李政道研究所
TSUNG-DAO LEE INSTITUTE

Phase Transitions, anomalous baryon number violation and electroweak multiplet dark matter

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In cooperation with Michael Ramsey-Musolf, Xu-Xiang Li, Tuomas Tenkanen, Wenxing Zhang

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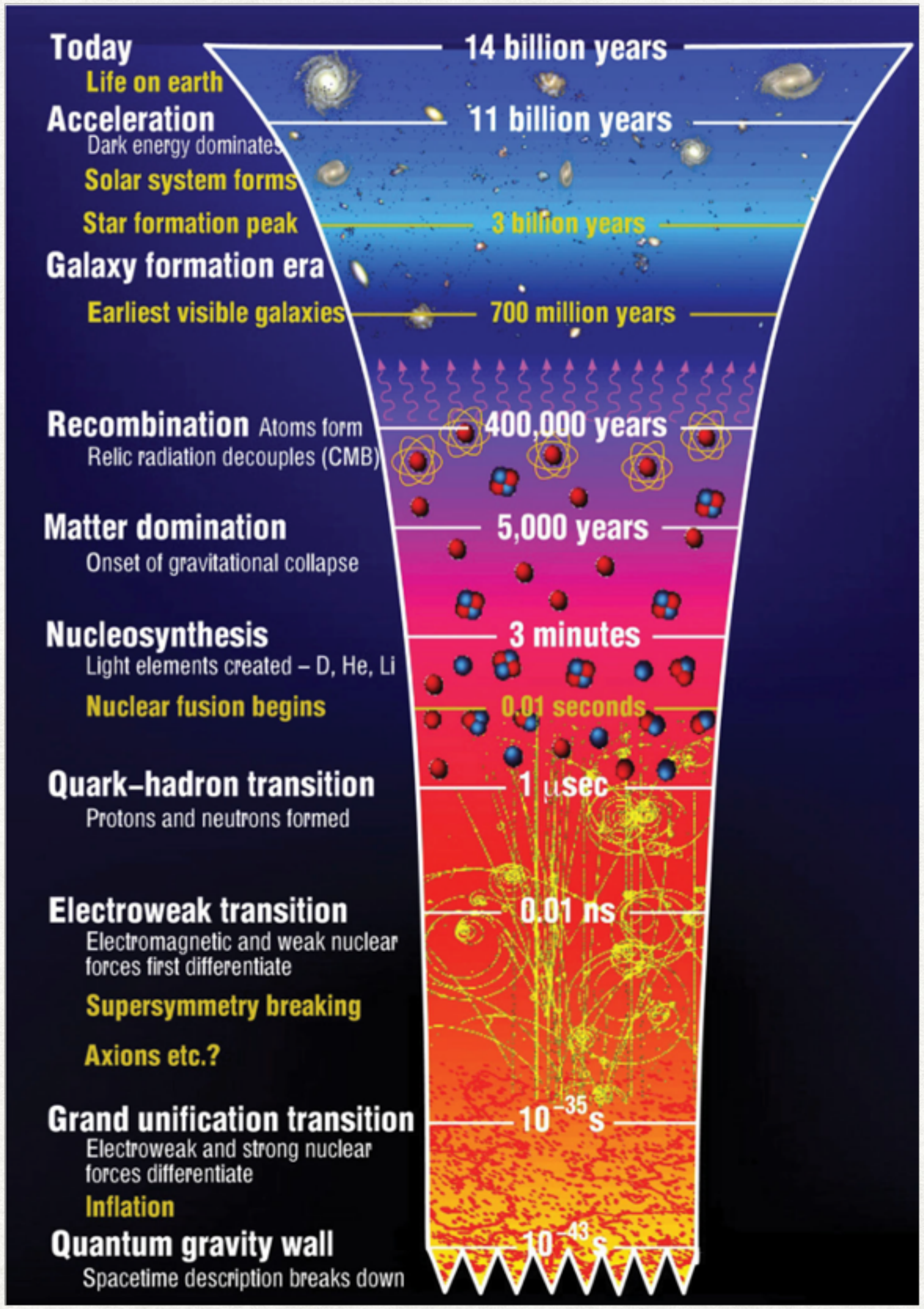
Matter-antimatter asymmetry of our universe



Credit: PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH 17, 013104 (2021)

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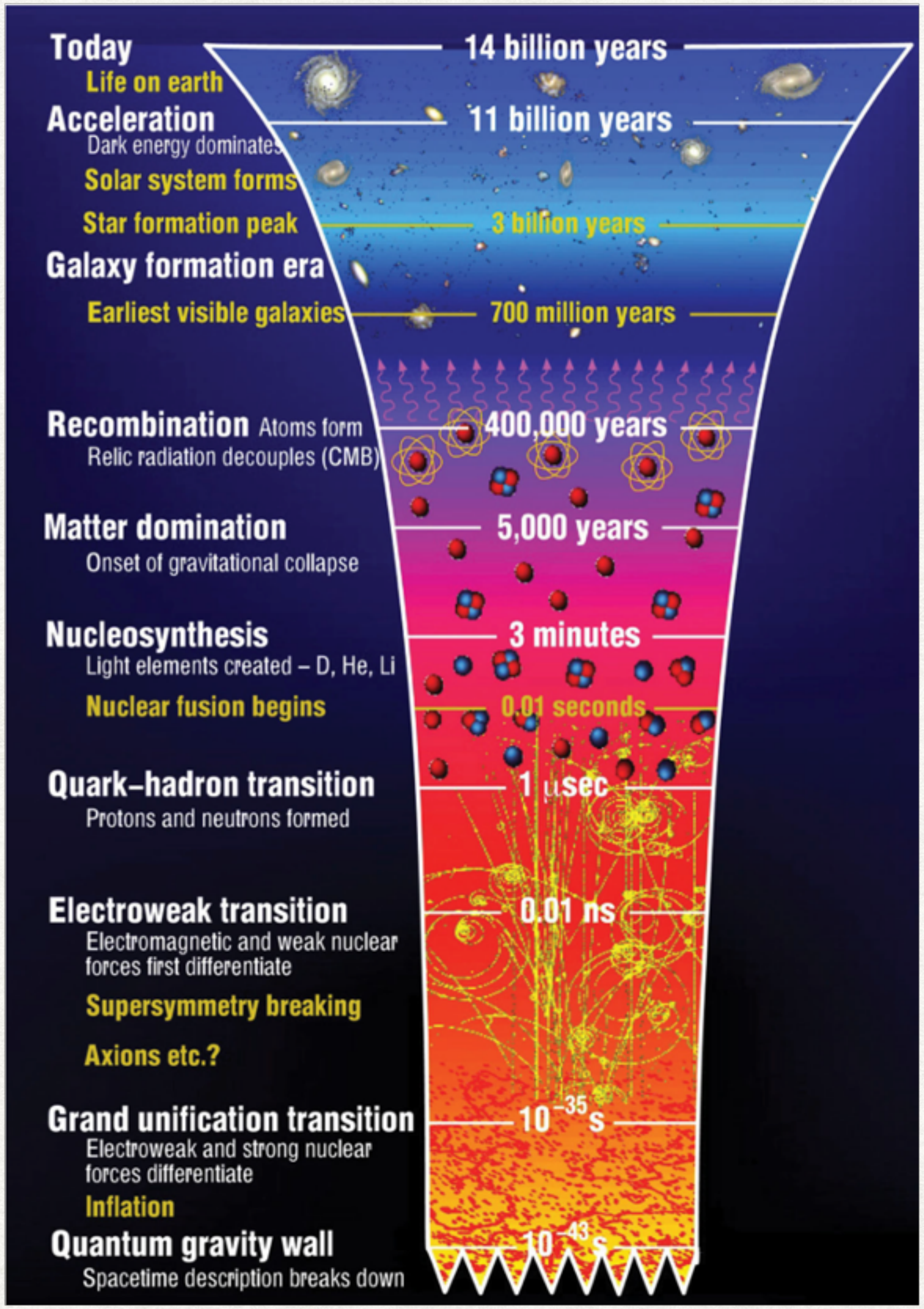
$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 6.1 \times 10^{-10}$$



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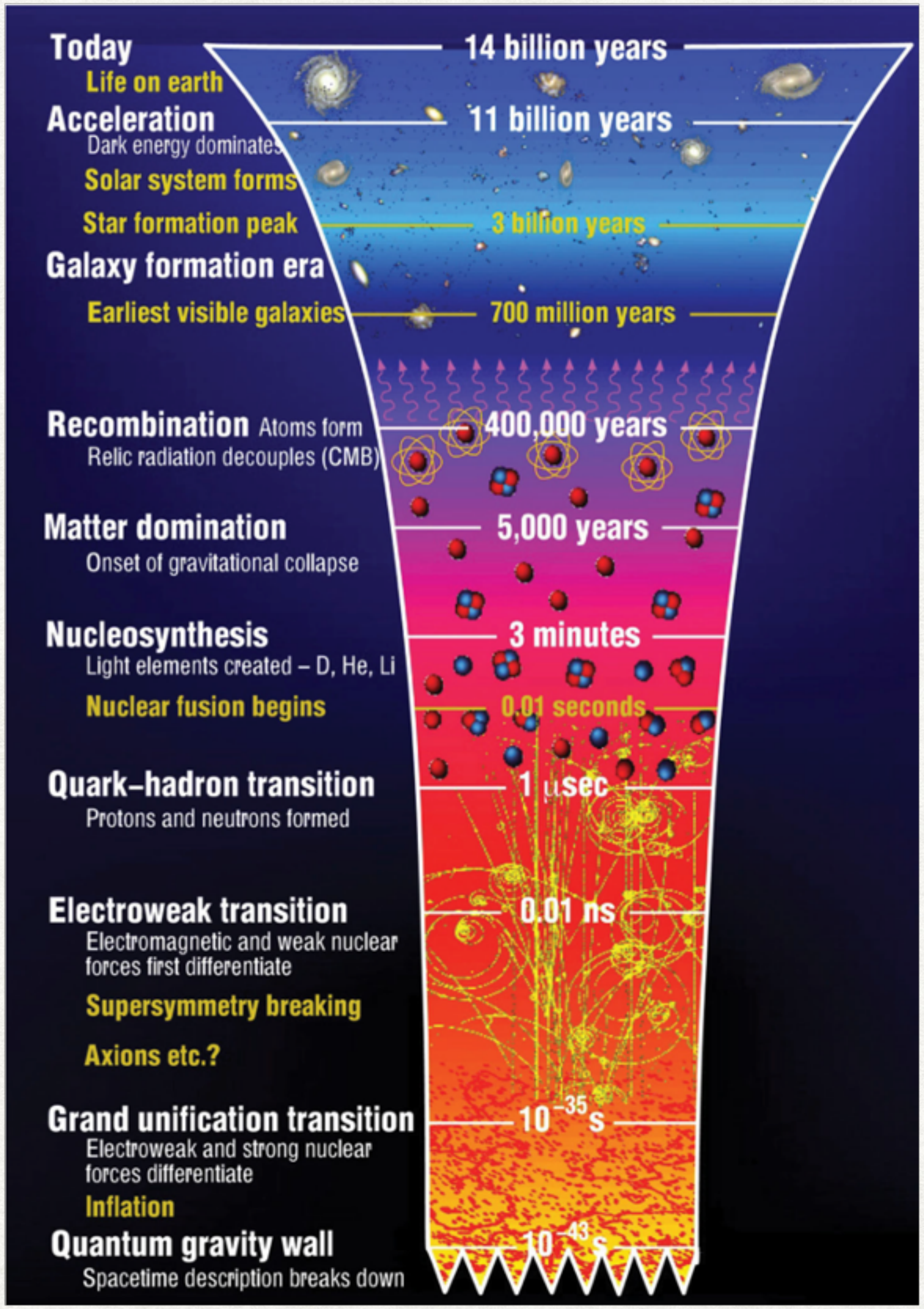

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 0$$

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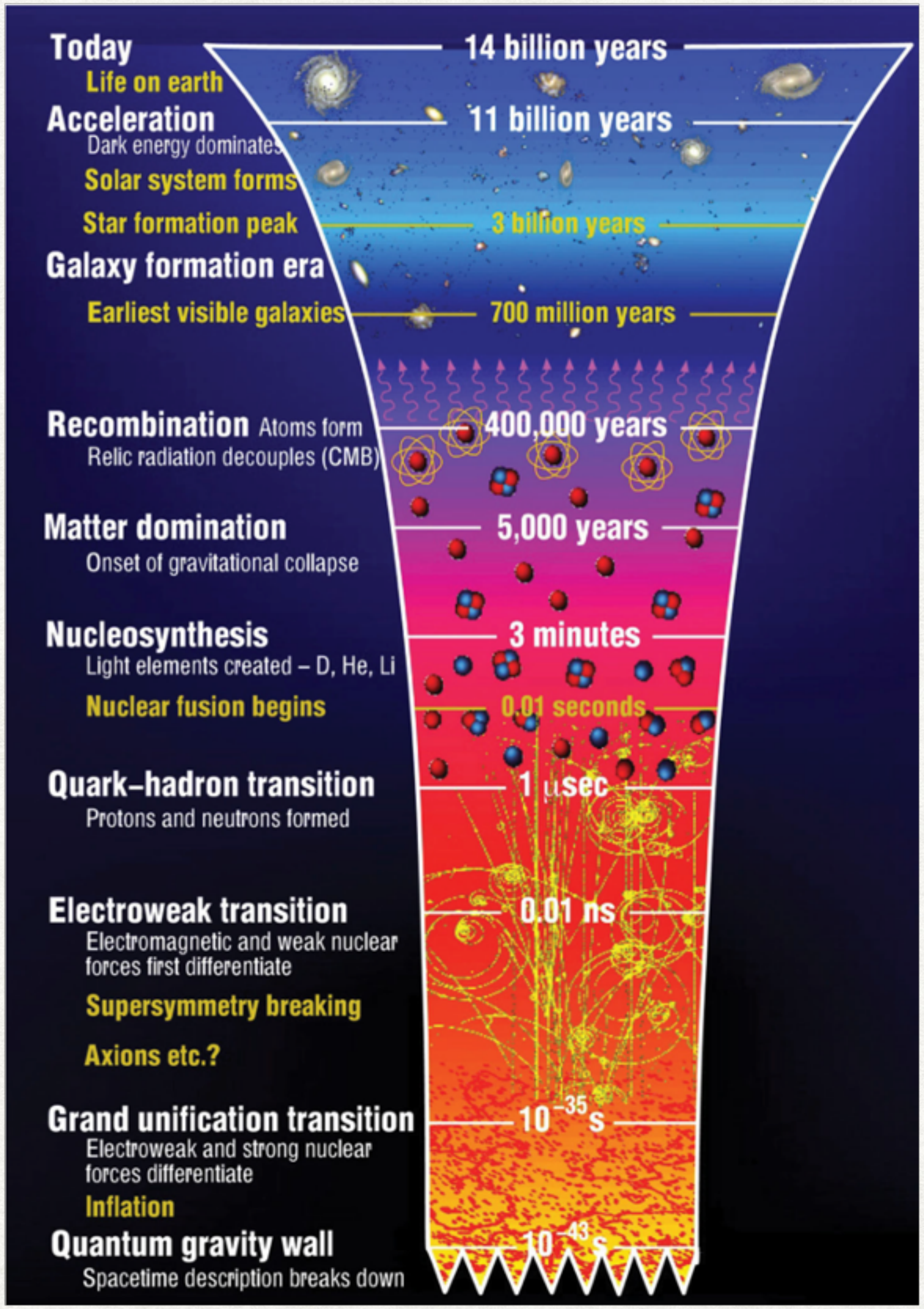
Matter-antimatter asymmetry of our universe

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 6.1 \times 10^{-10}$$

$$t_{EW} \simeq 10^{-11}s,\ T_{EW} \simeq 100\ GeV$$

At Electroweak epoch, generate the baryon asymmetry, η

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 0$$



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Electroweak Baryogenesis

Three necessary conditions for baryogenesis:

	SM	BSM
Baryon number violation		
Sufficient C & CP violation		
Departure from thermal equilibrium		

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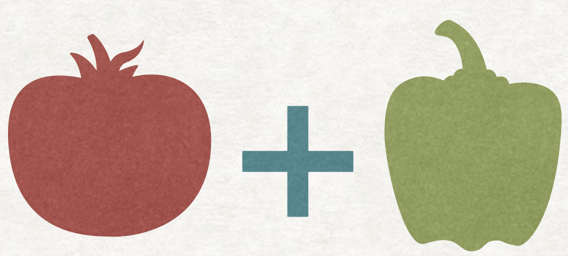

$m_\Phi \lesssim 700\ GeV$

LHC search!

Electroweak Baryogenesis

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My research!

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Departure from thermal equilibrium	✗	✓

My research!

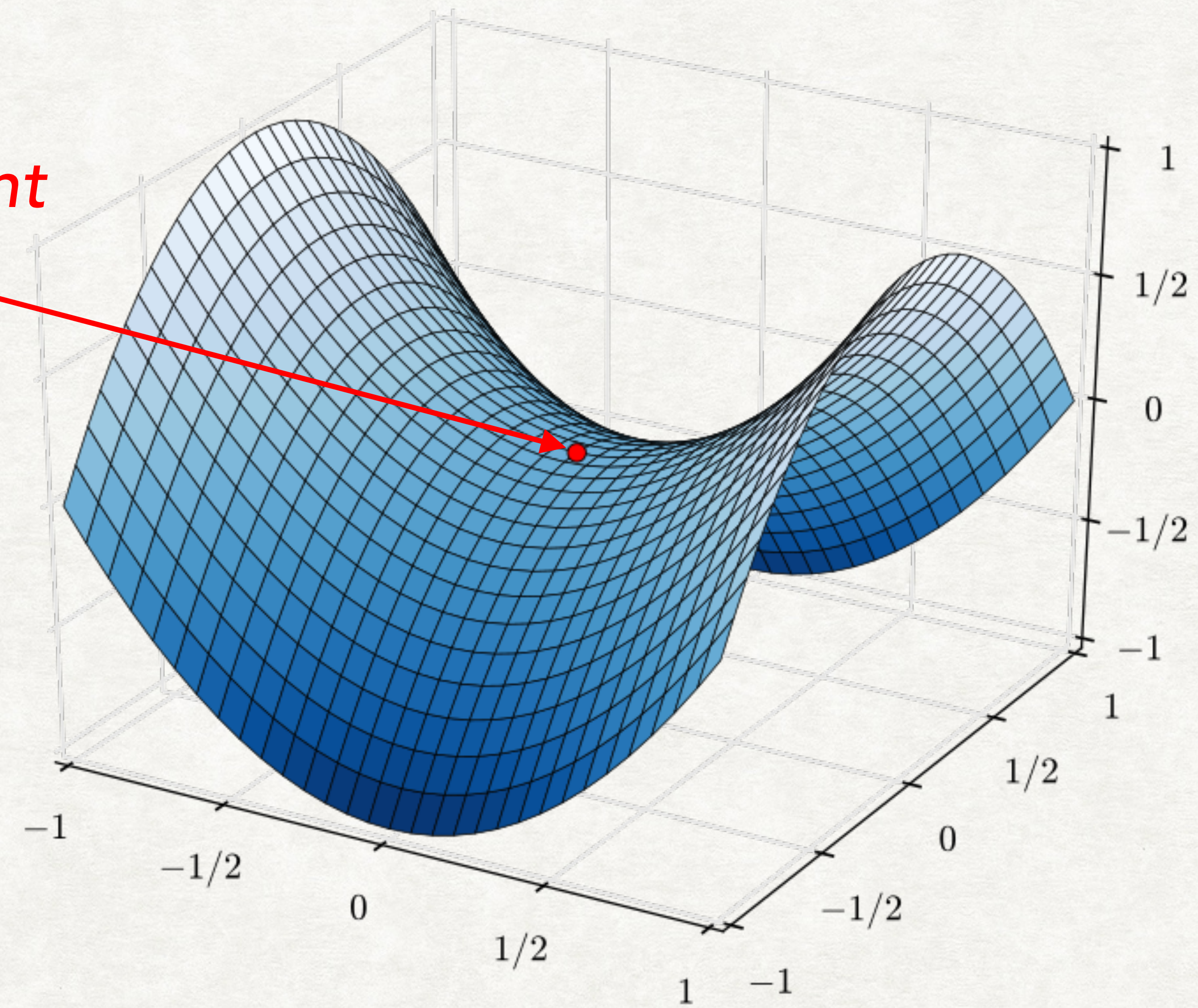
If the created baryon asymmetry can be preserved to today?

Computation of baryon number violation rate!

Baryon number violation: Sphaleron

Sphaleron: from Greek $\sigma\phi\alpha\lambda\epsilon\rho o\nu$, unreliable, ready to fall

Sphaleron, saddle point field solution (gauge+scalar)



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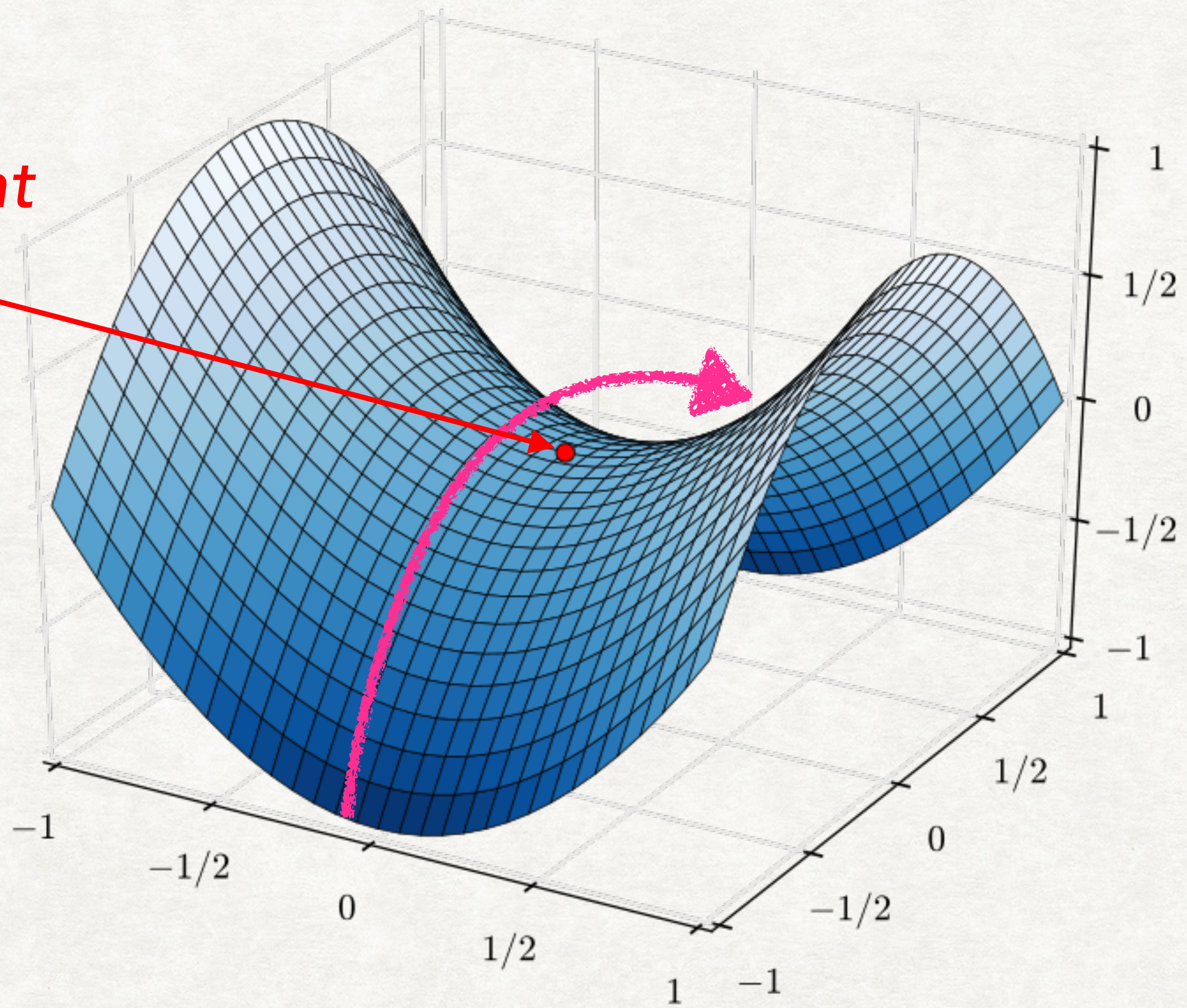

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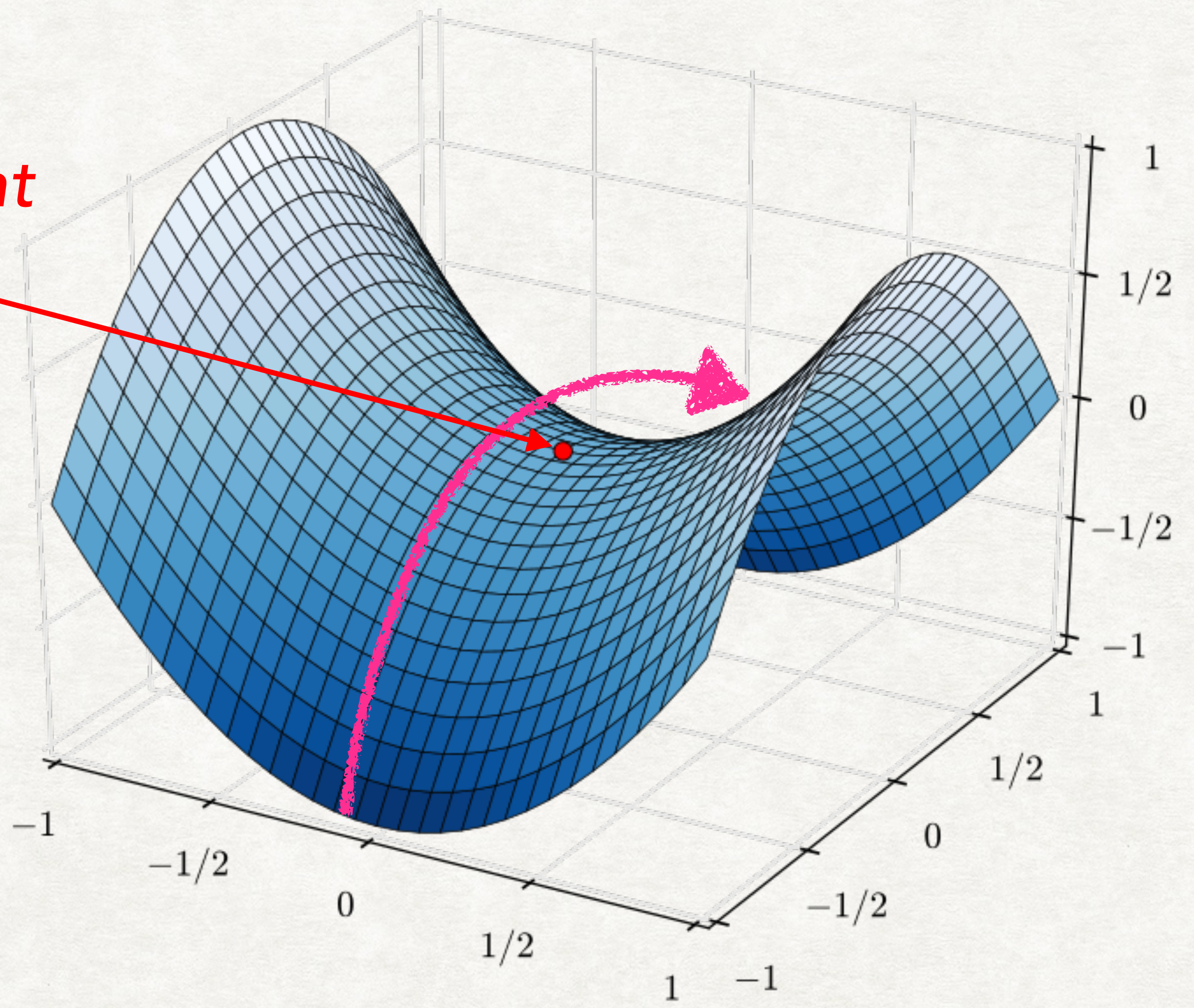

$\Delta B \neq 0$

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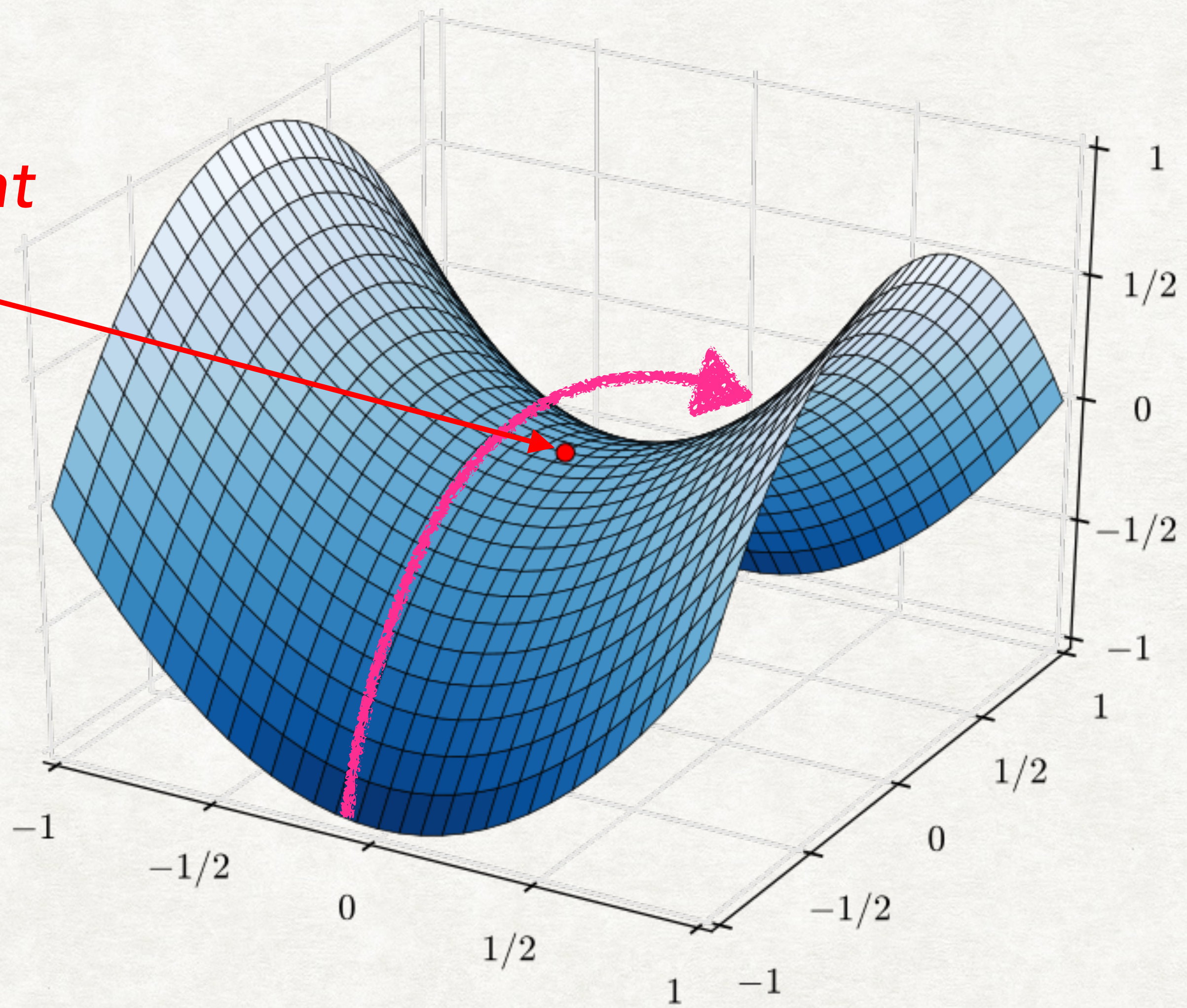

$\Delta B \neq 0$

$$\Gamma_B \simeq T^4 e^{-E_{sph}/T}$$

Novel aspects

- Topological classification : *sphaleron or monopole*

- Baryon number violation rate:

  - Monopole mass / Sphaleron energy in BSM

  - Sphaleron rate under gauge invariant 3D effective field theory — SM crossover application

Novel aspects

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DARK MATTER

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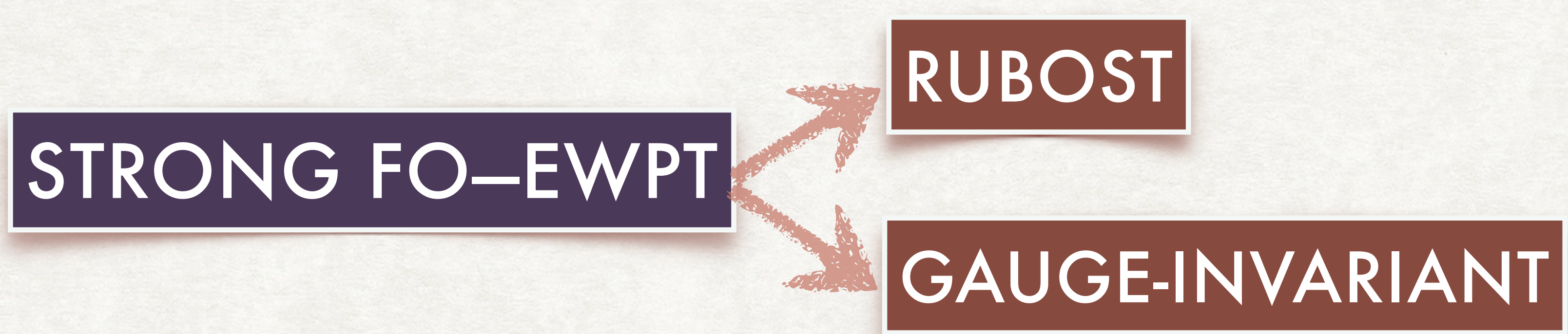

Topological classification of field solution

***YW**, Ramsey-Musolf, Zhang, 2307.02187v1,v2 will appear soon*

- If Multiplet Φ with $Y \neq 0$:

- If Multiplet Φ with $Y = 0$:

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- If Multiplet Φ with $Y \neq 0$:

$$\frac{G}{H} \simeq S^3, \quad \pi_2(G/H) = 0$$ **Sphaleron topology**

- If Multiplet Φ with $Y = 0$:

$$\frac{G}{H} \simeq S^2, \quad \pi_2(G/H) \neq 0$$ **Monopole topology**

$Y = 0$ multiplet can contribute to DM relic density

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$Y = 0$ **multiplet can contribute to DM relic density**

Topological analysis can tell us whether sphaleron or monopole solution emerges during the EWPT.

Baryon number violation rate

To the one-loop (determinant) order:

$$\Gamma_{sph}/\Gamma_{mon} = \omega_- \, N_{tr}(NV)_{rot} \, \kappa \, e^{-E/T}$$

Baryon number violation rate

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↑

LO effect

Baryon number violation rate

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Zero modes

LO effect

SM crossover sphaleron rate

Li, Ramsey-Musolf, Tenkanen, YW, in prep.

$$y = \frac{\mu_3^2}{g_3^4},\ x = \frac{\lambda_3}{g_3^2}$$

$$\Gamma_{sph,LO}(x,y) = T^4 \times e^{-\sqrt{-y}\mathcal{F}(x)}$$

$$\Gamma_{sph,NLO}(x,y) = T \times \mathcal{N}_{tr}(\mathcal{N}\mathcal{V})_{rot}(-y)^3 g_3^6 e^{-\sqrt{-y}\mathcal{F}(x)}$$

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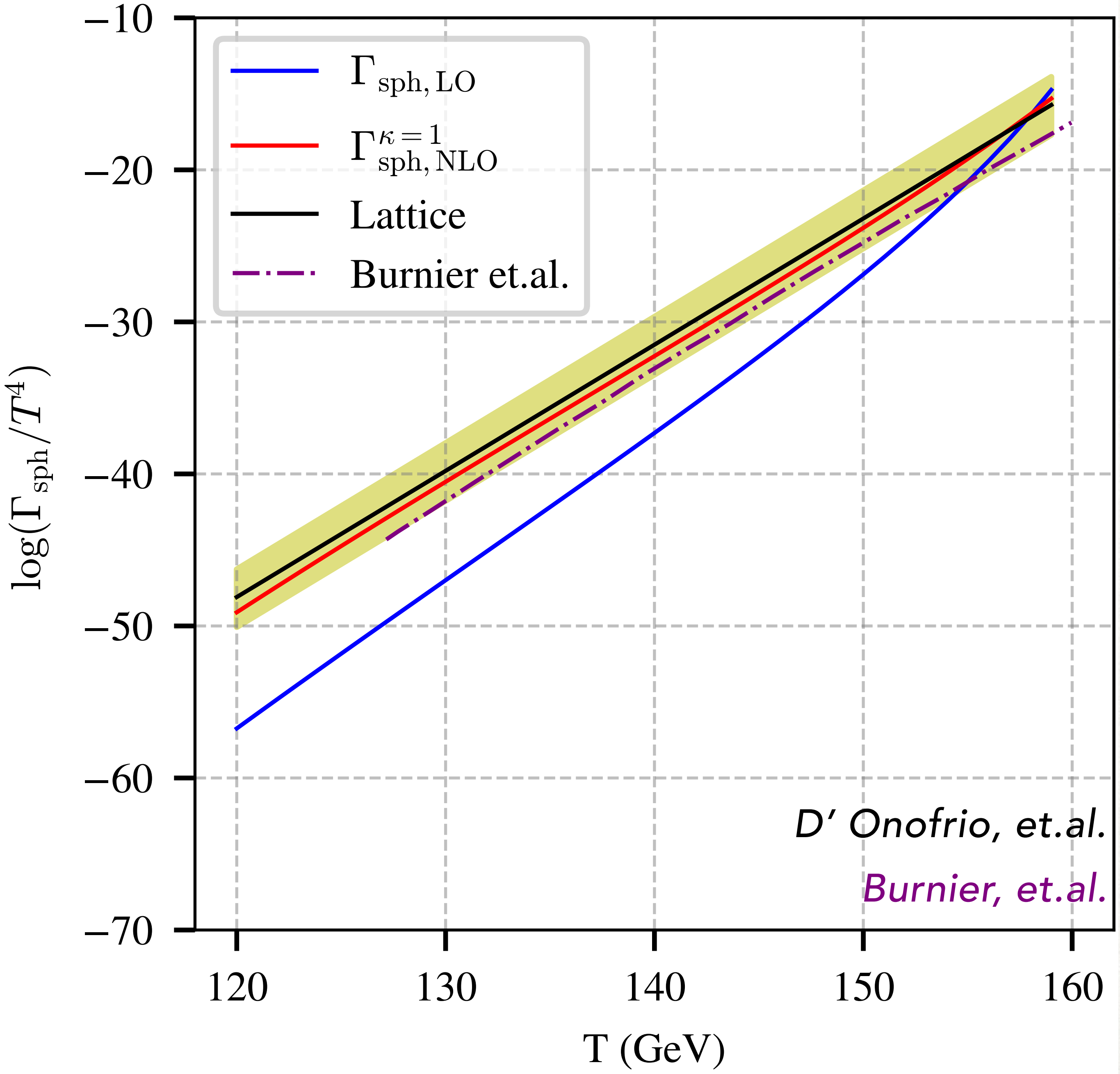

D' Onofrio, et.al. Phys.Rev.Lett. 113 (2014) 14

Burnier, et.al. JCAP 02 (2006) 007

Conclusion

- The **electroweak baryogenesis** is a promising mechanism to explain the **baryon asymmetry** of our universe, which is **testable** in the **future collider** experiments.

- The **topological classification** of field solutions for EW multiplet Φ:
  - $Y = 0$: **Monopole** solution - Dark Matter
  - $Y \neq 0$: **Sphaleron** solution

- The **sphaleron rate** can be formulated in a **rubout** and **gauge-invariant** way under 3D EFT, which is crucial for **"strong"** first order EWPT.

Thanks!

Strong first order EWPT

$$\frac{v_c}{T_c} > 1$$

How to reformulate this into a gauge invariant and rubout way?

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$$\frac{v_c}{T_c} > 1$$

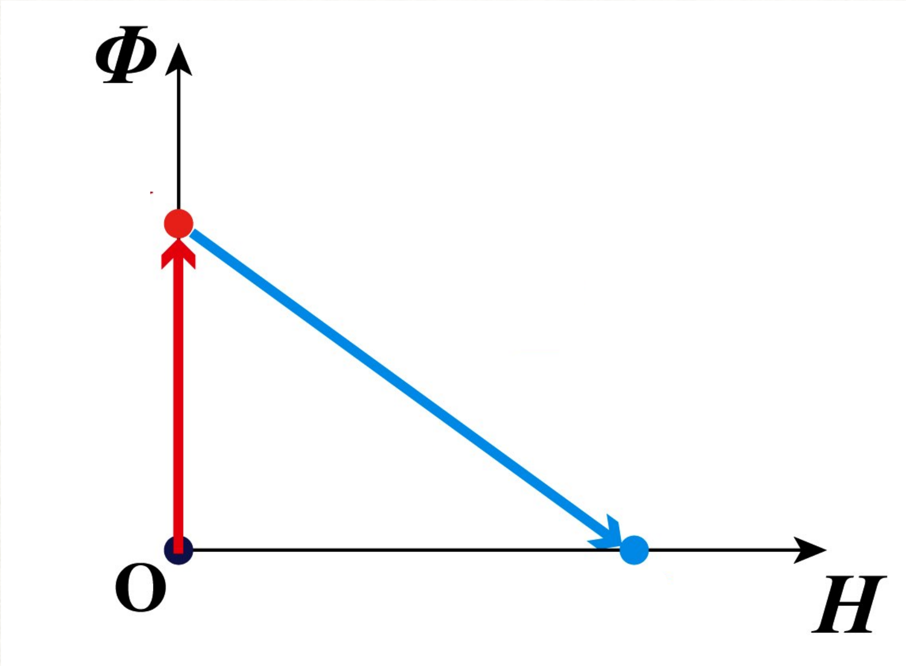
How to reformulate this into a gauge invariant and rubout way?

Gauge invariant and rubout Sphaleron /Monopole rate!

Complex SU(2) multiplet ($Y = 0$) extension

- Monopole mass in the broken phase

YW, Ramsey-Musolf, Zhang, 2307.02187v1,v2 will appear soon



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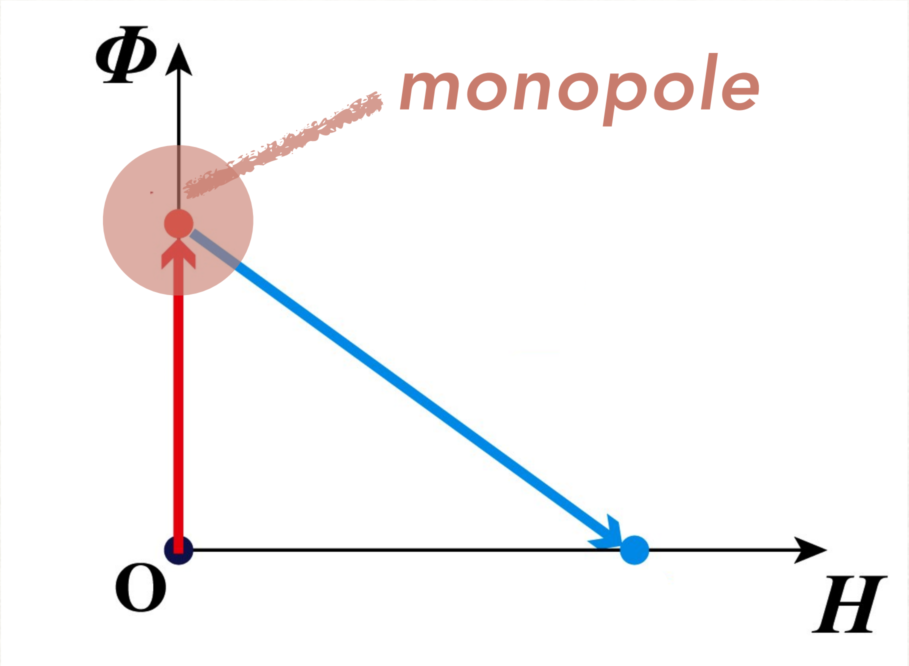

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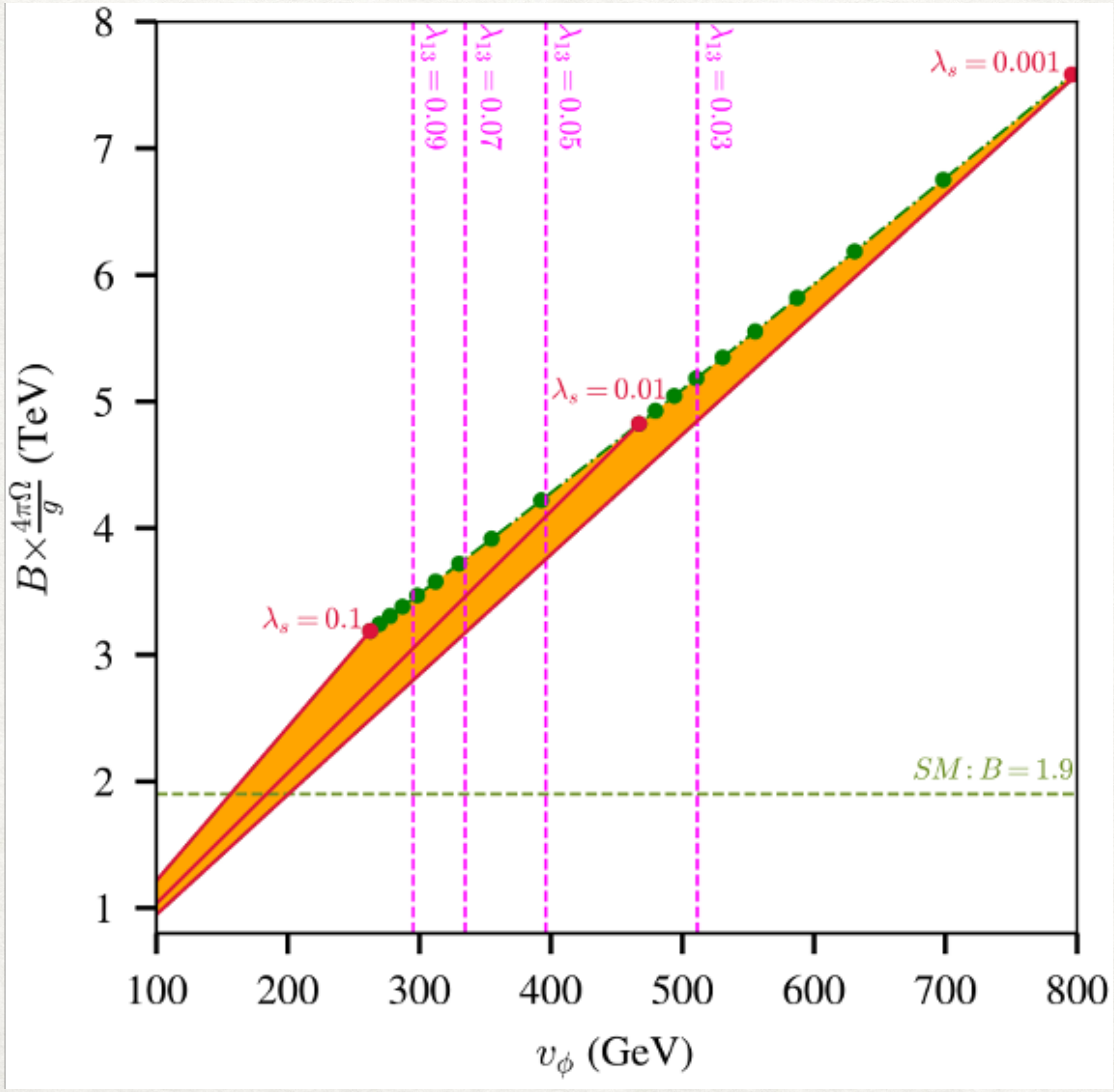

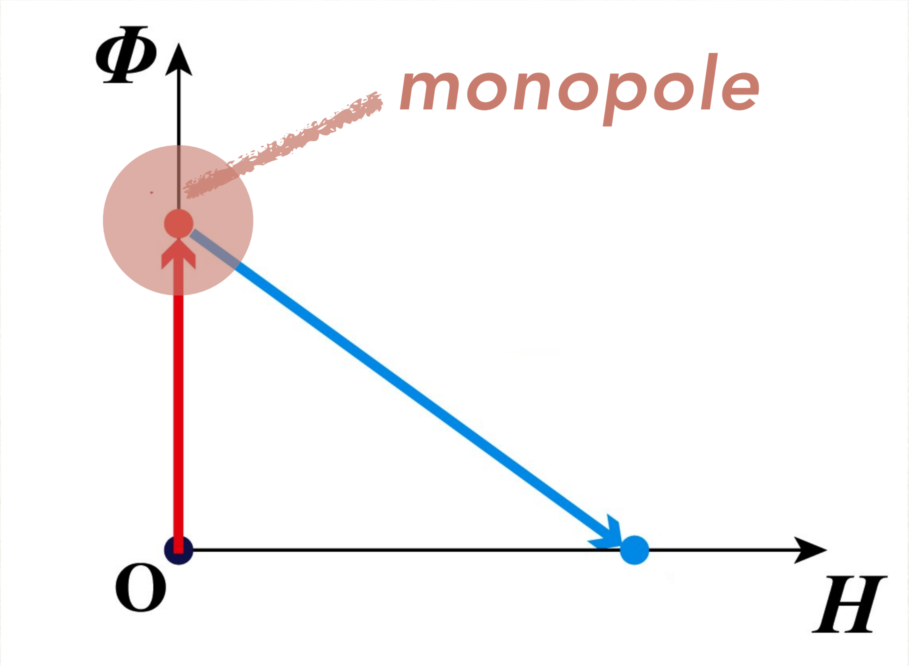

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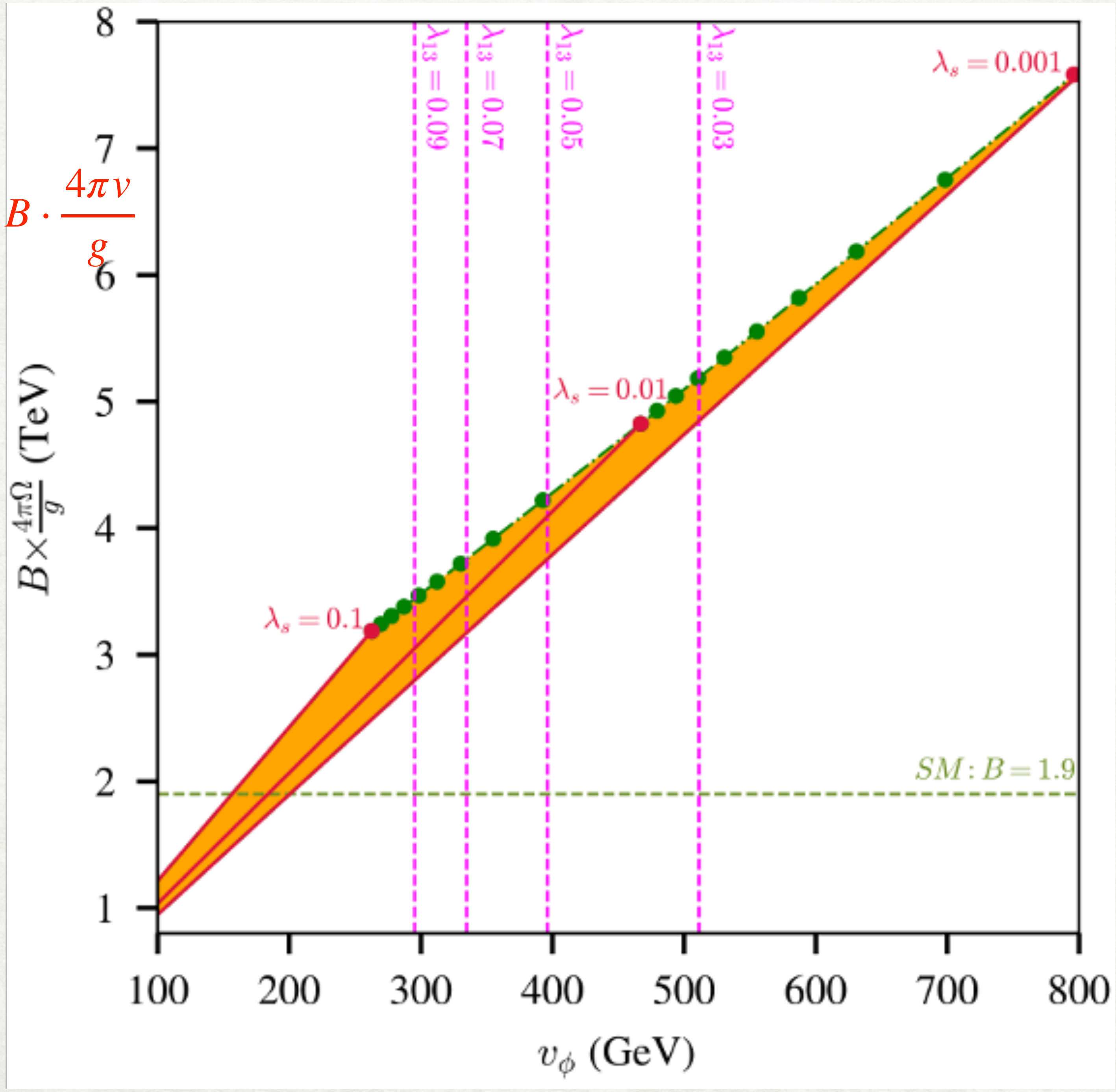

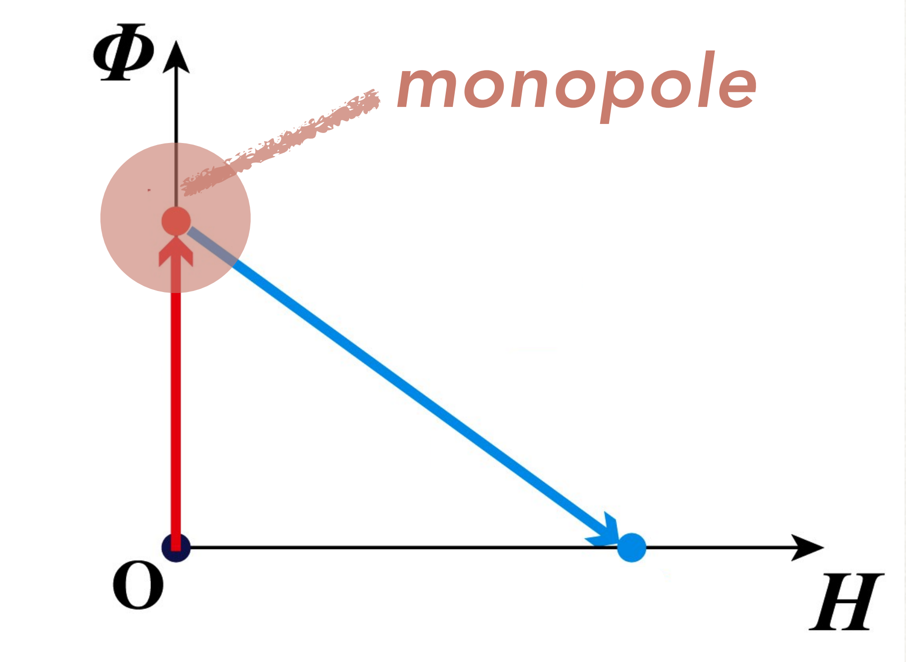

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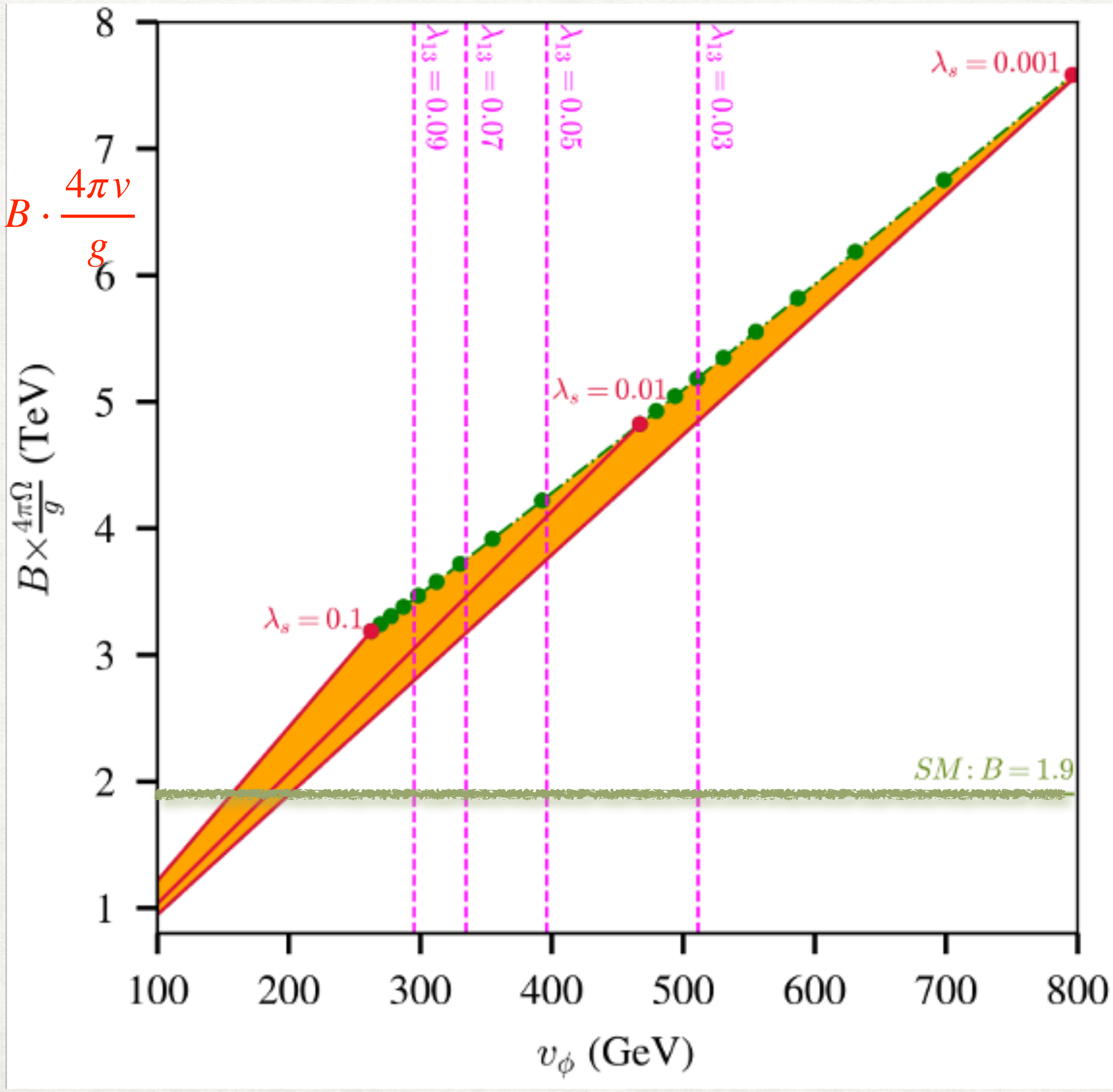

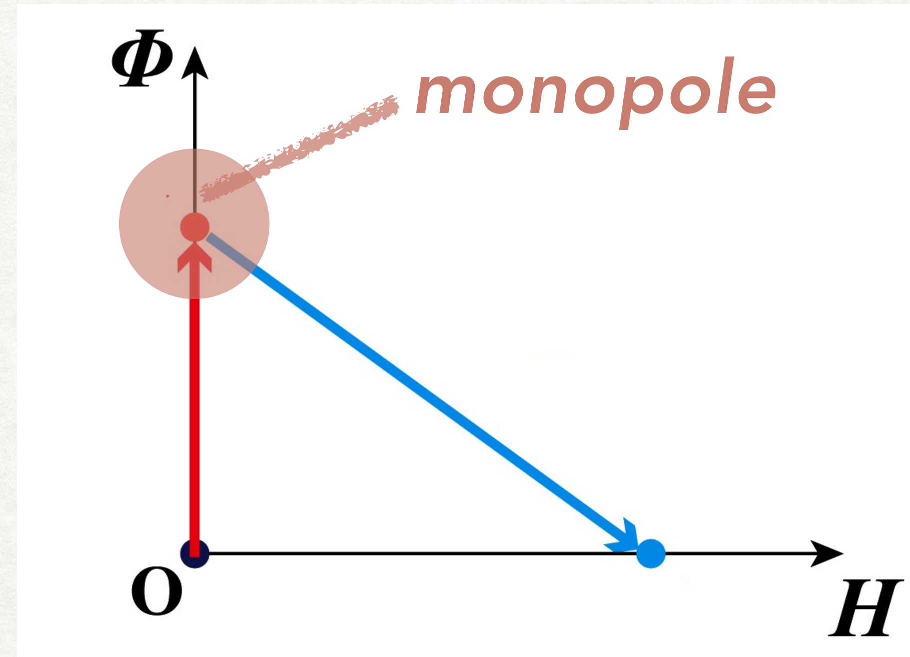

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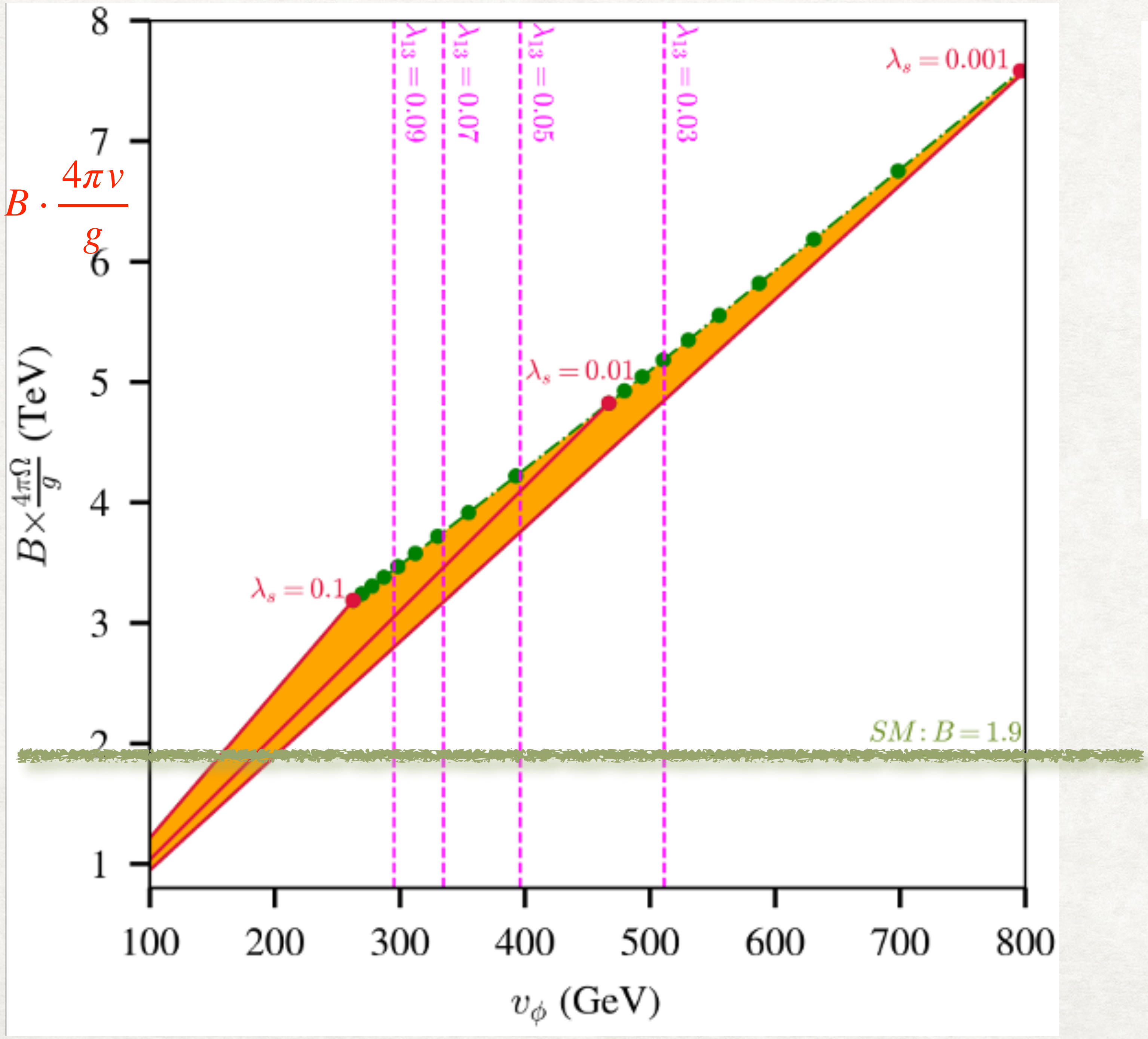

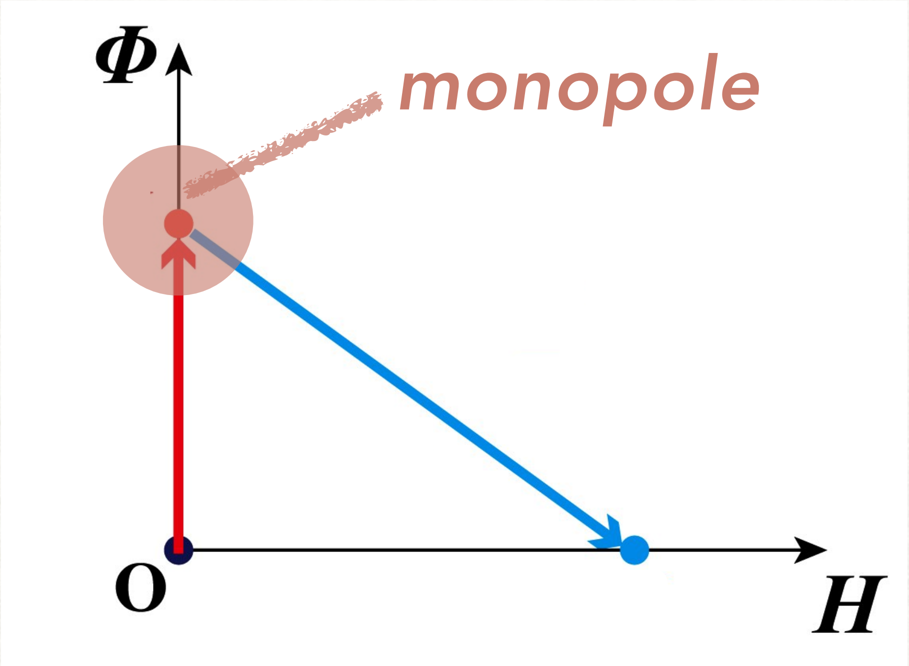

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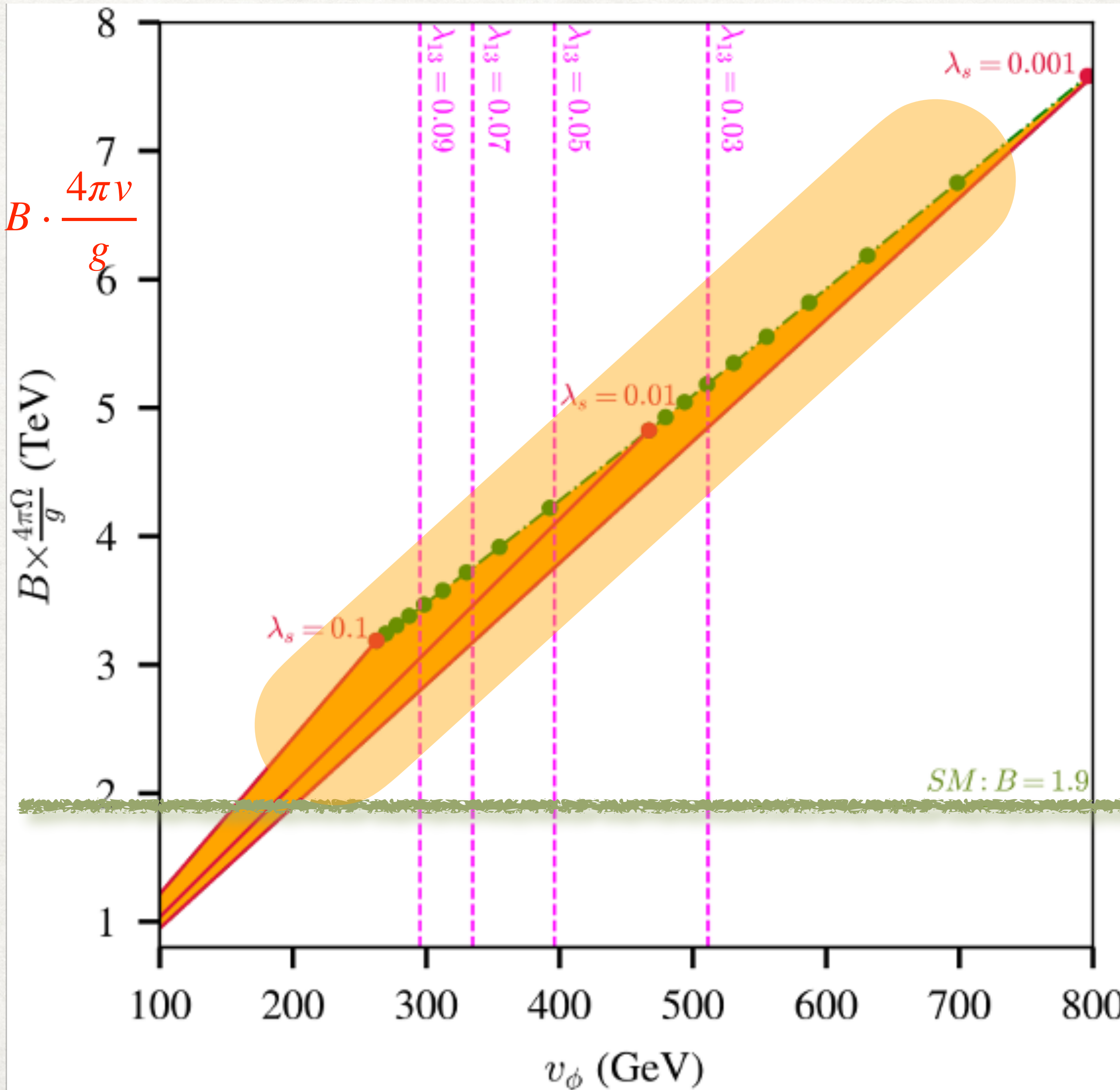

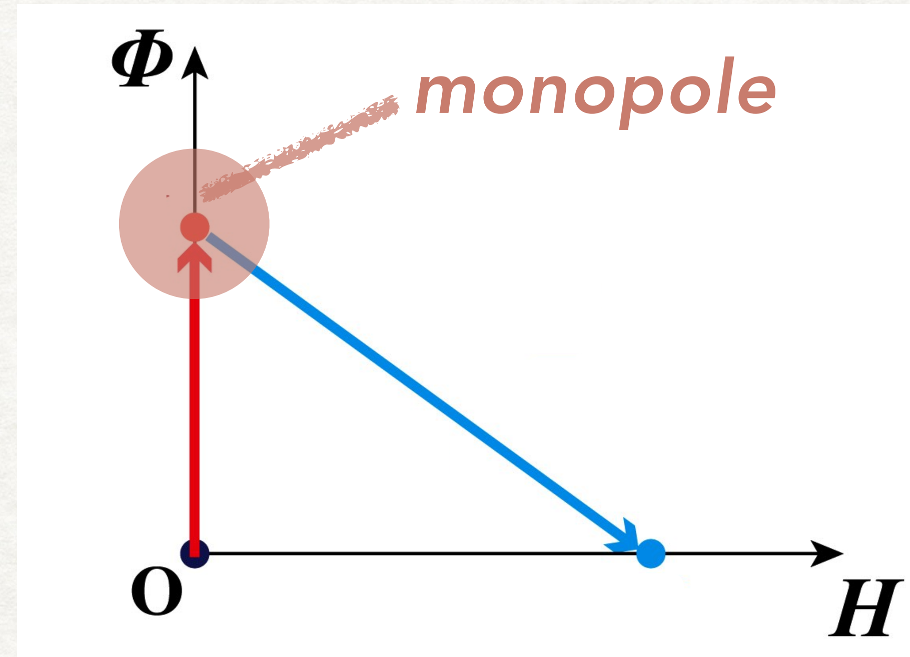

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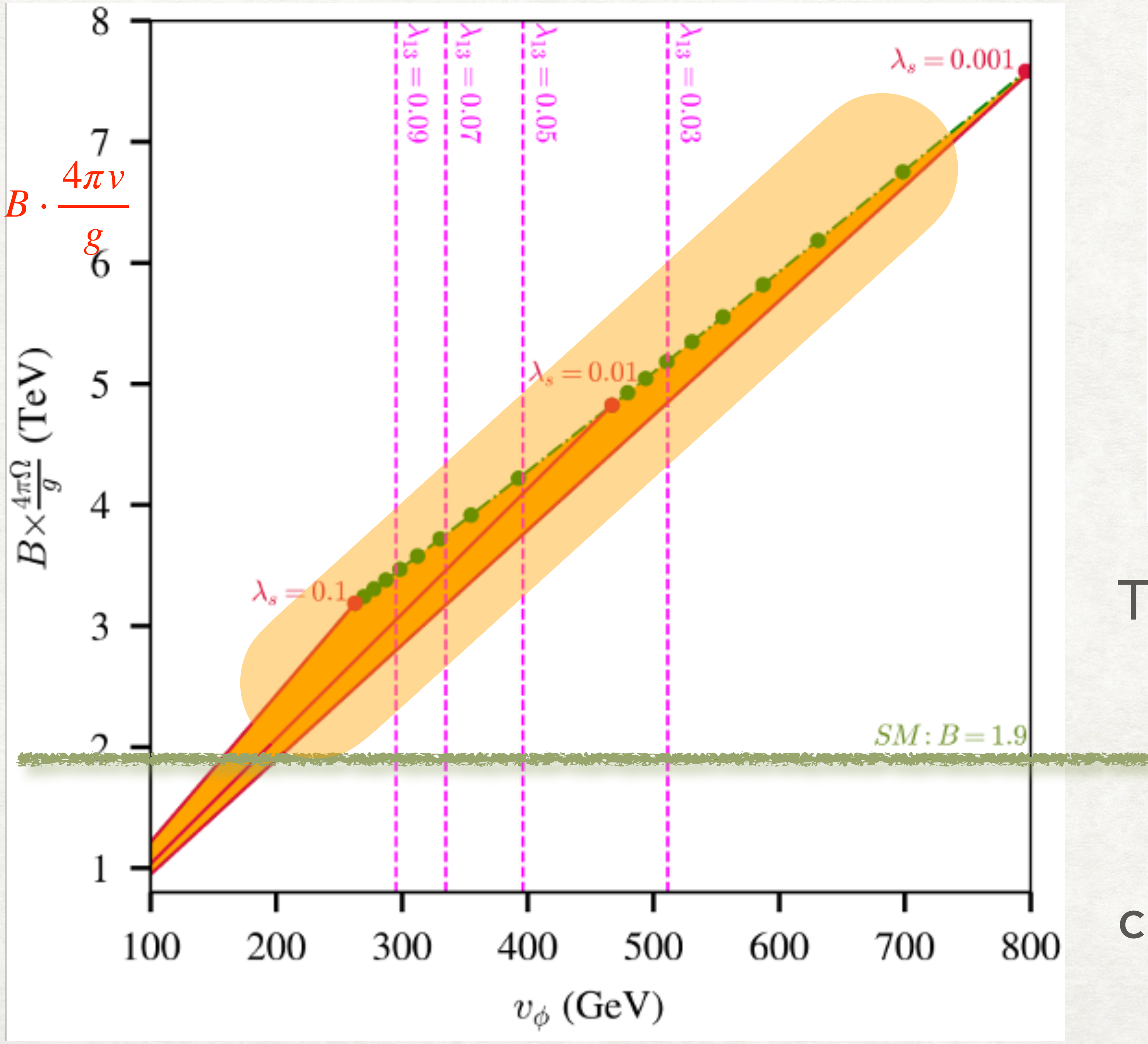
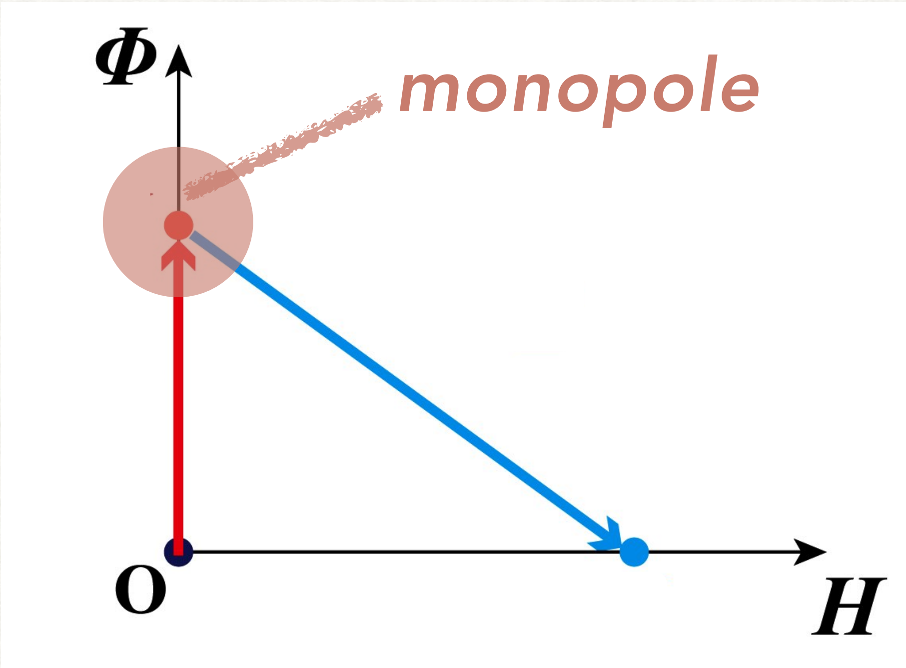


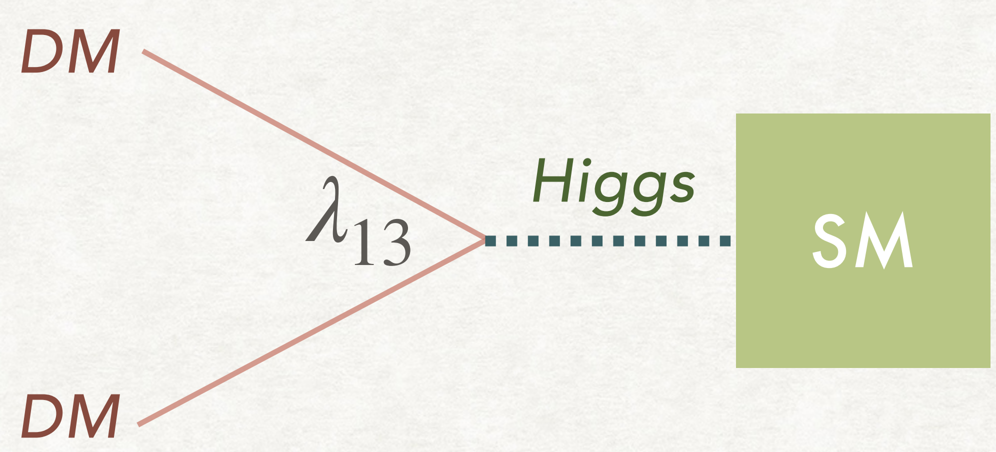


Traditional Strong FOEWPT:

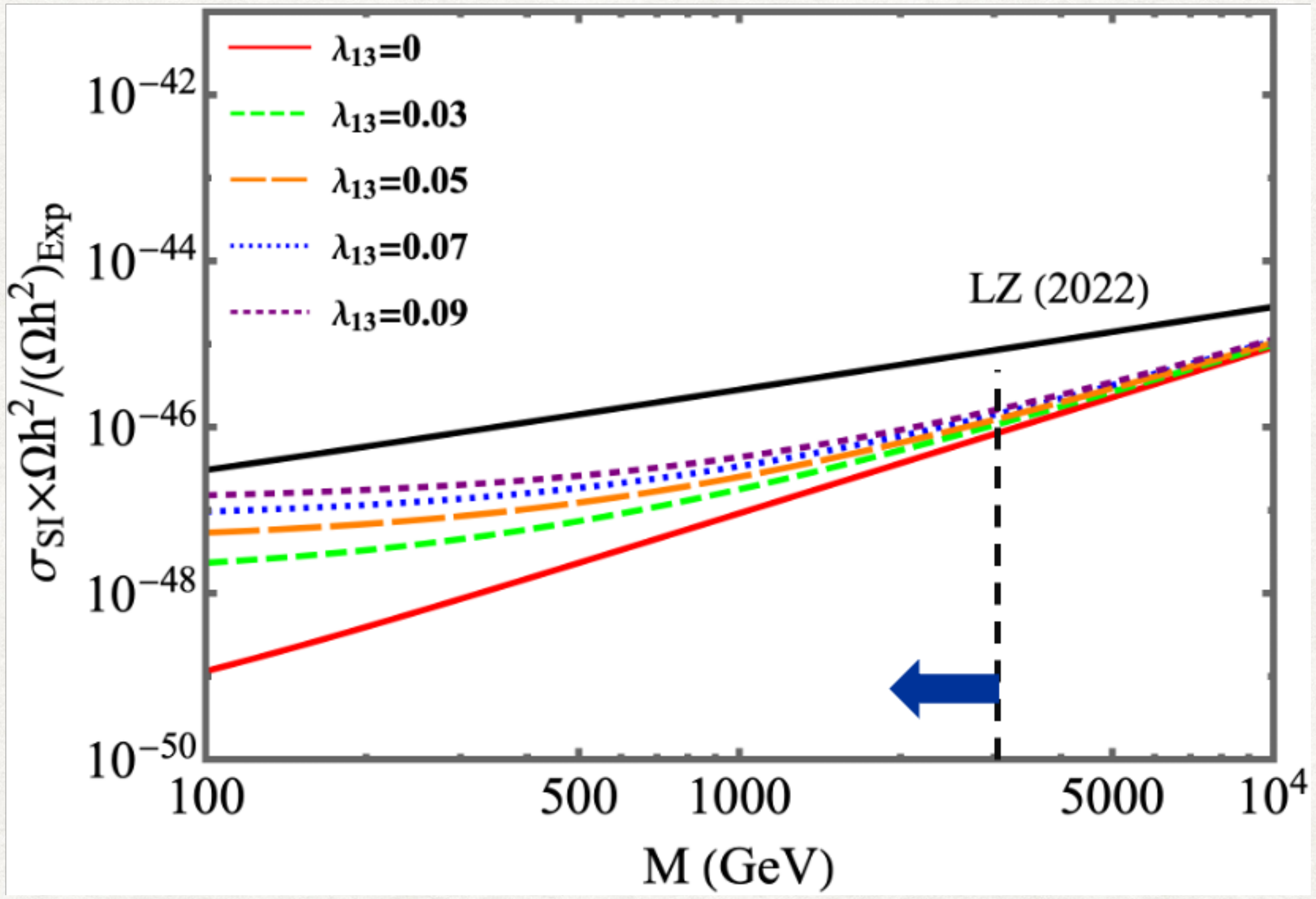
$$\frac{v_c}{T_c} > 1$$

can be relaxed!

SU(2) $Y = 0$ mutiplet contributes to DM relic density



Scaled cross section



Complex septuplet ($Y = 0$) extension to the SM

- The Higgs field and septuplet field

1812.07829

$$V = V_0(H) + V_{portal}(H, \Phi) + V_{self}(\Phi)$$

$$H = \begin{pmatrix} \omega^+ \\ \frac{1}{\sqrt{2}}(v + h + i\pi) \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \phi_{3,3} \\ \phi_{3,2} \\ \phi_{3,1} \\ \frac{1}{\sqrt{2}}(v_\phi + \phi + i\pi_\phi) \\ \phi_{3,-1} \\ \phi_{3,-2} \\ \phi_{3,-3} \end{pmatrix}$$

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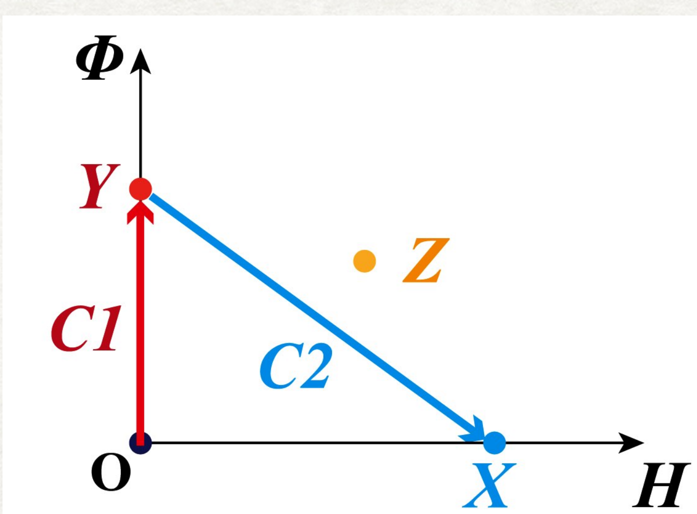


Baryon asymmetry could be made in $C1$, so it's important to know the Baryon number dilution rate (**monopole mass**) in broken phase at Y.

Topological classification of field solution

Before EWSB: $G = SU(2)_L \times U(1)_Y$

After EWSB:

- If Multiplet Φ with $Y \neq 0$: $H = U(1)_{em}$

$$\frac{G}{H} = \frac{SU(2)_L \times U(1)_Y}{U(1)_{em}} \simeq S^3$$

Sphaleron topology

i.e. SM Higgs

- If Multiplet Φ with $Y = 0$: $H = U(1)_{em} \times U(1)_Y$

$$\frac{G}{H} = \frac{SU(2)_L \times U(1)_Y}{U(1)_{em} \times U(1)_Y} \simeq S^2$$

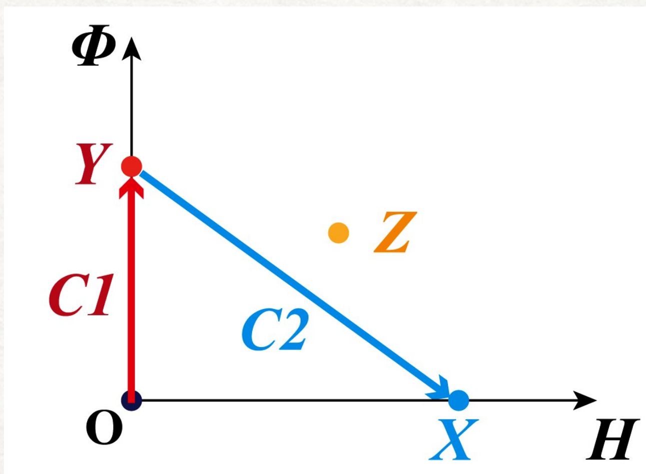
Monopole topology

i.e. real triplet

$Y = 0$ **multiplet can contribute to DM relic density**

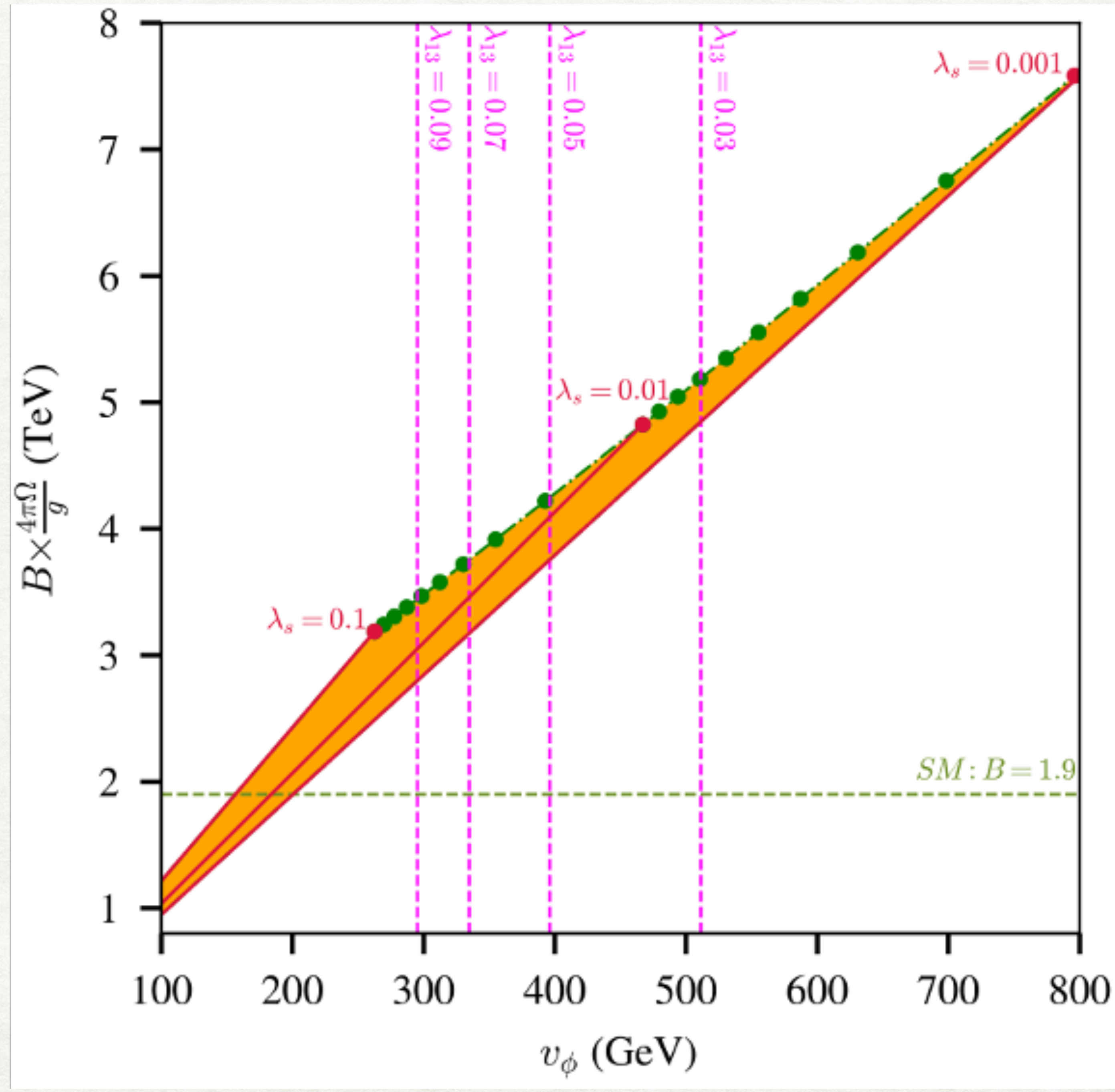
Complex septuplet ($Y = 0$) extension to the SM

- Monopole mass in the broken phase



$$\frac{4\pi B}{g}\frac{\bar{v}\left(T_C\right)}{T_C} - 6\ln\frac{\bar{v}\left(T_C\right)}{T_C} > f(X, \frac{\Delta t_{EW}}{t_H}, \mathscr{X}, \kappa)$$

BNPC CAN BE SATISFIED DURING THE FIRST BROKEN PHASE FOR LARGE MULTIPLET VEV



Patel, Ramsey-Musolf, JHEP 07 (2011) 029

Invariance property of the 1-form

- Electroweak scalar multiplet

$$E_{sph} = \frac{4\pi v}{g}\mathscr{F}(A_i^a, H, \Phi) \quad A_i^a T^a \sim f(\xi)(\partial_i U^\infty)U^{\infty-1} \quad i(U^{\infty-1})dU^\infty = \sum_{a=1}^{3} F_a T_a$$

Ahriche et.al. (2014) use the invariance property of F_a without proof

- Construction of general dimensional sphaleron unitary matrix

Express U^∞ as the multiplication of two Wigner-D matrices

$$U^\infty_{mn}(\mu,\theta,\phi) = \sum_{m'} D^J_{mm'}(\omega_-, -\theta, \mu) D^J_{m'n}(\mu, \theta, \omega_+),$$

$$\omega_\pm = -\mu \pm (\phi - \frac{\pi}{2}) \quad .$$

We demonstrate that F_a is invariant when $J = \left[\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right]$.

The general sphaleron energy expression

$$
\begin{aligned}
E_{sph} &= E(\mu = \frac{\pi}{2}) - E(\mu = -\frac{\pi}{2}) \\
&= \frac{4\pi\Omega}{g}\int d\xi \left[\frac{1}{4}F^a_{ij}F^a_{ij}(\xi,\mu=\frac{\pi}{2}) + \frac{1}{4}f_{ij}f_{ij}(\xi,\mu=\frac{\pi}{2}) + (D_iH)^\dagger(D_iH)(\xi,\mu=\frac{\pi}{2}) \right. \\
&\quad \left. +(D_i\Phi)^\dagger(D_i\Phi)(\xi,\mu=\frac{\pi}{2}) + V(H,\Phi)(\xi,\mu=\frac{\pi}{2}) - V(H,\Phi)(\xi,\mu=-\frac{\pi}{2})\right].
\end{aligned}
$$

Yanda Wu (TDLI/SJTU)

The field EOMs

$$f'' + \frac{2}{\xi^2}(1-f)\Big[f(f-2) + f_3\left(1+f_3\right)\Big] + (1-f)(\frac{v^2h^2}{4\Omega^2} + \alpha\phi^2) = 0,$$

$$f_3'' - \frac{2}{\xi^2}\left[3f_3 + f(f-2)\left(1+2f_3\right)\right] + (\frac{v^2}{4\Omega^2}h^2 + \beta\phi^2)(f_0 - f_3) = 0,$$

$$f_0'' + \frac{2}{\xi^2}\left(1-f_0\right) - \frac{g'^2}{g^2}(\frac{v^2}{4\Omega^2}h^2 + \beta\phi^2)(f_0 - f_3) = 0,$$

$$h'' + \frac{2}{\xi}h' - \frac{2}{3\xi^2}h[2(1-f)^2 + (f_0 - f_3)^2] - \frac{1}{g^2v^2\Omega^2}\frac{\partial V[h,\phi]}{\partial h} = 0,$$

$$\phi'' + \frac{2}{\xi}\phi' - \frac{8\Omega^2\phi}{3v_\phi^2\xi^2}[2\alpha(1-f)^2 + \beta(f_0 - f_3)^2] - \frac{1}{g^2v_\phi^2\Omega^2}\frac{\partial V[h,\phi]}{\partial \phi} = 0,$$

$$\alpha = \frac{[J(J+1) - J_3^2]v_\phi^2}{2\Omega^2}, \quad \beta = \frac{J_3^2 v_\phi^2}{\Omega^2}\,.$$