Precision QCD and Flavor physics

focus on the B meson decays

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Overview

I B physics and CP Violation

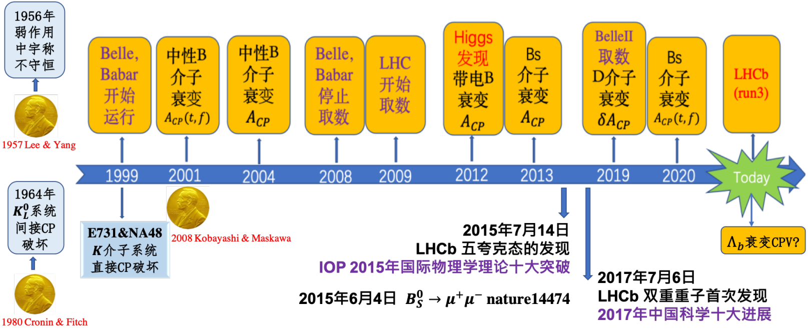
II Factorization approaches
- Low energy effective hamiltonian
- Factorization approaches and precise QCD

III Toward to high accuracy LCDAs
- $B_{(s)}$ LCDAs
- Light mesons LCDAs

IV Conclusion

B physics and CP Violation

- "Matter $\neq$ Antimatter" indicates the interaction with CPV
- Heavy flavour physics (HFP) provides many processes with CPV
  although it is inadequate and new mechanism of CPV is imminent
- Great running of B factories(1999-2008) and LHC(2009-)



CP 破坏测量 60 年

LHCb has become a general purpose detector nowadays

- CKM, CPV and rare decays in b and c hadrons
  - Indirect search of BSM via precise measurements
- this talk focus on precise QCD prediction of hadronic B decays

- presize QCD + EW
- hadron spectroscopy
- heavy-ion and fixed-target physics
- direct search of new physics   dark sector, $\cdots$

Factorization approaches

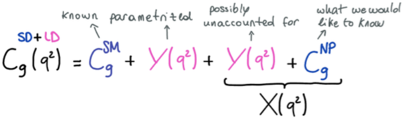
i low energy effective hamiltonian

ii factorization approaches and precise QCD

Low energy effective hamiltonian

- No direct evidence of BSM due to the mass gap
  - NP is either very heavy or light and weakly coupled to the SM

- Use Effective Field Theories and Data to bridge the gap
  - describe heavy NP via high dimension operators
  - use data (electroweak, flavor) to constrain the Wilson coefficients



Disentangling long-distance and NP in $b \to s\mu\mu$

- Roadmap of the low energy EFT of heavy flavor decays

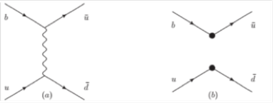
$$\text{New physics: } \mathcal{L}_{NP}$$
$$\downarrow$$
$$\text{Electroweak scale } (m_W)\text{: } \mathcal{L}_{EW} + \mathcal{L}_{D>4}$$
$$\downarrow$$
$$\text{Heavy quark scale } (m_b)\text{: } \mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{\rm CKM} \sum_i C_i(\mu) O_i(\mu) + \mathcal{L}_{{\rm eff},D>6}$$
$$\downarrow$$
$$\text{Hadron scale } (\Lambda_{\rm QCD})\text{: LCDAs, PDF, PDA}$$

Low energy effective hamiltonian

- derive the low energy effective Hamiltonian by integrating over m_W

[Buchalla 1996]

$$\mathcal{A} = -\frac{G_F}{\sqrt{2}} V^\star_{ud} V_{ub} (\bar{u}b)_{V-A} \frac{M_W^2}{k^2 - M_W^2} (\bar{d}u)_{V-A}$$
$$= \frac{G_F}{\sqrt{2}} V^\star_{ud} V_{ub} (\bar{u}b)_{V-A} (\bar{d}u)_{V-A} \left(1 + \mathcal{O}(\frac{k^2}{M_W^2})\right).$$
$$\mathcal{H}^0_{eff} = \frac{G_F}{\sqrt{2}} V^\star_{ud} V_{ub} (\bar{u}b)_{V-A} (\bar{d}u)_{V-A} + \text{High D Operatoes},$$



$b \to \bar{u}ud$

- product of two charged currents $\sim$ a series of local operators O_i with the weighted coefficients C_i

$$\mathcal{H} \to \mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ \xi_u \sum_{i=1,2} C_i(\mu) O_i^u(\mu) - \xi_t \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right\},$$
$$\mathcal{A}(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_i C_i V_i \langle M_1 M_2 | O_i | B \rangle,$$

- 弱作用标度 $\mathcal{O}(m_Z)$ 的动力学由 $C_i(\mu)$ 描述
- $\mathcal{O}(m_b)$ 标度的动力学
  由四费米有效算符 $O_i(\mu)$ 描述
- 强子能标 $\mathcal{O}(\Lambda_{\text{QCD}})$ 的动力学由强子波函数描述

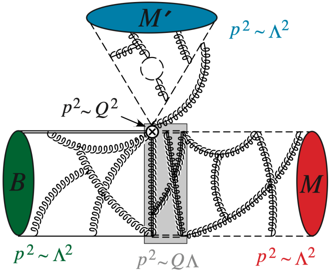
- dynamics at the scale $\mathcal{O}(m_W)$ is absorbed into Wilson coefficients $C_i(\mu)$
  - $C_i \sim$ match the $\mathcal{L}_{\text{eff}}$ with the full theory of weak decays [Ma '80, Inami&Lim '81, Clements '83]
  - the NLO QCD/QED corrections to C_i has been finished [Buchalla, 1996, Rev. Mod. Phys]
  - the NNLL program [Gorbahn, Haisch '04; Misiak, Steinhauser '04]

Factorization approaches $\langle MM'|O_i|B\rangle$

- the rest goes into the four fermion effective operators $O_i(\mu)$
- the key is to calculate the hadron matrix element $\langle MM'|O_i|B\rangle$

it is definitely a QCD problem

- a multi-scale QCD & QED problem

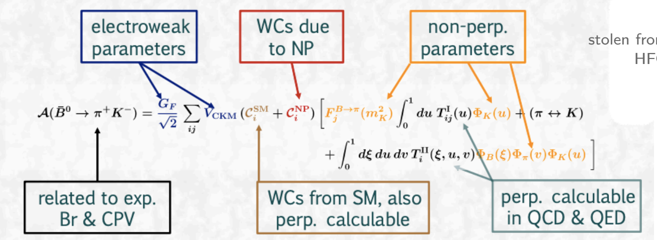


- ○ **Naive factorization**: $\sim F_{B\to M} \otimes f_{M'}$ [Bauer, Stech, Wirbel '85,'87]
- ○ **Generalized factorization**: QCD corrections from $O_{i=1,\cdots,10}$ [Ali, Kramer, Lü '98,'99]
- ○ **QCD factorization (QCDF)**: VC to $\mathcal{M}_{t,p}$ + corrections to spectator scattering [Benele, Buchalla, Neubert, Sachrajda '01]
- ○ **Soft-collinear effective theory (SCET)**: introduces different fields in different energy regions, simple kinematics but complicated dynamics with several typical scales [Bauer, Flemming, Pirjol, Stewart '01, Beneke, Chapovsky, Diehl, Feldmann, '02]
- ○ **Light-cone sum rules (LCSRs)**: redundant formulism, hard for phenomena [Khodjamirian '01,'03,'05]
- ○ **Perturbative QCD (PQCD)**: pick up the transversal momentum to regularate end-point singularity, resummations, prediction of CPV [Keum, Li '01, Sanda, Lü, Yang '01]

- ○ **QCD symmetry**: isospin, U spin, V spin, $SU(3)_F$
  [Zeppenfeld, '81]
  [London, Gronau, Rosner, He, Chiang, Cheng et al.]
- ○ **Factorization-assisted topological-amplitude approach (FAT)** universal decay amplitudes to be determined by data
  [Li, Lü, Yu '12, Qin, Li, Lü, Yu '14]

Factorization approaches $\langle MM'|O_i|B\rangle$

- **QCDF**: systematic framework to all α_s orders, but limited by $1/m_b$ corr.
  [Beneke '06,'07, Jain '07, Benele '10, Bell '15, '20, Huber '16]



stolen from X.Q Li
HFCPV2024

- **SCET**: $\mathcal{A} \simeq F_{BM} T_i^1 \otimes \phi_{M'} + T_i^{II} \otimes \phi_B \otimes \phi_M \otimes \phi_{M'}$ [Bauer '01, Chay '04, Becher '15]
- " QCD - SCET = short distant coefficients T^I, T^{II} "
  SCET reproduces precise **QCDF** in collinear and soft regions [Beneke '15]
- F_{BM} form factors from **LCSRs**: high order & power corrections
  [HPQCD 2013], [Bharucha '16, Wang '15,'16,;20, Lü '19, Beneke '17, Gubernari '19, SC '17,'19]

Factorization approaches $\langle MM'|O_i|B\rangle$

- Full N^2LO QCDF calculation $(\mathcal{O}(\alpha_s^2))$ [Bell, Beneke, Huber, Li '20]
- ◦ combining $1/m_b$ expansion with light-cone expansion for hard processes

I: $\lambda_B = 0.35 \pm 0.15$ GeV, $F_{B\pi} = 0.25 \pm 0.05$
II: $\lambda_B = 0.20 \pm 0.05$ GeV, $F_{B\pi} = 0.23 \pm 0.03$

branching fraction

Modes	QCDF NNLO-I (10^{-6})	QCDF NNLO-II (10^{-6})	DATA
$B^- \to \pi^-\pi^0$	$5.43^{+0.06+1.45}_{-0.06-0.84}$	$5.82^{+0.07+1.42}_{-0.06-1.35}$	$5.59^{+0.41}_{-0.40}$
$B^0 \to \pi^+\pi^-$	$7.37^{+0.86+1.22}_{-0.69-0.97}$	$5.70^{+0.70+1.16}_{-0.55-1.97}$	5.12 ± 0.19
$B^0 \to \pi^0\pi^0$	$0.33^{+0.11+0.42}_{-0.08-0.17}$	$0.63^{+0.12+0.64}_{-0.10-0.42}$	1.59 ± 0.26

direct CPV

Modes	QCDF NNLO	QCDF NNLO+LD	DATA
$B^- \to K^-\pi^0$	$10.18^{+1.91+2.03}_{-1.90-2.02}$	$-1.17^{+0.22+20.0}_{-0.22-6.62}$	3.7 ± 2.1
$B^0 \to K^-\pi^+$	$8.08^{+1.52+2.52}_{-1.51-2.65}$	$-3.23^{+0.61+19.6}_{-0.61-3.36}$	-8.3 ± 0.4
$B^0 \to \bar{K}^0\pi^0$	$-4.33^{+0.84+3.29}_{-0.78-2.32}$	$-1.41^{+0.27+5.54}_{-0.25-6010}$	0 ± 13

LD: power suppressed spectator and annihilation terms

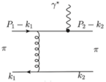
- large uncertainties comes from hadronic parameters
- annihilation diagram is calculable, finite, and contains strong phase
  [Lu, Shen, Wang[2] '22]

Factorization approaches $\langle MM'|O_i|B\rangle$

- **PQCD**: pick up the transversal momentum in the hard scattering amplitudes to regulate the end-point singularity [Huang 1991]

- end-point singularities appear in exclusive QCD processes

  † $m^2_{1,2} \ll Q^2$, light-cone coordinate $p_2 = (\frac{Q}{\sqrt{2}}, 0, 0_{\mathrm{T}})$, $p_3 = (0, \frac{Q}{\sqrt{2}}, 0_{\mathrm{T}})$, (anti-)valence quarks: $k_2 = x_2 p_2$, $\bar{k}_2 = \bar{x}_2 p_2$



$\phi \propto u(1-u)$, $m_0^\pi \phi^{P,\sigma} \propto m_0^\pi$

$$\propto \sum_t \int du_1 du_2 \kappa_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{u_1 u_2 Q^2 u_2 Q^2}$$

- pick up k_T in the internal propagators

$$\mathcal{M} \propto \sum_{t=2,3,4} \int du_1 du_2 dk_{1T} dk_{2T} \kappa_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{u_1 u_2 Q^2 - (k_{1T} - k_{2T})^2}$$

- end-point singularity at leading and subleading powers

$$\mathcal{H} \propto \frac{\alpha_s(\mu)}{u_1 u_2 Q^2 - k_T^2} \sim \frac{\alpha_s(\mu)}{u_1 u_2 Q^2} - \frac{\alpha_s(\mu) k_T^2}{(u_1 u_2 Q^2)^2} + \cdots$$

- the power suppressed TMD terms becomes important at the end-points

- ○ scales of transversal momentum $(Q, \sqrt{\Lambda Q}, \Lambda)$ and the large logarithms
- ○ hard scattering amplitude should not sensitive to $k_T \sim \Lambda_{\mathrm{QCD}}$
- ○ k_T resummation for $\mathcal{H}$ to obtain $S(x_i, b_i, Q)$ [Botts 1989, Li 92]
- ○ integrating over k_T, large log $\ln^2(x_i)$ when intermediate gluon is on shell
- ○ threshold resummation for Φ to obtain $S_t(x_i, Q)$ [Li 1999]
- ○ dynamics with $k_T < \sqrt{Q\Lambda}$ is organized into $S(x, b, Q)$
- ○ dynamics in small x is suppressed by $S_t(x, Q)$

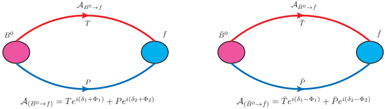
$$\mathcal{M} = \sum_t \phi^t(u_1, b_1) \otimes \mathcal{H}_i(t, b) \otimes \phi^t(u_2, b_2)\, Exp\left[-s(p^+, b) - \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu}))\right]$$

Factorization approaches $\langle MM'|O_i|B\rangle$

- **LO PQCD**: both F_{BM} and annihilation amplitudes are calculable
- Sources of strong phase difference to generate $\not{C}P$

○ emission and annihilation ($\delta_2 \neq 0$)

$$\frac{1}{k_T^2 - xm_B^2 - i\epsilon} = \mathcal{P}\left(\frac{1}{k_T^2 - xm_B^2}\right) + i\delta(k_T^2 - xm_B^2)$$



- ○ Sudakov expanent (NLO)
- ○ NLO corrections to spectator emission amplitude from Glauber gluon
- ○ on shell charm quark loop correction (NLO), leading source in QCDF

- successful in phenomena
  - ⋆ predicted a large CPV in $\pi^- K^+, \pi^+\pi^-$ (2000), confirmed by the B factories (2004)
  - ⋆ predicted $\mathcal{B}(B_s \to \pi^+\pi^-) \sim 6 \times 10^{-6}$ (2007), confirmed by CDF (2011)
  - ⋆ predicted $f_L \sim 0.7$ in penguin dominated $B \to VV$ by annihilation mechanism (2002), before the observation of "polarization puzzle"

- improved-PQCD with relativistic potential model of B-meson WF and soft form factor [Lü, Wang, Yang, '23,'24]

Factorization approaches $\langle MM'|O_i|B\rangle$

- ○ **dominate NLO PQCD** ($\mathcal{O}(\alpha_s^2)$): factorizable amplitudes [SC & Xiao '21], effective operators [Mishima '03, Li '05], hard scattering [Li '12, '13, '14; SC '14, '15, Hua '18, Liu '15,'16]

- • state-of-the-art PQCD calculation [Chai, SC, Ju, Yan, Lü, Xiao CPC 46.12(2022)123103]

⋆ $S_{\pi^0 \bar{K}^0} = 0.72^{+0.05}_{-0.05}$ is confirmed by Belle [PRL.131.111803(2023)]

⋆ $\mathcal{B}_{\omega\omega} = \left(1.21^{+0.45}_{-0.35}\right) \times 10^{-6}$, $f_L = 88.4 \pm 1.1$ are confirmed by Belle-II [PRL.133.1081801(2024)]

$\lambda_B = 0.40 \pm 0.04$ GeV and parameters of light mesons

branching fraction

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$B^0 \to \bar{K}^0\pi^0$	-2.63	$-7.70^{+0.12+0.17}_{-0.09-0.09}$	0 ± 13

Factorization approaches $\langle MM' | O_i | B \rangle$

- $K\pi$ puzzle: $\delta A_{CP}(K\pi) = 0.11 \pm 0.01$, $\sim 9\sigma$ deviation from SM prediction
- $\pi^0\pi^0$ puzzle: theoretical prediction gives a much smaller $\mathcal{B}$

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- high order, power corrections significantly improve the accuracy, reduce the μ dependence

Our results for $\mathcal{B}$ and f_L agree well with predictions from next-to-leading-order (NLO) perturbative QCD (PQCD) [23], but not from leading-order (LO) PQCD [19]. This indicates that NLO corrections and power-suppressed terms play an important role in color-suppressed $b \to (u, d)$ decays. Such a role would help clarify the puzzle in $B^0 \to \rho^0\rho^0$, where the measured f_L is significantly higher than the LO PQCD prediction [19]. Our result for $\mathcal{B}(B^0 \to \omega\omega)$ is significantly higher than the prediction from soft collinear effective theory [22]. Our result for A_{CP} shows no significant CP violation, consistent within uncertainties with CKM unitarity. Belle, PRL.133.1081801(2024)

[23] Chai, SC, Ju, Yan, Lu, Xiao, CPC 46.12(2022)123103
[19] Zou, Ali, Lu, Liu, Li, PRD 91.054033(2015)

- high accuracy B meson LCDAs is imperative in the high precision era

Towards to high accuracy LCDAs

i $B_{(s)}$ LCDAs

see talks from Jun Zeng, 14:00, Nov 14th and Yan-bin Wei, 14:00, Nov 16th

see talk from Xue-ying Han, 14:20, Nov 15th (Theory/CEPC/Computing/Performance Parallel)

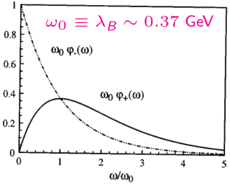
ii Light mesons LCDAs

Leading twist B-meson LCDA $\phi_+(\omega)$

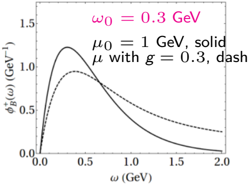
- Two-particle (2p) distribution amplitudes up to twist five [Grozin, Neubert '97]

$$\langle 0|\bar{d}_\alpha(x) h_{v\beta}(0)|\bar{B}_v\rangle = -\frac{i f_B m_B}{4}\int_0^\infty d\omega\, e^{-i\omega v\cdot x}\Big[(1+\not{v})\Big\{\big[\phi_+(\omega) + x^2 g_+(\omega)\big] - \frac{\big[\phi_+(\omega)-\phi_-(\omega) + x^2\left(g_+(\omega)-g_-(\omega)\right)\big]}{2v\cdot x}\not{x}\Big\}\gamma_5\Big]_{\beta\alpha}$$

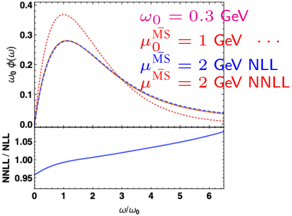
- Exponential models $\phi_+(\omega) = \frac{\omega}{\omega_0^2} e^{\frac{-\omega}{\omega_0}}$ inspired by QCD sum rules



[Grozin, Neubert '96]



[Lange, Neubert '03]
[Bell, Feldmann '08]
[Bell, Feldmann, Wang and Yip '13]



[Braun, Ji, Manashov '19]

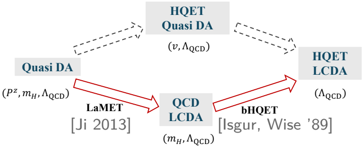
- a new representation in dual spectral functions $\phi^+(\omega) = \frac{\omega}{2\bar{\Lambda}^2}\Theta(2\bar{\Lambda}-\omega)$
  - the eigenfunctions of the LN renormalization kernel, an alternative representation of the RG solution
  - convolution integrals with $J_1(\sqrt{\omega/\omega'})$ to reproduce the exponent expression

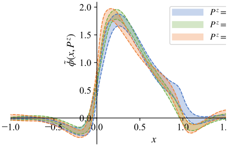
Leading twist B-meson LCDA $\phi_+(\omega)$

- various models of ϕ_B^+ [Wang, Shen, '15]
- large uncertainty of first inverse moment $\lambda_B = \int_{-\infty}^{+\infty} d\omega \phi_B^+(\omega, \mu) \in [0.2, 0.6]$ GeV

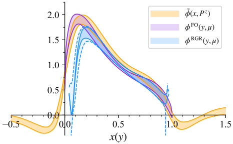
- the lattice simulation [LPC 2410.18658]

  - LCDAs can not be directly simulated on the lattice
  - Light-cone can be accessed by simulating correlation functions with a large but finite P_z
  - HQET can be accessed by simulating correlation functions with a large but finite m_Q

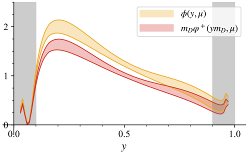




Quasi-DA
with boosted momenta



matched QCD LCDAs
from NLO kernel



matched HQET LCDAs
from multi-scale factorization

Leading twist B-meson LCDA $\phi_+(\omega)$

- consists with the existing model parametrizations

$$\varphi_{\rm I}^+(\omega,\mu_0)=\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0},$$

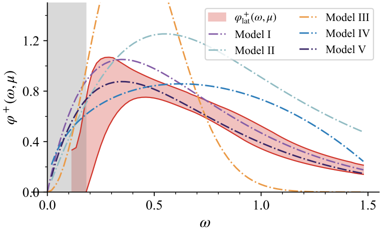
$$\varphi_{\rm II}^+(\omega,\mu_0)=\frac{4}{\pi\omega_0}\frac{k}{k^2+1}\left[\frac{1}{k^2+1}-\frac{2\left(\sigma_B^{(1)}-1\right)}{\pi^2}\ln k\right],$$

$$\varphi_{\rm III}^+(\omega,\mu_0)=\frac{2\omega^2}{\omega_0\omega_1^2}e^{-(\omega/\omega_1)^2},$$

$$\varphi_{\rm IV}^+(\omega,\mu_0)=\frac{\omega}{\omega_0\omega_2}\frac{\omega_2-\omega}{\sqrt{\omega\left(2\omega_2-\omega\right)}}\theta\left(\omega_2-\omega\right),$$

$$\varphi_{\rm V}^+(\omega,\mu_0)=\frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}U(\beta-\alpha,3-\alpha,\omega/\omega_0).$$

various commonly used models



LPC result in comparison to models

- extracted $\lambda_B(1\,\mathrm{GeV})=0.38\pm0.04$ GeV

  - the simulation of quasi DAs with a single lattice spacing and perturbative calculation at leading power
  - various systematic uncertainties from both lattice and analytical sides are not considered yet

Leading twist $B_{(s)}$-meson LCDA $\phi_+(\omega)$

- estimation $\lambda_{B_{(s)}}$ from HQET sum rule [Khodjamirian, Mandal, Mannel 2008.03935]

$$\lambda_{B_s} = 438 \pm 150\,\mathrm{MeV}, \quad \lambda_B = 383 \pm 153\,\mathrm{MeV}$$
$$\lambda_{B_s}/\lambda_B = 1.19 \pm 0.14$$

  - $SU(3)_f$ violation in the λ_{B_s} is an appreciable effect, in the same ballpark as for $f_{B_{(s)}}$
  - HQET sum rules is an independent (approximate) tool to investigate the heavy-meson DAs

- constraining λ_{B_s} by $B_s \to \gamma^*(\phi)$ form factors [Ivanov, Melikhov, Simula, 2407.13498]

  - define correlation function to calculate $B_s \to \gamma^*$ form factors $F_i(q^2, q'^2)$ from B_s LCSRs

$$i\int dx e^{iq'x}\langle 0|\{Tj_\alpha^{\mathrm{e.m.}}(x), \bar{s}\gamma_\mu q^\nu b(0)\}|\bar{B}_s(p)\rangle = ie\,\epsilon_{\mu\alpha q q'}\frac{F_V(q^2,q'^2)}{M_{B_s}},$$
$$i\int dx e^{iq'x}\langle 0|\{Tj_\alpha^{\mathrm{e.m.}}(x), \bar{s}\sigma_{\mu\nu} q^\nu b(0)\}|\bar{B}_s(p)\rangle = ie\,\epsilon_{\mu\alpha q q'}F_{TV}(q^2,q'^2).$$

  - interpolate the numerical results using a simple analytic fit formula

$$F(y_1,y_2) = f_0\left(1+a_0\frac{y_1}{1-y_1}\right) + R_\phi F_0\left(1+a_1\frac{y_1}{1-y_1}\right)\frac{y_2}{1-y_2} + R_\phi F_1\left(1+a_2\frac{y_1}{1-y_1}\right)\frac{y_2}{1-y_2/r^2}$$

  - consider a dispersion representation in q'^2 by the sum of two poles, m_ϕ^2 and an effective heavier pole
  - obtain $B_s \to \phi$ form factor $V(q^2)$, $T_1(q^2)$ without involving the systematic uncertainties of sum rules
  - constraint λ_{B_s} by comparing with the result obtained from approaches which do not use the $\phi_{B_s}^+$

$$\lambda_{B_s}(1\,\mathrm{GeV}) = 490 \pm 70\,\mathrm{MeV}$$

High twist B-meson LCDAs

- relations of 2p and 3p LCDAs via EOM and HQS [Kawamura, Kodaira, Qiao, Tanaka '01]
- **transversal momentum dependence** $\chi(\omega, \mathrm{k}_T)$ of B meson WFs [Huang, Qiao, Wu '06]
  - $\sim$ hyperbolalike curve, much different from Wandzura-Wilczek $\chi^{\mathrm{WW}}(\omega, \mathrm{k}_T)$ ($\sim$ delta function)
- **evolution equation** for high twist B meson DAs [Braun, Manashov, Offen, '15]
  - twist-three three-particle DA $\phi_3 = \Psi_A - \Psi_V$ is related to two particle DAs ϕ_-
- complete three-particle higher-twist DAs of the B-meson [Braun, Ji, Manashov '17]
  - eight independent three-particle DAs classified by collinear twist and chirality (conformal transformations)
  - **evolution equations** of three twist-four DAs $\Psi_A + \Psi_V, \Psi_A + X_A, \Psi_V - \tilde{X}_A$
  - **simple models of DAs** with different large-energy behavior that satisfy all tree-level EOM constraints

$$
\begin{aligned}
\langle 0|\bar{s}_\alpha(x) G_{\rho\delta}(ux) h_{v\beta}(0)|\bar{B}_{s,v}\rangle = \frac{f_{B_s} m_{B_s}}{4} \int_0^\infty d\omega \int_0^\infty d\zeta\, e^{-i(\omega+u\zeta)v\cdot x} \Big[(1+\not{v})\Big\{ - i\sigma_{\rho\delta}\Psi_V(\omega,\zeta) \\
+(v_\rho\gamma_\delta - v_\delta\gamma_\rho)\left[\Psi_A(\omega,\zeta) - \Psi_V(\omega,\zeta)\right] - \left(\frac{x_\rho v_\delta - x_\delta v_\rho}{v\cdot x}\right) X_A(\omega,\zeta) + i\epsilon_{\rho\delta\alpha\beta}\frac{x^\alpha v^\beta}{v\cdot x}\gamma_5 \tilde{X}_A(\omega,\zeta) \\
+\left(\frac{x_\rho\gamma_\delta - x_\delta\gamma_\rho}{v\cdot x}\right)\left[Y_A(\omega,\zeta) + W(\omega,\zeta)\right] - i\epsilon_{\rho\delta\alpha\beta}\frac{x^\alpha\gamma^\beta}{v\cdot x}\gamma_5 \tilde{Y}_A(\omega,\zeta) \\
-\left(\frac{x_\rho v_\delta - x_\delta v_\rho}{v\cdot x}\right)\frac{\not{x}}{v\cdot x} W(\omega,\zeta) + \left(\frac{x_\rho\gamma_\delta - x_\delta\gamma_\rho}{v\cdot x}\right)\frac{\not{x}}{v\cdot x} Z(\omega,\zeta)\Big\}\gamma_5\Big]_{\beta\alpha}
\end{aligned}
$$

- updated calculations of $B \to \pi, K, \rho, \phi, f_0$ form factors from B-meson LCSRs
  [Grbernari '19, Lü '19, Descotes-Genon '19, Cheng '20, $\cdots$]

Leading twist π-meson LCDA $\phi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(u)$

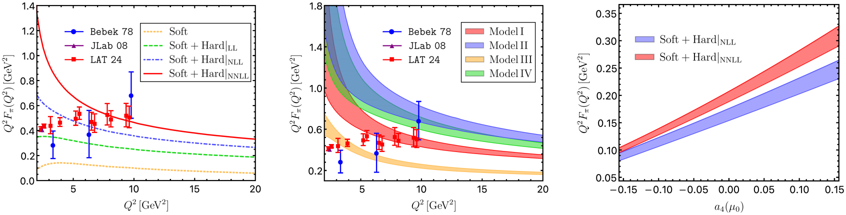
- QCD definition $a_n^\pi(\mu) = \langle \pi | q(z)\bar{q}(z) + z_\rho \partial_\rho q(z)\bar{q}(z) + \cdots | 0 \rangle$
- **QCDSR**: 0.19 ± 0.06[Chernyak '84], $0.26^{+0.21}_{-0.09}$[Khodjamirian '04], $0.28^{+0.08}_{-0.08}$[Ball '06]
  - nonlocal vacuum condensate is introduced and modeled for $a^\pi_{n>2}$ [Bakulev '01]
- **LQCD**: 0.334 ± 0.129 [UKQCD '10], 0.135 ± 0.032 [RQCD '19], $0.258^{+0.079}_{-0.052}$[LPC '22]
  - a_4^π is not available so far, what's the convergence ? $\Leftarrow$ the growing number of derivatives in $q\bar{q}$ operator
  - a_2, a_4, a_6 (QCD DAs) $\Leftarrow$ fitting to the quasi-DAs evaluated from LAMET
- dispersion relation as an **Inverse problem**[Li '20, Yu '22]
  - *quark-hadron duality* $\rightarrow$ *Laguerre Polynomials spectral density*: $\{a_2, a_4\} = \{0.249, 0.134\}$
- study status of $a_2^\pi(2\,\mathrm{GeV})$

Method	$a_2^\pi(2$ GeV$)$	Refs.
LO QCDSR, CZ model	0.39	[30,31]
QCDSR	$0.18^{+0.15}_{-0.26}$	[32]
QCDSR	0.19 ± 0.06	[33]
QCDSR, NLC	0.13 ± 0.04	[34,35]
$F_{\pi\gamma\gamma^*}$, LCSRs	0.12 ± 0.04 (2.4 GeV)	[36]
$F_{\pi\gamma\gamma^*}$, LCSRs	0.21 (2.4 GeV)	[37]
$F_{\pi\gamma\gamma^*}$, LCSRs, R	0.19	[38]
$F_{\pi\gamma\gamma^*}$, LCSRs, R	0.31	[39]
$F_{\pi\gamma\gamma^*}$, LCSRs, NLO	0.096	[40]
$F_{\pi\gamma\gamma^*}$, LCSRs, NLO	0.068	[41]
F_π^{em}, LCSRs	$0.17 \pm 0.10 \pm 0.05$	[42]
F_π^{em}, LCSRs, R	0.14 ± 0.02	[43]
$F_{B\to\pi}$, LCSRs	0.13 ± 0.13	[44]
$F_{B\to\pi}$, LCSRs	0.11	[45,46]
LQCD, TWST, $N_f = 2$, CW	0.201 ± 0.114	[47]
LQCD, TWST, $N_f = 2+1$, DWF	0.233 ± 0.088	[48]
LQCD, MST, $N_f = 2$	0.136 ± 0.03	[27]
LQCD, MST, $N_f = 2+1$, CW	0.0762 ± 0.0127	[29]

[SC, 1901.06071, Dipion LCDAs]

Leading twist π-meson LCDA $\phi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(u)$

- N^2LO hard-collinear factorization of $F_\pi(Q^2)$ [Ji, Shi, Wang[3], Yu 2411.03658]
  - rigorous two-loop computation of leading-twist contribution in the hard-collinear factorization
  - N^2LO factorization at leading power of Λ^2_{QCD}/Q^2 expansion by employing the light-cone projections on the leading-twist-collinear operators [Chen[2], Feng, Jia 2312.17228]



Model I: anti-de Sitter-QCD $\sim (u\bar{u})^{\alpha_\pi}$ with $\alpha_\pi = 0.585 \pm 0.06$ [Khodjamirian, et al 2011.11275]

Model II: $\{a_2, a_4\} = \{0.181(32), 0.107(36)\}$ from data-driven with the modular DR [SC, et al 2007.05550]

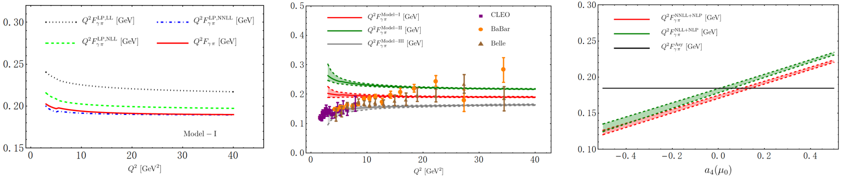
Model III: $\{a_2, a_4\} = \{0.149(50), -0.096(60)\}$ from QCD sum rules [Stefanis 2006.10576]

Model IV: $\{a_2, a_4, a_6\} = \{0.196(32), 0.085(26), 0.056(15)\}$ from LAMET [Cloet, et al 2407.00206]

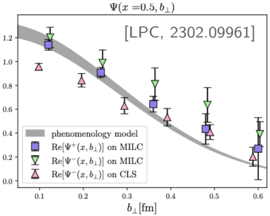
- the N^2LO QCD correction to the short-distance coefficient function is enormous
- an improved extraction of a_2, a_4 dictating the intricate profile of the pion DAs

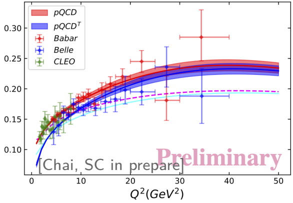
Leading twist π-meson LCDA $\phi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(u)$

- data-driven study of a_2 from $F_{\pi\gamma\gamma^*}$ by LCSRs
  - 0.14 [Agaev 2010, BABAR+CLEO], 0.10 [Agaev 2012, Belle+CLEO]
  - large uncertainty of $a^\pi_{n>2}$, discrepancy data at large Q^2
- N^{2}LO hard-collinear factorization of $F_{\pi\gamma\gamma^*}(Q^2)$ [Gao, Huber, Ji, Wang 2160.01390]



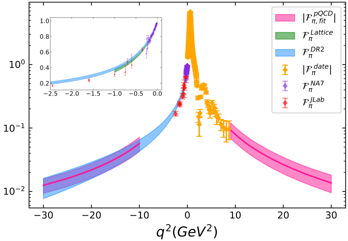
- intrinsic transverse momentum function (iTMD)





High twist π-meson LCDA chiral mass m_0^π

- **higher twist contributions** to exclusive QCD processes are commonly power suppressed $\mathcal{O}(1/Q)$
- twist 3 contribution is dominate in F_π due to chiral enhancement $\mathcal{O}(\frac{m_0}{u_i Q})$
  - odd-twist LCDA DO NOT contribute in LCSRs with the chiral symmetry limit
- this effect is greatly enhanced by the end-point problem
- the joint study of F_π from the NLO PQCD up to twist four and the modular DR
- a comprehensive description of F_π in the whole kinematics [Chai, SC, Hua 2209.13312]



- fitting the PQCD prediction to the result obtained from modular DR
- the obtained chiral mass $m_0^\pi = 1.37 \pm 0.30$ GeV
- much smaller than the ChPT value 1.89 GeV [Leutwyler '96]
- also smaller than the value obtained with $\overline{\mathrm{MS}}$ current quark masses $m_0^\pi = \frac{m_\pi^2}{m_u + m_d}$
- N^2LO correction or iTMD effect ?

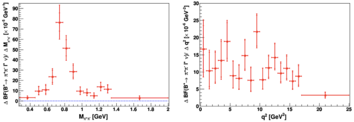
Width effects of ρ LCDAs DiPion LCDAs

- B_{l4} decays have rich observables, nontrivial tests of SM [Faller '14]
- Different exclusive $b \to u$ processes help in the $|V_{ub}|$ determination
  - V_{ub} extracted from $B^0 \to \pi^- l^+ \nu$ and $B^+ \to \rho^0 l^+ \nu$ has $\sim 3\sigma$ deviation [Belle II 2407.17403]

$$|V_{ub}|_{B\to\pi l\nu} = (3.93 \pm 0.19 \pm 0.13 \pm 0.19(\text{theo})) \times 10^{-3} \quad [\text{LQCD}]$$
$$|V_{ub}|_{B\to\pi l\nu} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16(\text{theo})) \times 10^{-3} \quad [\text{LQCD + LCSRs}]$$
$$|V_{ub}|_{B\to\rho l\nu} = (3.19 \pm 0.12 \pm 0.18 \pm 0.26(\text{theo})) \times 10^{-3} \quad [\text{LCSRs}]$$

  - the theoretical uncertainty does not consider the width effect of ρ in the $\pi\pi$ invariant mass spectral

- $B \to \rho l \bar{\nu}_l$ $(1.63 \pm 0.20) \times 10^{-4}$
  [BABAR '11, Belle '13, Belle II '24]
- first measurement of $B^+ \to \pi^+\pi^- l^+ \bar{\nu}_l$
  $(2.3 \pm 0.4) \times 10^{-4}$ [Belle '20]



[Belle 2005.07766]

- First measurement of $D^0 \to \pi^+\pi^- e^+ e^-$ [LHCb-PAPER-2024-047, prelim.]
  - $(4.53 \pm 1.38) \times 10^{-7}$ in ρ/ω and $(3.84 \pm 0.96) \times 10^{-7}$ in ϕ
  - $c \to u$-typed FCNC upper limit 0.7×10^{-5} by [BES III '18]
- $D^0 \to K^- \pi^0 \mu^+ \nu$ $(0.729 \pm 0.014 \pm 0.011)\,\%$ [BESIII 2403.10877]
  - in which the S-wave accounts $(2.06 \pm 0.05)\,\%$
- First Lattice QCD study of the $B \to \pi\pi l \bar{\nu}$ transition amplitude in the region of large q^2 and $\pi\pi$ invariant mass near the ρ resonance [Leskovec et.al. 2212.08833[hep-lat]]

Width effects of ρ LCDAs DiPion LCDAs

- The study of DiPion distribution amplitude will shine a light on **the width effect encountered in Flavor Physics** (multibody decays, $B \to [\pi\pi] l\nu$, $b \to sll$, $c \to ull$, $D\pi$ system $\cdots$) and **the controversial structure of scalar meson ?**

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov '99, Diehl '98]

$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u, \zeta, k^2)$$

- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z, \zeta, k^2, \mu) = 6z(1-z) \sum_{n=0,\mathrm{even}}^{\infty} \sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2, \mu) C_n^{3/2}(2z-1) C_\ell^{1/2}(2\zeta-1)$$

- $B_{n\ell}(k^2, \mu)$ have similar scale dependence as the a_n of π, ρ, f_0 mesons
- **Evolution from $4m_\pi^2$ to large invariant mass** $k^2 \sim \mathcal{O}(m_c^2)$ and furtherly to $\mathcal{O}(m_b\lambda_{\rm QCD})$

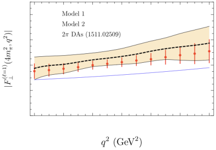
$$B_{n\ell}^I(k^2) = B_{n\ell}^I(0)\, \mathrm{Exp}\left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^m}{dk^{2m}} \ln B_{n\ell}^I(0) + \frac{k^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds\, \frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

- 2πDAs LCSRs results in [SC, Khodjamirian, Virto 1709.0173, SC 1901.06071]

  - high partial waves give few percent contributions to $B \to \pi\pi$ form factors
  - ρ', ρ'' and NR background contribute $\sim 20\% - 30\%$ to P-wave

Width effects of ρ LCDAs DiPion LCDAs

- 30% smaller than it obtained from B-meson LCSRs [SC, Khodjamirian and Virto 1701.01663]
  - high twist contributions ?
  - uncertainty of B-meson LCDAs



- At the current accuracy, they give same order plots of $B \to \pi^+\pi^0$ FFs
- For the $P-$wave FFs, they both predict sizable non-ρ contribution ($\sim 15\%$)
  - qualitatively explain the $|V_{ub}|$ obtained from $B \to \pi l\nu$ and $B \to \rho l\nu$
- B-meson LCSRs can not predict the contributions from higher partial waves, but it indeed exist in the DiPion LCSRs, althogh small
- B-meson LCSRs rely on the resonance model (an inverse problem), DiPion LCSRs is currently limited by the poor knowledge of DiPion LCDAs
- high twist DiPion LCDAs is a big task for the practitioners of QCD in HP

Conclusion

Much more topics of the precise QCD in flavor sector, including but not limited to

- CPV in baryon decays [He, Liu 2404.19196], [Jia, Jiang, Wang, Yu 2408.14959], [Wang, Yu 2407.04110]
- Double mixing CPV $B_s^0(\bar{B}_s^0) \to \rho^0 \bar{K}_0(K_0) \to \rho^0 \pi^- e^+ \nu$ [Shen, Song, Qin, 2301.05848, 2403.01904]
- CPV in D meson from LCSRs [Lenz, Piscopo, Rusov 2403.02267]
- see talks from Zheng-hua Zhang 16:20, Nov 14th for CPV from the interference terms in cascade decays

- RG evolution of three-particle B-meson soft function [Huang, Ji, Shen, Wang^2 2312.15439]
- LCDAs of heavy mesons from bHQET [Beneke, et al 2305.06401, Deng, et al 2409.00632]
- LD penguin contribution to $B_{d,s} \to \gamma\gamma$ decays [Qin, Shen, Wang^2 2207.02691]
- the weak annihilation contribution in $B \to \{K, \pi\} l^+ l^-$ [Shen, Huang, Wang^2 2403.11258]
- QED effects in $B \to \tau\tau$ [Zhou, et al 2301.00697], in LCDAs [Beneke, Böer, et al 2204.09091, 2108.05589]

- $\mathcal{B}(B^+ \to K^+ \nu\bar{\nu}) = (2.3 \pm 0.7) \times 10^{-5}$, 2.7σ from the SM [Belle II 2311.14647]
  no hadronic uncertainty beyond FF [HPQCD 2207.13371]
- FCNC in Kaons $\mathcal{B}(K_L^0 \to \pi^0 \nu\bar{\nu}) < 2 \times 10^{-9}$ [New preliminary result from KOTO]

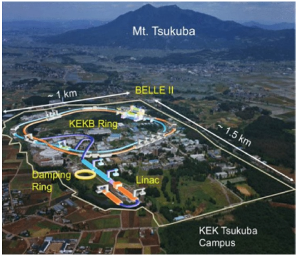
- form factors of kaon and nucleon [Huang, et al 2407.18724], [Chen, Feng, Jia 2406.19994]
- gravitational form factors and the conformal anomaly [Corianó, et al 2409.19586]

- gravitational form factors of proton from lattice [Hackett, et al 2310.08484]
- lattice revisiting of D_s^* radiative decay and the width [Meng, et al 2401.13475]
- lattice evaluation of $B \to D^*$ form factor [HPQCD 2304.03137, JLQCD 2306.05657]
- lattice evaluation of leading twist η_c DA [Blossier, et al 2406.04668]

- ......

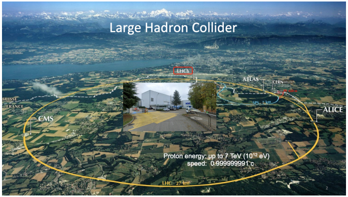
- **SuperKEKB(2018-2026)** [E. Kou et al. [Belle-II], 2019]

  - Belle II has collected 531 fb^{-1} data so far with record peak luminosity $4.7{\times}10^{34}cm^{-2}s^{-1}$
  - Goal: 50 ab^{-1} data and peak luminosity at $6.5{\times}10^{35}cm^{-2}s^{-1}$
  - $|V_{ub}|$ to 1.2% in $B \to \phi, \rho l\nu$, δA_{CP} in $B \to K^*\pi, K\rho, K^*\rho$, $B \to VV$, α from $B \to \pi^0\pi^0, \cdots$
  - † $B^+ \to \rho^+\rho^0, B^0 \to K^0\pi^0$ [Belle-II, 2021]
  - † First measurement of CP asymmetry parameters in $B^0 \to K_S^0\pi^0, \omega\omega$ [Belle-(II), 2023,04].



- **HL-LHC(2030-2033)** [CERN Yellow Rep. Monogr, 2019]

  - $\mathcal{L} = 23(300)\mathrm{fb}^{-1}$ in phase 1(2), 2 order larger than LHC, $2 \times 10^{33(34)}\mathrm{cm}^{-2}\mathrm{s}^{-1}$
  - $|V_{ub}|$ to $0.7\%(0.4\%)$ in $B \to \pi\pi, \pi\rho, \rho\rho$, $C_{\pi^+\pi^-}, S_{\pi^+\pi^-}$ (one order improvement), α from $B \to \rho\rho, \rho\pi, \cdots$.
  - † $A^{\rm dir}_{\pi\pi} = (2.32 \pm 0.61) \times 10^{-3}$ [LHCb, 2023]
  - † $\delta A_{CP}(\Lambda_b^0 \to \Lambda K^+K^-) = 0.083 \pm 0.028$ [LHCb-Paper-2024-043]



重味物理研究即将全面进入精确测量的时代，这为深入理解基本相互作用和探索新物理机制提供了极佳的机遇。

Thank you for your patience.