# The On-Shell Method of Effective Field Theory

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# Outline

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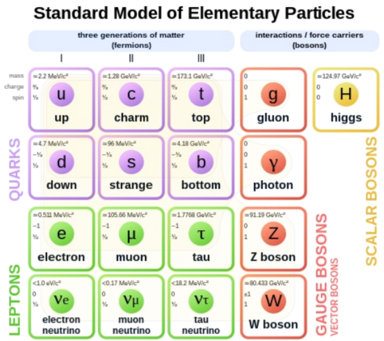
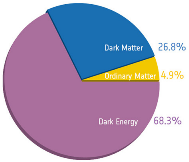
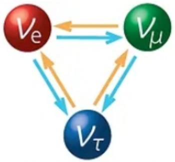
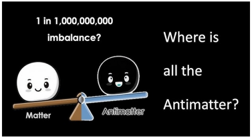
# New Physics Beyond the Standard Model



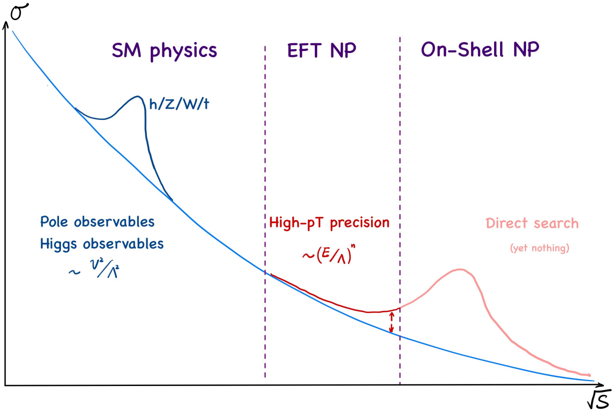

BUT...



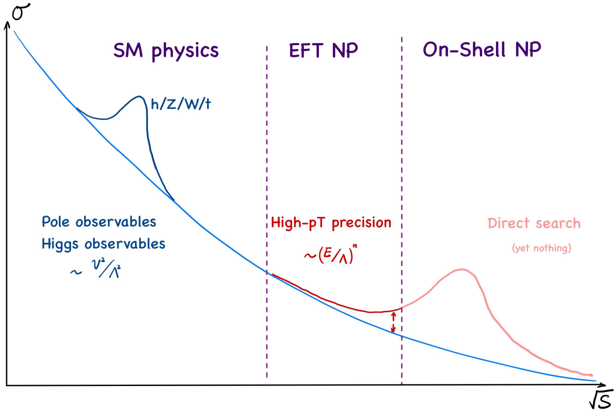



# The Approach of Effective Field Theory




| Paradigm shift | Experimentalist | Theorist |
|---|---|---|
| On-Shell New Physics | bump hunting | model building |
| EFT New Physics | precision measurement | effective operators |

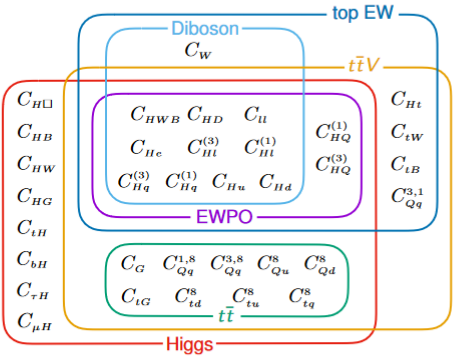
# The Approach of Effective Field Theory




$$\mathcal{L}_{\mathrm{EFT}}(\Lambda) = \mathcal{L}_{\mathrm{SM}} + \sum_{d>4} \frac{c_i}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$

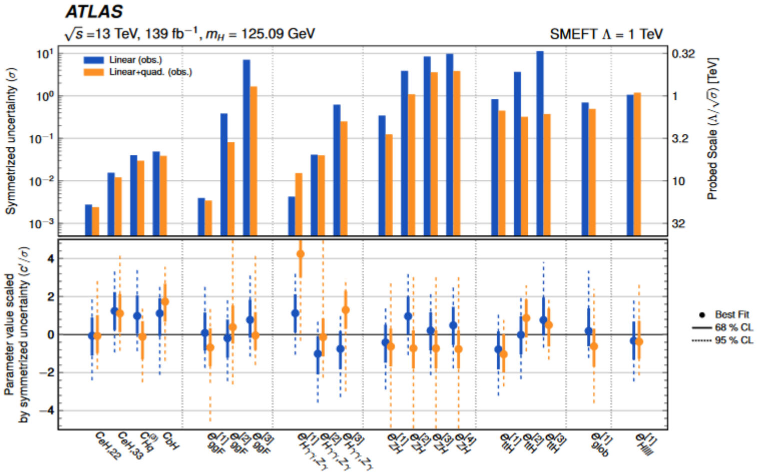
# SMEFT Phenomenology

The Dim-6 operators in SMEFT are well studied in phenomenology for a long time.

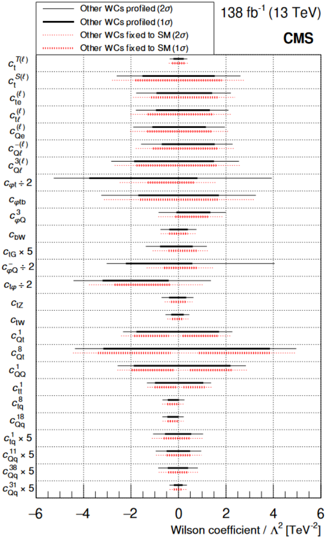



$\Rightarrow$ need wide combination of measurement [Ellis,Madigan,Mimasu,Sanz,You,2021]

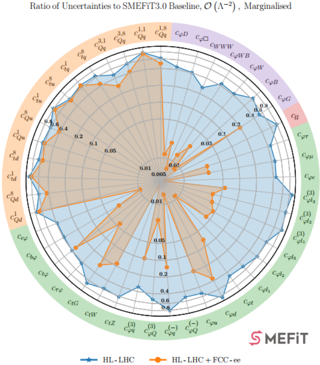
# LHC Global Fit



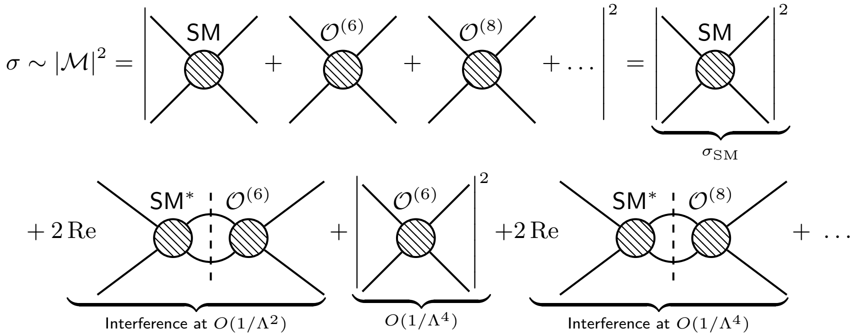
# LHC Global Fit




- No significant deviation from SM yet!
- Future colliders will improve the precision



# Dim$>6$ effective operators in SMEFT




Linear v.s. Linear+Quadratic model

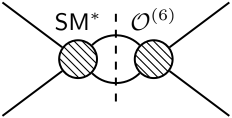
Why not include $\mathcal{O}^{(8)}$?

| Wilson coefficient | Includes $\lvert\mathcal{M}_{d6}\rvert^2$ | 95% confidence interval [TeV$^{-2}$] Expected | Observed | $p$-value (SM) |
|---|---|---|---|---|
| $c_W/\Lambda^2$ | no | [−0.30, 0.30] | [−0.19, 0.41] | 45.9% |
| | yes | [−0.31, 0.29] | [−0.19, 0.41] | 43.2% |
| $\tilde{c}_W/\Lambda^2$ | no | [−0.12, 0.12] | [−0.11, 0.14] | 82.0% |
| | yes | [−0.12, 0.12] | [−0.11, 0.14] | 81.8% |
| $c_{HWB}/\Lambda^2$ | no | [−2.45, 2.45] | [−3.78, 1.13] | 29.0% |
| | yes | [−3.11, 2.10] | [−6.31, 1.01] | 25.0% |
| $\tilde{c}_{HWB}/\Lambda^2$ | no | [−1.06, 1.06] | [0.23, 2.34] | 1.7% |
| | yes | [−1.06, 1.06] | [0.23, 2.35] | 1.6% |

[ATLAS, 2006.15458]

# Dim$>6$ effective operators in SMEFT

- Non-interference processes:



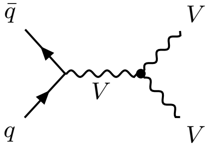

$= 0$

When the phase space integration vanishes/suppressed:

1. helicity selection
2. angular momentum selection
3. other symmetry (gauge group, CP, *etc.* )

# Dim$>6$ effective operators in SMEFT

- Non-interference processes:
- Processes that dim-6 does not contribute: *e.g.* nTGC



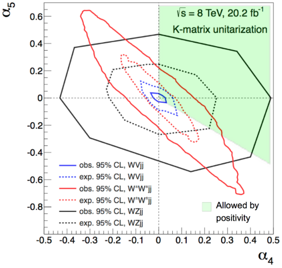

Diboson searches:

$$g\mathcal{O}_{G+} = \widetilde{B}_{\mu\nu}W^{a\mu\rho}(D_\rho D_\lambda W^{a\nu\lambda} + D^\nu D^\lambda W^a_{\lambda\rho}),$$
$$g\mathcal{O}_{G-} = \widetilde{B}_{\mu\nu}W^{a\mu\rho}(D_\rho D_\lambda W^{a\nu\lambda} - D^\nu D^\lambda W^a_{\lambda\rho}),$$
$$\mathcal{O}_{\widetilde{B}W} = \mathrm{i}\, H^\dagger \widetilde{B}_{\mu\nu}W^{\mu\rho}\{D_\rho, D^\nu\}H + \mathrm{h.c.},$$

[Ellis, He, Xiao, 2020]

# Dim$>6$ effective operators in SMEFT

- Non-interference processes:
- Processes that dim-6 does not contribute: *e.g.* nTGC
- Positivity Bound: theoretical constraints on Wilson coefficients from unitarity and locality of S-matrix, starting from dim-8



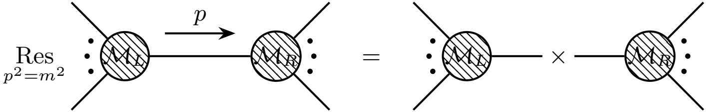

[Zhang,Zhou,1808.00010]

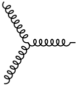
- For theorists: unitarity bound from RG running
- For experimentalists: Bayesian priors in fits

# Outline

# Inspiration from Scattering Amplitude

On-shell amplitudes can be constructed from lower-point building blocks:




The tree-level amplitudes of Yang-Mills theory can be constructed from 

**Lesson Learned:** independent parameters $\longleftrightarrow$ amplitude building blocks:

- local amplitudes without poles
- all legs are on-shell
- analytic functions of kinematic variables (up to crossing symmetry)
- satisfy all the symmetry (can exist independently)

# Amplitude/Operator Correspondence

- The independent parameters of EFT are the Wilson coefficients of an independent basis of effective operators.
- They correspond to the leading on-shell amplitudes

$$\mathcal{O} = (\bar{\psi}_1\gamma^\mu\psi_2)(\bar{\psi}_3\gamma_\mu\psi_4) \quad \simeq \quad \mathcal{B} = (\bar{u}(p_1)\gamma^\mu u(p_2))(\bar{u}(p_3)\gamma_\mu u(p_4))$$
$$p_1 + p_2 + p_3 + p_4 = 0 \ ,$$
$$p_i^2 = m_i^2 \ , \quad \not{p}_i u(p_i) = m_i u(p_i) \ .$$

- What's the difference with Feynman vertices?
  1. Some vertices are not gauge invariant, which thus do not exist independently.
  2. Some vertices have legs that vanish on shell, and hence do not count.

# Field Redefinition

What if an operator only has vertices that vanish on-shell?

$$\mathcal{O}' = X^a(\Box + m^2)\phi^a \quad \simeq \quad \mathcal{B}'(X,\phi) \sim -p^2 + m^2 = 0 \ .$$

The description of effective operators is redundant due to field redefinition:

$$\phi^a \to \phi^a + X^a \quad \simeq \quad \mathcal{L} \to \mathcal{L}' = \mathcal{L} + \frac{\partial \mathcal{L}}{\partial \phi^a} X^a + O(X^2)$$

$$\text{where} \quad \frac{\partial \mathcal{L}}{\partial \phi^a} = -(\Box + m^2)\phi^a + J^a$$

The operator $\mathcal{O}'$ can be replaced (usually referred to as EOM at $O(X^1)$)

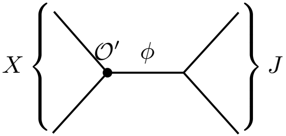
$$\mathcal{L}' = (\mathcal{L} \supset \cancel{\mathcal{O}'}) \cancel{-\mathcal{O}'} + X^a J^a + O(X^2)$$

The on-shell interpretation: $\mathcal{O}'$ is irrelevant to the field/particle $\phi^a$.

# On-Shell Types of Effective Operators

Effective operators should be categorized by what on-shell amplitudes they contribute to

$$\mathcal{O}' \quad \notin \text{type}\{X,\phi\} \quad \text{but} \quad \in \text{type}\{X,J\}$$




$$\sim \quad X(\cancel{-p^2+m^2})\frac{i}{\cancel{p^2-m^2}}J$$

NO $\phi$ pole!

Example:
$$O_W = \frac{ig}{2}(H^\dagger \tau^a \overleftrightarrow{D}^\mu H) D^\nu W^a_{\mu\nu}$$
$$\rightarrow \frac{g^2}{2}(H^\dagger \tau^a \overleftrightarrow{D}^\mu H)\underbrace{\left[H^\dagger \tau^a \overleftrightarrow{D}^\mu H + \bar{Q}\tau^a\gamma^\mu Q + \bar{L}\tau^a\gamma^\mu L\right]}_{J^{a\mu}}$$
$$\in \text{type}\{H^4D^2\} \oplus \text{type}\{H^2Q^2D\} \oplus \text{type}\{H^2L^2D\}$$

When complete sets of operators are considered in those types, $O_W$ should NOT be considered again!

# Independent Operator Basis of an On-Shell Type

An independent basis of effective operators is crucial in the global analysis of EFT.

- Global fit with experiment
- Matching with UV theory
- Renormalization Group equations

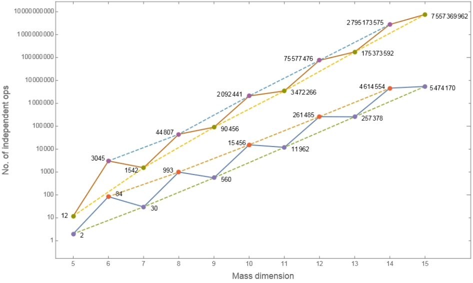
$\Rightarrow$ **To find the independent basis of the corresponding amplitudes $\mathcal{B}$ type by type!**

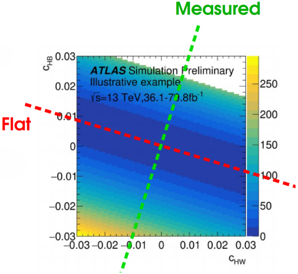
1. no redundancy from field redefinition
2. directly connected with on-shell physical observables

Recent developments on operator basis (Warsaw, Hilbert Series, Young tensor *etc.* )
are all (whether explicitly or not) based on this philosophy!

# Outline

# Progress of Operator Basis Enumeration




- Field Redefinition (on-shell condition)
- IBP (momentum conservation)
- $D = 4$ (Fierz, Schouten, Gram Det...)
- other Group Identities
- Repeated Fields (flavor relations)

- Preferred Basis for NP Search
- Reduction of Operators
- Conversion between Operator Bases

# Young Tensor Basis

On-shell amplitudes and the corresponding effective operators (in $SU(2)_l \times SU(2)_r$ form):

$$\mathcal{B}_N(\{h_i\}) \sim \epsilon^{\otimes n} \tilde{\epsilon}^{\otimes \tilde{n}} \prod_{i=1}^{N} \lambda_i^{r_i - h_i} \tilde{\lambda}_i^{r_i + h_i} \quad \simeq \quad \epsilon^{\otimes n} \tilde{\epsilon}^{\otimes \tilde{n}} \prod_{i=1}^{N} (D^{\lfloor r_i \rfloor} \Psi_i) \sim \mathcal{O}_N(\{\Psi_{h_i}\})$$

## Young Tensor Method

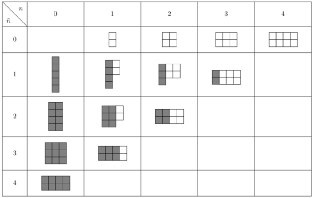
The independent set of operators forms primary irrep. of $SU(N)$:

$$\epsilon^{\alpha_i \alpha_j} \to \mathcal{U}^i{}_k \mathcal{U}^j{}_l \epsilon^{\alpha_k \alpha_l} \;, \quad \tilde{\epsilon}_{\dot{\alpha}_i \dot{\alpha}_j} \to \mathcal{U}^\dagger{}_i{}^k \mathcal{U}^\dagger{}_j{}^l \tilde{\epsilon}_{\dot{\alpha}_k \dot{\alpha}_l}$$

[Henning, Melia, 1902.06754; Li, Ren, Shu, **Xiao**, Yu, Zheng, 2005.00008]

| $\tilde{n}$ \ $n$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | $\phi^8$ | $\psi^2\phi^5$ | $\psi^4\phi^2$, $F_{\rm L}\psi^2\phi^3$, $F_{\rm L}^2\phi^4$ | $F_{\rm L}\psi^4$, $F_{\rm L}^2\psi^2\phi$, $F_{\rm L}^3\phi^2$ | $F_{\rm L}^4$ |
| 1 | $\psi^{\dagger 2}\phi^5$ | $\psi^{\dagger 2}\psi^2\phi^2$, $\psi^\dagger\psi\phi^4 D$, $\phi^6 D^2$ | $F_{\rm L}\psi^{\dagger 2}\psi^2$, $F_{\rm L}^2\psi^{\dagger 2}\phi$, $\psi^\dagger\psi^3\phi D$, $F_{\rm L}\psi^\dagger\psi\phi^2 D$, $\psi^2\phi^3 D^2$, $F_{\rm L}\phi^4 D^2$ | $F_{\rm L}^2\psi^\dagger\psi D$, $\psi^4 D^2$, $F_{\rm L}\psi^2\phi D^2$, $F_{\rm L}^2\phi^2 D^2$ | |
| 2 | $\psi^{\dagger 4}\phi^2$, $F_{\rm R}\psi^{\dagger 2}\phi^3$, $F_{\rm R}^2\phi^4$ | $F_{\rm R}\psi^{\dagger 2}\psi^2$, $F_{\rm R}^2\psi^2\phi$, $\psi^{\dagger 3}\psi\phi D$, $F_{\rm R}\psi^\dagger\psi\phi^2 D$, $\psi^{\dagger 2}\phi^3 D^2$, $F_{\rm R}\phi^4 D^2$ | $F_{\rm R}^2 F_{\rm L}^2$, $F_{\rm R}F_{\rm L}\psi^\dagger\psi D$, $\psi^{\dagger 2}\psi^2 D^2$, $F_{\rm R}\psi^2\phi D^2$, $F_{\rm L}\psi^{\dagger 2}\phi D^2$, $F_{\rm R}F_{\rm L}\phi^2 D^2$, $\phi^4 D^4$, $\psi^\dagger\psi\phi^2 D^3$ | | |
| 3 | $F_{\rm R}\psi^{\dagger 4}$, $F_{\rm R}^2\psi^{\dagger 2}\phi$, $F_{\rm R}^3\phi^2$ | $F_{\rm R}^2\psi^\dagger\psi D$, $\psi^{\dagger 4} D^2$, $F_{\rm R}\psi^{\dagger 2}\phi D^2$, $F_{\rm R}^2\phi^2 D^2$ | | | |
| 4 | $F_{\rm R}^4$ | | | | |

$\Leftrightarrow$




[Li, Ren, **Xiao**, Yu, Zheng, 2201.04639]

# Reduction of Effective Operators

It is common to get operators different from the constructed basis in matching or RGE.

**Step 1** Label the constituting fields in order and put the operator in the standard form with spinor indices (fermions and gauge bosons in chiral basis);

**Step 2** Move derivatives backward with IBP *as much as possible*;

**Step 3** Apply Schouten Identities to the Lorentz contractions

$$\epsilon^{\alpha_i\alpha_l}\epsilon^{\alpha_k\alpha_j} \to \epsilon^{\alpha_i\alpha_j}\epsilon^{\alpha_k\alpha_l} + \epsilon^{\alpha_i\alpha_k}\epsilon^{\alpha_l\alpha_j}\ , \qquad i<j<k<l\ .$$

**Step 4** When repeated fields are present, check the independence of various irreducible tensors (flavor relation) under the permutation group.

**Step 5** Retain the "off-shell" pieces in each of the above steps (Green's Basis), and perform field redefinition to convert them to the other on-shell types in the end.

[Li, Ren, **Xiao**, Yu, Zheng, 2201.04639; work in progress]

# Non-Linear Symmetry and Adler Zero Condition

How about Goldstone bosons with non-linear constraints?

$$\overbrace{\pi \to \pi + \xi}^{\text{shift symmetry}} \quad \Rightarrow \quad \overbrace{\lim_{p_i \to 0} \mathcal{B}(\cdots \pi_i \cdots) = 0}^{\text{Adler's zero condition}}$$

$$\textit{e.g. } \psi^2 F_R \pi^2 D^2 \quad \sim \quad \left\{ \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 4 \\ \hline 2 & 2 & 2 & 5 \\ \hline 4 & 5 \\ \cline{1-2} \end{array}\ , \ \underbrace{\cancel{\begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 5 & 5 \\ \hline 4 & 4 \\ \cline{1-2} \end{array}}\ , \ \cancel{\begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 4 & 4 \\ \hline 5 & 5 \\ \cline{1-2} \end{array}}}_{\text{not satisfying Adler's zero}}\ , \ \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 4 & 5 \\ \hline 4 & 5 \\ \cline{1-2} \end{array} \right\}$$

# Non-Linear Symmetry and Adler Zero Condition

Systematic construction of NL$\sigma$M corrections at $O(p^6)$ and $O(p^8)$, $P$-even and odd

| $SU(N_f)$ | Operator Basis | Amplitude Basis |
|---|---|---|
| $SU(2)$ | $\mathcal{O}_1 = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\mu u_\nu u_\rho u_\sigma\rangle$ | $\mathcal{B}_1 = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{14}s_{15}s_{26}$ |
| | $\mathcal{O}_2 = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\mu u_\nu u_\sigma u_\rho\rangle$ | $\mathcal{B}_2 = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{14}s_{16}s_{25}$ |
| | $\mathcal{O}_3 = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u^\sigma u_\nu u_\sigma u_\rho\rangle$ | $\mathcal{B}_3 = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{14}s_{16}s_{35}$ |
| | $\mathcal{O}_4 = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\mu u_\nu u_\rho u_\sigma\rangle$ | $\mathcal{B}_4 = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{14}s_{25}s_{26}$ |
| | $\mathcal{O}_5 = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\mu u_\rho u_\nu u_\sigma\rangle$ | $\mathcal{B}_5 = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{15}s_{24}s_{26}$ |
| | $\mathcal{O}_6 = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\rho u_\mu u_\sigma u_\nu\rangle$ | $\mathcal{B}_6 = \mathcal{Y}\circ\text{tr}[123456]s_{14}s_{16}s_{23}s_{25}$ |
| | $\mathcal{O}_7 = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma u_\nu u_\rho u_\sigma\rangle$ | $\mathcal{B}_7 = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{14}s_{25}s_{36}$ |
| | $\mathcal{O}_8 = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma u_\nu u_\sigma u_\rho\rangle$ | $\mathcal{B}_8 = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{14}s_{26}s_{35}$ |
| | $\mathcal{O}_9 = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma u_\rho u_\nu u_\sigma\rangle$ | $\mathcal{B}_9 = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{15}s_{24}s_{36}$ |
| $SU(3)$ | $\mathcal{O}_{10} = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\sigma u_\mu u_\nu u_\rho\rangle$ | $\mathcal{B}_{10} = \mathcal{Y}\circ\text{tr}[123456]s_{14}s_{15}s_{16}s_{23}$ |
| | $\mathcal{O}_{11} = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\mu u_\sigma u_\nu u_\rho\rangle$ | $\mathcal{B}_{11} = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{15}s_{16}s_{24}$ |
| | $\mathcal{O}_{12} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\rho u_\sigma u_\mu u_\nu\rangle$ | $\mathcal{B}_{12} = \mathcal{Y}\circ\text{tr}[123456]s_{15}s_{16}s_{23}s_{24}$ |
| | $\mathcal{O}_{13} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\mu u_\rho u_\sigma u_\nu\rangle$ | $\mathcal{B}_{13} = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{16}s_{24}s_{25}$ |
| | $\mathcal{O}_{14} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\rho u_\mu u_\nu u_\sigma\rangle$ | $\mathcal{B}_{14} = \mathcal{Y}\circ\text{tr}[123456]s_{14}s_{15}s_{23}s_{26}$ |
| | $\mathcal{O}_{15} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u^\sigma u_\sigma u_\nu u_\rho\rangle$ | $\mathcal{B}_{15} = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{15}s_{16}s_{34}$ |
| | $\mathcal{O}_{16} = \langle\nabla^\mu u^\nu u^\rho u^\sigma\nabla_\rho u_\sigma u_\mu u_\nu\rangle$ | $\mathcal{B}_{16} = \mathcal{Y}\circ\text{tr}[123456]s_{15}s_{16}s_{24}s_{34}$ |
| | $\mathcal{O}_{17} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma u_\sigma u_\rho u_\nu\rangle$ | $\mathcal{B}_{17} = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{16}s_{25}s_{34}$ |
| | $\mathcal{O}_{18} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma u_\sigma u_\nu u_\rho\rangle$ | $\mathcal{B}_{18} = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{15}s_{26}s_{34}$ |
| | $\mathcal{O}_{19} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u^\sigma u_\sigma u_\nu u_\rho\rangle$ | $\mathcal{B}_{19} = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{15}s_{26}s_{34}$ |
| | $\mathcal{O}_{20} = \langle\nabla^\mu u^\nu u^\rho u^\sigma u_\sigma\nabla_\mu u_\nu u_\rho\rangle$ | $\mathcal{B}_{20} = \mathcal{Y}\circ\text{tr}[123456]s_{15}^2 s_{26}s_{34}$ |
| | $\mathcal{O}_{21} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma u_\rho u_\sigma u_\nu\rangle$ | $\mathcal{B}_{21} = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{16}s_{24}s_{35}$ |
| | $\mathcal{O}_{22} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u^\sigma u_\nu u_\sigma u_\rho\rangle$ | $\mathcal{B}_{22} = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{14}s_{26}s_{35}$ |
| | $\mathcal{O}_{23} = \langle\nabla^\mu u^\nu u^\rho u^\sigma\nabla_\mu u_\nu u_\sigma u_\rho\rangle$ | $\mathcal{B}_{23} = \mathcal{Y}\circ\text{tr}[123456]s_{14}^2 s_{26}s_{35}$ |
| | $\mathcal{O}_{24} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u^\sigma u_\nu u_\rho u_\sigma\rangle$ | $\mathcal{B}_{24} = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{14}s_{15}s_{36}$ |
| | $\mathcal{O}_{25} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u^\sigma u_\rho u_\nu u_\sigma\rangle$ | $\mathcal{B}_{25} = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{15}s_{24}s_{36}$ |
| | $\mathcal{O}_{26} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u^\sigma u_\nu u_\rho u_\sigma\rangle$ | $\mathcal{B}_{26} = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{14}s_{25}s_{36}$ |
| | $\mathcal{O}_{27} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u_\rho u^\sigma u_\sigma u_\nu\rangle$ | $\mathcal{B}_{27} = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{16}s_{23}s_{45}$ |
| | $\mathcal{O}_{28} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u_\nu u^\sigma u_\rho u_\sigma\rangle$ | $\mathcal{B}_{28} = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{13}s_{15}s_{46}$ |
| | $\mathcal{O}_{29} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u_\nu u_\rho u^\sigma u_\sigma\rangle$ | $\mathcal{B}_{29} = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{13}s_{14}s_{56}$ |
| | $\mathcal{O}_{30} = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\mu u_\sigma\rangle\langle u_\nu u_\rho\rangle$ | $\mathcal{B}_{30} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}s_{15}s_{16}s_{24}$ |
| | $\mathcal{O}_{31} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\rho u_\sigma\rangle\langle u_\mu u_\nu\rangle$ | $\mathcal{B}_{31} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{15}s_{16}s_{23}s_{24}$ |
| | $\mathcal{O}_{32} = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\mu u_\nu\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{32} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}s_{14}s_{15}s_{26}$ |
| | $\mathcal{O}_{33} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\mu u_\rho\rangle\langle u_\nu u_\sigma\rangle$ | $\mathcal{B}_{33} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}s_{15}s_{24}s_{26}$ |
| | $\mathcal{O}_{34} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\mu u_\nu\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{34} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}s_{14}s_{25}s_{26}$ |
| | $\mathcal{O}_{35} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma u_\sigma\rangle\langle u_\nu u_\rho\rangle$ | $\mathcal{B}_{35} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{12}s_{15}s_{26}s_{34}$ |
| | $\mathcal{O}_{36} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma u_\rho\rangle\langle u_\nu u_\sigma\rangle$ | $\mathcal{B}_{36} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{12}s_{15}s_{24}s_{36}$ |
| | $\mathcal{O}_{37} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u^\sigma u_\rho\rangle\langle u_\nu u_\sigma\rangle$ | $\mathcal{B}_{37} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}s_{15}s_{24}s_{36}$ |
| | $\mathcal{O}_{38} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma u_\nu\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{38} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{12}s_{14}s_{25}s_{36}$ |
| | $\mathcal{O}_{39} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u^\sigma u_\nu\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{39} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}s_{14}s_{25}s_{36}$ |
| | $\mathcal{O}_{40} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u^\nu u^\rho\rangle\langle u^\sigma u_\sigma\rangle$ | $\mathcal{B}_{40} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{12}s_{13}s_{14}s_{56}$ |

| $SU(N_f)$ | Operator Basis | Amplitude Basis |
|---|---|---|
| $SU(4)$ | $\mathcal{O}_{41} = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\sigma u_\mu\rangle\langle u_\nu u_\rho\rangle$ | $\mathcal{B}_{41} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{14}s_{15}s_{16}s_{23}$ |
| | $\mathcal{O}_{42} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\rho u_\mu\rangle\langle u_\nu u_\sigma\rangle$ | $\mathcal{B}_{42} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{14}s_{15}s_{23}s_{26}$ |
| | $\mathcal{O}_{43} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u^\sigma u_\sigma\rangle\langle u_\nu u_\rho\rangle$ | $\mathcal{B}_{43} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{12}s_{15}s_{16}s_{34}$ |
| | $\mathcal{O}_{44} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u^\sigma u_\sigma\rangle\langle u_\nu u_\rho\rangle$ | $\mathcal{B}_{44} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}s_{15}s_{26}s_{34}$ |
| | $\mathcal{O}_{45} = \langle\nabla^\mu u^\nu u^\rho u^\sigma u_\sigma\rangle\langle\nabla_\mu u_\nu u_\rho\rangle$ | $\mathcal{B}_{45} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{15}^2 s_{26}s_{34}$ |
| | $\mathcal{O}_{46} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u^\sigma u_\nu\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{46} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{12}s_{14}s_{15}s_{36}$ |
| | $\mathcal{O}_{47} = \langle\nabla^\mu u^\nu u^\rho\nabla_\rho u^\sigma u_\mu\rangle\langle u_\nu u_\sigma\rangle$ | $\mathcal{B}_{47} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{14}s_{15}s_{23}s_{36}$ |
| | $\mathcal{O}_{48} = \langle\nabla^\mu u^\nu u^\rho u^\sigma\nabla_\rho u_\mu\rangle\langle u_\nu u_\sigma\rangle$ | $\mathcal{B}_{48} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{14}s_{15}s_{24}s_{36}$ |
| | $\mathcal{O}_{49} = \langle\nabla^\mu u^\nu u^\rho u^\sigma\nabla_\mu u_\nu\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{49} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{14}^2 s_{25}s_{36}$ |
| | $\mathcal{O}_{50} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u_\nu u^\sigma\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{50} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{12}s_{13}s_{15}s_{46}$ |
| | $\mathcal{O}_{51} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u_\rho u^\sigma\rangle\langle u^\nu u_\sigma\rangle$ | $\mathcal{B}_{51} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{12}s_{15}s_{23}s_{46}$ |
| | $\mathcal{O}_{52} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u_\nu u^\sigma\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{52} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}^2 s_{25}s_{46}$ |
| | $\mathcal{O}_{53} = \langle u^\mu u^\nu u^\rho u^\sigma\rangle\langle\nabla_\mu\nabla_\nu u_\rho u_\sigma\rangle$ | $\mathcal{B}_{53} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{15}s_{25}s_{35}s_{46}$ |
| | $\mathcal{O}_{54} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u_\nu u_\rho\rangle\langle u^\sigma u_\sigma\rangle$ | $\mathcal{B}_{54} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{12}s_{13}s_{24}s_{56}$ |
| | $\mathcal{O}_{55} = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\sigma\rangle\langle u_\mu u_\nu u_\rho\rangle$ | $\mathcal{B}_{55} = \mathcal{Y}\circ\text{tr}[123\|456]s_{14}s_{15}s_{16}s_{23}$ |
| | $\mathcal{O}_{56} = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\mu\rangle\langle u_\nu u_\rho u_\sigma\rangle$ | $\mathcal{B}_{56} = \mathcal{Y}\circ\text{tr}[123\|456]s_{13}s_{14}s_{15}s_{26}$ |
| | $\mathcal{O}_{57} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\rho\rangle\langle u_\mu u_\nu u_\sigma\rangle$ | $\mathcal{B}_{57} = \mathcal{Y}\circ\text{tr}[123\|456]s_{14}s_{15}s_{23}s_{26}$ |
| | $\mathcal{O}_{58} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\mu\rangle\langle u_\nu u_\rho u_\sigma\rangle$ | $\mathcal{B}_{58} = \mathcal{Y}\circ\text{tr}[123\|456]s_{13}s_{14}s_{25}s_{26}$ |
| | $\mathcal{O}_{59} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma\rangle\langle u_\nu u_\sigma u_\rho\rangle$ | $\mathcal{B}_{59} = \mathcal{Y}\circ\text{tr}[123\|456]s_{12}s_{14}s_{26}s_{35}$ |
| | $\mathcal{O}_{60} = \langle\nabla^\mu u^\nu u^\rho u^\sigma\rangle\langle\nabla_\mu u_\nu u_\sigma u_\rho\rangle$ | $\mathcal{B}_{60} = \mathcal{Y}\circ\text{tr}[123\|456]s_{14}^2 s_{26}s_{35}$ |
| | $\mathcal{O}_{61} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma\rangle\langle u_\nu u_\rho u_\sigma\rangle$ | $\mathcal{B}_{61} = \mathcal{Y}\circ\text{tr}[123\|456]s_{12}s_{14}s_{25}s_{36}$ |
| | $\mathcal{O}_{62} = \langle\nabla^\mu u^\nu u^\rho u^\sigma\rangle\langle\nabla_\mu u_\nu u_\rho u_\sigma\rangle$ | $\mathcal{B}_{62} = \mathcal{Y}\circ\text{tr}[123\|456]s_{14}^2 s_{25}s_{36}$ |
| | $\mathcal{O}_{63} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u_\nu\rangle\langle u^\sigma u_\rho u_\sigma\rangle$ | $\mathcal{B}_{63} = \mathcal{Y}\circ\text{tr}[123\|456]s_{12}s_{13}s_{15}s_{46}$ |
| | $\mathcal{O}_{64} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u_\rho\rangle\langle u^\sigma u_\nu u_\sigma\rangle$ | $\mathcal{B}_{64} = \mathcal{Y}\circ\text{tr}[123\|456]s_{12}s_{15}s_{23}s_{46}$ |
| | $\mathcal{O}_{65} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u_\nu\rangle\langle u^\sigma u_\rho u_\sigma\rangle$ | $\mathcal{B}_{65} = \mathcal{Y}\circ\text{tr}[123\|456]s_{12}s_{13}s_{25}s_{46}$ |
| | $\mathcal{O}_{66} = \langle\nabla^\mu u^\nu u^\rho u_\mu\rangle\langle\nabla_\nu u^\sigma u_\rho u_\sigma\rangle$ | $\mathcal{B}_{66} = \mathcal{Y}\circ\text{tr}[123\|456]s_{13}s_{14}s_{25}s_{46}$ |
| | $\mathcal{O}_{67} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma\rangle\langle u_\mu u_\nu\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{67} = \mathcal{Y}\circ\text{tr}[12\|34\|56]s_{13}s_{14}s_{25}s_{26}$ |
| | $\mathcal{O}_{68} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho\rangle\langle u^\sigma u_\nu\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{68} = \mathcal{Y}\circ\text{tr}[12\|34\|56]s_{12}s_{14}s_{25}s_{36}$ |
| $SU(5)$ | $\mathcal{O}_{69} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u^\sigma\rangle\langle u_\nu u_\rho u_\sigma\rangle$ | $\mathcal{B}_{69} = \mathcal{Y}\circ\text{tr}[123\|456]s_{12}s_{14}s_{15}s_{36}$ |
| | $\mathcal{O}_{70} = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma\rangle\langle u_\mu u_\nu\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{70} = \mathcal{Y}\circ\text{tr}[12\|34\|56]s_{13}s_{14}s_{15}s_{26}$ |
| | $\mathcal{O}_{71} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma\rangle\langle u_\mu u_\rho\rangle\langle u_\nu u_\sigma\rangle$ | $\mathcal{B}_{71} = \mathcal{Y}\circ\text{tr}[12\|34\|56]s_{13}s_{15}s_{24}s_{26}$ |
| | $\mathcal{O}_{72} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho\rangle\langle u^\sigma u_\sigma\rangle\langle u_\nu u_\rho\rangle$ | $\mathcal{B}_{72} = \mathcal{Y}\circ\text{tr}[12\|34\|56]s_{12}s_{15}s_{26}s_{34}$ |
| | $\mathcal{O}_{73} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu\rangle\langle u^\sigma u_\nu\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{73} = \mathcal{Y}\circ\text{tr}[12\|34\|56]s_{12}s_{14}s_{15}s_{36}$ |
| | $\mathcal{O}_{74} = \langle\nabla^\mu u^\nu u^\rho\rangle\langle\nabla_\mu u^\sigma u_\nu\rangle\langle u_\rho u_\sigma\rangle$ | $\mathcal{B}_{74} = \mathcal{Y}\circ\text{tr}[12\|34\|56]s_{13}s_{14}s_{25}s_{36}$ |
| $SU(N_f\geq 6)$ | $\mathcal{O}_{75} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu\rangle\langle u^\sigma u_\sigma\rangle\langle u_\nu u_\rho\rangle$ | $\mathcal{B}_{75} = \mathcal{Y}\circ\text{tr}[12\|34\|56]s_{12}s_{15}s_{16}s_{34}$ |
| | $\mathcal{O}_{76} = \langle\nabla^\mu u^\nu u^\rho\rangle\langle\nabla_\mu u^\sigma u_\rho\rangle\langle u_\nu u_\sigma\rangle$ | $\mathcal{B}_{76} = \mathcal{Y}\circ\text{tr}[12\|34\|56]s_{13}s_{15}s_{24}s_{36}$ |

| $SU(N_f)$ | Operator Basis | Amplitude Basis |
|---|---|---|
| $SU(2)$ | $\mathcal{O}_1 = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u_\nu u^\sigma u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_1 = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{13}\epsilon(2,4,5,6)$ |
| | $\mathcal{O}_2 = \langle\nabla^\mu u^\rho\nabla^\sigma u^\nu u_\mu u_\nu u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_2 = \mathcal{Y}\circ\text{tr}[123456]s_{14}s_{23}\epsilon(1,2,5,6)$ |
| $SU(3)$ | $\mathcal{O}_3 = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\mu u^\eta u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_3 = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{15}\epsilon(1,2,4,6)$ |
| | $\mathcal{O}_4 = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\mu u_\nu u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_4 = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{14}\epsilon(1,2,5,6)$ |
| | $\mathcal{O}_5 = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u^\sigma u^\eta u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_5 = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{15}\epsilon(1,3,4,6)$ |
| | $\mathcal{O}_6 = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u^\sigma u_\nu u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_6 = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{14}\epsilon(1,3,5,6)$ |
| | $\mathcal{O}_7 = \langle\nabla^\mu u^\rho\nabla_\nu u^\sigma u_\mu u^\eta u^\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_7 = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{24}\epsilon(1,3,5,6)$ |
| | $\mathcal{O}_8 = \langle\nabla^\mu u^\nu u^\rho u_\mu u^\sigma u_\nu u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_8 = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{13}\epsilon(1,4,5,6)$ |
| | $\mathcal{O}_9 = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma u^\eta u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_9 = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{15}\epsilon(2,3,4,6)$ |
| | $\mathcal{O}_{10} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u^\sigma u^\eta u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{10} = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{15}\epsilon(2,3,4,6)$ |
| | $\mathcal{O}_{11} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u^\sigma u_\nu u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{11} = \mathcal{Y}\circ\text{tr}[123456]s_{12}s_{14}\epsilon(2,3,5,6)$ |
| | $\mathcal{O}_{12} = \langle\nabla^\mu u^\nu u^\rho\nabla_\nu u^\sigma u_\mu u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{12} = \mathcal{Y}\circ\text{tr}[123456]s_{13}s_{14}\epsilon(2,3,5,6)$ |
| | $\mathcal{O}_{13} = \langle\nabla^\mu u^\nu u^\rho u^\sigma\nabla_\mu u_\nu u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{13} = \mathcal{Y}\circ\text{tr}[123456]s_{14}^2\epsilon(2,3,5,6)$ |
| | $\mathcal{O}_{14} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u_\nu u^\sigma u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{14} = \mathcal{Y}\circ\text{tr}[123456]s_{13}^2\epsilon(2,4,5,6)$ |
| | $\mathcal{O}_{15} = \langle\nabla^\mu u^\nu\nabla_\mu u_\nu u^\rho u^\sigma u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{15} = \mathcal{Y}\circ\text{tr}[123456]s_{12}^2\epsilon(3,4,5,6)$ |
| | $\mathcal{O}_{16} = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u^\eta u_\mu u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{16} = \mathcal{Y}\circ\text{tr}[123456]s_{14}s_{15}\epsilon(1,2,3,6)$ |
| | $\mathcal{O}_{17} = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u_\mu u^\eta\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{17} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}s_{25}\epsilon(1,2,4,6)$ |
| | $\mathcal{O}_{18} = \langle\nabla^\mu\nabla^\nu u^\rho u_\mu u^\sigma u^\eta\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{18} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{12}s_{15}\epsilon(1,3,4,6)$ |
| | $\mathcal{O}_{19} = \langle\nabla^\mu u^\nu\nabla_\mu u^\rho u^\sigma u^\eta\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{19} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{12}s_{15}\epsilon(2,3,4,6)$ |
| | $\mathcal{O}_{20} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u^\sigma u^\eta\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{20} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}s_{15}\epsilon(2,3,4,6)$ |
| $SU(4)$ | $\mathcal{O}_{21} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u^\eta u_\mu\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{21} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{14}s_{15}\epsilon(1,2,3,6)$ |
| | $\mathcal{O}_{22} = \langle\nabla^\mu u^\rho\nabla^\nu u^\sigma u^\eta u_\mu\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{22} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{14}s_{25}\epsilon(1,2,3,6)$ |
| | $\mathcal{O}_{23} = \langle\nabla^\mu u^\nu\nabla^\rho u^\sigma u_\mu u^\eta\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{23} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}s_{15}\epsilon(1,2,4,6)$ |
| | $\mathcal{O}_{24} = \langle\nabla^\mu u^\rho u^\nu\nabla_\mu u^\sigma u^\eta\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{24} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{13}s_{25}\epsilon(1,3,4,6)$ |
| | $\mathcal{O}_{25} = \langle\nabla^\mu u^\nu u^\rho u^\sigma\nabla_\mu u^\eta\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{25} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{14}s_{25}\epsilon(1,3,4,6)$ |
| | $\mathcal{O}_{26} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu u^\sigma u^\eta\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{26} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{14}s_{25}\epsilon(2,3,4,6)$ |
| | $\mathcal{O}_{27} = \langle u^\mu\nabla^\nu u^\rho u^\sigma\nabla_\mu u^\eta\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{27} = \mathcal{Y}\circ\text{tr}[1234\|56]s_{14}s_{25}\epsilon(2,3,4,6)$ |
| | $\mathcal{O}_{28} = \langle\nabla^\mu u^\nu u^\rho u^\sigma\nabla_\mu\rangle\langle u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{28} = \mathcal{Y}\circ\text{tr}[123\|456]s_{14}s_{25}\epsilon(1,3,4,6)$ |
| | $\mathcal{O}_{29} = \langle\nabla^\mu u^\nu u^\rho u_\nu\rangle\langle u^\sigma u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{29} = \mathcal{Y}\circ\text{tr}[123\|456]s_{12}s_{13}\epsilon(1,4,5,6)$ |
| | $\mathcal{O}_{30} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu\rangle\langle u_\nu u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{30} = \mathcal{Y}\circ\text{tr}[123\|456]s_{14}^2\epsilon(2,3,5,6)$ |
| | $\mathcal{O}_{31} = \langle\nabla^\mu u^\nu\nabla_\mu u_\nu\rangle\langle u^\sigma u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{31} = \mathcal{Y}\circ\text{tr}[123\|456]s_{12}s_{13}\epsilon(2,4,5,6)$ |
| | $\mathcal{O}_{32} = \langle\nabla^\mu u^\nu\nabla_\mu u_\nu u^\rho\rangle\langle u^\sigma u^\eta u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{32} = \mathcal{Y}\circ\text{tr}[123\|456]s_{12}^2\epsilon(3,4,5,6)$ |
| | $\mathcal{O}_{33} = \langle\nabla^\mu u^\nu u^\rho\nabla_\mu\rangle\langle u^\sigma u^\eta\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{33} = \mathcal{Y}\circ\text{tr}[12\|34\|56]s_{13}s_{15}\epsilon(2,3,4,6)$ |
| $SU(N_f\geq 5)$ | $\mathcal{O}_{34} = \langle\nabla^\mu\nabla^\nu u^\rho u^\sigma u^\eta\rangle\langle u_\mu u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{34} = \mathcal{Y}\circ\text{tr}[123\|456]s_{14}s_{15}\epsilon(1,2,3,6)$ |
| | $\mathcal{O}_{35} = \langle\nabla^\mu u^\rho\nabla^\nu u^\sigma\rangle\langle u_\mu u^\eta\rangle\langle u_\nu u^\lambda\rangle\epsilon_{\rho\sigma\eta\lambda}$ | $\mathcal{B}_{35} = \mathcal{Y}\circ\text{tr}[12\|34\|56]s_{13}s_{25}\epsilon(1,2,4,6)$ |

[Low, Shu, **Xiao**, Zheng, 2209.00198]

# Electroweak Chiral Effective Theory

It is also easy to add external sources and construct **Chiral Effective Theory (ChEFT)**

| Classes | $\mathcal{N}_{\rm type}$ | $\mathcal{N}_{\rm term}$ | $\mathcal{N}_{\rm operator}$ |
|---|---|---|---|
| $UhD^4$ | $3+6+0+0$ | 15 | 15 |
| $X^2Uh$ | $6+4+0+0$ | 10 | 10 |
| $XUhD^2$ | $2+6+0+0$ | 8 | 8 |
| $X^3$ | $4+2+0+0$ | 6 | 6 |
| $\psi^2UhD$ | $4+8+0+0$ | 13(16) | $13{n_f}^2 \quad (16{n_f}^2)$ |
| $\psi^2UhD^2$ | $6+10+0+0$ | 60(80) | $60{n_f}^2 \quad (80{n_f}^2)$ |
| $\psi^2UhX$ | $7+7+0+0$ | 22(28) | $22{n_f}^2 \quad (28{n_f}^2)$ |
| $\psi^4$ | $12+24+4+8$ | 117(160) | $\frac{1}{4}{n_f}^2(31-6n_f+335{n_f}^2) \quad ({n_f}^2(9-2n_f+125{n_f}^2))$ |
| Total | 123 | 261(313) | $\frac{335{n_f}^4}{4}-\frac{3{n_f}^3}{2}+\frac{411{n_f}^2}{4}+39 \quad (39+133{n_f}^2-2{n_f}^2-2{n_f}^3+125{n_f}^4)$<br>$\mathcal{N}_{\rm operatrs}(n_f=1)=224(295), \quad \mathcal{N}_{\rm operatrs}(n_f=3)=7704(11307)$ |

Missing operators in previous literature $\Rightarrow$

- gray operators with right-handed neutrinos
- Goldstone matrix $\mathbf{U}$ and spurion $\mathbf{T}$
- Young symmetrizer $\mathcal{Y}$ for repeated fields
- Coefficient function of higgs $\mathcal{F}(h)$

$$\mathcal{O}_{33}^{Uh\psi^4} = (\bar{q}_{Ls}\gamma_\mu\tau^I\mathbf{T}q_{Lp})(\bar{q}_{Rr}\gamma^\mu\mathbf{U}^\dagger\tau^I\mathbf{U}q_{Rt})\mathcal{F}_{33}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{34}^{Uh\psi^4} = (\bar{q}_{Ls}\gamma_\mu\lambda^A\tau^I\mathbf{T}q_{Lp})(\bar{q}_{Rr}\gamma^\mu\lambda^A\mathbf{U}^\dagger\tau^I\mathbf{U}q_{Rt})\mathcal{F}_{34}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{89}^{Uh\psi^4} = (\bar{l}_{Ls}\gamma_\mu\tau^I l_{Lp})(\bar{l}_{Rt}\sigma^\mu\tau^I\mathbf{U}^\dagger\mathbf{T}\mathbf{U}l_{Rs})\mathcal{F}_{89}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{107}^{Uh\psi^4} = (\bar{l}_{Ls}\gamma_\mu\tau^I\mathbf{T}l_{Lp})(\bar{q}_{Lt}\gamma^\mu\tau^I q_{Lr})\mathcal{F}_{107}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{113}^{Uh\psi^4} = (\bar{l}_{Rs}\gamma_\mu\tau^I\mathbf{T}l_{Rp})(\bar{q}_{Rt}\gamma^\mu\tau^I q_{Rr})\mathcal{F}_{113}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{119}^{Uh\psi^4} = (\bar{l}_{Rs}\gamma_\mu\mathbf{U}^\dagger\tau^I\mathbf{T}\mathbf{U}l_{Rp})(\bar{q}_{Lt}\gamma^\mu\tau^I q_{Lr})\mathcal{F}_{119}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{125}^{Uh\psi^4} = (\bar{l}_{Ls}\gamma_\mu\tau^I\mathbf{T}l_{Lp})(\bar{q}_{Rt}\gamma^\mu\mathbf{U}^\dagger\tau^I\mathbf{U}q_{Rr})\mathcal{F}_{125}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{140}^{Uh\psi^4} = \mathcal{Y}[\tiny\begin{array}{|c|c|}\hline r & s \\ \hline t & \\ \hline\end{array}]\epsilon^{abc}\epsilon^{ln}\epsilon^{km}((\mathbf{T}l_L{}^T)_{pm}C(\mathbf{T}q_L)_{ran})(q_{L}{}^T_{rak}Cq_{Ltcl})\mathcal{F}_{159}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{160}^{Uh\psi^4} = \mathcal{Y}[\tiny\begin{array}{|c|c|}\hline r & s \\ \hline t & \\ \hline\end{array}]\epsilon^{abc}\epsilon^{km}\epsilon^{ln}((\mathbf{T}l_R{}^T)_{pm}C(\mathbf{T}q_R)_{ran})(q_{R}{}^T_{sbk}Cq_{Rtcl})\mathcal{F}_{160}^{Uh\psi^4}(h).$$

[Sun, **Xiao**, Yu, 2206.07722, 2210.14939]

# SMEFT vs HEFT

The E(lectro)W(eak)ChEFT is also known as the Higgs EFT (HEFT)

**SMEFT**

$$\mathcal{F}(h) = 1 + 2\frac{h}{v} + \frac{h^2}{v^2}$$

Dim-6

$$\widetilde{B}_{\mu\nu}\langle D^\mu D_\alpha W^{\alpha\beta} W_\beta{}^\nu\rangle$$
$$\widetilde{B}_{\mu\nu}\langle D_\beta D_\alpha W^{\alpha\mu} W^{\beta\nu}\rangle$$
$$\widetilde{B}_{\mu\nu}\langle D_\alpha W^{\alpha\mu} D_\beta W^{\beta\nu}\rangle$$
$$\widetilde{B}_{\mu\nu}\langle D_\alpha W^{\alpha\beta} D_\beta W^{\mu\nu}\rangle$$
$$\widetilde{B}_{\mu\nu}\langle D_\alpha W^{\alpha\beta} D^\mu W_\beta{}^\nu\rangle$$

Dim-8

$$\mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$[B_{\mu\nu}B^{\mu\nu}] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$\left[(D_\mu\Phi)^\dagger \widehat{W}_{\beta\nu} D^\mu\Phi\right] \times B^{\beta\nu}$$
$$\left[(D_\mu\Phi)^\dagger \widehat{W}_{\beta\nu}\widehat{W}^{\beta\mu} D^\nu\Phi\right]$$
$$\mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$\left[B_{\mu\nu}B^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$\left[(D_\mu\Phi)^\dagger \widehat{W}_{\beta\nu} D^\nu\Phi\right] \times B^{\beta\mu} + \text{h.c.}$$

⟷

**HEFT**

$$\mathcal{F}(h) = 1 + a\frac{h}{v} + b\frac{h^2}{v^2} + \cdots$$

P4

$$\mathrm{Tr}[V^\mu V_\mu]\mathrm{Tr}[V^\nu V_\nu]\mathcal{F}_6(h)$$
$$\mathrm{Tr}[V^\mu V^\nu]\mathrm{Tr}[V_\mu V_\nu]\mathcal{F}_{11}(h)$$
$$\mathrm{Tr}[V^\mu V_\mu](\mathrm{Tr}[TV_\nu])^2\mathcal{F}_{23}(h)$$
$$\mathrm{Tr}[V^\mu V^\nu]\mathrm{Tr}[TV_\mu]\mathrm{Tr}[TV_\nu]\mathcal{F}_{24}(h)$$
$$(\mathrm{Tr}[TV_\mu]\mathrm{Tr}[TV_\nu])^2\mathcal{F}_{26}(h)$$

P6

$$\mathrm{Tr}[T\mathcal{D}_{\mu\nu}]\mathrm{Tr}[T\mathcal{D}^{\mu\nu}]\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha]$$
$$\mathrm{Tr}[T\mathcal{D}^\mu_\mu]\mathrm{Tr}[T\mathcal{D}^\nu_\nu]\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha]$$
$$-g^2\,\mathrm{Tr}[T\widehat{W}_{\mu\nu}]\mathrm{Tr}[T\widehat{W}^{\mu\nu}]\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha]$$
$$i\,g\,Tr[T\widehat{W}_{\mu\nu}]\mathrm{Tr}[T\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[TV^\nu]\mathrm{Tr}[TV_\alpha]$$
$$-g'^2\,B_{\mu\nu}B^{\mu\alpha}\mathrm{Tr}[TV^\nu]\mathrm{Tr}[TV_\alpha]$$
$$-gg'\,B_{\mu\nu}\mathrm{Tr}[T\widehat{W}^{\mu\nu}]\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha]$$

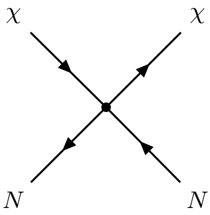
- HEFT has more general couplings for Higgs physics
- HEFT has different power counting than SMEFT

# Dark Matter Effective Field Theory

Direct search of dark matter:

- Non-relativistic DM with arbitrary spin
- Small momentum exchange $t \ll m_\chi$
- Local interaction (effective operator)

$\Rightarrow$ Heavy Particle Effective Theory (HPET)




Lorentz invariance is "spontaneously broken" by the DM momentum $p_\chi^\mu \sim m_\chi v^\mu$:

Reparameterization Invariance (RPI): $\delta \mathcal{B}_v = 0 \quad \simeq \quad \mathcal{L}_{\text{HPET}}$

| | ChPT | HPET |
|---|---|---|
| Non-linear symmetry | $SU(N)$ | Lorentz |
| Constraints on $\mathcal{B}$ | Adler's Zero | RPI |

[Li, Low, **Xiao**, work in progress]

# Outline

# Operators that Produce Partial Wave Amplitudes

The on-shell correspondence is not just useful for basis construction

$$\text{4-fermion couplings}\begin{cases} \mathcal{O}^{(S)} = (\bar{\psi}\psi)(\bar{\chi}\chi) & \simeq \quad \mathcal{B}^{(S)} \sim d^{J=0}_{0,0}(\theta) \\ \mathcal{O}^{(V)} = (\bar{\psi}\gamma^{\mu}\psi)(\bar{\chi}\gamma_{\mu}\chi) & \simeq \quad \mathcal{B}^{(V)} \sim d^{J=1}_{1,\pm 1}(\theta) \\ \mathcal{O}^{(T)} = (\bar{\psi}\sigma^{\mu\nu}\psi)(\bar{\chi}\sigma_{\mu\nu}\chi) & \simeq \quad \mathcal{B}^{(T)} \sim d^{J=1}_{0,0}(\theta) \end{cases}$$

The operators are classified according to the angular momentum in certain channel
the same way that the amplitudes are decomposed into partial waves!

Generalized partial waves: $\mathbf{W^2}\mathcal{B}^J = -J(J+1)P^2\mathcal{B}^j \simeq \mathcal{O}^J$

Pauli-Lubanski $\mathbf{W}^{\mu} = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\mathbf{P}_{\nu}\mathbf{J}_{\rho\lambda}$

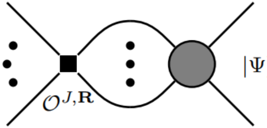
Can define partial waves for arbitrary number of particles with arbitrary spins!

[Shu, **Xiao**, Zheng, 2111.08019]

# Applications to Pheno Study

Define J-basis operators by fixing angular momentum $J$ and gauge rep $\mathbf{R}$:

- Angular momentum conservation: [Jiang, Shu, **Xiao**, Zheng, 2001.04481]




$$|\Psi\rangle^{J',\mathbf{R}'} \sim \delta^{JJ'}\delta^{\mathbf{R}\mathbf{R}'}$$

1. Non-interference
2. Vanishing loop diagrams (multi-loop)
3. Vanishing ADM element

**Non-Abelian**

| | | (4,0) | | | | (4,2) | |
|---|---|---|---|---|---|---|---|
| | | $V^+V^+V^-V^-$ | $V^+V^-\psi^+\psi^-$ | $V^+V^-\phi\phi$ | $V^+\psi^-\psi^-\phi$ | $V^+V^+V^+V^-$ | $V^+V^+\psi^+\psi^-$ |
| (4,0) | $\psi^2\bar\psi^2$ | × | 0 | × | 0* | × | R |
| | $\phi^4D^2$ | × | × | 0 | × | × | × |
| | $\phi^2\psi\bar\psi D$ | × | 0 | 0 | 0 | × | R |
| (4,2) | $F\psi^2\phi$ | × | R | R | R | × | 0 |
| | $F^2\phi^2$ | R | 0 | R | R | 0* | 0* |
| | $\psi^4$ | × | 0 | × | 0 | × | 0 |
| (4,−2) | $\bar F\bar\psi^2\phi$ | × | R | R | R | × | 0 |
| | $\bar F^2\phi^2$ | R | 0 | R | R | 0 | 0 |
| | $\bar\psi^4$ | × | 0 | × | R | × | 0 |

**Abelian**

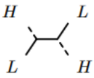
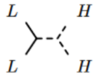
| | (4,0) | | | | (4,2) | |
|---|---|---|---|---|---|---|
| | $V^+V^+V^-V^-$ | $V^+V^-\psi^+\psi^-$ | $V^+V^-\phi\phi$ | $V^+\psi^-\psi^-\phi$ | $V^+V^+V^+V^-$ | $V^+V^+\psi^+\psi^-$ |
| $\psi^2\bar\psi^2$ | × | 0 | × | 0* | × | 0 |
| $\phi^4D^2$ | × | × | 0 | × | × | × |
| $\phi^2\psi\bar\psi D$ | × | 0 | 0 | 0 | × | 0 |
| $F\psi^2\phi$ | × | R | R | R | × | 0 |
| $F^2\phi^2$ | R | 0 | R | R | 0 | 0 |
| $\psi^4$ | × | 0 | × | 0 | × | 0 |
| $\bar F\bar\psi^2\phi$ | × | R | R | R | × | 0 |
| $\bar F^2\phi^2$ | R | 0 | R | R | 0 | 0 |
| $\bar\psi^4$ | × | 0 | × | R | × | 0 |

- Perturbative Unitarity Bound, Positivity Bound [Yang, Ren, Yu, 2312.04663]

- Implication of UV physics: $\mathcal{O}^{J,\mathbf{R}} \sim$ resonance with $(J,\mathbf{R})$

## Implication of UV Resonances

Analysing J-basis in all channels, get all tree-level UV origin:

| Topology | j-basis | Quantum numbers $\{J, \mathbf{R}, Y\}$ | Model |
|---|---|---|---|
|   | $\mathcal{B}^{J=1/2,\mathbf{R}=1}_{\{13\}} = \mathcal{B}^p_1 + \mathcal{B}^p_2.$ | $\{\frac{1}{2}, 1, 0\}$ | Type I |
| | $\mathcal{B}^{J=1/2,\mathbf{R}=3}_{\{13\}} = -\mathcal{B}^p_1 + 3\mathcal{B}^p_2,$ | $\{\frac{1}{2}, 3, 0\}$ | Type III |
|   | $\mathcal{B}^{J=0,\mathbf{R}=3}_{\{12\}} = -2\mathcal{B}^p_1,$ | $\{0, 3, -1\}$ | Type II |
| | $\mathcal{B}^{J=0,\mathbf{R}=1}_{\{12\}} = 2\mathcal{B}^p_2.$ | $\{0, 1, -1\}$ | N/A |

$\Rightarrow$ Only three types of seesaw models for $\mathcal{O}^{(5)} = (HL)^T \mathcal{C} (HL) \supset v^2 (\nu^T \mathcal{C} \nu)$

- Completely bottom-up search
- Does NOT apply to loop-level origins

# Implication of UV Resonances

47 UV resonances responsible for Dim-6 SMEFT! [Li, Ni, **Xiao**, Yu, 2204.03660]

19 scalars

| Notation | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $S_7$ | $S_8$ |
|---|---|---|---|---|---|---|---|---|
| Name | $\mathcal{S}$ | $\mathcal{S}_1$ | $\mathcal{S}_2$ | $\varphi$ | $\Xi$ | $\Xi_1$ | $\Theta_1$ | $\Theta_3$ |
| Irrep | $(\mathbf{1},\mathbf{1})_0$ | $(\mathbf{1},\mathbf{1})_1$ | $(\mathbf{1},\mathbf{1})_2$ | $(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$ | $(\mathbf{1},\mathbf{3})_0$ | $(\mathbf{1},\mathbf{3})_1$ | $(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$ | $(\mathbf{1},\mathbf{4})_{\frac{3}{2}}$ |
| Notation | $S_9$ | $S_{10}$ | $S_{11}$ | $S_{12}$ | $S_{13}$ | $S_{14}$ | | |
| Name | $\omega_4$ | $\omega_1$ | $\omega_2$ | $\Pi_1$ | $\Pi_7$ | $\zeta$ | | |
| Irrep | $(\mathbf{3},\mathbf{1})_{-\frac{4}{3}}$ | $(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$ | $(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$ | $(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$ | $(\mathbf{3},\mathbf{2})_{\frac{7}{6}}$ | $(\mathbf{3},\mathbf{3})_{-\frac{1}{3}}$ | | |
| Notation | $S_{15}$ | $S_{16}$ | $S_{17}$ | $S_{18}$ | $S_{19}$ | | | |
| Name | $\Omega_2$ | $\Omega_1$ | $\Omega_4$ | $\Upsilon_1$ | $\Phi$ | | | |
| Irrep | $(\mathbf{6},\mathbf{1})_{-\frac{2}{3}}$ | $(\mathbf{6},\mathbf{1})_{\frac{1}{3}}$ | $(\mathbf{6},\mathbf{1})_{\frac{4}{3}}$ | $(\mathbf{6},\mathbf{3})_{\frac{1}{3}}$ | $(\mathbf{8},\mathbf{2})_{\frac{1}{2}}$ | | | |

14 fermions

| Notation | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ |
|---|---|---|---|---|---|---|---|
| Name | $N$ | $E^c$ | $\Delta_1^c$ | $\Delta_3^c$ | $\Sigma$ | $\Sigma_1^c$ | |
| Irrep | $(\mathbf{1},\mathbf{1})_0$ | $(\mathbf{1},\mathbf{1})_1$ | $(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$ | $(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$ | $(\mathbf{1},\mathbf{3})_0$ | $(\mathbf{1},\mathbf{3})_1$ | $(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$ |
| Notation | $F_8$ | $F_9$ | $F_{10}$ | $F_{11}$ | $F_{12}$ | $F_{13}$ | $F_{14}$ |
| Name | $D$ | $U$ | $Q_5$ | $Q_1$ | $Q_7$ | $T_1$ | $T_2$ |
| Irrep | $(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$ | $(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$ | $(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$ | $(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$ | $(\mathbf{3},\mathbf{2})_{\frac{7}{6}}$ | $(\mathbf{3},\mathbf{3})_{-\frac{1}{3}}$ | $(\mathbf{3},\mathbf{3})_{\frac{2}{3}}$ |

14 vectors

| Notation | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_6$ | $V_7$ |
|---|---|---|---|---|---|---|---|
| Name | $\mathcal{B}$ | $\mathcal{B}_1$ | $\mathcal{L}_3^\dagger$ | $\mathcal{W}$ | $\mathcal{U}_2$ | $\mathcal{U}_5$ | $\mathcal{Q}_5$ |
| Irrep | $(\mathbf{1},\mathbf{1})_0$ | $(\mathbf{1},\mathbf{1})_1$ | $(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$ | $(\mathbf{1},\mathbf{3})_0$ | $(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$ | $(\mathbf{3},\mathbf{1})_{\frac{5}{3}}$ | $(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$ |
| Notation | $V_8$ | $V_9$ | $V_{10}$ | $V_{11}$ | $V_{12}$ | $V_{13}$ | $V_{14}$ |
| Name | $\mathcal{Q}_1$ | $\mathcal{X}$ | $\mathcal{Y}_1^\dagger$ | $\mathcal{Y}_5^\dagger$ | $\mathcal{G}$ | $\mathcal{G}_1$ | $\mathcal{H}$ |
| Irrep | $(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$ | $(\mathbf{3},\mathbf{3})_{\frac{2}{3}}$ | $(\mathbf{6},\mathbf{2})_{-\frac{1}{6}}$ | $(\mathbf{6},\mathbf{2})_{\frac{5}{6}}$ | $(\mathbf{8},\mathbf{1})_0$ | $(\mathbf{8},\mathbf{1})_1$ | $(\mathbf{8},\mathbf{3})_0$ |

# Implication of UV Resonances

7+59 UV tree-level models for Dim-7 seesaw! [Li, Ni, **Xiao**, Yu, 2204.03660]

- type $\{L^2H^4\}$ $\rightarrow$
- type $\{L^2H^2D^2\}$

$\downarrow$

| Topology | j-basis | Quantum numbers $\{J,\mathbf{R},Y\}$ |
|---|---|---|
|  | $\mathcal{O}_{\{13\},1} = 3\mathcal{O}^p_1 + 6\mathcal{O}^p_2 - 9\mathcal{O}^p_3 - 2\mathcal{O}^p_4$ | $\{\frac{3}{2},3,0\}$ |
| | $\mathcal{O}_{\{13\},2} = 3\mathcal{O}^p_2 - \mathcal{O}^p_4$ | $\{\frac{1}{2},3,0\}$ |
| | $\mathcal{O}_{\{13\},3} = -3\mathcal{O}^p_1 + 2\mathcal{O}^p_2 - 3\mathcal{O}^p_3 + 2\mathcal{O}^p_4$ | $\{\frac{3}{2},1,0\}$ |
| | $\mathcal{O}_{\{13\},4} = \mathcal{O}^p_2 + \mathcal{O}^p_4$ | $\{\frac{1}{2},1,0\}$ |
|  | $\mathcal{O}_{\{12\},1} = 2\mathcal{O}^p_1 - 4\mathcal{O}^p_4$ | $\{1,3,-1\}$ |
| | $\mathcal{O}_{\{12\}} = -2\mathcal{O}^p_1$ | $\{0,3,-1\}$ |
| | $\mathcal{O}_{\{12\}} = 4\mathcal{O}^p_2 - 2\mathcal{O}^p_3$ | $\{1,1,-1\}$ |
| | $\mathcal{O}_{\{12\}} = 2\mathcal{O}^p_3$ | $\{0,1,-1\}$ N/A |

| Topology | j-basis | Quantum numbers $\{J,\mathbf{R},Y\}$ |
|---|---|---|
|  | $\mathcal{O}_{\{12\|34\|56\},1} = 2\mathcal{O}^p_1 - 4\mathcal{O}^p_2$ | $\{0,3,-1\},\{0,3,1\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{12\|34\|56\},2} = 2\mathcal{O}^p_2 + 4\mathcal{O}^p_3$ | $\{0,3,-1\},\{0,1,1\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{12\|34\|56\},3} = 12\mathcal{O}^p_4$ | $\{0,1,-1\},\{0,3,1\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{12\|34\|56\},4} = 4\mathcal{O}^p_1 + 4\mathcal{O}^p_2$ | $\{0,3,-1\},\{0,3,1\},\{0,1,0\}$ |
| | $\mathcal{O}_{\{12\|34\|56\},5} = 2\mathcal{O}^p_4 + 4\mathcal{O}^p_5$ | $\{0,1,-1\},\{0,1,1\},\{0,1,0\}$ |
|  | $\mathcal{O}_{\{13\|24\|56\},1} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},3,0\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{13\|24\|56\},2} = -\mathcal{O}^p_1 + 3\mathcal{O}^p_2 + 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},1,0\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{13\|24\|56\},3} = -\mathcal{O}^p_1 + \mathcal{O}^p_2 - 2\mathcal{O}^p_3 - 3\mathcal{O}^p_4$ | $\{\frac{1}{2},1,0\},\{\frac{1}{2},3,0\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{13\|24\|56\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_2 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},3,0\},\{0,1,0\}$ |
| | $\mathcal{O}_{\{13\|24\|56\},5} = \mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5$ | $\{\frac{1}{2},1,0\},\{\frac{1}{2},1,0\},\{0,1,0\}$ |
|  | $\mathcal{O}_{\{16\|23\|45\},1} = 2\mathcal{O}^p_1 - 2\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 6\mathcal{O}^p_5$ | $\{\frac{1}{2},3,-1\},\{\frac{1}{2},3,0\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{16\|23\|45\},2} = -2\mathcal{O}^p_1 - \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$ | $\{\frac{1}{2},3,-1\},\{\frac{1}{2},1,0\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{16\|23\|45\},3} = 3\mathcal{O}^p_2 + 3\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$ | $\{\frac{1}{2},1,-1\},\{\frac{1}{2},3,0\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{16\|23\|45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$ | $\{\frac{1}{2},3,-1\},\{\frac{1}{2},3,0\},\{0,1,1\}$ |
| | $\mathcal{O}_{\{16\|23\|45\},5} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5$ | $\{\frac{1}{2},1,-1\},\{\frac{1}{2},1,0\},\{0,1,1\}$ |
|  | $\mathcal{O}_{\{12\|125\|34\},1} = \mathcal{O}^p_1 + 4\mathcal{O}^p_2$ | $\{0,3,-1\},\{0,4,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{12\|125\|34\},2} = -8\mathcal{O}^p_1 + 4\mathcal{O}^p_2$ | $\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{12\|125\|34\},3} = -12\mathcal{O}^p_4$ | $\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{12\|125\|34\},4} = -2\mathcal{O}^p_2 - 4\mathcal{O}^p_3$ | $\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$ |
| | $\mathcal{O}_{\{12\|125\|34\},5} = -2\mathcal{O}^p_4 - 4\mathcal{O}^p_5$ | $\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$ |
|  | $\mathcal{O}_{\{12\|126\|34\},1} = 3\mathcal{O}^p_1$ | $\{0,3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{12\|126\|34\},2} = 12\mathcal{O}^p_2$ | $\{0,3,-1\},\{0,2,-\frac{3}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{12\|126\|34\},3} = -12\mathcal{O}^p_4$ | $\{0,1,-1\},\{0,2,-\frac{3}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{12\|126\|34\},4} = -2\mathcal{O}^p_2 - 4\mathcal{O}^p_3$ | $\{0,3,-1\},\{0,2,-\frac{3}{2}\},\{0,1,1\}$ |
| | $\mathcal{O}_{\{12\|126\|34\},5} = -2\mathcal{O}^p_4 - 4\mathcal{O}^p_5$ | $\{0,1,-1\},\{0,2,-\frac{3}{2}\},\{0,1,1\}$ |
|  | $\mathcal{O}_{\{12\|124\|36\},1} = -\mathcal{O}^p_1 - 4\mathcal{O}^p_3$ | $\{0,3,-1\},\{0,4,-\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{12\|124\|36\},2} = 2\mathcal{O}^p_1 + 6\mathcal{O}^p_2 + 2\mathcal{O}^p_3$ | $\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{12\|124\|36\},3} = -6\mathcal{O}^p_4 - 6\mathcal{O}^p_5$ | $\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{12\|124\|36\},4} = -2\mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 2\mathcal{O}^p_3$ | $\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$ |
| | $\mathcal{O}_{\{12\|124\|36\},5} = -2\mathcal{O}^p_4 + 2\mathcal{O}^p_5$ | $\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$ |
|  | $\mathcal{O}_{\{13\|135\|24\},1} = \mathcal{O}^p_1 - 2\mathcal{O}^p_3 - 6\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},4,\frac{1}{2}\},\{\frac{1}{2},3,0\}$ |
| | $\mathcal{O}_{\{13\|135\|24\},2} = -\mathcal{O}^p_1 - 3\mathcal{O}^p_2 - 4\mathcal{O}^p_3 + 9\mathcal{O}^p_4 + 6\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},3,0\}$ |
| | $\mathcal{O}_{\{13\|135\|24\},3} = \mathcal{O}^p_1 - \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4$ | $\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},3,0\}$ |
| | $\mathcal{O}_{\{13\|135\|24\},4} = \mathcal{O}^p_1 - 3\mathcal{O}^p_2 - 2\mathcal{O}^p_3 - 3\mathcal{O}^p_4$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},1,0\}$ |
| | $\mathcal{O}_{\{13\|135\|24\},5} = \mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5$ | $\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},1,0\}$ |
|  | $\mathcal{O}_{\{16\|146\|23\},1} = -2\mathcal{O}^p_1 + 4\mathcal{O}^p_3 - 12\mathcal{O}^p_5$ | $\{\frac{1}{2},3,-1\},\{\frac{1}{2},4,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$ |
| | $\mathcal{O}_{\{16\|146\|23\},2} = 2\mathcal{O}^p_1 - 3\mathcal{O}^p_2 - \mathcal{O}^p_3 + 9\mathcal{O}^p_4 + 3\mathcal{O}^p_5$ | $\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$ |
| | $\mathcal{O}_{\{16\|146\|23\},3} = 3\mathcal{O}^p_2 + 3\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$ | $\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$ |
| | $\mathcal{O}_{\{16\|146\|23\},4} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 - 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5$ | $\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},1,0\}$ |
| | $\mathcal{O}_{\{16\|146\|23\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 - \mathcal{O}^p_4 + \mathcal{O}^p_5$ | $\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},1,0\}$ |

| | | |
|---|---|---|
|  | $\mathcal{O}_{\{13\|123\|45\},1} = \mathcal{O}^p_1 - 4\mathcal{O}^p_2 - 4\mathcal{O}^p_3$ | $\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{13\|123\|45\},2} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 - 9\mathcal{O}^p_4 - 9\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{13\|123\|45\},3} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$ | $\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{13\|123\|45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$ |
| | $\mathcal{O}_{\{13\|123\|45\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5$ | $\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$ |
|  | $\mathcal{O}_{\{13\|123\|46\},1} = \mathcal{O}^p_1 - 4\mathcal{O}^p_2 - 4\mathcal{O}^p_3$ | $\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{13\|123\|46\},2} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 9\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{13\|123\|46\},3} = -\mathcal{O}^p_1 - 2\mathcal{O}^p_2 + \mathcal{O}^p_3 - 3\mathcal{O}^p_5$ | $\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{13\|123\|46\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 3\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,1,0\}$ |
| | $\mathcal{O}_{\{13\|123\|46\},5} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 - 2\mathcal{O}^p_4 - \mathcal{O}^p_5$ | $\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$ |
|  | $\mathcal{O}_{\{16\|126\|34\},1} = 6\mathcal{O}^p_1$ | $\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{16\|126\|34\},2} = -3\mathcal{O}^p_2 + 9\mathcal{O}^p_4$ | $\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{16\|126\|34\},3} = -3\mathcal{O}^p_2 - 3\mathcal{O}^p_4$ | $\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{16\|126\|34\},4} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5$ | $\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{16\|126\|34\},5} = \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5$ | $\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$ |
|  | $\mathcal{O}_{\{23\|235\|46\},1} = \mathcal{O}^p_1 - 2\mathcal{O}^p_3 + 6\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},4,\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{23\|235\|46\},2} = \mathcal{O}^p_1 - 6\mathcal{O}^p_2 - 5\mathcal{O}^p_3 - 3\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{23\|235\|46\},3} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_5$ | $\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{23\|235\|46\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 - 6\mathcal{O}^p_4 - 3\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$ |
| | $\mathcal{O}_{\{23\|235\|46\},5} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 + 2\mathcal{O}^p_4 + \mathcal{O}^p_5$ | $\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$ |
|  | $\mathcal{O}_{\{13\|136\|45\},1} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 2\mathcal{O}^p_3 - 6\mathcal{O}^p_4 - 6\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},4,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{13\|136\|45\},2} = 2\mathcal{O}^p_1 - 5\mathcal{O}^p_2 - 5\mathcal{O}^p_3 - 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{13\|136\|45\},3} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$ | $\{\frac{1}{2},1,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{13\|136\|45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5$ | $\{\frac{1}{2},3,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$ |
| | $\mathcal{O}_{\{13\|136\|45\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5$ | $\{\frac{1}{2},1,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$ |
|  | $\mathcal{O}_{\{16\|156\|34\},1} = 2\mathcal{O}^p_1 - 4\mathcal{O}^p_2 + 12\mathcal{O}^p_4$ | $\{\frac{1}{2},3,-1\},\{\frac{1}{2},4,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{16\|156\|34\},2} = 4\mathcal{O}^p_1 + \mathcal{O}^p_2 - 3\mathcal{O}^p_4$ | $\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{16\|156\|34\},3} = -3\mathcal{O}^p_2 - 3\mathcal{O}^p_4$ | $\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$ |
| | $\mathcal{O}_{\{16\|156\|34\},4} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5$ | $\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$ |
| | $\mathcal{O}_{\{16\|156\|34\},5} = \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5$ | $\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$ |
|  | $\mathcal{O}_{\{34\|134\|56\},1} = -\mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 6\mathcal{O}^p_4$ | $\{0,3,1\},\{\frac{1}{2},4,\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{34\|134\|56\},2} = 4\mathcal{O}^p_1 - 8\mathcal{O}^p_2 + 12\mathcal{O}^p_4$ | $\{0,3,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{34\|134\|56\},3} = 2\mathcal{O}^p_2 + 4\mathcal{O}^p_3$ | $\{0,1,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$ |
| | $\mathcal{O}_{\{34\|134\|56\},4} = -4\mathcal{O}^p_1 - 4\mathcal{O}^p_2$ | $\{0,3,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$ |
| | $\mathcal{O}_{\{34\|134\|56\},5} = 2\mathcal{O}^p_4 + 4\mathcal{O}^p_5$ | $\{0,1,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$ |
|  | $\mathcal{O}_{\{16\|126\},1} = 6\mathcal{O}^p_1$ | $\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\}$ |
| | $\mathcal{O}_{\{16\|126\},2,3} = \begin{pmatrix} \mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5 \\ -2\mathcal{O}^p_2 - \mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 3\mathcal{O}^p_5 \end{pmatrix}$ | $\{\frac{1}{2},3,-1\},\{0,2,-\frac{3}{2}\}$ |
| | $\mathcal{O}_{\{16\|126\},4,5} = \begin{pmatrix} -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 - \mathcal{O}^p_4 - 2\mathcal{O}^p_5 \\ -2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 2\mathcal{O}^p_4 - \mathcal{O}^p_5 \end{pmatrix}$ | $\{\frac{1}{2},1,-1\},\{0,2,-\frac{3}{2}\}$ |

# Outline

## Summary and Outlook

- Effective field theory is an essential constituent for new physics searches.
- The era of precision measurements (HL-LHC, future colliders) calls for the systematic study of higher dimensional effective operators.
- On-shell method provides efficient algorithms to tackle with effective operators.
  1. Independent operator bases, operator reduction, EFT in various scenarios. . .
  2. New structures from generalized partial waves – J-basis operators.
- Applied to EFT calculations: matching, running. . .
  1. Theoretically interesting, but challenging (on-shell constructibility)
  2. Systematic and efficient, but large scale (44807 operators at dim-8)!

**Thank you for your attention!**