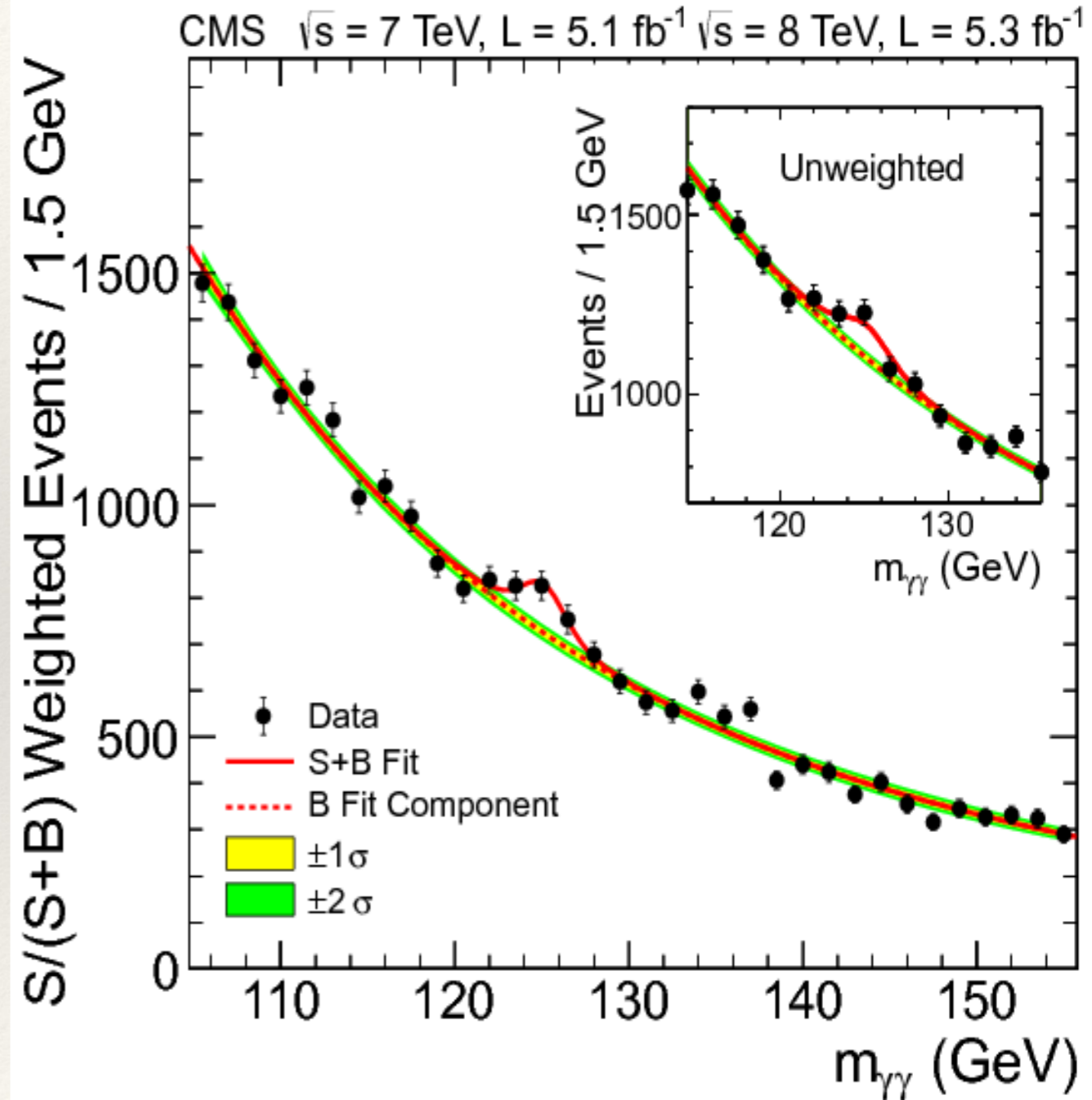
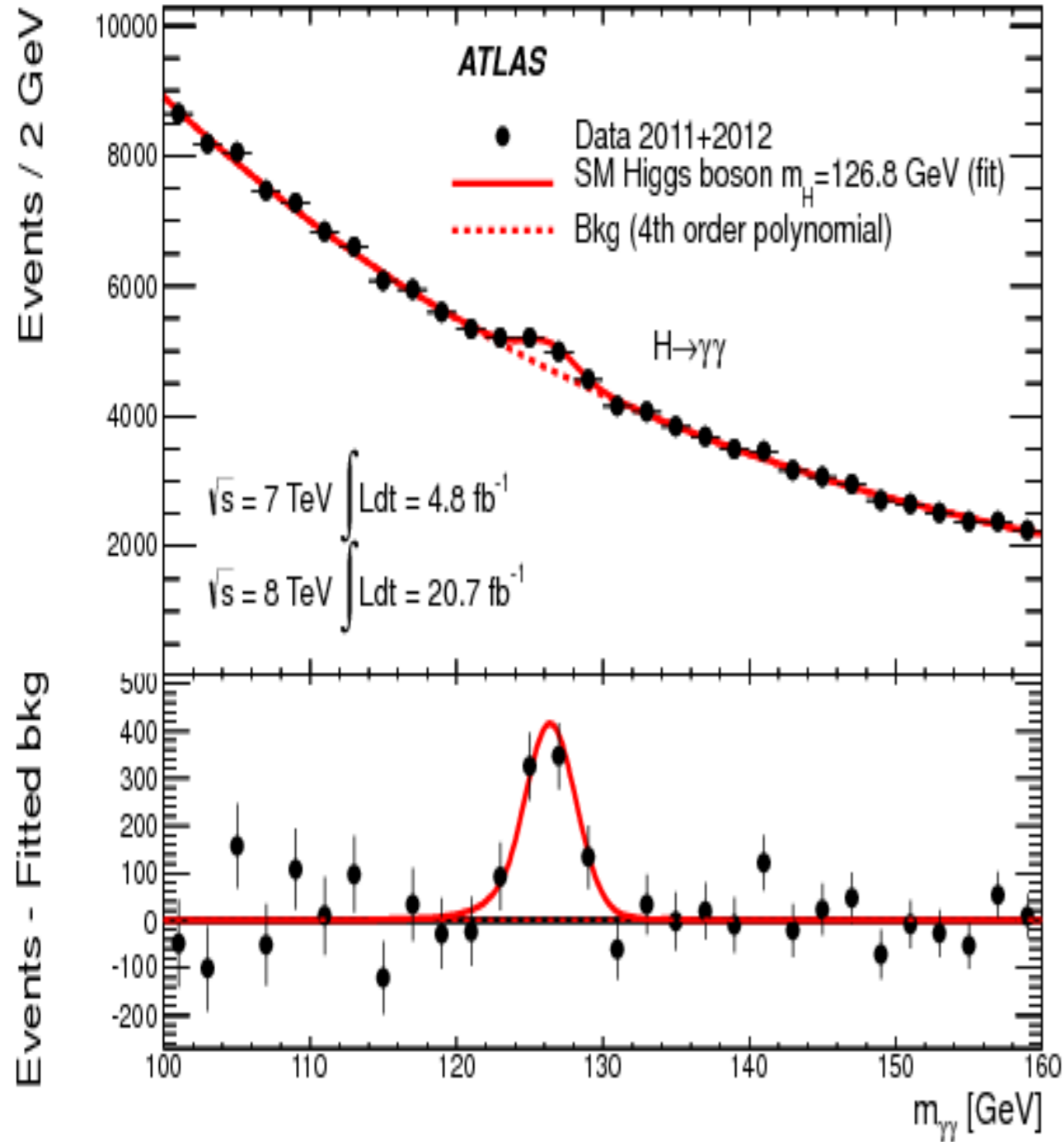


# Improved constraint on Higgs boson self-couplings with quartic and cubic power dependence in the cross section

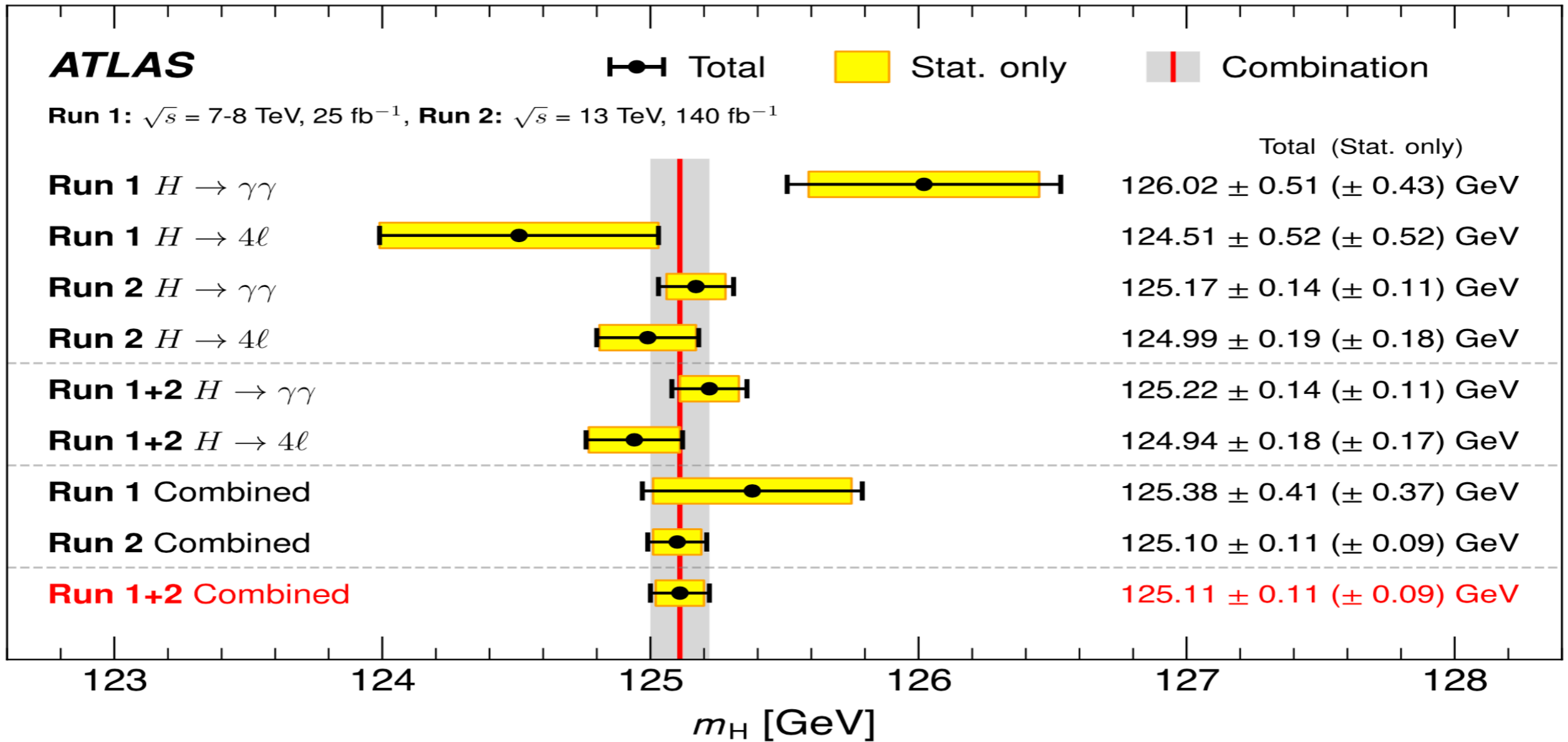
Jian Wang (王健)  
Shandong University

CLHCP, Qingdao  
Nov 14, 2024

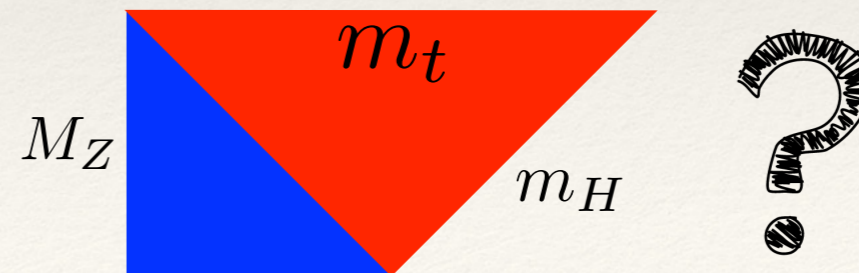
# Discovery in 2012

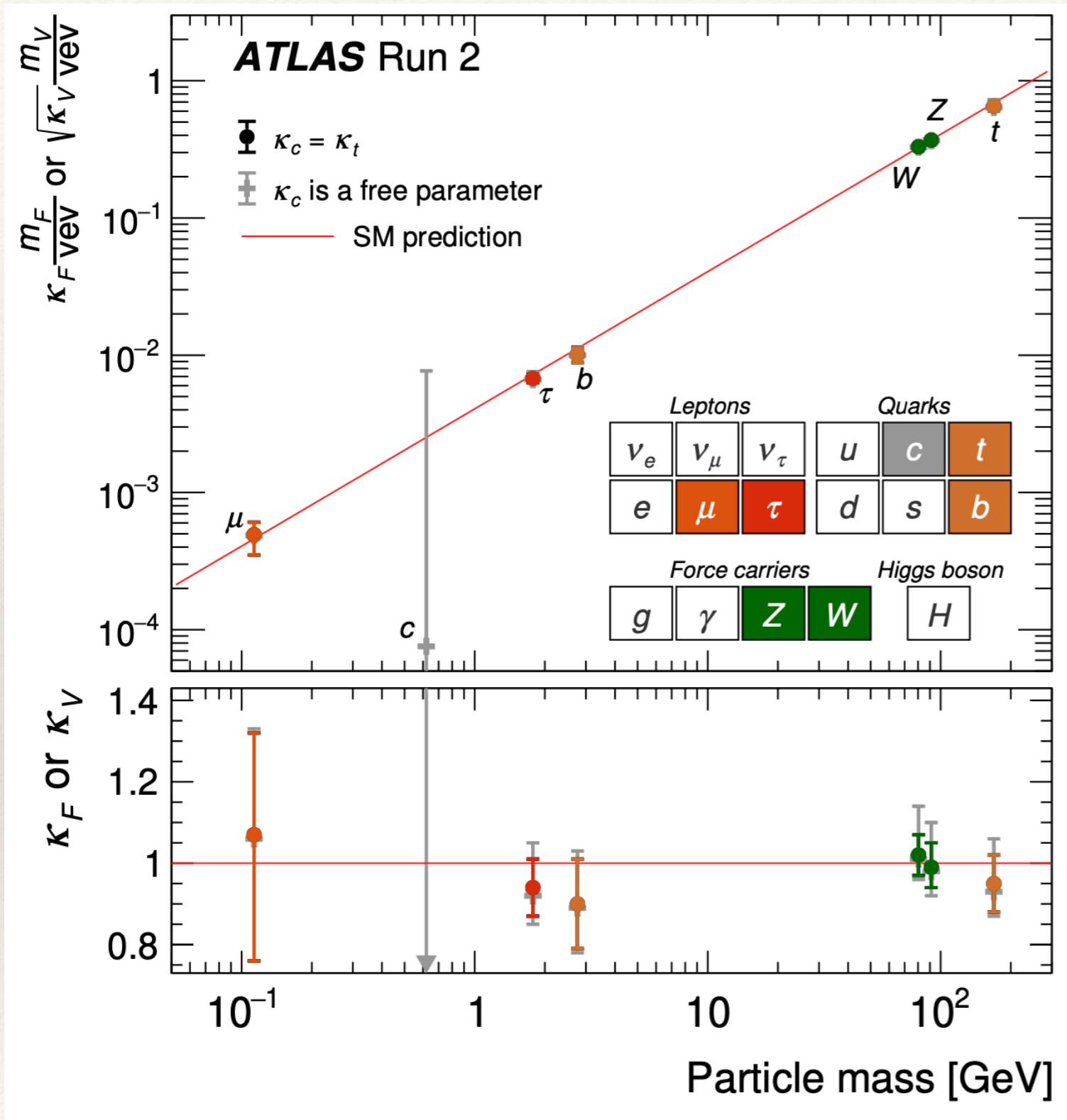


# How precise is our understanding?



$$\frac{m_t}{m_H} = \frac{m_H}{M_Z} \approx \sqrt{2} \pm 0.04$$



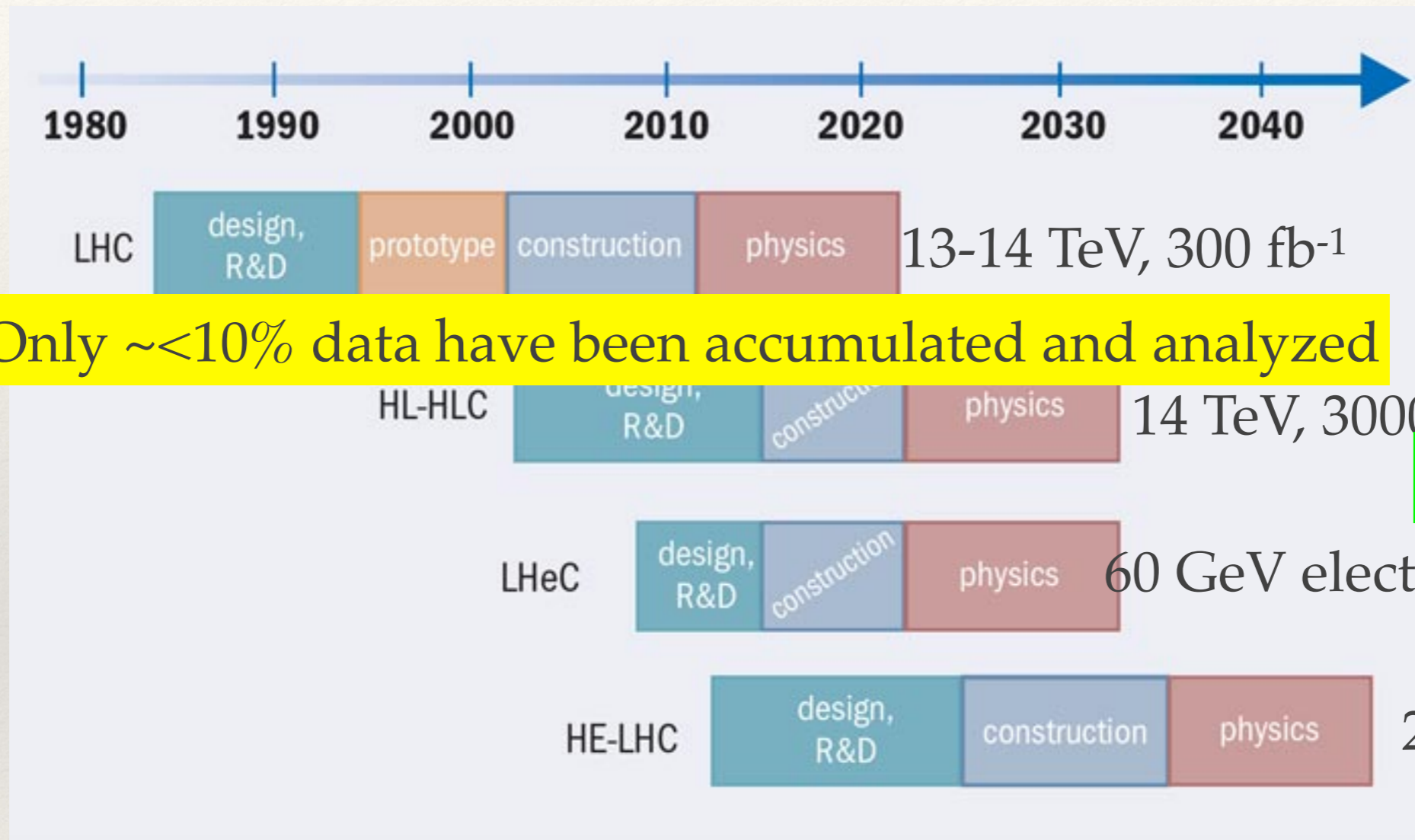


So far, the H(125) properties are consistent with the SM expectation!

Its couplings with SM particles are proportional to their masses;  
Higgs mechanism

existence of a “fifth force”  
 different from gravity

# Future plan of LHC



Only  $\sim < 10\%$  data have been accumulated and analyzed

150000000 H

High precision theoretical calculation is important for measuring SM parameters, and thus for predicting backgrounds for new physics searches. It is also needed to detect new physics that appears as a small deviation.

# Higgs self-coupling in the SM

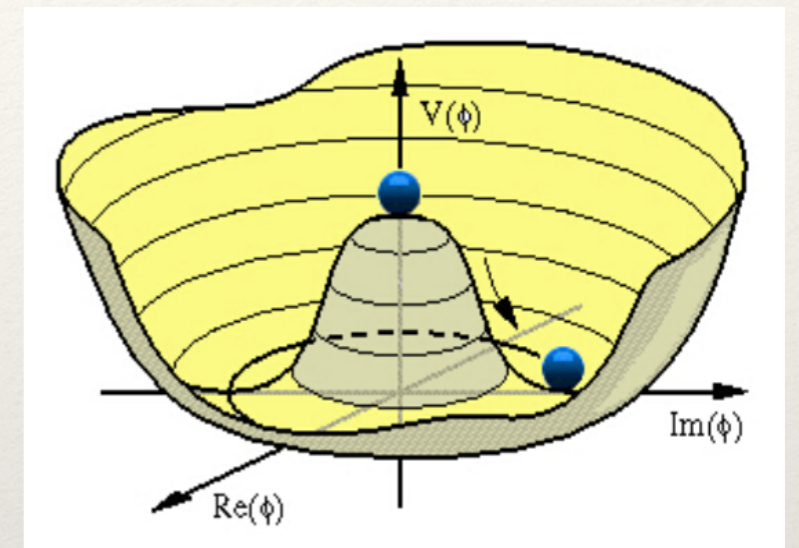
$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$



$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$V(h) = \frac{1}{2} m_h^2 h^2 + \sqrt{\frac{\lambda}{2}} m_H h^3 + \frac{1}{4} \lambda h^4$$

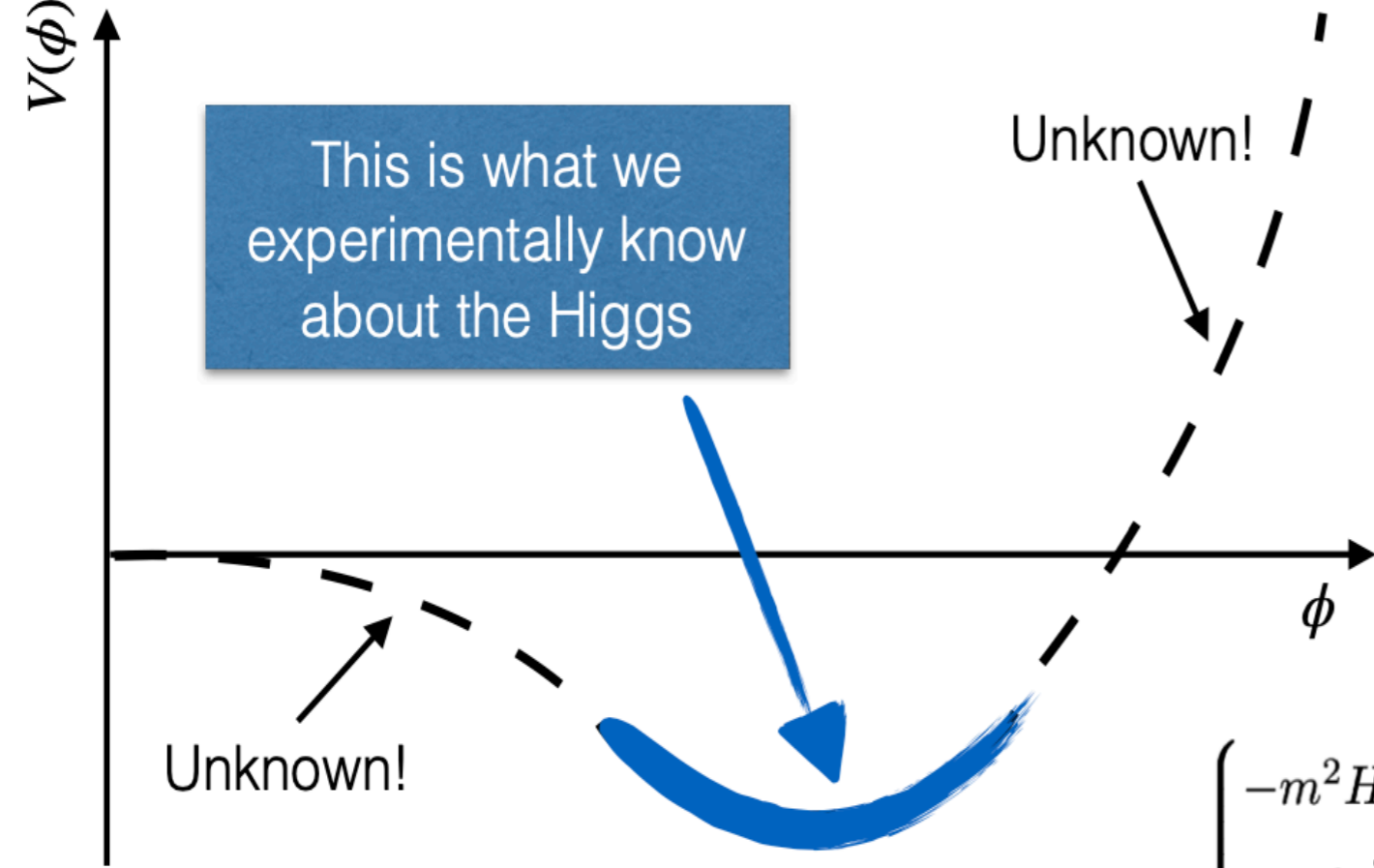
$$\lambda = m_H^2 / 2v^2$$



In some new physics models, the trilinear Higgs self-coupling may change by O(100)%, while the couplings with gauge bosons and fermions are still in agreement with SM.

S.Kanemura, et al, PLB558,157

**We need to measure the trilinear self coupling directly.**

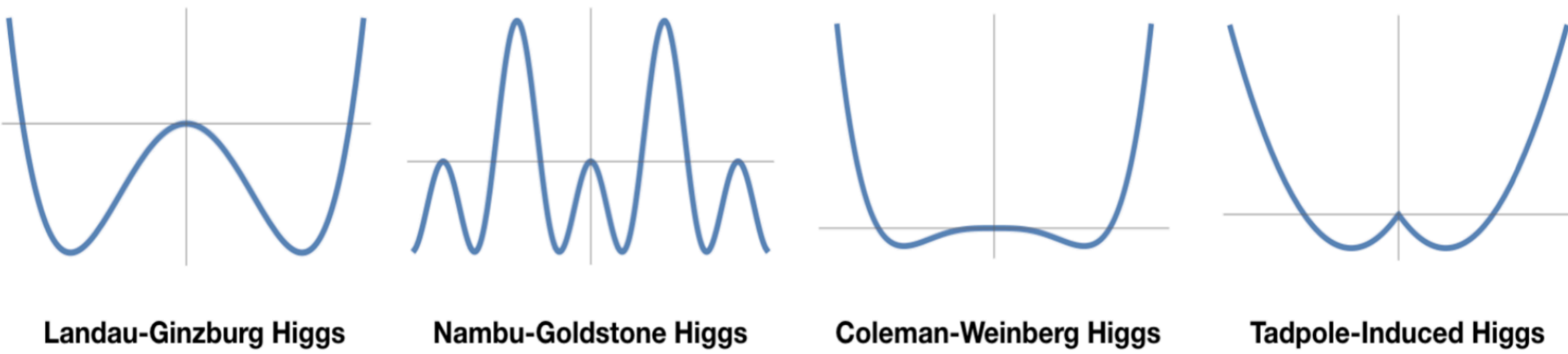


**Different potential shapes could explain the same physics we see now!**

$$V(H) \simeq \begin{cases} -m^2 H^\dagger H + \lambda (H^\dagger H)^2 + \frac{c_6 \lambda}{\Lambda^2} (H^\dagger H)^3, & \text{Elementary Higgs} \\ -a \sin^2(\sqrt{H^\dagger H}/f) + b \sin^4(\sqrt{H^\dagger H}/f), & \text{Nambu-Goldstone Higgs} \\ \lambda (H^\dagger H)^2 + \epsilon (H^\dagger H)^2 \log \frac{H^\dagger H}{\mu^2}, & \text{Coleman-Weinberg Higgs} \\ -\kappa^3 \sqrt{H^\dagger H} + m^2 H^\dagger H, & \text{Tadpole-induced Higgs} \end{cases}$$

Agrawal, Saha, L.X.Xu, J.-H. Yu, C.-P. Yuan

[arxiv:1907.02078](https://arxiv.org/abs/1907.02078)



$\kappa_\lambda = \kappa_t = 1$   
 $\kappa_V = \kappa_{2V} = 1$

—●— Observed

----- Median expected

■ 68% expected

■ 95% expected

bb ZZ  
 Expected: 40  
 Observed: 32

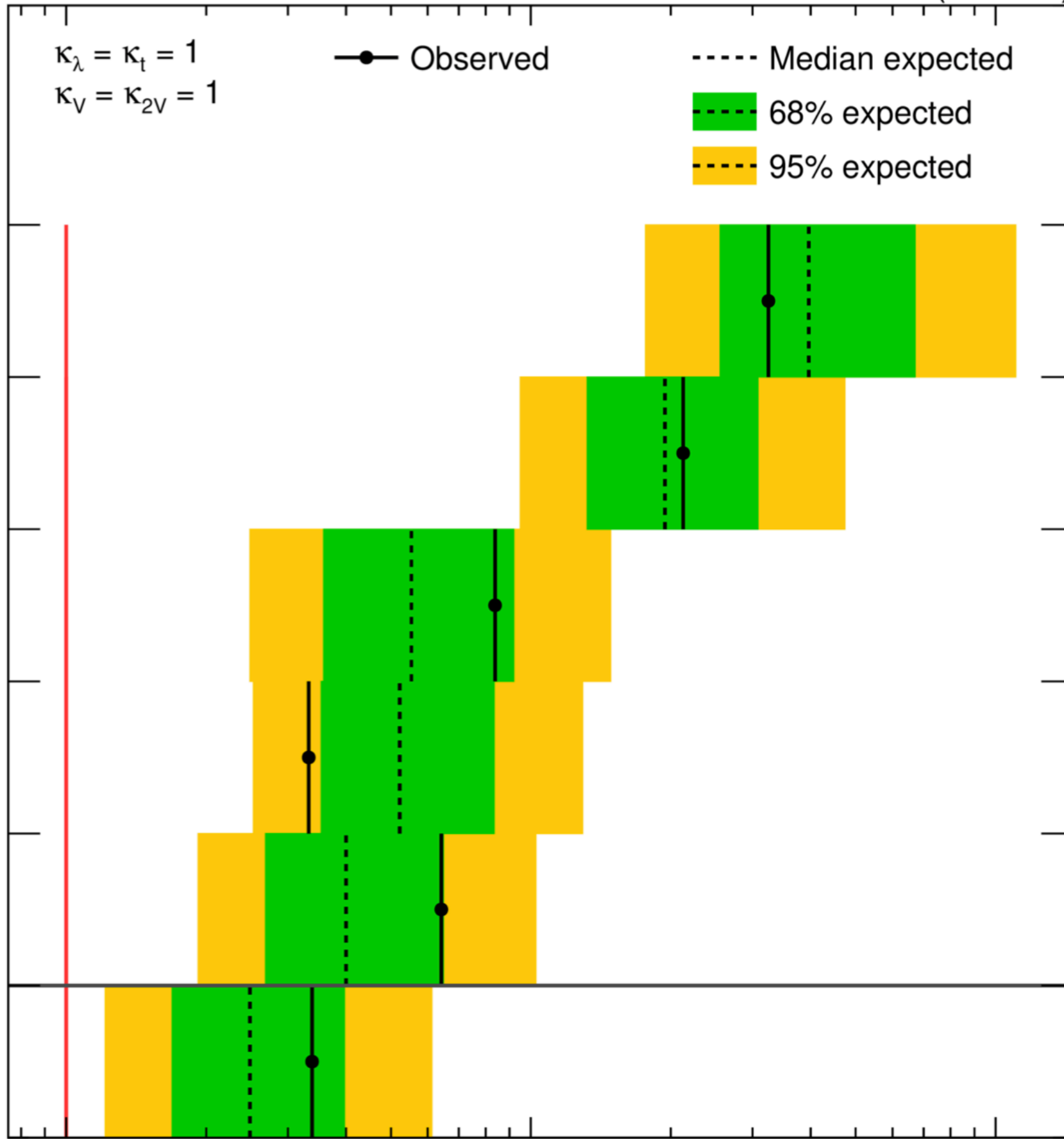
Multilepton  
 Expected: 19  
 Observed: 21

bb  $\gamma\gamma$   
 Expected: 5.5  
 Observed: 8.4

bb  $\tau\tau$   
 Expected: 5.2  
 Observed: 3.3

bb bb  
 Expected: 4.0  
 Observed: 6.4

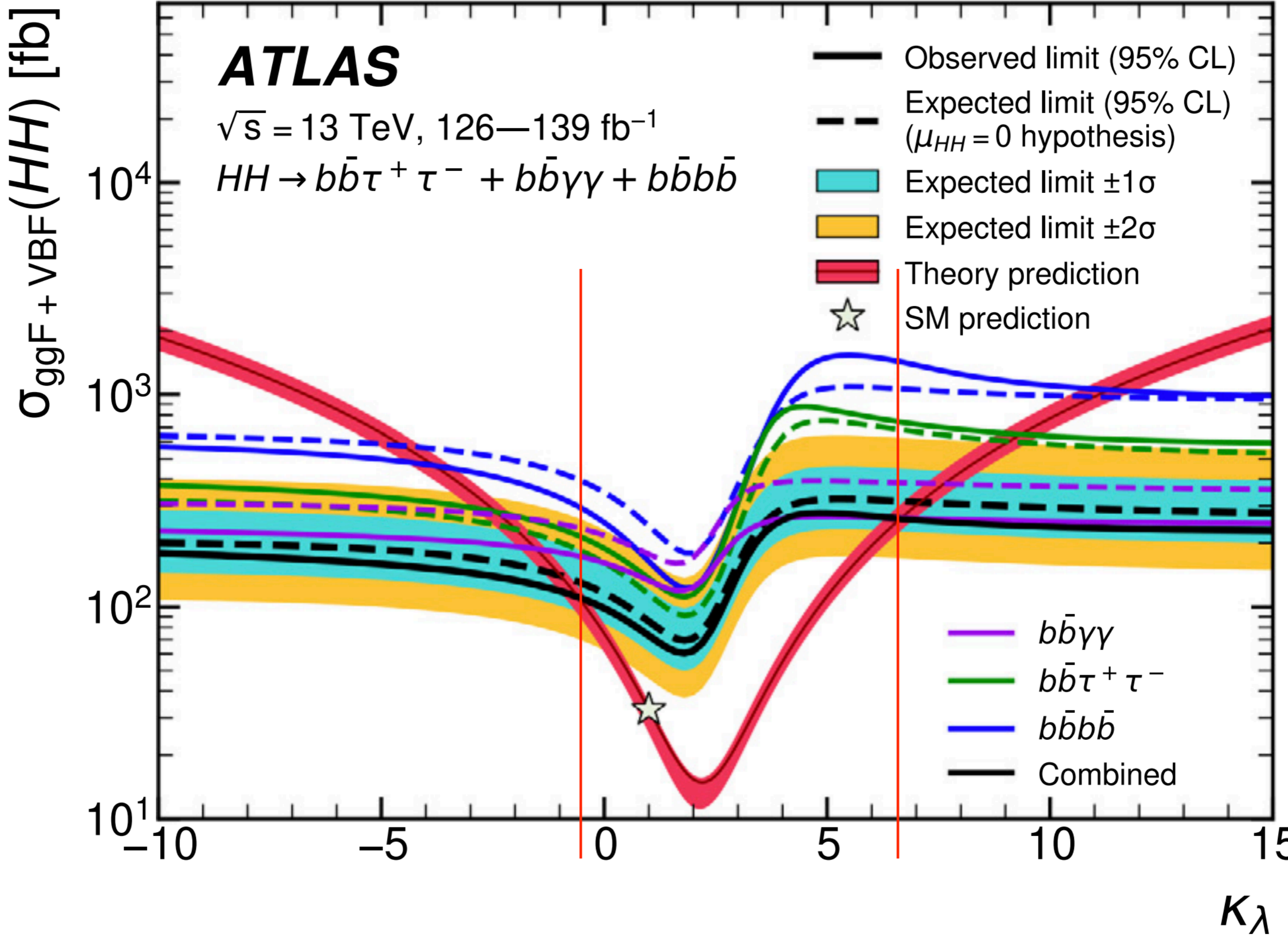
Combined  
 Expected: 2.5  
 Observed: 3.4

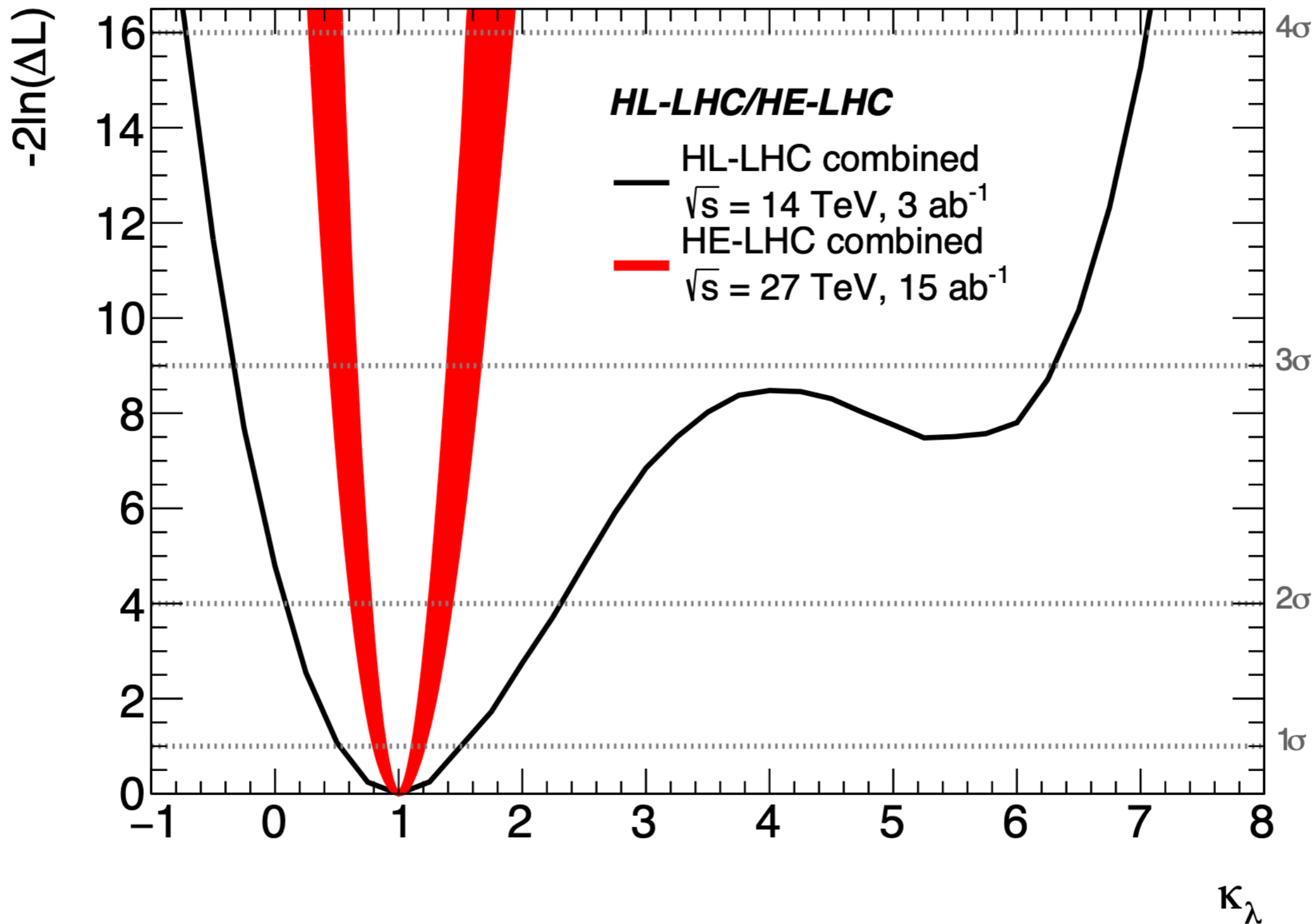


$$-1.2 < \kappa_\lambda < 6.5$$

95% CL limit on  $\sigma(pp \rightarrow HH) / \sigma_{\text{Theory}}$



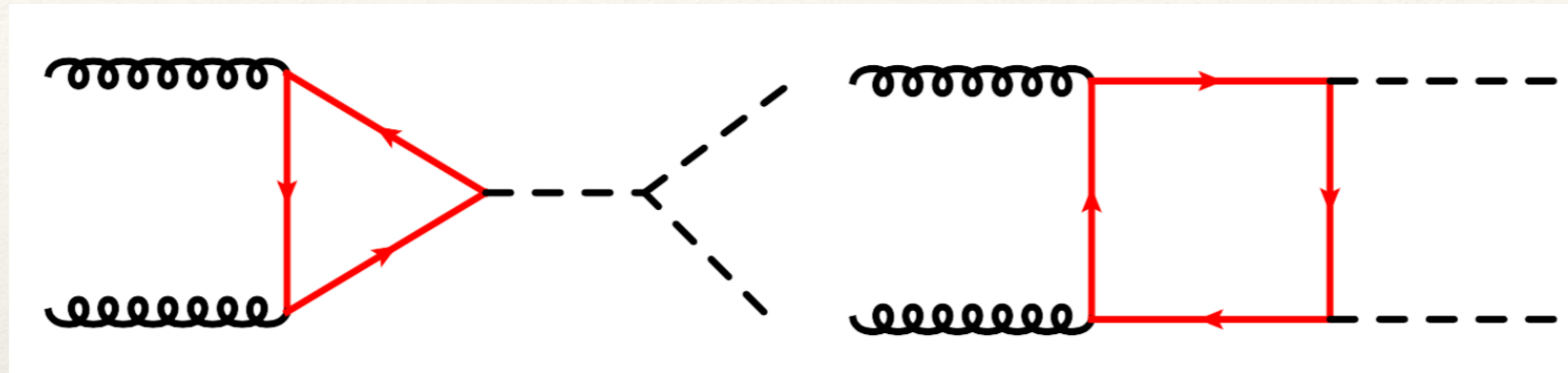




Expected accuracy:  
 ~50% at HL-LHC  
 ~10% at HE-LHC

1902.00134

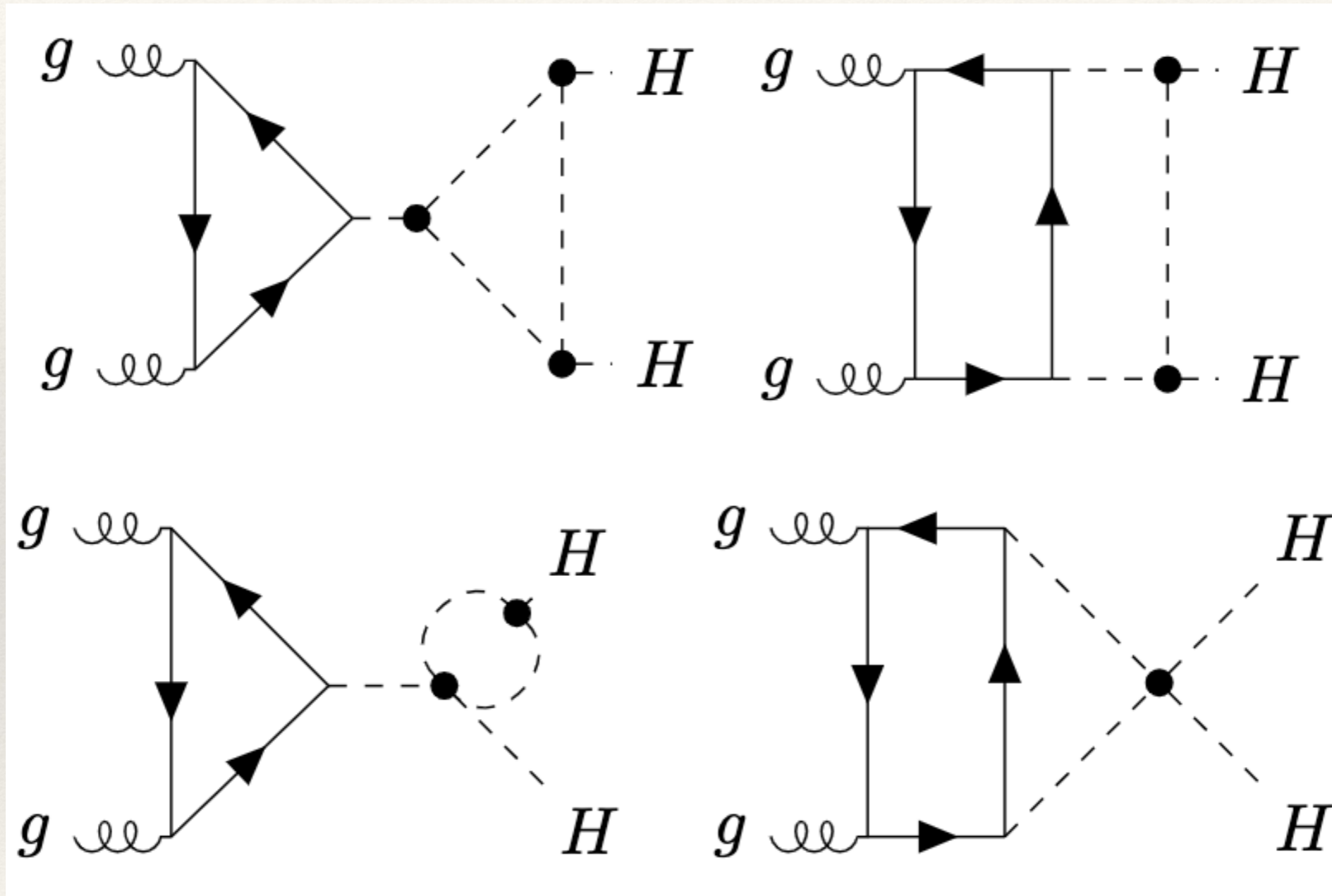
# pp $\rightarrow$ HH as a function of $\kappa$



$$\sigma_{HH} = A + B\kappa + C\kappa^2$$

computation	A [fb]	A/A(LO)	B [fb]	B/B(LO)	C [fb]	C/C(LO)
LO $m_t$ fin	35.0		-23.0		4.73	
NLO $m_t$ fin	62.6	1.79	-44.4	1.93	9.64	2.04
NLO $m_t$ fin $\times$ NNLO SM FTApprox	70.0	2.00	-49.6	2.16	10.8	2.28
NNLO + NNLL $m_t \rightarrow \infty \times$						
NNLO+NLL SM (partial $m_t$ fin)	71.3	2.04	-47.7	2.08	9.93	2.10

# A more realistic function form



$$\sigma_{HH} = A + B\kappa + C\kappa^2 + D\kappa^3 + E\kappa^4$$

---

# Non-trivial task

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Input parameters of conventional EW calculations:

$$e, m_H, m_t, m_W, m_Z$$

If one takes the Higgs self-coupling  $\lambda$  as an input, the correction would be proportional to  $\lambda$ .

Performing rescaling  $\lambda \rightarrow \kappa\lambda$  before or after substituting  $m_H^2 = 2\lambda v^2$  gives different results.

# Renormalization

The renormalized Lagrangian in the  $\kappa$  framework after EW gauge symmetry breaking:

$$\begin{aligned} \mathcal{L}_H^\kappa = & \frac{1}{2} Z_\phi (\partial_\mu H)^2 - \left( -\frac{1}{2} Z_{\mu^2} Z_\phi Z_\nu^2 \mu^2 v^2 + \frac{1}{4} Z_\lambda Z_\phi^2 Z_\nu^4 \lambda v^4 \right) - (Z_\lambda Z_\phi^2 Z_\nu^3 \lambda v^3 - Z_{\mu^2} Z_\phi Z_\nu \mu^2 v) H \\ & - \left( \frac{3}{2} Z_\lambda Z_\phi^2 Z_\nu^2 \lambda v^2 - \frac{1}{2} Z_{\mu^2} Z_\phi \mu^2 \right) H^2 - Z_{\kappa_{3H}} Z_\lambda Z_\phi^2 Z_\nu \lambda_{3H} v H^3 - \frac{1}{4} Z_{\kappa_{4H}} Z_\lambda Z_\phi^2 \lambda_{4H} H^4 + \dots \end{aligned}$$

The linear term is

$$(\mu^2 v - \lambda v^3) H + [(\delta Z_{\mu^2} + \delta Z_\phi + \delta Z_\nu) \mu^2 v - (\delta Z_\lambda + 2\delta Z_\phi + 3\delta Z_\nu) \lambda v^3] H$$

We choose the renormalization scheme in which there is no tadpole contributions.

$\mu^2 = \lambda v^2$  and  $(\delta Z_{\mu^2} - \delta Z_\lambda - \delta Z_\phi - 2\delta Z_\nu) \mu^2 v + T = 0$  with T the one-loop diagrams.

$$T = \frac{3\lambda_{3H} v}{16\pi^2} m_H^2 \left( \frac{1}{\epsilon} + \ln \frac{\mu_R^2}{m_H^2} + 1 \right)$$

# Renormalization

The quadratic term is

$$\begin{aligned} & \frac{1}{2}(\partial_\mu H)^2 - \mu^2 H^2 + \frac{1}{2}\delta Z_\phi(\partial_\mu H)^2 - \left( \frac{3}{2}\delta Z_\lambda + \frac{5}{2}\delta Z_\phi - \frac{1}{2}\delta Z_{\mu^2} + 3\delta Z_\nu \right) \mu^2 H^2 \\ & \equiv \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}m_H^2 H^2 + \frac{1}{2}\delta Z_\phi(\partial_\mu H)^2 - \frac{1}{2}(\delta Z_{m_H^2} + \delta Z_\phi)m_H^2 H^2 \end{aligned}$$

We choose the on-shell renormalization scheme.

$$\begin{aligned} \delta Z_{m_H^2} &= \frac{3\lambda_{4H}}{16\pi^2} \left( \frac{1}{\epsilon} + \ln \frac{\mu_R^2}{m_H^2} + 1 \right) + \frac{9\lambda_{3H}^2 v^2}{m_H^2} \frac{1}{8\pi^2} \left( \frac{1}{\epsilon} + \ln \frac{\mu_R^2}{m_H^2} + 2 - \frac{\pi}{\sqrt{3}} \right) \\ \delta Z_\phi &= \frac{9\lambda_{3H}^2 v^2}{8\pi^2} \frac{\sqrt{3} - 2\pi/3}{\sqrt{3}m_H^2} \end{aligned}$$

We also have to introduce the renormalization factor for the coupling modifier.

$$\delta Z_{\kappa_{3H}} = -\frac{3}{16\pi^2} \frac{1}{\epsilon} \left( -2\lambda_{4H} - \lambda_{3H} + 6\lambda_{3H}^2 \frac{v^2}{m_H^2} \right)$$

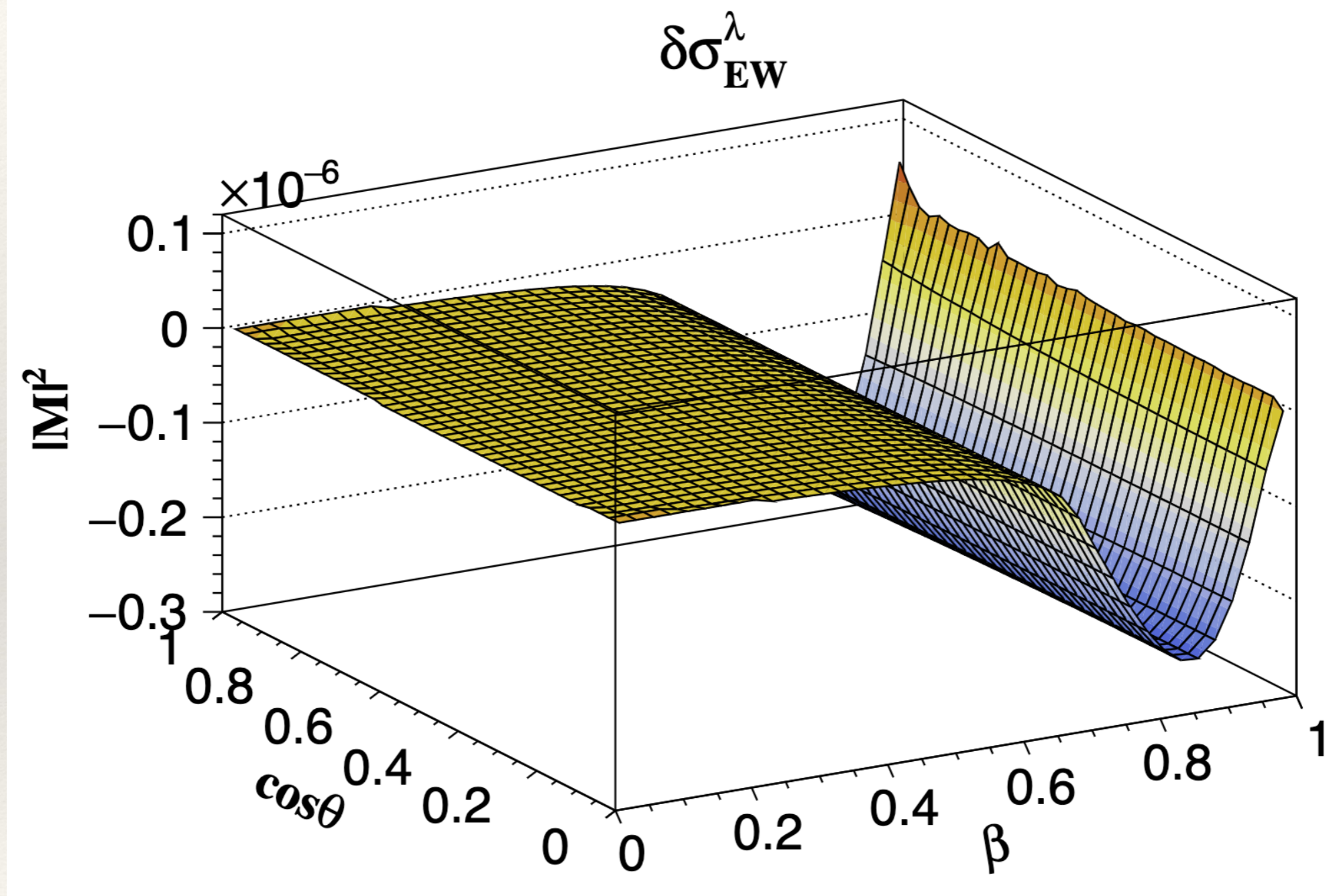
# Renormalization

The result of one-particle reducible diagrams and counter-terms:

$$\begin{aligned}
 & \mathcal{M}_{gg \rightarrow H^* \rightarrow HH}^{\text{LO}} \times \left\{ \frac{3}{16\pi^2} \frac{1}{\epsilon} \left( -2\lambda_{4H} - \lambda_{3H} + 6\lambda_{3H}^2 \frac{v^2}{m_H^2} \right) + \delta Z_{\kappa_{3H}} \right. \\
 & + \frac{3}{16\pi^2} \ln \frac{\mu_R^2}{m_H^2} \left[ -2\lambda_{4H} - \lambda_{3H} + 6\lambda_{3H}^2 \frac{v^2}{m_H^2} \right] \\
 & - \frac{9\lambda_{3H}^2}{8\pi^2} \frac{v^2}{s - m_H^2} \left[ \beta \left( \ln \left( \frac{1 - \beta}{1 + \beta} \right) + i\pi \right) + \frac{s}{m_H^2} \left( 1 - \frac{2\pi}{3\sqrt{3}} \right) + \frac{5\pi}{3\sqrt{3}} - 1 \right] \\
 & + \frac{3\lambda_{3H}^2}{16\pi^2} \frac{v^2}{m_H^2} (21 - 4\sqrt{3}\pi) - \frac{9\lambda_{3H}^2 v^2}{4\pi^2} C_0[m_H^2, m_H^2, s, m_H^2, m_H^2, m_H^2] \\
 & \left. - \frac{3\lambda_{4H}}{16\pi^2} \left[ \beta \left( \ln \left( \frac{1 - \beta}{1 + \beta} \right) + i\pi \right) + 5 - \frac{2\pi}{\sqrt{3}} \right] - \frac{3\lambda_{3H}}{16\pi^2} \right\},
 \end{aligned}$$



# Squared matrix elements



H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

# Updated function form

The  $\lambda$  dependent correction is

$$\delta\sigma_{\text{ggF,EW}}^{\kappa\lambda} = (0.075\kappa_{\lambda_{3H}}^4 - 0.158\kappa_{\lambda_{3H}}^3 - 0.006\kappa_{\lambda_{3H}}^2 \kappa_{\lambda_{4H}} - 0.058\kappa_{\lambda_{3H}}^2 + 0.070\kappa_{\lambda_{3H}} \kappa_{\lambda_{4H}} - 0.149\kappa_{\lambda_{4H}}) \text{ fb}$$

$$\delta\sigma_{\text{VBF,EW}}^{\kappa\lambda} = (0.0215\kappa_{\lambda_{3H}}^4 - 0.0324\kappa_{\lambda_{3H}}^3 - 0.0019\kappa_{\lambda_{3H}}^2 \kappa_{\lambda_{4H}} - 0.0043\kappa_{\lambda_{3H}}^2 + 0.0151\kappa_{\lambda_{3H}} \kappa_{\lambda_{4H}} - 0.0211\kappa_{\lambda_{4H}}) \text{ fb}$$

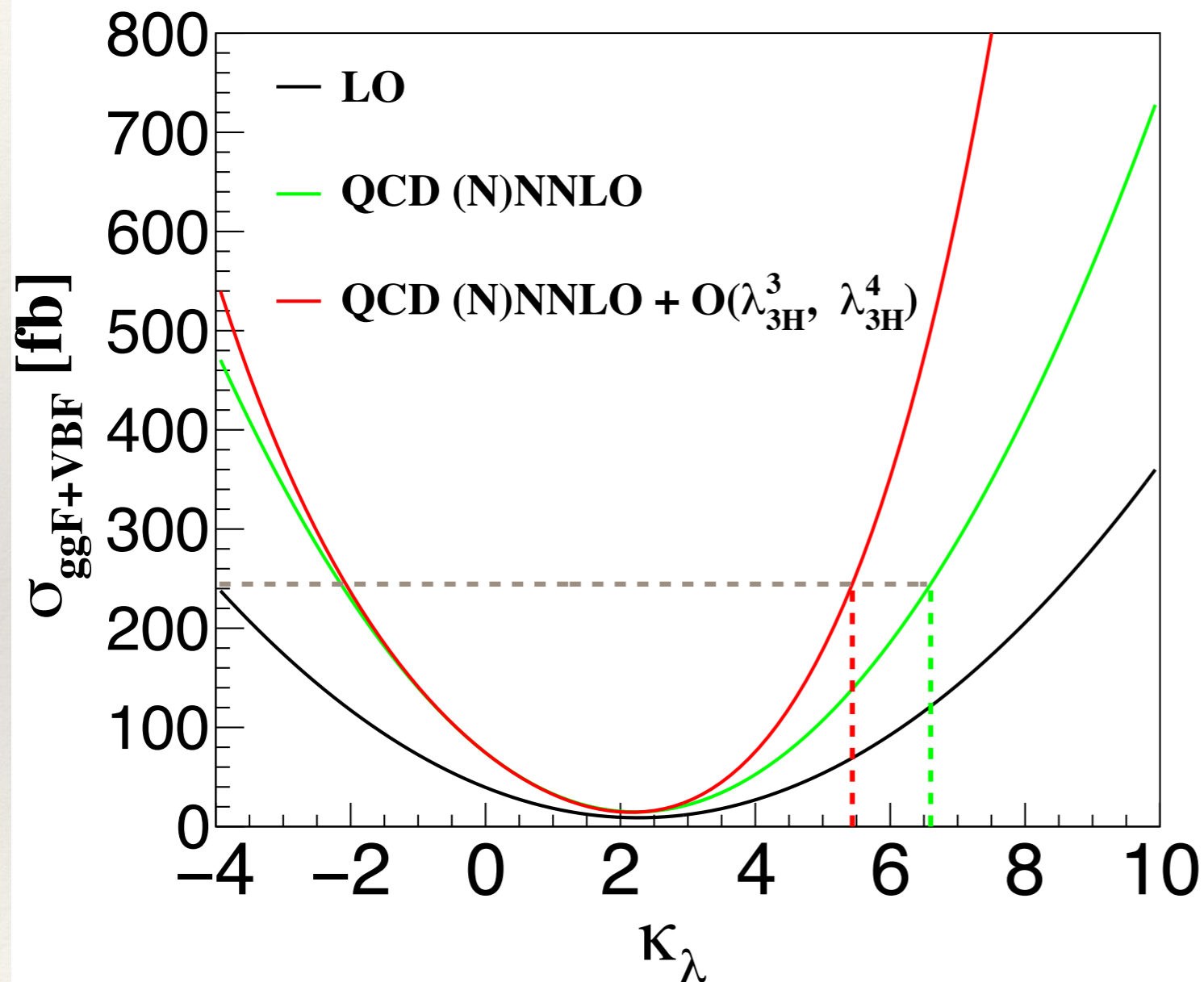
$\kappa_{\lambda_{3H}}$	$\kappa_{\lambda_{4H}}$	ggF			VBF		
		$\sigma_{\text{LO}}^{\kappa\lambda}$	$\sigma_{\text{NNLO-FT}}^{\kappa\lambda}$	$\delta\sigma_{\text{EW}}^{\kappa\lambda}$	$\sigma_{\text{LO}}^{\kappa\lambda}$	$\sigma_{\text{NNLO}}^{\kappa\lambda}$	$\delta\sigma_{\text{EW}}^{\kappa\lambda}$
1	1	16.7	31.2	-0.225	1.71	1.69	$-2.30 \times 10^{-2}$
3	1	8.59	18.4	1.28	3.59	3.53	$8.35 \times 10^{-1}$
6	1	67.3	161	60.6	25.1	24.6	20.7
1	3	16.7	31.2	-0.393	1.71	1.69	$-3.89 \times 10^{-2}$
1	6	16.7	31.2	-0.646	1.71	1.69	$-6.27 \times 10^{-2}$
3	3	8.59	18.4	1.30	3.59	3.53	$8.50 \times 10^{-1}$
6	6	67.3	161	61.0	25.1	24.6	20.7

The QCD corrections are significant in ggF, but not sensitive to  $\kappa_{3H}$ .

The EW corrections are 91% (82%) in ggF (VBF) for  $\kappa_{3H} = 6$ .

The dependence on  $\lambda_{4H}$  is weak.

# More stringent constraint



ATLAS (CMS) limit

6.6 (6.49)



5.4 (5.37)

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# Summary

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The SM is a master piece in human history. It has been tested by a lot of experiments at very high precision level.

However, the Higgs sector still needs more precise comparison between theories and experiments.

Higher-order quantum corrections provide more precise estimate of the dependence on Higgs self-couplings.

**Thanks a lot for your attention!**