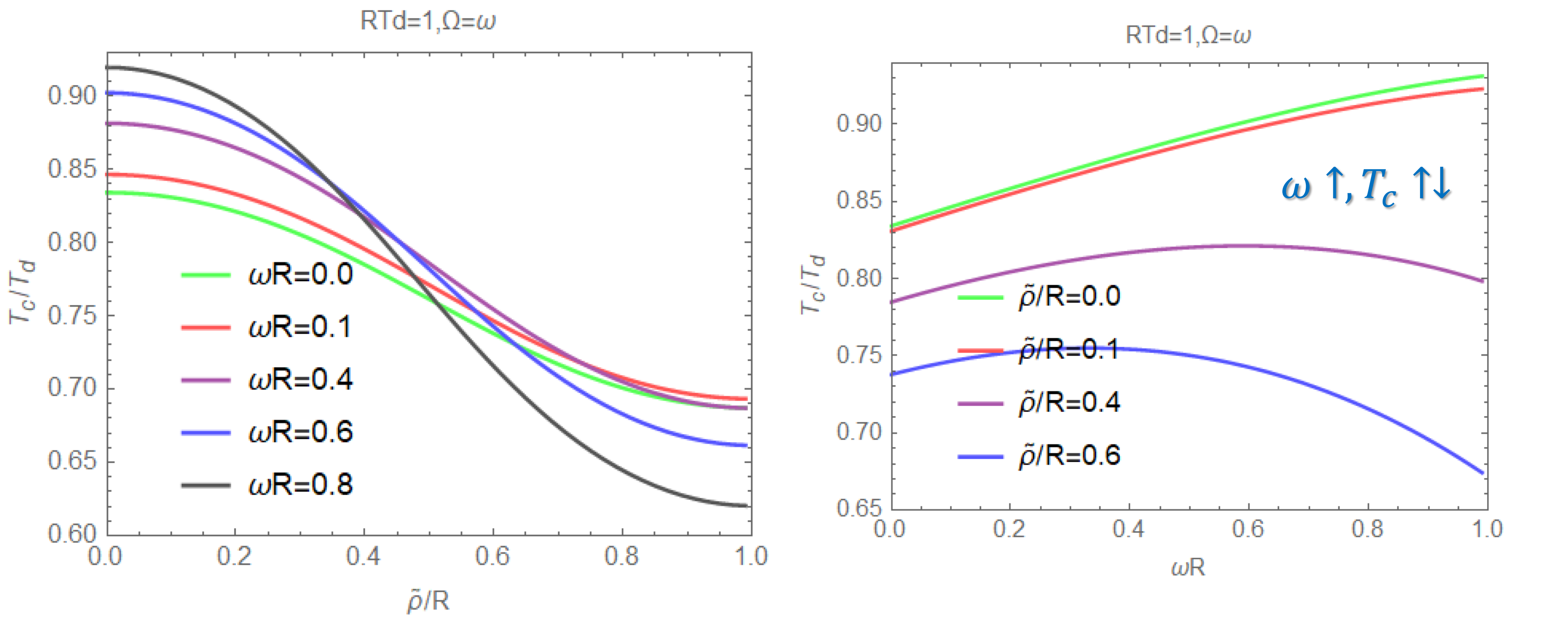


Fig. 7 Dimensionless phase transition temperature T_c/T_d as a function of $\tilde{\rho}/R$, and ωR.

1) First-order phase transition, $T < T_d$, $q = \frac{1}{3}$, $\ell = 0$;

2) Tolman-Ehrenfest (TE) law $T(r) = \dfrac{T_0}{\sqrt{1 - \Omega^2 r^2}} = \dfrac{T_0}{\sqrt{1 + \Omega_I^2 r^2}}$.

Summary

- We constructed a matrix model under rotation using the effective theory of background fields.
- We studied the deconfinement phase transition under SU(3) gauge group.

Outlook

- Figure out why the phase transition temperature at each position is still changing when setting $\omega=0$.
- Figure out why roots cannot be found for imaginary Ω when w is relatively large.
- Considering the influence of boundary effects on our results.
- Compute the bulk average result of the entire system.
- Optimize the numerical calculation methods.
- Considering the impact of rotation on thermodynamics.

Thanks for your attention!

Back up

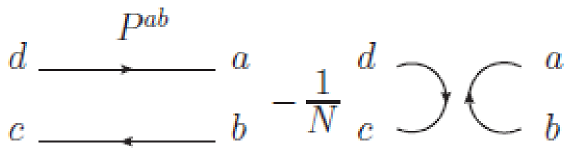
Background and Motivations

Feynman rules in the double line basis
(propagators as an example)

Quark propagator

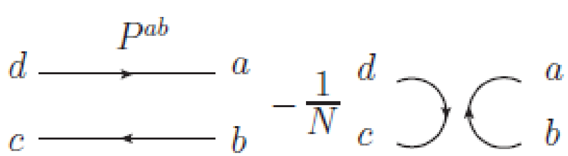
$$= \langle \psi^a(P)\bar{\psi}^b(-P)\rangle = \frac{\delta^{ab}}{-i\not{P}^a + m}$$

Ghost propagator



$$= \langle \eta^{ab}(P)\bar{\eta}^{cd}(-P)\rangle = \frac{\mathcal{P}^{ab,cd}}{(P^{ab})^2}$$

Gluon propagator



$$= \langle B^{ab}_{\mu}(P)B^{cd}_{\nu}(-P)\rangle = \left(\delta_{\mu\nu} - (1-\xi)\,\frac{P^{ab}_{\mu}P^{ab}_{\nu}}{(P^{ab})^2}\right)\frac{\mathcal{P}^{ab,cd}}{(P^{ab})^2}$$

where: $\mathcal{P}^{ab}_{cd} = \delta^a_c\delta^b_d - \frac{1}{N}\delta^{ab}\delta_{cd}$ with $a, b, c, d = 1 \ldots N$

Generators of the fundamental representation $\quad (t^{ab})_{cd} = \frac{1}{\sqrt{2}}\mathcal{P}^{ab}_{cd}$