CRIME SCENE INVESTATION AT JUNO

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OUTLINE

- ❖Introduction: what/why/how…
- ❖Reconstruction of reactor anti-neutrinos
  - ❖PMT waveform
  - ❖Energy
  - ❖Vertex
- ❖Reconstruction of atmospheric neutrinos
  - ❖Directionality
  - ❖Particle Identification
- ❖Summary

Ballistics test in THE DARK KNIGHT

1

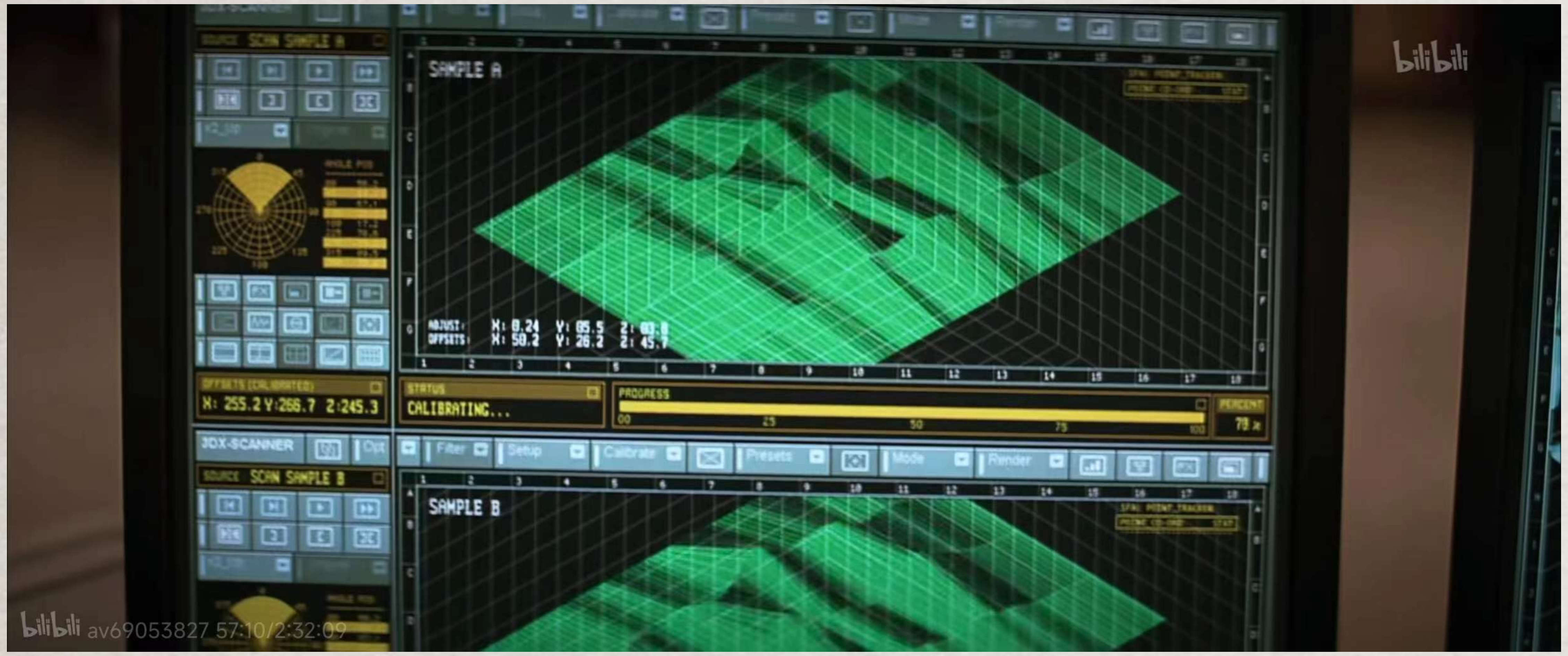
2

3

4

Ballistics test in THE DARK KNIGHT

5



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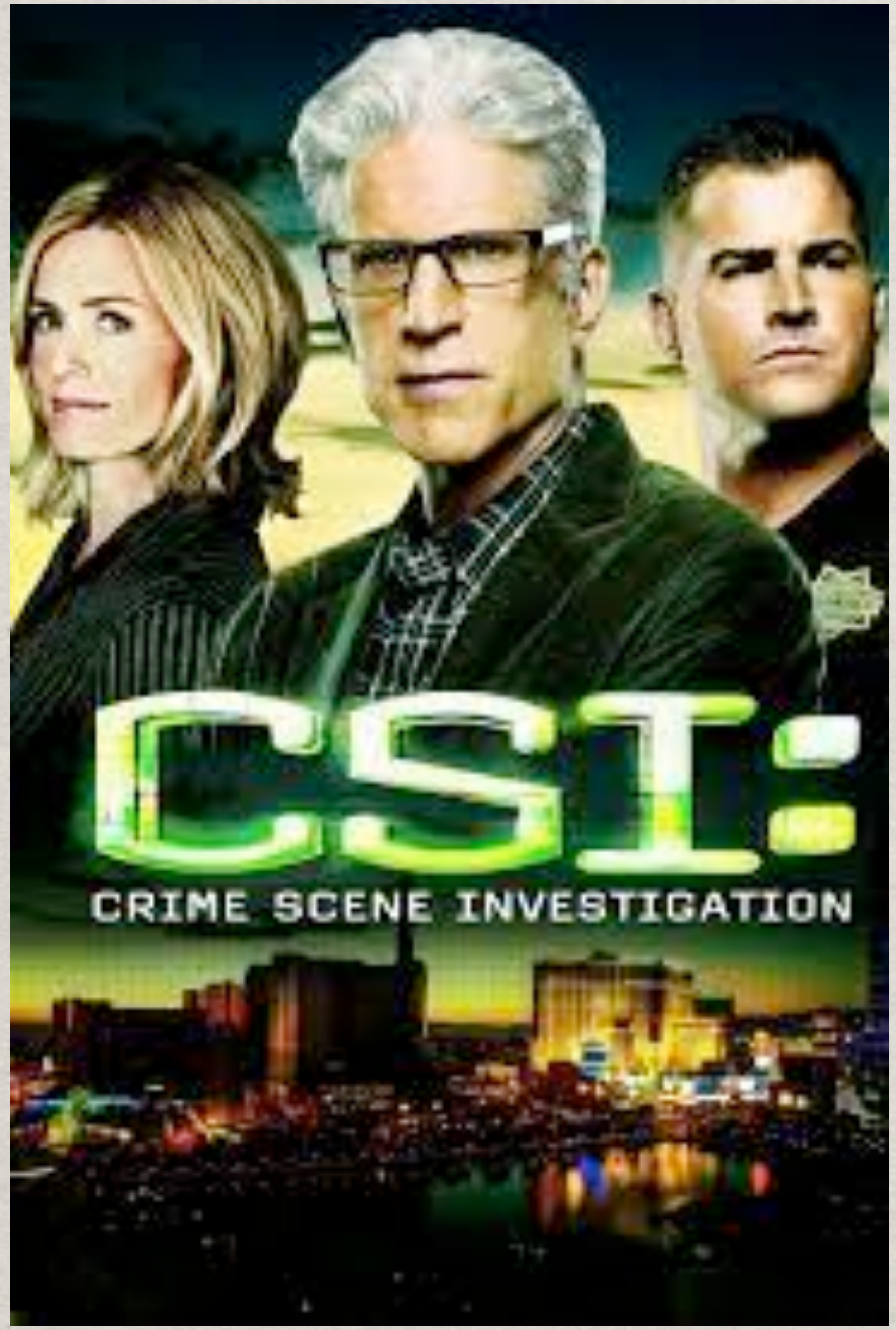
RECO IS COOL



height
weight
skin color
…

⟷

Particle?

energy
type
…

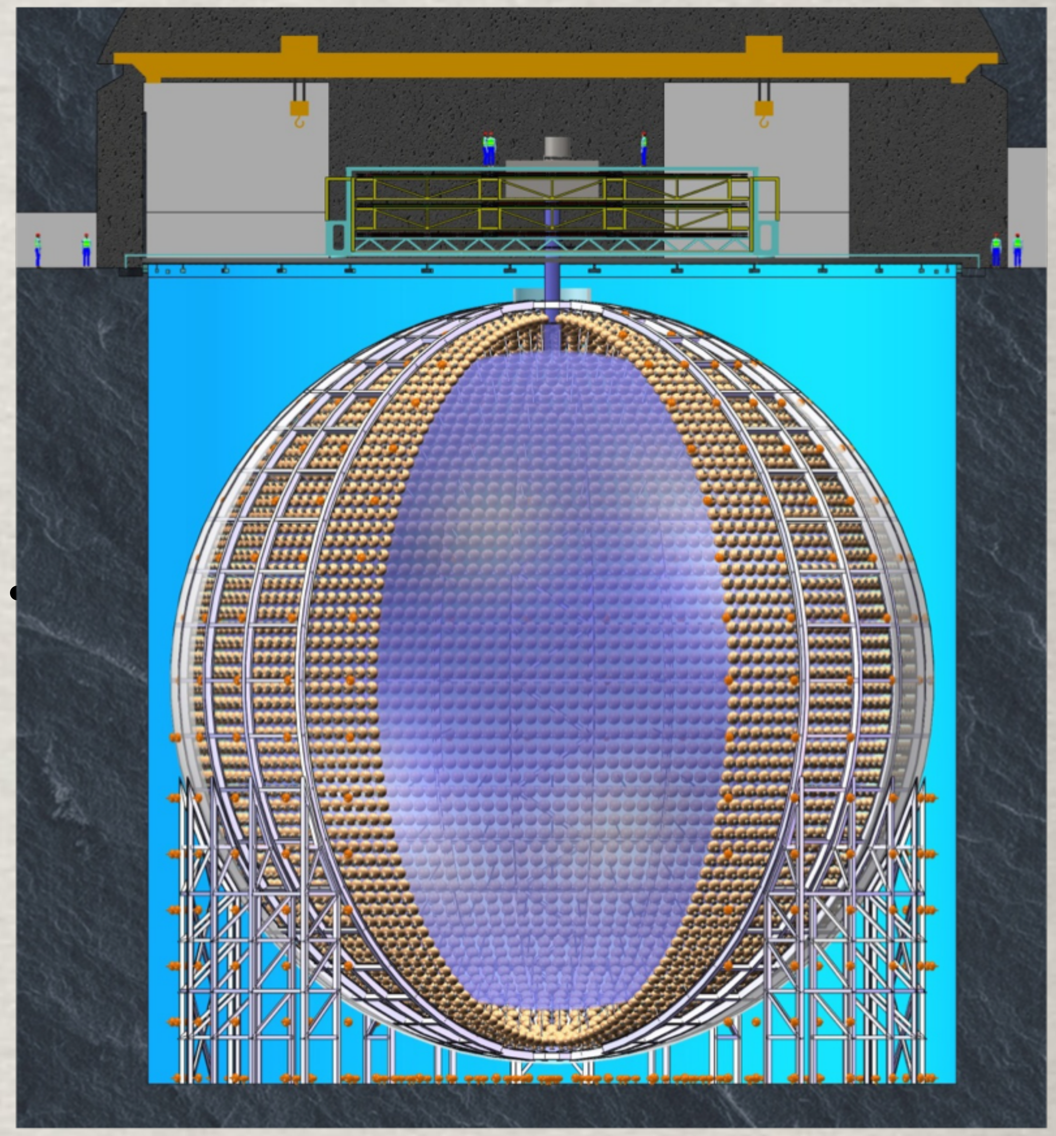
where ⟷ vertex/track

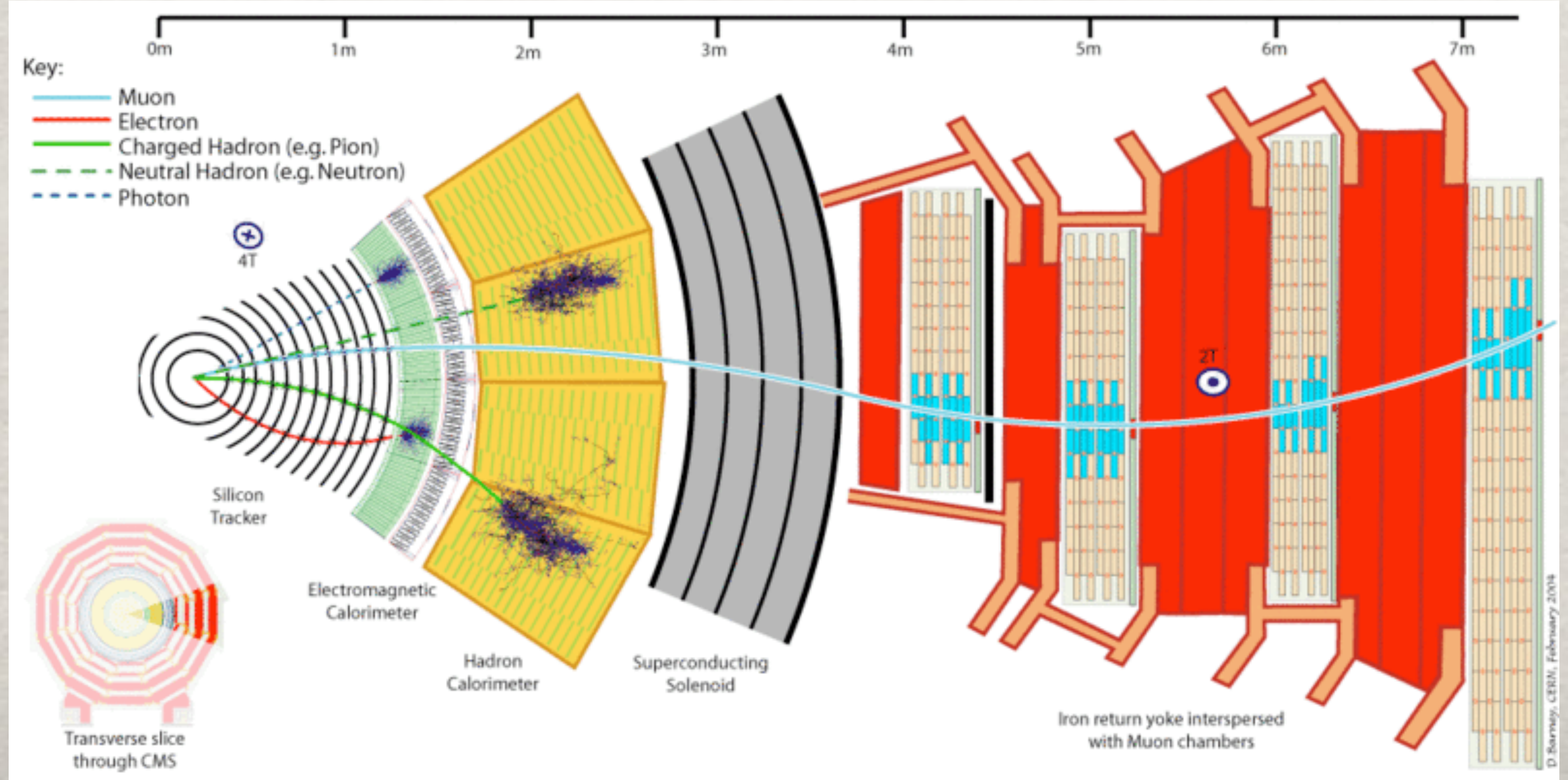
when ⟷ time

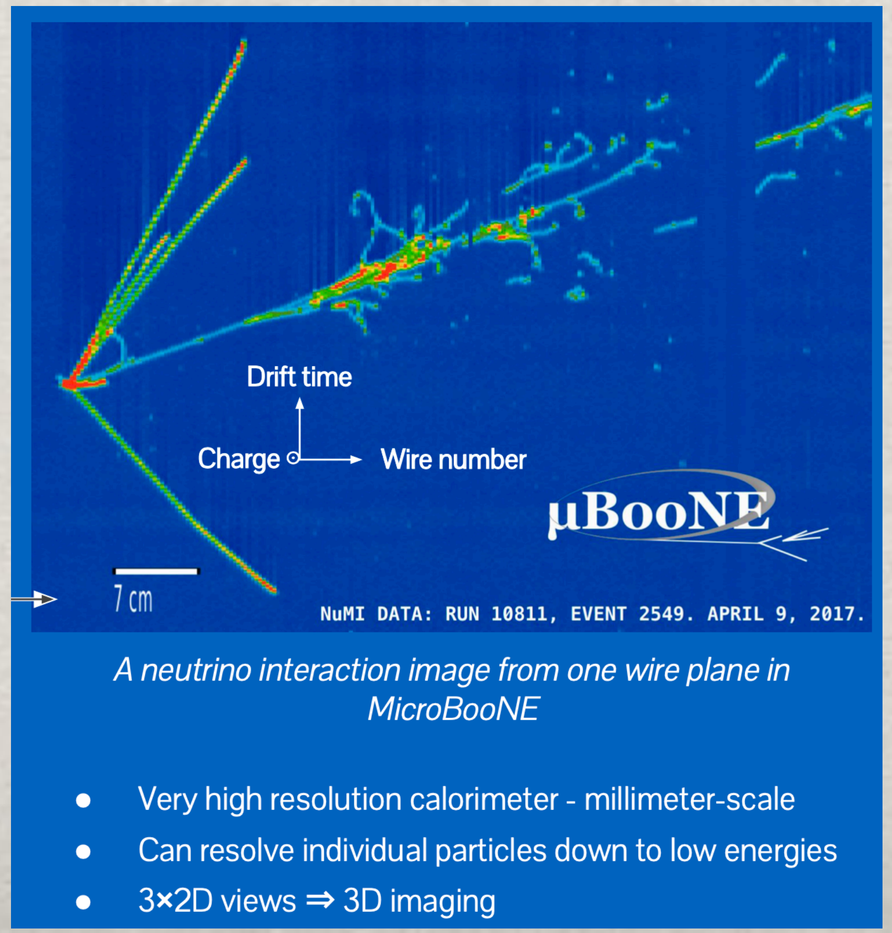
what ⟷ physics process

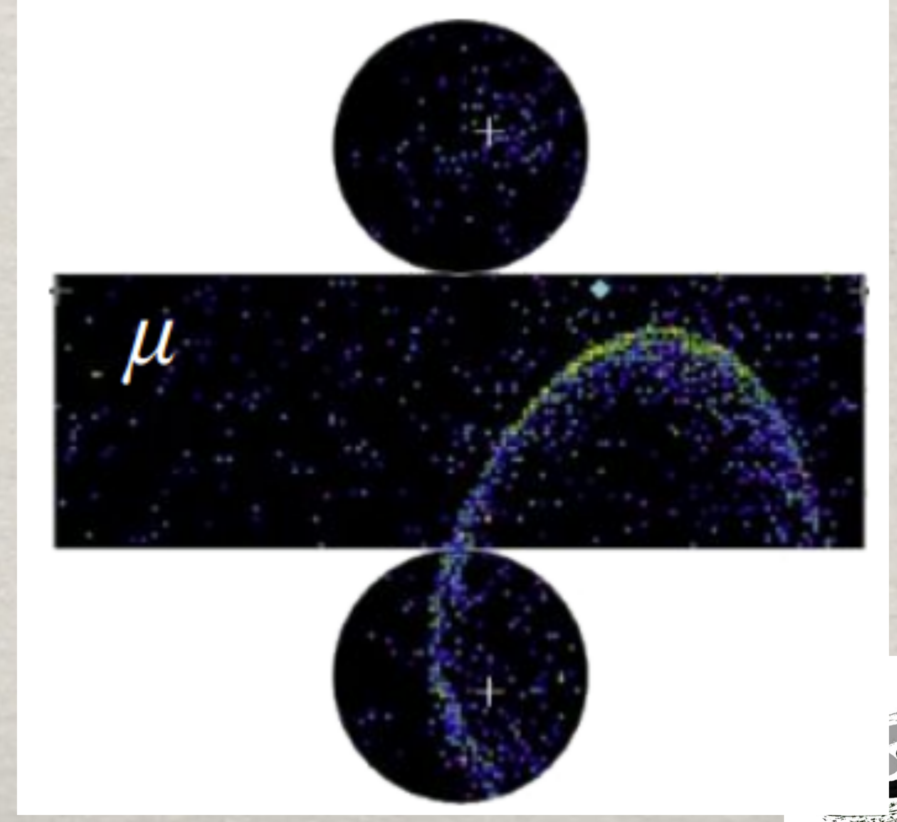
RECO IS CHALLENGING

- CSI@JUNO objectives:
  - Identify particle type: e⁺, α, β, γ, neutron/proton, μ…
  - Determine particle properties: position, energy, track, direction…
- Know yourself, Liquid Scintillator detector
  - pros: low energy threshold, high energy resolution and ? …
  - cons: unsegmented, neither track info, nor Cherenkov rings…







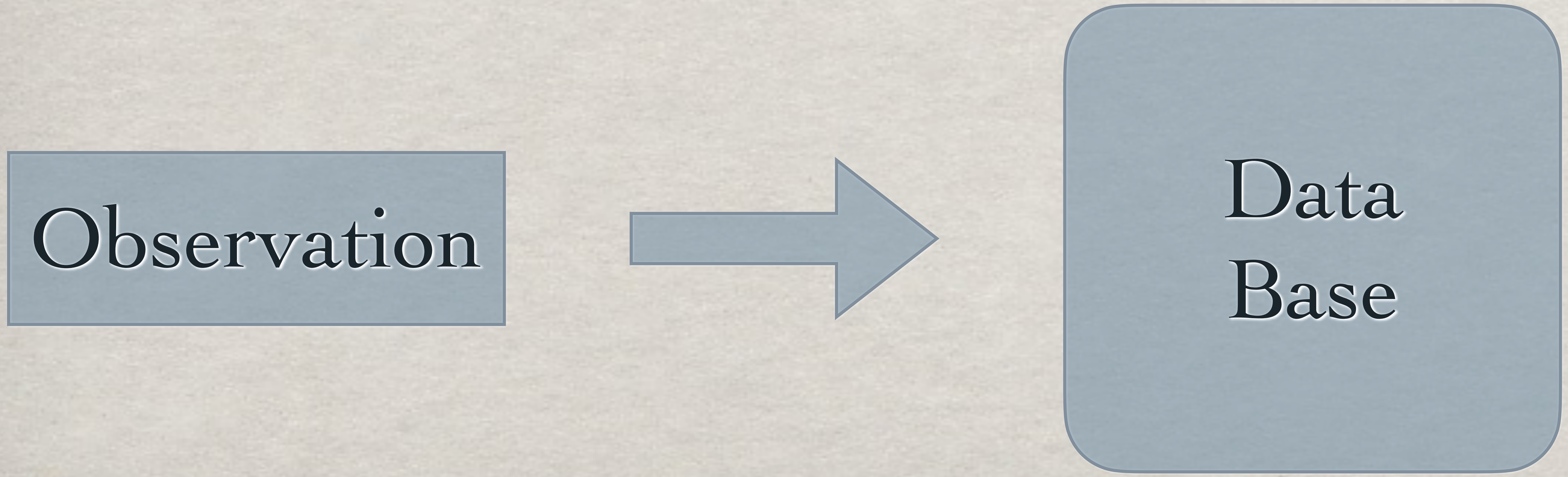


RECO IS CHALLENGING

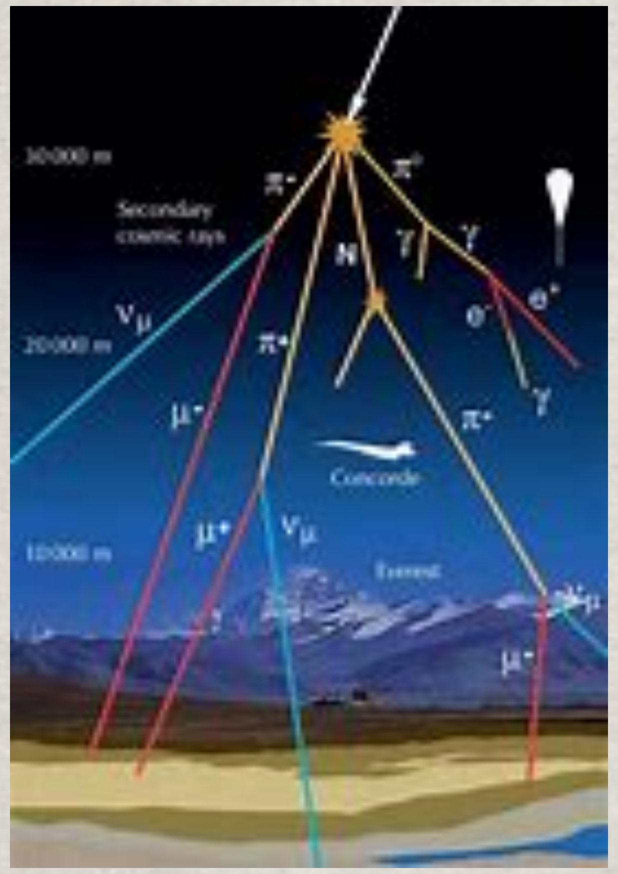
- CSI@JUNO objectives:
  - Identify particle type: e^+, α, β, γ, neutron/proton, μ…
  - Determine particle properties: position, energy, track, direction…
- Know your suspect, particle behavior in JUNO:
  - charged particles deposit energy and emit scintillation photons, together with negligible Cherenkov light
  - particle topology: point/ball-like source(MeV region); track or shower(GeV region)
  - ~~multiple crime scenes <—> coincident signals~~

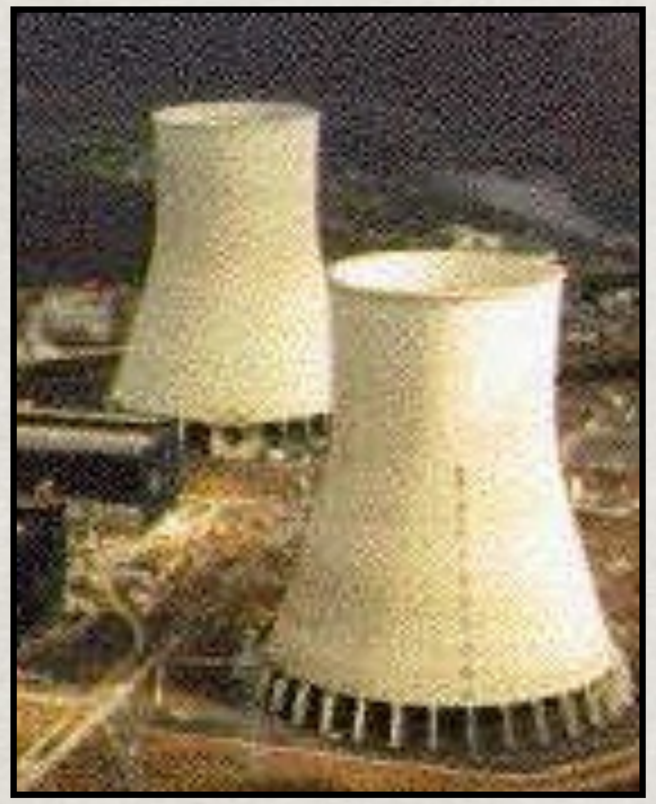
THE QUANTUM WORLD

- ❖For the Macro world: distinctive/unique evidence(fingerprint/DNA…)
- ❖For the Micro world: One to Many due to the quantum nature
  - ❖"identical particle" —> different detector signal
  - ❖fixed detector signal —> could originate from different particles
- ❖Strategies: matching, likelihood method…



RECO IS IMPORTANT





Neutrino Mass Ordering

reactor $\bar{\nu}$
MeV

- PMT waveform
  - non-uniformity
  - PMT dark noise
  - PMT charge smear
  - ^{14}C pileUp
- energy
- vertex

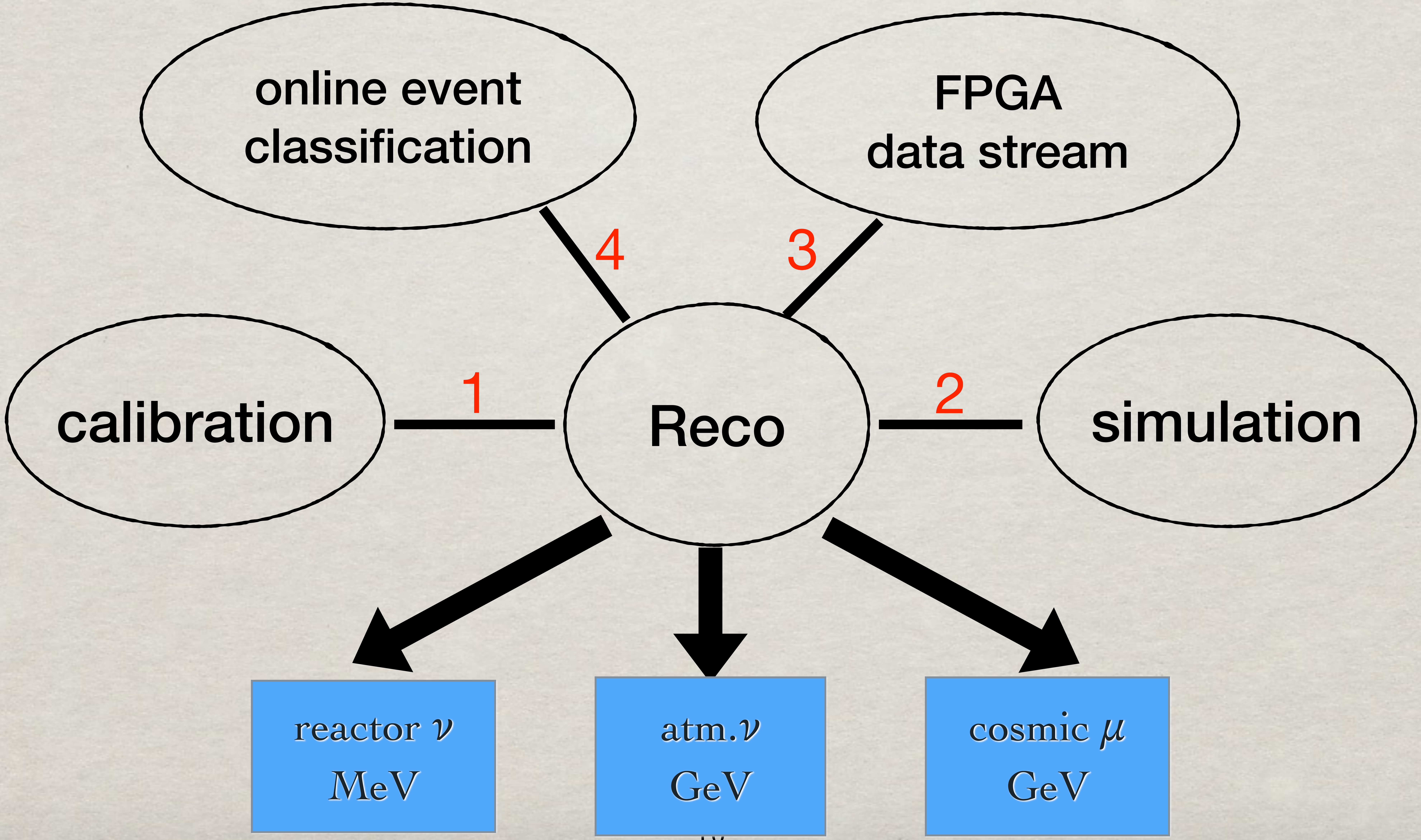
atm.ν
GeV

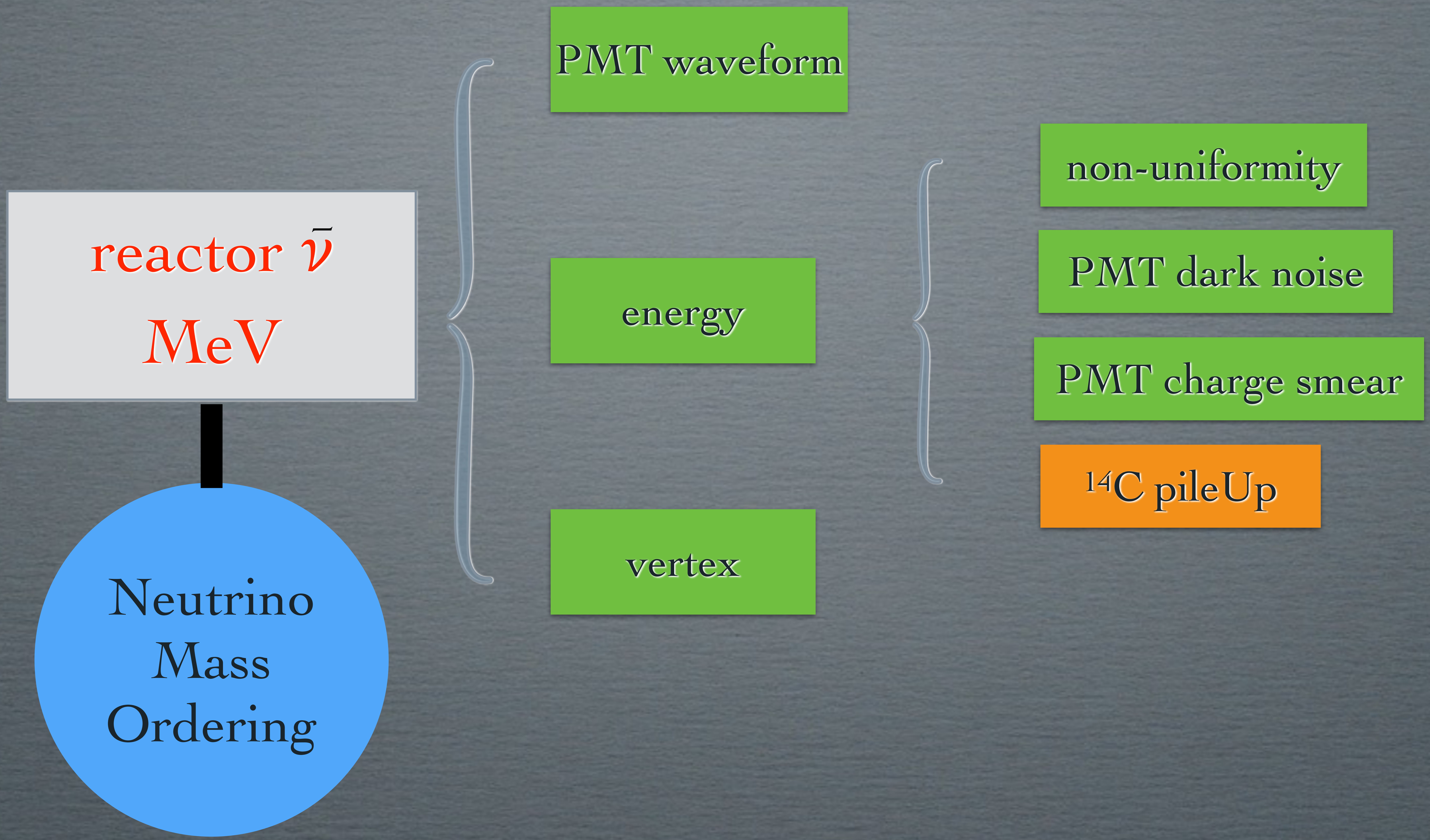
- Direction
- Particle ID
  - ν_μ vs ν_e vs NC
  - ν vs $\bar{\nu}$
- Energy

cosmic μ
GeV

- muon classification
- multi-detector combined reco
- track
- shower vertex

RECO IS IMPORTANT



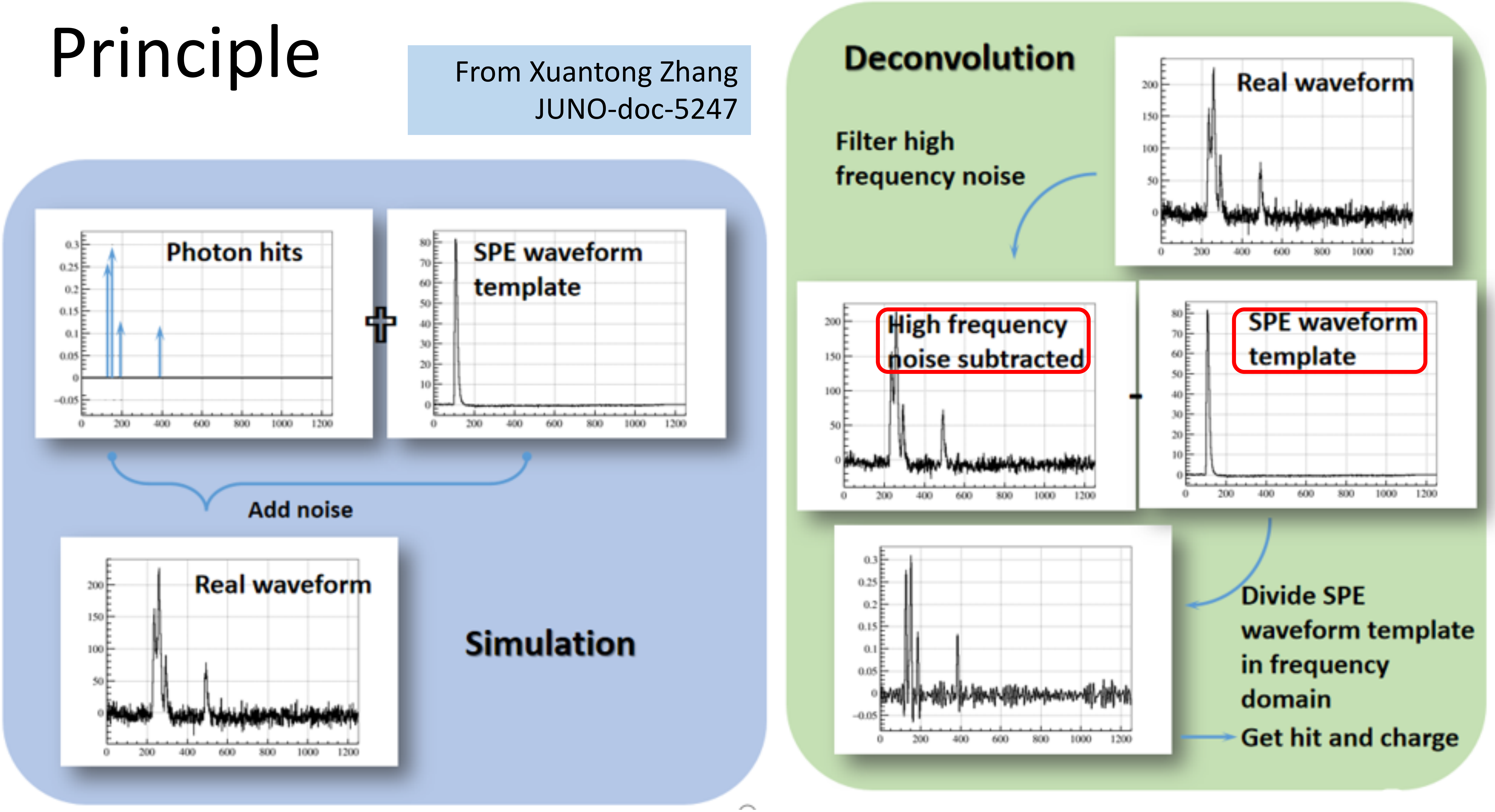


reactor $\bar{\nu}$
MeV
Neutrino Mass Ordering
PMT waveform
energy
vertex
non-uniformity
PMT dark noise
PMT charge smear
^{14}C pileUp

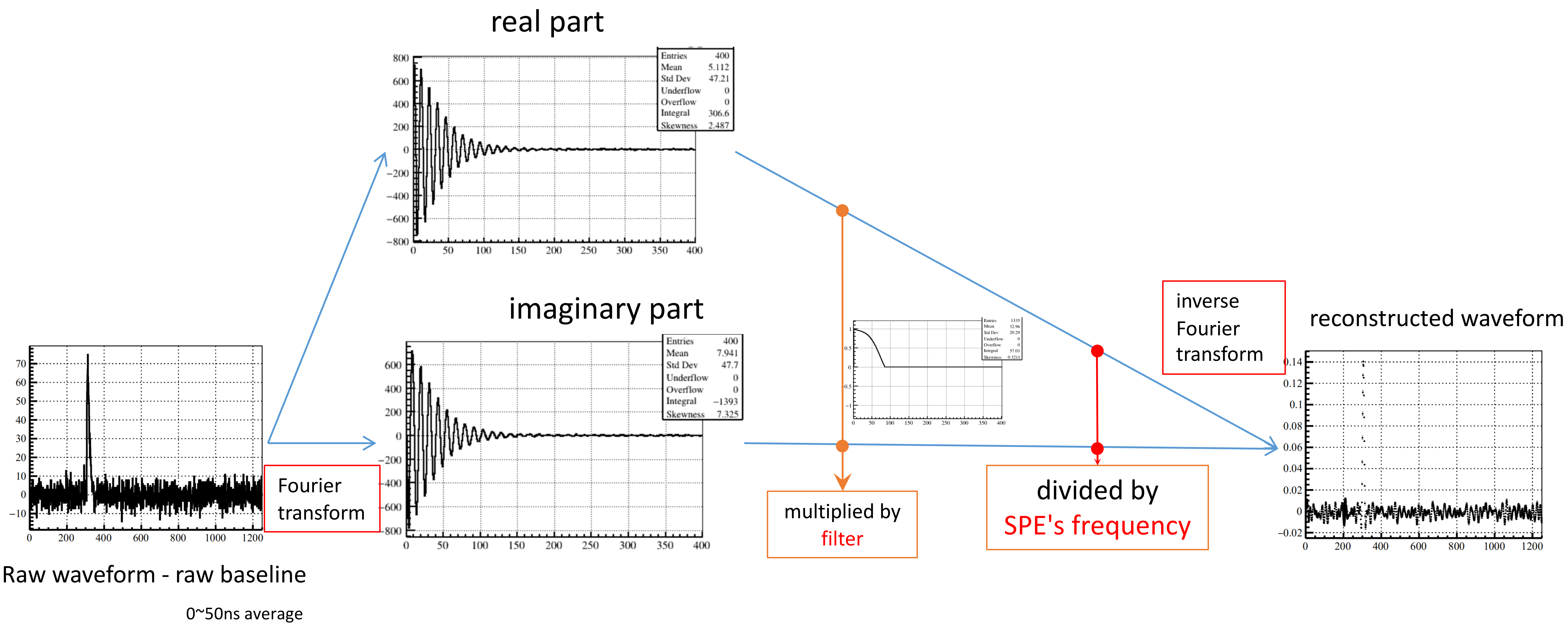
PMT WAVEFORM RECO

Principle

From Xuantong Zhang
JUNO-doc-5247



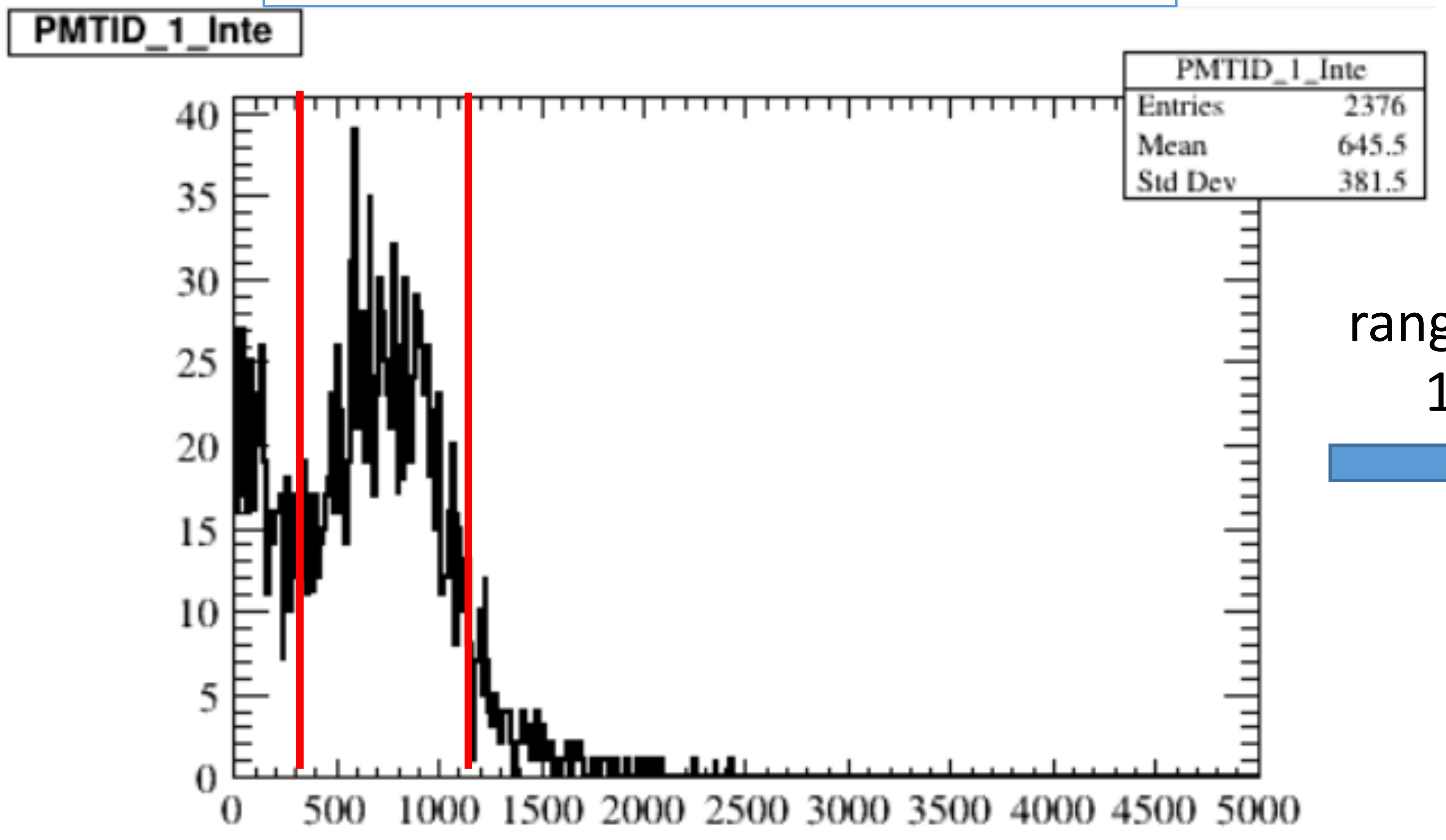
Deconvolution Workflow



Extract SPE waveforms

- Not all waveforms from calibration sources are of SPE, but for each PMT, its 1PE charge distribution is certain.

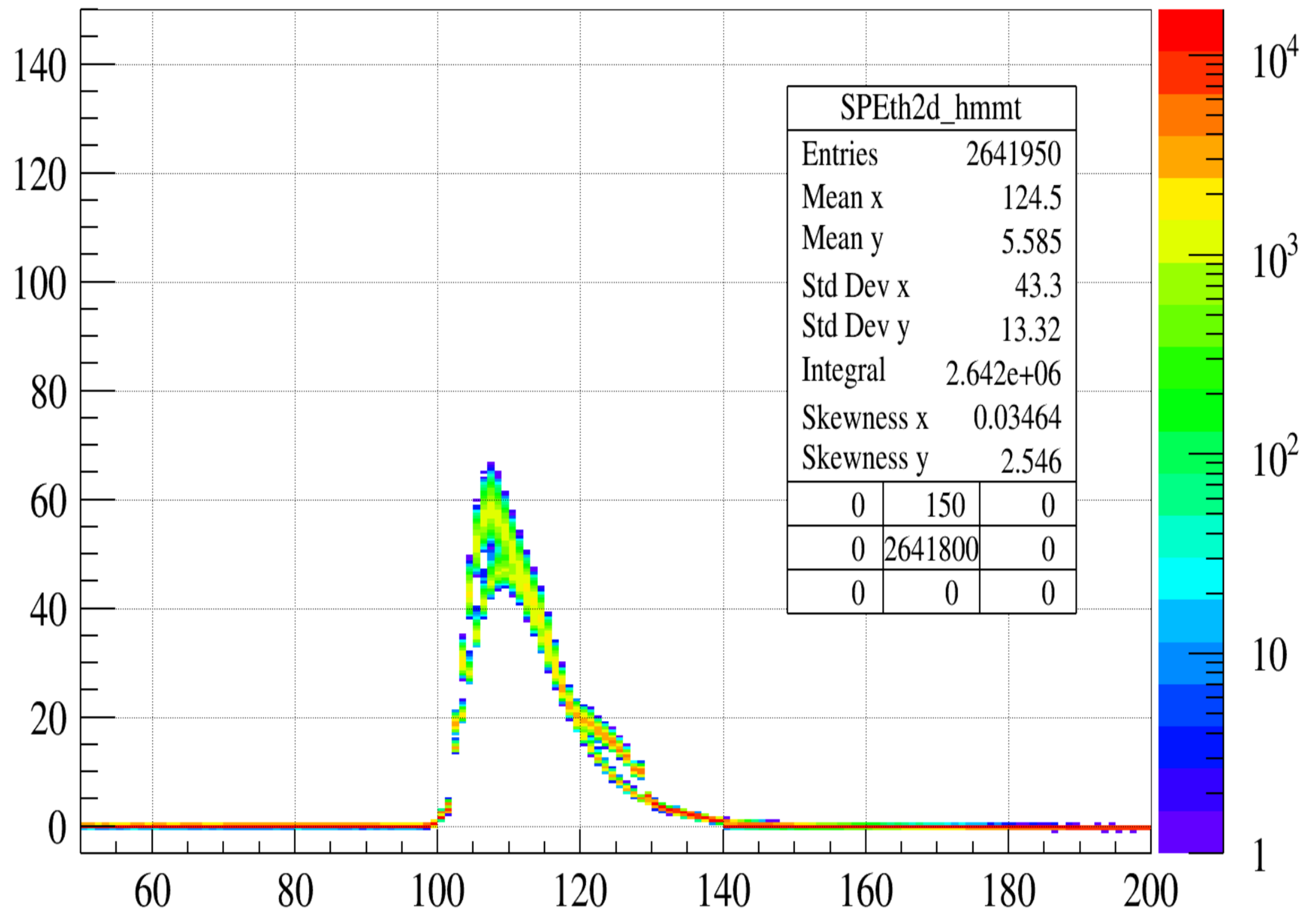
Fill integral charge into histogram using simple integral method



find 1PE cut with rangedGaussian fit

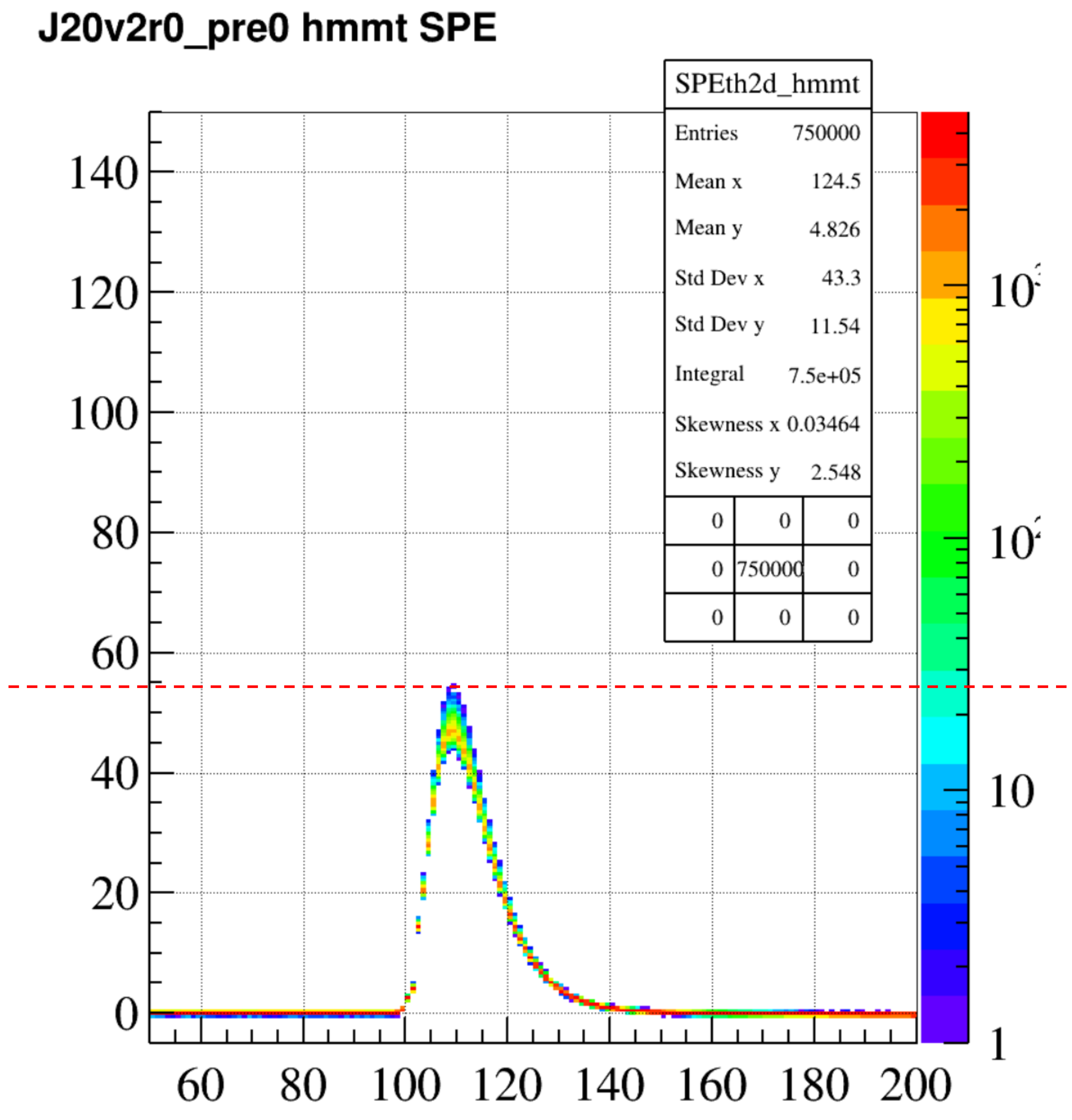
ranged cut 1.5σ

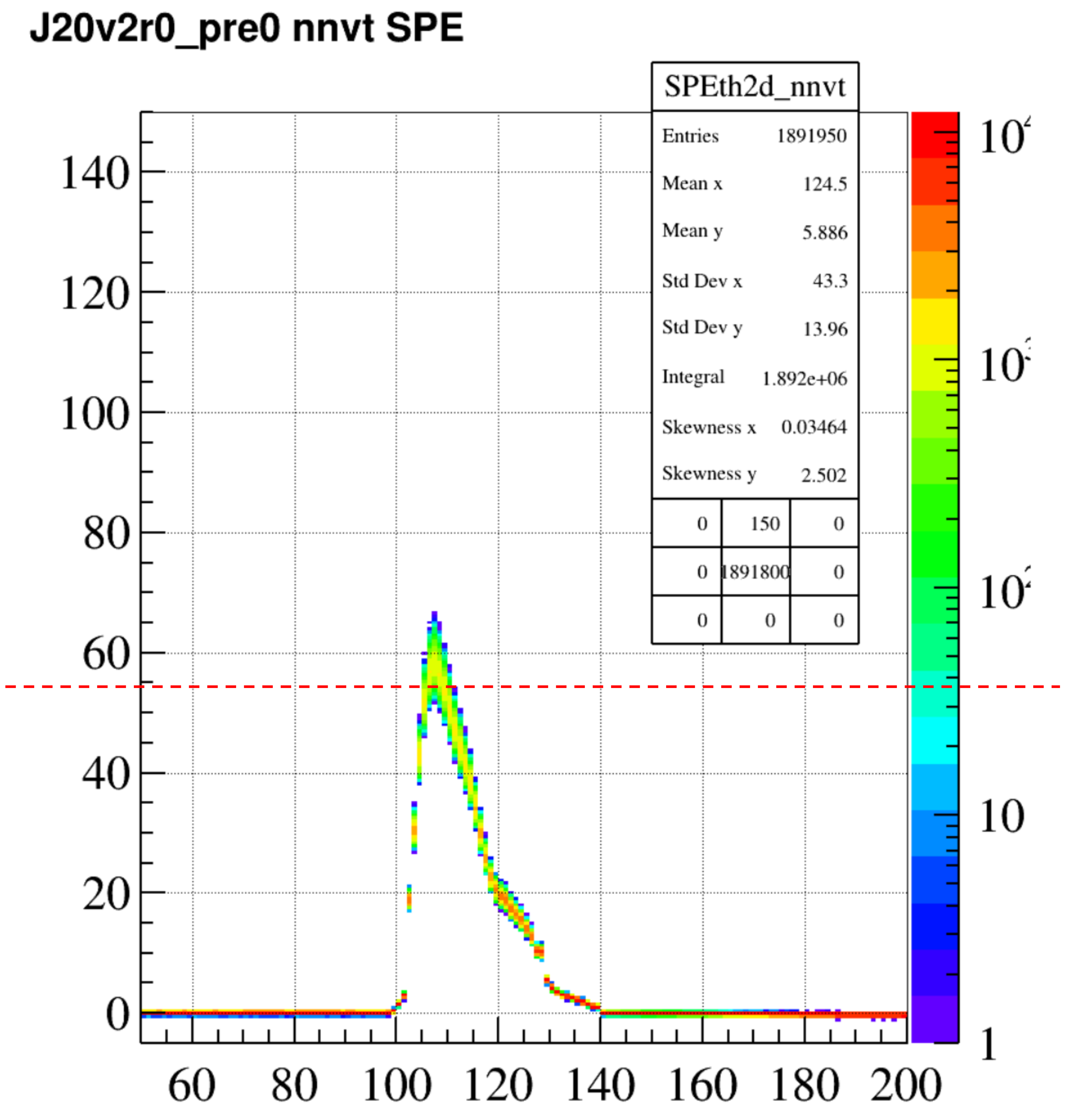
SPE spectum



SPE Spectra

PMTs of the same type get similar SPE spectrum
So I generate different filter for hmmt and nnvt PMT.





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Construct filter

Wiener filter definition:

$$F(f) = \frac{wave^2 - noise^2}{wave^2}$$

extract waveforms and do Fourier transform to get modulus distribution

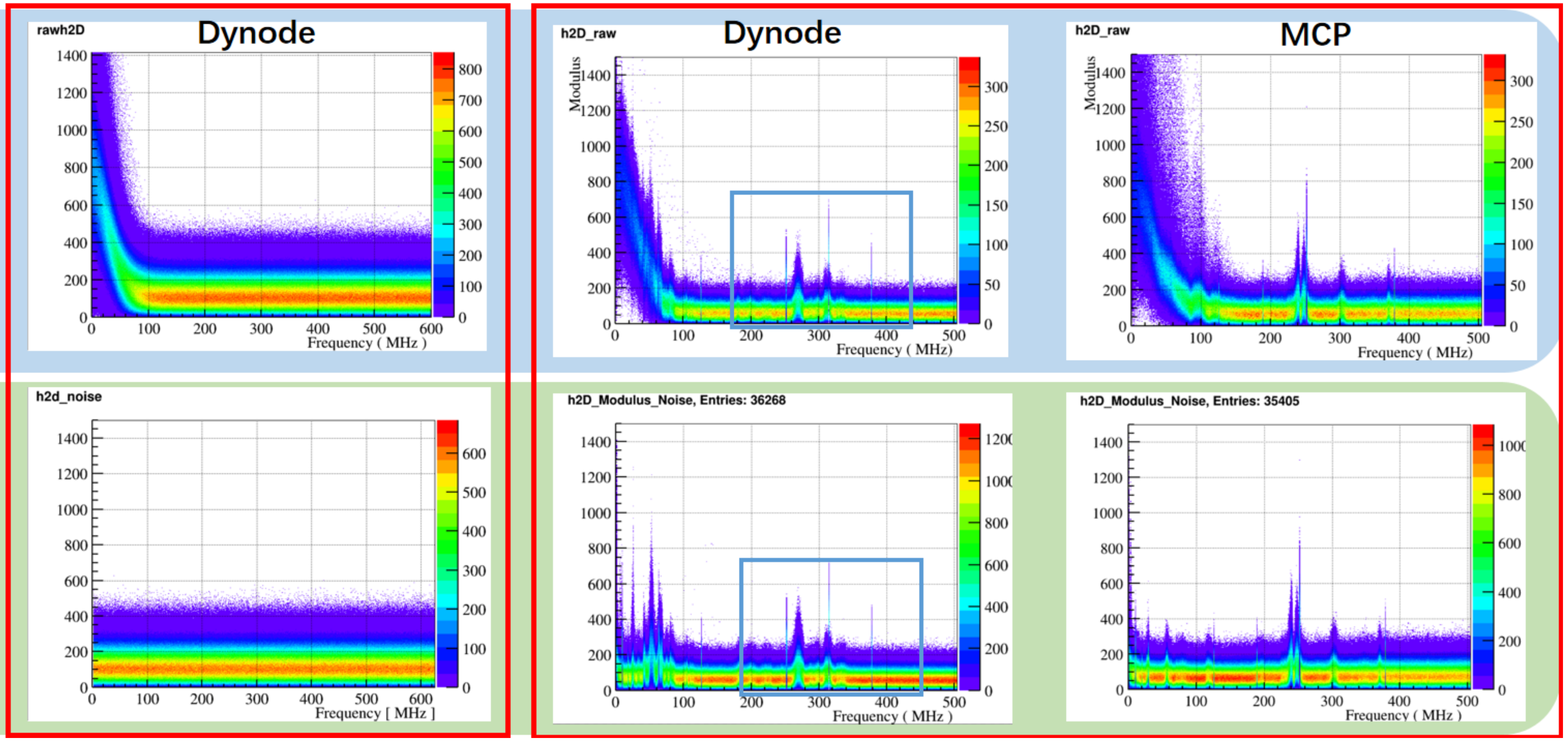
From Xiaojie Luo
JUNO-doc-6558

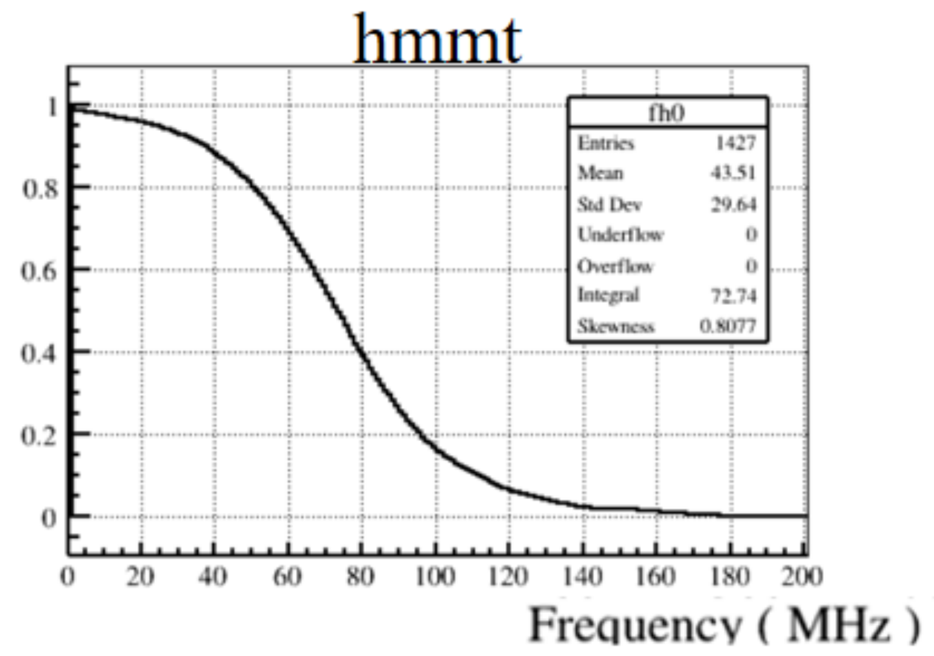
Waves of frequency over 450 are considered to be noise frequency, and noise is assumed to be flat in frequcey domain.

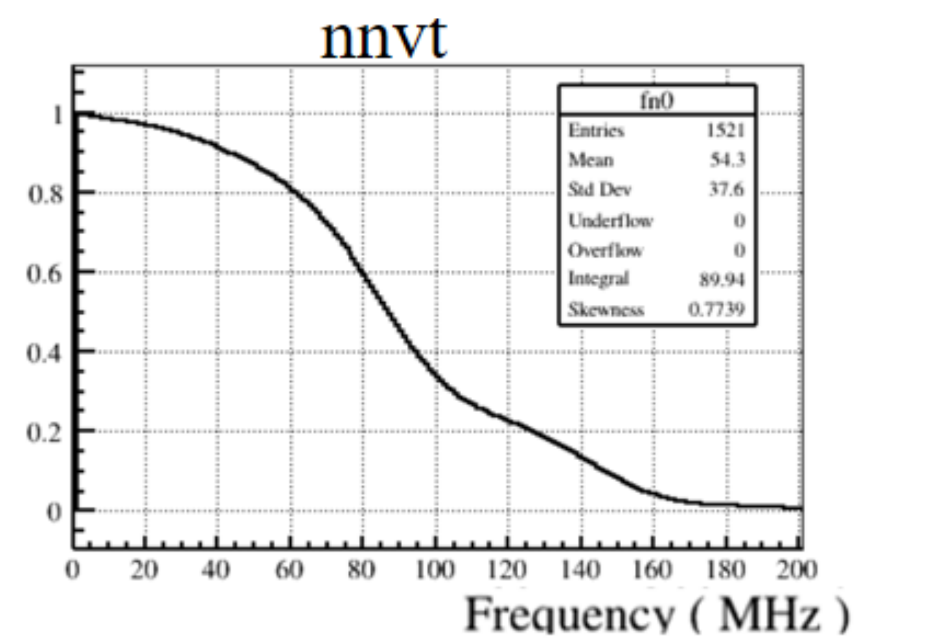
Simulation **Realistic**

Signal Spectrum

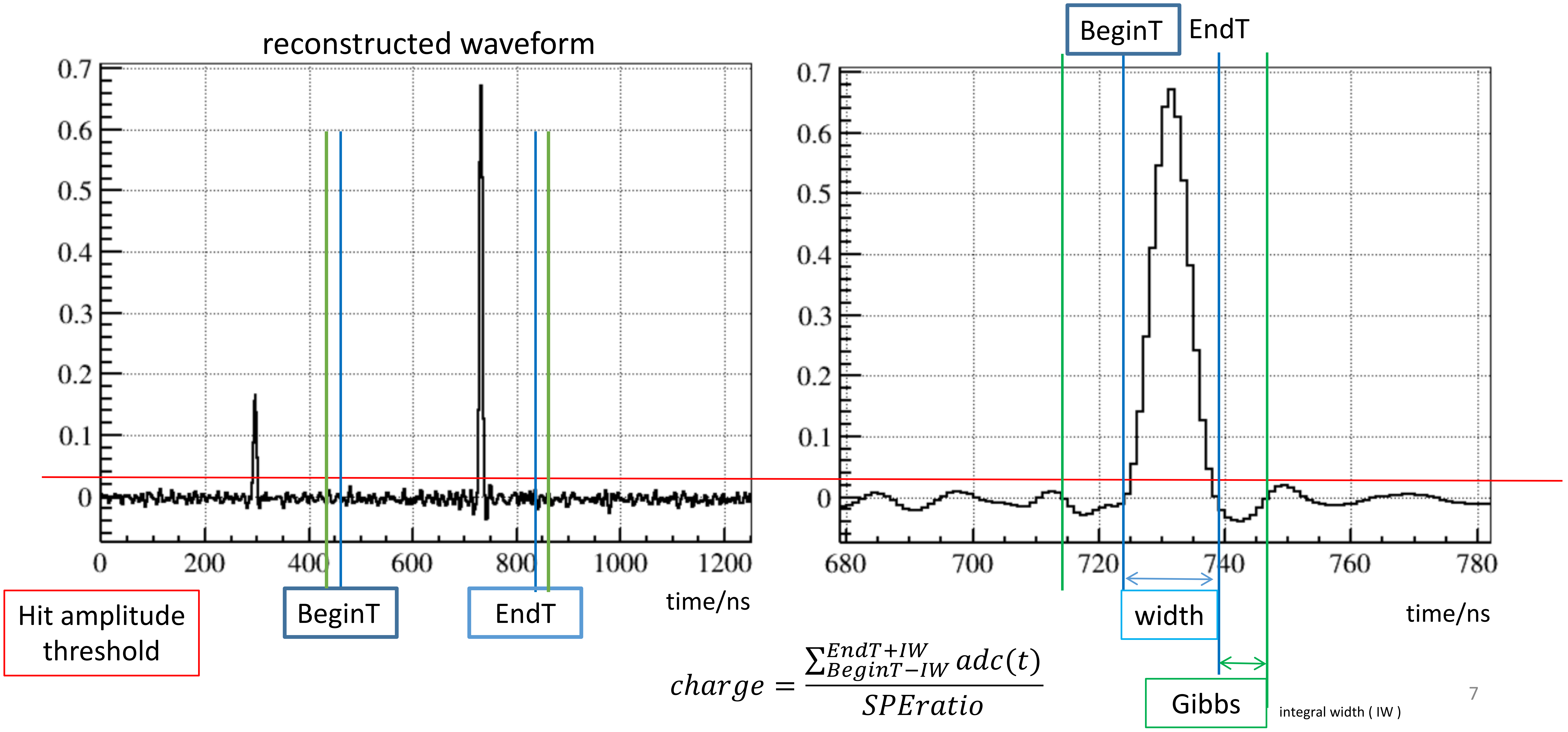
Noise Spectrum







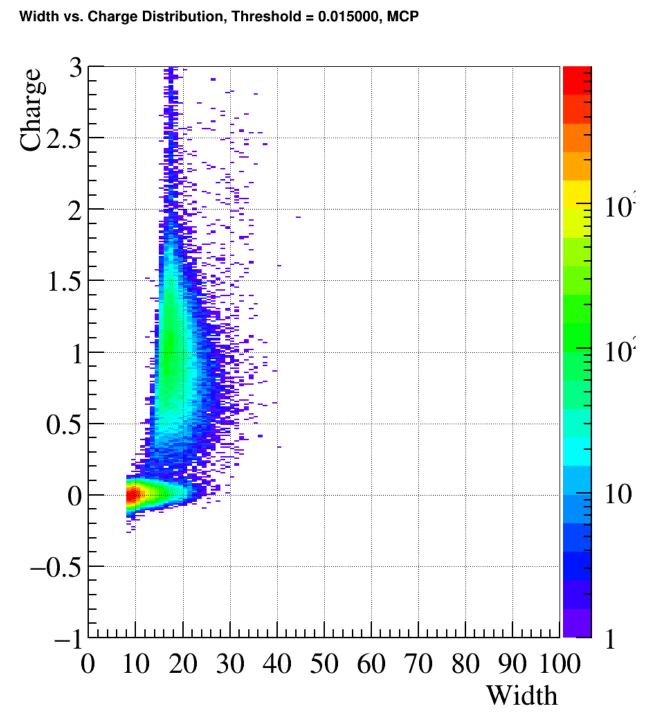
Threshold, width and Gibbs effect

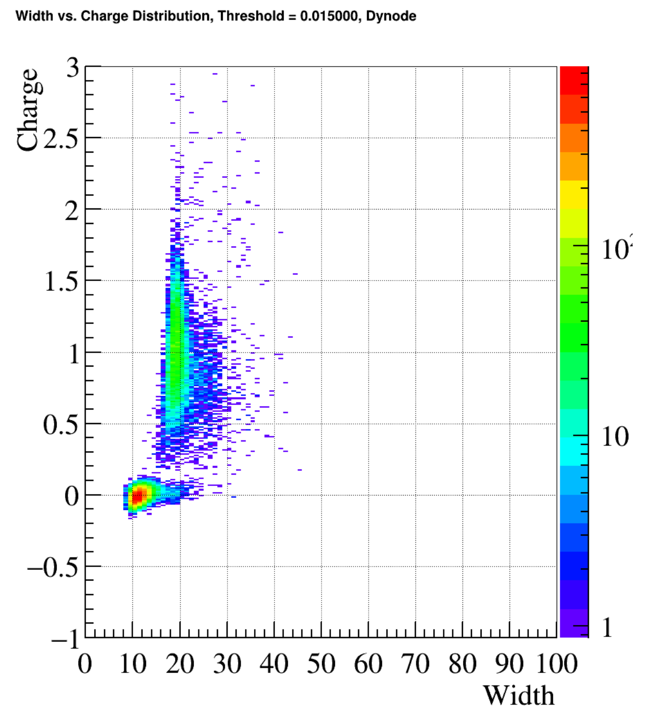


$$charge = \frac{\sum_{BeginT-IW}^{EndT+IW} adc(t)}{SPEratio}$$

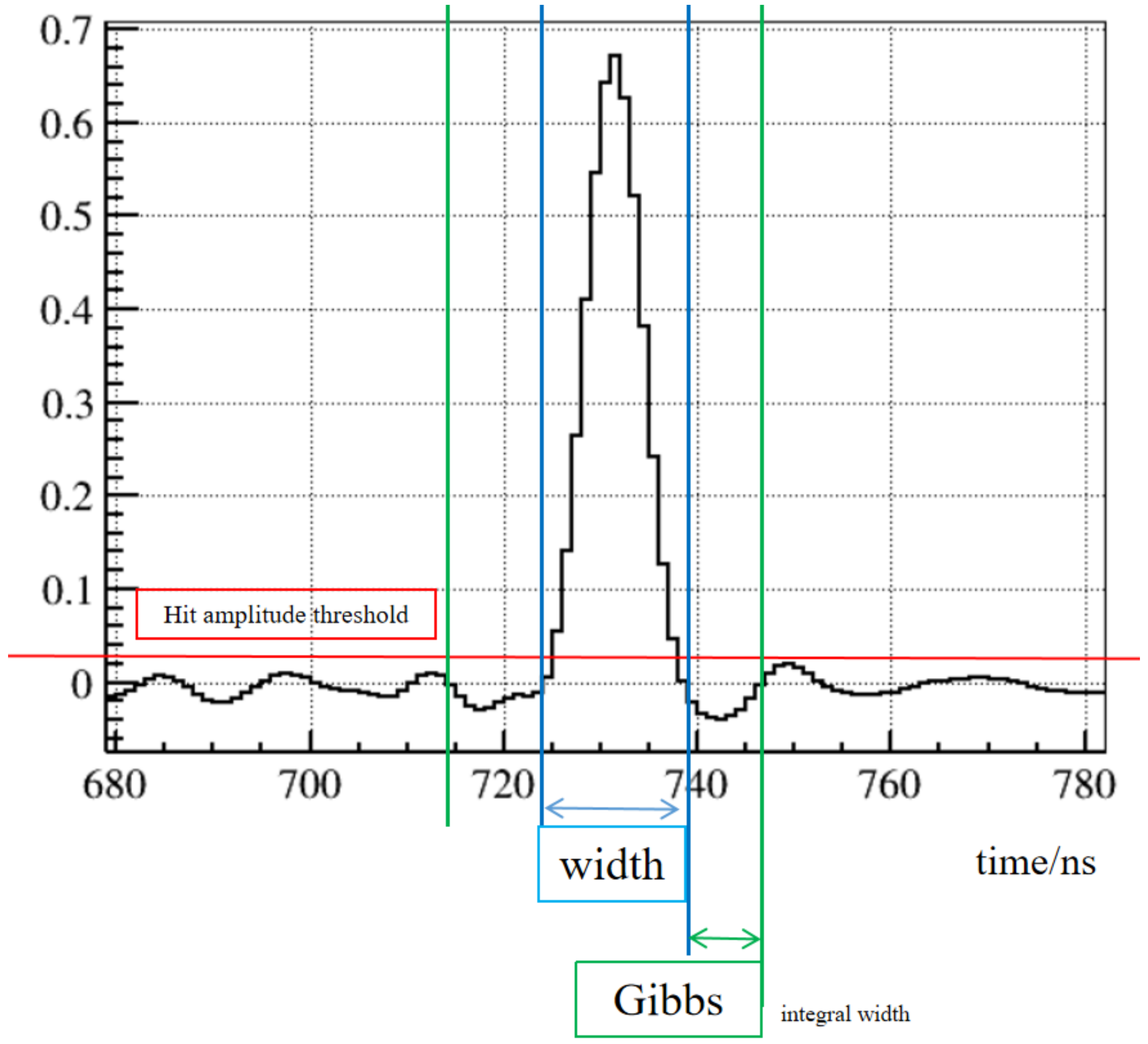
integral width (IW)

Filter relative parameters

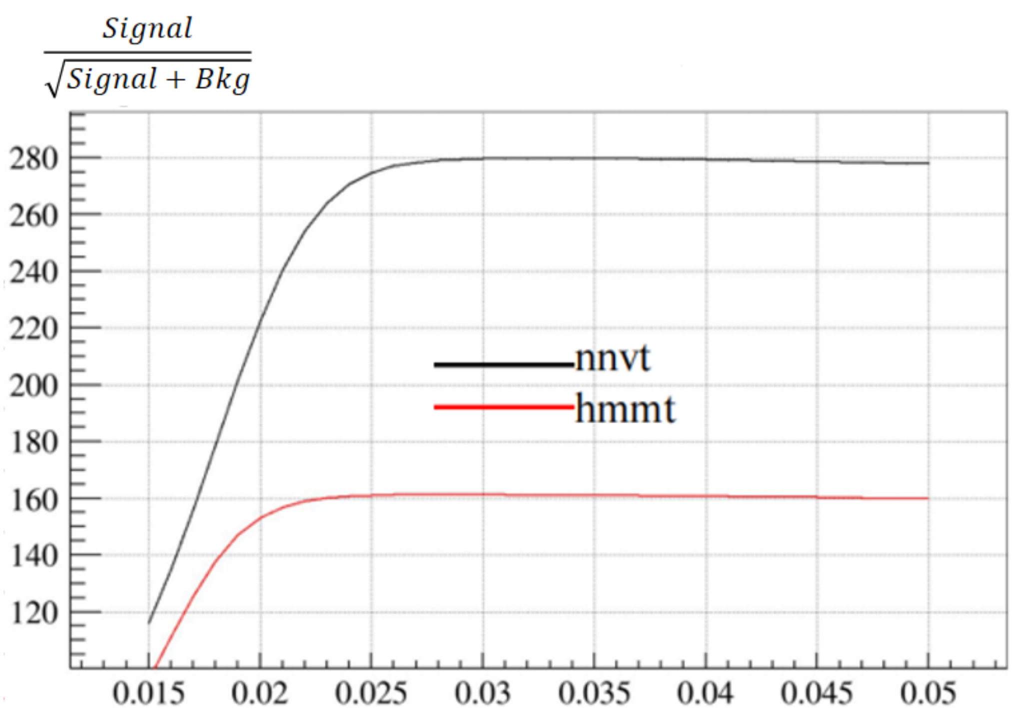


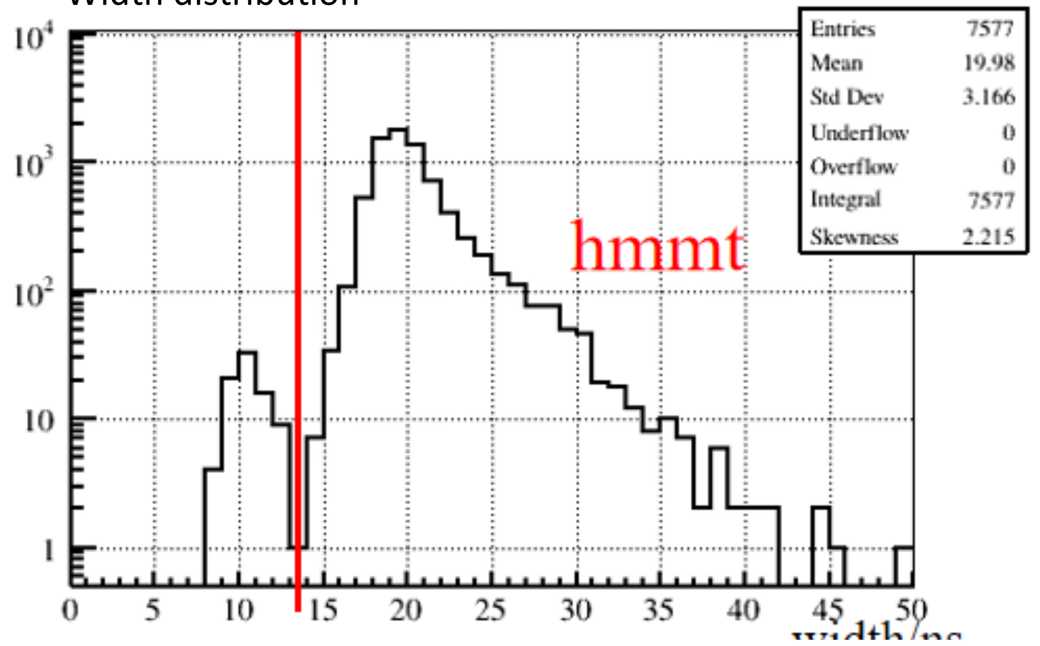


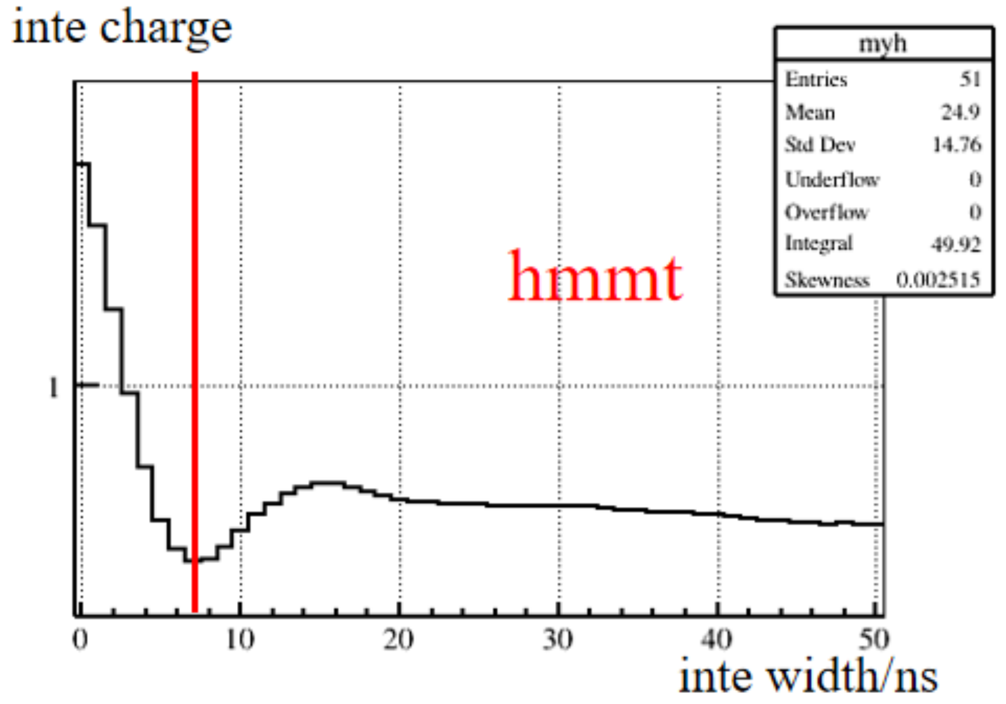
reconstructed waveform



Threshold, width and Gibbs effect







hit amplitude threshold

nnvt: 0.030

hmmt: 0.025

hit width threshold(ns)

nnvt: 12

hmmt: 12

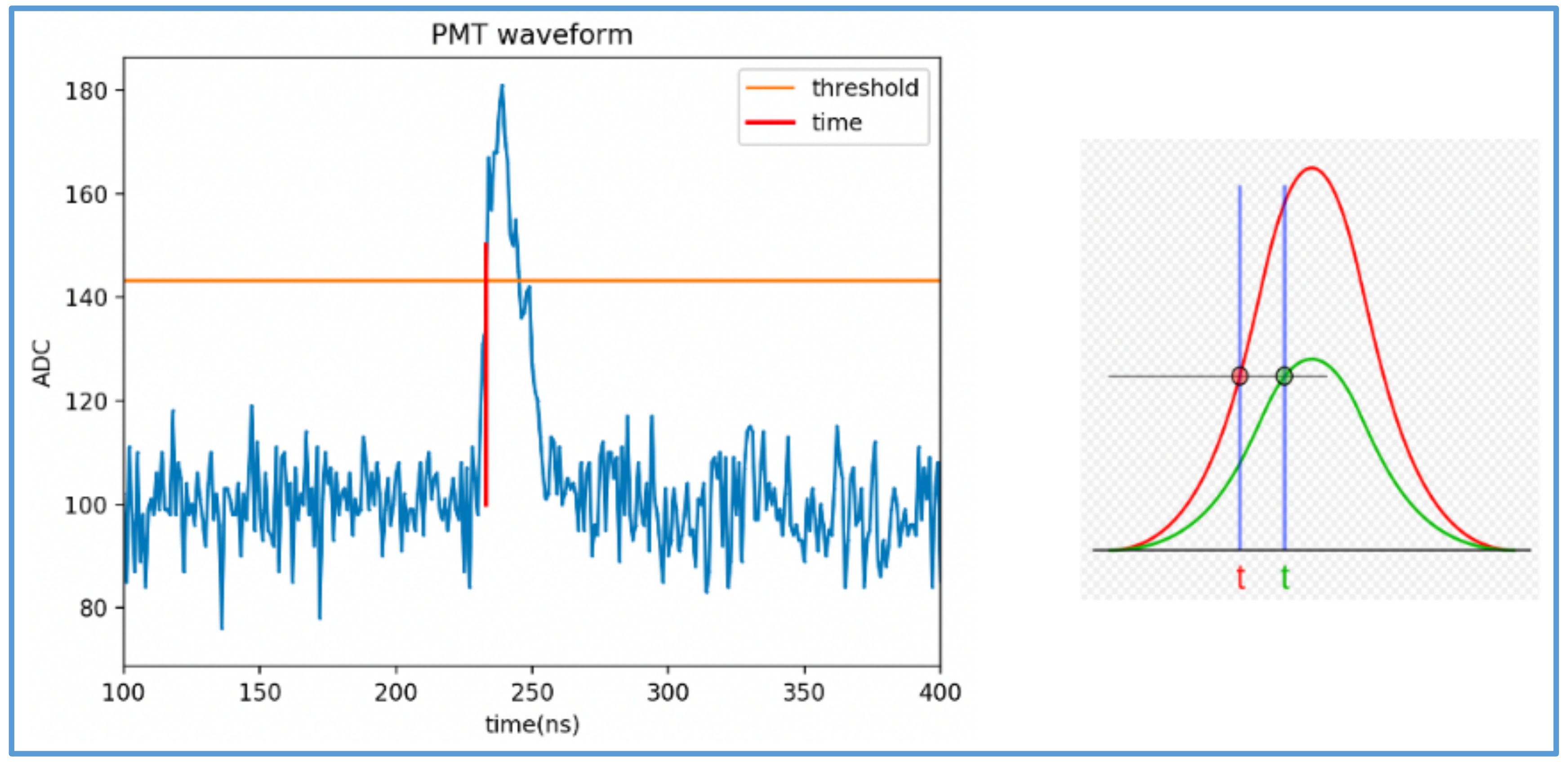
Integral width(ns)

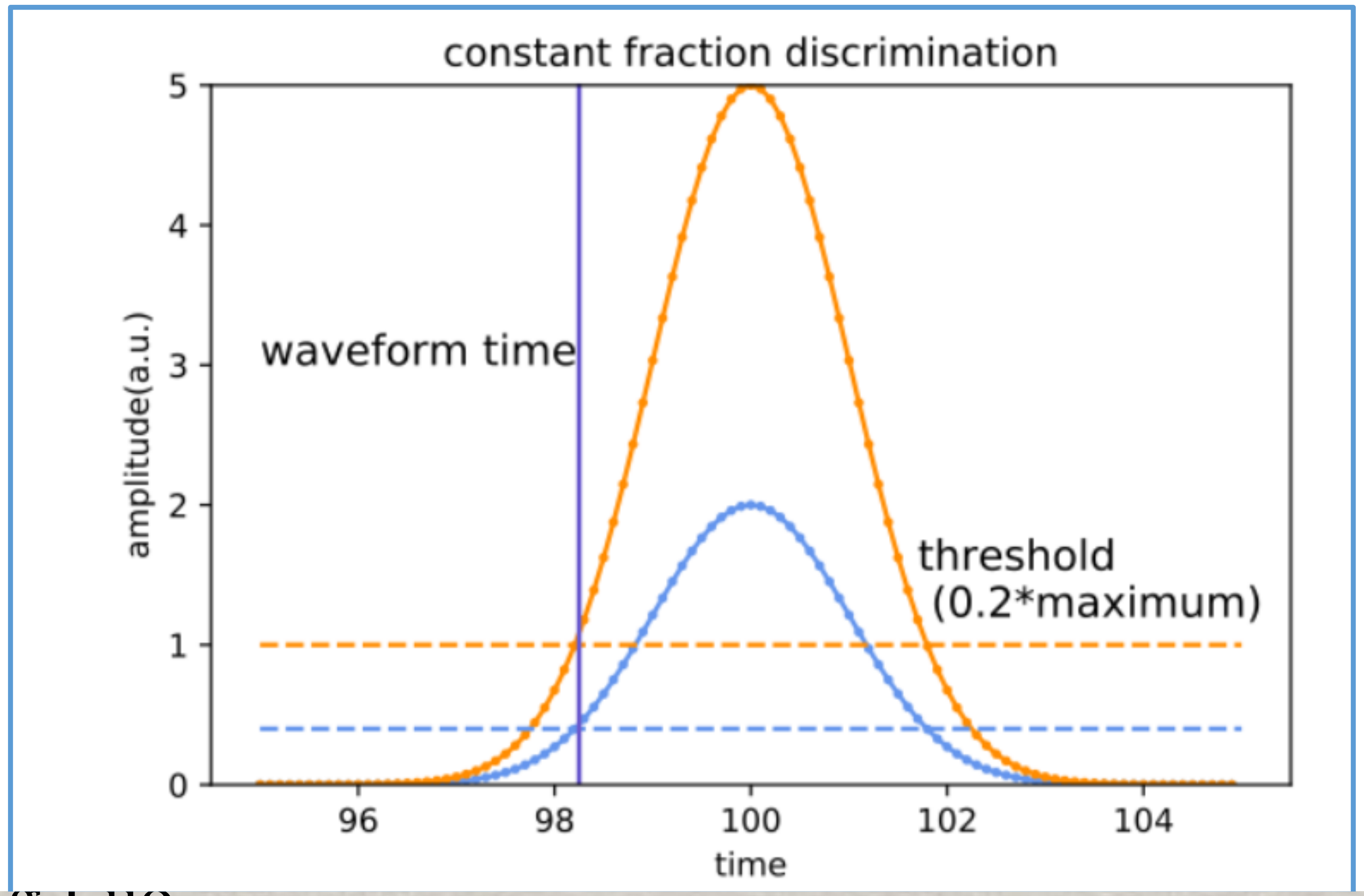
nnvt: 7

hmmt: 8

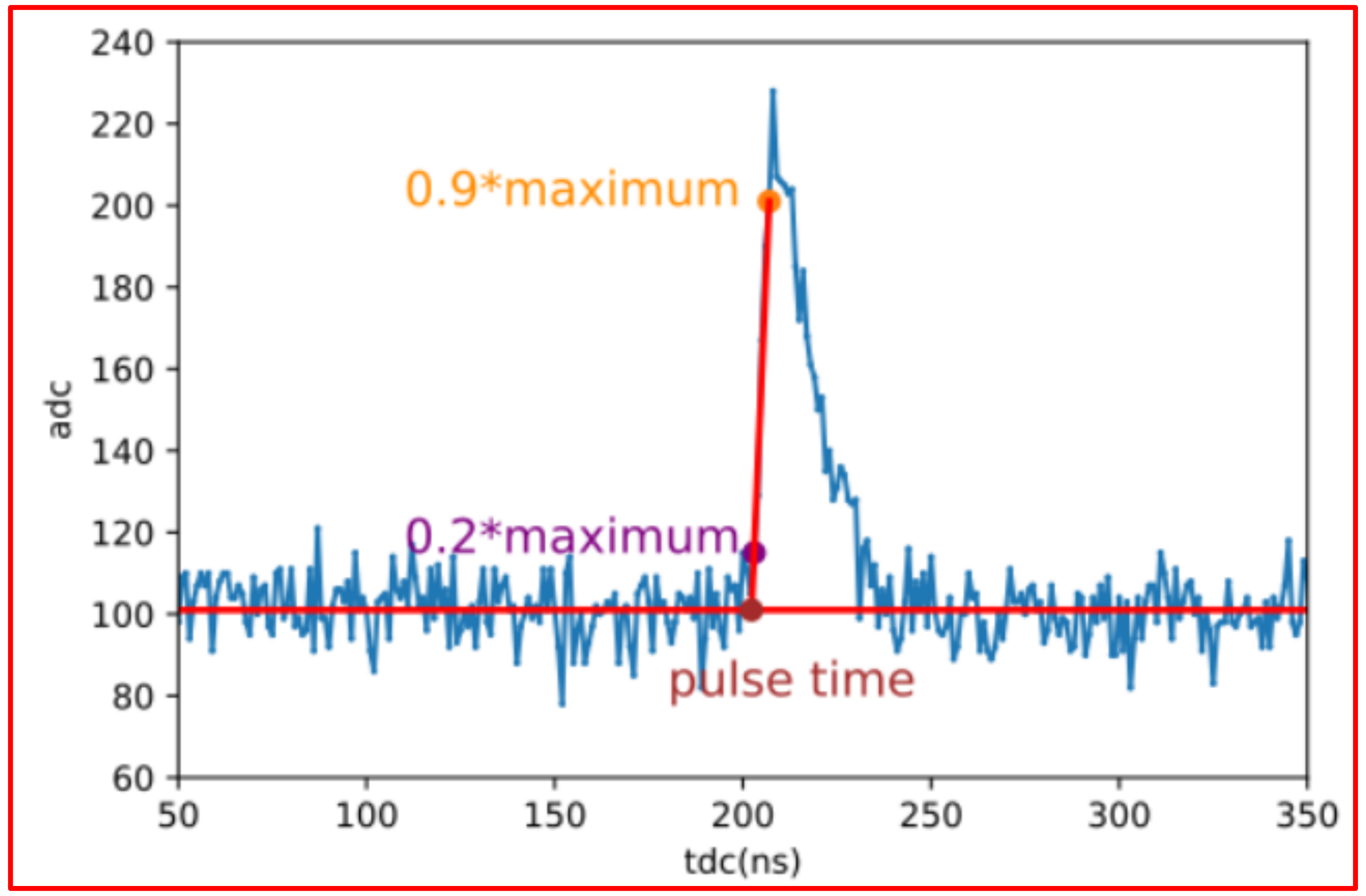
Time Rec (Miao Yu)

fixed threshold

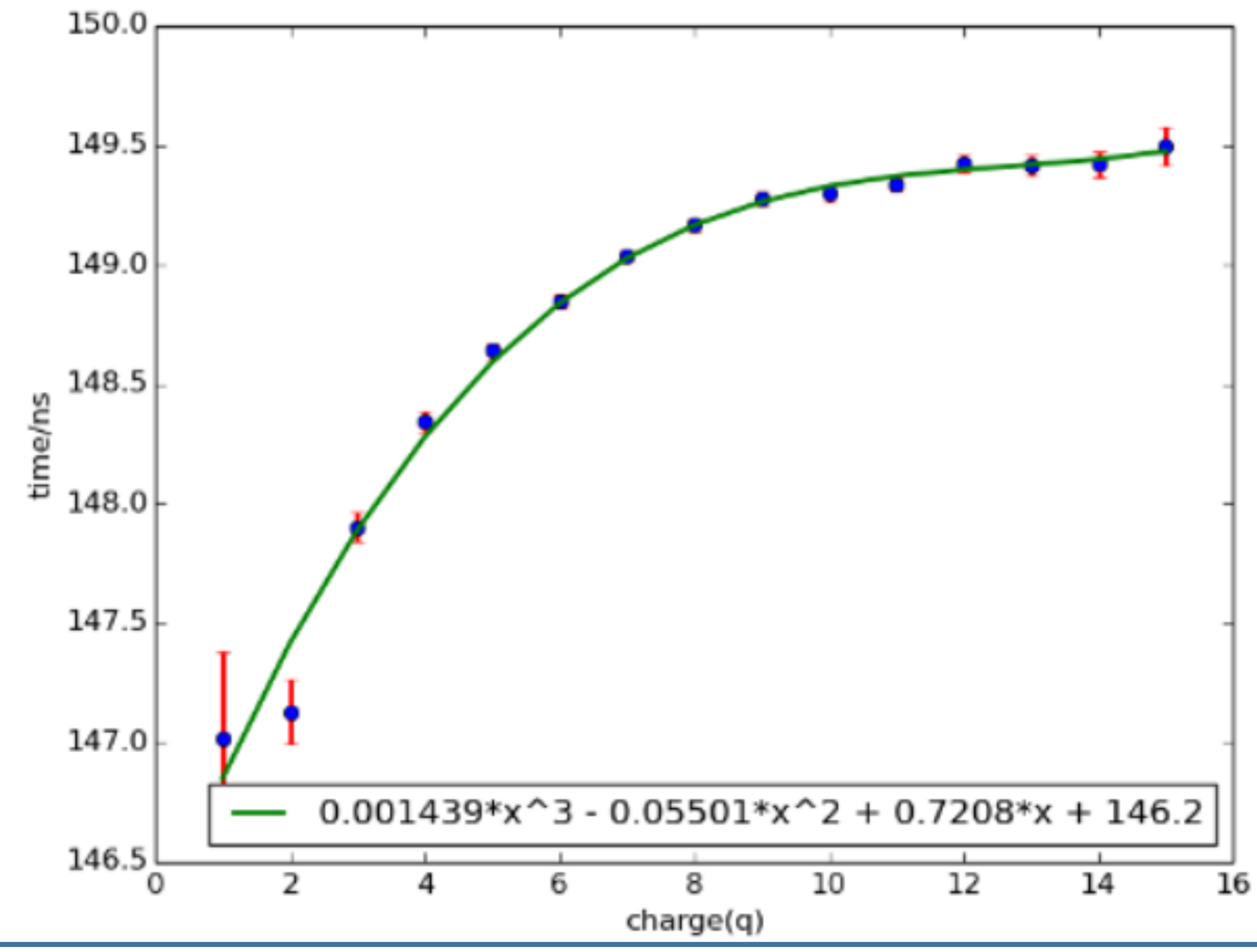




linear fit



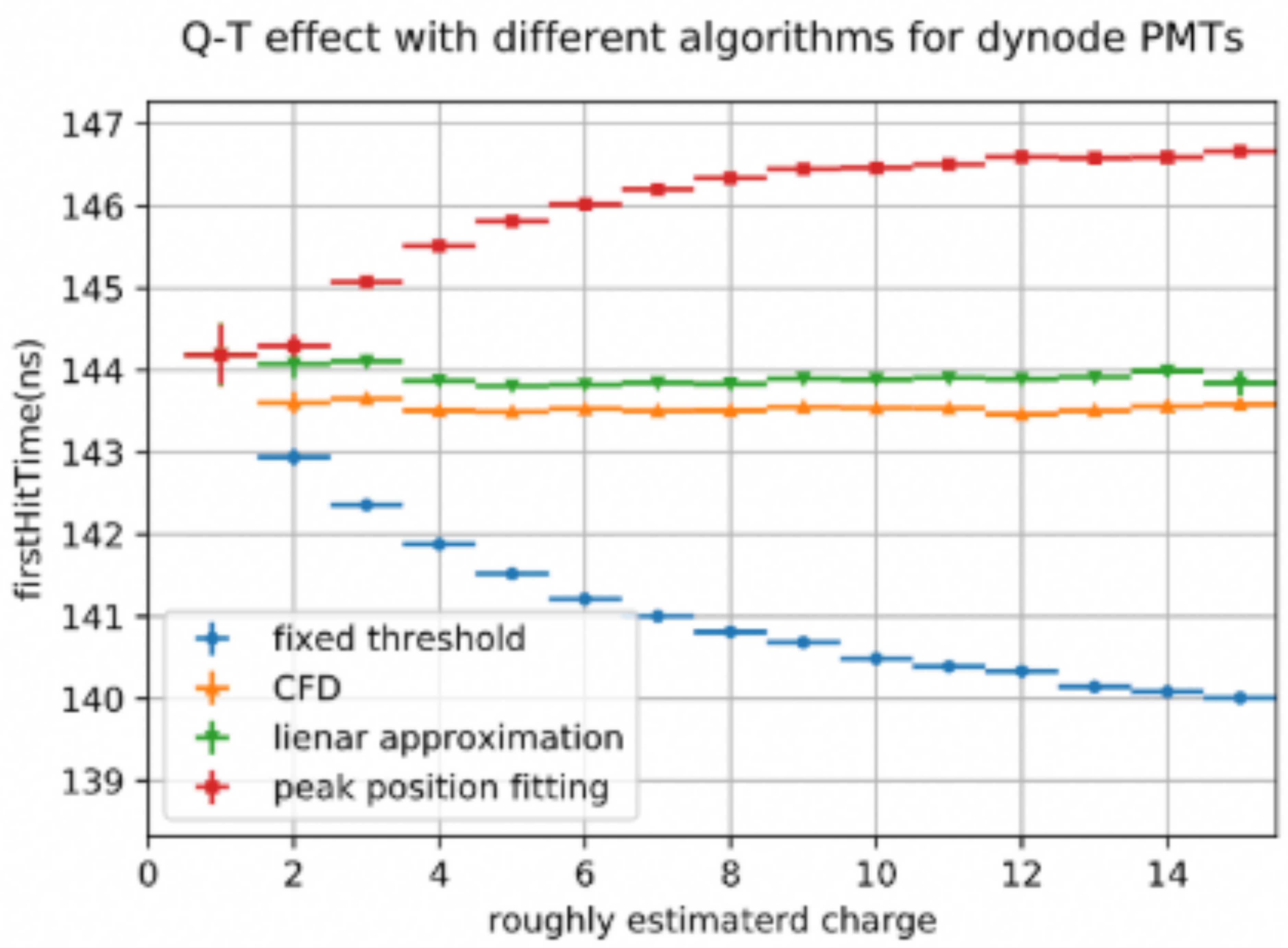
peak position time as hit time, use curve to reduce bias(need to know Q)

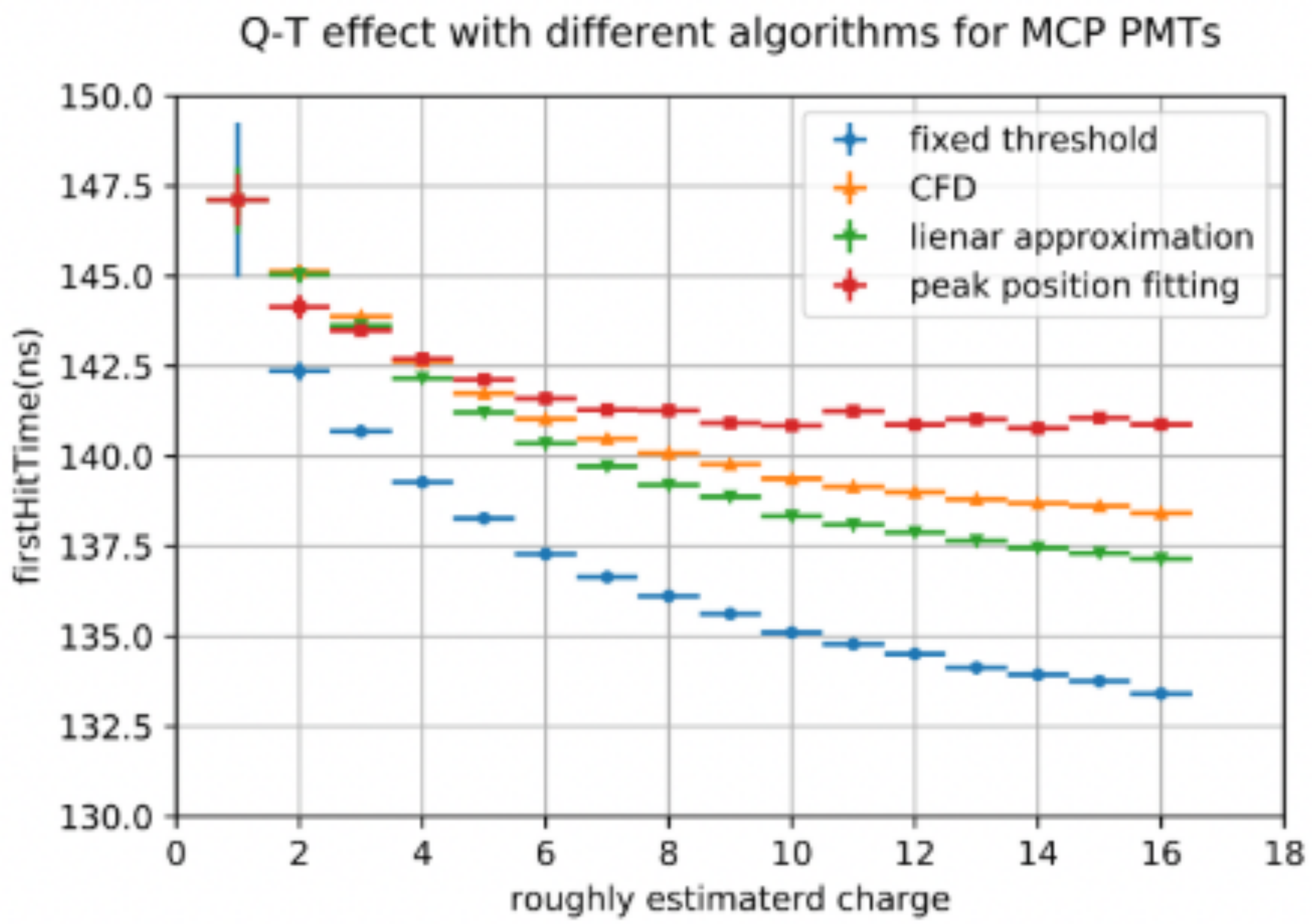


TimeRec's FHT will overWrite the FHT of Deconvolution

First Hit Time given by TimeRec

Linear Fit





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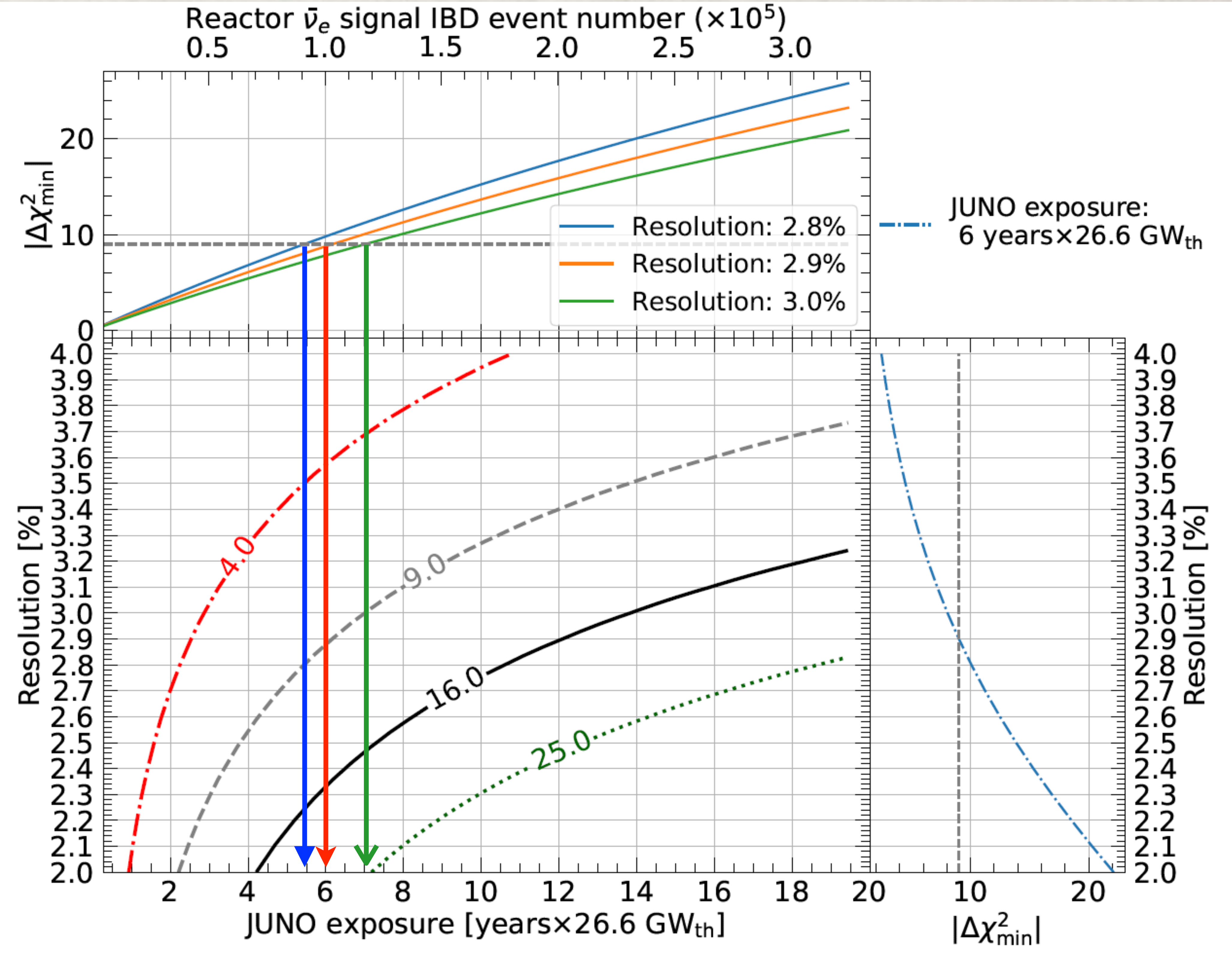
ENERGY RECO

REACTOR NEUTRINOS

	DETECTOR TARGET MASS	ENERGY RESOLUTION
KamLAND	1000 t	6%/√E
D. Chooz	8+22 t	8%/√E
RENO	16 t	8%/√E
Daya Bay	20 t	8%/√E
Borexino	300 t	5%/√E
JUNO	**20000 t**	**3%/√E**



❖Challenging： un-precedented energy resolution

❖Important： better resolution —> larger sensitivity

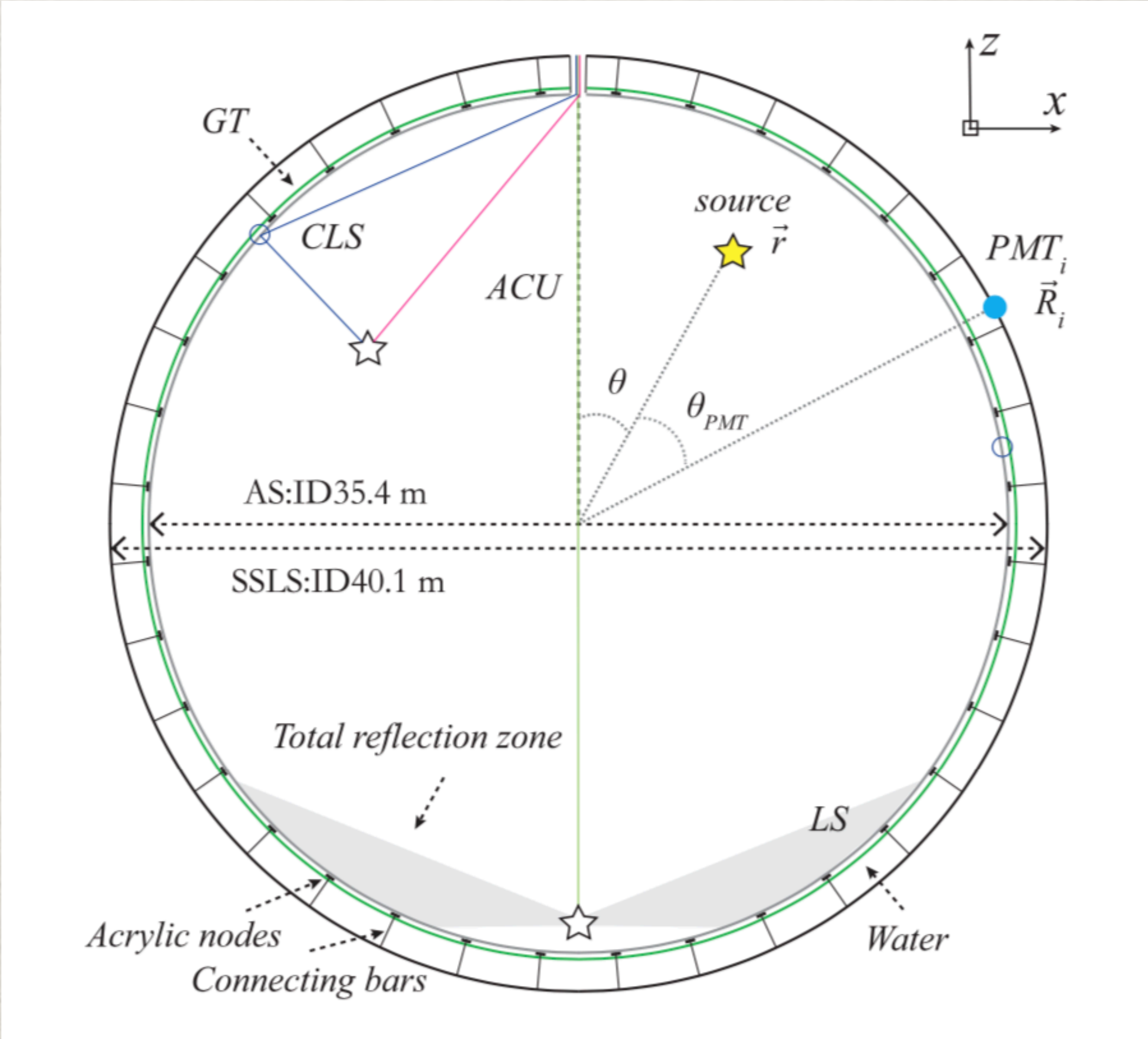
Energy Reco.

NIMA 1001 (2021) 165287

- ❖Simple total PE method: E ~ total PE
- ❖Maximum likelihood method*
  - ❖optical model independent
  - ❖calibration data driven
  - ❖taking into account differences among PMTs
- ❖Main factors for energy resolution:
  - ❖photon statistics
  - ❖energy non-uniformity*
  - ❖PMT dark noise

METHOD PRINCIPLE

- ❖Step1: use calibration data to construct the expected number of PhotoElectron per unit E $\hat{\mu}(r, \theta, \theta_{PMT})$ for PMTs
- ❖Step2: maximize the likelihood function



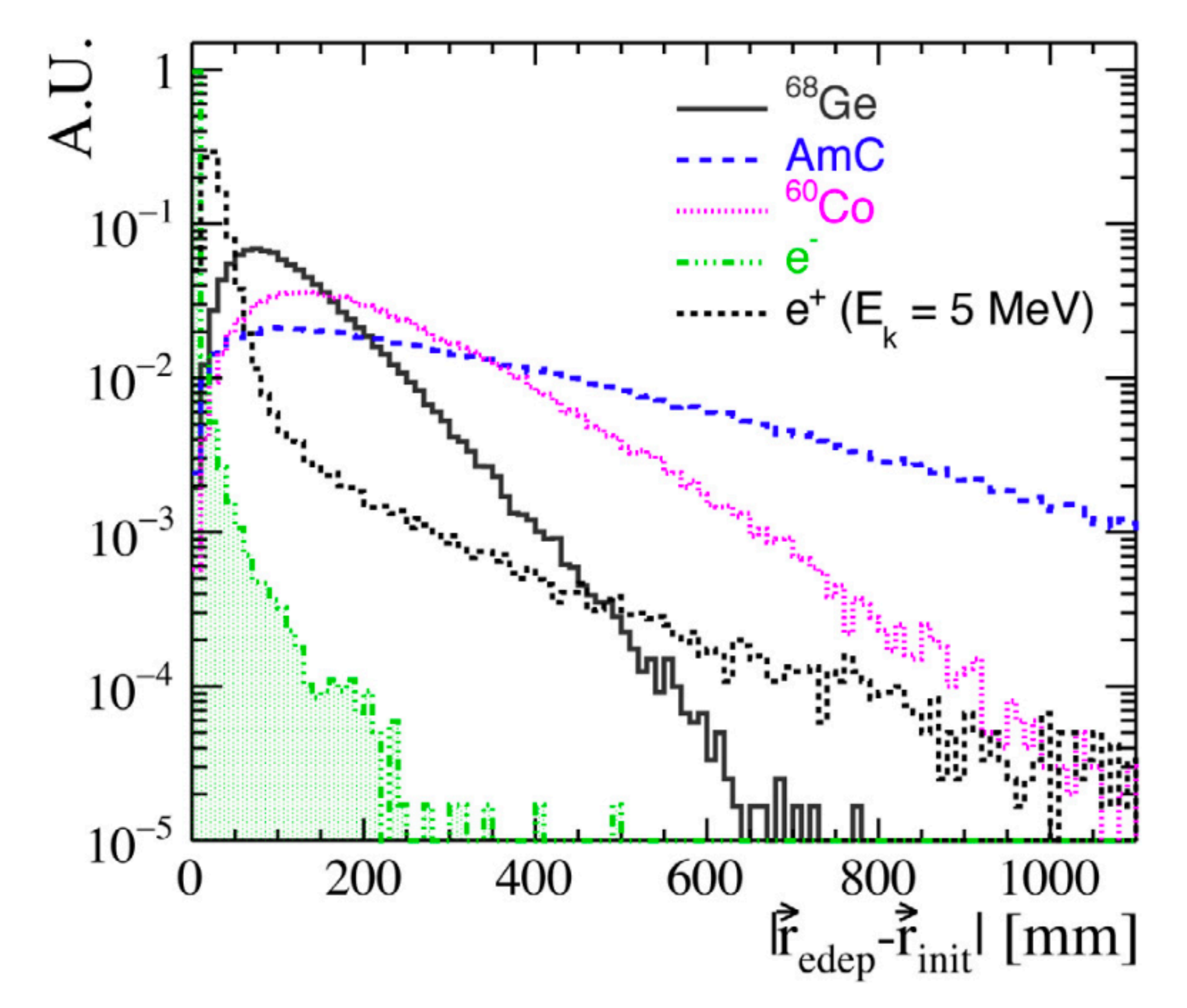
$$\mathcal{L}(\{k_i\}|r, \theta, \phi, E_{vis}) = \prod_i \mathcal{L}(k_i|r, \theta, \phi, E_{vis}) = \prod_i \frac{e^{-\mu_i} \cdot \mu_i^{k_i}}{k_i}$$

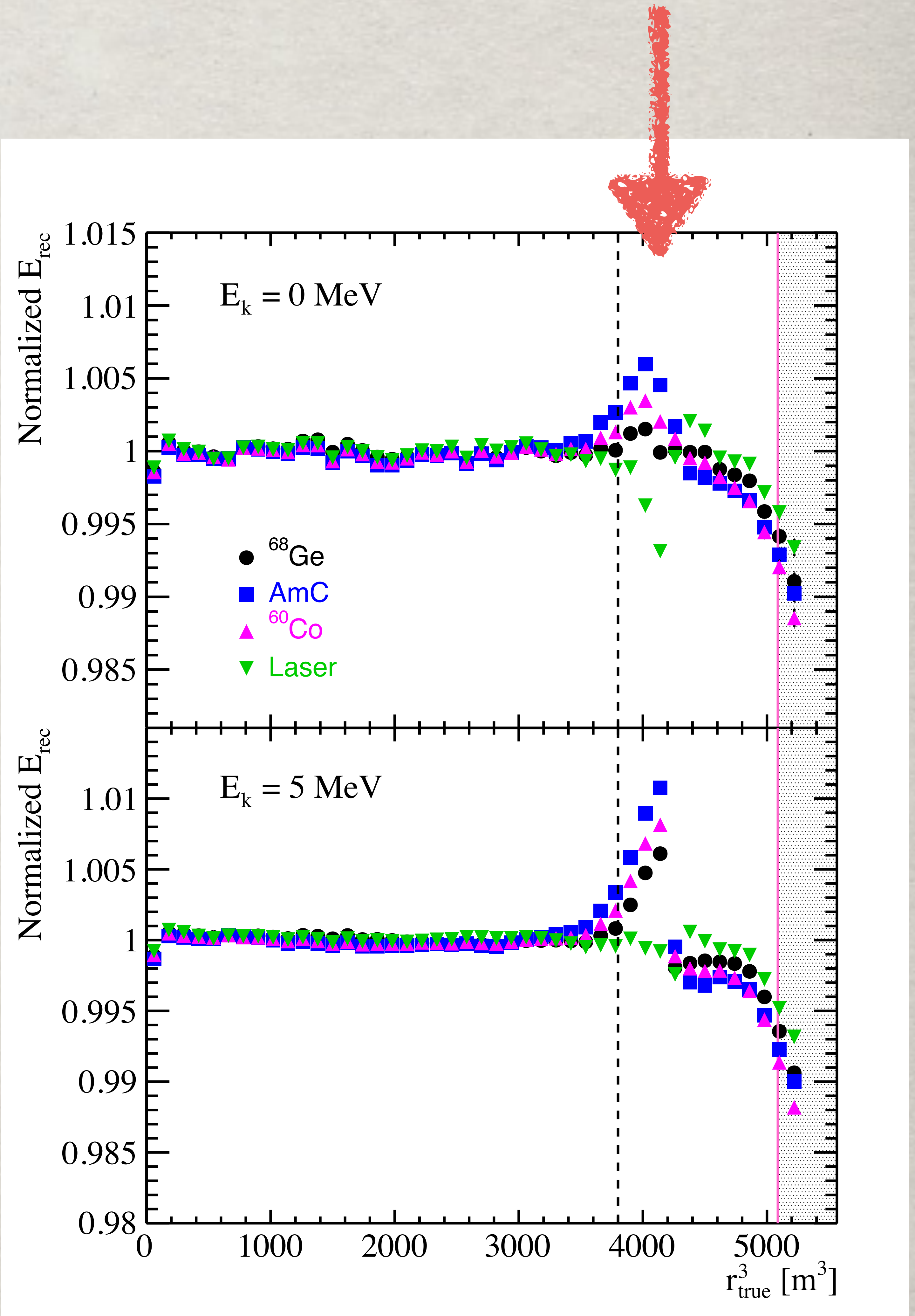
$$\mu_i = E_{vis} \cdot \hat{\mu}_i$$

*{k_i} — detected PE for PMTs
E_{vis} — visible energy

SOURCE CHOICE

Source	Type	Energy [MeV]
^{68}Ge	γ	2×0.511
^{60}Co	γ	$1.173 + 1.333$
AmC	(n,H)γ	2.22
Laser	op	1

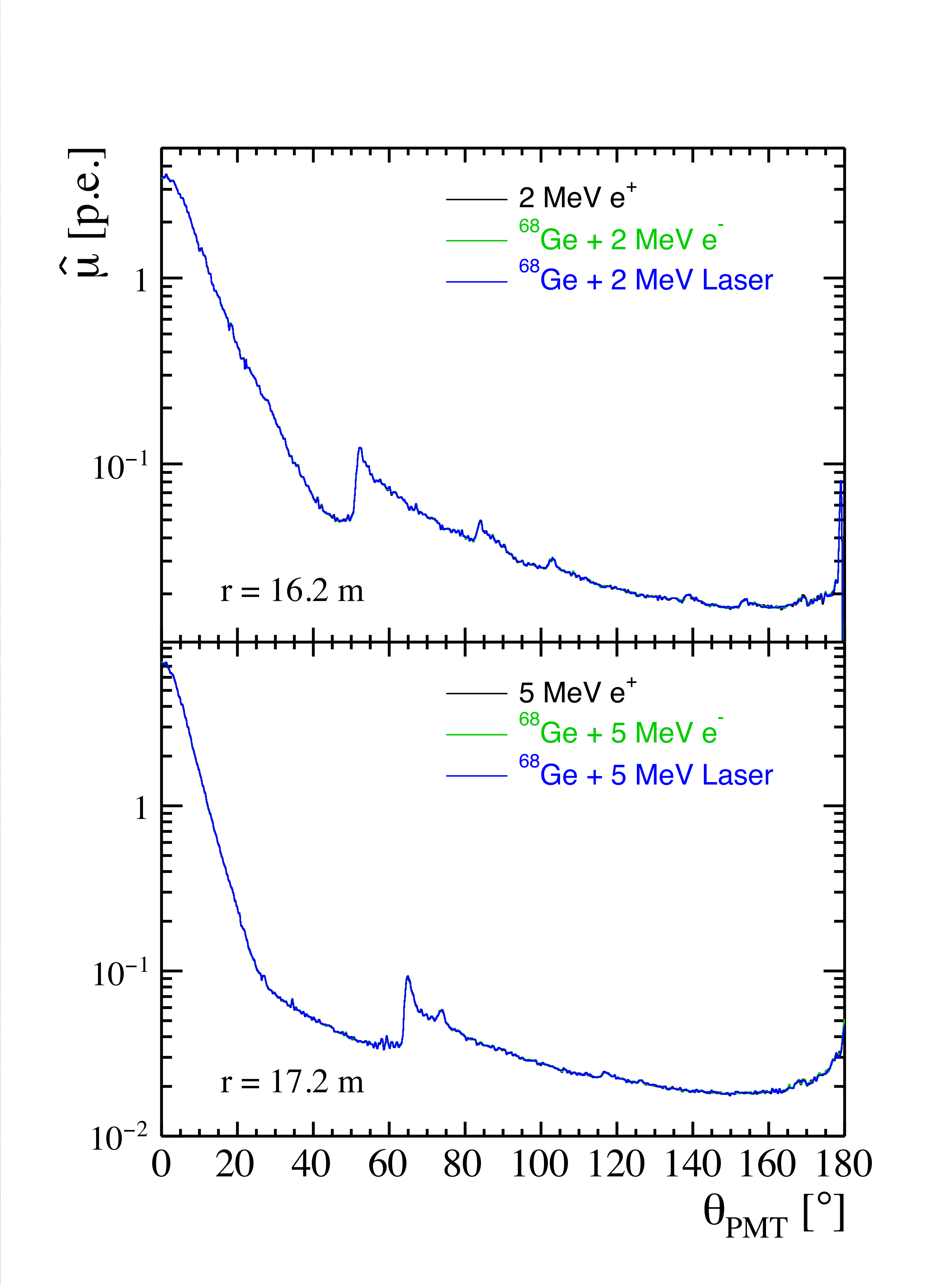




- ❖Obvious energy non-uniformity in the total reflection region
- ❖Laser(^{68}Ge) is better at high(low) energy

COMBINED SOURCE

- ❖Energy deposition of positron in LS
  - ❖kinetic part: point-like
  - ❖annihilation part: ball-like
- ❖Use combined source Laser+^{68}Ge to mimic positron



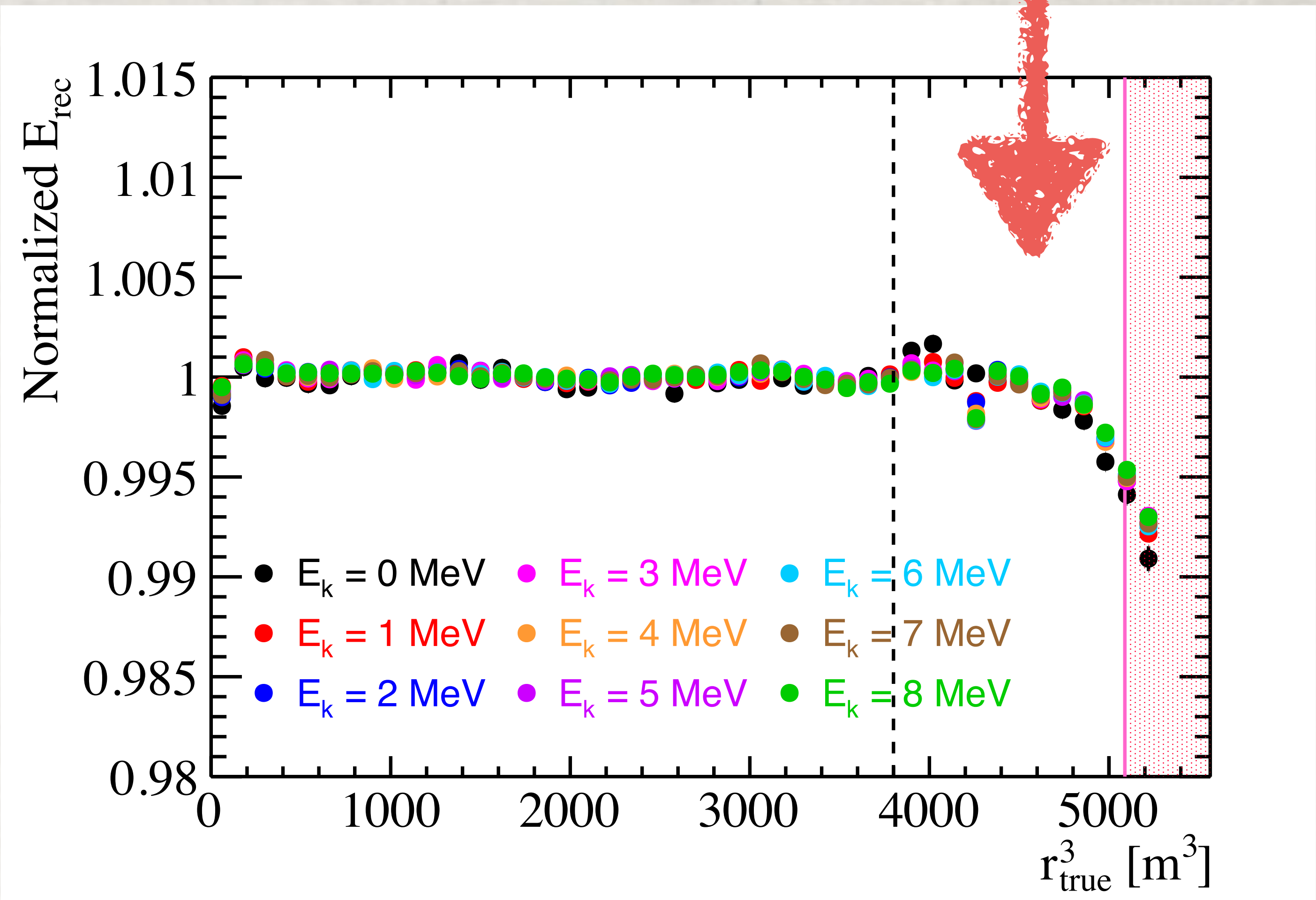
$$\hat{\mu}^{comb} = \frac{1}{E_{vis}} \cdot (E_{vis}^{Ge} \cdot \hat{\mu}^{Ge}(r, \theta, \theta_{PMT}) + E_k \cdot \hat{\mu}^{L}(r, \theta, \theta_{PMT}))$$

$$E_{vis} = E_{vis}^{Ge} + E_k$$

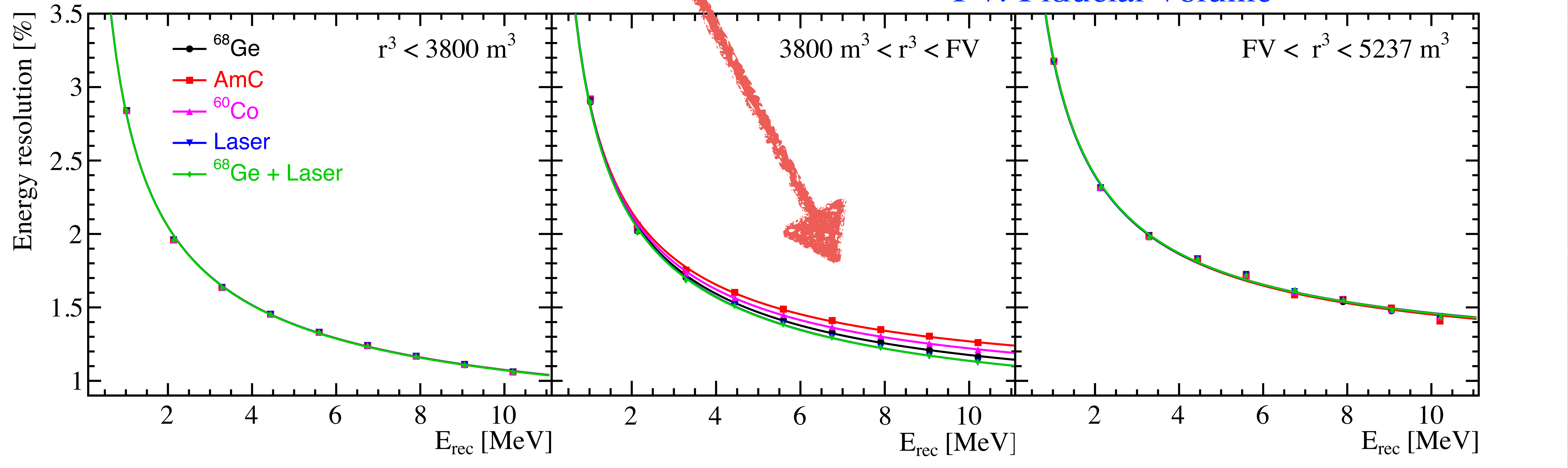
*E_k — kinetic energy of e^+

PERFORMANCE

❖Combined source improves the energy-uniformity (consequently energy resolution) in the total reflection region



FV: Fiducial Volume



VERTEX/ENERGY COMBINED RECO

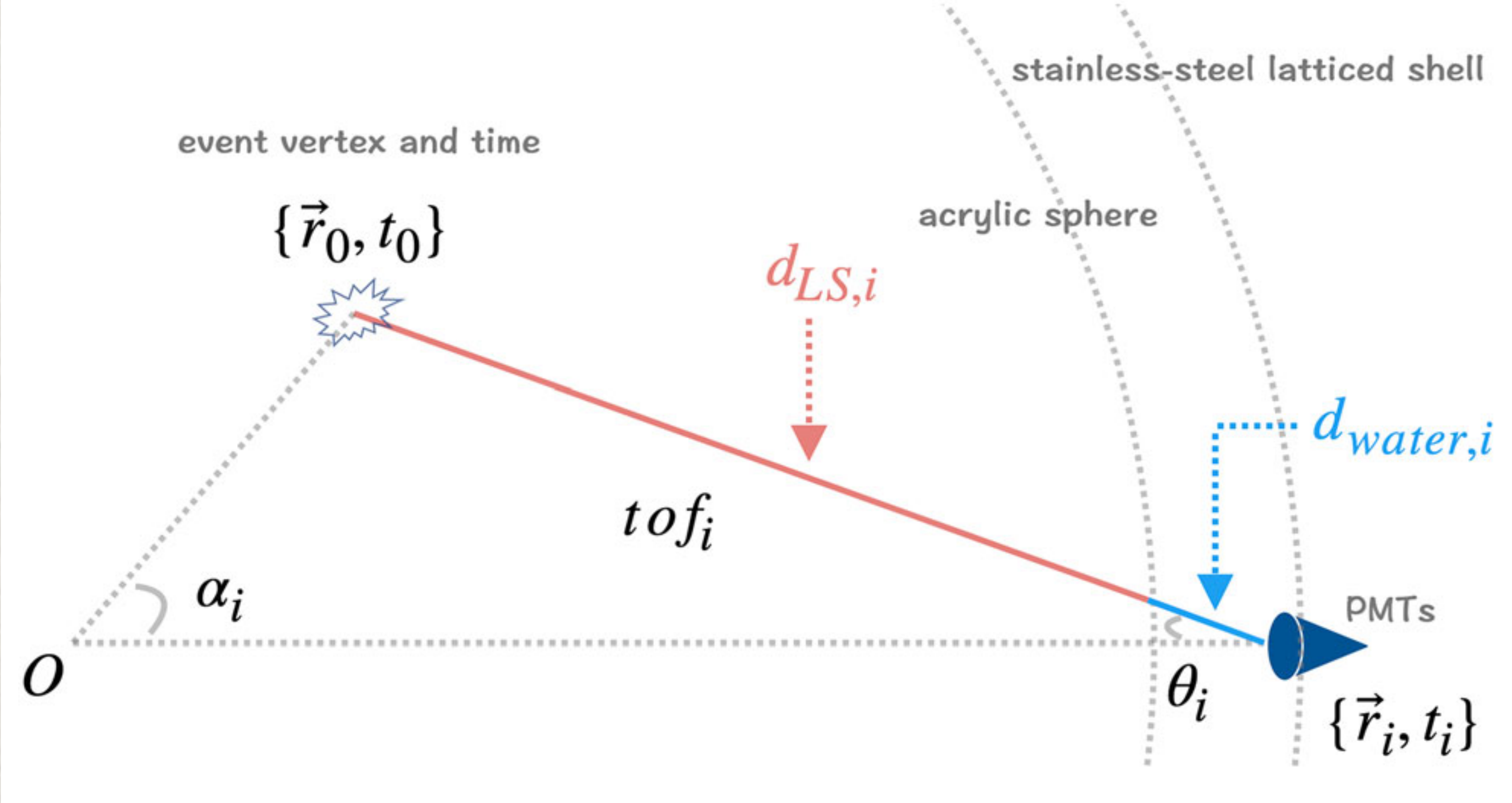
Time Likelihood

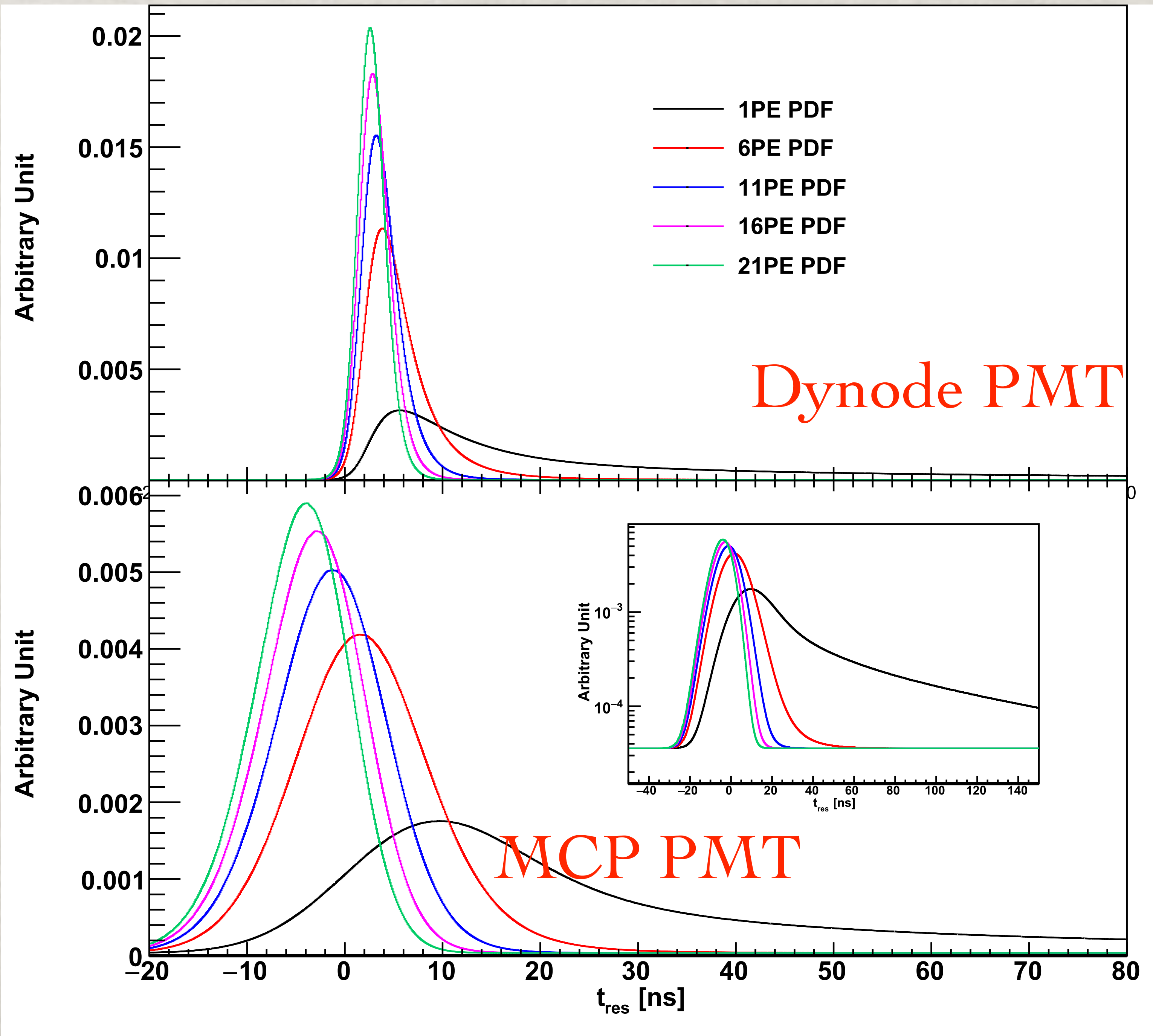
- ❖Define residual time

$$t^i_{\text{res}}(\vec{r}_0, t_0) = t_i - \text{tof}_i - t_0,$$

- ❖Construct pdf $p(t_{res})$
- ❖Minimize likelihood function

$$\mathcal{L}(\vec{r}_0, t_0) = -\ln\left(\prod_i p(t^i_{\text{res}})\right).$$





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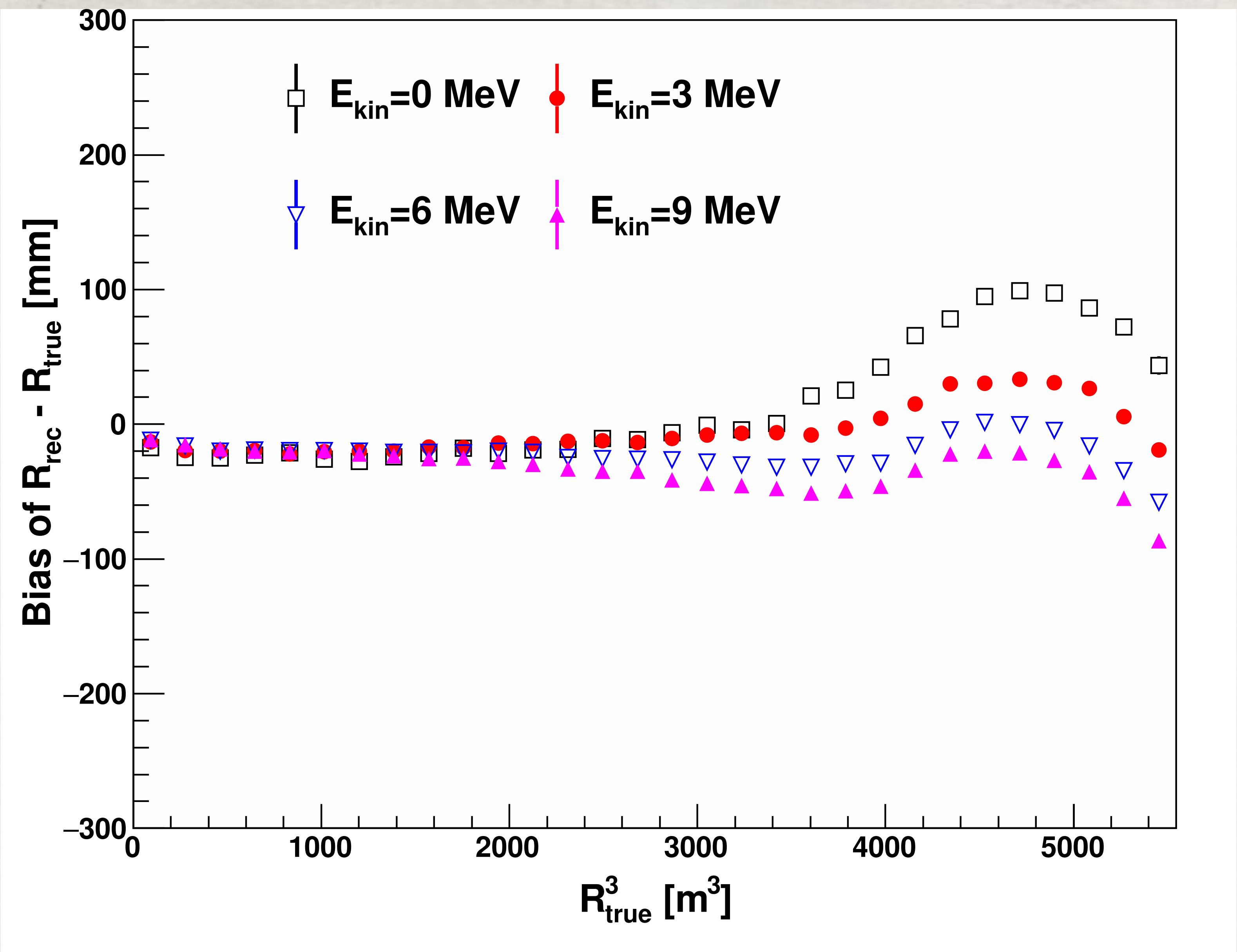
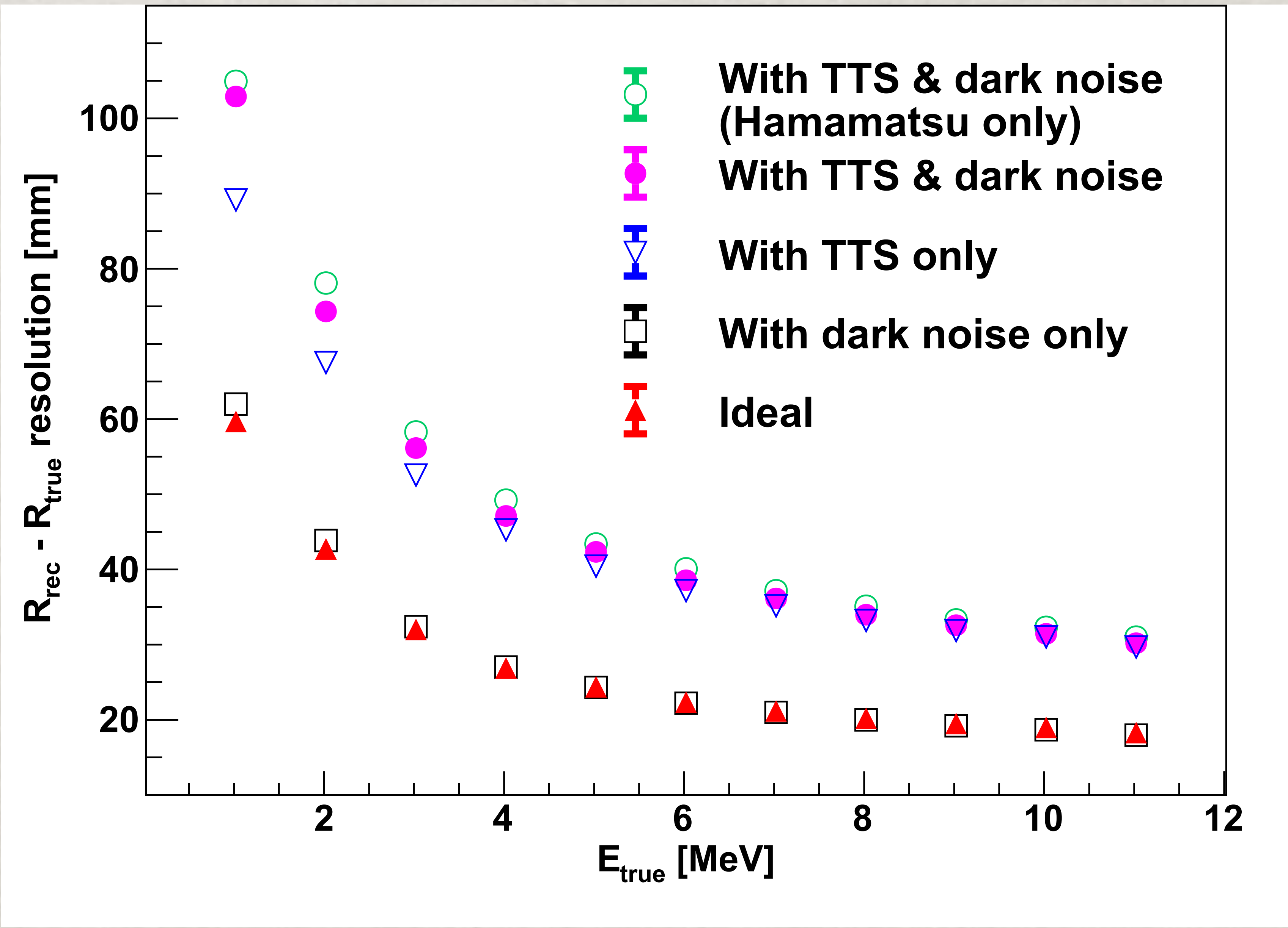
PERFORMANCE

- ❖Bias near the detector edge
- ❖PMT Transit Time Spread(TTS) is the dominant factor





LIKELIHOOD METHOD

Nucl.Sci.Tech. 34 (2023) 6, 83

Method	PMT input info	reco target
QMLE	charge only	r, E
TMLE	time only	r
QTMLE	charge & time	r, E

$$\mathcal{L}(q_1, q_2, ..., q_N|\mathbf{r}, E_{\text{vis}}) = \prod_{unfired} e^{-\mu_j} \prod_{fired} \left(\sum_{k=1}^{+\infty} P_Q(q_i|k) \times P(k, \mu_i) \right), \quad (15)$$

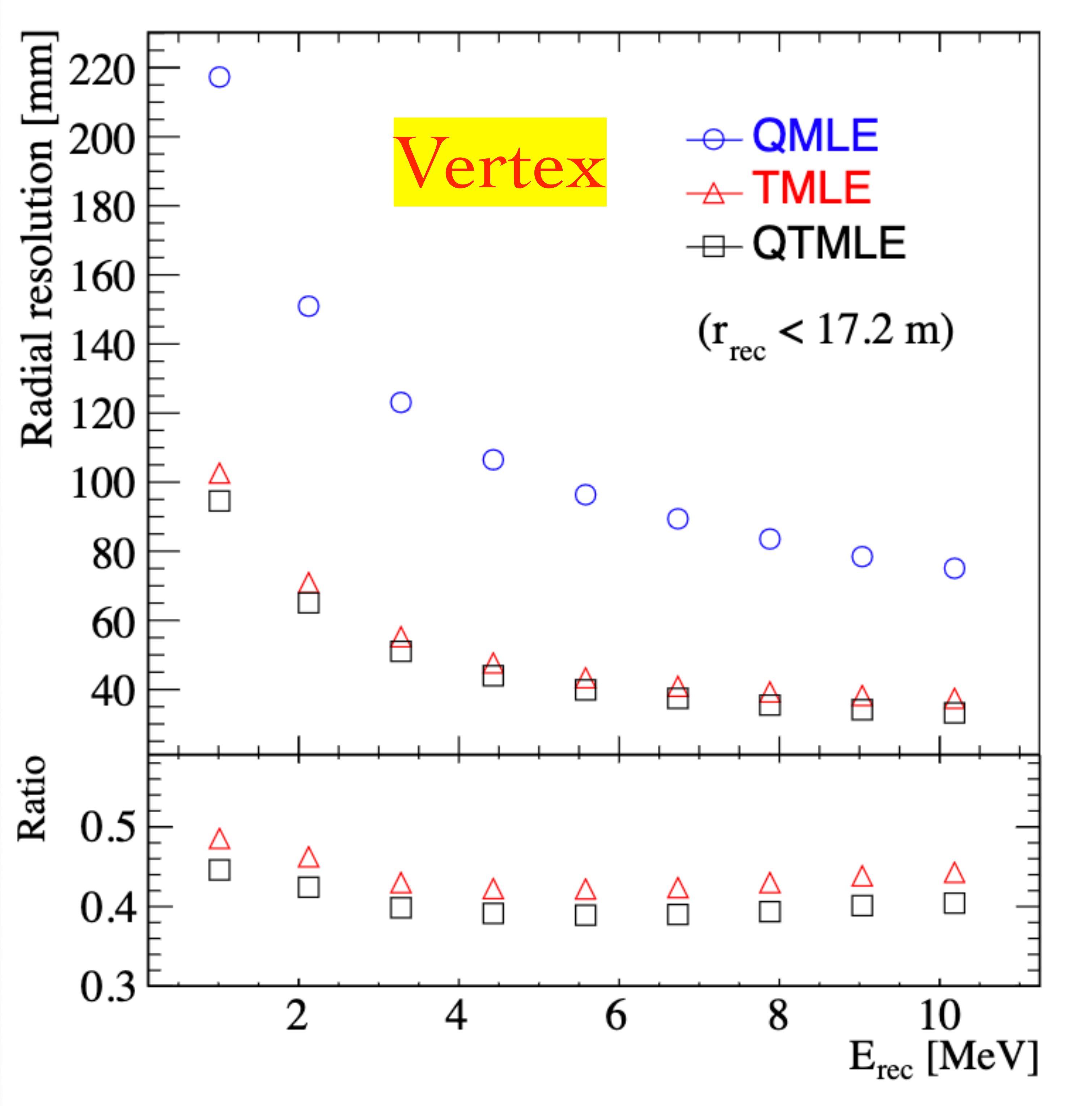
$$\mathcal{L}(t_{1,r}, t_{2,r}, ..., t_{N,r}|\mathbf{r}, t_0) = \prod_{T-\text{valid}\ hit} \frac{\sum_{k=1}^{K} P_T(t_{i,r}|r, d_i, \mu_i^l, \mu_i^d, k) \times P(k, \mu_i^l + \mu_i^d)}{\sum_{k=1}^{K} P(k, \mu_i^l + \mu_i^d)}, \quad (16)$$

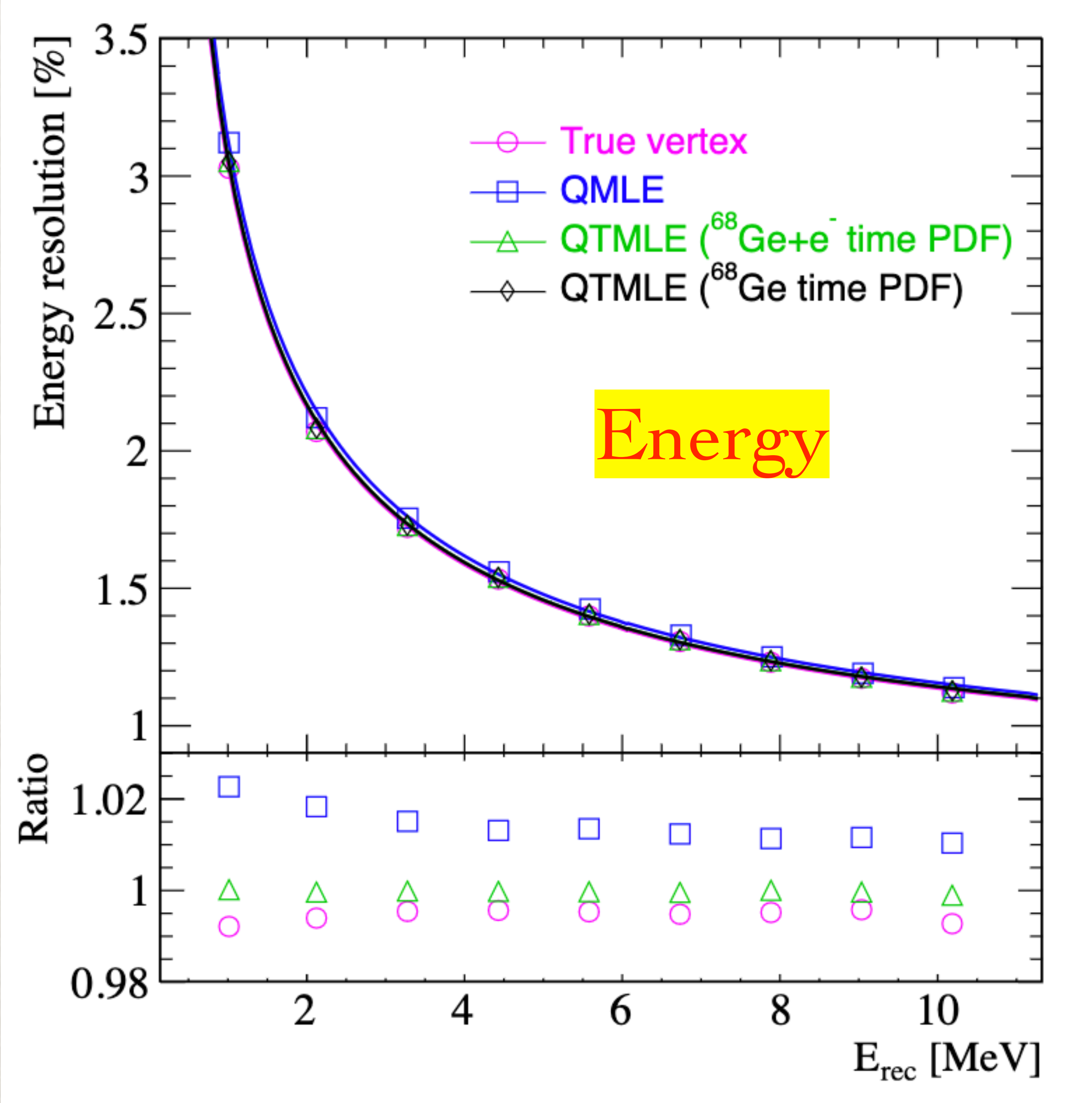
$$\mathcal{L}(q_1, q_2, ..., q_N; t_{1,r}, t_{2,r}, ..., t_{N,r}|\mathbf{r}, t_0, E_{\text{vis}}) = \prod_{unfired} e^{-\mu_j} \prod_{fired} \left(\sum_{k=1}^{+\infty} P_Q(q_i|k) \times P(k, \mu_i) \right) \prod_{T-\text{valid}\ hit} \left(\frac{\sum_{k=1}^{K} P_T(t_{i,r}|r, d_i, \mu_i^l, \mu_i^d, k) \times P(k, \mu_i^l + \mu_i^d)}{\sum_{k=1}^{K} P(k, \mu_i^l + \mu_i^d)} \right). \quad (17)$$

PERFORMANCE COMPARISON

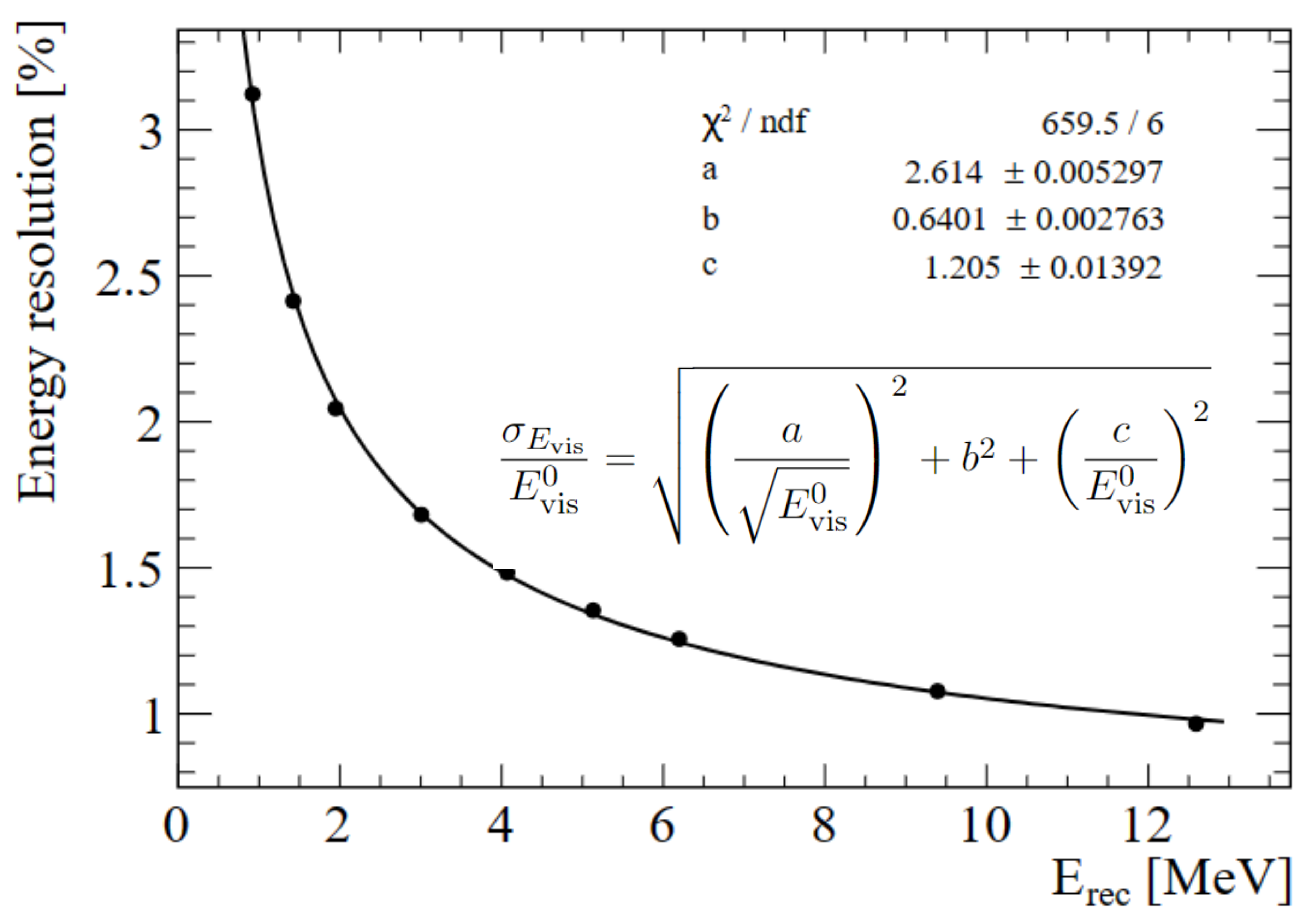
Nucl.Sci.Tech. 34 (2023) 6, 83

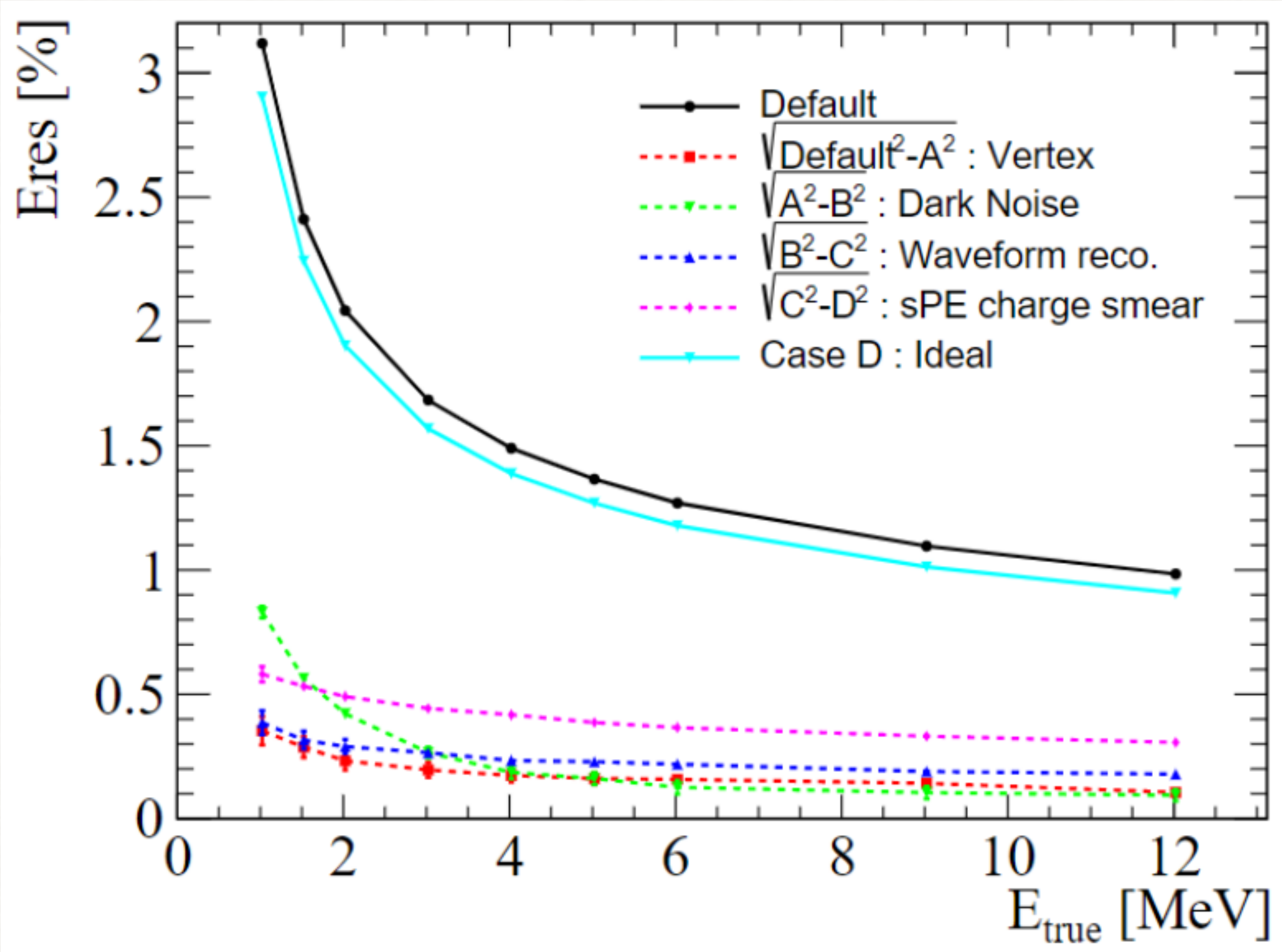
- ❖ R_{res} depends mainly on PMT time info, charge info also helps
- ❖ Impact of R_{res} on E_{res} is ≤ 0.6% for QTMLE





REACTOR NEUTRINOS

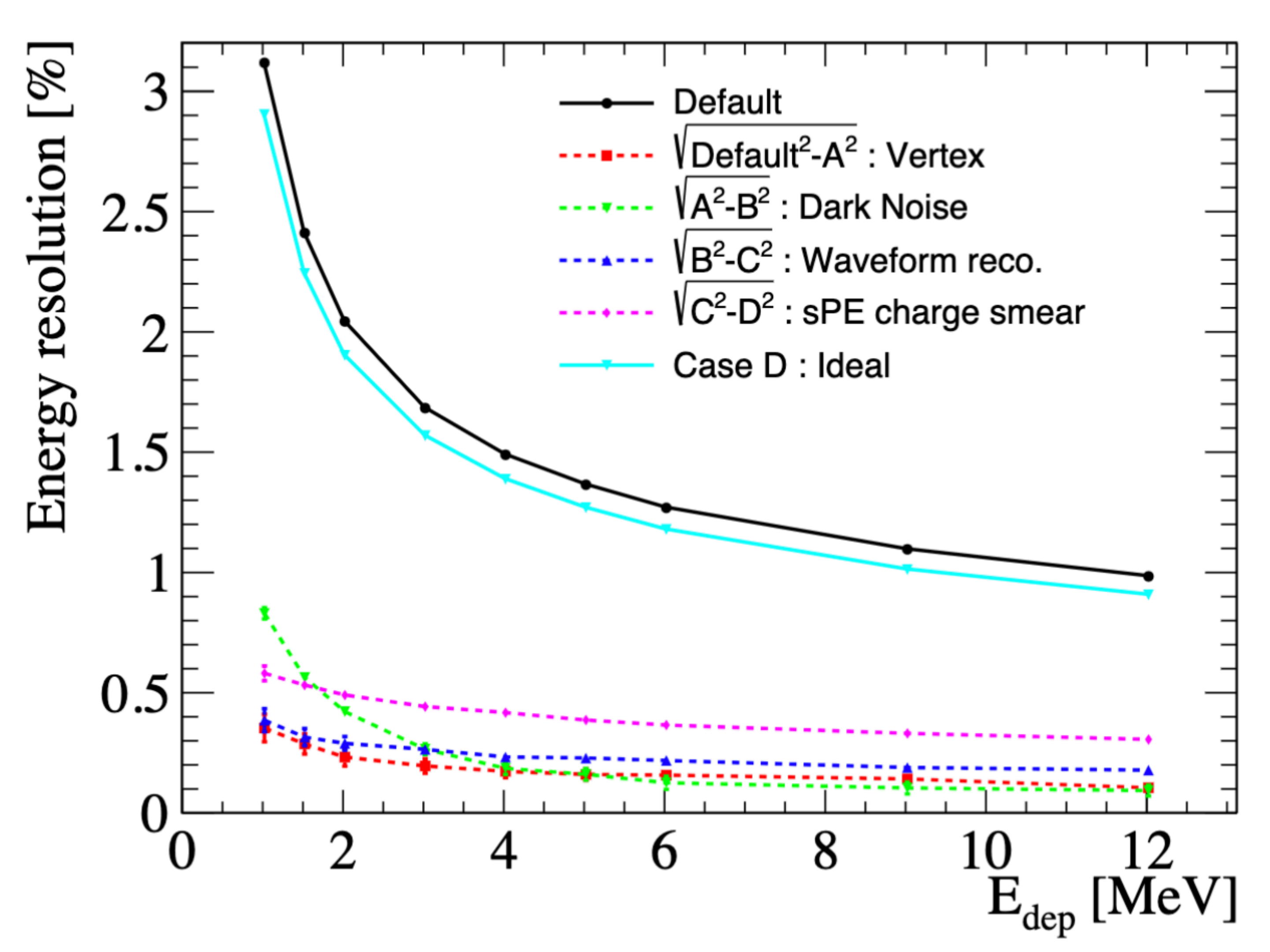




- Latest predicted Eres 2.95%@1MeV
- Decomposition of the Energy Resolution
  - PMT dark noise, charge smearing

PMT WAVEFORM PHOTON COUNTING

PMT WAVEFORM PHOTON COUNTING



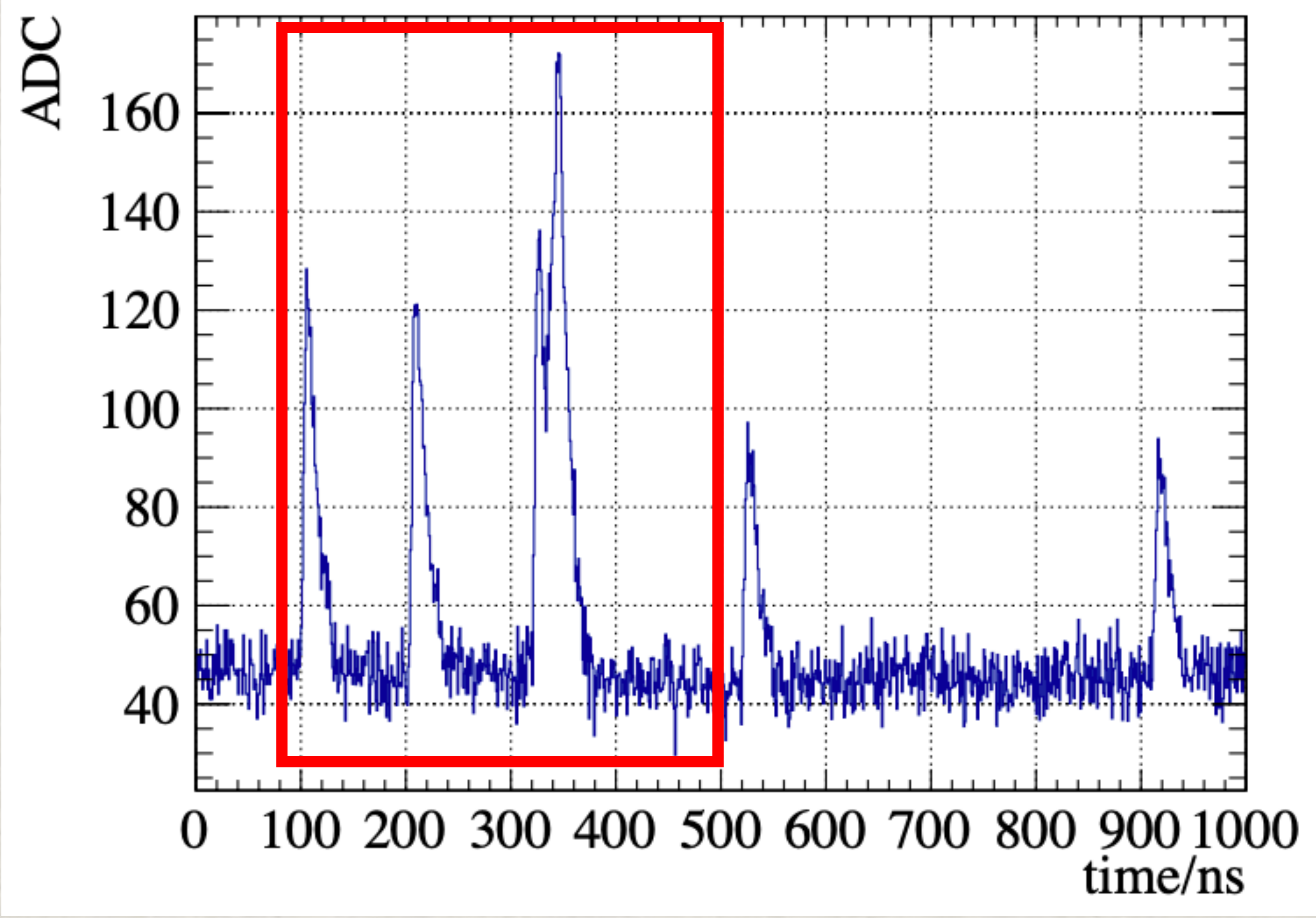
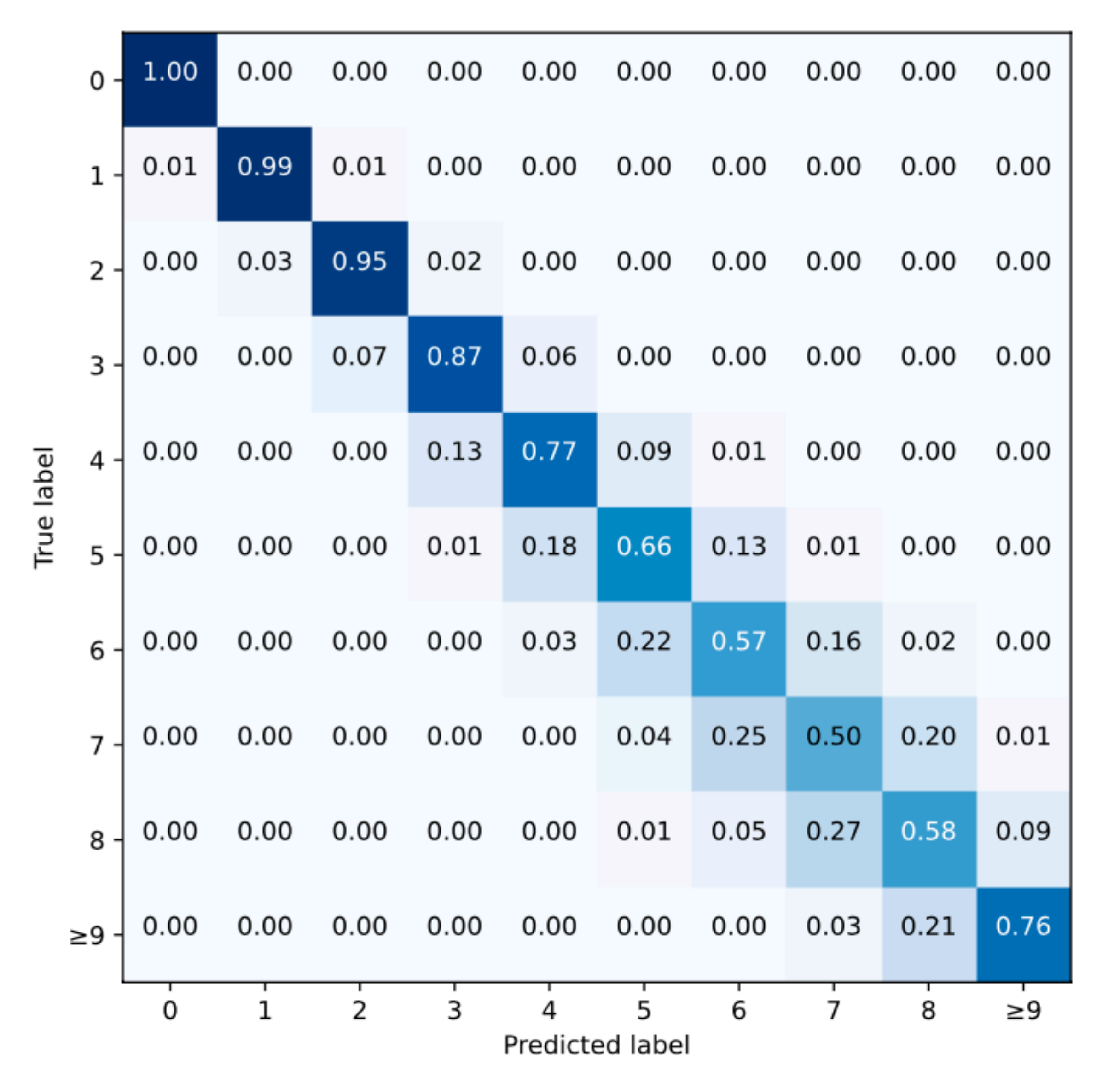


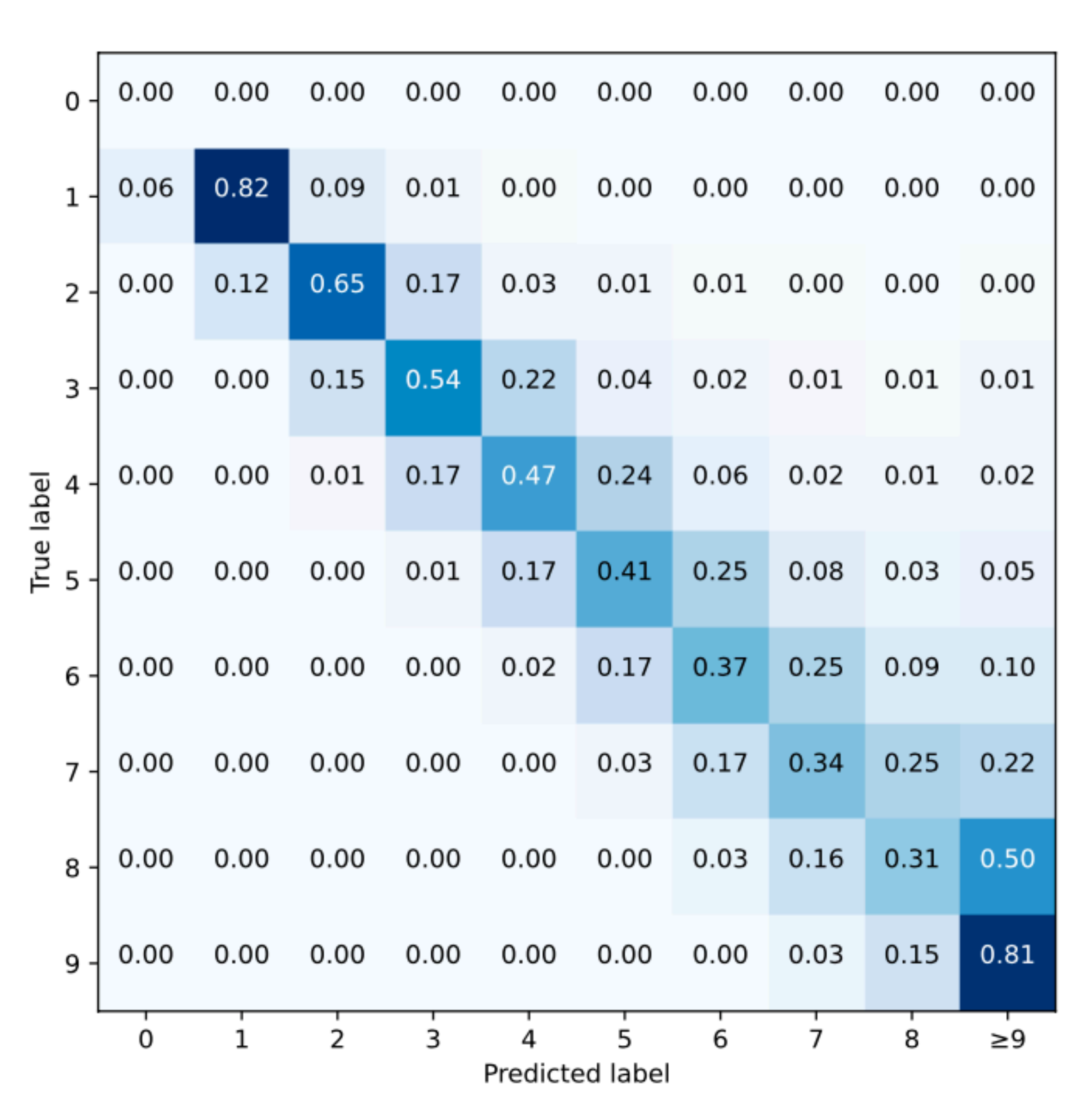
Table 2: Modified RawNet architecture. For convolutional layers, numbers inside parentheses refer to filter length, stride size, and number of filters. For gated recurrent unit (GRU) and fully-connected layers, numbers inside the parentheses indicate the number of nodes.

Layer	Input	Output shape
Strided -conv	Conv(3,3,128) BN LeakyReLU	(128, 140)
Res block	{ Conv(3,1,128) BN LeakyReLU Conv(3,1,128) BN - - - - - - LeakyReLU MaxPool(3) } ×2	(128, 46)
Res block	{ Conv(3,1,256) BN LeakyReLU Conv(3,1,256) BN - - - - - - LeakyReLU MaxPool(3) } ×2	(256, 1)
GRU	GRU(1024)	(1024,)
Speaker embedding	FC(128)	(128,)
Output	FC(10)	(10,)

- ❖*Input*: pre-processed PMT waveform within 420ns signal window
- ❖*Model*: Customized RawNet
- ❖*Output*: $\{p_k\}$ the probability for predicting ($k=0,1, \ldots \geq 9$) PEs

PHOTON COUNTING PERFORMANCE





- ❖ Left: Confusion matrix of RawNet
  - ❖99% (95%, 87%) accuracy for 1PE (2PEs, 3PEs)
  - ❖Accuracy decreases rapidly as nPEs increases
- ❖ Right: Confusion matrix based on charge classification
  - ❖The accuracy is markedly inferior to that of RawNet

ENERGY RECONSTRUCTION

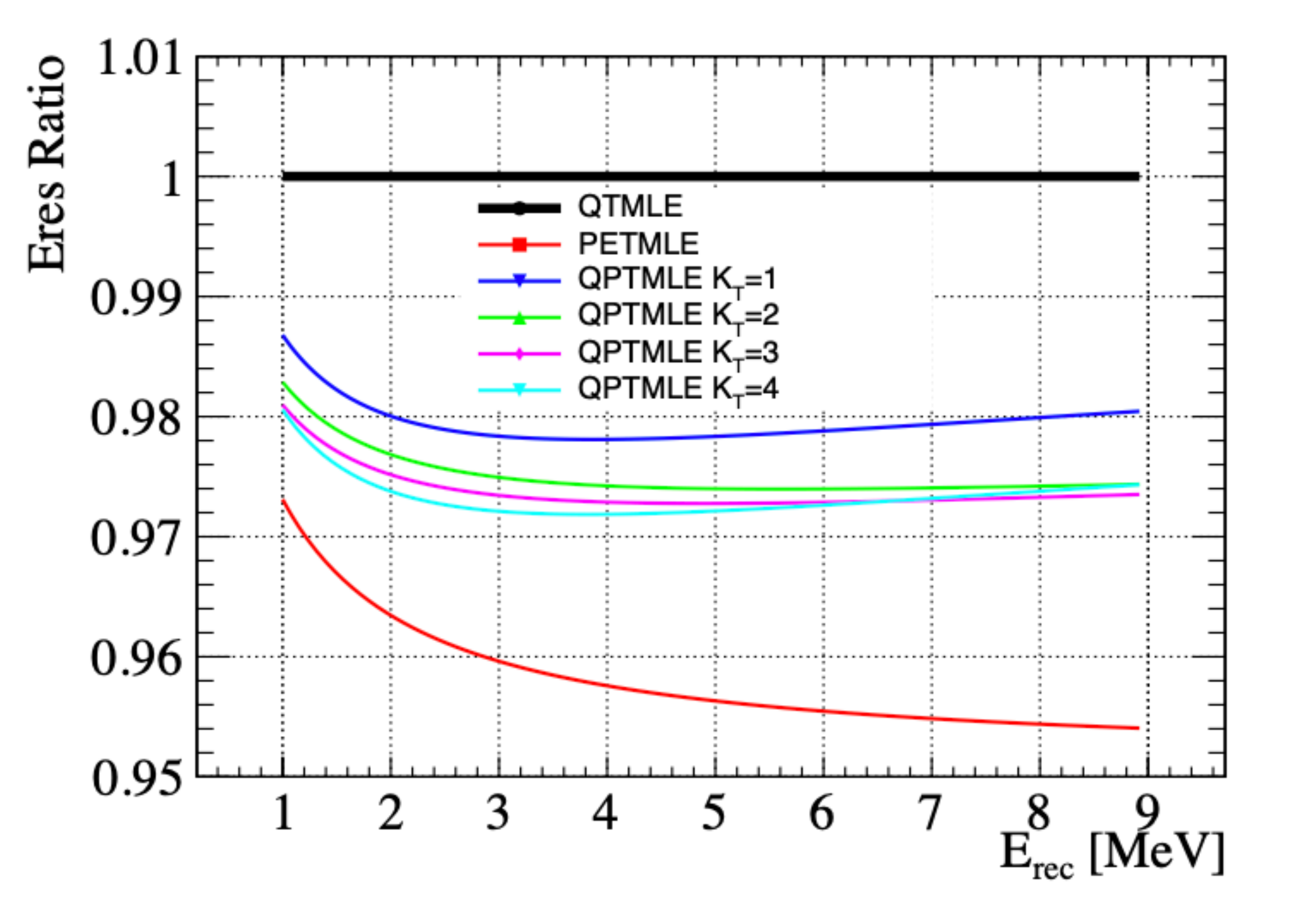
Algo. Name	Observable	Likelihood: $\kappa \le K_T$	Likelihood: $\kappa > K_T$
QTMLE (reference)	q (charge)	$\mathcal{L}(q_i\|\mu_i) = \sum_{k=1}^{+\infty} P_Q(q_i\|k)P(k,\mu_i)$	
PETMLE (ideal)	k (true PEs)	$\mathcal{L}(k_i\|\mu_i) = P(k_i,\mu_i)$	
QPTMLE (realistic)	$\{p_k\}$, q	$\mathcal{L}(\{p_k^i\}\|\mu_i) = \sum_{k=0}^{9} R_{K_T k} p_k^i P(k,\mu_i)$,	
QPETMLE (100% accuracy)	$k(p_k=1)$, q	$\mathcal{L}(k_i\|\mu_i) = P(k_i,\mu_i)$	$\mathcal{L}(q_i\|\mu_i) = \sum_{k=1}^{+\infty} P_Q(q_i\|k)P(k,\mu_i)$
QCTMLE	κ (p_κ:max), q	$) = \sum_{k=0}^{9} C_{k\kappa_i} \times P(k,\mu_i)$	

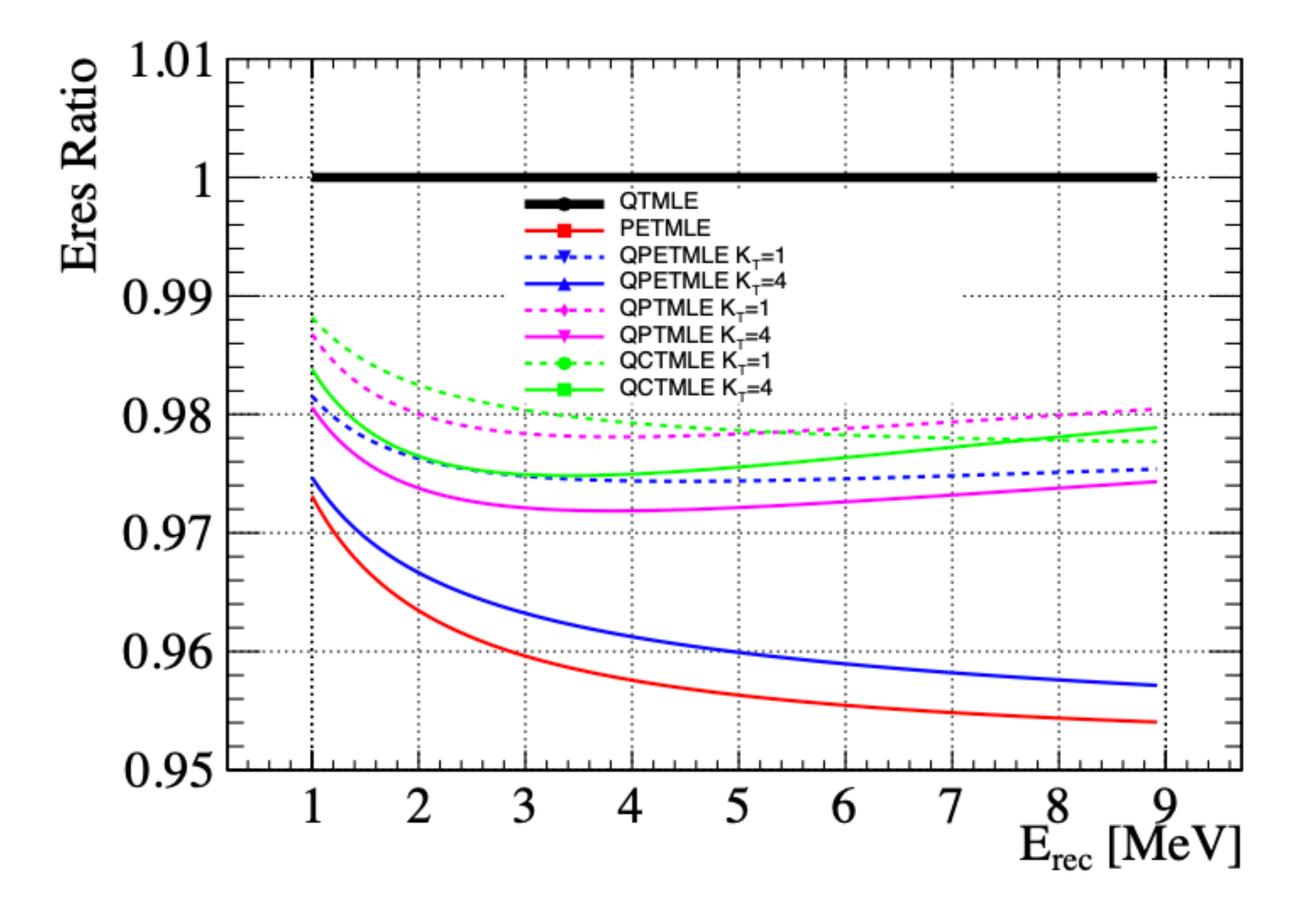
where μ_i is the expected nPEs for the i-th PMT, $P(k,\mu_i)$ is just the Poission probability of observing k p.e. given μ_i and $P_Q(q_i|k)$ is the charge pdf for k p.e..

$$R_{K_T k} = \sum_{\kappa=0}^{K_T} C_{k\kappa}$$

confusion matrix $C_{kk'}$

ENERGY RESOLUTION





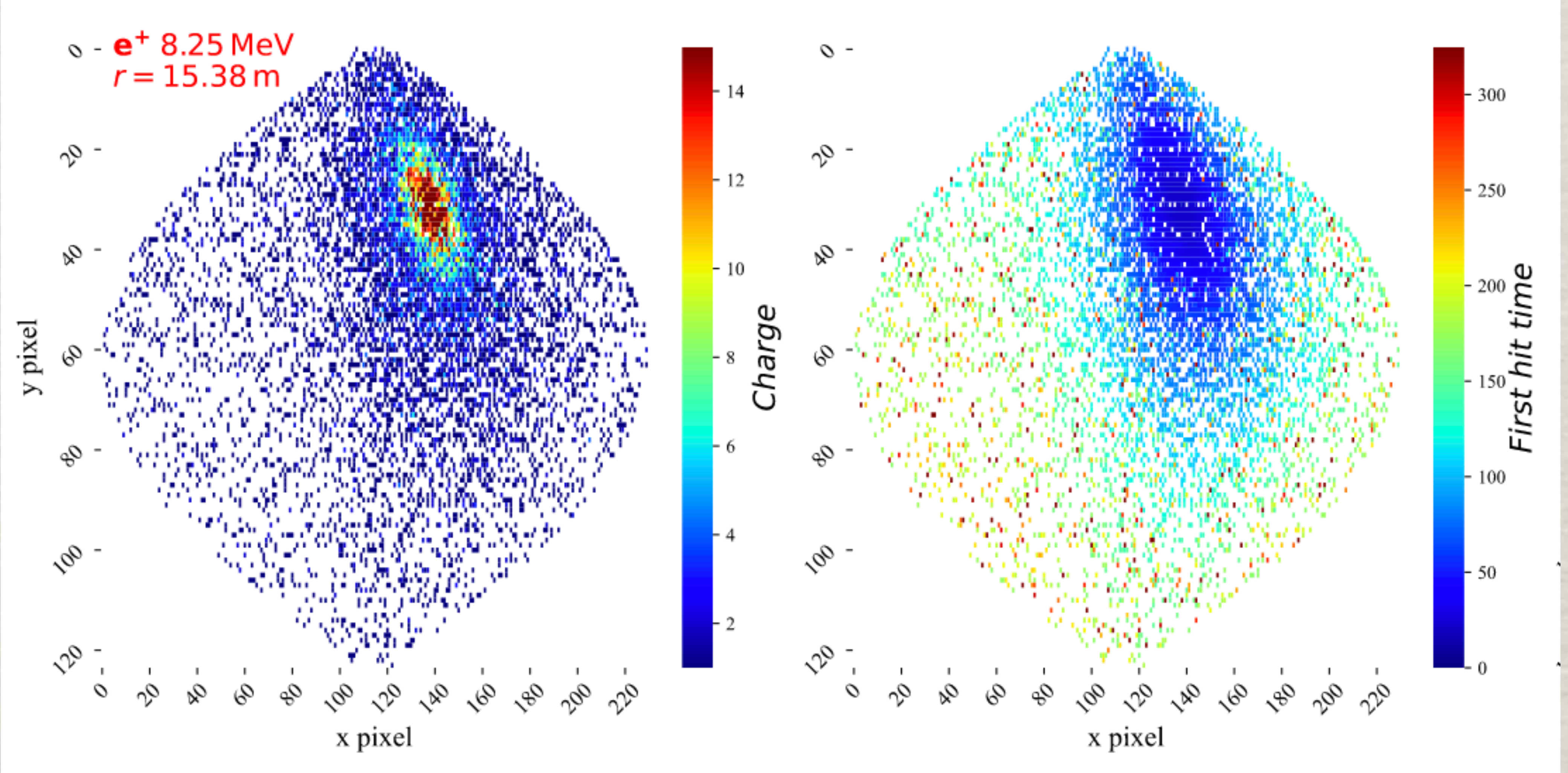
- ❖Using the photon counting info for PMTs with ($\kappa \leq K_T$) PEs can improve the energy resolution
- ❖The improvement becomes smaller as K_T increases due to the dropping accuracy for high PEs
- ❖Additional checks were done to validate the results

Vertex reco with ML

PRINCIPLES

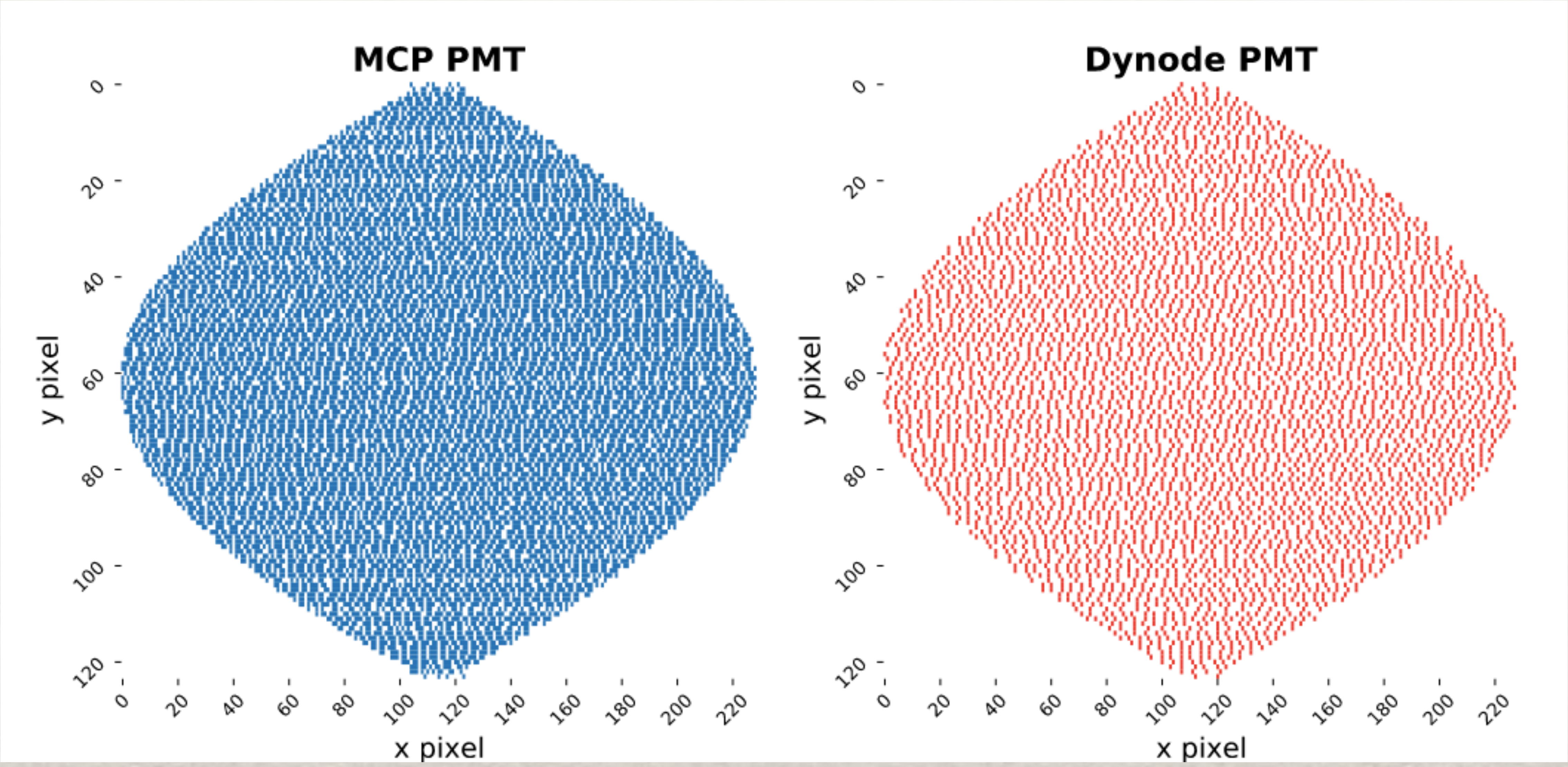
NIMA 1010 (2021) 165527

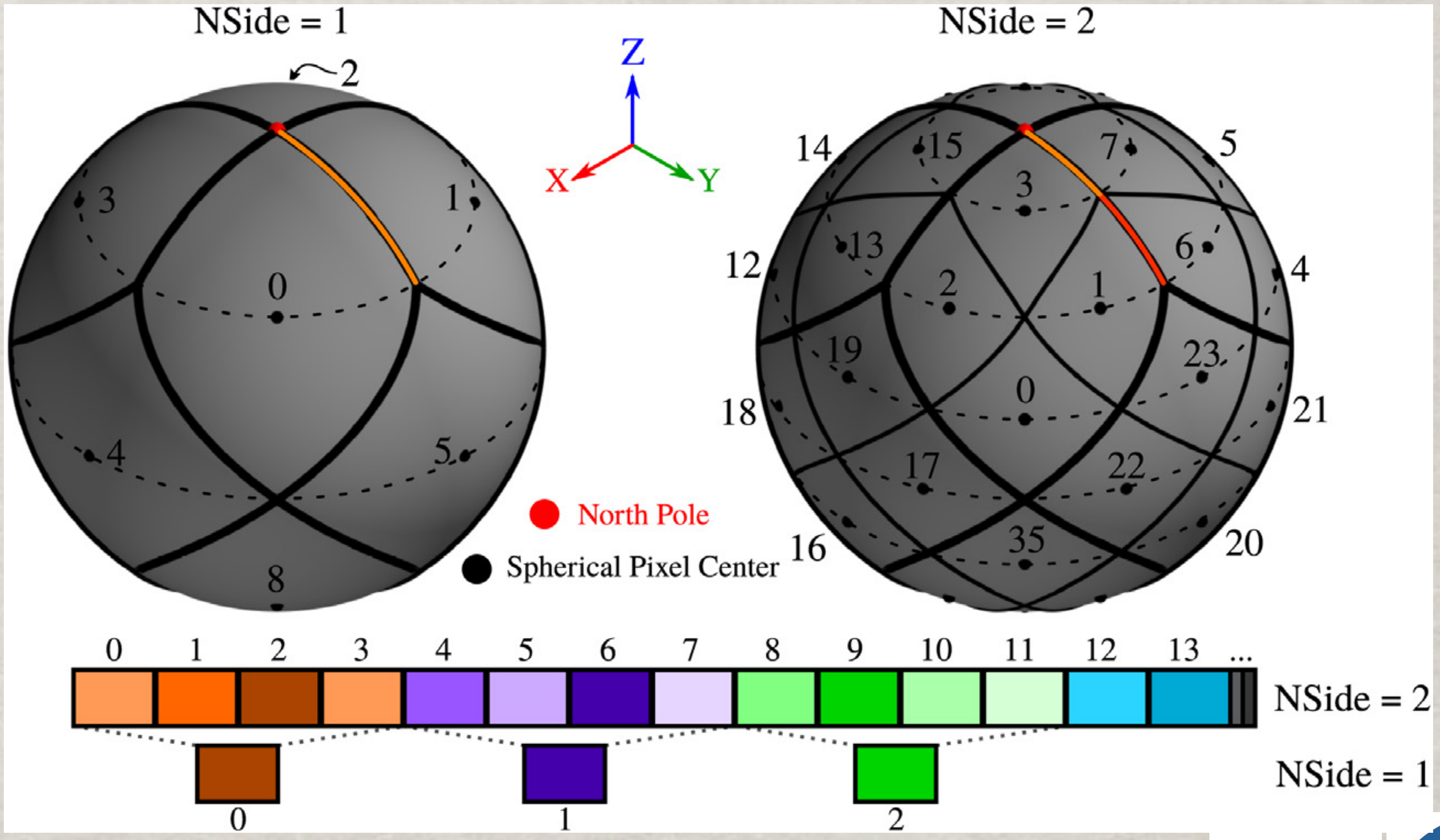
- Large number of PMTs O(10^5) installed on a sphere
  - each PMT as a pixel —> JUNO as a Camera
  - ensemble of PMTs charge/time form an image
- Image is highly vertex and energy dependent
- Vertex/energy reconstruction <—> Image recognition



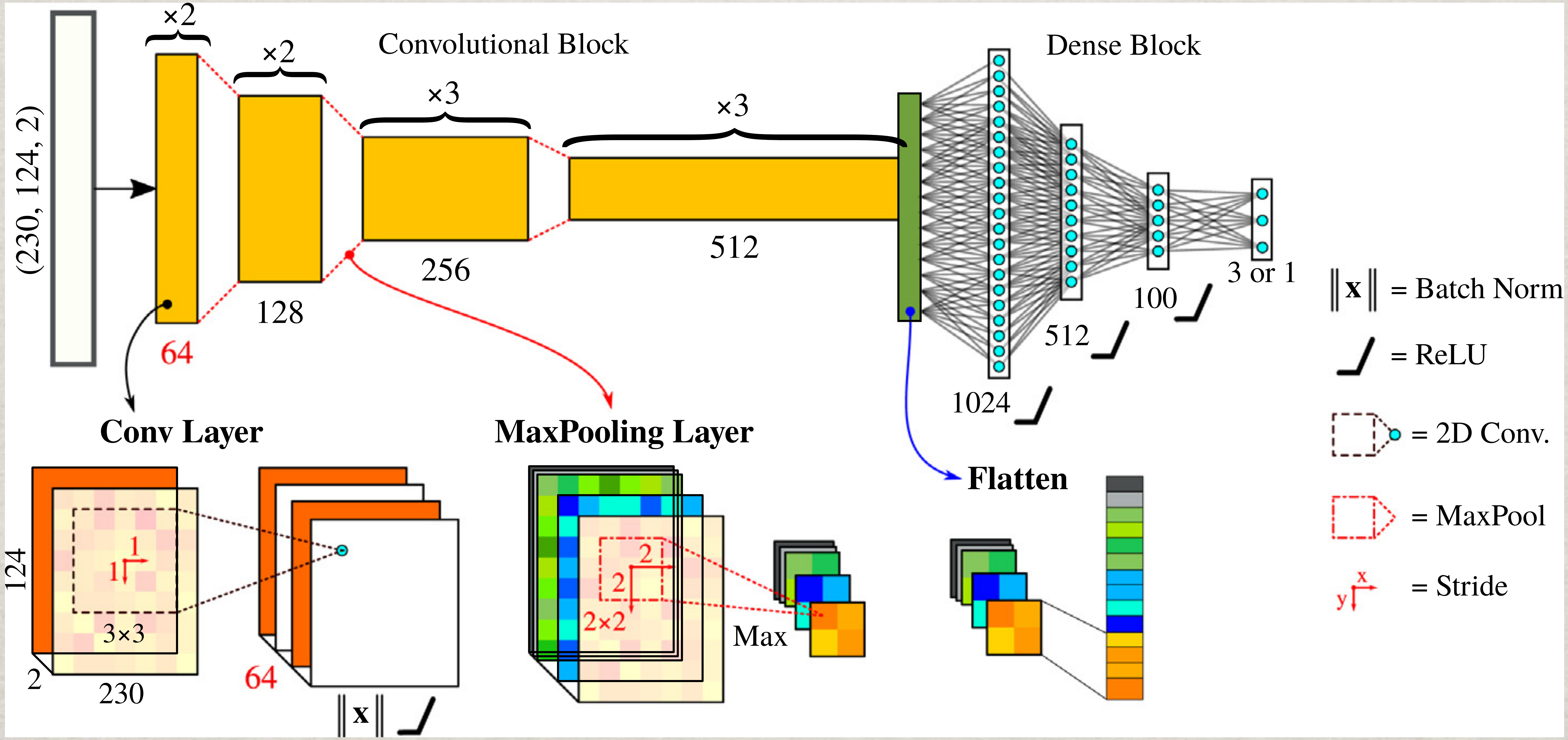
INPUTS

- ❖Large number of PMTs O(10^5) installed on a sphere
- ❖Method 1: projection to 2D plane —>Plane CNN
- ❖Method 2: HEALPix —> plane/spherical CNN
- ❖Method 3: 3D models such as pointNet++/Transformer



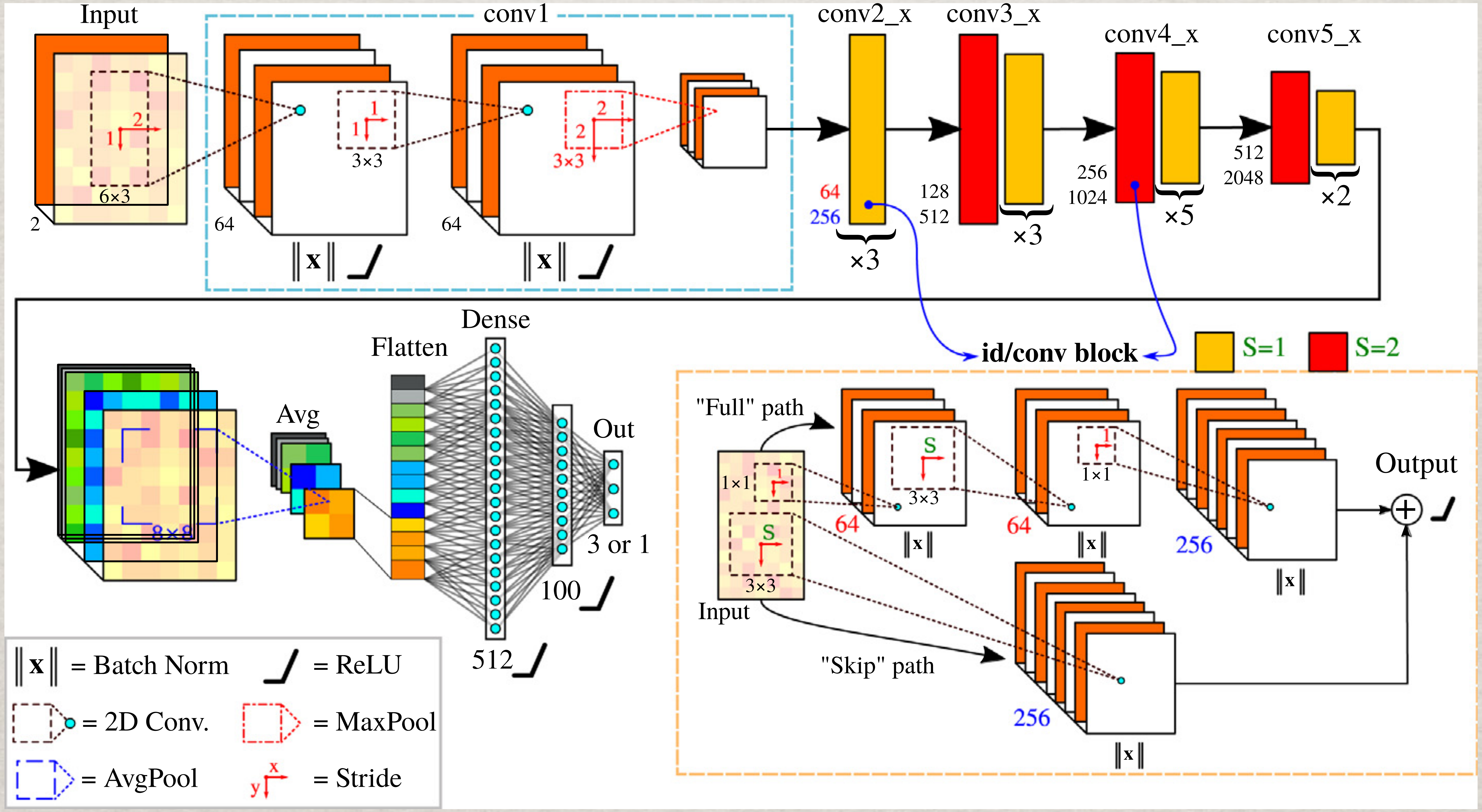


MODELS: VGG-J

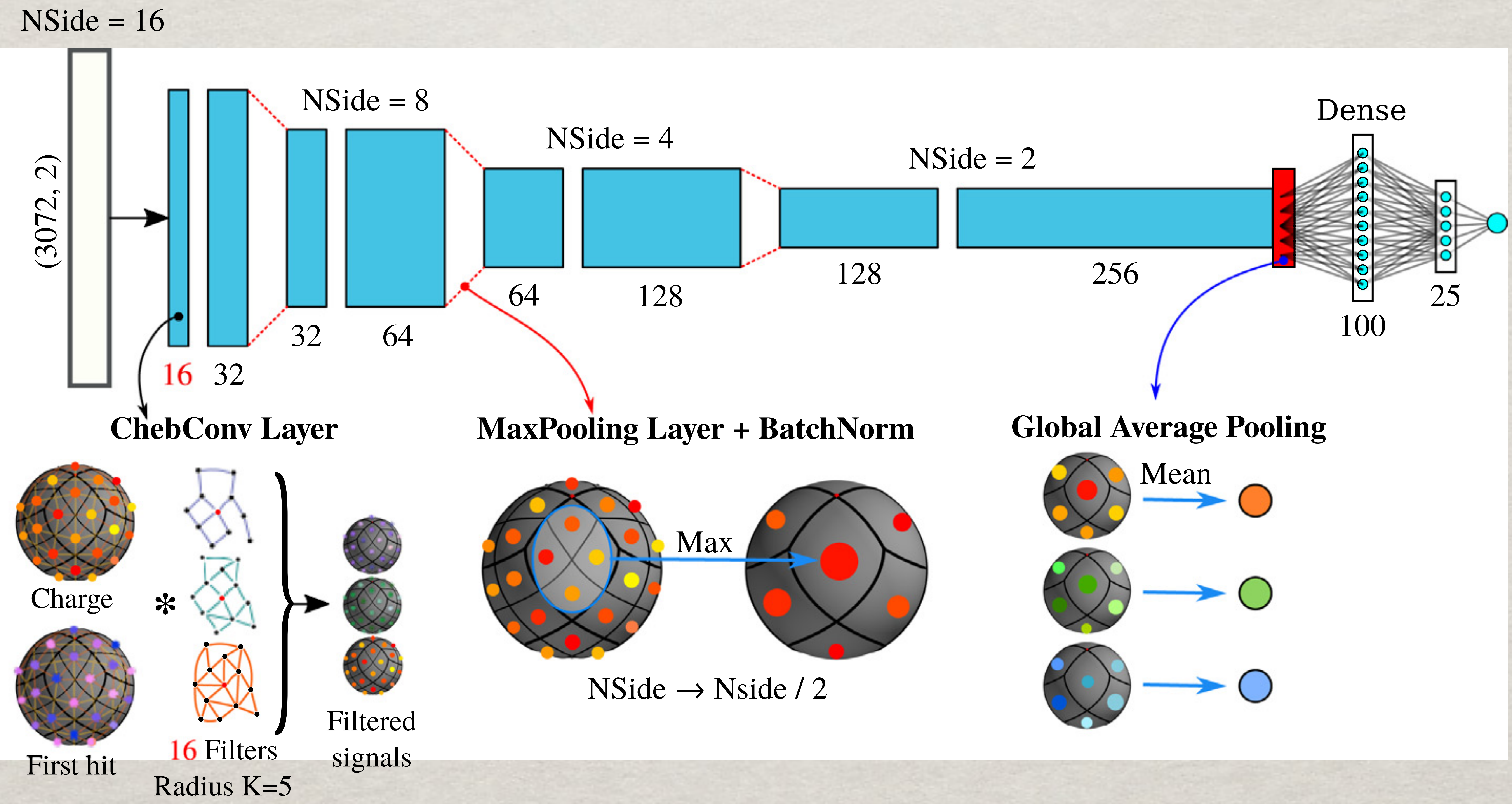


(230, 124, 2)
×2
×2
Convolutional Block
×3
×3
64
128
256
512
Dense Block
1024
512
100
3 or 1
Conv Layer
MaxPooling Layer
Flatten
124
2
230
3×3
64
2×2
Max
x
= Batch Norm
= ReLU
= 2D Conv.
= MaxPool
= Stride

MODELS: RESNET-J

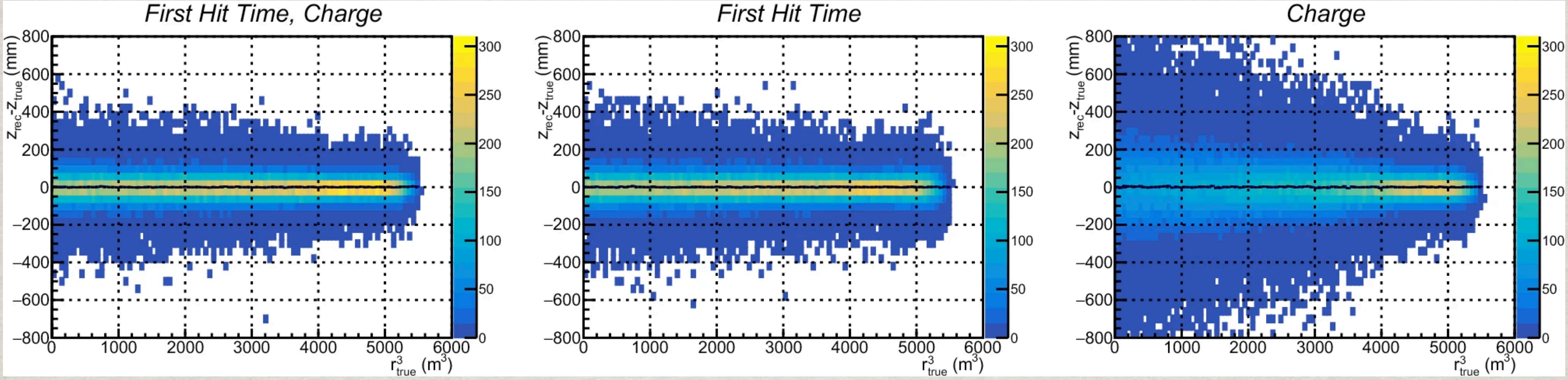


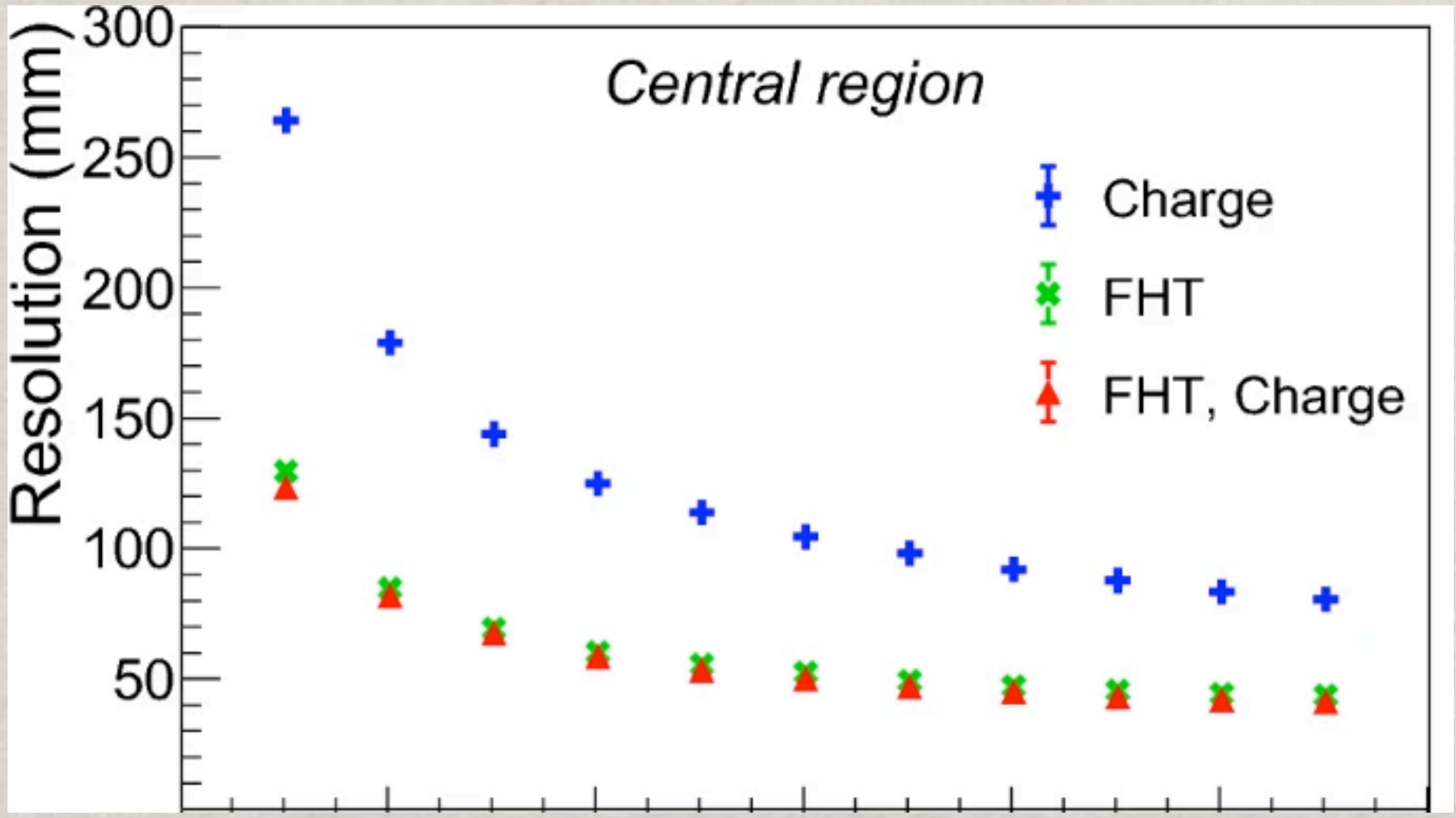
MODELS: GNN-J

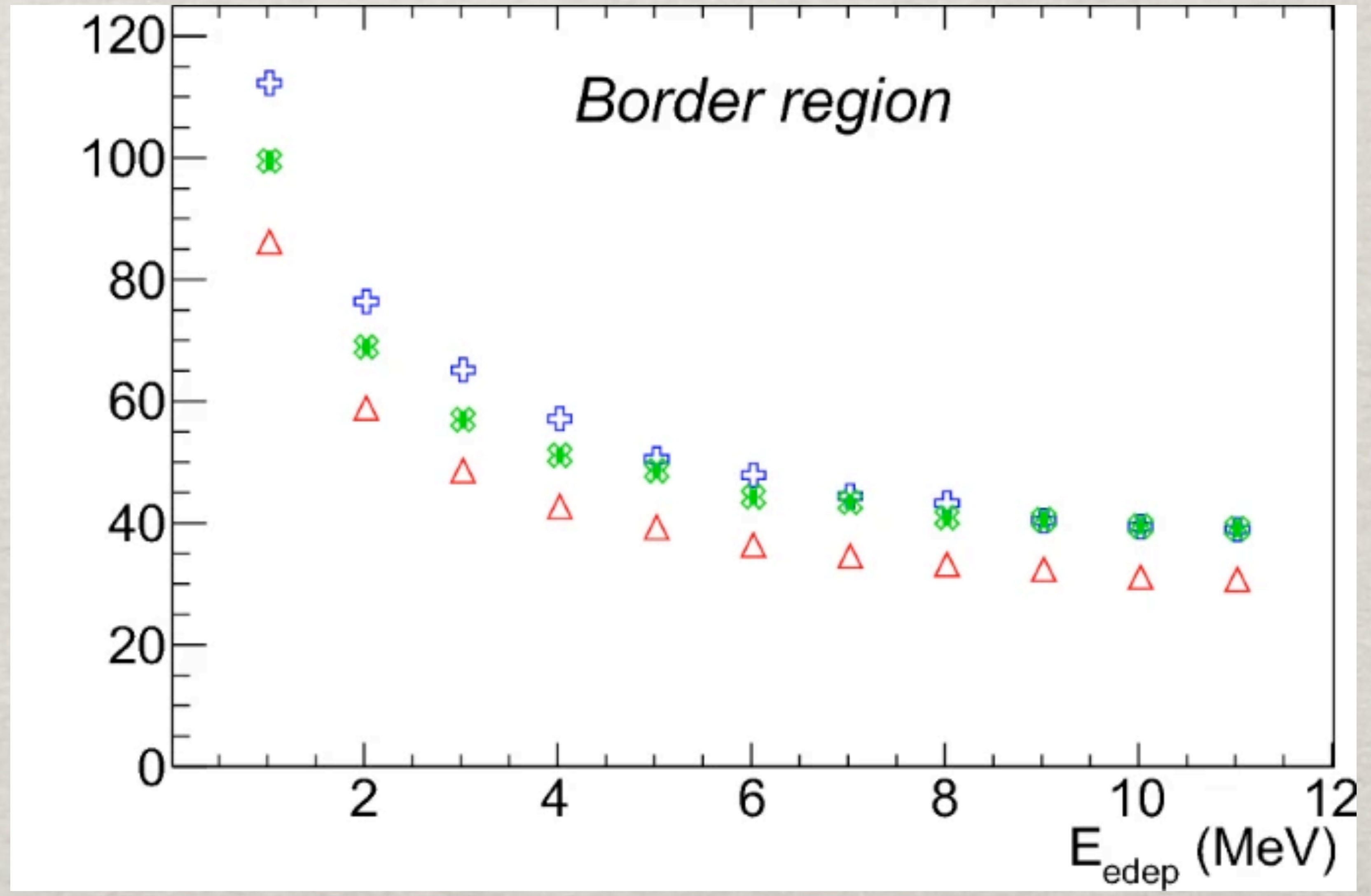

NSide = 16
(3072, 2)
16
32
NSide = 8
32
64
NSide = 4
64
128
NSide = 2
128
256
Dense
100
25
ChebConv Layer
MaxPooling Layer + BatchNorm
Global Average Pooling
Charge
First hit
*
16 Filters
Radius K=5
Filtered signals
Max
NSide → Nside / 2
Mean

Charge vs Time

Nucl.Sci.Tech. 33 (2022) 7, 93



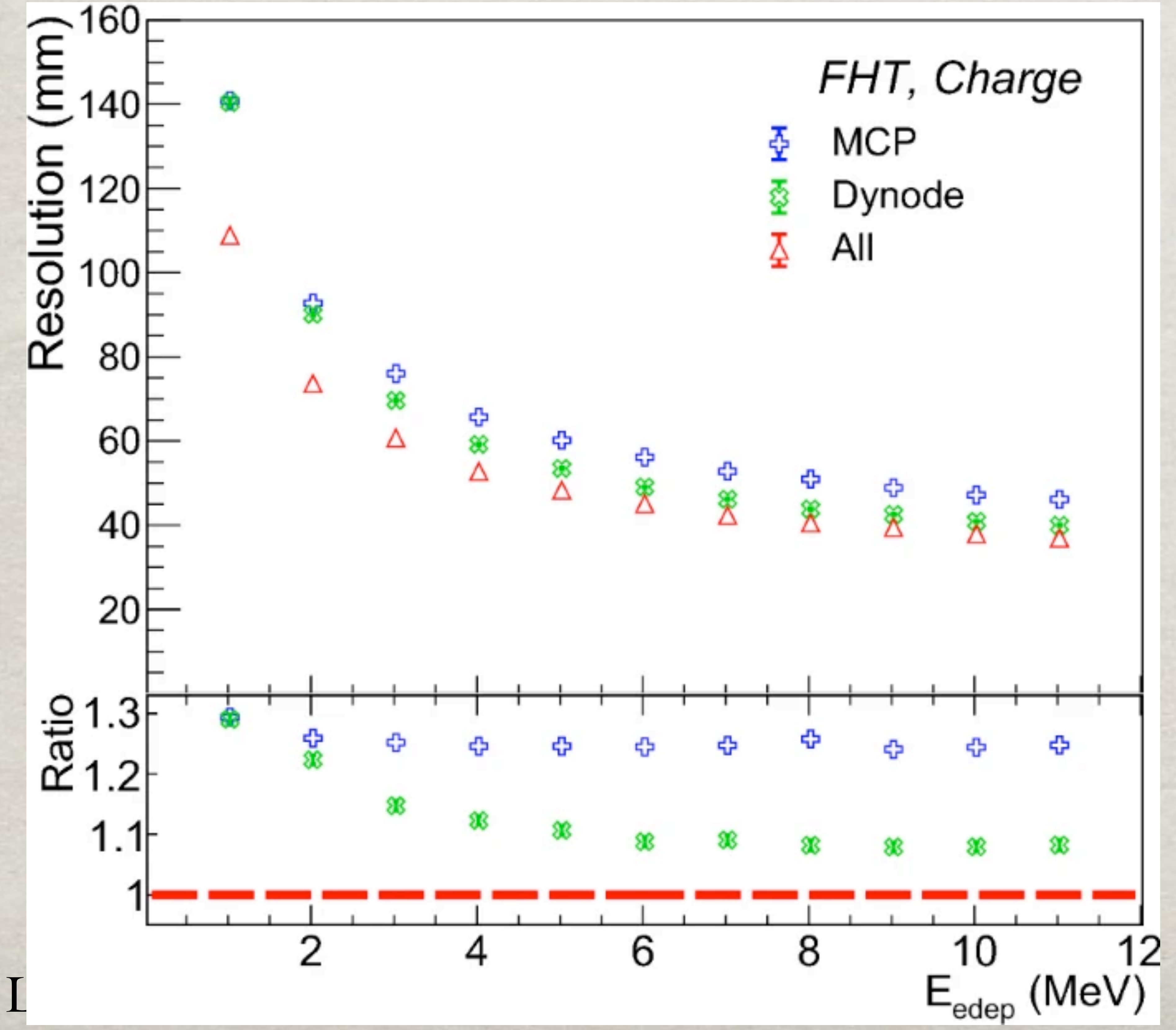


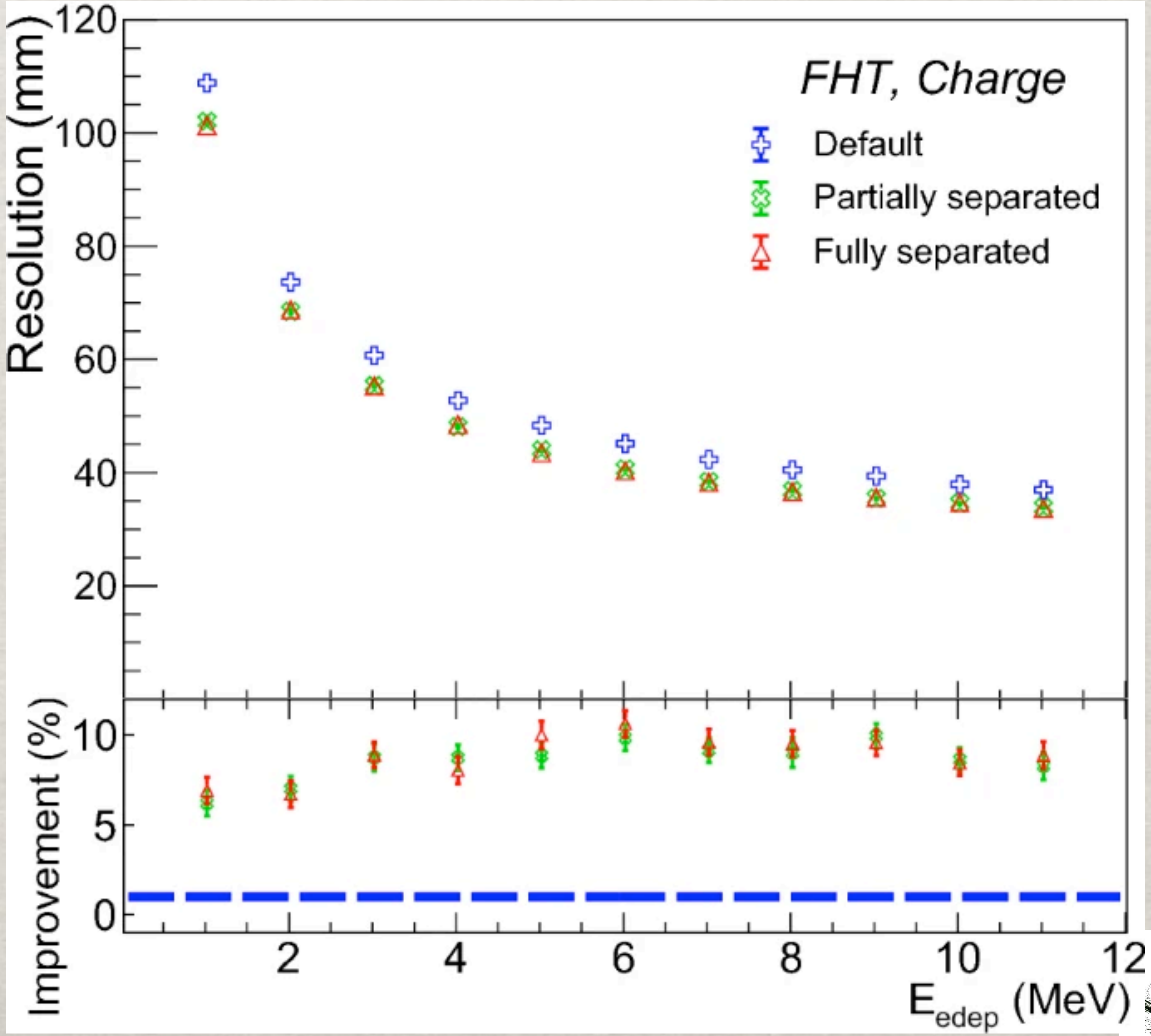


MCP vs Dynode

Nucl.Sci.Tech. 33 (2022) 7, 93

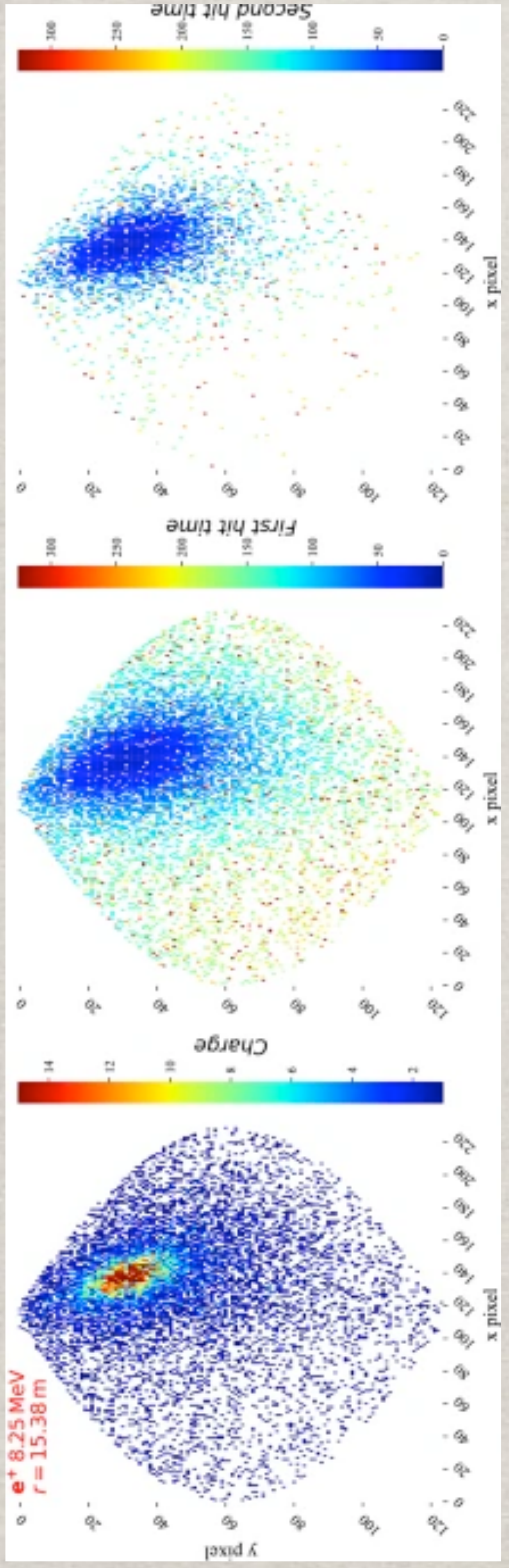
- ❖Number: 12612 vs 5000
- ❖Time resolution(σ_{tts}): 12 ns vs 2.8 ns

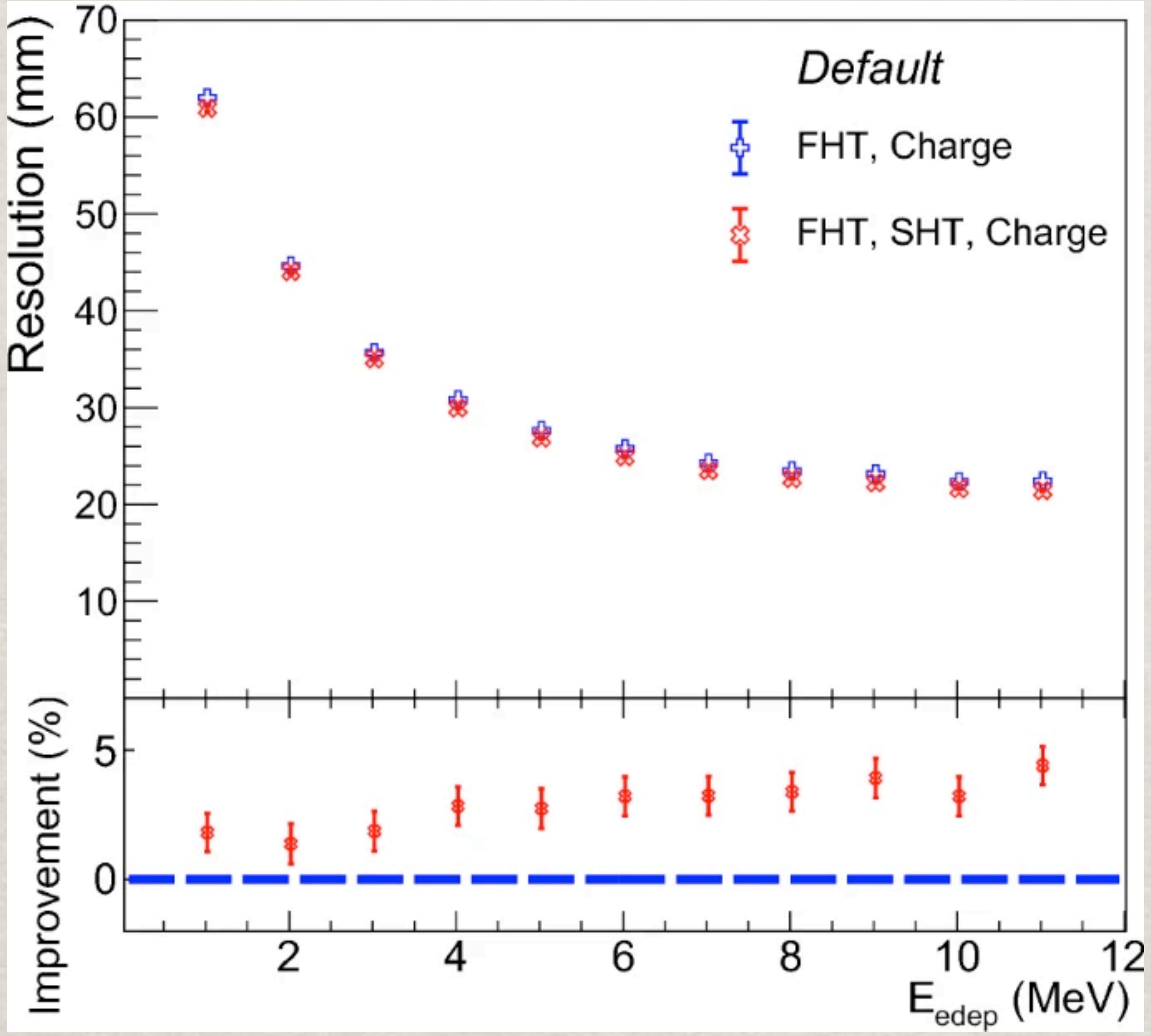


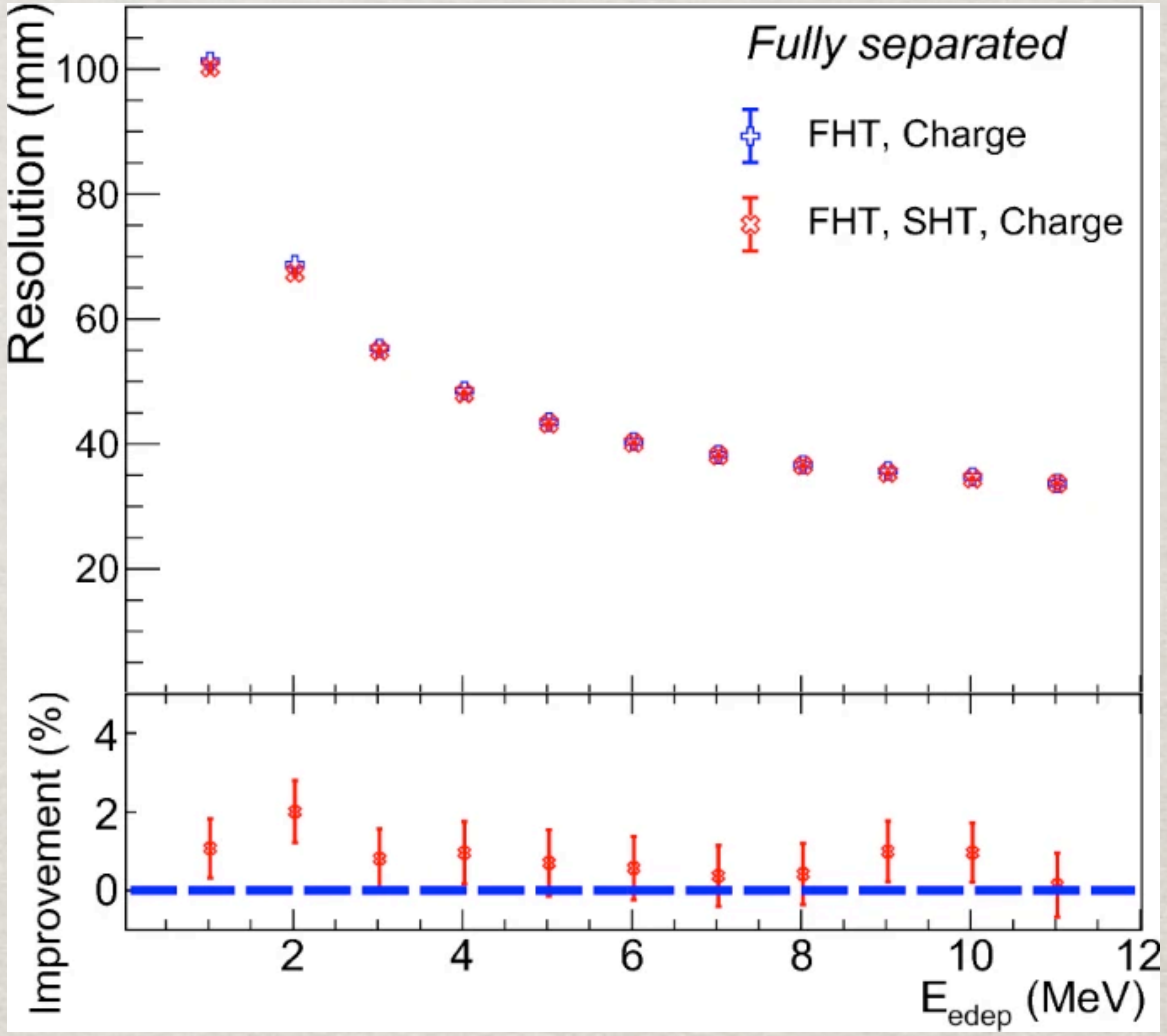


Addition of 2nd Hit

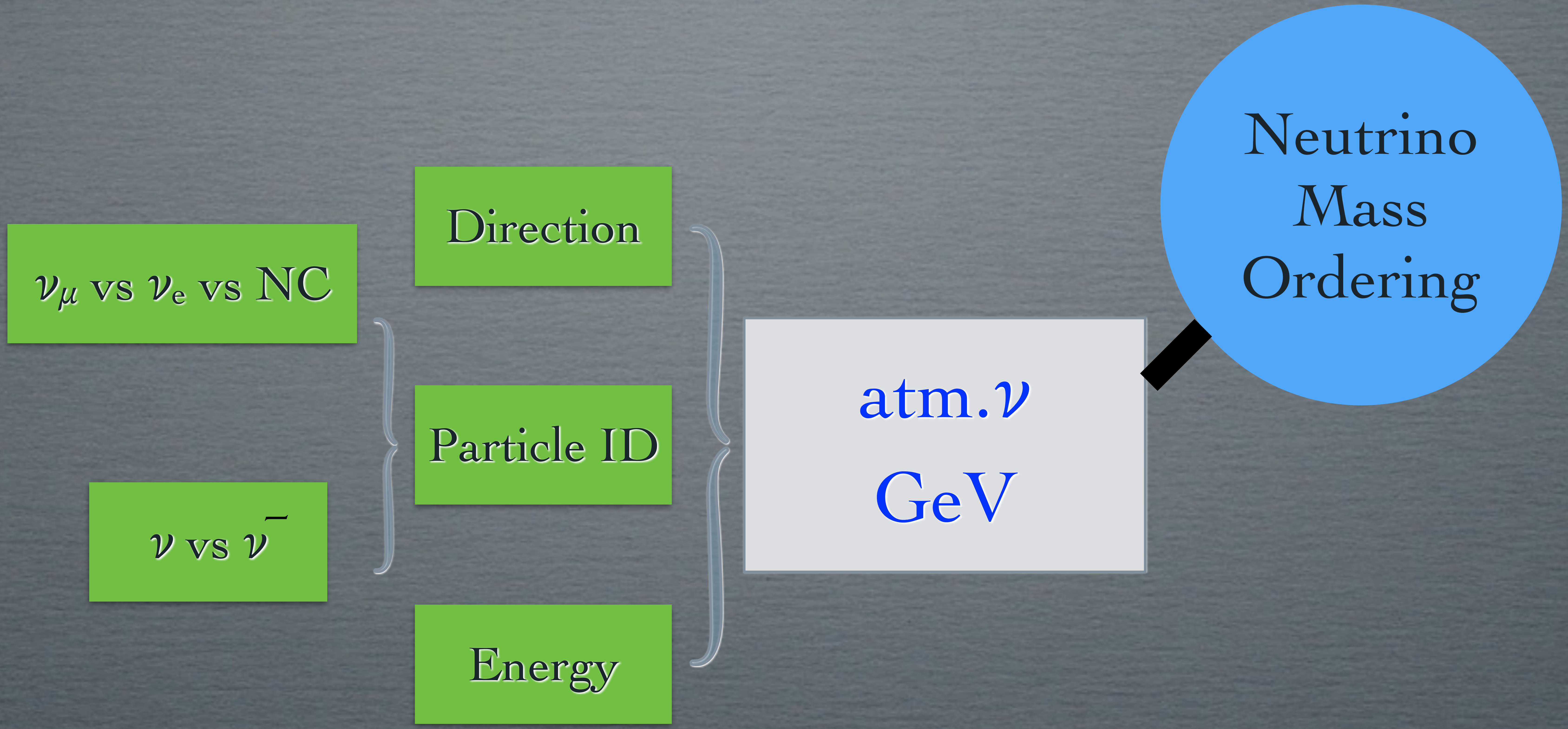
Nucl.Sci.Tech. 33 (2022) 7, 93







- ❖Later hits are also useful in principle
- ❖PMT Time resolution is the key

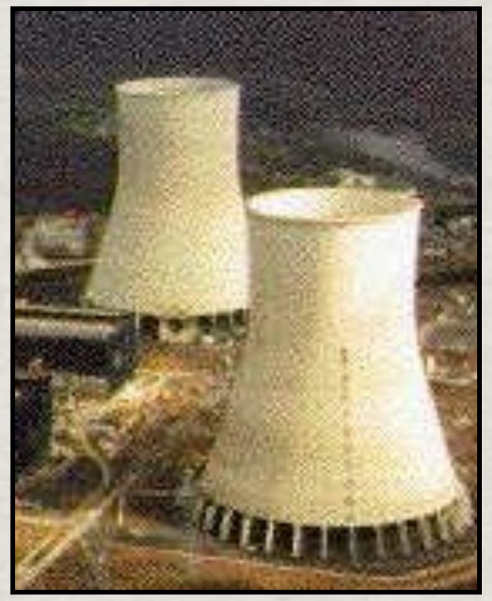

ν_μ vs ν_e vs NC
ν vs $\bar{\nu}$
Direction
Particle ID
Energy
atm.ν
GeV
Neutrino
Mass
Ordering

JUNO NMO SYNERGY

- NMO @ 6years $\Delta\chi^2$: Reactor(~ 9), atm.(~ 1.96), $|\Delta m^2_{ee}|$(4|1.5% or 9|1%)
- 1.96 of atm. was estimated with assumptions
- Can we do better than Yellow Book?

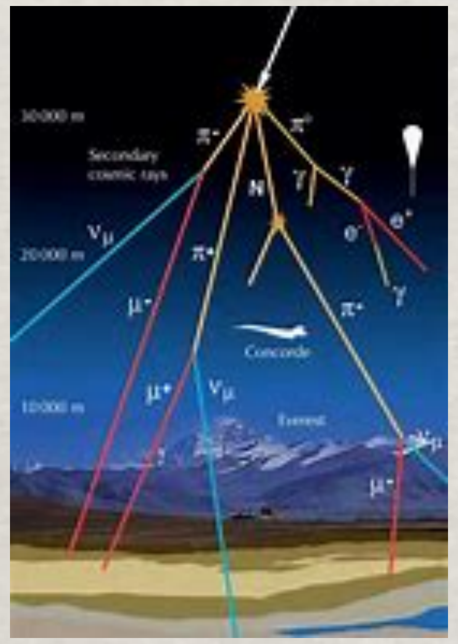
	Yellow Book assumptions
Event Selection $\nu_e/\bar{\nu}_e$	E_{vis} > 1GeV $Y_{vis}=E_h/E_{vis}$ < 0.5
Directionality	$\sigma_{\theta\mu} = 1^\circ$ $\sigma_{\theta\nu} = 10^\circ$
Classification	CC-e vs CC-μ vs NC: 100% eff.
	ν vs $\bar{\nu}$: Ne, Y_{vis},
Energy	σ_{Evis} = 1%/√E

reactor ν
MeV

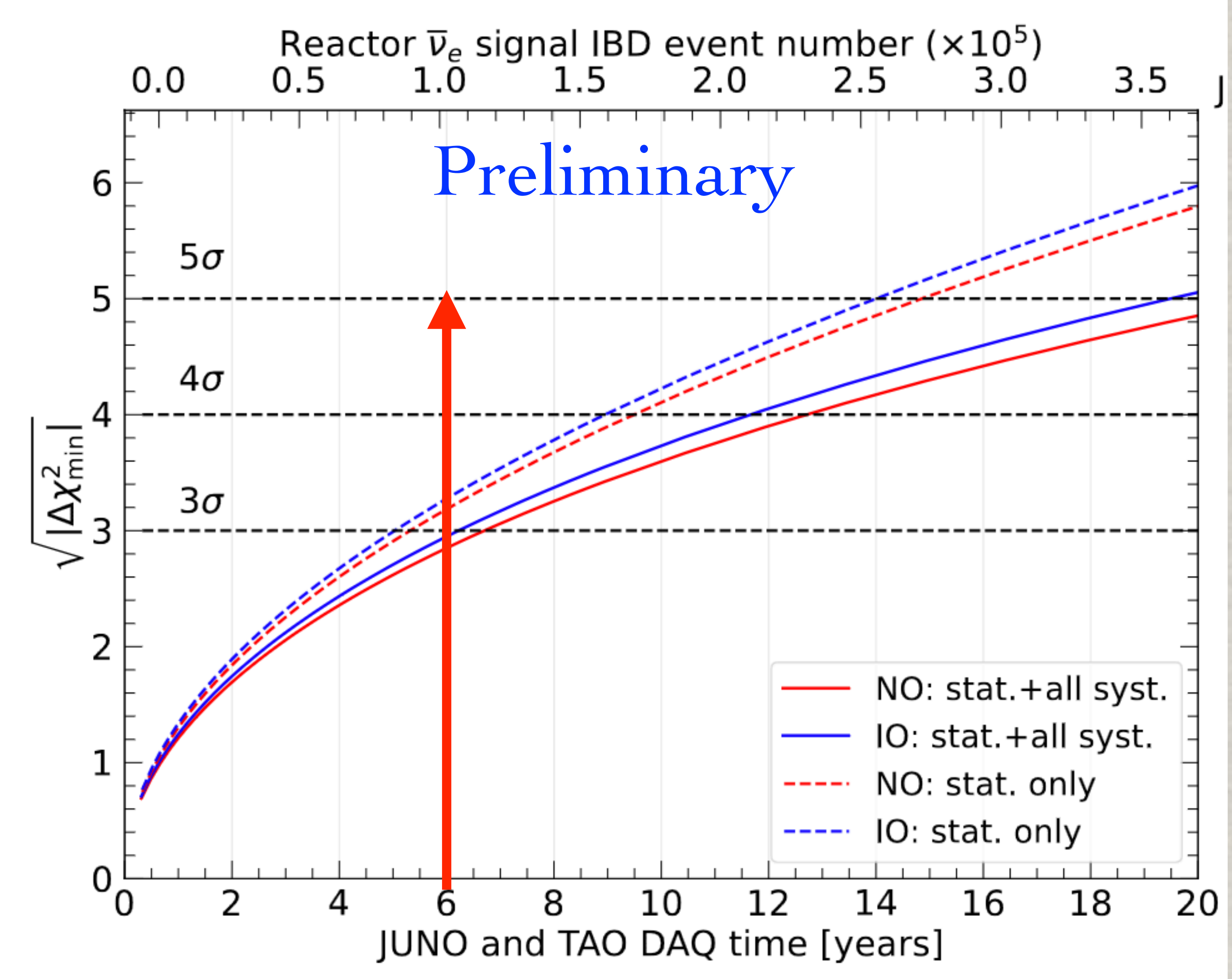


Neutrino Mass Ordering

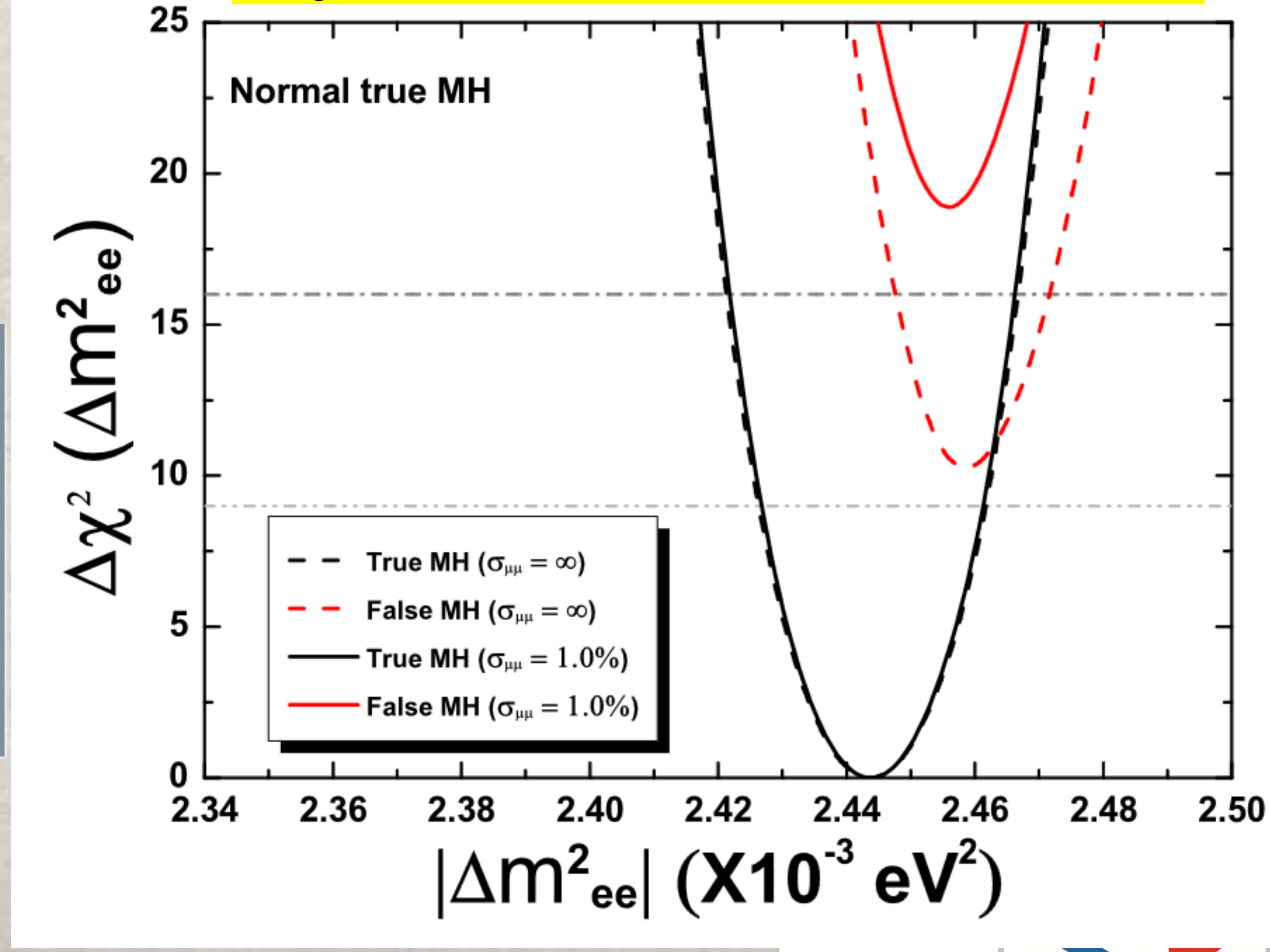
atm.ν
GeV



$|\Delta m^2_{\alpha\alpha}|$

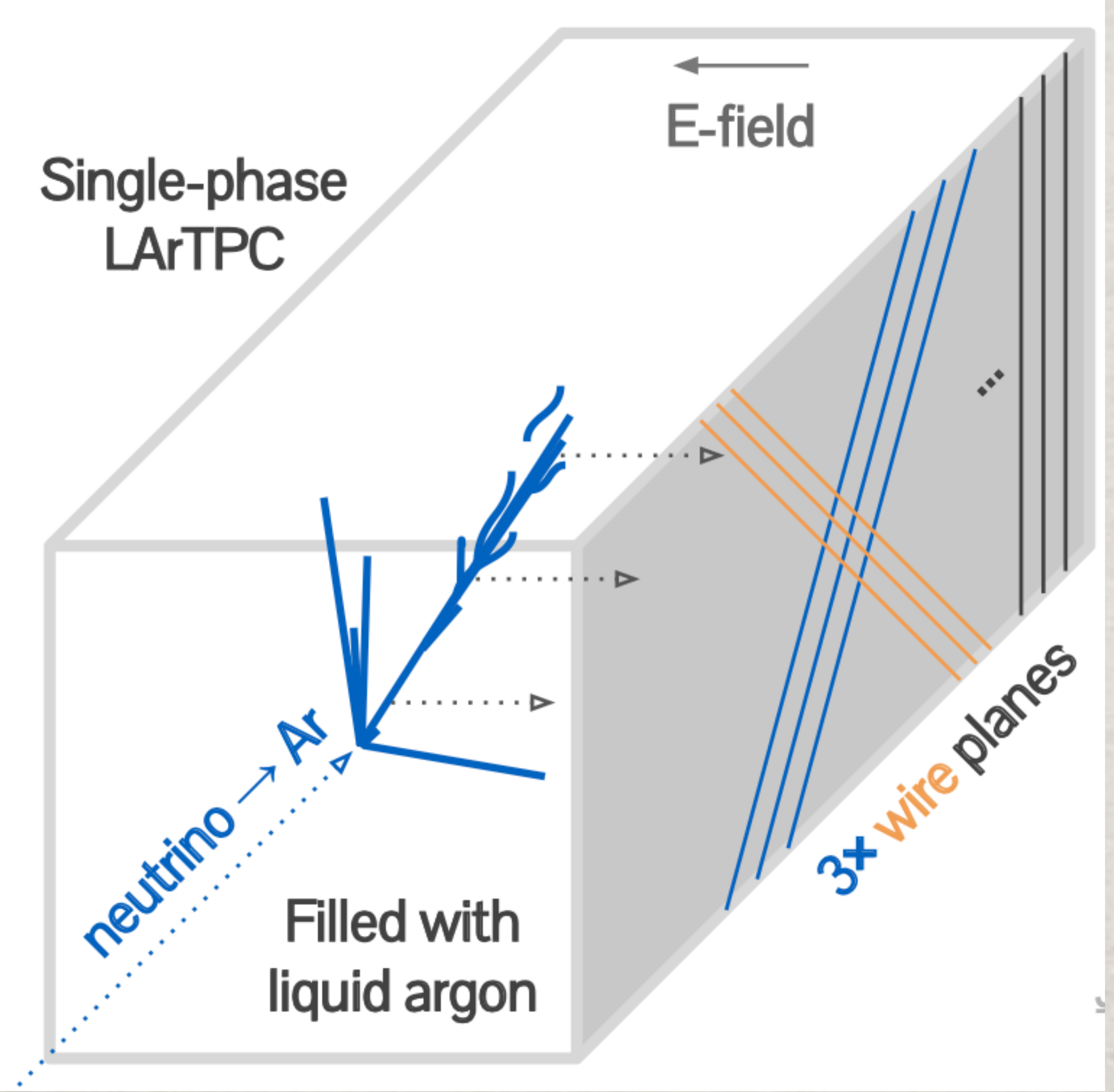


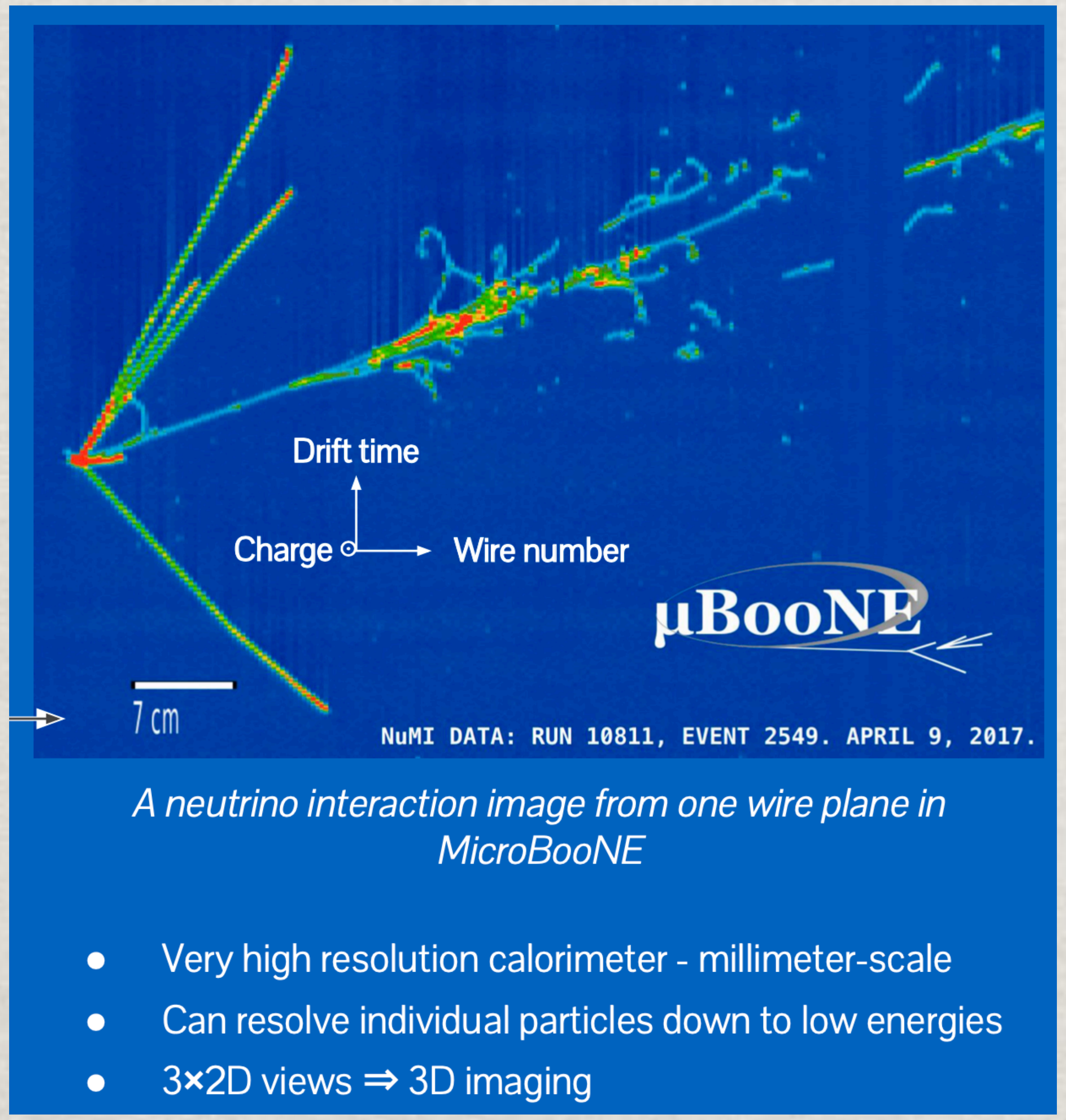
Yufeng Li et al.
Phys.Rev.D 88 (2013) 013008



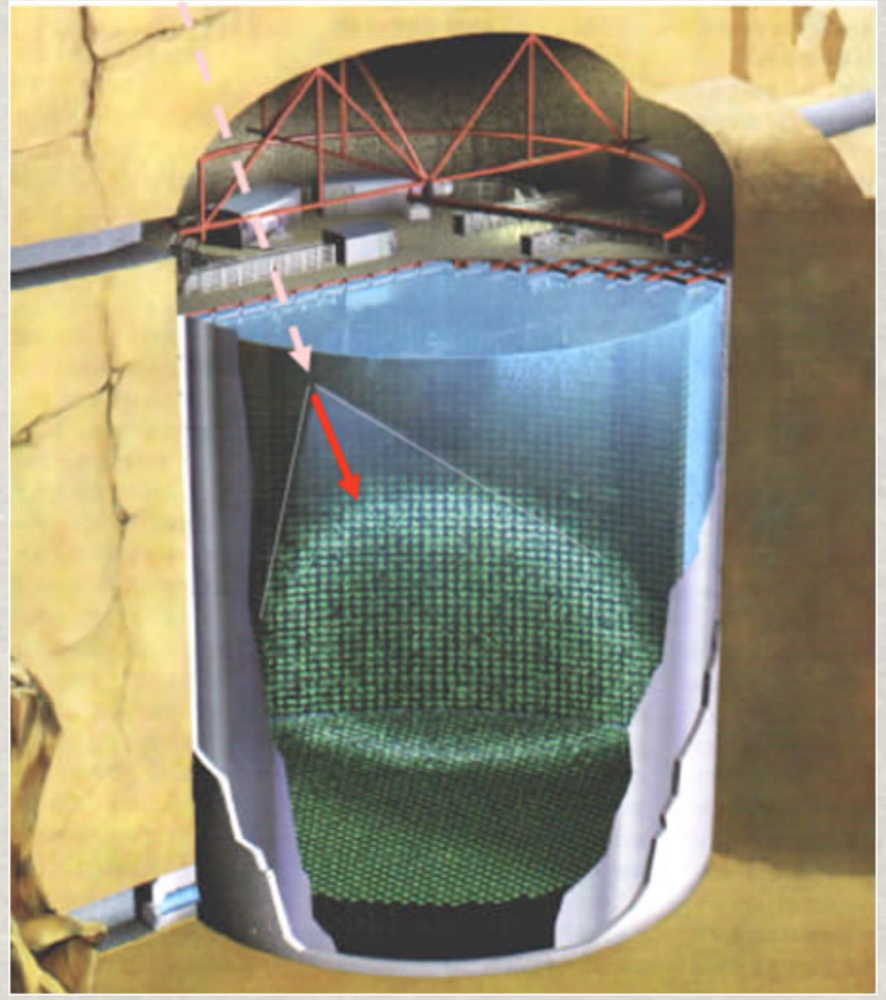
ATM. NEUTRINOS: CHALLENGES

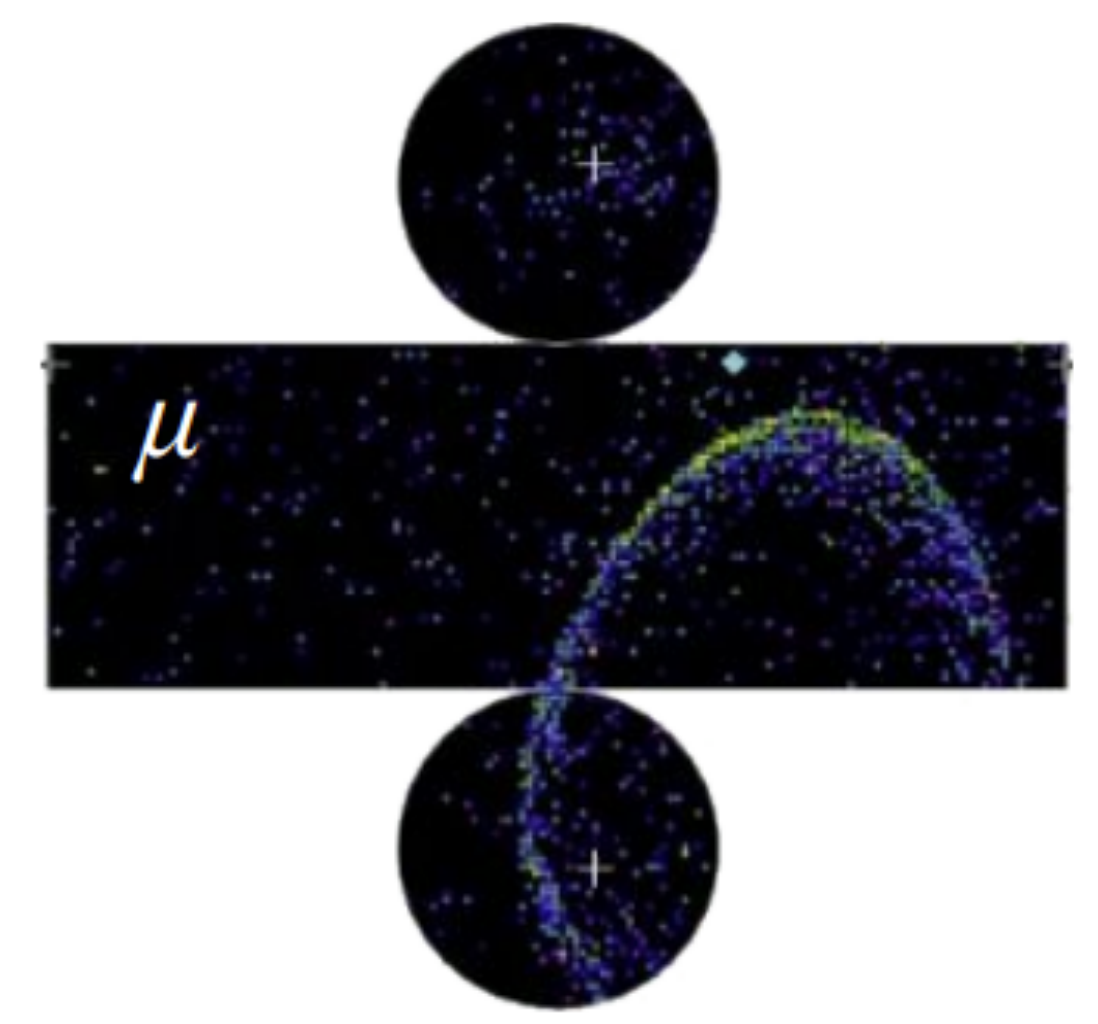
LArTPC

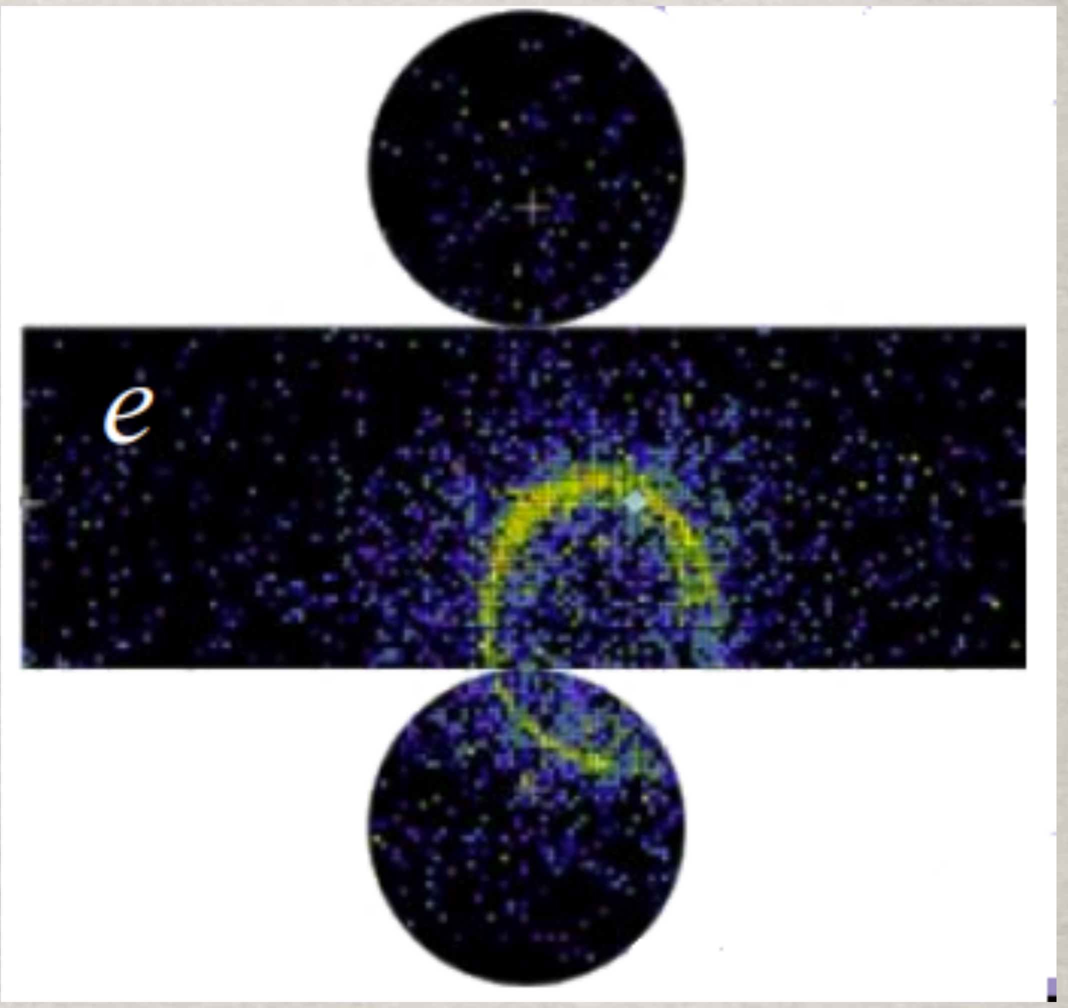




Water
Cherenkov



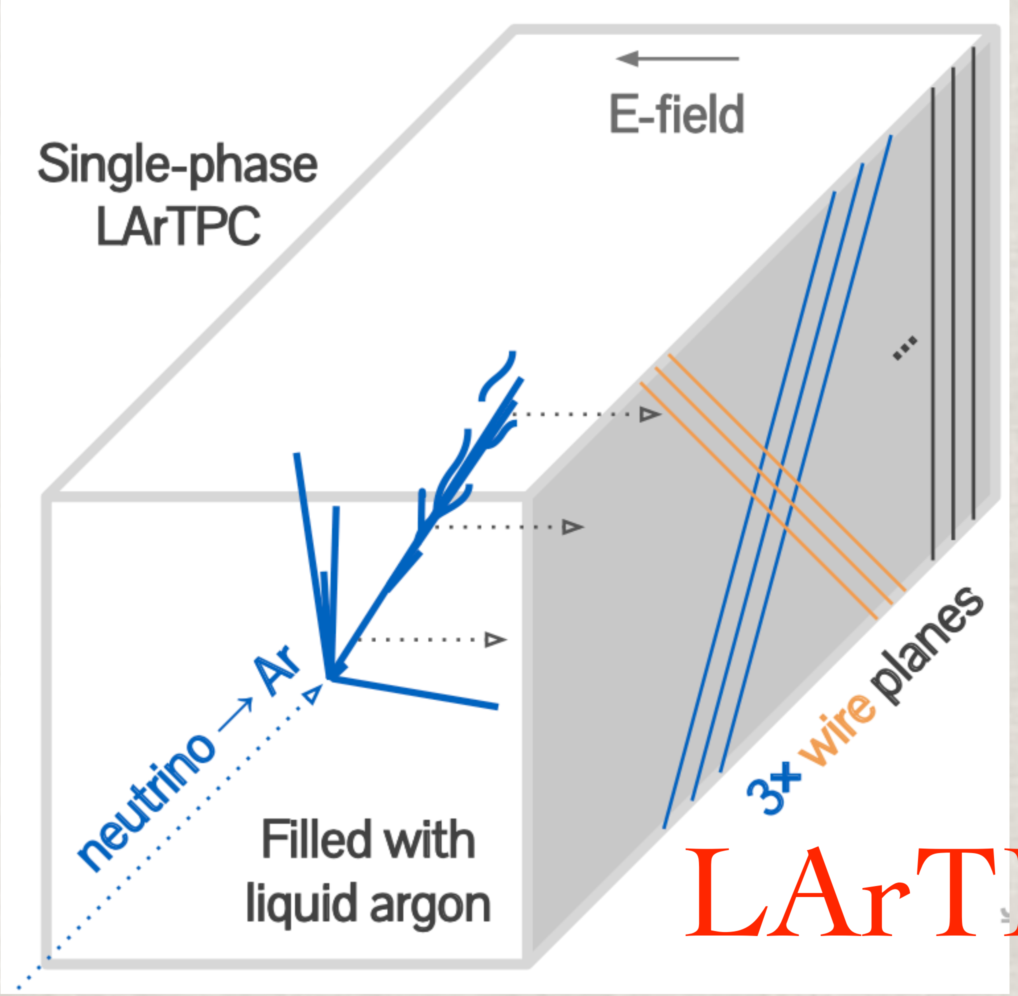


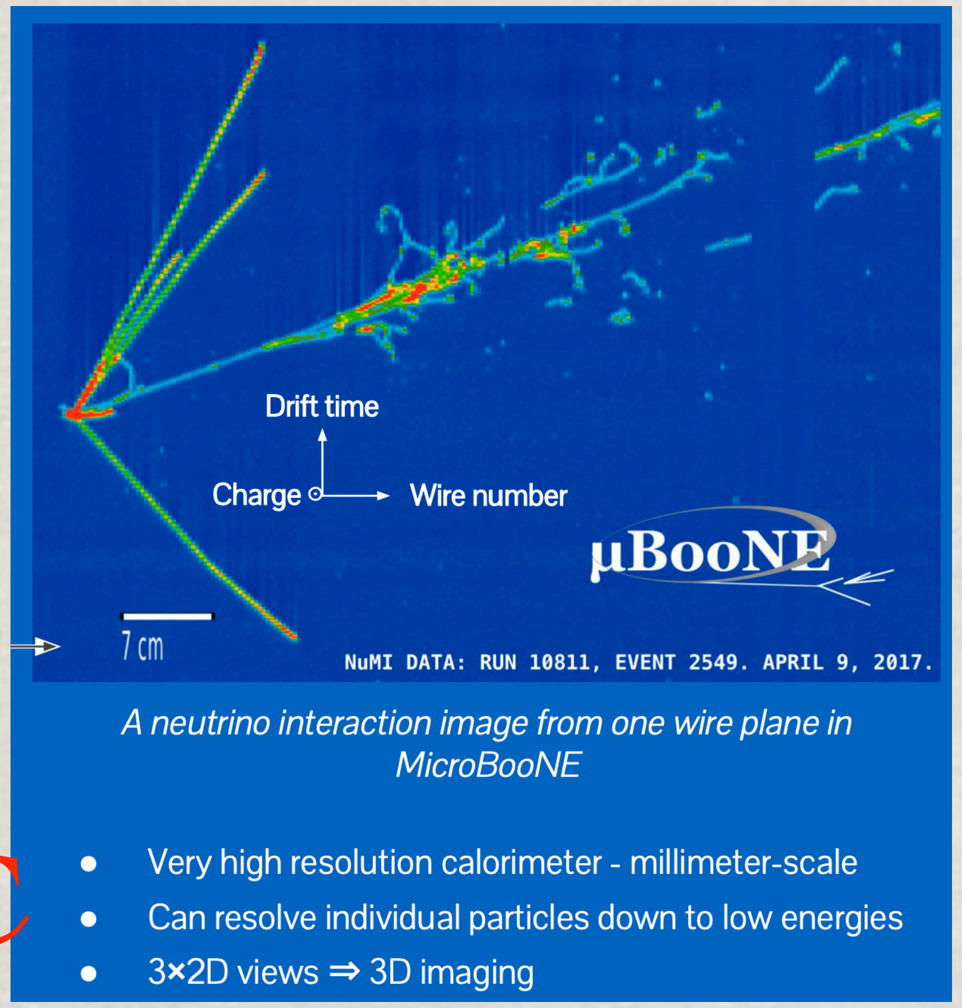


❖Neither track info, nor Cherenkov rings

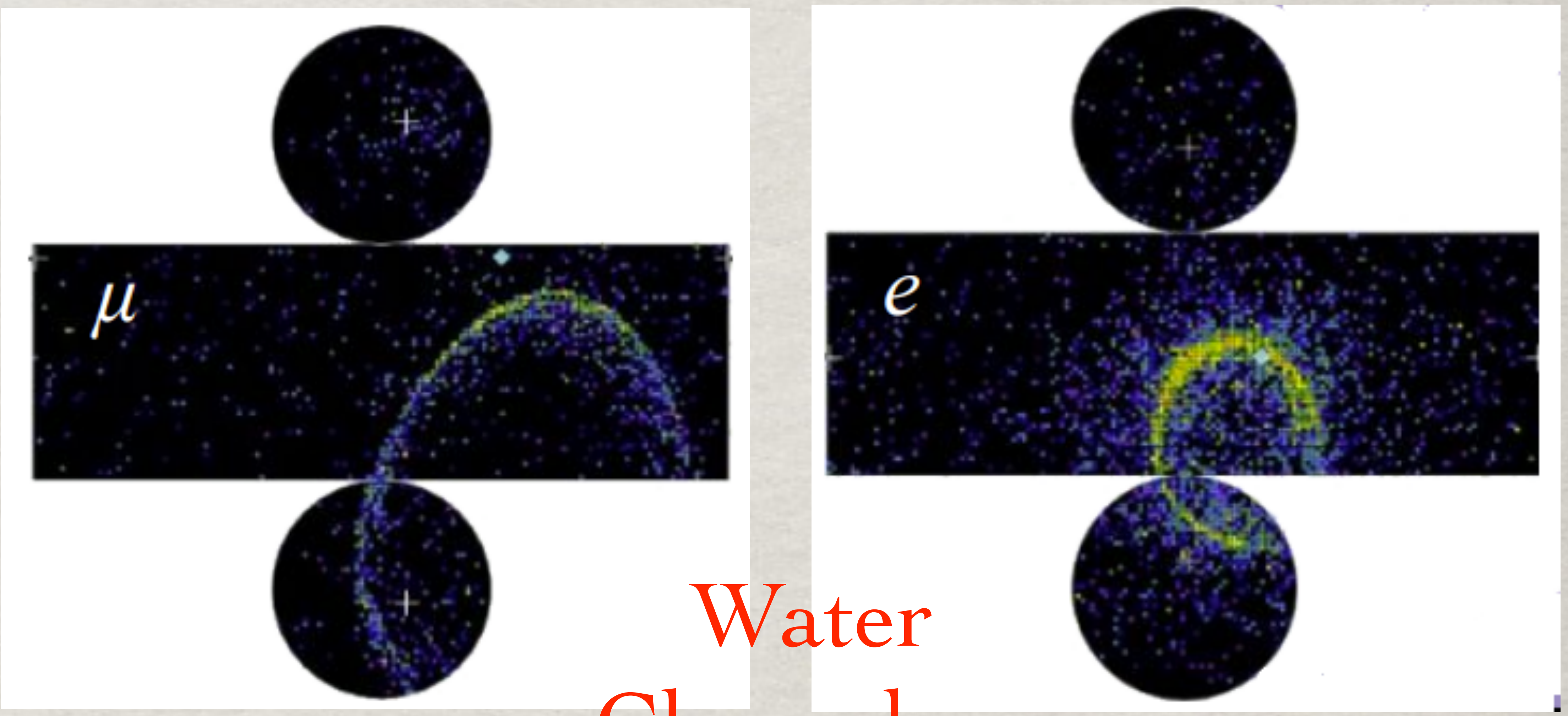
❖Can we still do <u>Direction reco and PID</u> for JUNO?

CHALLENGES AND OPPORTUNITIES





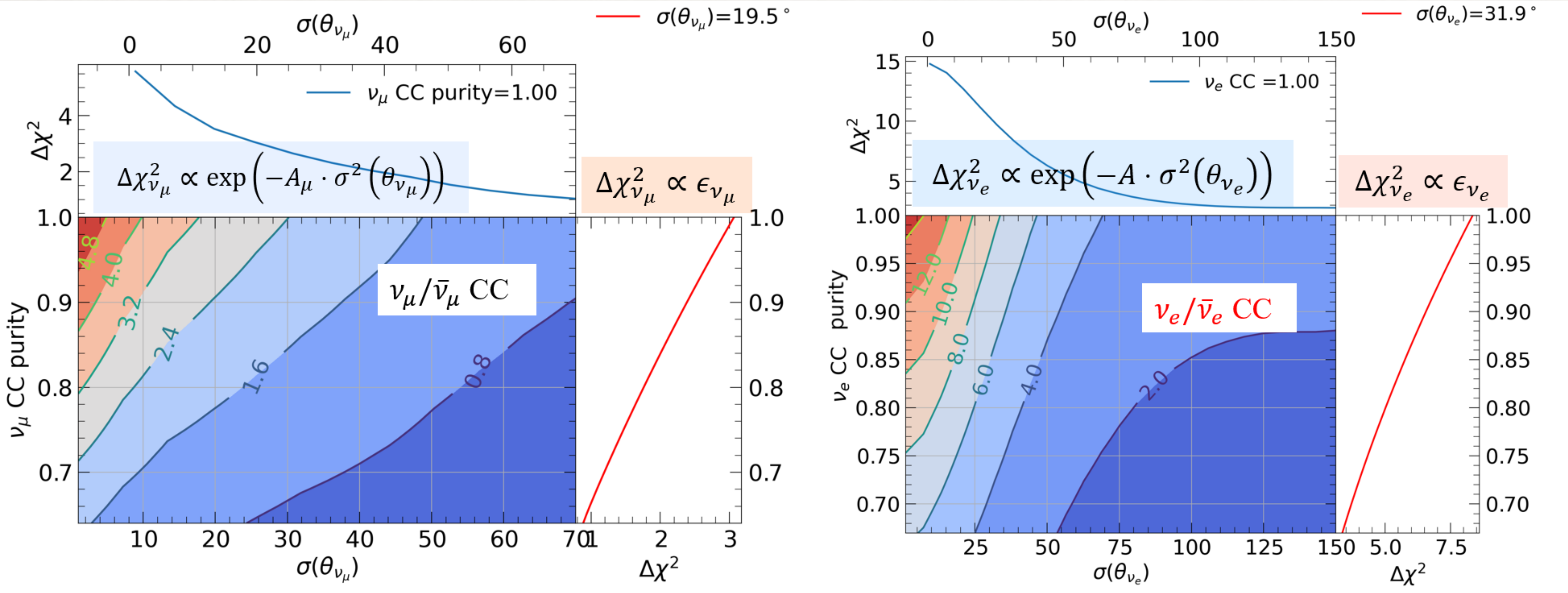
LArTPC

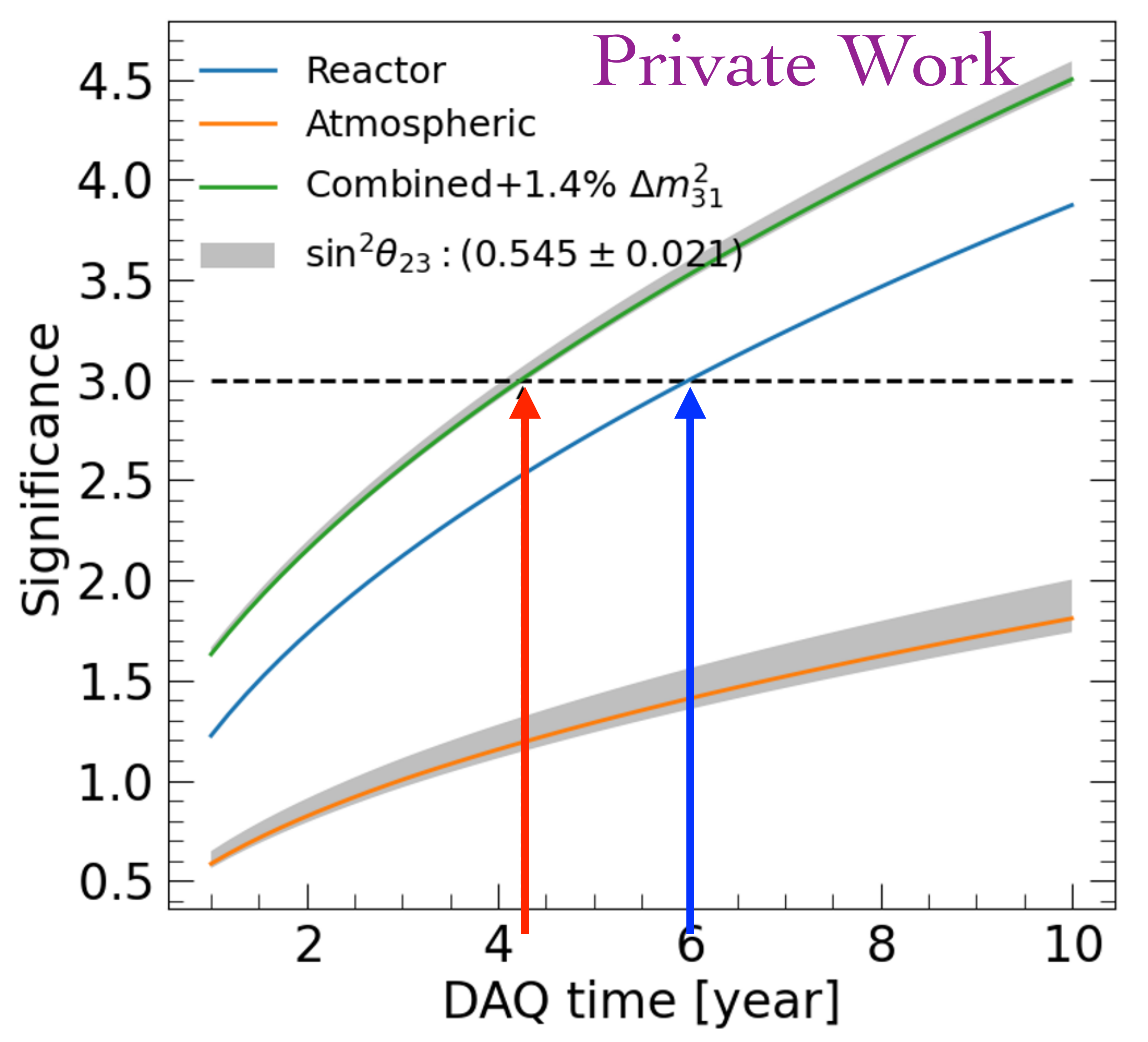


Water Cherenkov

- Neither track information, nor Cherenkov rings for JUNO
- Advantages of JUNO: 1. large PMT coverage(78%), large volume; 2. excellent neutron tagging; 3. hadronic component visible in LS; 4. can measure distinctive isotopes

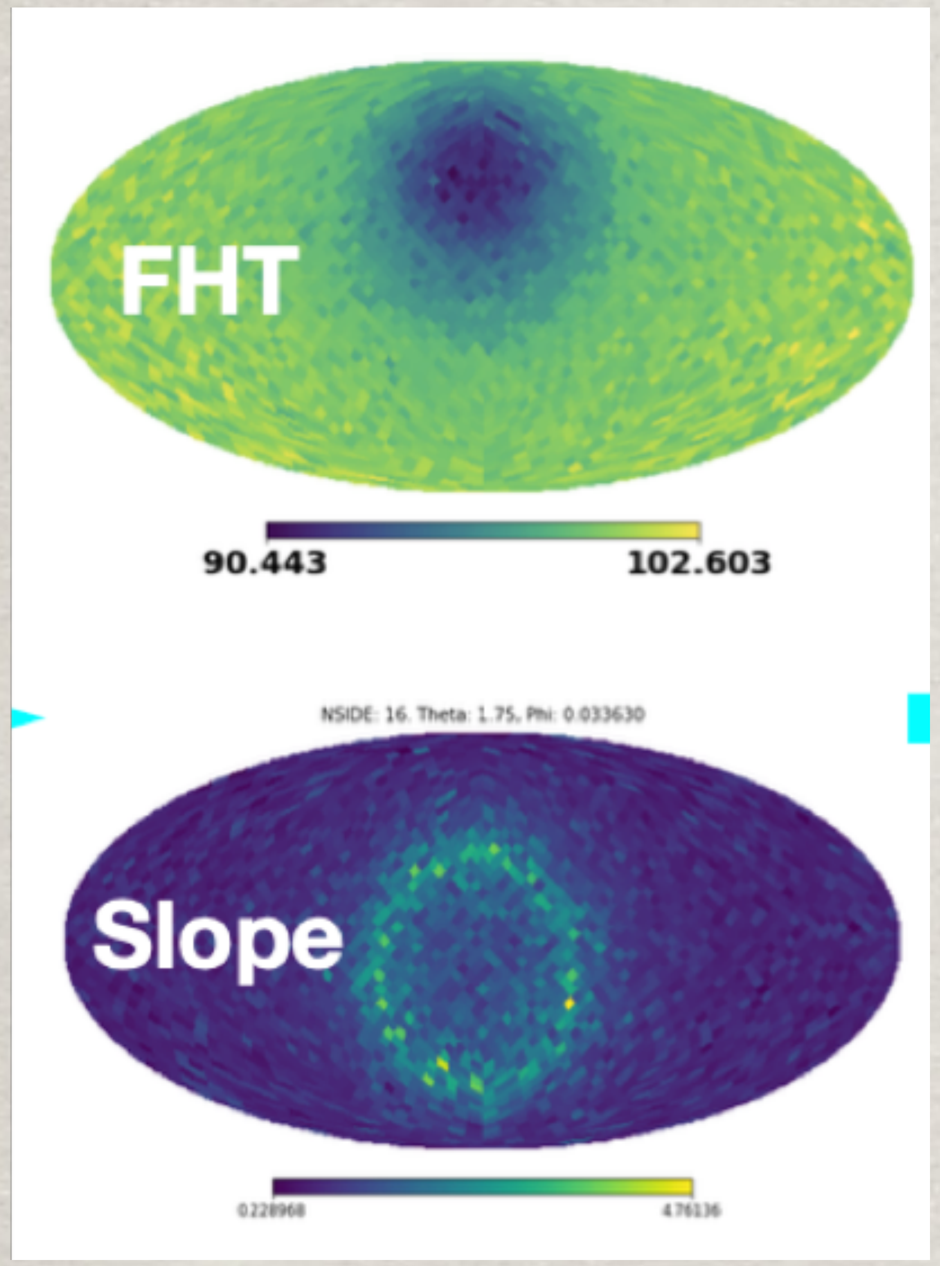
ATM. NEUTRINOS: IMPORTANCE



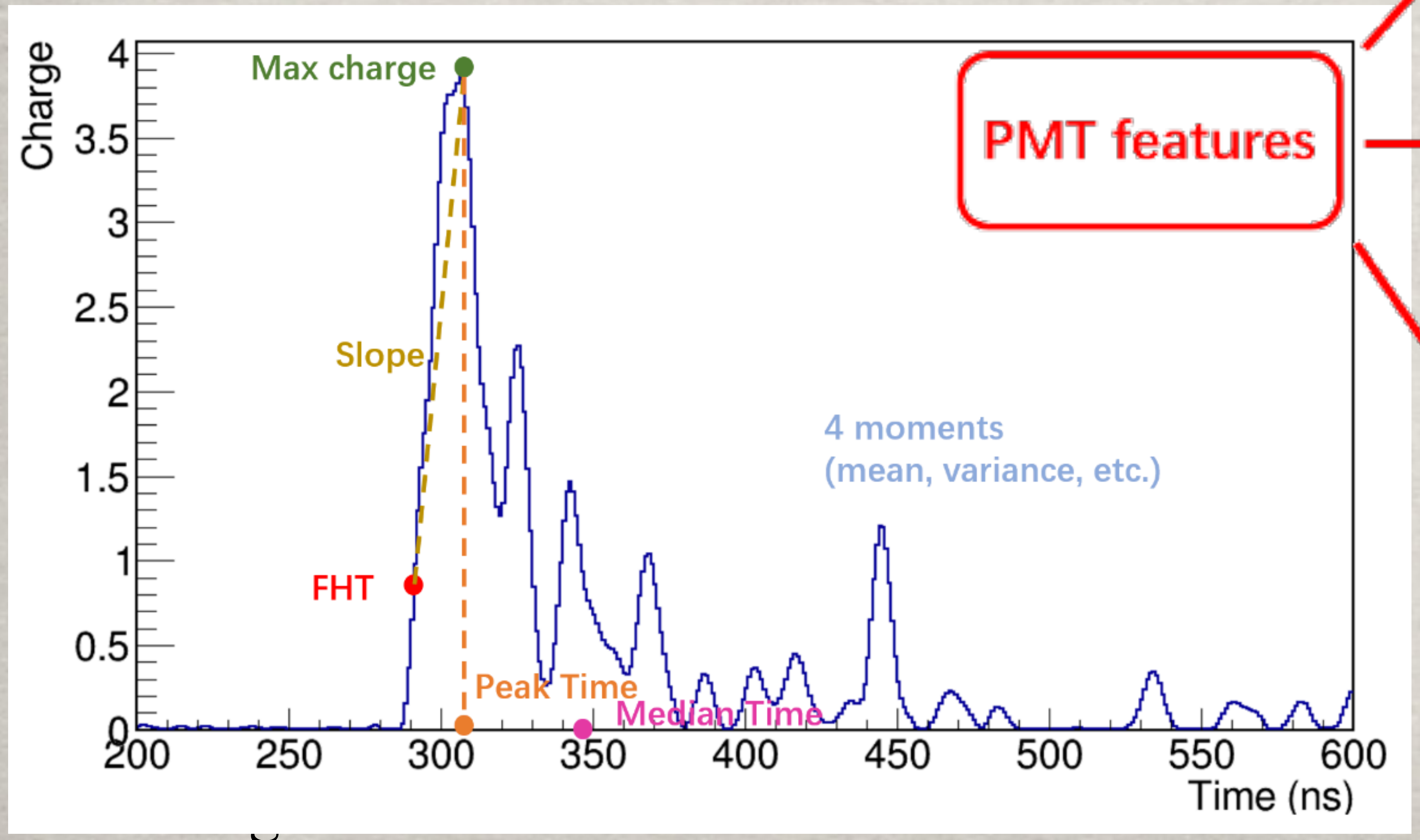


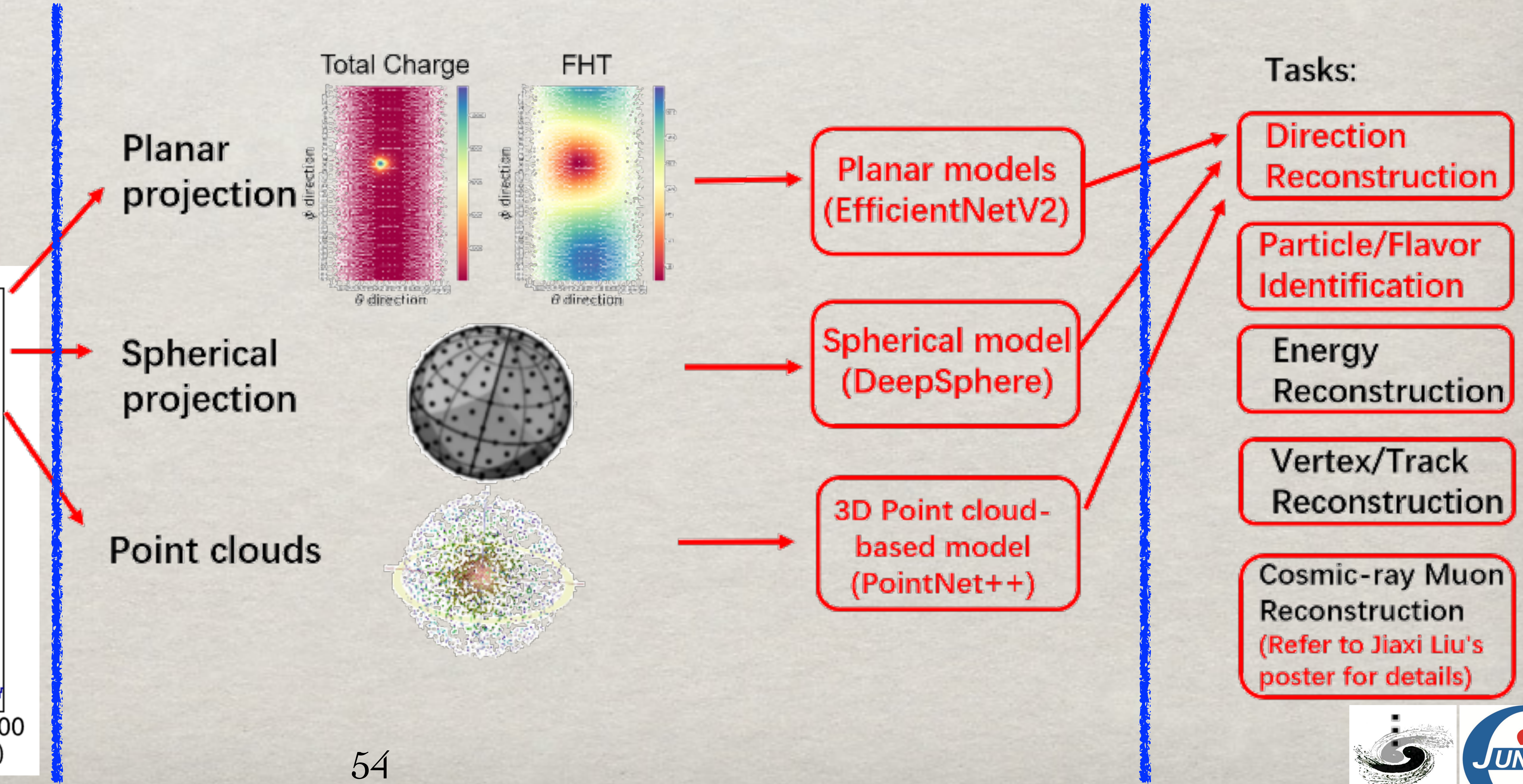
- Atm. NMO sensitivity largely depends on angular resolution and flavor identification
- NMO 3σ: reactor alone(6y) —> reactor + atm. (~4.2y)
- Major background for lots of analyses

RECO/PID METHODOLOGY



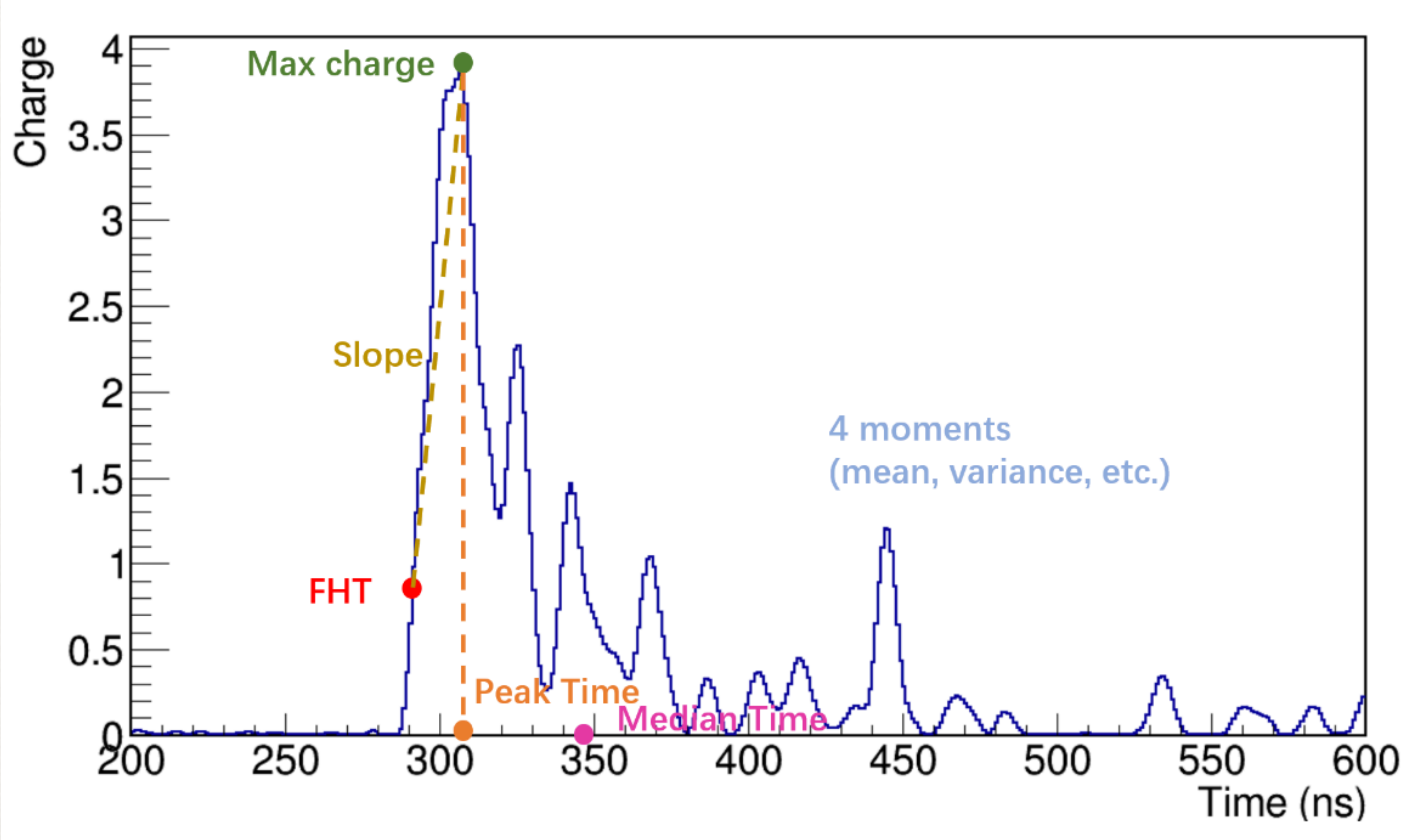
- Step 1: feature extraction from PMT waveforms
- Step 2: model building
- Step 3: optimization and validation



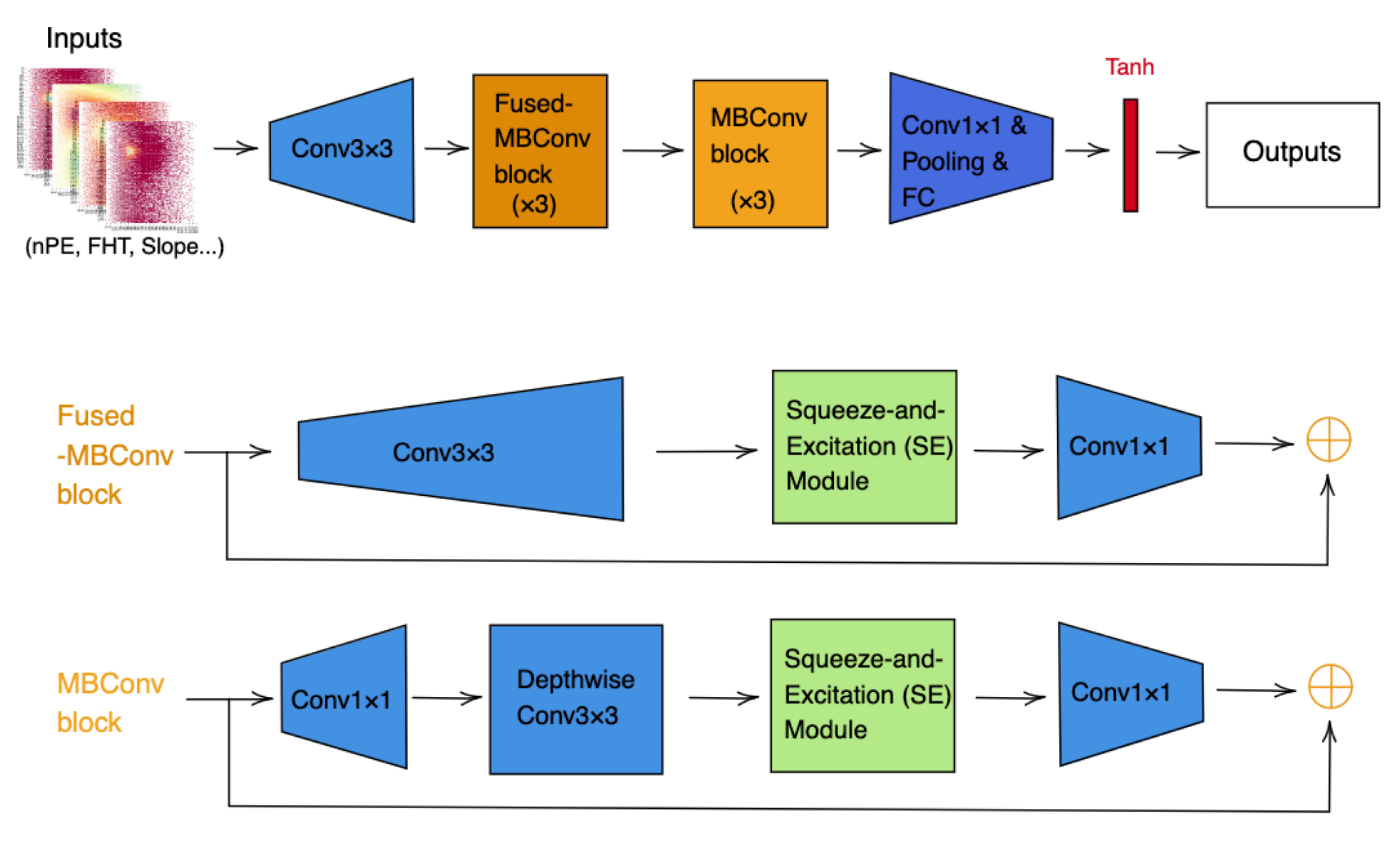


INPUTS: PMT FEATURES

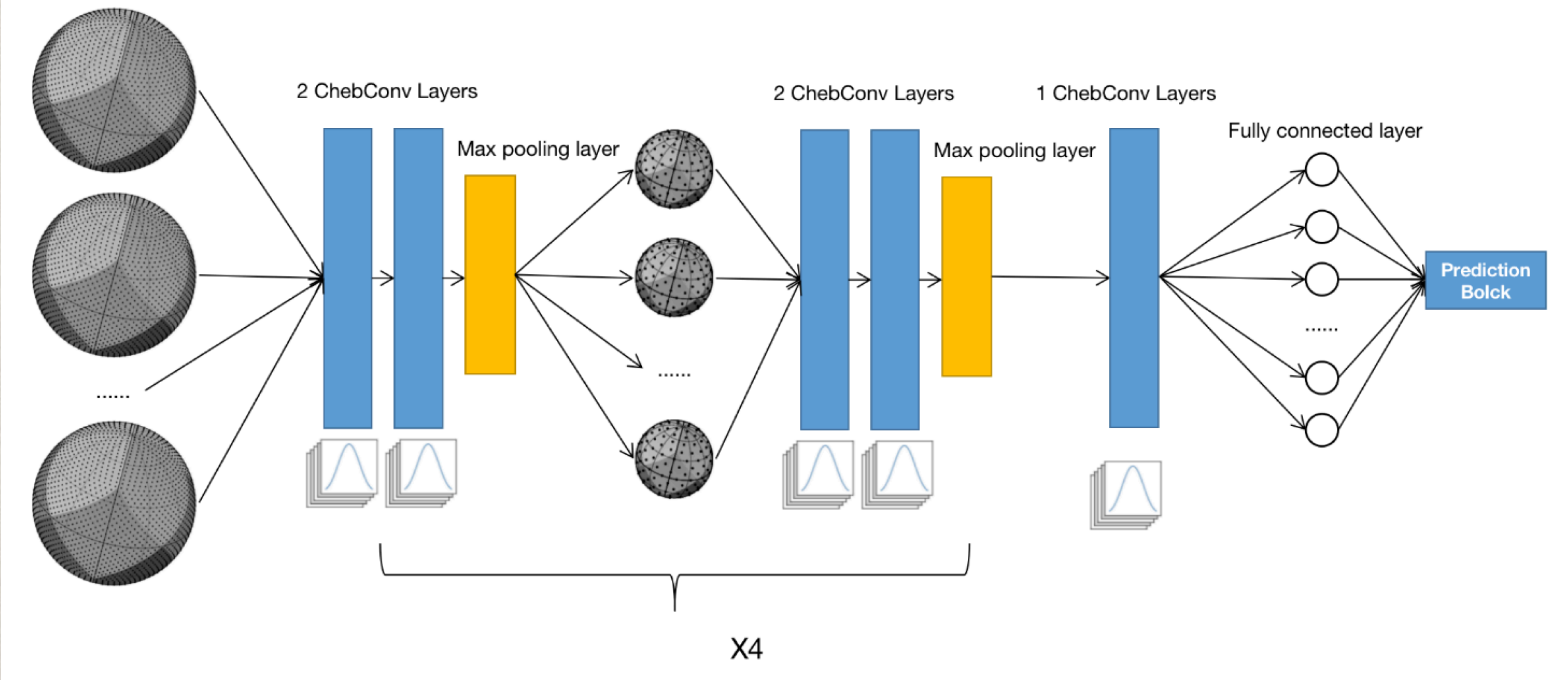
❖Feature variables extracted from PMT waveforms



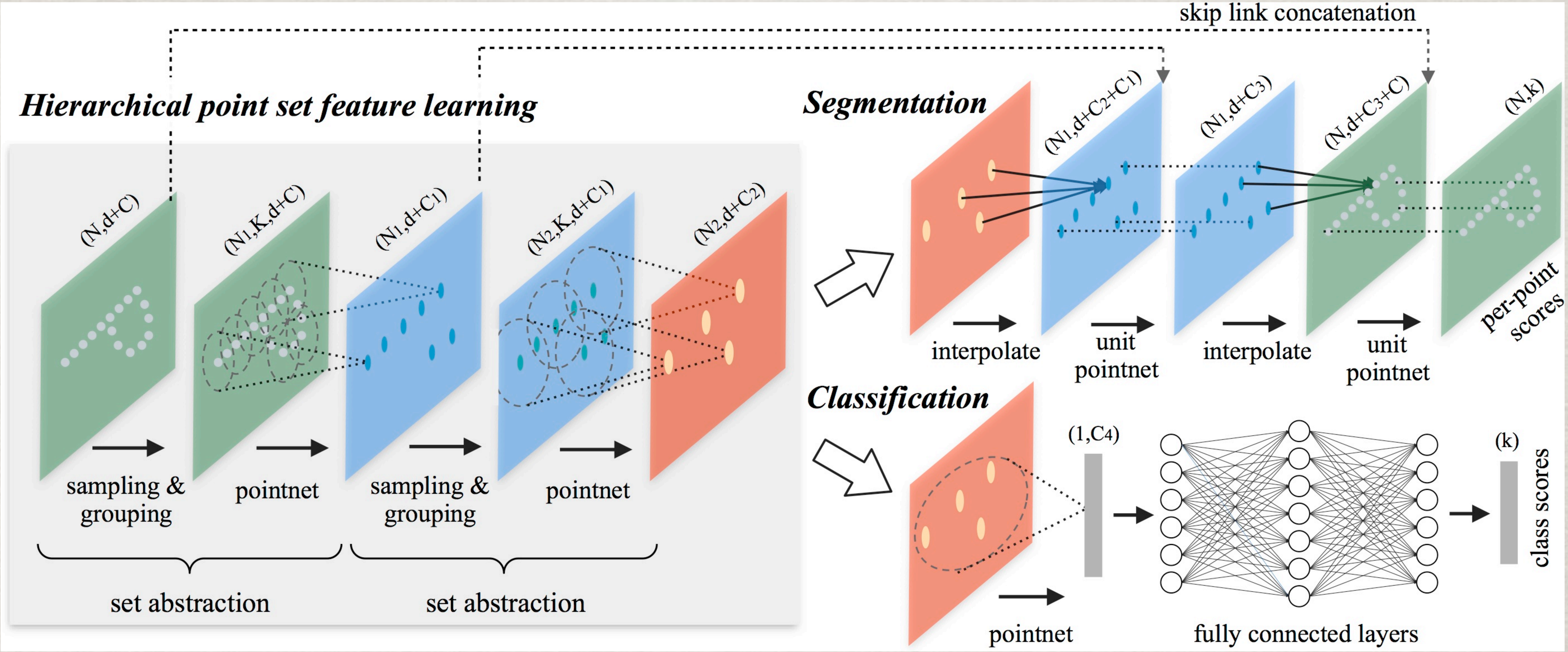
plane model: EfficientNetV2-S



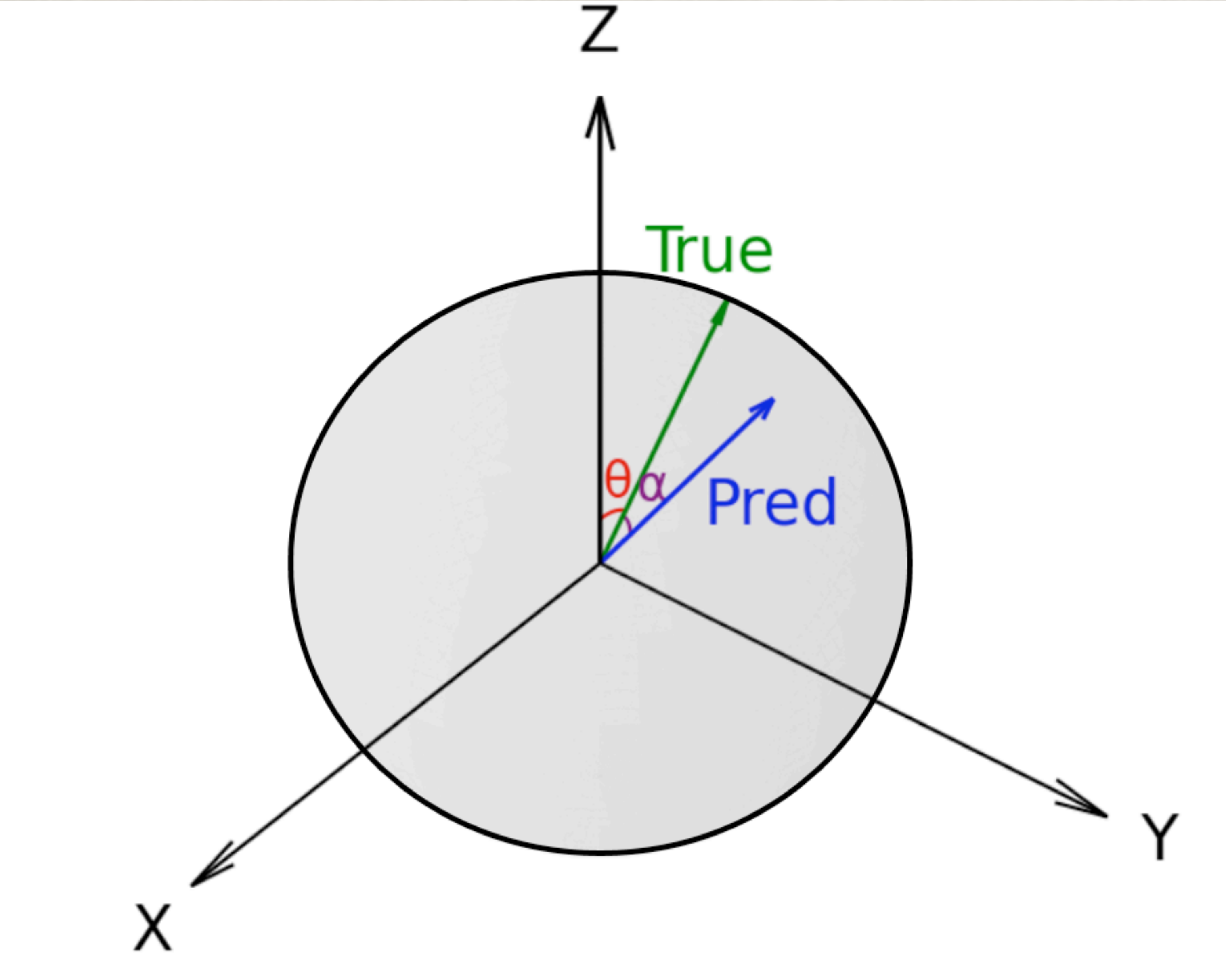
spherical model: Deepsphere

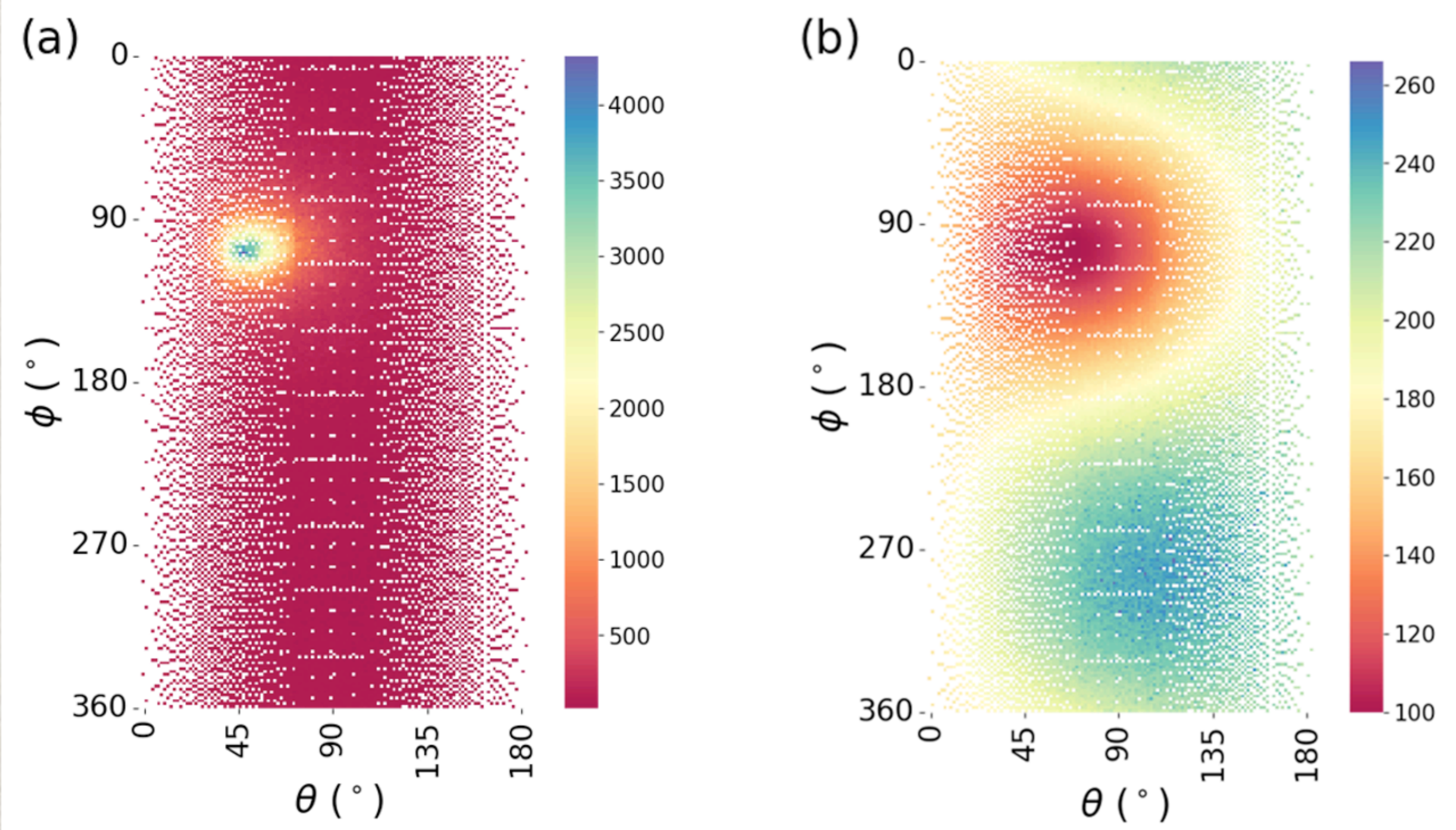


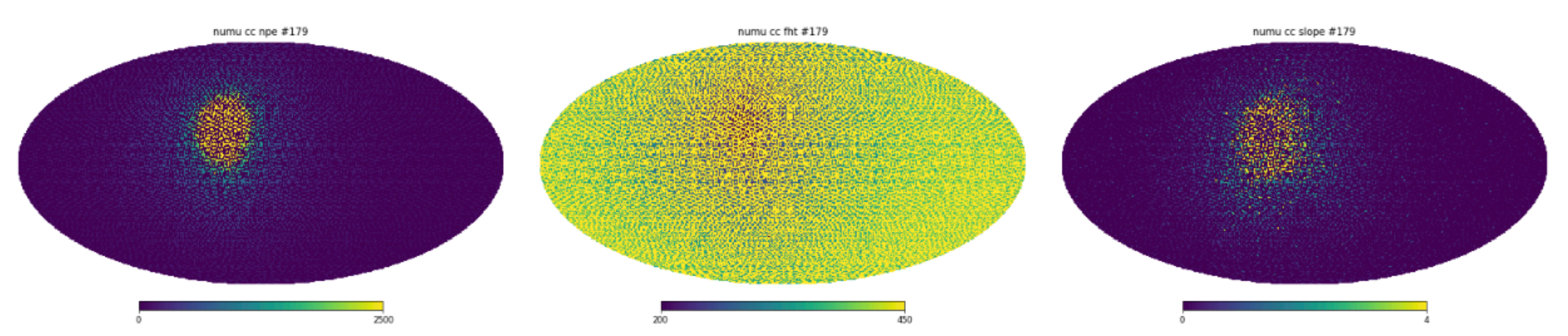
3D model: PointNet++



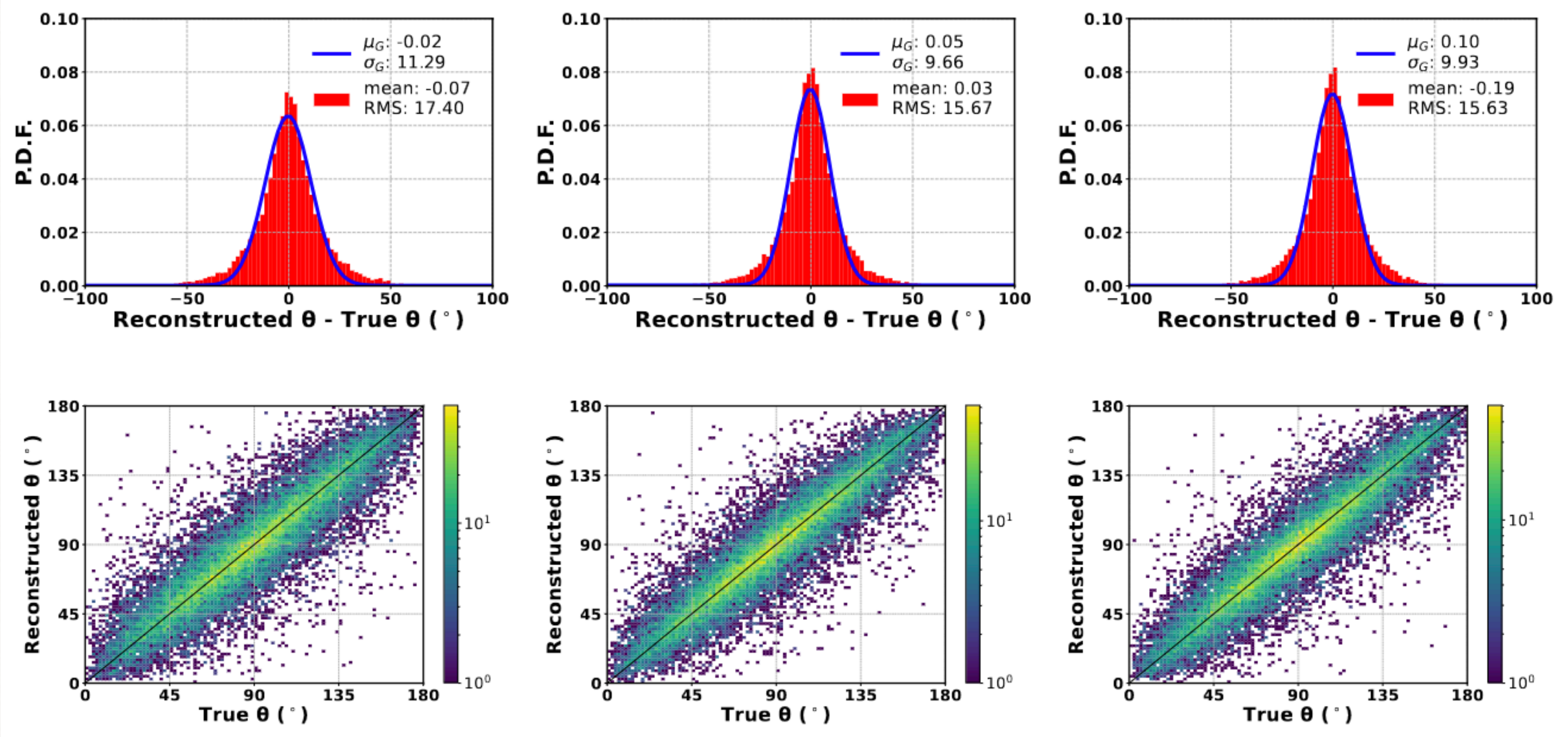
ATM. NEUTRINOS: DIRECTIONALITY







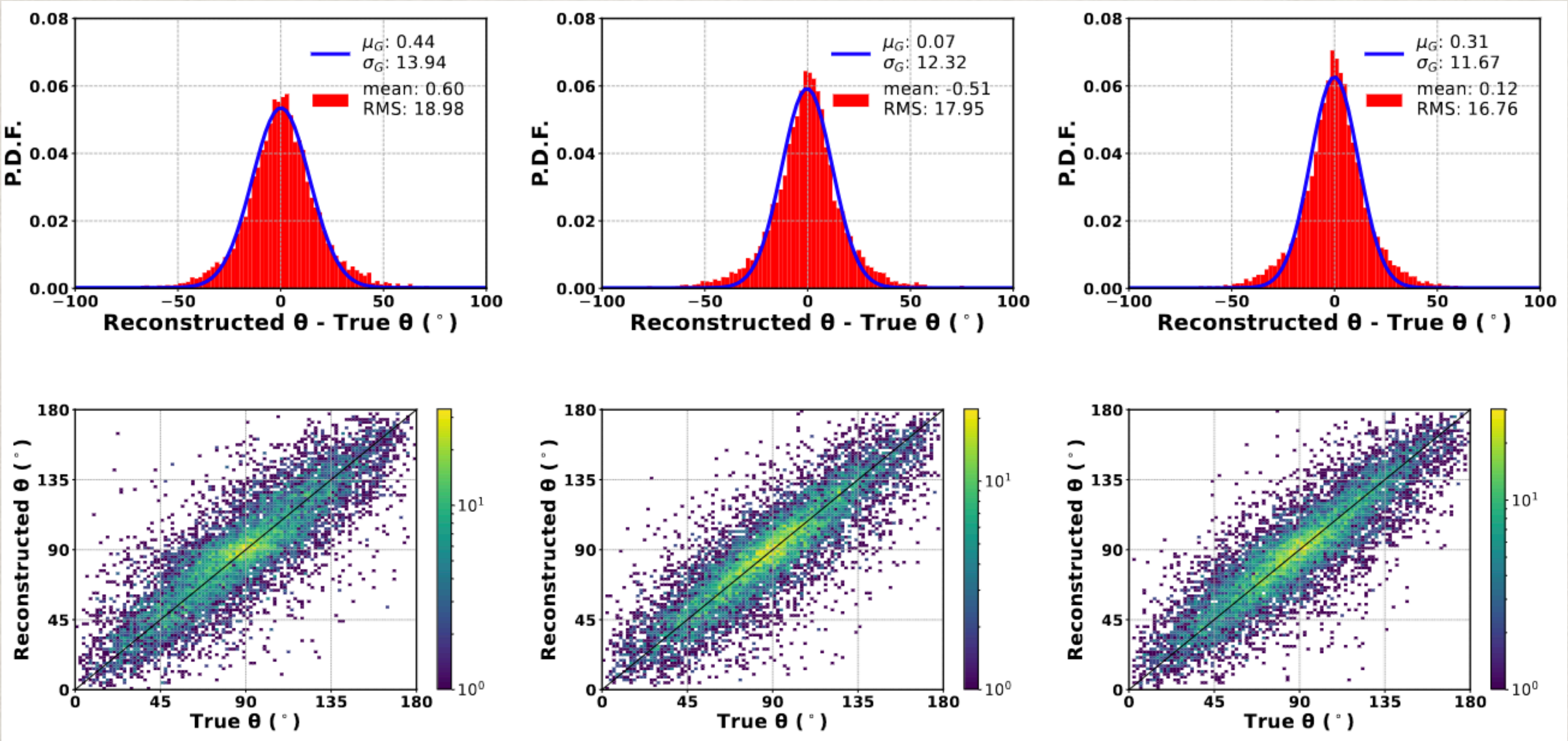
ATM. NEUTRINOS: DIRECTIONALITY



❖Average angular resolution around 10° for ν_μ

❖Consistent performance among three models

ATM. NEUTRINOS: DIRECTIONALITY

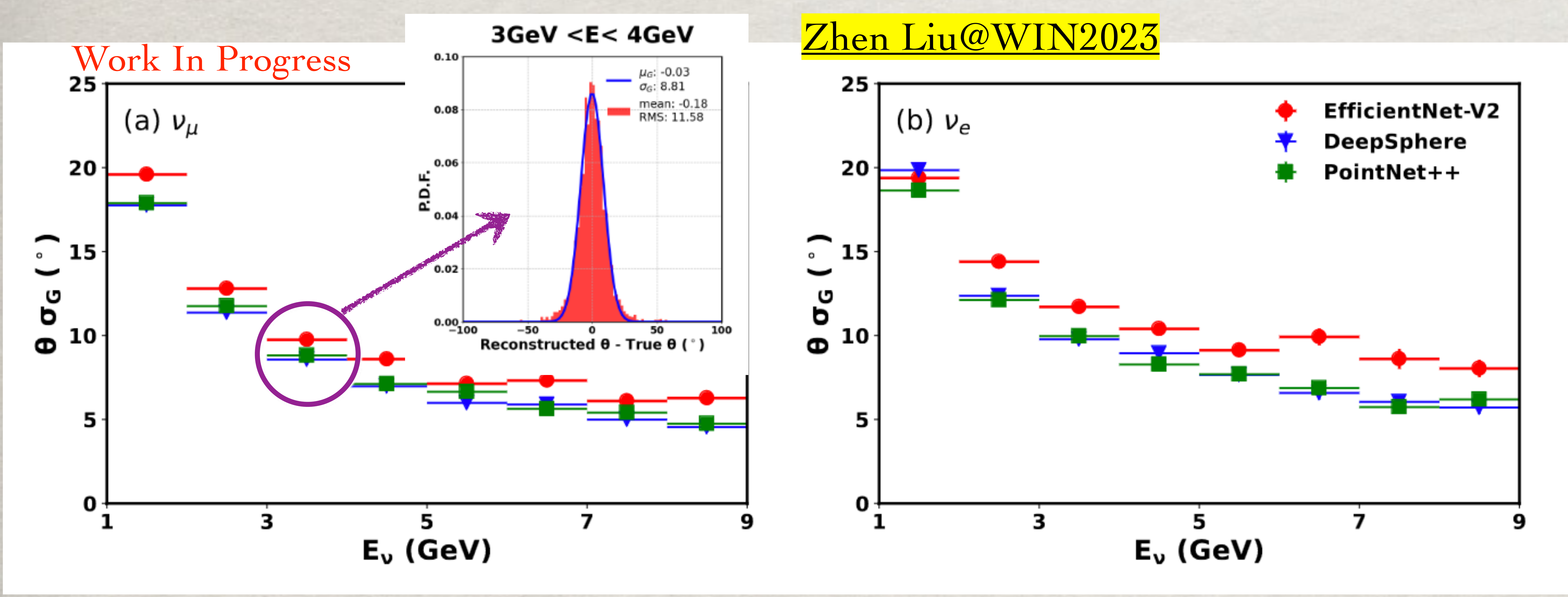


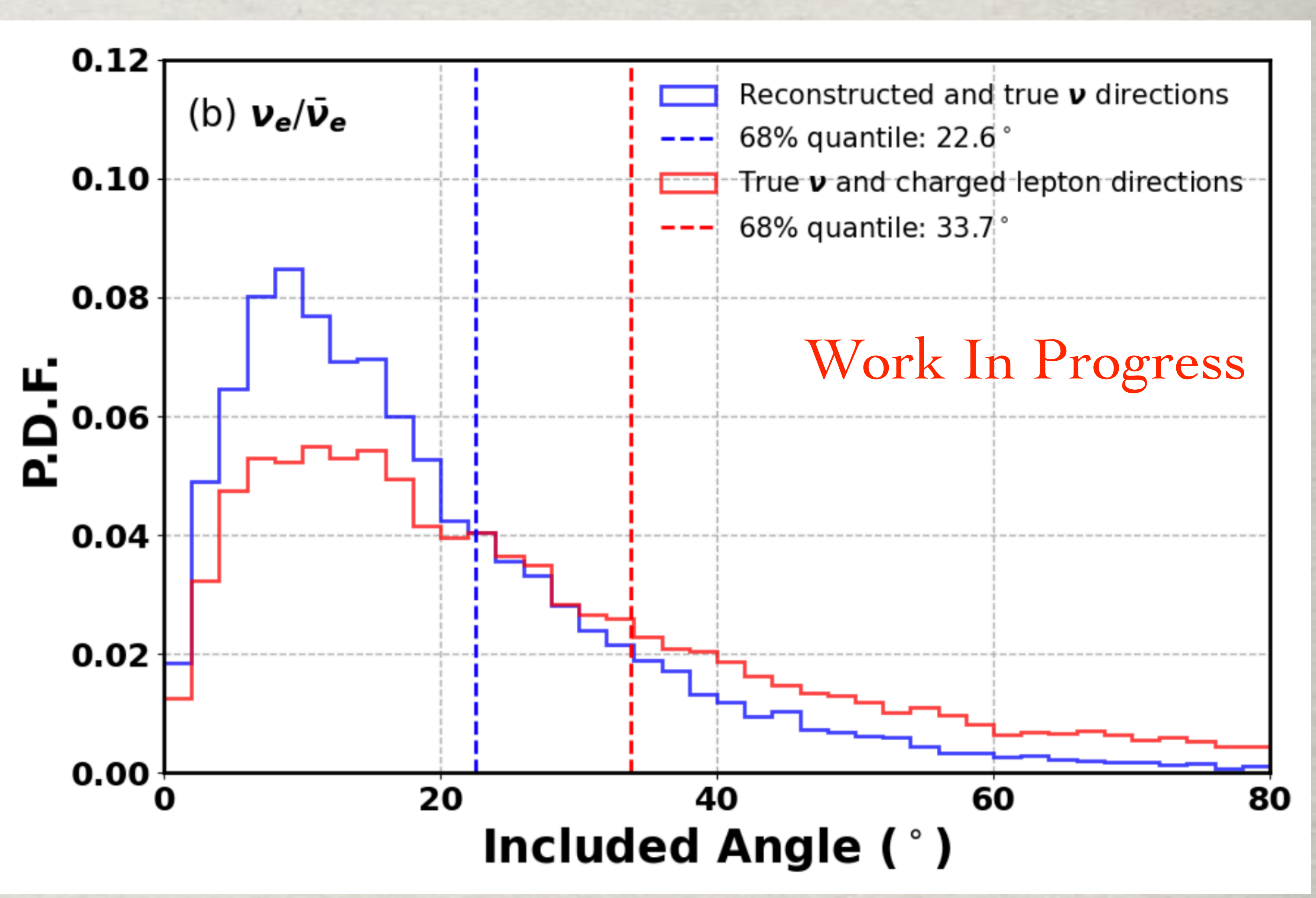
❖Average angular resolution around 12° for ν_e

❖Consistent performance among three models

DIRECTIONALITY

Zhen Liu@WIN2023





- Directly reconstruct the direction of ν instead of the charged lepton
  - mitigate the intrinsic large uncertainty between the two
  - hadronic component in LS also helps, advantageous w.r.t. Water Cerenkov
- Energy dependent Zenith Angle resolution, less than 10° for E>3GeV

J. Phys. G: 43 (2016) 030401

Yellow Book

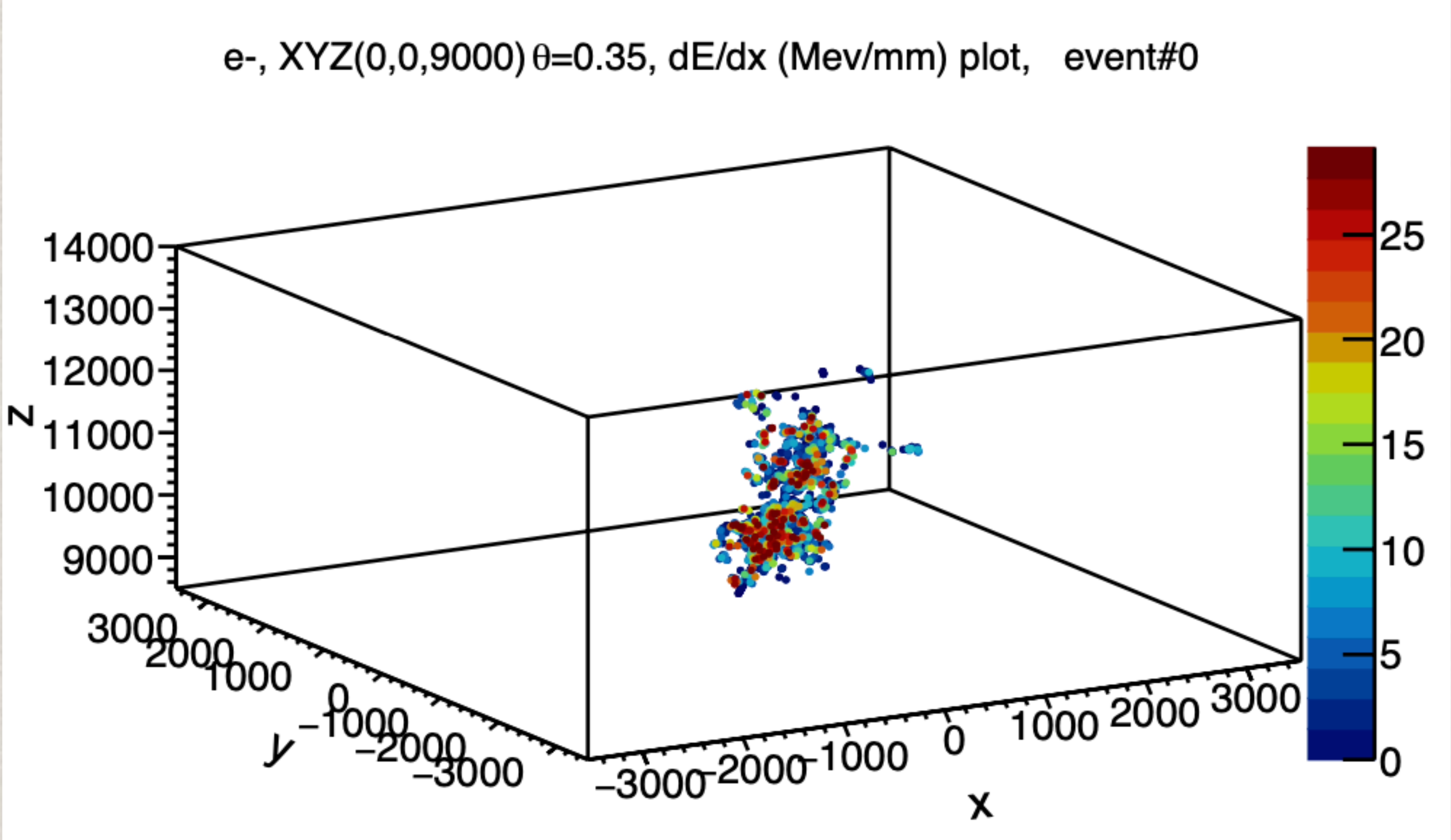
$\sigma_{\theta\mu} = 1^\circ$

$\sigma_{\theta\nu} = 10^\circ$

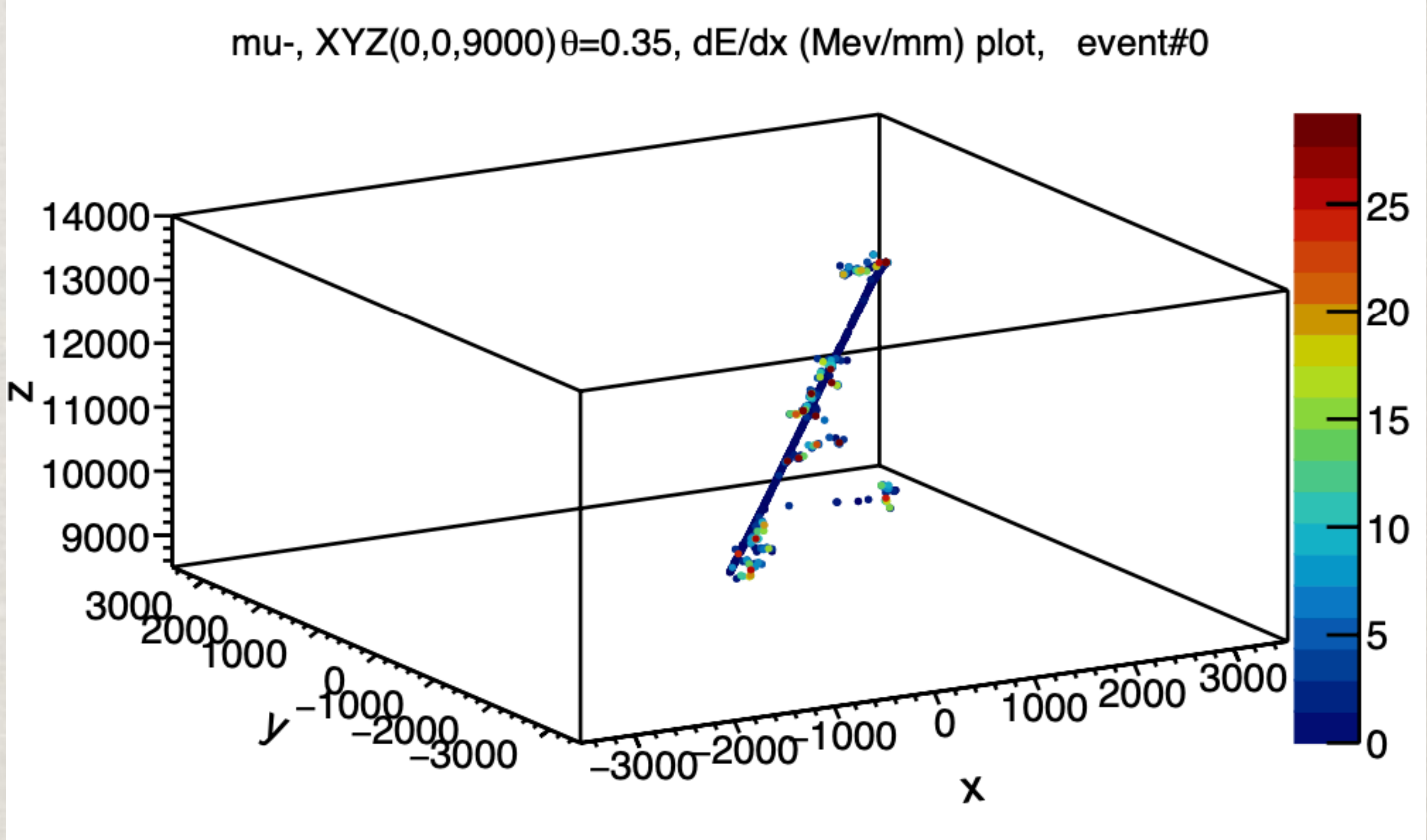
PARTICLE TOPOLOGY

Energy deposition topology in LS for different type of particles

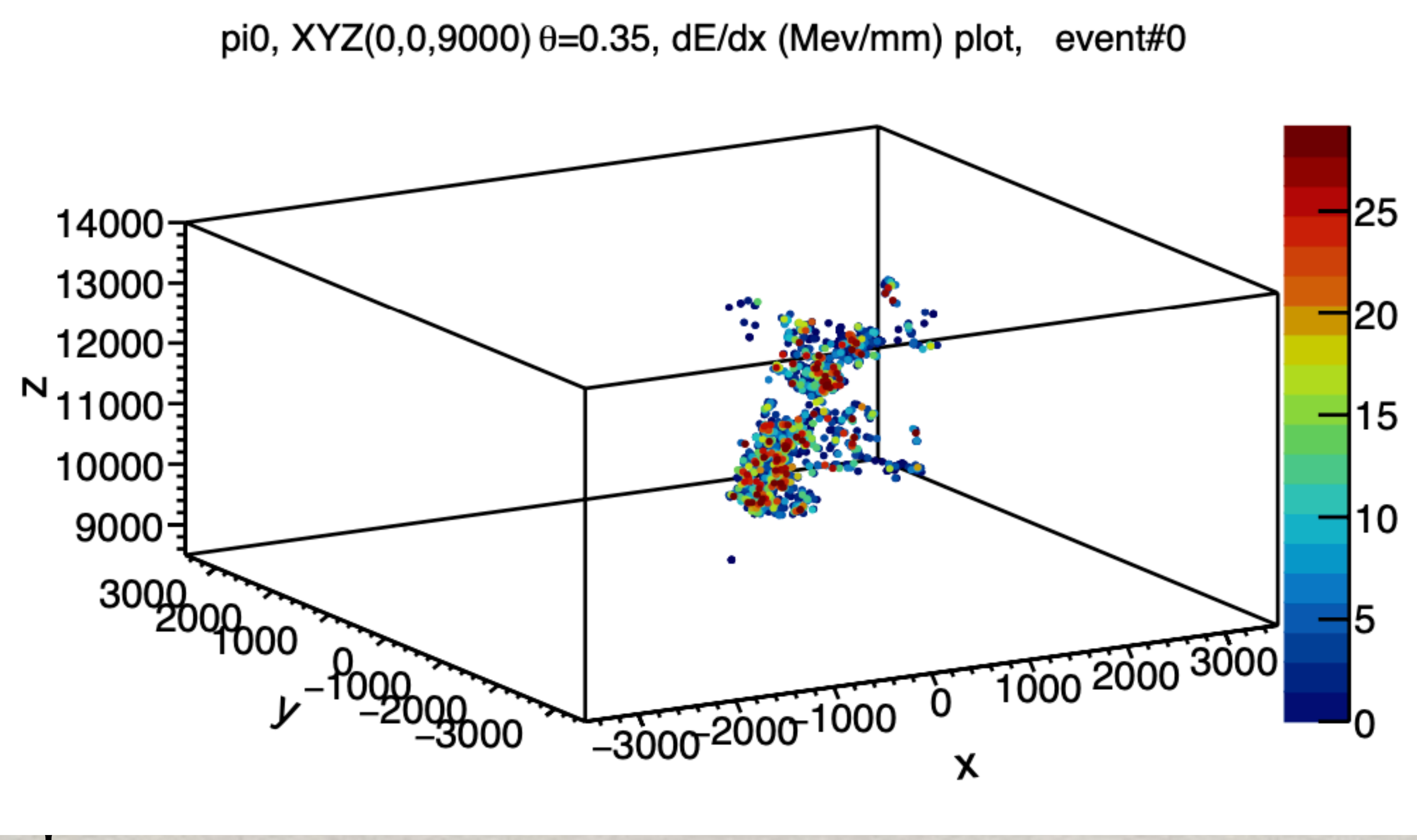
e



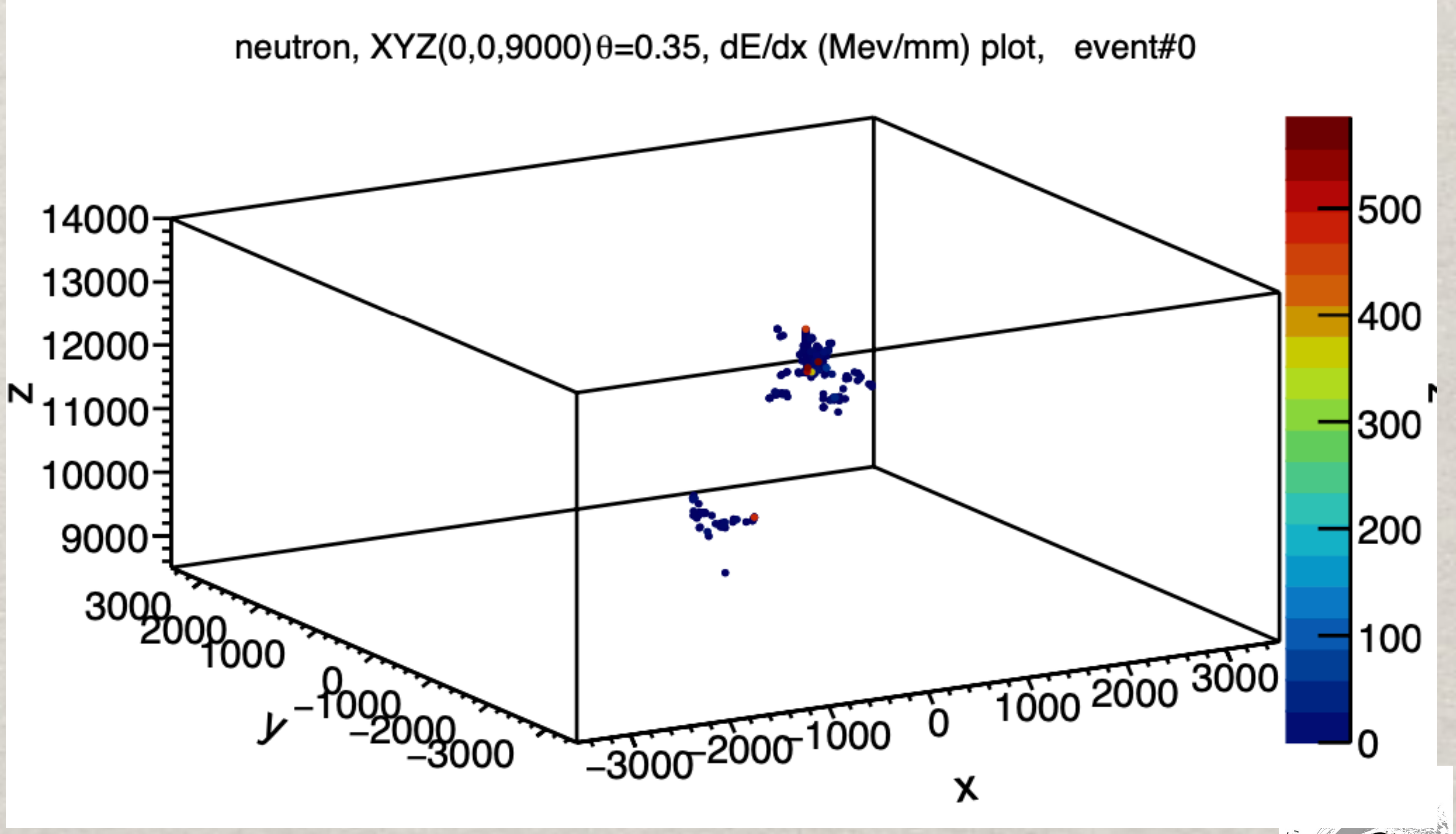
μ



π^0



n



PID STRATEGY

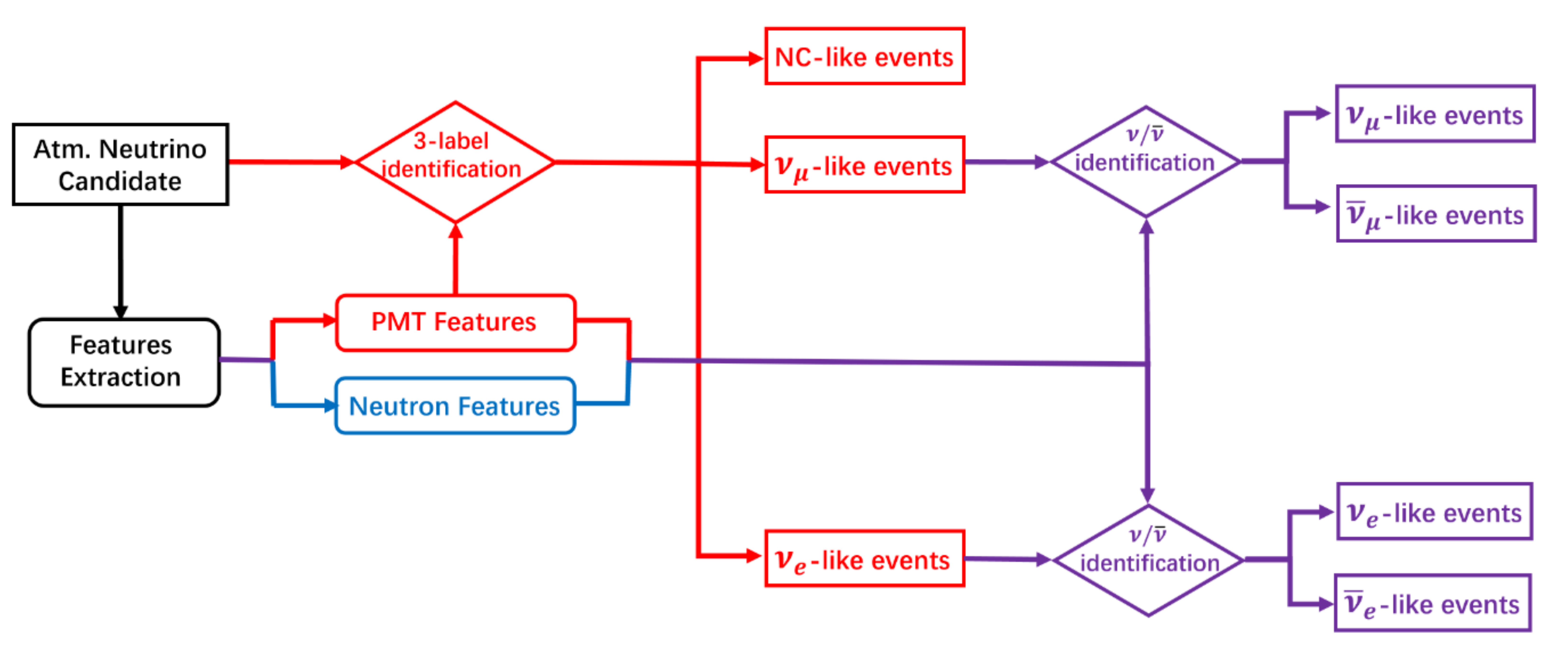
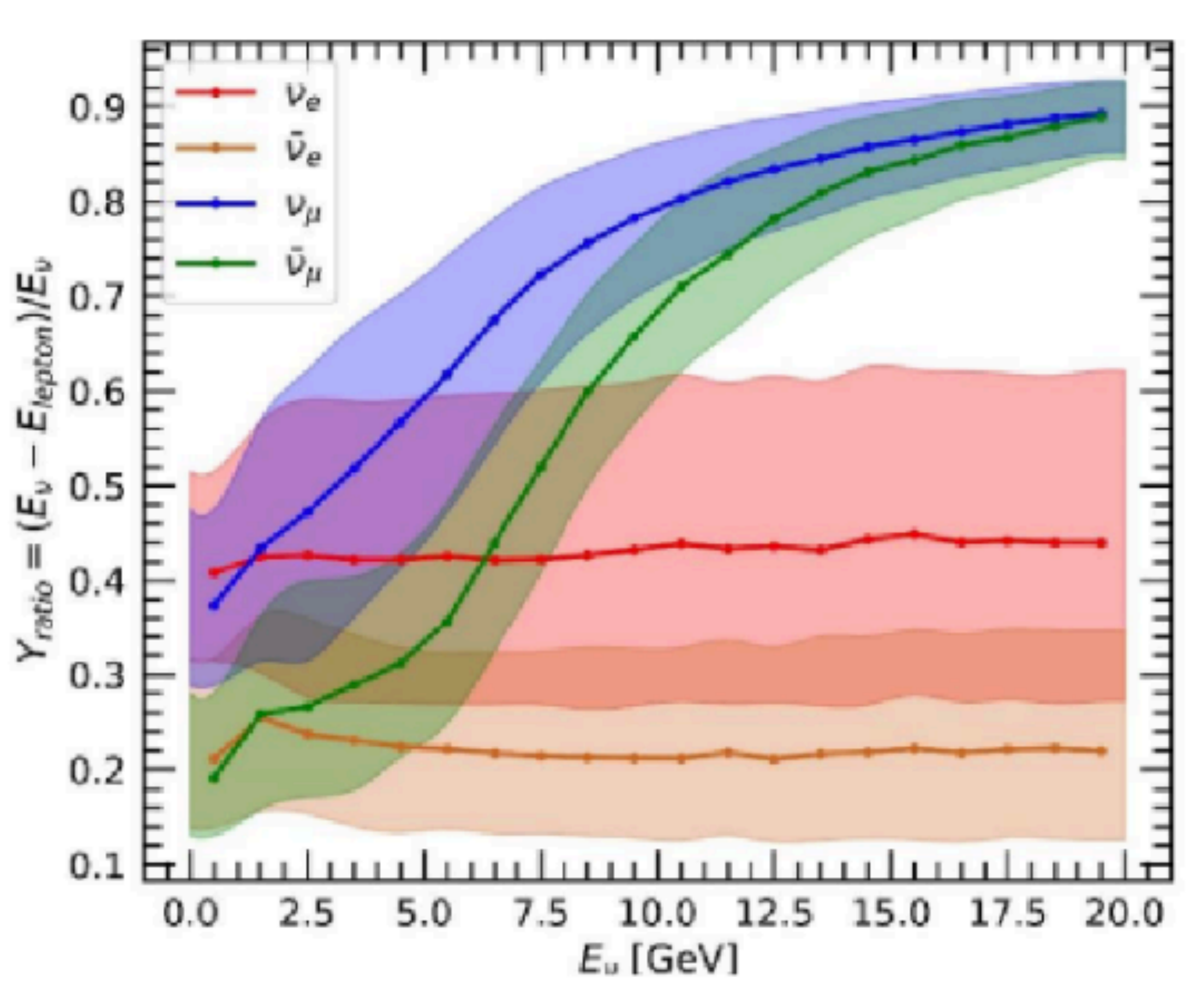
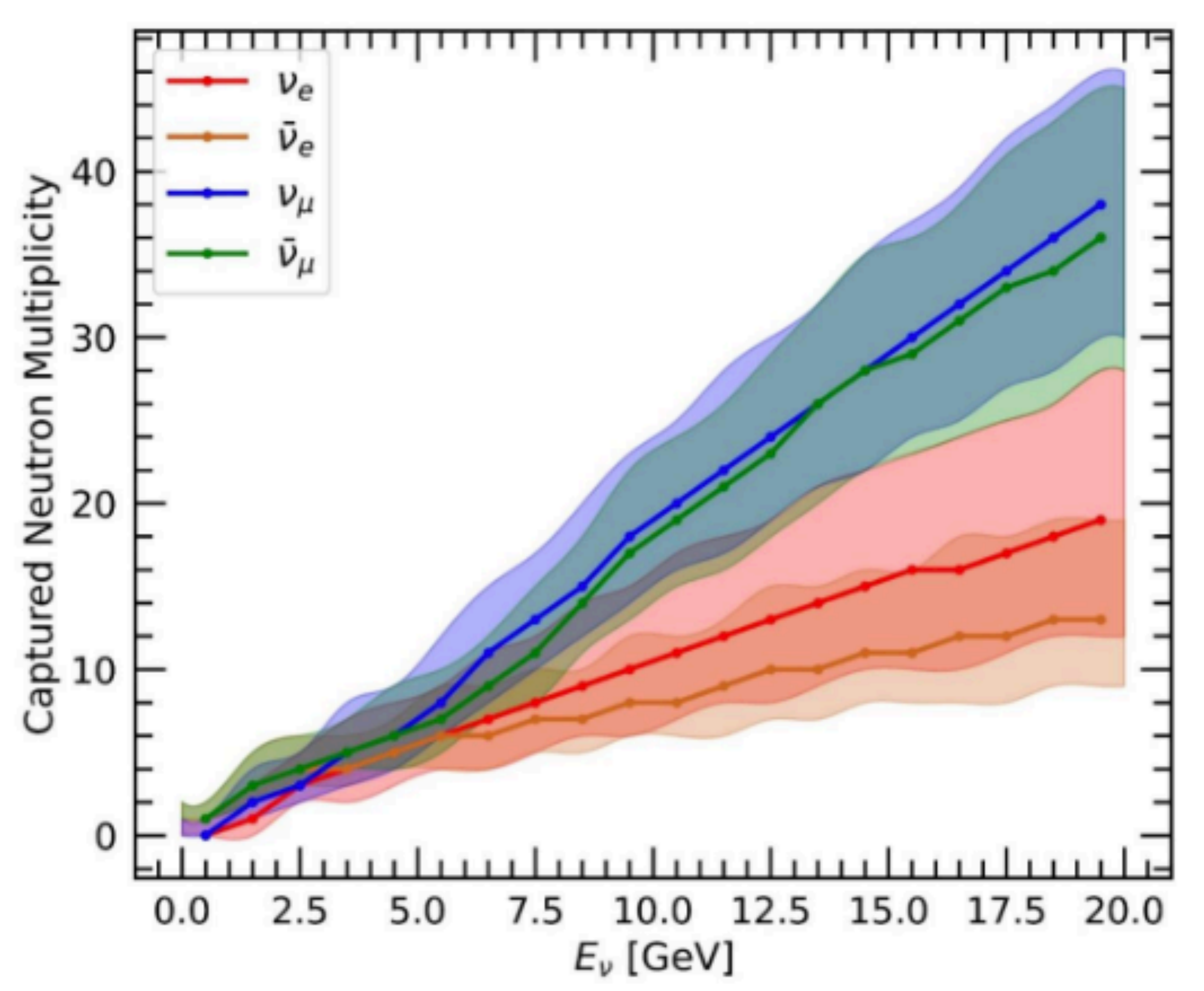


Figure 4. The schematic workflow of atmospheric neutrino classification.



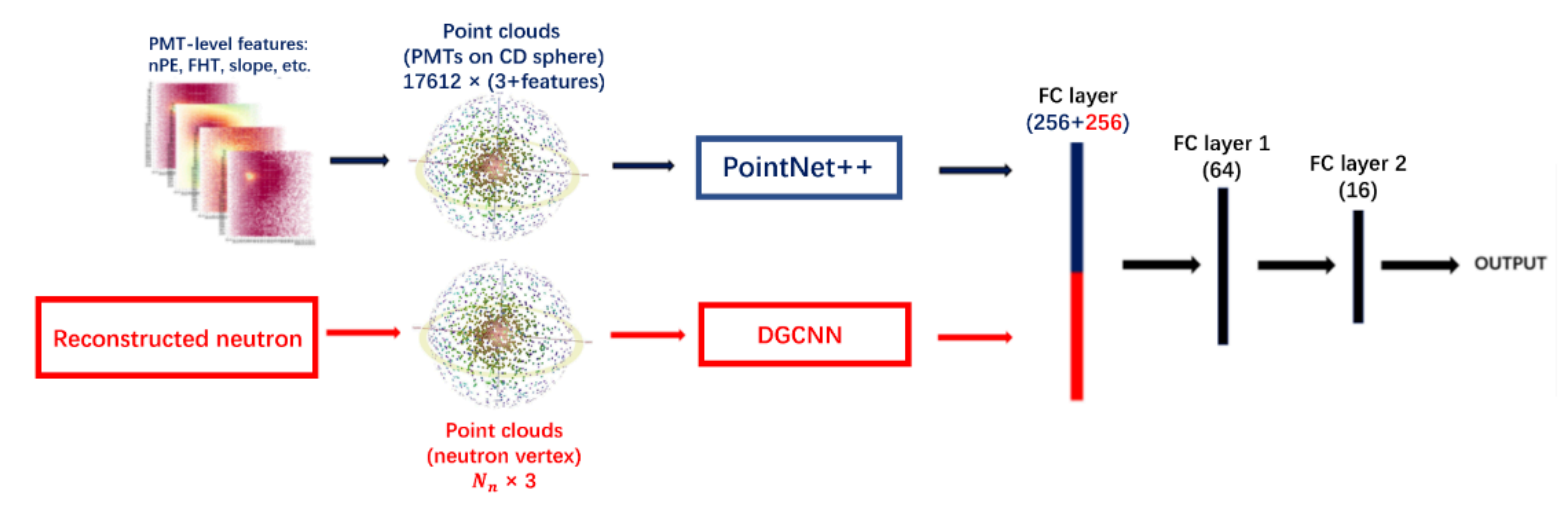
(a) Y_{ratio}



(b) Neutron multiplicity

- Both leptons&hadrons visible, different topology
- step1: CC-e/CC-mu/NC classification
- step2: $\bar{\nu}$ vs ν

PID ML INPUT & MODEL



- ❖PMT features —> PointNet++ (x, y, z, feature_i…)
- ❖Neutron candidates —> DGCNN (x, y, z)

PID PERFORMANCE EVALUATION

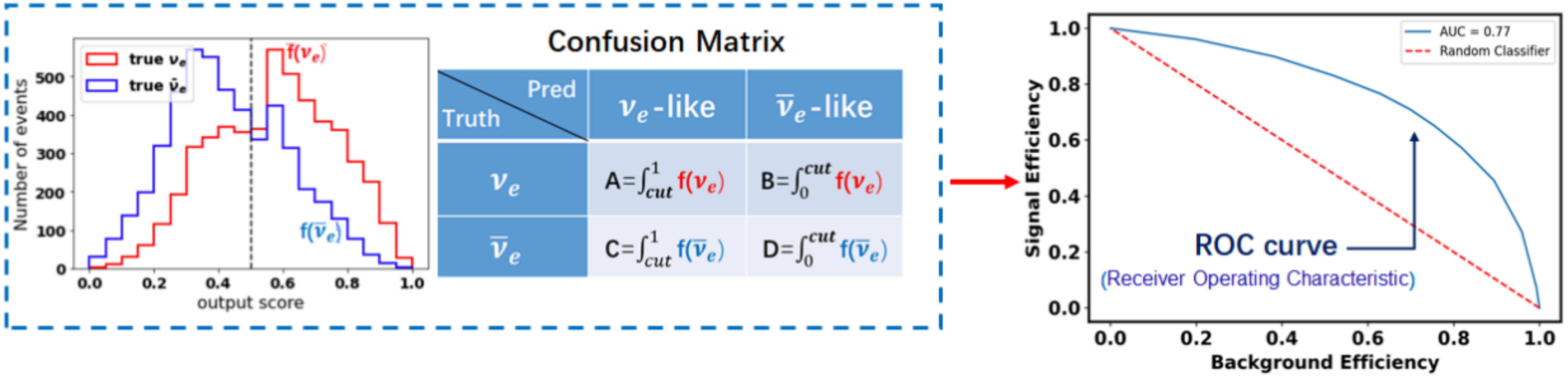
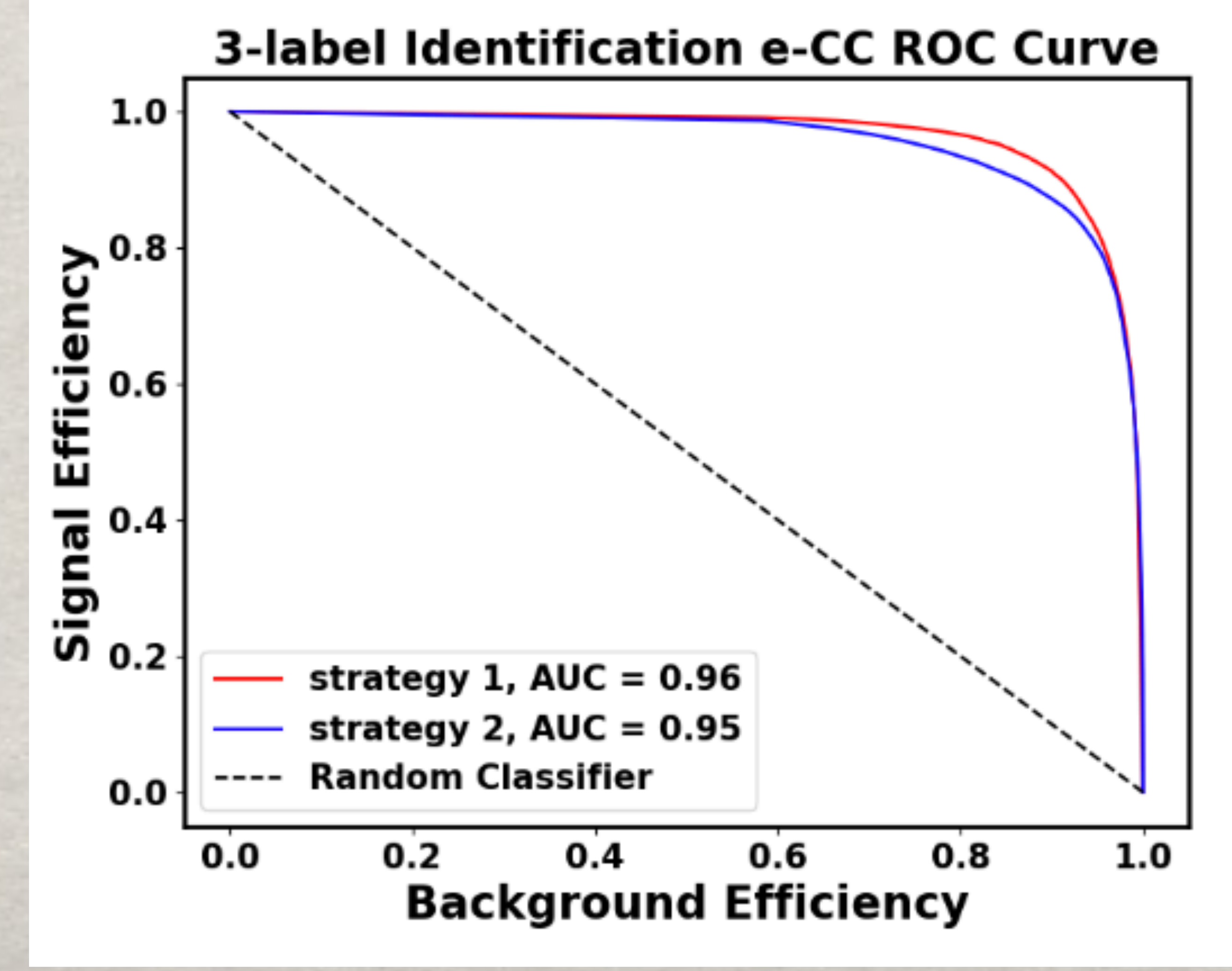
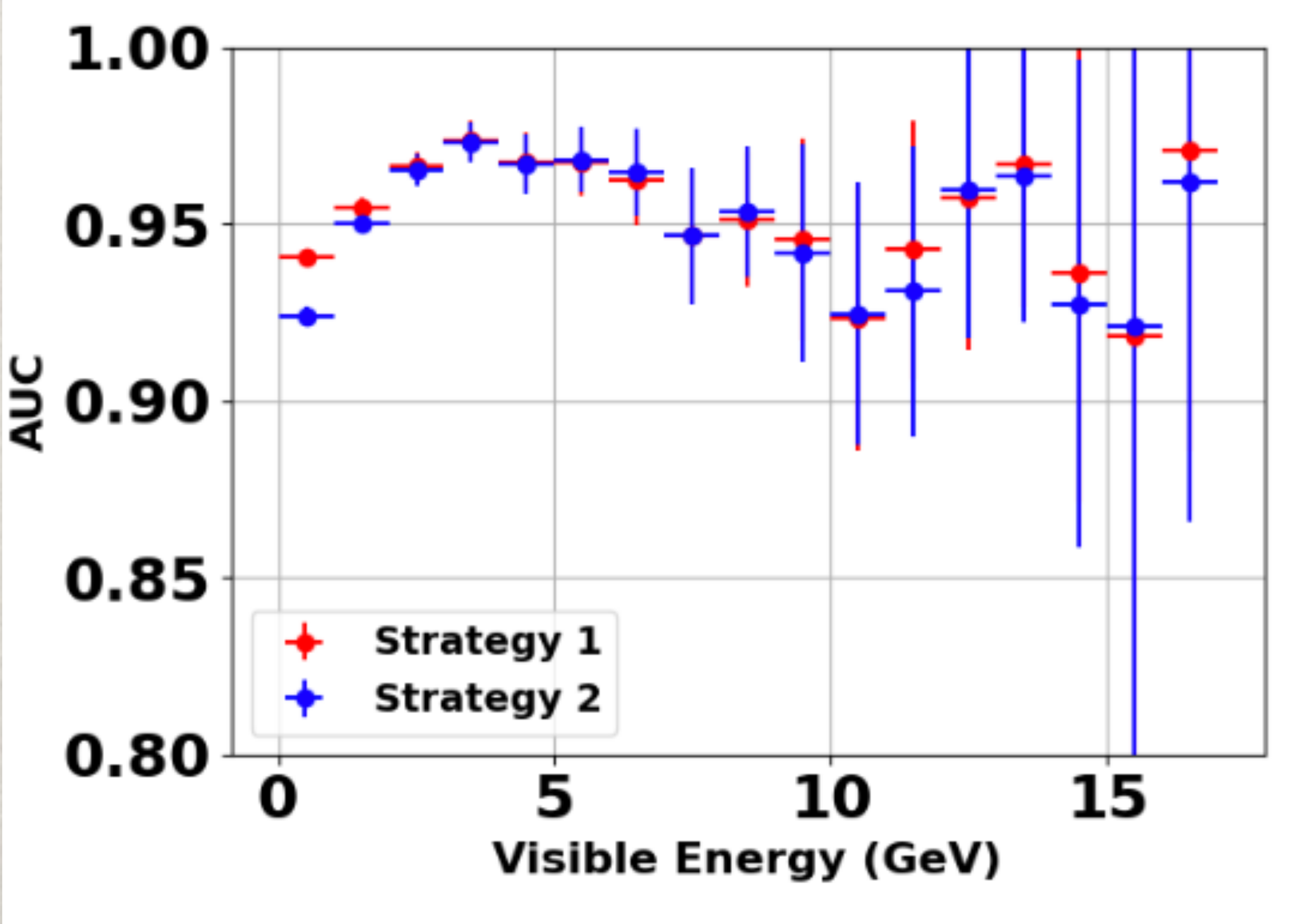


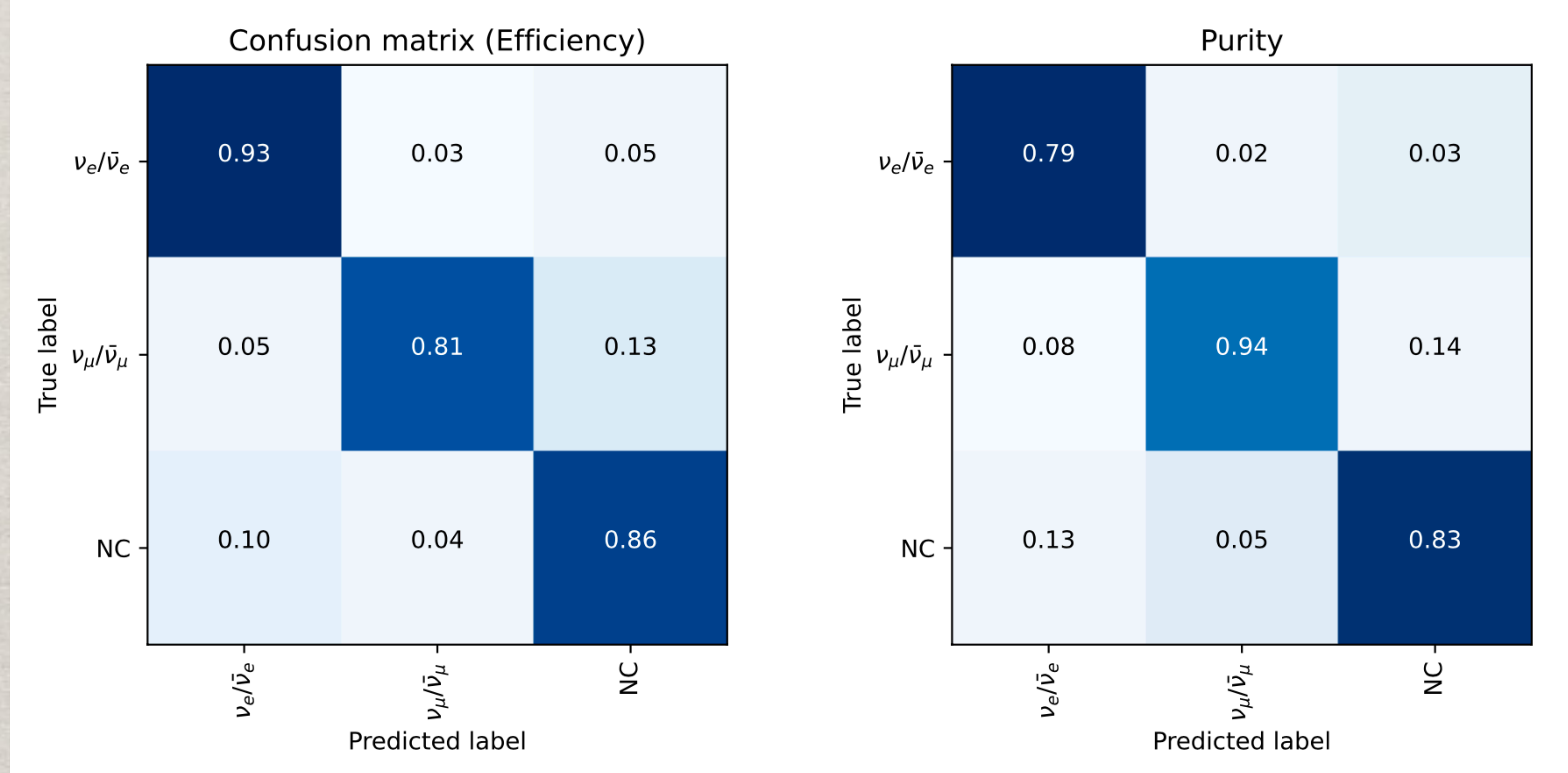
Fig. 8: Illustration of the AUC score using $\nu_e\,/\bar{\nu}_e$ classification as an example. The AUC score can be viewed as an optimisation of $\nu_e\,/\bar{\nu}_e$ efficiencies.

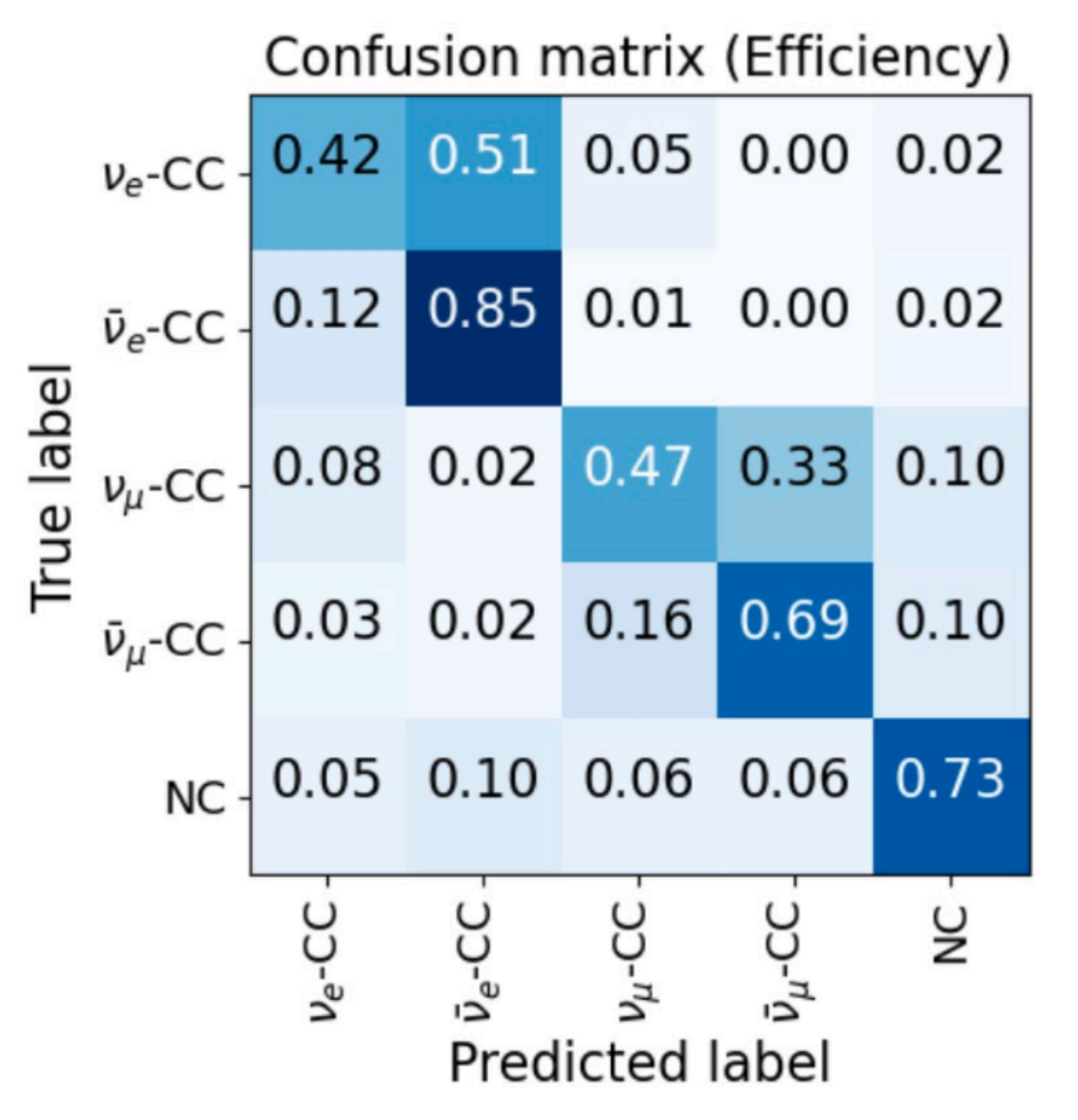




ATM. NEUTRINOS: PID(1)

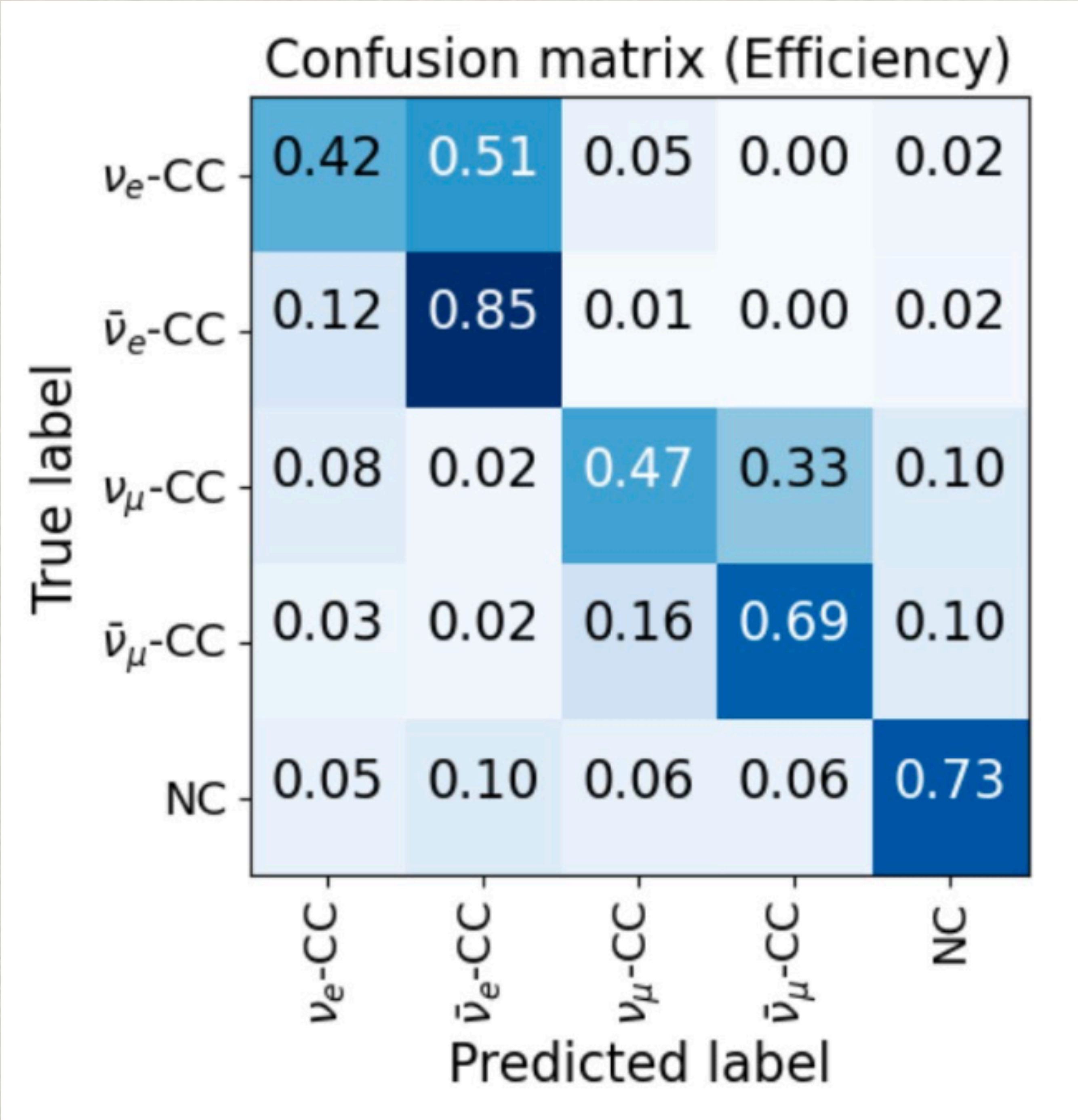
- 3-label classification: $\nu_\mu/\bar{\nu}_\mu$ vs $\nu_e/\bar{\nu}_e$ vs NC
  - same inputs and models as Directionality Reco

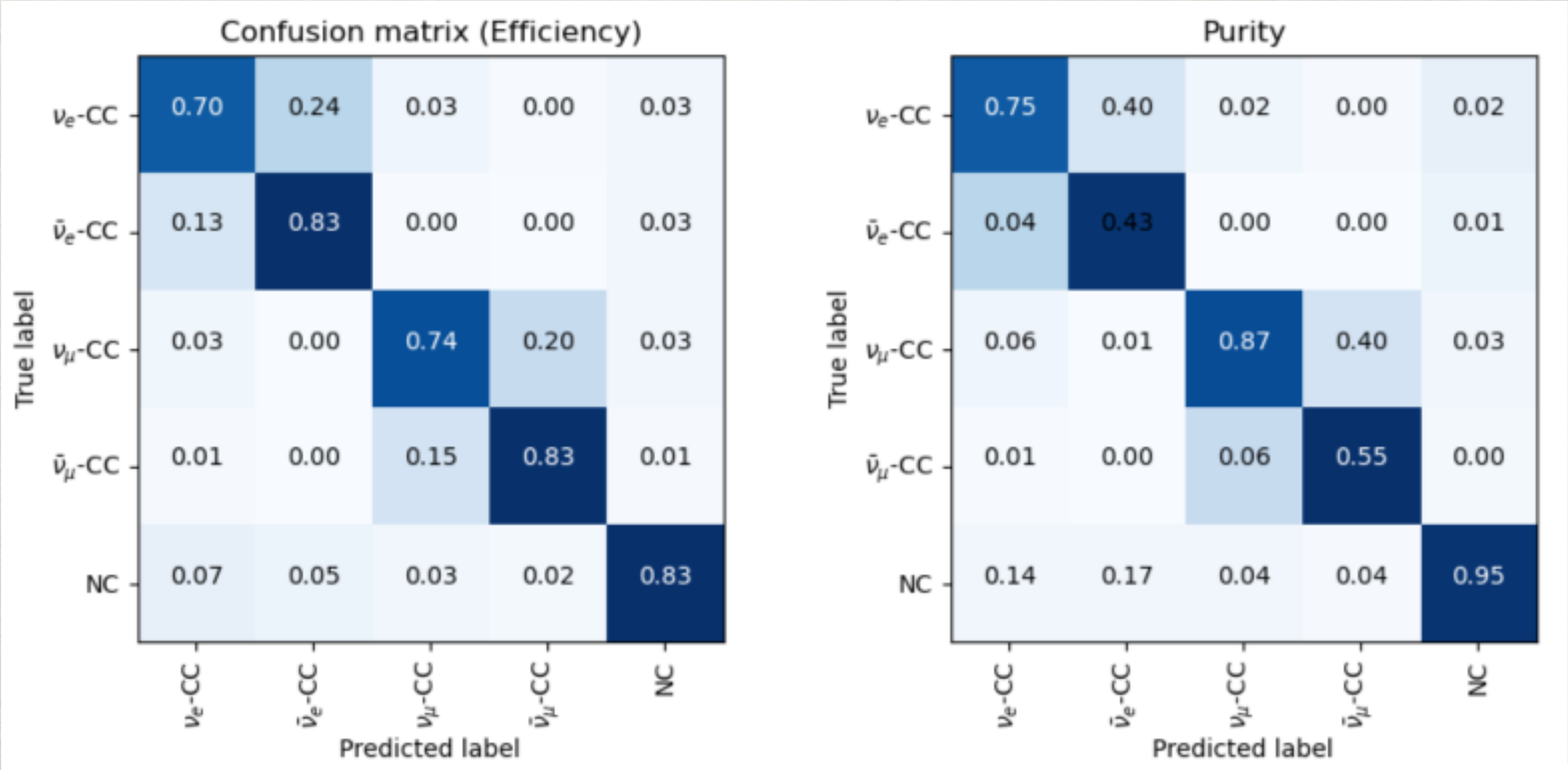




ATM. NEUTRINOS: PID(2)

- ❖ 5-label classification: ν_μ vs $\bar{\nu}_\mu$ vs ν_e vs $\bar{\nu}_e$ vs NC
  - ❖ PMT features + event level variables (neutron/micheal electron…)





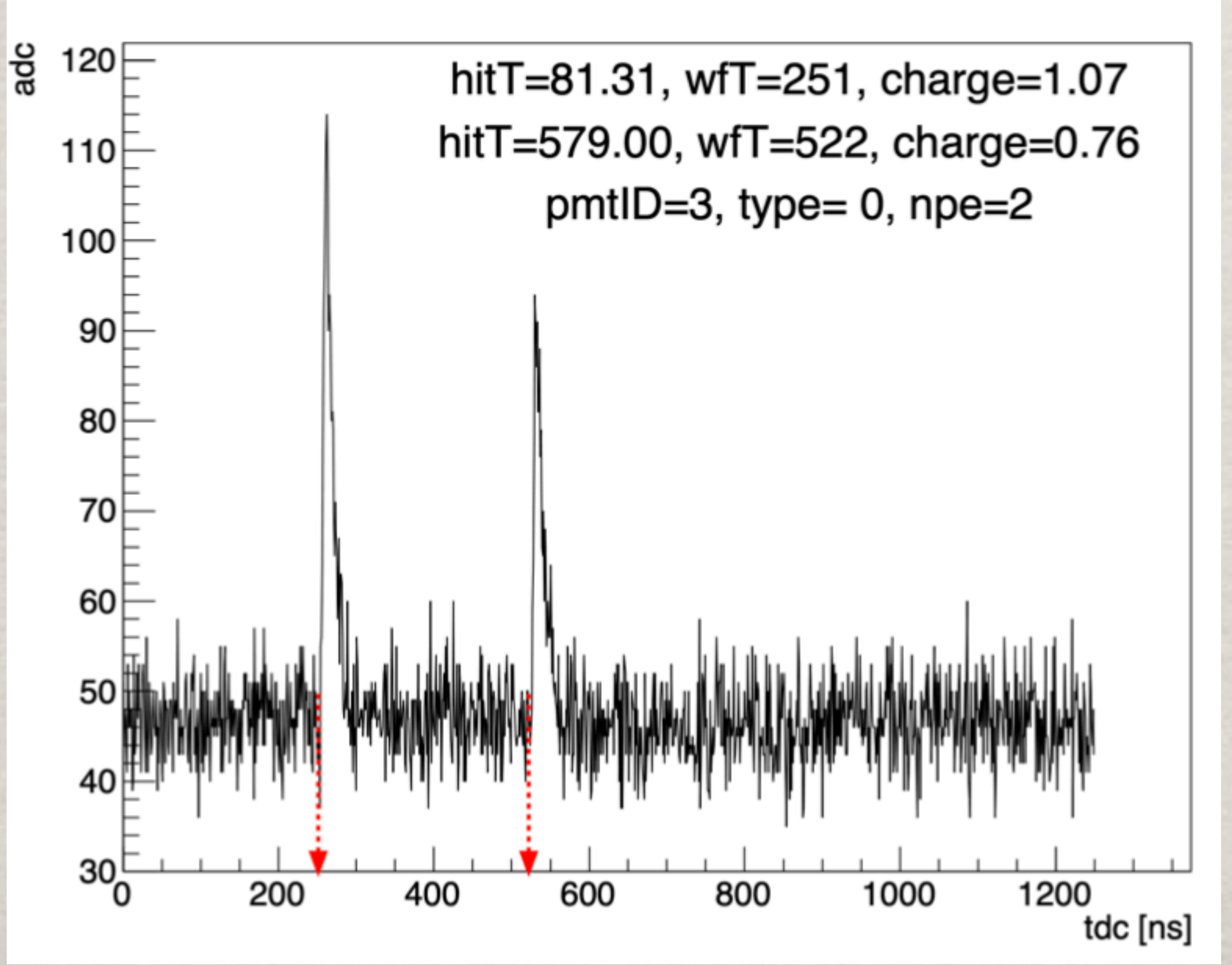
OTHER INTERESTING TOPICS

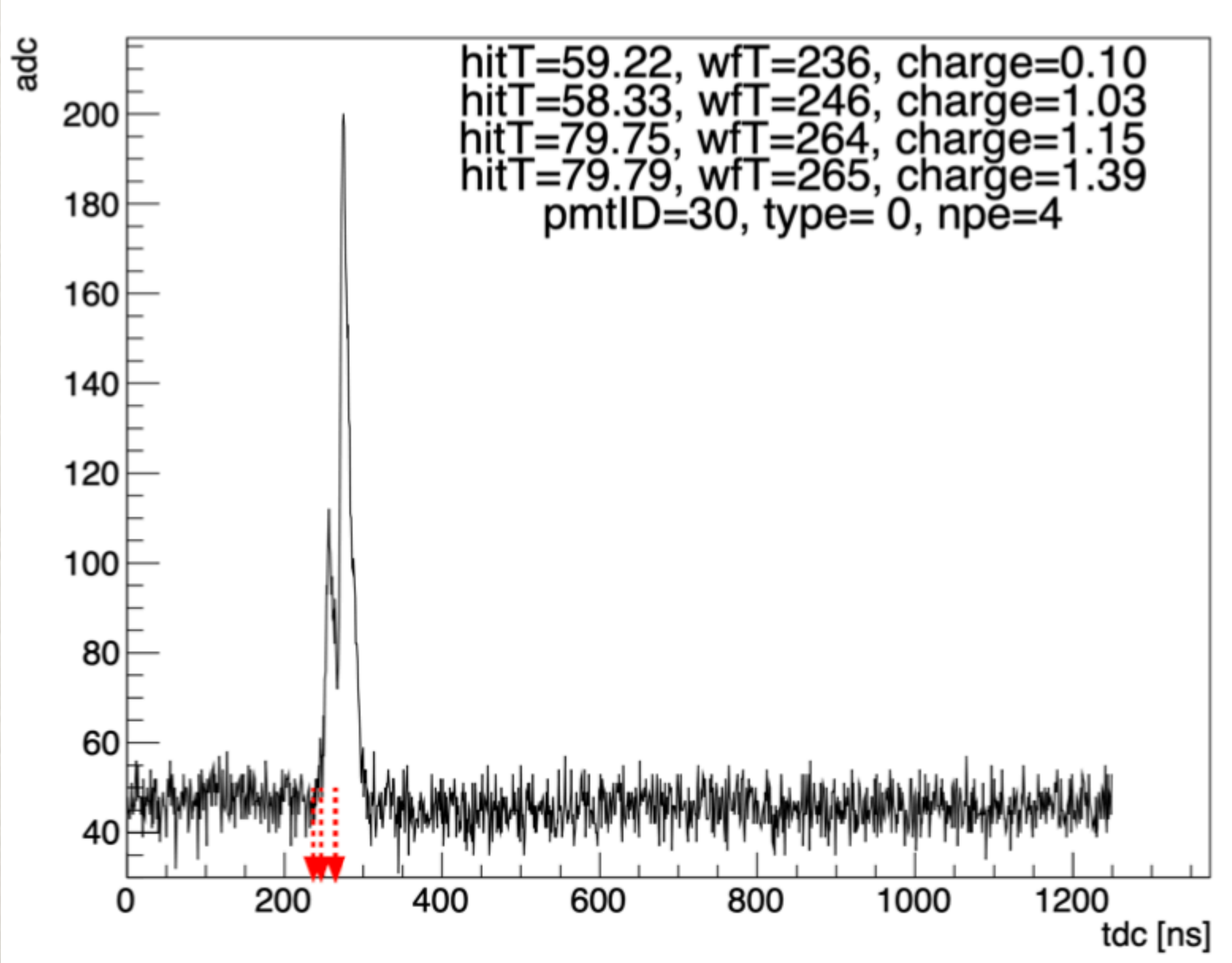
MORE INTERESTING TOPICS

- ❖PMT de-noising, waveform reco
- ❖14C pileUp identification
- ❖Muon classification/combined reco
- ❖Seperation of Scintillation and Cherenkov photons?
- ❖Multi-target reco?
- ❖And more…

PMT WAVEFORM RECO II

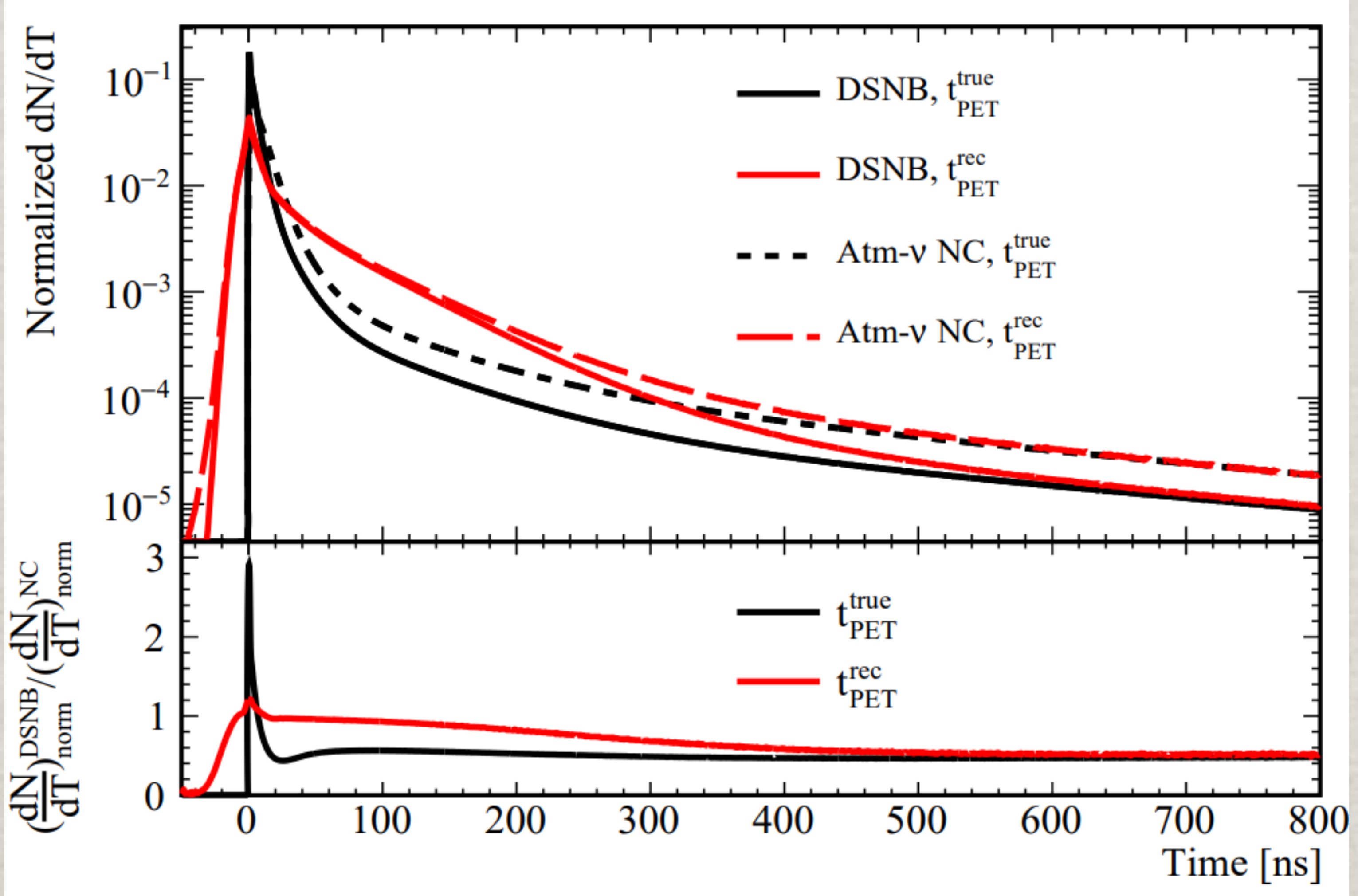
- Regression:
  - easy: total charge or first hit time 😀
  - difficult: charge and time for the first 5 or 10 pulses 😵‍💫
  - super difficult: charge and time for each pulse 😱
- Method: 1D waveform + CNN

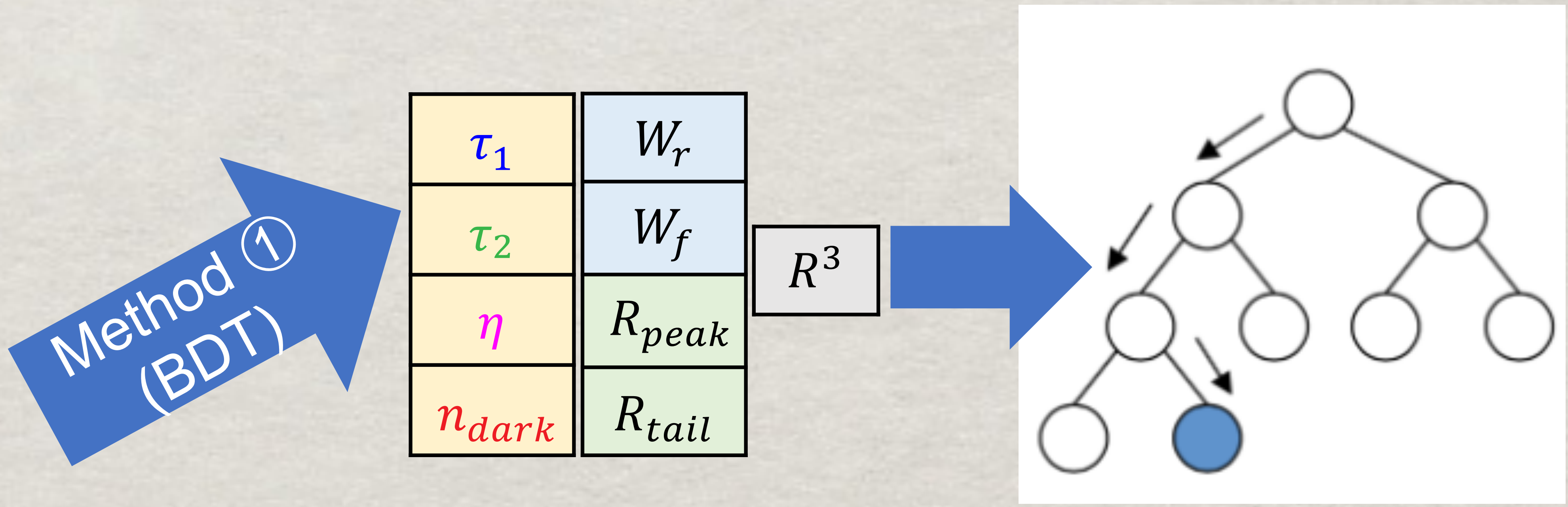


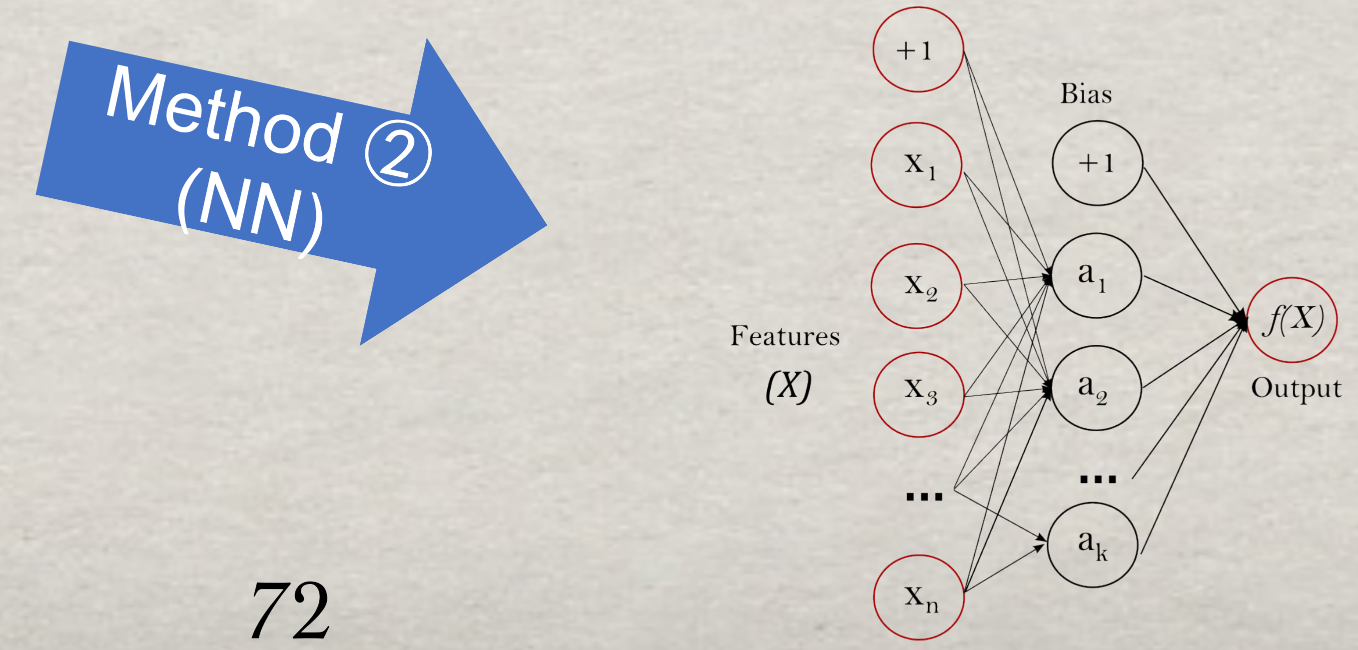


PARTICLE IDENTIFICATION

- **Goal**: Pulse Shape Discrimination (γ/e/e$^+$, vs proton/neutron)
- **Principle**: different scintillation timing profile
- Method: BDT or NN







SUMMARY

- ❖Reco@JUNO is cool
- ❖Reco@JUNO is crucial
- ❖Lots of interesting/challenging problems

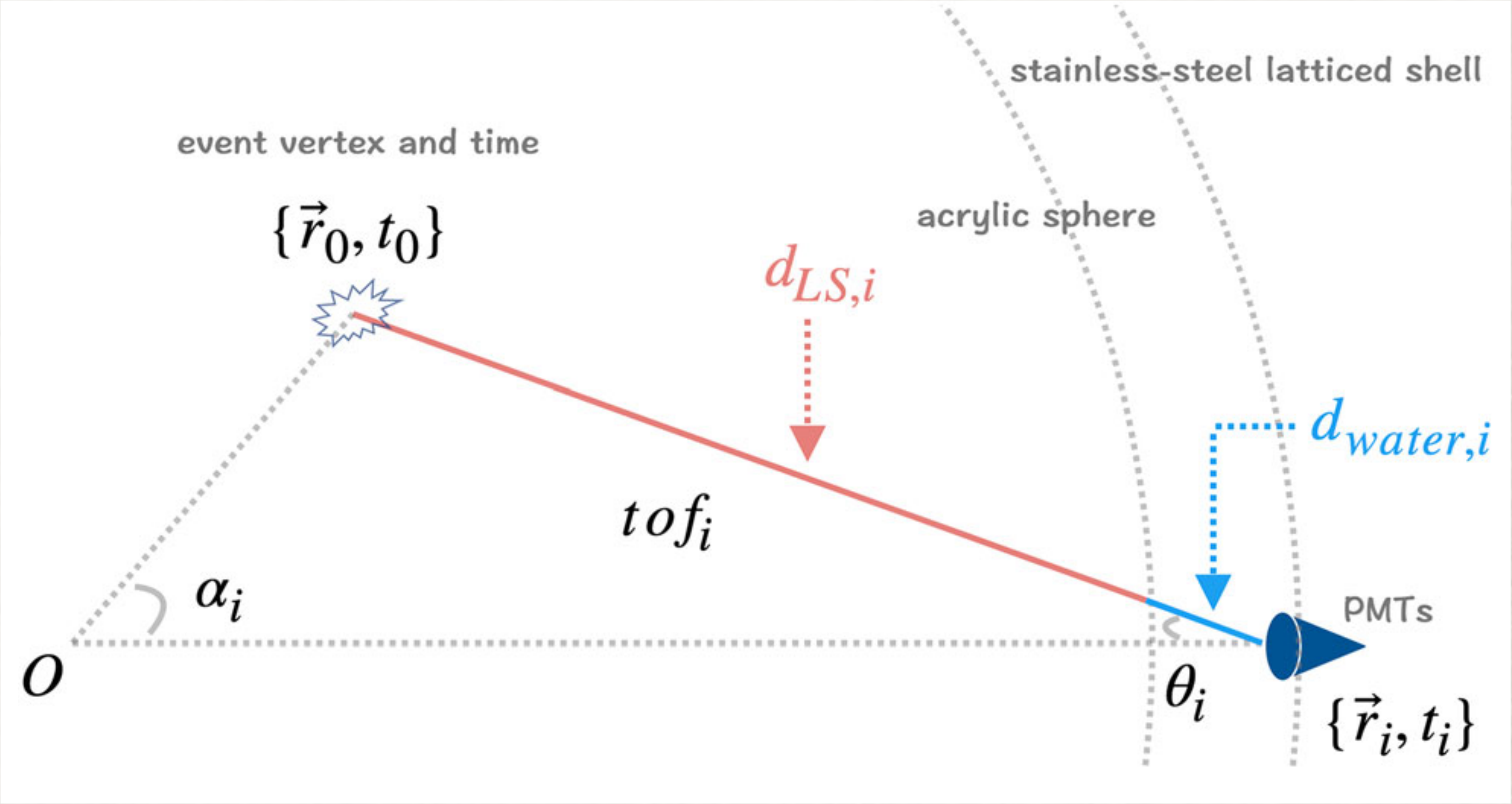
BACK UP

Vertex Reco.

Nucl.Sci.Tech. 32 (2021) 5, 49

Methods	PMT info.	pros&cons	Usage
Charge Center	charge	simple and fast less accurate	initial value
Peak Time Fitter	time	simple more accurate	more accurate initial value
Time Likelihood	time	complex and most accurate	final value

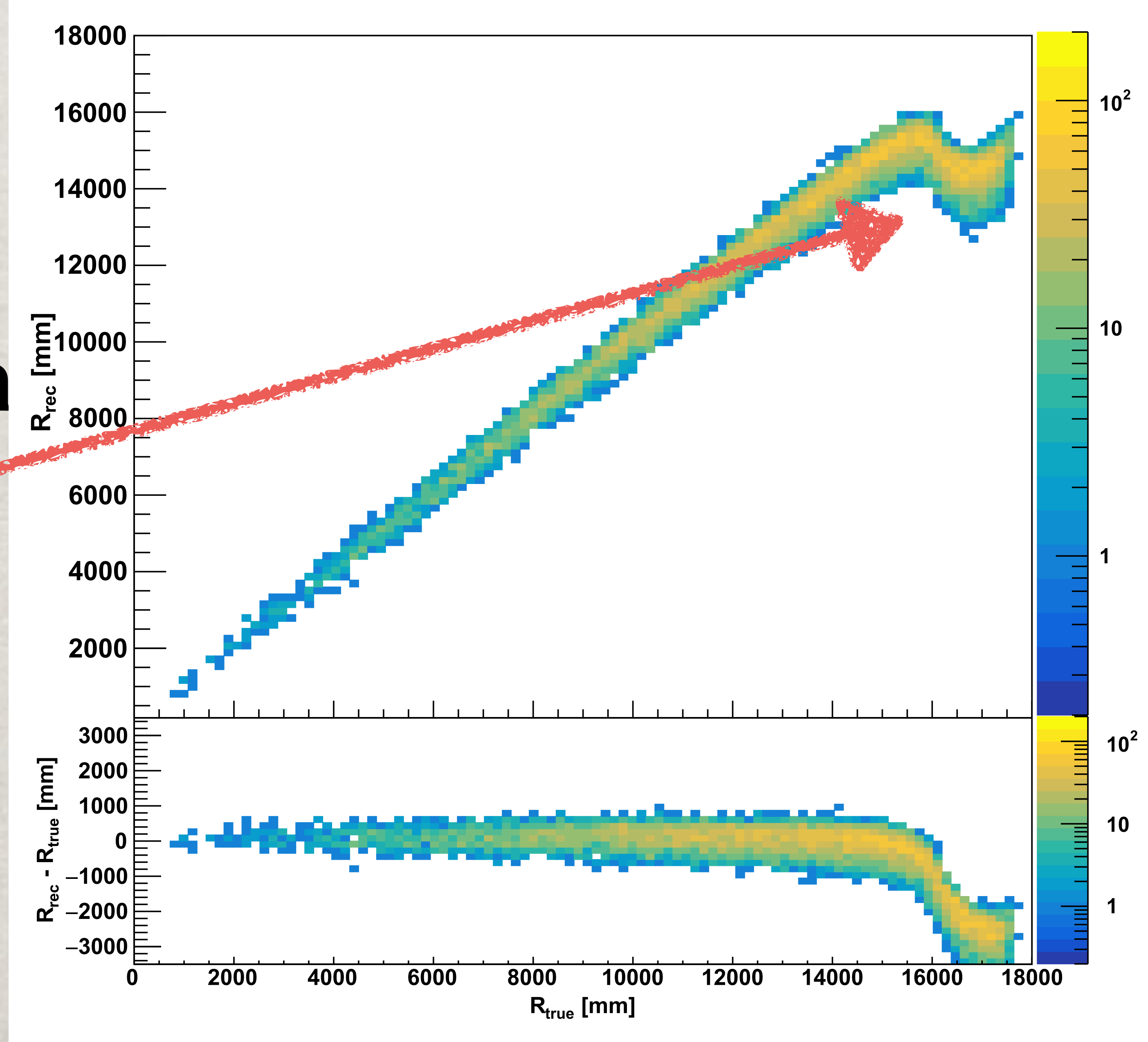
Charge Center



❖Charge weighted average position of

$$\vec{r}_0 = a \cdot \frac{\sum_i q_i \cdot \vec{r}_i}{\sum_i q_i},$$

❖Large bias near the edge due to photon



Wuming Luo

Peak Time Fitter

❖Define "residual time"

$$\Delta t_i(j) = t_i - \mathrm{tof}_i(j),$$ j-th iteration

❖Apply correction to the vertex

$$\vec{\delta}[\vec{r}(j)] = \frac{\sum_i \left(\frac{\Delta t_i(j) - \Delta t^{\mathrm{peak}}(j)}{\mathrm{tof}_i(j)}\right) \cdot (\vec{r}_0(j) - \vec{r}_i)}{N^{\mathrm{peak}}(j)},$$

❖Iterate until Δt shape converges