

Probing heavy meson lightcone distribution amplitudes with heavy quark spin symmetry

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饮水思源 · 爱国荣校



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Summary and outlook

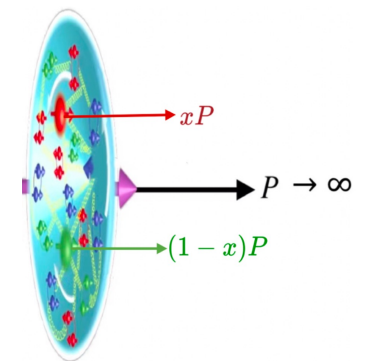
Heavy Meson LCDAs

- The heavy meson LCDA describes the distribution of momentum of the quark-antiquark pair.
- Crucial for calculating decay processes of mesons, providing insights into their dynamics.
- An important tool for understanding non-perturbative aspects of QCD, especially in the strong coupling regime of heavy quarks.
- It forms the basis for developing HQET, aiding in the study of heavy quark properties.



I	II	III
$\approx 2.2 \text{ MeV}/c^2$ 2/3 1/2 u 上	$\approx 1.28 \text{ GeV}/c^2$ 2/3 1/2 c 粲	$\approx 173.1 \text{ GeV}/c^2$ 2/3 1/2 t 顶
$\approx 4.7 \text{ MeV}/c^2$ -1/3 1/2 d 下	$\approx 96 \text{ MeV}/c^2$ -1/3 1/2 s 奇	$\approx 4.18 \text{ GeV}/c^2$ -1/3 1/2 b 底
轻夸克		重夸克

Fermilab@1977





Why is LCDA important?



Understand the strong interactions of heavy quark decay

$B \rightarrow \pi \pi$ Phys. Rev. Lett. 83, 1914 (1999)

$B \rightarrow \pi K$ Nucl. Phys. B 606, 245 (2001)

$B \rightarrow \pi D$ Phys. Rev. D 69, 112002 (2004)

Accurate measurement of SM parameters : $V_{ub} V_{cs}$

$B \rightarrow \pi \ell \nu$ PLB, 633(2006)61

$D \rightarrow K \ell \nu$ ZPC, 29 (1985) 637, 1862 citations

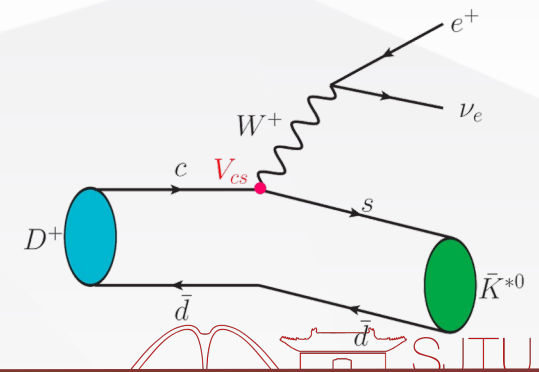
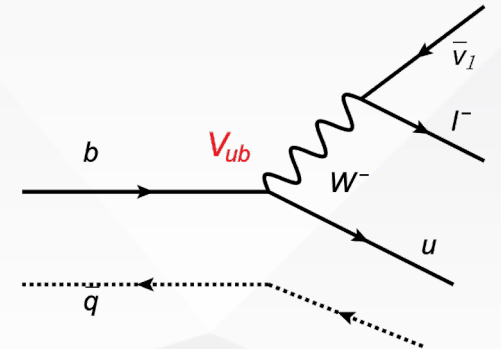
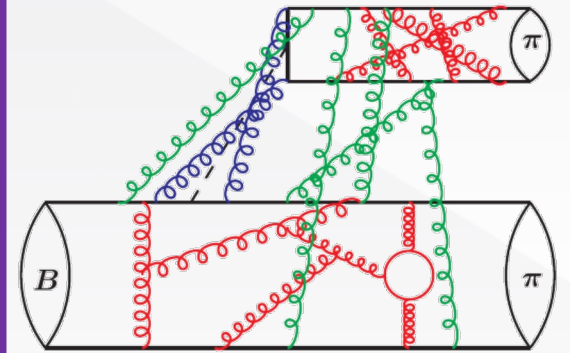
Precise measurement of CP violation parameters : A_{CP}

$$A_{CP}(B^+ \rightarrow \pi^+ \pi^0)$$

$$A_{CP}(B^+ \rightarrow D^0 \ell^+ \nu_\ell)$$

$$A_{CP}(B^+ \rightarrow K^+ \pi^0)$$

$$A_{CP}(B^+ \rightarrow \bar{D}^0 \pi^+)$$



Why is LCDA important?

Test of lepton universality with $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ decays

LHCb Collaboration · R. Aaij (CERN) et al. (May 16, 2017)

Published in: *JHEP* 08 (2017) 055 · e-Print: [1705.05802](#) [hep-ex]

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[reference search](#) [↻](#) 1,344 citations

Angular analysis of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay using 3 fb^{-1} of integrated luminosity

LHCb Collaboration · Roel Aaij (CERN) et al. (Dec 14, 2015)

Published in: *JHEP* 02 (2016) 104 · e-Print: [1512.04442](#) [hep-ex]

[pdf](#) [links](#) [DOI](#) [cite](#) [datasets](#) [claim](#)

[reference search](#) [↻](#) 985 citations

Measurement of the Differential Branching Fraction and Forward-Backward Asymmetry for $B \rightarrow K^{(*)} \ell^+ \ell^-$

Belle Collaboration · J.-T. Wei (Taiwan, Natl. Taiwan U.) et al. (Apr, 2009)

Published in: *Phys.Rev.Lett.* 103 (2009) 171801 · e-Print: [0904.0770](#) [hep-ex]

[pdf](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [↻](#) 630 citations

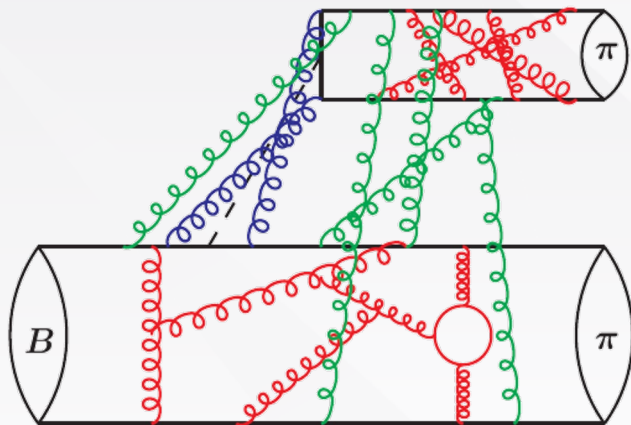
Lepton Flavor Universality

Angular Analysis and P'_5

Forward-backward Asymmetry



Without reliable (precise) knowledge on LCDAs, it is hard to probe NP



$$\langle \pi(p') \pi(q) | Q_i | \bar{B}(p) \rangle = f^{B \rightarrow \pi}(q^2) \int_0^1 dx T_i^I(x) \phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{II}(\xi, x, y) \phi_B(\xi) \phi_\pi(x) \phi_\pi(y)$$

$B \rightarrow \pi$ form factor

Hard kernel

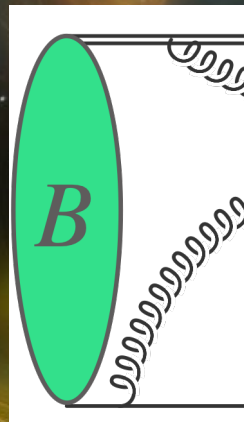
B-meson LCDA

QCD Factorization: BBNS, PRL 83, 1914 (1999)

For PQCD, See: Keum, Li, Sanda PRD 63, 054008 (2001)



Heavy quark spin symmetry





The heavy meson QCD LCDA

$$\mathcal{O}_C^P(u) = \int \frac{dt}{2\pi} e^{-iutn_+ \cdot p} \bar{\xi}_C^{(Q)}(0) \not{n}_+ \gamma_5 [0, tn_+] \xi_C(tn_+) = \int \frac{dt}{2\pi} e^{-iutn_+ \cdot p} \bar{Q}(0) \not{n}_+ \gamma_5 [0, tn_+] q(tn_+),$$

$$\mathcal{O}_C^{\parallel}(u) = \int \frac{dt}{2\pi} e^{-iutn_+ \cdot p} \bar{\xi}_C^{(Q)}(0) \not{n}_+ [0, tn_+] \xi_C(tn_+) = \int \frac{dt}{2\pi} e^{-iutn_+ \cdot p} \bar{Q}(0) \not{n}_+ [0, tn_+] q(tn_+),$$

$$\mathcal{O}_C^{\perp\mu}(u) = \int \frac{dt}{2\pi} e^{-iutn_+ \cdot p} \bar{\xi}_C^{(Q)}(0) \not{n}_+ \gamma_{\perp}^{\mu} [0, tn_+] \xi_C(tn_+) = \int \frac{dt}{2\pi} e^{-iutn_+ \cdot p} \bar{Q}(0) \not{n}_+ \gamma_{\perp}^{\mu} [0, tn_+] q(tn_+),$$

$$\langle H(p_H) | \mathcal{O}_C^P(u) | 0 \rangle = -if_P \phi_P(u),$$

$$\langle H^*(p_H, \eta) | \mathcal{O}_C^{\parallel}(u) | 0 \rangle = f_{\parallel} \frac{m_H}{n_+ \cdot p_H} n_+ \cdot \eta^* \phi_{\parallel}(u),$$

$$\langle H^*(p_H, \eta) | \mathcal{O}_C^{\perp\mu}(u) | 0 \rangle = f_{\perp}(\mu) \eta_{\perp}^{*\mu} \phi_{\perp}(u).$$

$$q(x) = \xi_C(x) + \eta_C(x),$$

$$Q(x) = \xi_C^{(Q)}(x) + \eta_C^{(Q)}(x),$$

$$\xi_C(x) = \frac{\not{n}_- \not{n}_+}{4} q(x),$$

$$\xi_C^{(Q)}(x) = \frac{\not{n}_- \not{n}_+}{4} Q(x),$$

$$\eta_C(x) = \frac{\not{n}_+ \not{n}_-}{4} q(x),$$

$$\eta_C^{(Q)}(x) = \frac{\not{n}_+ \not{n}_-}{4} Q(x).$$





In heavy-quark limit:

$$\mathcal{L}_{\text{HQET}} = \bar{h}_v(x) i v \cdot D h_v(x)$$

The leading-twist heavy-meson LCDA in HQET [Grozin and Neubert, 96]

$$\langle H(p_H) | O_v^P(tn_+) | 0 \rangle = -i \tilde{f}_H m_H n_+ \cdot v \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega; \mu),$$

$$\langle H^*(p_H, \eta) | O_v^{\parallel}(tn_+) | 0 \rangle = \tilde{f}_H m_H n_+ \cdot \eta^* \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega; \mu),$$

$$\langle H^*(p_H, \eta) | O_v^{\perp\mu}(tn_+) | 0 \rangle = \tilde{f}_H m_H n_+ \cdot v \eta_\perp^{*\mu} \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega; \mu),$$

$$O_v^P(tn_+) = \bar{h}_v(0) \not{n}_+ \gamma_5 [0, tn_+] q_s(tn_+),$$

$$O_v^{\parallel}(tn_+) = \bar{h}_v(0) \not{n}_+ [0, tn_+] q_s(tn_+),$$

$$O_v^{\perp\mu}(tn_+) = \bar{h}_v(0) \not{n}_+ \gamma_\perp^\mu [0, tn_+] q_s(tn_+),$$

$$h_v(x) = e^{im_Q v \cdot x} \frac{1 + \not{v}}{2} Q(x)$$





HQET LCDAs is boost invariant and can also be defined in boosted HQET (bHQET)

$$\langle H(p_H) | \mathcal{O}_b^P(\omega) | 0 \rangle = -i \tilde{f}_H \varphi_+(\omega; \mu),$$

$$\langle H^*(p_H, \eta) | \mathcal{O}_b^{\parallel}(\omega) | 0 \rangle = \tilde{f}_H \frac{n_+ \cdot \eta^*}{n_+ \cdot v} \varphi_+(\omega; \mu)$$

$$\langle H^*(p_H, \eta) | \mathcal{O}_b^{\perp\mu}(\omega) | 0 \rangle = \tilde{f}_H \eta_{\perp}^{*\mu} \varphi_+(\omega; \mu).$$

$$h_n(x) \equiv \sqrt{\frac{2}{n_+ \cdot v}} e^{im_Q v \cdot x} \frac{\not{n}_- \not{n}_+}{4} Q(x)$$

The soft-collinear field describes the light anti-quark in the heavy meson in the boosted frame.

$$\mathcal{O}_b^P(\omega) = \frac{1}{m_H} \int \frac{dt}{2\pi} e^{-it\omega n_+ \cdot v} \sqrt{\frac{n_+ \cdot v}{2}} \bar{h}_n(0) \not{n}_+ \gamma_5 [0, tn_+] \xi_{sc}(tn_+),$$

$$\mathcal{O}_b^{\parallel}(\omega) = \frac{1}{m_H} \int \frac{dt}{2\pi} e^{-it\omega n_+ \cdot v} \sqrt{\frac{n_+ \cdot v}{2}} \bar{h}_n(0) \not{n}_+ [0, tn_+] \xi_{sc}(tn_+),$$

$$\mathcal{O}_b^{\perp\mu}(\omega) = \frac{1}{m_H} \int \frac{dt}{2\pi} e^{-it\omega n_+ \cdot v} \sqrt{\frac{n_+ \cdot v}{2}} \bar{h}_n(0) \not{n}_+ \gamma_{\perp}^{\mu} [0, tn_+] \xi_{sc}(tn_+),$$

$$\xi_{sc} = \frac{\not{n}_- \not{n}_+}{4} q_{sc}(x)$$

The heavy meson QCD LCDA

$$\mathcal{O}_{\text{QCD}}^i(u) = \int \frac{dt}{2\pi} e^{-iutn_+ \cdot p_H} \bar{Q}(0) \not{n}_+ \Gamma_i [0, tn_+] q(tn_+)$$

u : The light quark momentum fraction

The heavy meson HQET LCDA

$$\Gamma_i = \gamma^5, 1, \gamma_\perp^\mu$$

$$\mathcal{O}_v^i(\omega) = \int \frac{dt}{2\pi} e^{-it\omega n_+ \cdot v} \bar{h}_v(0) \not{n}_+ \Gamma_i [0, tn_+] q_s(tn_+)$$

$$\langle Q(p_Q) \bar{q}(p_q) | \mathcal{O}_{\text{QCD}}^i(u) | 0 \rangle = \frac{1}{n_+ p_H} \int_0^\infty d\omega \mathcal{J}^i(u, \omega) \langle Q(p_Q) \bar{q}(p_q) | \mathcal{O}_v^i(\omega) | 0 \rangle$$

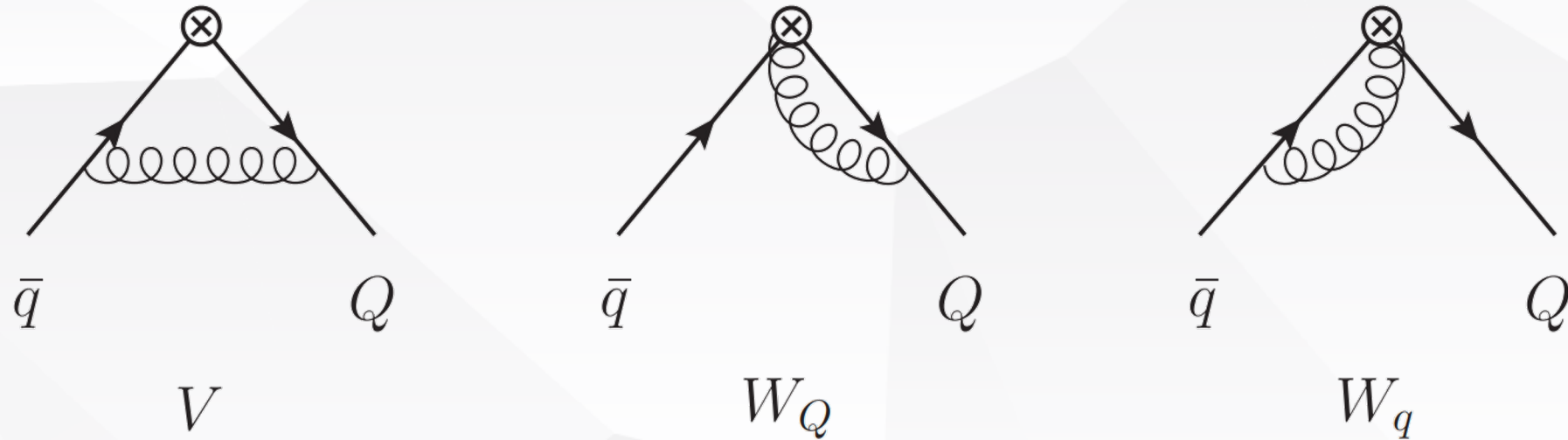


FIG. 1. The one-loop diagrams relevant for calculating the jet function.

$$\phi_i(u) = \frac{\tilde{f}_H}{f_i} \mathcal{J}^i(u, \omega) \otimes \varphi_+(\omega), \quad u \sim \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_Q}\right) \text{ peak region}$$

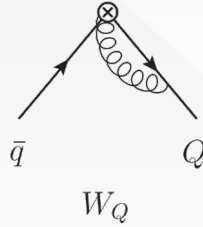


Heavy quark spin symmetry



$$W_{Q,i}^{\text{QCD}}(u, s) = \int \frac{D^d q}{(2\pi)^d} \frac{-ig_{\mu\nu} \delta^{ab} \delta_{ik} \bar{u}(p_Q) (igT_{ij}^a \gamma^\mu) \frac{i(p_Q - q + m_Q)}{(p_Q - q)^2 - m_Q^2 + i\epsilon} (ign_+^\nu T_{jk}^b) \frac{i}{n_+ \cdot q + i\epsilon} \Gamma_i v(p_q)}{(\delta(u - s - \frac{n_+ \cdot q}{n_+ \cdot p_H}) - \delta(u - s)) \bar{u}(p_Q) \Gamma_i v(p_q)}$$

$$= -ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{2n_+ \cdot (p_Q - q) (\delta(u - s - \frac{n_+ \cdot q}{n_+ \cdot p_H}) - \delta(u - s))}{[(p_Q - q)^2 - m_Q^2 + i\epsilon] [n_+ \cdot q + i\epsilon] [q^2 + i\epsilon]}$$

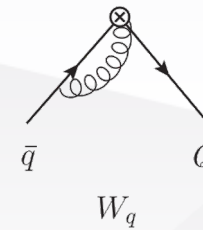


$$W_{Q,i}^{\text{QCD},h}(u, s) = ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{2n_+ \cdot (p_Q - q) \delta(s - u)}{[(p_Q - q)^2 - m_Q^2 + i\epsilon] [n_+ \cdot q + i\epsilon] [q^2 + i\epsilon]}$$

$$W_{Q,i}^{\text{QCD},s}(u, s) = -ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{2n_+ \cdot p_Q (\delta(u - s - \frac{n_+ \cdot q}{n_+ \cdot p_H}) - \delta(u - s))}{[-2p_Q \cdot q + i\epsilon] [n_+ \cdot q + i\epsilon] [q^2 + i\epsilon]} + \mathcal{O}(\lambda)$$

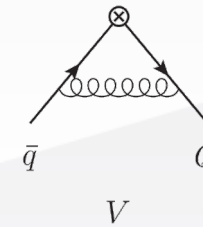
$$W_{q,i}^{\text{QCD}}(u, s) = \int \frac{D^d q}{(2\pi)^d} \frac{-ig_{\mu\nu} \delta^{ab} \delta_{ik} \bar{u}(p_Q) \frac{i}{n_+ \cdot q + i\epsilon} (-igT_{ij}^a n_+^\mu) \Gamma_i \frac{i(p_q - q)}{(p_q - q)^2 + i\epsilon} (ig\gamma^\nu T_{jk}^b) v(p_q)}{(\delta(s - u - \frac{n_+ \cdot q}{n_+ \cdot p_H}) - \delta(s - u)) \bar{u}(p_Q) \Gamma_i v(p_q)}$$

$$= ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{2n_+ \cdot (p_q - q) (\delta(s - u - \frac{n_+ \cdot q}{n_+ \cdot p_H}) - \delta(s - u))}{[(p_q - q)^2 + i\epsilon] [n_+ \cdot q + i\epsilon] [q^2 + i\epsilon]}$$



$$W_{q,i}^{\text{QCD},h}(u, s) = -\delta(u - s) ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{2n_+ \cdot (-q)}{[q^2 + i\epsilon] [n_+ \cdot q + i\epsilon] [q^2 + i\epsilon]} = 0$$

$$W_{q,i}^{\text{QCD},s}(u, s) = ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{2n_+ \cdot (p_q - q) (\delta(s - u - \frac{n_+ \cdot q}{n_+ \cdot p_H}) - \delta(s - u))}{[(p_q - q)^2 + i\epsilon] [n_+ \cdot q + i\epsilon] [q^2 + i\epsilon]} + \mathcal{O}(\lambda)$$



$$V_i^{\text{QCD}}(u, s) = \begin{cases} ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{\delta(u - s - \frac{n_+ \cdot q}{n_+ \cdot p_H}) [(d-2)q_\perp^2 + 2q^+ p_Q^- + 2p_q^+ p_Q^-]}{[(p_Q - q)^2 - m_Q^2 + i\epsilon] [(p_q + q)^2 + i\epsilon] [q^2 + i\epsilon]}, & \text{for } \Gamma_i = \not{n}_+ \gamma_5, \not{n}_+ \\ ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{\delta(u - s - \frac{n_+ \cdot q}{n_+ \cdot p_H}) [(d-4)q_\perp^2 + 2q^+ p_Q^- + 2p_q^+ p_Q^-]}{[(p_Q - q)^2 - m_Q^2 + i\epsilon] [(p_q + q)^2 + i\epsilon] [q^2 + i\epsilon]}, & \text{for } \Gamma_i = \not{n}_+ \gamma_\perp. \end{cases}$$

$$V_i^{\text{QCD},h}(u, s) = 0$$

$$V_i^{\text{QCD},s}(u, s) = ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{\delta(u - s - \frac{n_+ \cdot q}{n_+ \cdot p_H}) [2q^+ p_Q^- + 2p_q^+ p_Q^-]}{[-2p_Q \cdot q + i\epsilon] [(p_q + q)^2 + i\epsilon] [q^2 + i\epsilon]}$$





Heavy quark spin symmetry



$$\begin{aligned}
W_{Q,i}^{\text{HQET}}(\omega, \omega_0) &= \int \frac{D^d q}{(2\pi)^d} \frac{-ig_{\mu\nu} \delta^{ab} \delta_{ik} \bar{u}(p_Q) (igT_{ij}^a v^\mu)^{\frac{1+\not{p}}{2}} \frac{i\Gamma_i}{v \cdot k + i\epsilon} (ign_+^\nu T_{jk}^b)^{\frac{i}{n_+ \cdot q + i\epsilon}} v(p_q)}{q^2 + i\epsilon} \frac{\bar{u}(p_Q) \Gamma_i v(p_q)}{(\delta(\omega - \frac{n_+ \cdot (p_q + q)}{n_+ \cdot v}) - \delta(\omega - \frac{n_+ \cdot p_q}{n_+ \cdot v}))} \\
&= -ig_s^2 C_F n_+ \cdot v \int \frac{D^d q}{(2\pi)^d} \frac{(\delta(\omega - \frac{n_+ \cdot (p_q + q)}{n_+ \cdot v}) - \delta(\omega - \frac{n_+ \cdot p_q}{n_+ \cdot v}))}{[-v \cdot q + i\epsilon] [n_+ \cdot q + i\epsilon] [q^2 + i\epsilon]}.
\end{aligned}$$

$$\begin{aligned}
W_{q,i}^{\text{HQET}}(\omega, \omega_0) &= \int \frac{D^d q}{(2\pi)^d} \frac{-ig_{\mu\nu} \delta^{ab} \delta_{ik} \bar{u}(p_Q) \frac{i}{n_+ \cdot q + i\epsilon} (-igT_{ij}^a n_+^\mu) \Gamma_i \frac{i(\not{p}_q - \not{q})}{(p_q - q)^2 + i\epsilon} (ig\gamma^\nu T_{jk}^b) v(p_q)}{q^2 + i\epsilon} \frac{\bar{u}(p_Q) \Gamma_i v(p_q)}{(\delta(\omega - \frac{n_+ \cdot (p_q - q)}{n_+ \cdot v}) - \delta(\omega - \frac{n_+ \cdot p_q}{n_+ \cdot v}))} \\
&= ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{2n_+ \cdot (p_q - q) (\delta(\omega - \frac{n_+ \cdot (p_q - q)}{n_+ \cdot v}) - \delta(\omega - \frac{n_+ \cdot p_q}{n_+ \cdot v}))}{[(p_q - q)^2 + i\epsilon] [n_+ \cdot q + i\epsilon] [q^2 + i\epsilon]}.
\end{aligned}$$

$$\begin{aligned}
V_i^{\text{HQET}}(\omega, \omega_0) &= \int \frac{D^d q}{(2\pi)^d} \frac{-ig_{\mu\nu} \delta^{ab} \delta_{ik} \bar{u}(p_Q) (igT_{ij}^a v^\mu)^{\frac{1+\not{p}}{2}} \frac{i\Gamma_i}{v \cdot k + i\epsilon} \frac{i(\not{p}_q + \not{q})}{(p_q + q)^2 + i\epsilon} (ig\gamma^\nu T_{jk}^b) v(p_q) \delta(\omega - \frac{n_+ \cdot (p_q + q)}{n_+ \cdot v})}{q^2 + i\epsilon} \frac{\bar{u}(p_Q) \Gamma_i v(p_q)}{(\delta(\omega - \frac{n_+ \cdot (p_q + q)}{n_+ \cdot v}) - \delta(\omega - \frac{n_+ \cdot p_q}{n_+ \cdot v}))} \\
&= -ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{1}{q^2 + i\epsilon} \frac{1}{-v \cdot q + i\epsilon} \frac{1}{(p_q + q)^2 + i\epsilon} \frac{\bar{u}(p_Q) \Gamma_i (\not{p}_q + \not{q}) \not{v} v(p_q) \delta(\omega - \frac{n_+ \cdot (p_q + q)}{n_+ \cdot v})}{\bar{u}(p_Q) \Gamma_i v(p_q)} \\
&= ig_s^2 C_F \int \frac{D^d q}{(2\pi)^d} \frac{\delta(\omega - \frac{n_+ \cdot (p_q + q)}{n_+ \cdot v}) [2q^+ p_Q^- + 2p_q^+ p_Q^-]}{[-2p_Q \cdot q + i\epsilon] [(p_q + q)^2 + i\epsilon] [q^2 + i\epsilon]}.
\end{aligned}$$

QCD

HQET

$$i \frac{\not{p} + m_Q}{p^2 - m_Q^2 + i\epsilon} = i \frac{m_Q \not{v} + m_Q + \not{k}}{2m_Q v \cdot k + k^2 + i\epsilon} \rightarrow i \frac{1 + \not{v}}{2v \cdot k + i\epsilon}$$

$$\gamma^\mu \rightarrow \frac{1 + \not{v}}{2} \gamma^\mu \frac{1 + \not{v}}{2} = v^\mu \frac{1 + \not{v}}{2} \rightarrow v^\mu$$

The soft region contribution of QCD is diagram by diagram the same as the HQET results.

$$W_{Q,i}^{\text{QCD},s}(u, \frac{\omega}{m_H}) = m_H W_{Q,i}^{\text{HQET}}(um_H, \omega)$$

$$W_{q,i}^{\text{QCD},s}(u, \frac{\omega}{m_H}) = m_H W_{q,i}^{\text{HQET}}(um_H, \omega)$$

$$V_i^{\text{QCD},s}(u, \frac{\omega}{m_H}) = m_H V_i^{\text{HQET}}(um_H, \omega)$$





Heavy quark spin symmetry



$$\mathcal{J}^i(u, \omega) = \theta(m_H - \omega) \delta\left(u - \frac{\omega}{m_H}\right) \left(1 + \frac{\alpha_s C_F}{4\pi} \mathcal{J}_{\text{peak}}^{(1)}(m_H) + \mathcal{O}(\alpha_s^2)\right), \quad i = P, \parallel, \perp$$

$$\mathcal{J}_{\text{peak}}^{(1)}(m_H) = \frac{1}{2} \ln^2 \frac{\mu^2}{m_H^2} + \frac{1}{2} \ln \frac{\mu^2}{m_H^2} + \frac{\pi^2}{12} + 2. \quad \text{Independent of hadron spin!}$$

$$W_{Q, hc}^{\text{SCET}} = W_{Q, h}^{\text{QCD}} = 2ig_s^2 C_F \int \frac{d^D l}{(2\pi)^D} \frac{n_+ \cdot (p_Q - l) \delta(u - s)}{(l^2 - 2l \cdot p_Q + i\epsilon)(l^2 + i\epsilon)(n_+ \cdot l + i\epsilon)}$$

$$W_{q, hc}^{\text{SCET}} = W_{q, h}^{\text{QCD}} = 0$$

$$V_{hc}^{\text{SCET}} = V_h^{\text{QCD}} = 0.$$

More details, see Yan-Bing Wei's talk(16th afternoon)

$$W_{Q, i}^{\text{HQET}} = W_{Q, i}^{\text{bHQET}}, \quad W_{q, i}^{\text{HQET}} = W_{q, i}^{\text{bHQET}}, \quad \text{and} \quad V_i^{\text{HQET}} = V_i^{\text{bHQET}}$$





Quasi DAs to QCD LCDAs



$$\hat{\mathcal{O}}_i(x) = \int \frac{dz n_z \cdot P}{2\pi} e^{-ixzn_z \cdot P} \bar{Q}(0) \Gamma_i[0, zn_z] q(zn_z) \quad \Gamma_i = \gamma^{z/t} \gamma_5, \gamma^{z/t}, \gamma^{z/t} \gamma_\perp^\mu \text{ for } i = P, \parallel, \perp$$

$$\langle H(p_H) | \hat{\mathcal{O}}_P(x) | 0 \rangle = -i \hat{f}_P P^{z/t} \hat{\phi}_P(x),$$

$$\langle H^*(p_H, \eta) | \hat{\mathcal{O}}_{\parallel}(x) | 0 \rangle = \hat{f}_{\parallel} m_{H^*} \eta^{*,z/t} \hat{\phi}_{\parallel}(x),$$

$$\langle H^*(p_H, \eta) | \hat{\mathcal{O}}_{\perp}^\mu(x) | 0 \rangle = \hat{f}_{\perp} P^{z/t} \eta_{\perp}^{*\mu} \hat{\phi}_{\perp}(x).$$

Quasi DAs

Lattice QCD calculable!

$$\hat{\phi}_i(x) = \int dy \left[\left(\delta(x-y) + C_i^{(1)}(x, y, \mu) - C_{CT}^{(1)} \right) \phi_i(y, \mu) + \mathcal{O} \left(\frac{\Lambda_{QCD}^2}{(yPz)^2}, \frac{m_H^2}{(Pz)^2} \right) \right]$$

LaMET matching

J. Xu, Q. A. Zhang and S. Zhao, PRD 97 (2018) 114026

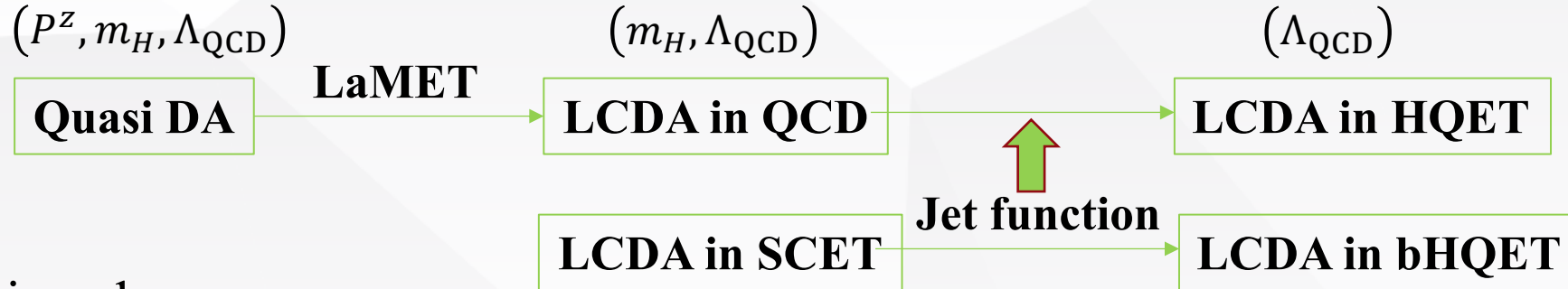
Y. S. Liu, W. Wang, J. Xu, Q. A. Zhang, S. Zhao and Y. Zhao, PRD, 99(2019) 094036

The recent work: 2403.17492 [hep-ph], 2410.18654 [hep-lat]





➤ Start from Quasi DA, calculable from LQCD

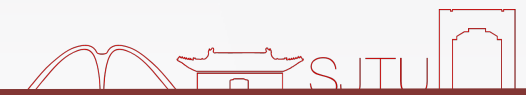
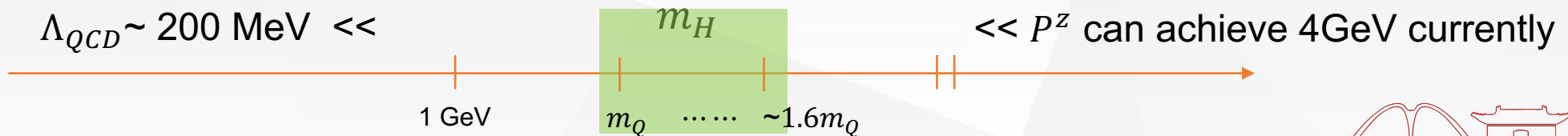


• A multi-scale processes:

1. LaMET requires $\Lambda_{\text{QCD}}, m_H \ll P^z$ and finally integrate out P^z ;

2. HQET requires $\Lambda_{\text{QCD}} \ll m_H$ and integrate out m_H ; *See Yueying Han's talk(15th afternoon)*

⇒ **Hierarchy $\Lambda_{\text{QCD}} \ll m_H \ll P^z$** : A big challenge for lattice simulation but still calculable on the lattice





- The LCDAs at for a heavy pseudoscalar and vector meson within the framework of HQET become indistinguishable.
- The leading-twist HQET LCDA can be determined through lattice simulations of quasi-DAs with a large momentum.
- One can make use of three different equal time matrix elements and determine the same HQET LCDA.

Thank you for your attention!





Back up

In the rest frame the external state momenta are

$$p_Q^\mu \sim (m_Q, \Lambda_{\text{QCD}}, m_Q), \quad p_q^\mu \sim (\Lambda_{\text{QCD}}, \Lambda_{\text{QCD}}, \Lambda_{\text{QCD}}).$$

The hard and soft region is

$$\text{hard} \sim (m_Q, m_Q, m_Q), \quad \text{soft} \sim (\Lambda_{\text{QCD}}, \Lambda_{\text{QCD}}, \Lambda_{\text{QCD}})$$

In the boost frame, the external state momenta are

$$p_Q^\mu \sim \text{hard collinear} \sim (Q, m_H, m_H^2/Q), \quad p_q^\mu \sim \text{soft collinear} \sim (Q, m_H, m_H^2/Q)\lambda.$$

$$|H^{(*)}(p_H)\rangle = \sqrt{m_H} [|H_v\rangle + \mathcal{O}(1/m_H)]$$