Lecture 1 Intro to Effective Field Theories

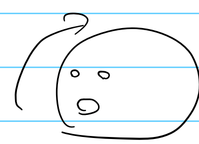
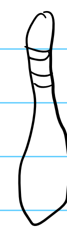
Big Picture

There is an interesting physics at all scales.

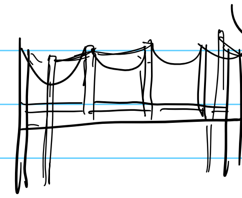
E(F)T tells you how to tease out interesting physics at particular scale

This idea of EFT is $\boxed{\text{obvious}}$

- We do not need to know the microscopic details of bowling ball when we play bowling. Just that it has certain mass, and that it spins.

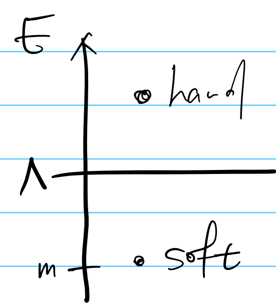



- Engineers do not need to know the mass of Higgs boson or strong interactions when constructing a building, only certain macroscopic properties, like elasticity of steel.



Decoupling Theorem

To describe physics at an IR scale m, we do not need to know the detailed dynamics of what is going w/ UV scale



Beta Function In QCD, resolution scale μ of a process plays a very important role

$$\sim g_s \qquad \frac{g_s^2(\mu)}{4\pi} = \alpha_s(\mu)$$

$-\alpha_s$, parameter of strong coupling constant that tells us about the "strength" of strong interactions

How strong do qks & gluons interact?

$$\mu \frac{d}{d\mu} \alpha_s(\mu) = \underbrace{\beta[\alpha_s]}_{\boxed{\text{beta-function}}} = \frac{-\beta_0 \alpha_s(\mu)^2}{2\pi} + \ldots$$

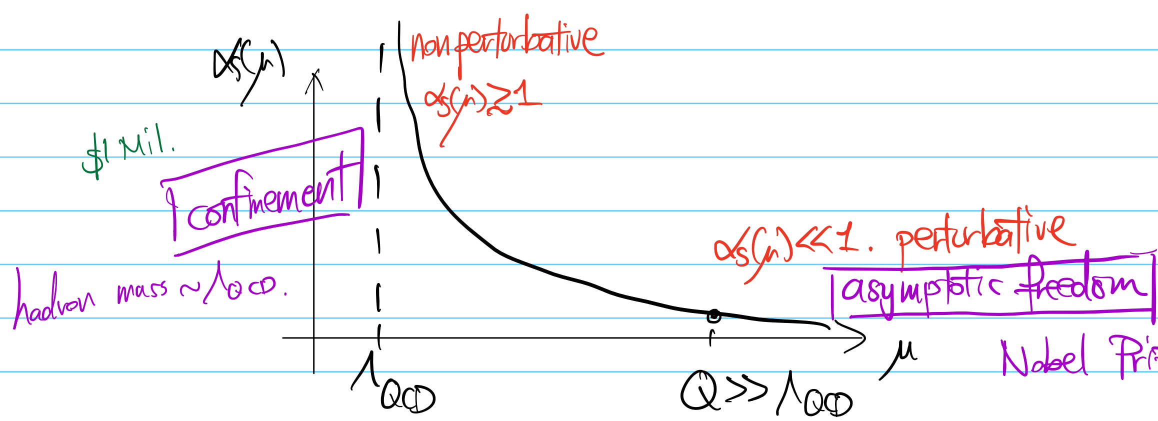
In general, $\frac{d}{d\ln\mu^2} a_s = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + \ldots$

$$\beta_0 = \frac{11}{3} C_A - \frac{2}{3} n_f = 11 - \frac{2}{3} n_f \geq 0 \text{ if } n_f \leq 16.$$

strong coupling constants decrease with energy scale.

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{\beta_0}{2\pi}\alpha_s(\mu)\ln\frac{\mu}{\mu_0}} = \frac{2\pi}{\beta_0 \ln\frac{\mu}{\Lambda_{QCD}}}, \qquad \Lambda_{QCD} \sim 0.25 \text{ GeV}$$

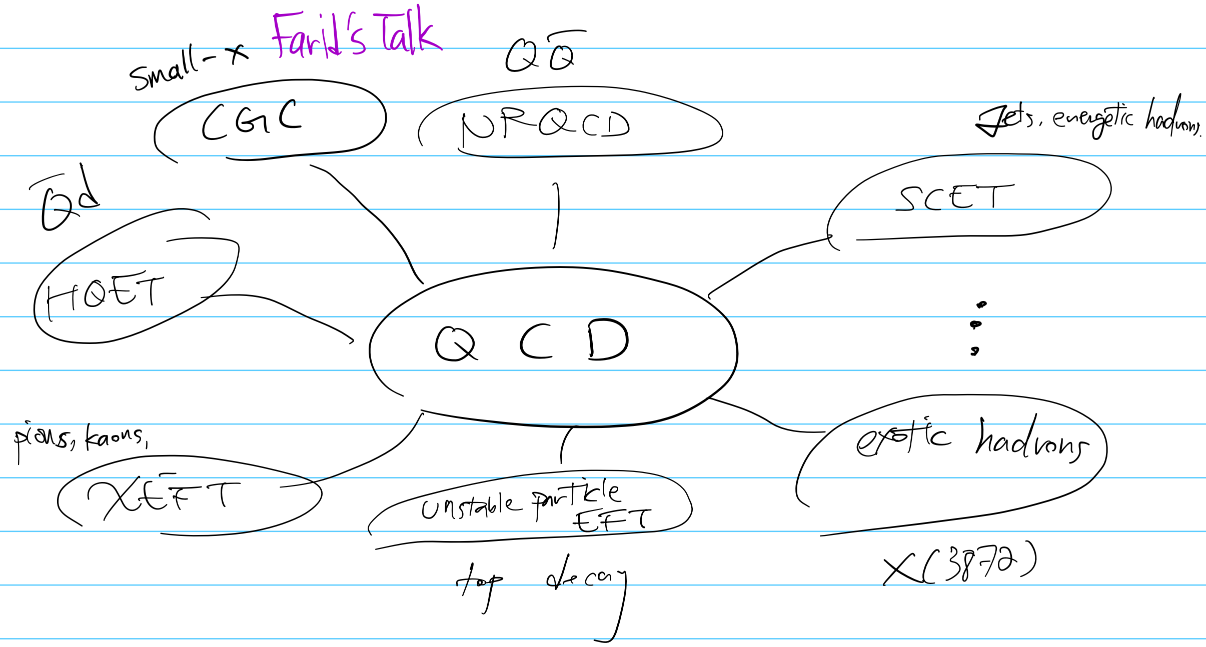
dimensional transmutation



Nobel Prize 2004

QCD and Effective Theories

QCD is the richest known QFT of nature:

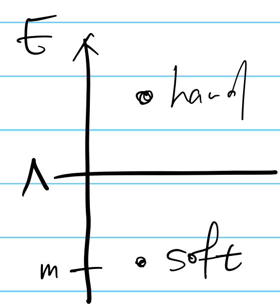


top-down and bottom-up EFTs

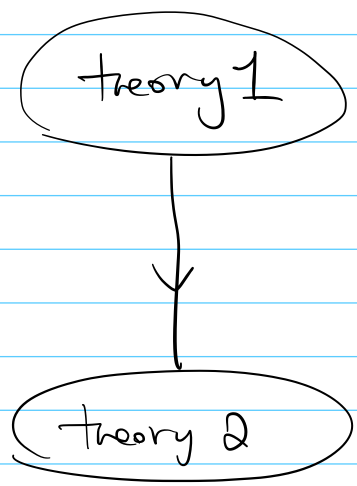
$$\mathcal{L} = \sum_i C_i(\Lambda)\, O_i(m,\Lambda)$$

long-distance physics

encodes short-distance detail that is matched indep of long distance dynamics.



top-down:



Theory 1 is known, but one "integrates out" the undesired details to construct theory 2 to describe the IR details.
(or at same energy theory w/ not all d.o.f)

NRQCD, SCET, HQET

bottom up: underlying theory is known or is known but matching is too difficult.

So construct them out of expected symmetries and d.o.f. ⇒ writing down most general operators

Chiral perturbation theory, CGC

can fix coef. by experiment, for ex.

Fundamental ingredients of EFT

a) Relevant d.o.f ⇒ what fields?

b) Symmetries ⇒ what interactions?

c) Systematic expansion of EFT ⇒ power counting, expansion parameters.

simple example of top down EFT

Take a theory w/ a heavy scalar ϕ of mass M and light fermion of mass m.

$$\mathcal{L}_{\text{theory}1} = \bar{\psi}(i\not{\partial} - m)\psi + \frac{1}{2}\left[(\partial_\mu\phi)^2 - M^2\right] + g\phi\bar{\psi}\psi$$

with $m^2 \ll M^2$

theory 1

$\downarrow$

theory 2

Let us "integrate out" heavy scalar ϕ and only consider physics at the scale $\ll M$.

$$\mathcal{L}_{\text{theory } 2} = \bar{\psi}(i\cancel{\partial} - m)\psi + \frac{a}{M^2}\underbrace{(\bar{\psi}\psi)^2}_{\text{dim-6 operator.}} + \cdots$$

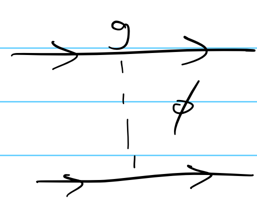
d.o.f

- ψ is the d.o.f for the lower energy theory.

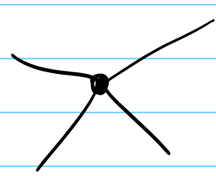
"a" is the coefficient which contains information about the short-distance dynamics.

In the IR, the two theory needs to agree: i.e. their S-matrix needs to be the same.

At tree-level

Theory 1: 

$$= \frac{i(ig)^2}{q^2 - M^2} = \frac{ig^2}{M^2}\left(1 + \mathcal{O}\left(\frac{q^2}{M^2}\right)\right)$$

Theory 2: 

$$= \frac{ia}{M^2}$$

Thus, $a = g^2$; tree-level matching

In this classic example, we integrate out the high energy modes $\sim M$ are integrated out relative to the low-energy mode $\sim q$

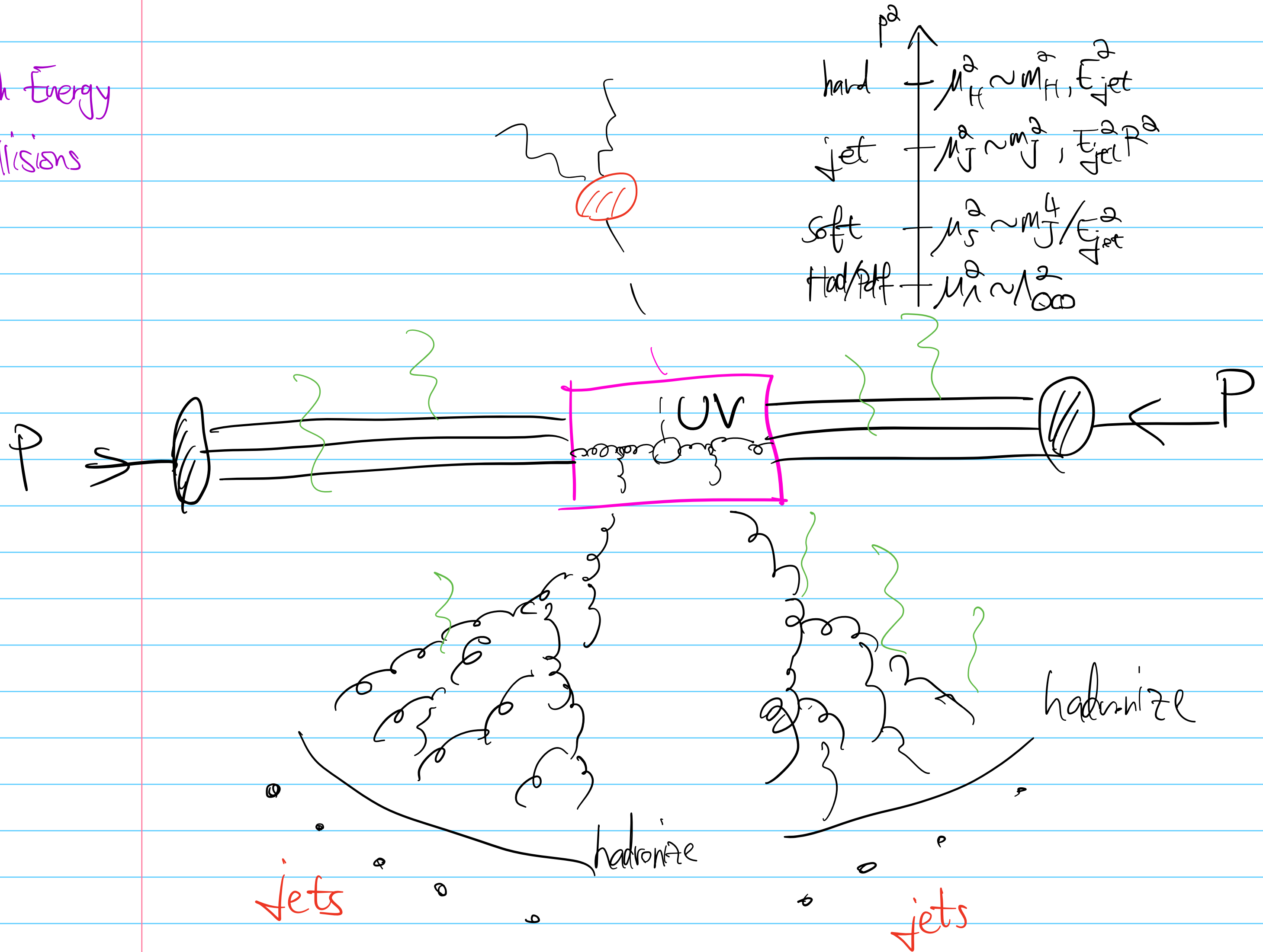
Thus, expansion parameter $\lambda = \frac{q}{M}$ } P.C.

One can systematically construct EFTs

$$\mathcal{L} = \mathcal{L}^{(0)} + \lambda \mathcal{L}^{(1)} + \lambda^2 \mathcal{L}^{(2)} + \ldots$$

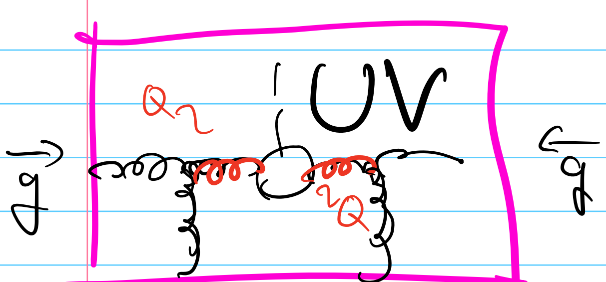
$$\mathcal{L}^{(n)} = \sum_i \underset{\text{UV}}{C_i^{(n)}} \underset{\text{IR}}{\mathcal{O}_i^{(n)}} .$$

High Energy Collisions



$$\delta x_{had} \sim \frac{\hbar}{1\,\text{GeV}} \sim 10^{-15}\,m$$

$$\delta x_{hard} \sim \frac{\hbar}{1\,\text{TeV}} \sim 10^{-18}\,m$$



Hard scattering is quite rare and usually appear only once

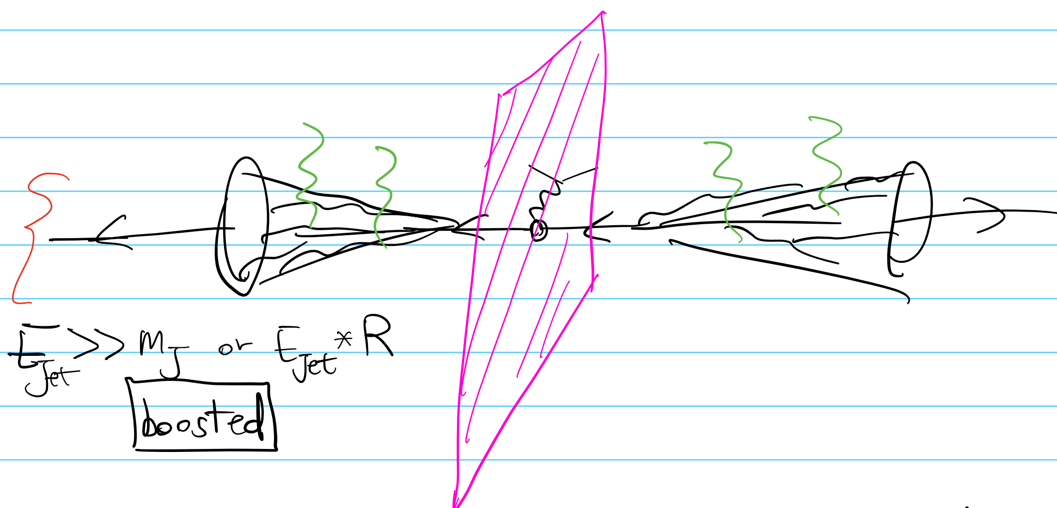
Lecture 2 Intro to Soft-Collinear Effective Field Theory

- What are the d.o.f of SCET, EFT for jets?
- " symmetries " "
- " power correction parameters " "

Today, we discuss d.o.f.

dijet production

Illustrative example is to consider $e^+e^- \to$ dijets



$E_{Jet} \gg m_J$ or $E_{Jet} * R$

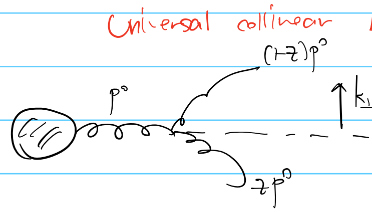
$\boxed{\text{boosted}}$

Collinear & Soft radiations are enhanced.

- Landau equation (pinch surfaces, etc).
- Method of Regions.

Universal collinear behavior of the scattering amplitude.

collinear/soft emissions are enhanced



$$\frac{|A_{N+1}(\cdots)|^2}{|A_N(\cdots)|^2} \simeq \frac{\alpha_s C_F}{\pi}\frac{dk_\perp}{k_\perp}dz\, P_{gg}(z), \quad \text{where} \quad P_{gg}(z) = \frac{z}{1-z} + \frac{1-z}{z} + z(1-z)$$

prob. of these collinear splitting.

strong enhancement in $k_\perp \to 0$ and $z \to 0, 1$.

Aside: Light-Cone Coords

basis vectors $n^\mu, \bar{n}^\mu$ ← auxilary

with $n^2, \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$

Notation

$p^+ = n \cdot p$, $p^- = \bar{n} \cdot p$

vectors $$p^\mu = \frac{n^\mu}{2} \bar{n} \cdot p + \frac{\bar{n}^\mu}{2} n \cdot p + p_\perp^\mu = \frac{n^\mu}{2} p^- + \frac{\bar{n}^\mu}{2} p^+ + p_\perp^\mu$$

$$p^2 = n \cdot p \, \bar{n} \cdot p + p_\perp^2 = p^+ p^- + p_\perp^2 = p^+ p^- - \vec{p}_\perp^{\,2}$$

metric $$g^{\mu\nu} = \frac{n^\mu \bar{n}^\nu}{2} + \frac{\bar{n}^\mu n^\nu}{2} + g_\perp^{\mu\nu}$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\alpha\rho} \frac{\bar{n}_\alpha n_\rho}{2}$$

- $n^2 = 0$ requires complementary vector $\bar{n}^\mu$ for decomposition (dual vector orthogonality)

choice $n^\mu = (1,0,0,1)$, $\bar{n}^\mu = (1,0,0,-1)$ works.

note: Farid's convention $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$, here $x^\pm = x^0 \mp x^3$.

Thus, jet momenta $p^\mu \sim 2E_{Jet}\, n^\mu$ + smaller components.

where $n^\mu = (1, \hat{n})$, $n^2 = 0$.

small mass measurement

$$p^2 = p^+ p^- - \vec{p}_\perp^{\,2} \sim m_J^2 \Rightarrow p^\mu \sim \left(\frac{m_J^2}{p^-}, p^-, m_J\right)$$

$$\sim p^- \left(\left(\frac{m_J}{p^-}\right)^2, 1, \frac{m_J}{p^-}\right)$$

$$\sim p^- (\lambda^2, 1, \lambda)$$

collinear mode

Jet w/ small jet radius R

$$p^2 \sim p^{-2}(1-\cos R) \sim p^{-2}R^2 \sim m_J^2 \Rightarrow p^\mu \sim p^-(R^2, 1, R)$$

collinear mode

⇒ similar collinear mode for the jet in the opposite direction.

$\boxed{\text{soft mode}}$ soft radiation follows homogeneous scaling

$$p_s^\mu \sim p^-(\lambda^\kappa, \lambda^\kappa, \lambda^\kappa)$$

jet mass measurement

$$p^2 = (p_c+p_s)^2 \sim \underset{\sim\lambda^2}{p_c^2} + \underset{\sim\lambda^{2\kappa}}{p_s^2} + \underset{\sim\lambda^{2+\kappa}}{p_c^+ p_s^-} + \underset{\sim\lambda^{\kappa}}{p_c^- p_s^+} + \underset{\sim\lambda^{1+\kappa}}{\vec{p}_{c\perp}\cdot\vec{p}_{s\perp}} \sim \underset{\sim\lambda^2}{m_J^2}$$

power-suppressed.

$\Longrightarrow \kappa = 2.$ Thus, $p_s^\mu \sim p^-(\lambda^2, \lambda^2, \lambda^2)$

"ultrasoft" mode.

- soft mode also exists for small-R dijet production, but story is more complicated (non-global logarithms, global-soft, soft-collinear mode etc.)

Expected d.o.f depends on the IR physics we want to probe.

In $\boxed{\text{jet mass}}$ $m_J^2 = p^2 = (p_n + p_s)^2 \sim p_s^+ p_n^-$

we see that '+' component soft momenta talks to n-coll jet mass.
'−' " " $\bar{n}$-coll " .

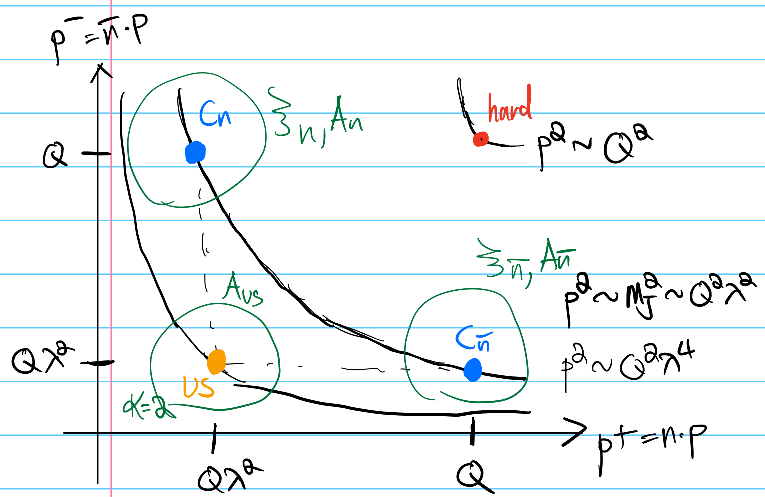
$$
\begin{array}{llll}
 & + & - & \perp \\
p_n \sim p^- (& \lambda^2, & 1, & \lambda) \\
p_s \sim p^- (& \lambda^2, & \lambda^2, & \lambda^2) \\
p_{\bar{n}} \sim p^- (& 1, & \lambda^2, & \lambda)
\end{array}
\qquad \lambda \sim \frac{m_J}{p^-}
$$

Can $\alpha = 1$? i.e. $p_s \sim Q(\lambda, \lambda, \lambda)$

$\boxed{\text{Jet broadening}}$ $\rho = \sum_{i \in \text{Jet}} |p_{i\perp}| \sim p_{s\perp} + p_{n\perp} \sim Q\lambda$.

(jet mass)

SCET$_I$



- many scenarios are covered by these two type of cases.

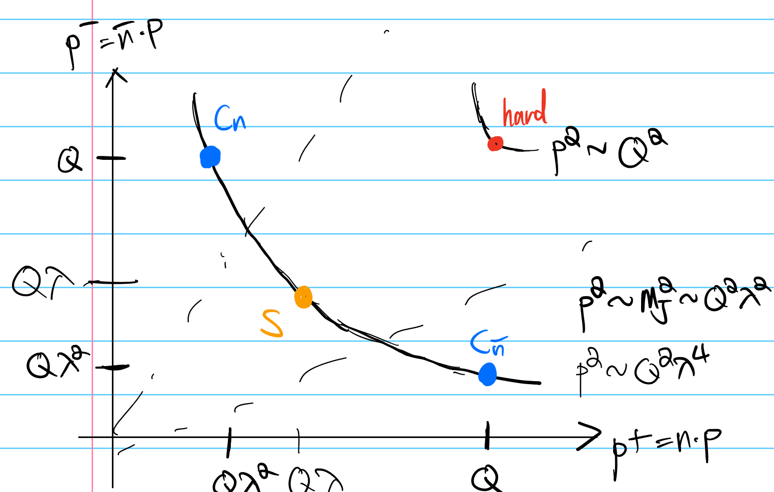
"jet mass is SCET$_I$ observable"

"jet broadening is SCET$_{II}$ observable".

- multiple fields needed for same particle

(jet broadening)

SCET$_{II}$



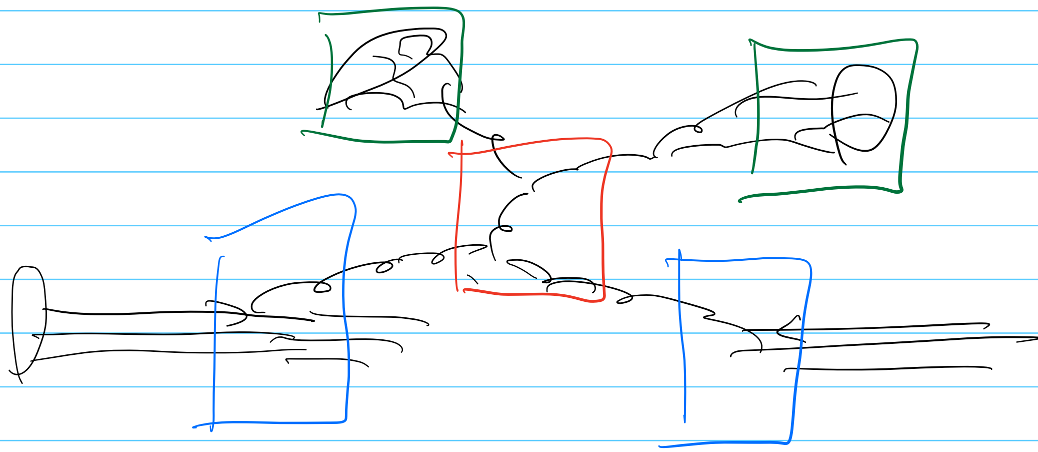
- many modes in same invariant mass scale. Thus different than the usual "simple" picture of integrating out higher energy modes.

$$m \ll \Lambda \qquad \Lambda,\ m$$

- Why use SCET?
  SCET has multiple copies of the QCD $\mathcal{L}$.
  It simplifies the "interaction" between the IR modes

QCD factorization

How is many different dynamics present in QCD going to talk to each other and factorize?



$d\sigma \sim$

$$d\sigma \sim \int dx_a\, dx_b\, \boxed{f_g(x_a,\mu)}\,\boxed{f_g(x_b,\mu)}\,\boxed{d\hat{\sigma}_{gg\to gg}(x_a,x_b,z_c,z_d,\mu)}\,\boxed{J_g(z_c,\mu)}\,\boxed{J_g(z_d,\mu)}$$

different dynamics of QCD can be separated, "factorized".

SCET automates many of these procedures.

SCET Lagrangian

$$\mathcal{L}_{SCET} = \mathcal{L}_{hard} + \mathcal{L}_{dyn}$$

Interactions between different sectors (collinear sectors or collinear-soft) which cause hard scattering

part which describes interactions between 1-collinear or ultrasoft-collinear sectors.