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Electroweak corrections to double Higgs production at the LHC

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In cooperation with:
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Introduction

- Introduction
- Calculation strategy
- Results
- Summary

Introduction to Higgs Boson

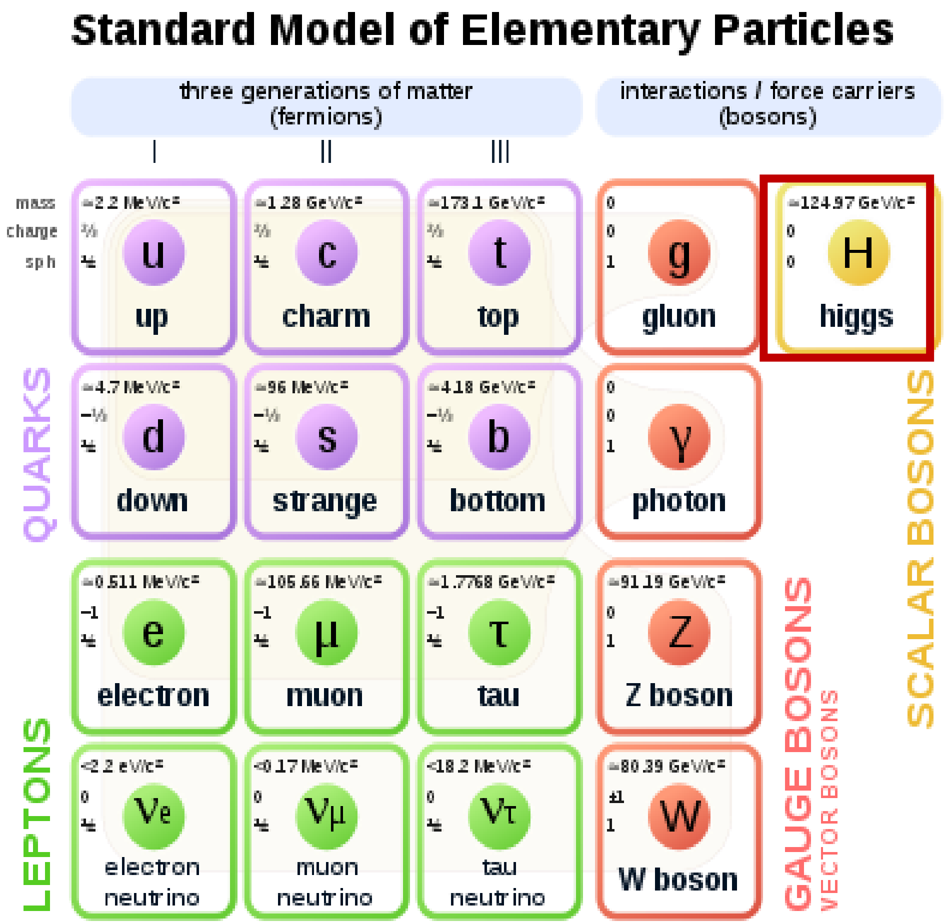
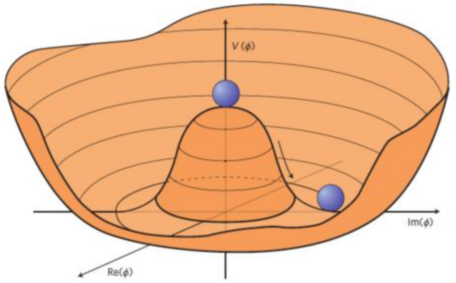


Figure taken from Wikipedia

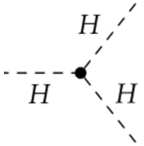
- Discovery of Higgs boson(2012,LHC): last fundamental particle in SM.
- Experiments at the ALTAS and CMS: agrees with result SM predicted.
- Problems not solved: electroweak symmetry breaking, Higgs coupling to SM particles/DM, hierarchy problem… Require new physics beyond SM.
- One promising way probing new physics: precision measurements of the properties of H (for e.g. Higgs self coupling).

Higgs self coupling

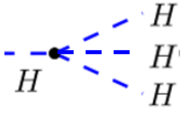


Plot taken from Ellis:1312.5672

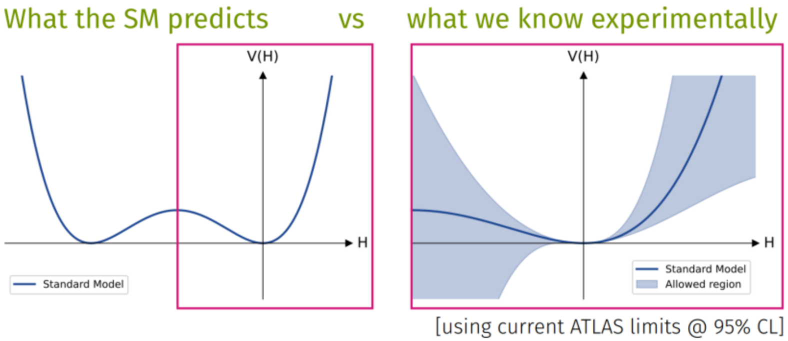
$$V(H) = \frac{m_H^2}{2}H^2 + \lambda_{HHH}^{SM} v H^3 + \frac{1}{4}\lambda_{HHHH}^{SM} H^4$$



Feasible at HL-LHC

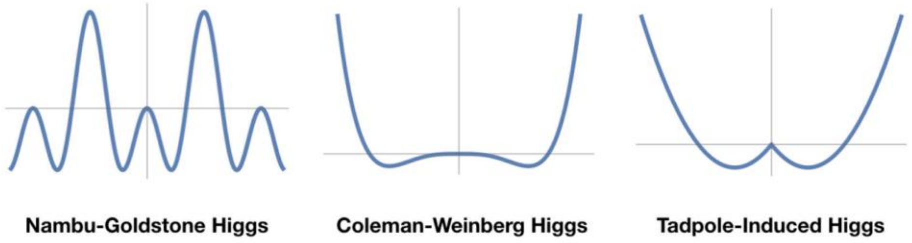


20??

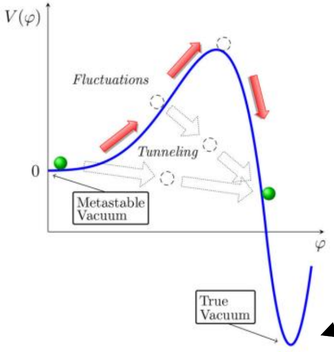


Plot taken from Moser: Higgs 2023

New physics



Agrawal et al: 1907.02078



What would Universe be like in such vacuum?

big consequences for the Universe

Markkanen et al: 1809.06923

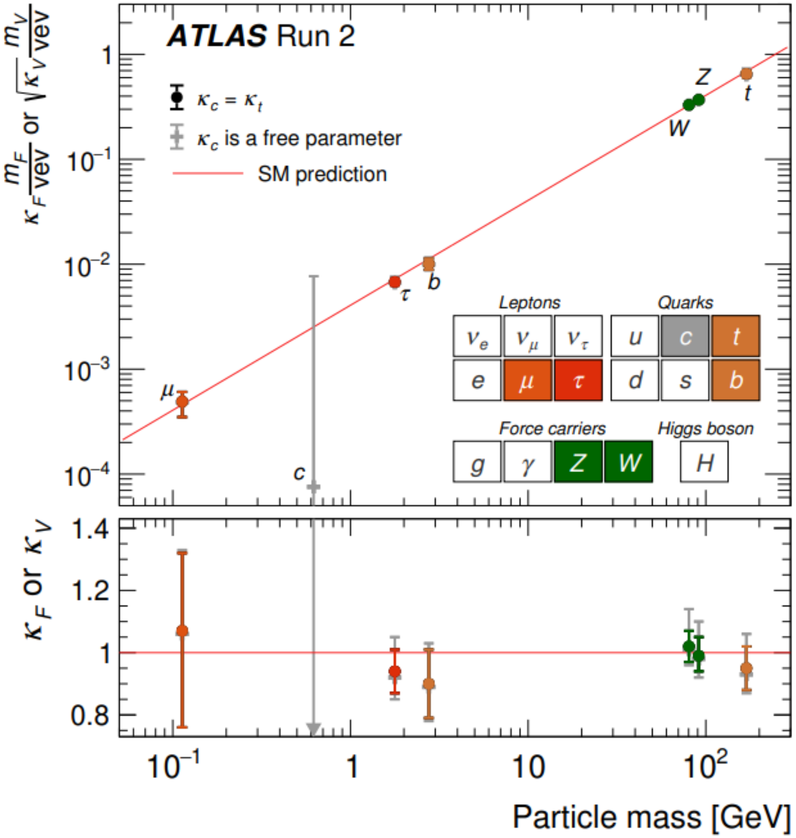
Measurements of Higgs boson coupling

😊 $g_{Hf\bar{f}}$, g_{HVV}

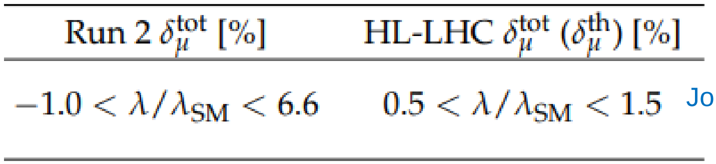
- ➢ can be measured with high precision.

☹ λ_{HHH}, λ_{HHHH}

- ➢ require multi-Higgs production, small cross sections.
- ➢ Mixed with complicated background.



ALTAS:2207.00092

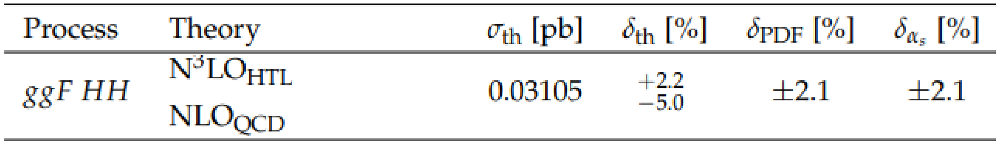


Run 2 δ_μ^{tot} [%]	HL-LHC δ_μ^{tot} (δ_μ^{th}) [%]
$-1.0 < \lambda/\lambda_{\text{SM}} < 6.6$	$0.5 < \lambda/\lambda_{\text{SM}} < 1.5$

Jones: LHEP 2023 (2023) 442

Status of QCD corrections

- NLO QCD
    - NLO QCD with full top-quark mass dependence, Borowka et al:1604.06447
    - NLO QCD matched to parton shower, Heinrich et al:1703.09252
    - NLO QCD with soft-gluon resummation, Ferrera et al: 1609.01691
- NNLO QCD
    - NNLO QCD in heavy-top limit (HTL) approximation, Florian et al:1305.5206
    - NNLO in HTL+ NLO with full top-quark mass dependence, Florian et al:2106.14050
    - NNLO QCD in HTL matched to parton shower, Alioli et al: 2212.10489
- NNNLO QCD
    - NNNLO QCD in HTL, Chen et al:1909.06808
    - NNNLO in HTL include the top-quark mass effects, Chen et al:1912.13001
    - NNNLO in HTL + NLO with full top-quark mass dependence + soft-gluon resummation, Ajjath et al:2209.03914

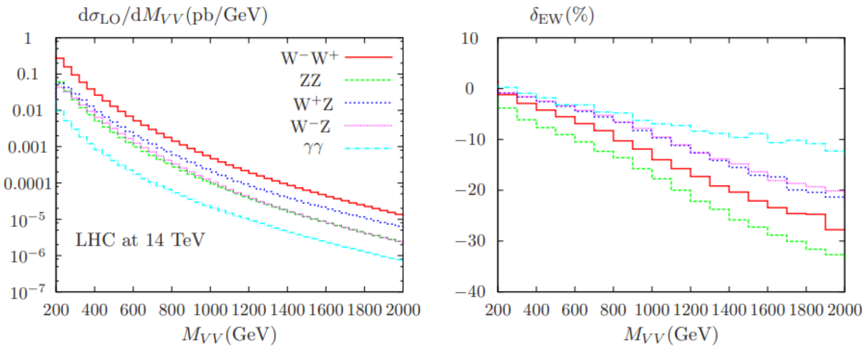


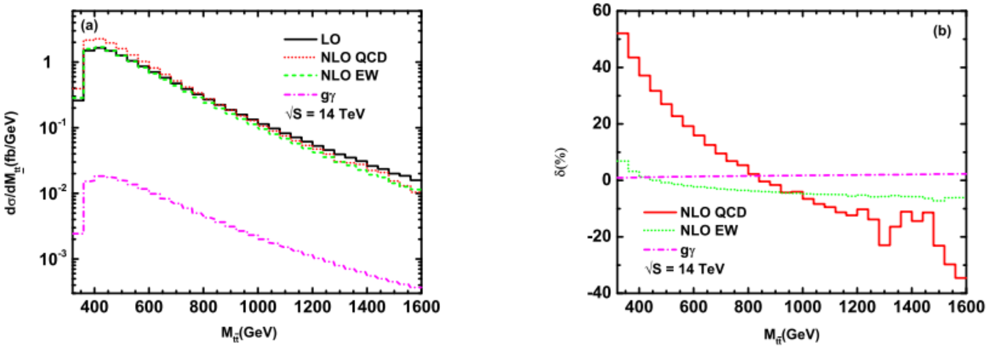
Process	Theory	σ_{th} [pb]	δ_{th} [%]	δ_{PDF} [%]	δ_{α_s} [%]
ggF HH	$\text{N}^3\text{LO}_{\text{HTL}}$ NLO_{QCD}	0.03105	$^{+2.2}_{-5.0}$	±2.1	±2.1

Jones: LHEP 2023 (2023) 442

Importance of EW corrections

- Unknown size of EW corrections
  - Biggest uncertainties from theoretical side
- NLO EW corrections are notably significant at high energy region
  - Sudakov enhancement: $\alpha \sim 0.7\% \rightarrow \frac{\alpha}{4\pi \sin\theta_W^2}\log^2(\frac{s}{m_Z^2})|_{s=2000^2} \sim 10\% \sim \alpha_s$

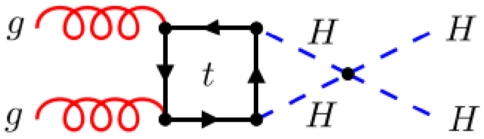




A Bierweiler et al:1305.5402

Zhang et al: 1407.1110

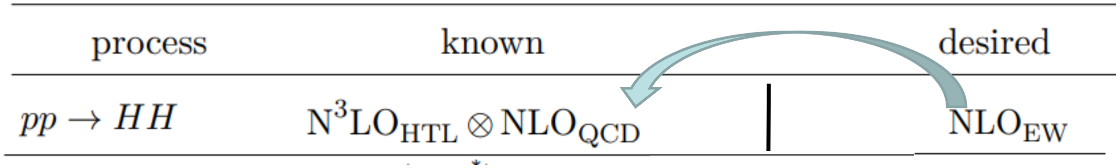
- Higgs quartic coupling only emerges at the NLO EW level
  - Constrained on λ_{HHHH}^{SM} indirectly from NLO EW correction



Status of NLO EW corrections

- Results in literature
  - Higgs self-coupling corrections, Borowka et al: 1811.12366
  - Two-loop box diagrams, Davies et al:2207.02587
  - Top-quark Yukawa corrections, Muhlleitner et al:2207.02524
  - HTL and Neglecting diagrams with massless fermion loops, Davies et al: 2308.01355
  - Top-Yukawa and Higgs self-coupling contributions: Heinrich et al:2407.04653
  - Higgs self-coupling contributions+QCD corrections: Li et al:2407.14716
- Our results
  - All two-loop diagrams and keeps mass effects, Bi et al:2311.16963

focal point in the 2015, 2017, 2019,
and 2021 Les Houches precision wish lists



process	known	desired
$pp \to HH$	$\mathrm{N^3LO_{HTL}} \otimes \mathrm{NLO_{QCD}}$	$\mathrm{NLO_{EW}}$

Calculation strategy

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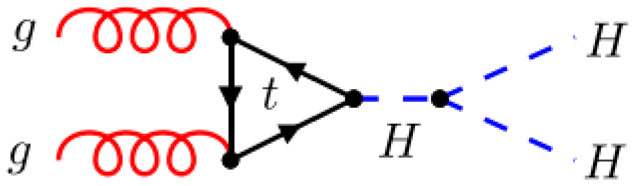
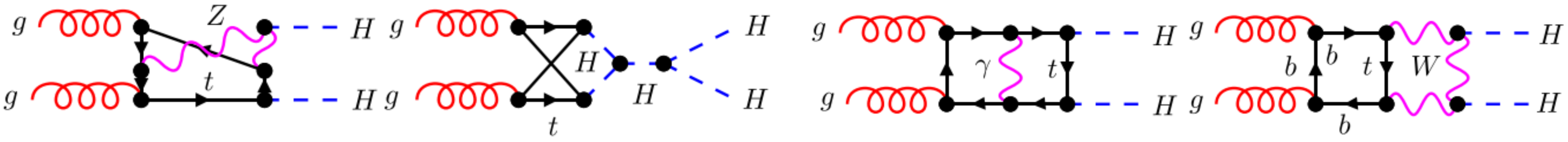
EW corrections to double H production at the LHC

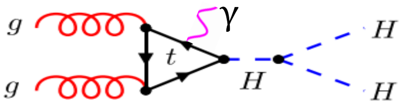
LO diagrams:



Typical Feynman diagrams at LO

NLO diagrams:

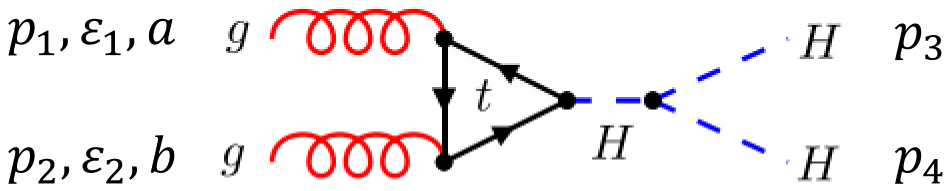




Forbidden due to Furry Theorem

Typical Feynman diagrams at NLO EW

Amplitudes of $gg \to HH$



- Amplitude Structure:

$$\mathcal{M}_{ab} = \delta_{ab}\epsilon_1^{\mu}\epsilon_2^{\nu}\mathcal{M}_{\mu\nu}$$

$$\mathcal{M}^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu} + \Delta_0^{\mu\nu} + \Delta_5^{\mu\nu}$$

- General decomposition at any number of loop.
- $\Delta_0^{\mu\nu}$: depends on p_1^{μ} or p_2^{ν}. No contribution at the matrix element level.
- $\Delta_5^{\mu\nu}$: depends on Levi-Civita tensor. No contribution at the matrix element squared level at NLO EW.
- F_1, F_2: Form factors.

Calculation of form factors

- Form factors can be expressed as:

$$F_{1,2}(x) = \sum_i d_i(x)\, FI_i(x)$$

$$x: \hat{s} = (p_1 + p_2)^2, \quad \hat{t} = (p_1 - p_3)^2.$$

- Reduce $FI_i(\hat{s})$ to master integrals (IBP):

$$\{FI_i(x)\} = \{\sum_k c_{i,k}(x) I_k(x)\}$$

  - ➢ $d_i(x)$ and $c_{i,k}(x)$ are analytic.
  - ➢ A huge number of I_k need to be calculated.
  - ➢ The number of $\{I_k\} < \{FI_i\}$.
  - ➢ The number of I_k is finite.
  - ➢ We can construct the different equations for I_k and solve them.

Different equations for I_k

Construct differential equations (DEs): $\vec{I}(x) = \{I_1(x), I_2(x) \dots I_N(x)\}$

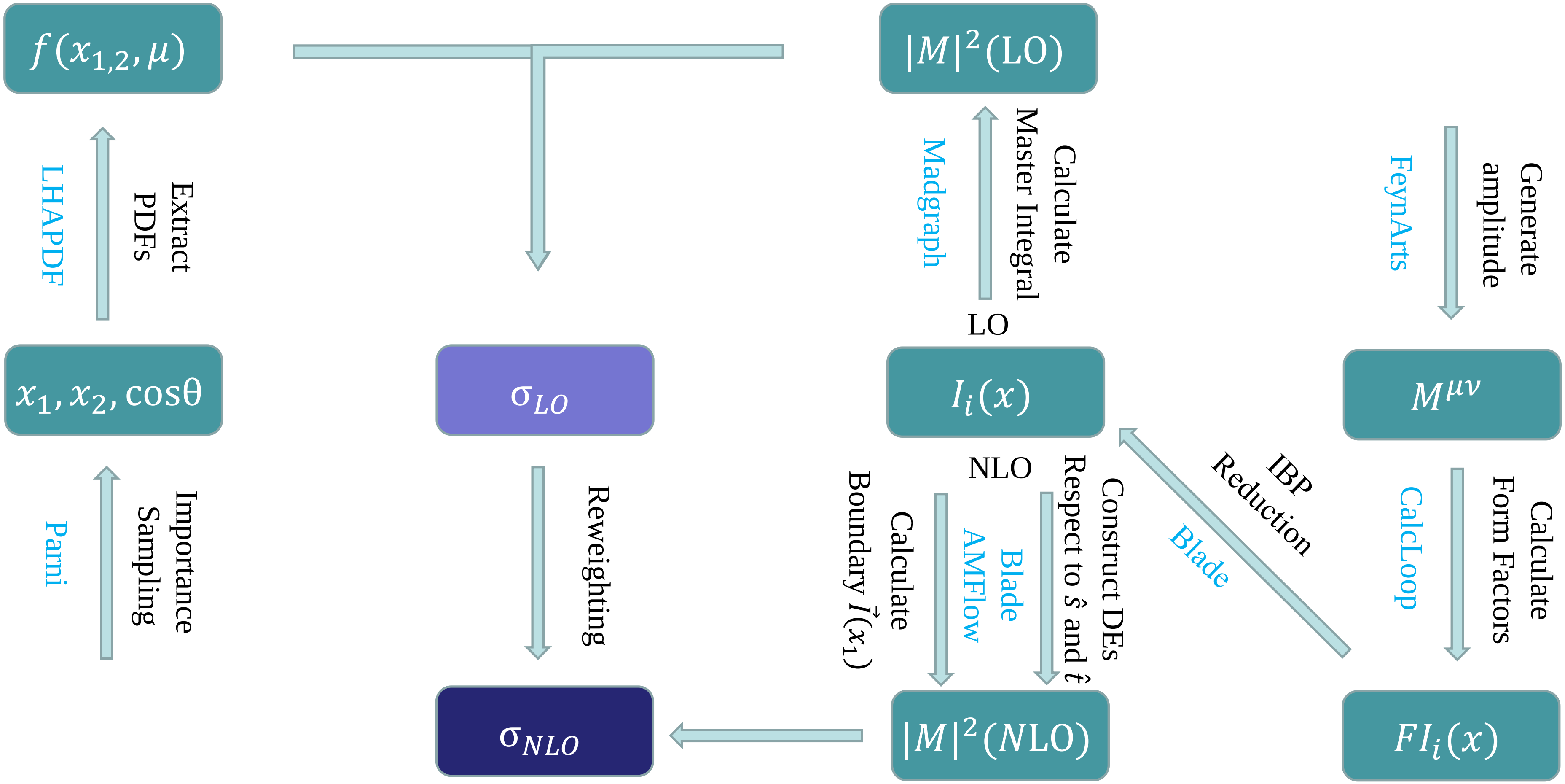
$$\frac{dI_m(x)}{dx} = \sum_n A_{m,n}(x)\, \mathrm{I}_\mathrm{n}'(x) \quad \xrightarrow{\text{IBP}} \quad \frac{d\vec{I}(x)}{dx} = A(x)\vec{I}(x)$$

- $\vec{I}(x)$ can be expanded as a power expansion near x_0,
  - regular:$S = \{0\}, k_0 = 0,$
  - singular:$S = \{-2\epsilon, 1 + \epsilon \dots\}, k_\mu \geq 0,$

$$I_i(x) = \sum_{\mu \in S} (x - x_0)^\mu \sum_{k=0}^{k_\mu} \log(x - x_0)^k \sum_{n=0}^{m} c_{i,\mu,k,n} (x - x_0)^n$$

- $c_{i,\mu,k,n}$ can be determined once any boundary $\vec{I}(x_1)$ are provided.
- $\vec{I}(x_1)$ can be determined by AMFlow Liu et al:2201.11669
- Taking adequate expansion order m, we can eventually achieve predictions with high precision.
- $\vec{I}(x)$ can be evaluated at any points of x efficiently.

Calculation flowchart



$f(x_{1,2},\mu)$

$|M|^2(\mathrm{LO})$

Extract PDFs

LHAPDF

Calculate Master Integral

Madgraph

Generate amplitude

FeynArts

LO

$x_1, x_2, \cos\theta$

σ_{LO}

$I_i(x)$

$M^{\mu\nu}$

Importance Sampling

Parni

Reweighting

NLO

Calculate Boundary $\vec{I}(x_1)$

Blade

AMFlow

Construct DEs Respect to $\hat{s}$ and $\hat{t}$

IBP Reduction

Blade

Calculate Form Factors

CalcLoop

σ_{NLO}

$|M|^2(\mathrm{NLO})$

$FI_i(x)$

Calculation strategy

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Input Paramaters

$$m_t = 172.69 \text{ GeV}$$

PDG2022

$$\frac{m_H^2}{m_t^2} = \frac{12}{23}, \quad \frac{m_Z^2}{m_t^2} = \frac{23}{83}, \quad \frac{m_W^2}{m_t^2} = \frac{14}{65},$$

$$G_\mu = 1.166378 \times 10^{-5} \ \text{GeV}^{-2}$$

$$\alpha = \frac{\sqrt{2}}{\pi} G_\mu m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right)$$

CKM=1

PDFs: NNPDF31_nlo_as_0118

on-shell renormalization: masses and fields; G_μ-scheme: Electromagnetic coupling

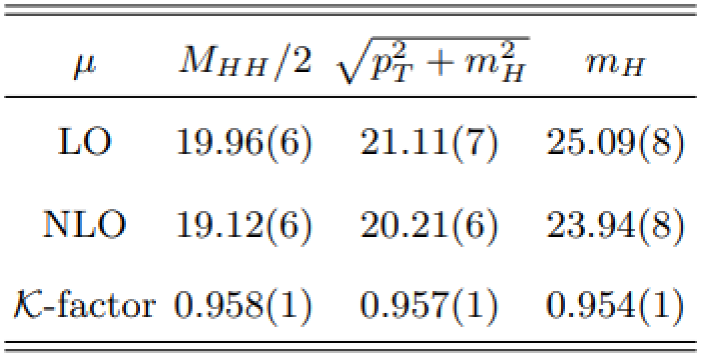
Denner et al:1912.06823

D=4-2ε ， ε =±1/1000

$$\sigma(\varepsilon)=a_0+ a_1\varepsilon + a_3\varepsilon^2 + \cdots$$

$$\sigma(0)\sim \frac{\sigma(+1/1000)+\sigma(-1/1000)}{2} = a_0+a_3\varepsilon^2+\ldots$$

Results: Total cross sections

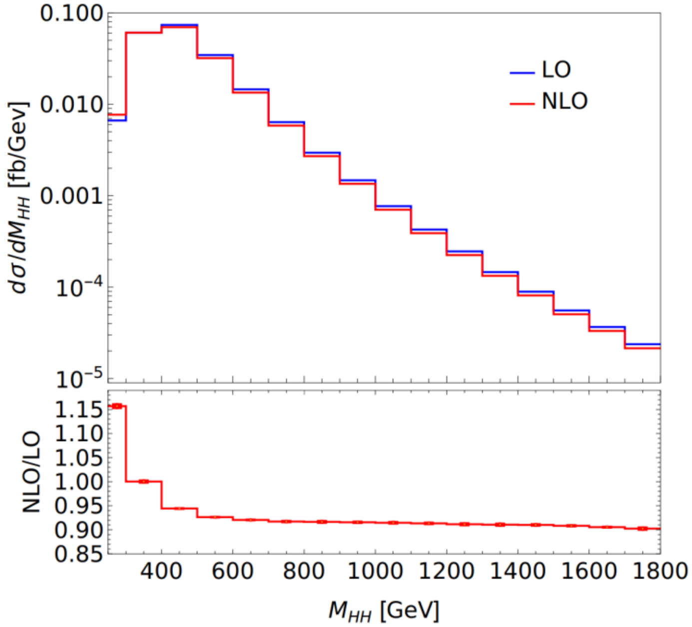


μ	$M_{HH}/2$	$\sqrt{p_T^2+m_H^2}$	m_H
LO	19.96(6)	21.11(7)	25.09(8)
NLO	19.12(6)	20.21(6)	23.94(8)
$\mathcal{K}$-factor	0.958(1)	0.957(1)	0.954(1)

LO and NLO EW corrected integrated cross sections (in fb) 14 TeV LHC.

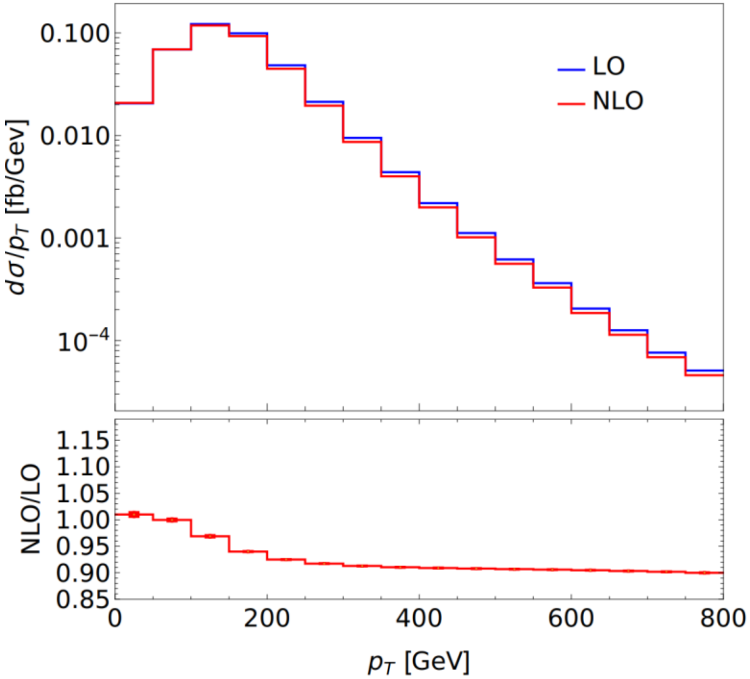
- Differences with varying scale choices are around 20%.
    - Huge scale uncertainties. Can be reduced by including QCD corrections.
- K-factor is insensitive to the scale choice.
    - EW corrections beyond NLO are on the order of a few thousandths.
- The statistical uncertainty for the K-factor is smaller than that of $\sigma_{LO,NLO}$.
    - K-factor can get a controllable error with far fewer events.

Results: Differential cross sections



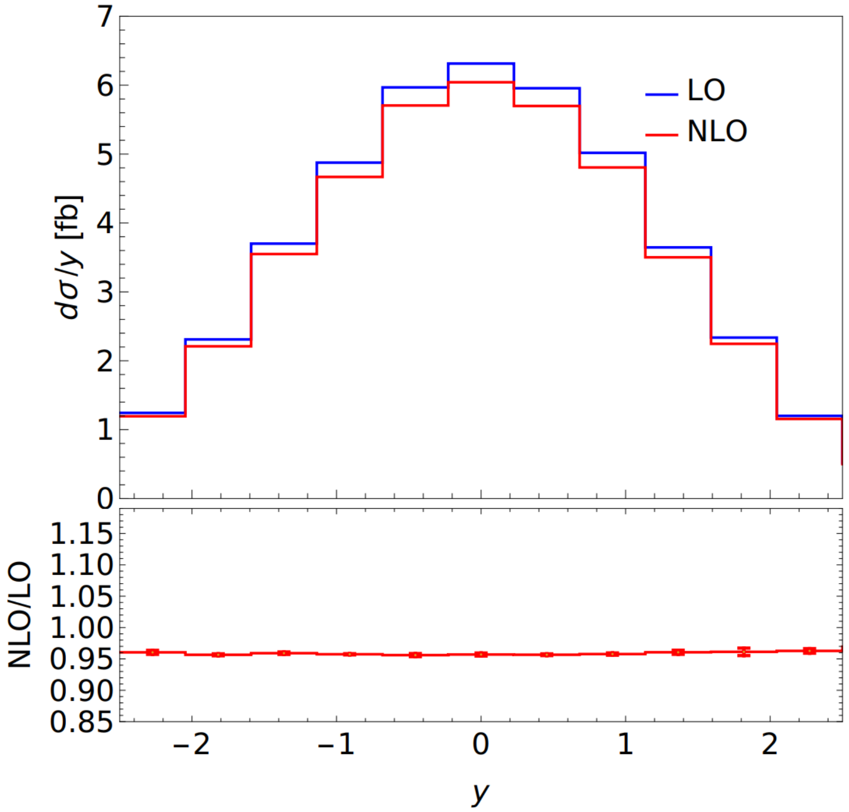
- Big positive corrections at the HH threshold.
  - Enhancement due to $\sigma_{LO}\left(\sqrt{\hat{s}} = 2m_H\right) \sim 0$.
- -10% correction at high energy region.
  - EW Sudakov effects.
- Tiny cross section at high energy region
  - Gluon PDFs are highly suppressed at high energy region.

Results: Differential cross sections



- Positive corrections at the beginning of the spectrum.
  - ➢ The events in this region are mixed with high $\sqrt{\hat{s}}$ and low $\sqrt{\hat{s}}$.
- -10% correction at high energy region.
  - ➢ EW Sudakov effects.

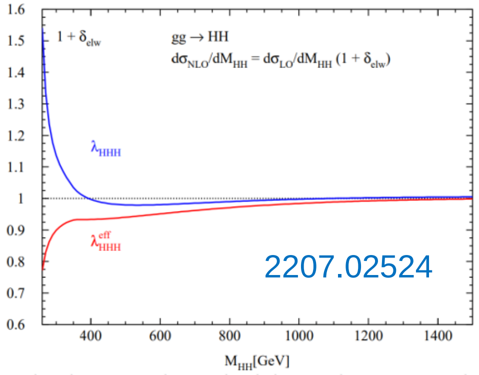
Results: Differential cross sections

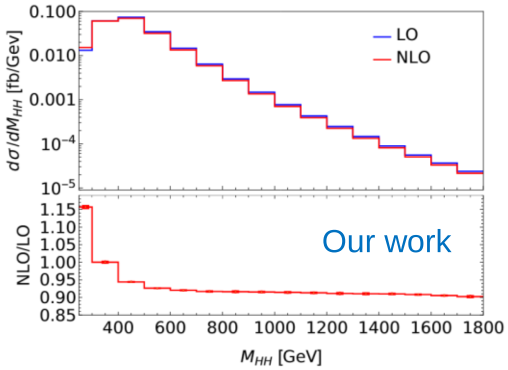


- Flat corrections at around -4%.
  - ➢ Similar to the total cross section

Results: comparisons with other publication

- Top-quark Yukawa corrections, 2207.02524

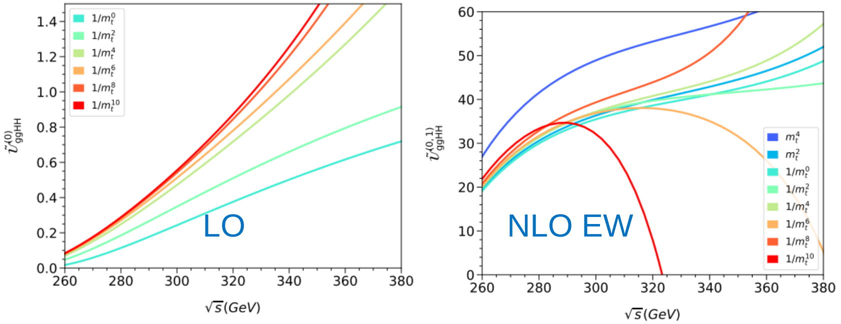




- ➢ Similar Enhancement at Threshold
- ➢ Differences appear at the tail

- HTL and neglecting diagrams with massless fermion loops, 2308.01355

2308.01355



- ➢ HTL doesn't show a convergent behaviour.
- ➢ At $\sqrt{\hat{s}}$=260 GeV, our full results revel the correction is 34% and 57% once neglect the diagram contains only mass less fermion

- Talk by prof. Wang, 2407.14716, 2407.04653
  - ➢ ~+1% corrections when only considering Top-Yukawa corrections and Higgs self coupling corrections.
  - ➢ Our full results revel the correction is ~-4%.

Summary

- Higgs self coupling is important to identify the Higgs potential and to probe new physics.
- The study of σ(HH) is the best way to extract the Higgs self coupling.
- Our full calculation includes all the diagrams and all the mass effects.
- -4% EW corrections at total cross section level.
- For dimensionful observables, EW corrections reach up to +15% at the beginning of the spectrum and −10% in the tail.
- Our results suggest that the remained uncertainties from theoretical side is overall about few percent and it's precise enough for the measurements at the HL-HLC.

Thanks for your attention!