Shedding Light on Hadronization by Quarkonium Energy Correlator

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Based on: Chen, Liu, Ma, Phys.Rev.Lett. 133 (2024) 19, 191901

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Outline

I. Motivation

II. Quarkonium energy correlator

III. e^+e^- annihilation

IV. pp collision

V. Summary

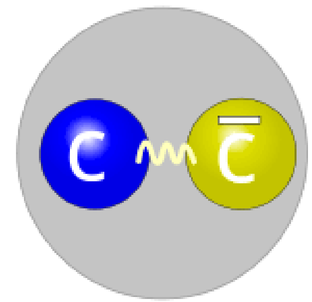
Motivation

➢ Heavy quarkonium

- **Bound state of $Q\bar{Q}$ pair under strong interaction**

  eg: $J/\psi\ \ \psi', \chi_{cJ}, \Upsilon(nS) \cdots$

  - The simplest system in QCD: two-body problem
  - “Hydrogen atom in QCD”, “an ideal laboratory in QCD”



- **A non-relativistic QCD system: $v^2 \ll 1$**
  - Charmonium: $\mathrm{m}{\sim}1.3\mathrm{Ge}V,\ v^2 \approx 0.3$
  - Bottomonium: $m{\sim}4.5GeV,\ v^2 \approx 0.1$

- **Multiple well-separated scales :**
  - Quark mass: M
  - Momentum: $\mathrm{M}v$
  - Energy: Mv^2

$$\mathrm{M} \gg \mathrm{Mv} \gg \mathrm{Mv}^2 \sim \Lambda_{\mathrm{QCD}}$$

- Involving both perturbative and nonperturbative physics
- Production is ideal to understand hadronization: why and how quarks become hadrons?
- NRQCD factorization Bodwin, Braaten, Lepage, 9407339

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_n d\hat{\sigma}_n(P_H)\langle\mathcal{O}_n^H\rangle$$

Production of a heavy quark pair Expansion in: α_s	Hadronization (LDMEs) Expansion in: v

- n: quantum numbers of the pair: ${}^{2S+1}L_J^{[c]}$
- Hadronization: remains largely unknown: amount of energy released? Energy Distribution?

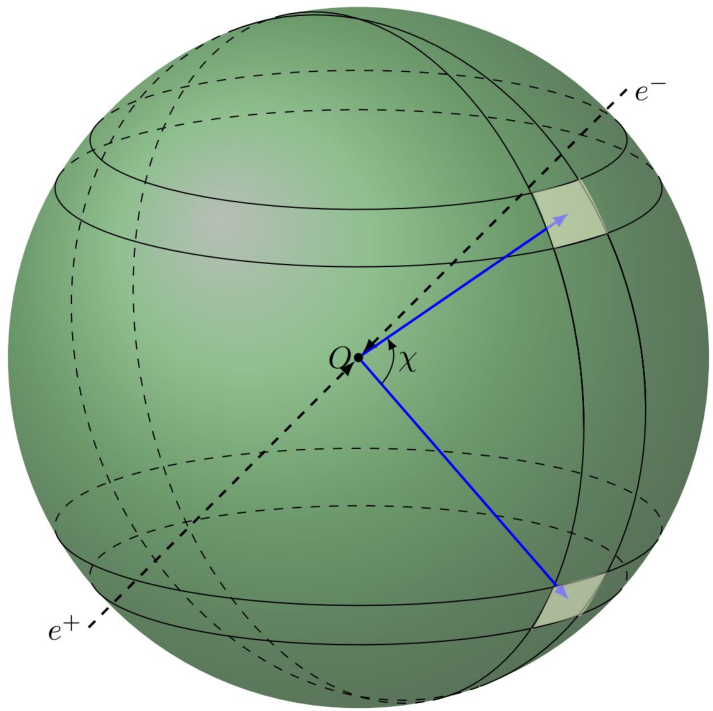
➢Energy-energy correlator (EEC)

☐ Angular correlation weighted by energy deposit in the Calorimeter

Basham, Brown, Ellis and Love, Phys. Rev. Lett. 41, 1585 (1978).

$$\frac{d\Sigma}{d\cos\chi} = \sum_{i,j} \int \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos\chi) d\sigma,$$

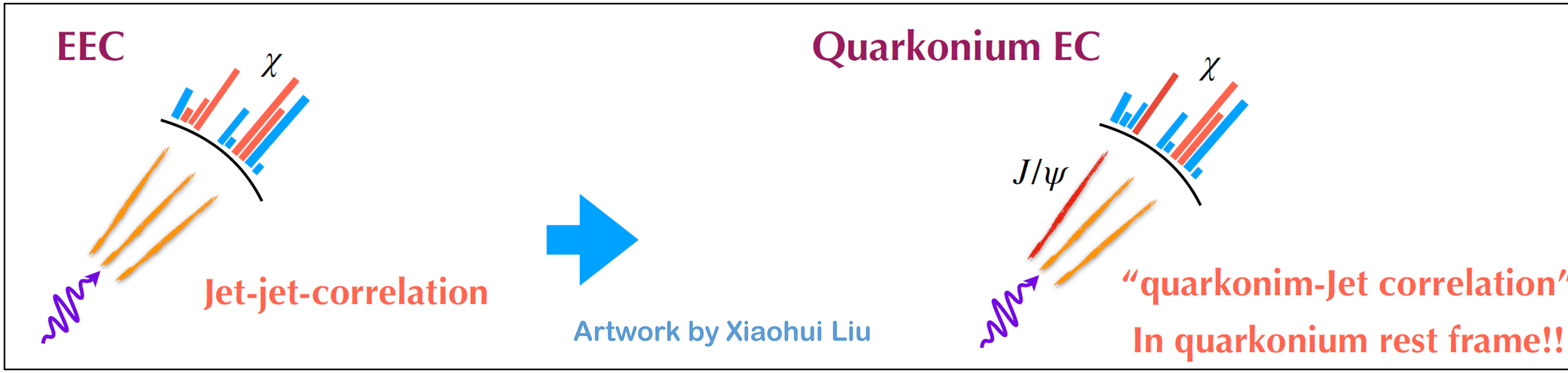
- An infrared safe observable
- Nonperturbative effect is supposed to be small
- Calculable in perturbative QCD



arXiv:1801.02627

➢Quarkonium Energy Correlator

Chen, Liu, Ma, arXiv:2405.10056



$$\Sigma_{\mathrm{EEC}} = \frac{1}{\sigma}\int d\sigma \sum_{i,j} \frac{E_i E_j}{Q^2}\delta(\cos\theta_{ij} - \cos\chi) \Rightarrow \Sigma_{\mathrm{QEC}} \propto \frac{1}{\sigma_{J/\psi}}\int d\sigma_{J/\psi} \sum_{i} \frac{E_i}{M}\delta(\cos\theta_{i} - \cos\chi)$$

~ average energy at the angle χ

- $\Sigma_{\mathrm{QEC}} = \Sigma_{\mathrm{QEC,P.T.}} + \Sigma_{\mathrm{QEC,N.P.}}$
- Hadronization could be large
- Probe the average energy emitted during the hadronization
- Distinguish between different production mechanisms (CS or CO mechanisms in the NRQCD)

Quarkonium energy correlator

➢Definition

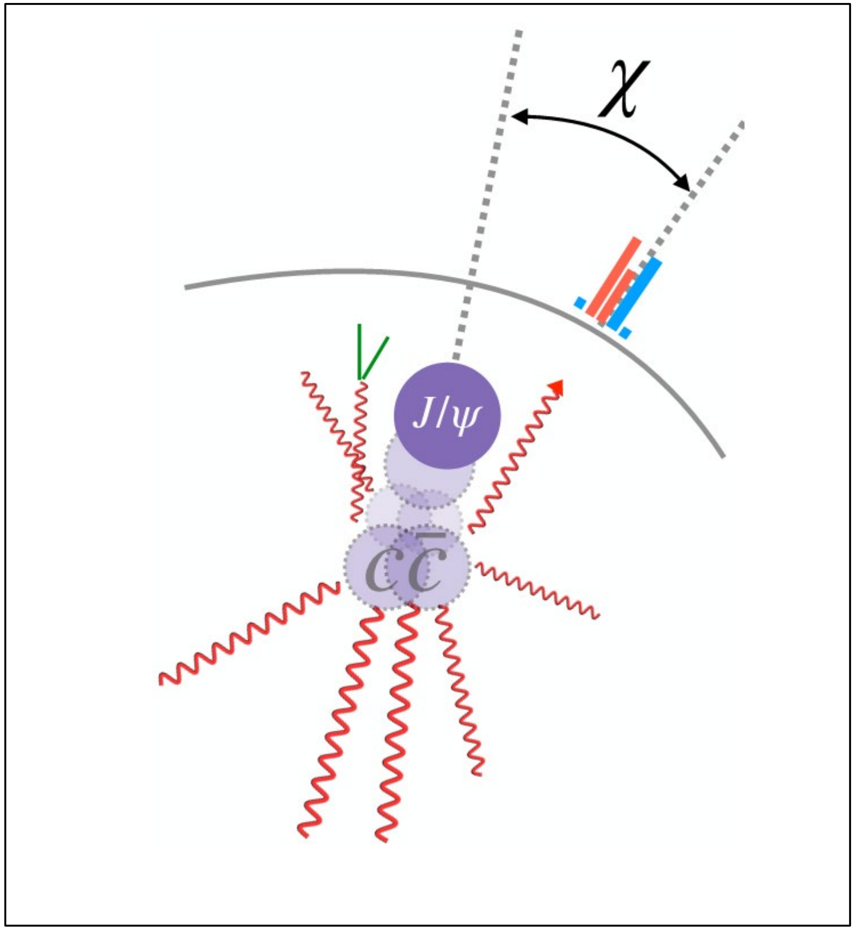
$$\Sigma(\cos\chi) = \int d\sigma \sum_i \frac{E_i}{M}\delta(\cos\chi - \cos\theta_i) ,$$

$d\sigma$: the differential cross section for generating J/ψ

χ : the angular of detector relative to the flying direction of J/ψ

E_i: the total energy carried by particles propagating at the angle χ

M: the mass of the J/ψ

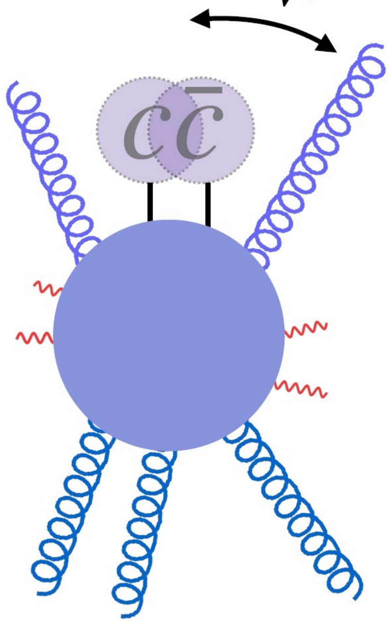


➢ Generic J/ψ production configuration

COM frame

$$r \equiv \frac{M^2}{\hat{s}} \ll 1$$

$$\theta \gtrsim 2\sqrt{r}$$



$$E_s \sim \mathcal{O}(M), E_{J_{near}} \sim E_{J_{away}} \sim \mathcal{O}\left(\frac{\sqrt{\hat{s}}}{2}\right)$$

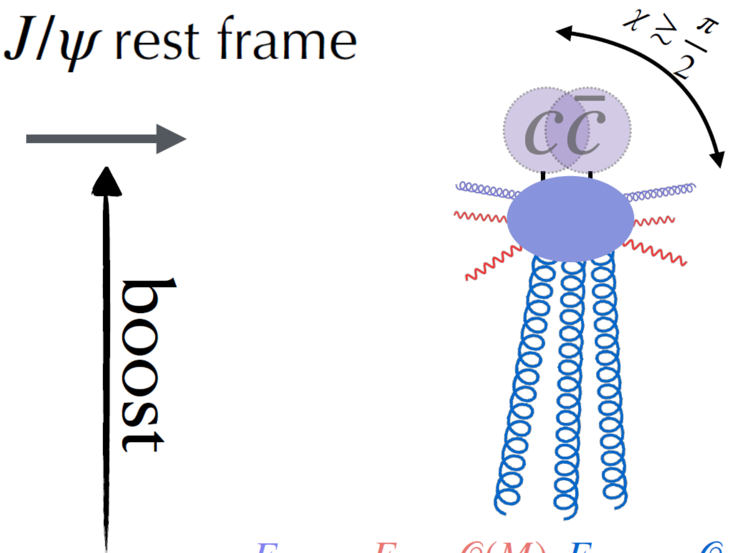
- **dead-cone effects**

Dokshitzer, Khoze and Troian, J. Phys. G 17 (1991)

$$d\sigma_{Q\to Qg} \sim \frac{\alpha_s C_F}{\pi} \frac{dE_g}{E_g} \frac{\theta^2 d\theta^2}{[\theta^2 + \theta_0^2]^2}$$

$$\theta_0 \sim \frac{M}{E_{J/\psi}} \sim \frac{2M}{\sqrt{\hat{s}}} = 2\sqrt{r}$$

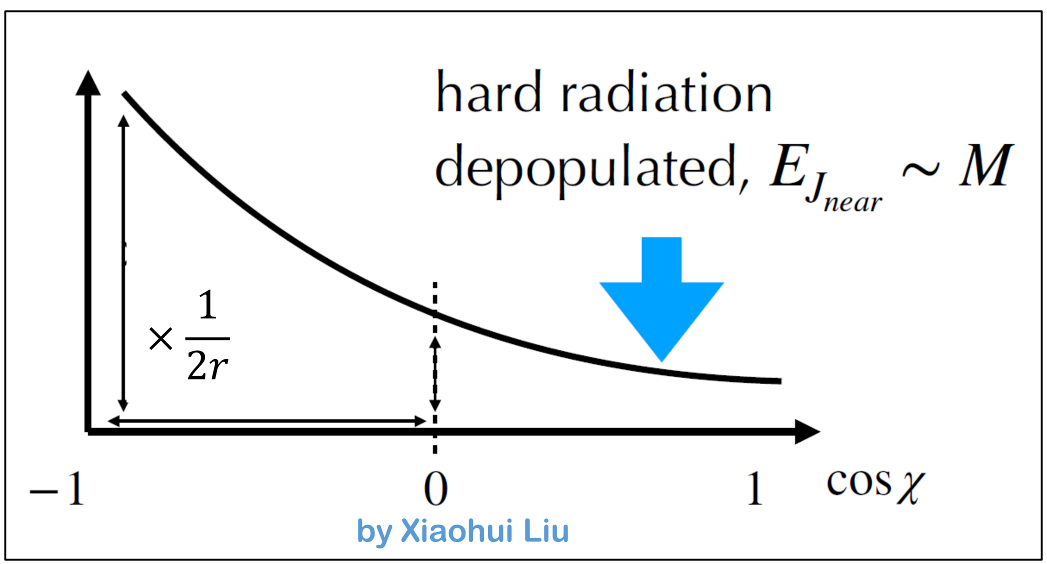
J/ψ rest frame



$E_{J_{near}} \sim E_s \sim \mathcal{O}(M), E_{J_{away}} \sim \mathcal{O}\left(\frac{\hat{s}}{2M}\right)$

- **boost factor** $a \sim \frac{\sqrt{\hat{s}}}{M} = \sqrt{\frac{1}{r}}$
- $E_{J_{away}} \sim a\frac{\sqrt{\hat{s}}}{2} = \frac{\hat{s}}{2M}$
- $E_{J_{near}} \sim O(M)$
- $\chi \geq O(\frac{\pi}{2})$

COM frame



- $\Sigma_{\mathrm{QEC,P.T.}}$ **drops rapidly by a factor of**

$$O\left(E_{J_{away}}/E_{J_{near}}\right) \sim O\left(\frac{1}{2r}\right)$$

e^+e^-annihilation

- $e^+e^- \to \gamma^* \to J/\psi + X$
  - Leading $\Sigma_{P.T.}$: $e^+e^- \to \gamma^* \to c\bar{c}[^3S_1^{[1]}]+g+g$

Defining: $r = \frac{4m_c^2}{s}, \quad z = \frac{2p_\psi \cdot q_{\gamma^*}}{s}, \quad x_i = \frac{2k_i \cdot q_{\gamma^*}}{s}, \quad z_i = \frac{2k_i \cdot p_\psi}{4m_c^2},$

$$\Rightarrow \quad \Sigma_{P.T.}^{^3S_1^{[1]}}(\cos\chi) = \frac{256\pi e_c^2 \alpha^2 \alpha_s^2}{81 s} \frac{\mathcal{O}^{J/\psi}(^3S_1^{[1]})}{(2m_c)^3} r^2 \times \int_{2\sqrt{r}}^{1+r} dz \int_{x_-}^{x_+} dx_1 f(z, x_1) \sum_{i=1}^{2} \frac{z_i}{2} \delta(\cos\chi - \cos\theta_i) \,,$$

with $z_1 = \frac{1}{2r}(x_1 - x_2 + z - 2r)$, $z_2 = \frac{1}{2r}(x_2 - x_1 + z - 2r)$,

$x_2 = 2 - x_1 - z$, $x_\pm = \frac{1}{2}\left(2 - z \pm \sqrt{z^2 - 4r}\right)$

$$f(z, x_1) = \frac{1}{(2-z)^2}\Big(\frac{1}{r} + \frac{(2+x_1)x_1}{(1-x_2-r)^2} + \frac{(2+x_2)x_2}{(1-x_1-r)^2} + \frac{2(1-z)(1-r)}{r(1-x_2-r)(1-x_1-r)} + \frac{(2-z)^2((z-r)^2-1)}{(1-x_2-r)^2(1-x_1-r)^2} + \frac{6(1+r-z)^2}{(1-x_2-r)^2(1-x_1-r)^2}\Big),$$

Yuan, Qiao, Chao, 9703438

☐ Leading $\Sigma_{N.P.}$: $e^+e^- \to \gamma^* \to c\bar{c}[^1S_0^{[8]}, ^3P_J^{[8]}] + g$

$$\Rightarrow \quad \Sigma^{ch}_{N.P.}(\cos\chi) = \int d\Phi_g \frac{k^0}{M} \delta(\cos\chi - \cos\theta) \sum_\lambda \hat{\sigma}^{ch}_\lambda A^{ch}_\lambda(\theta, \phi),$$

$A^{ch}_\lambda(\theta,\phi)$: **the transition amplitude for $c\bar{c}[ch] \to J/\psi + g$**

Φ_g : **the gluon phase space**

$$d\Phi_g = \frac{\mathrm{d}^4 k}{(2\pi)^4}(2\pi)\delta_+(k^2) = \frac{k^0 \mathrm{d}k^0 \mathrm{d}^2\Omega}{2(2\pi)^3},$$

$\hat{\sigma}^{ch}_\lambda$: **the total cross section that produces the $c\bar{c}[ch]$ pair with polarization λ**

For ${}^1S_0^{[8]}$ channel

- **The intermediate $c\bar{c}[{}^1S_0^{[8]}]$ state is unpolarized**
- **The soft gluon emission should be isotropic**
- $A^{{}^1S_0^{[8]}}(\theta,\phi) = \frac{1}{4\pi}G_0$, $\frac{1}{4\pi}\int d\Phi_g G_0 \sim \mathcal{O}(\langle\mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle)$

$$\Sigma_{N.P.}^{{}^1S_0^{[8]}}(\cos\chi) = \hat{\sigma}^{{}^1S_0^{[8]}} \int d\Phi_g \frac{k_0}{M}\frac{G_0}{4\pi}\delta(\cos\chi - \cos\theta) = \hat{\sigma}^{{}^1S_0^{[8]}} \langle\mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle \frac{m_c\bar{v}_0}{2M}$$

$$= \bar{v}_0 \frac{\sigma^{{}^1S_0^{[8]}}}{4},$$

average energy emitted during hadronization

$$\sigma^{{}^1S_0^{[8]}} = \frac{32\pi^2\alpha^2\alpha_s e_c^2}{3s^2 m_c}(1-r)\langle\mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle$$

For ${}^3P_J^{[8]}$ channel

- **The intermediate $c\bar{c}[{}^3P_J^{[8]}]$ state with S = 1 and L = 1 is transmitted into the J/ψ through one electrical dipole (E1) interaction,**

$$\Rightarrow \quad \Sigma_{N.P.}^{{}^3P_J^{[8]}}(\cos\chi) = \int d\Phi_2 \frac{k^0}{M}\delta(\cos\theta - \cos\chi)\sum_{\lambda,L_z}\hat{\sigma}_{\lambda,L_z}^{{}^3P_J^{[8]}}[\{S=1,L=1\}^{(8)}] \times A_{\lambda,L_z}^{{}^3P_J^{[8]}}[\{S=1,L=1\}^{(8)}](\theta,\phi).$$

$$A_{\lambda,L_z}^{{}^3P_J^{[8]}}[\{S=1,L=1\}^{(8)}](\theta,\phi) \propto \sum_{S_z} A[c\bar{c}[(1L_z;1\lambda)] \to J/\psi(S_z)+g]A^*[c\bar{c}[(1L_z;1\lambda)] \to J/\psi(S_z)+g],$$

$$\Sigma_{N.P.}^{{}^3P_J^{[8]}}(\cos\chi) = \int d\Phi_g \frac{k^0}{M}\frac{G_1}{2\pi}\delta(\cos\chi - \cos\theta) \times 3\left(\frac{1+\cos^2\theta}{8}\hat{\sigma}_T^{{}^3P_0^{[8]}} + \frac{1-\cos^2\theta}{4}\hat{\sigma}_L^{{}^3P_0^{[8]}}\right)$$

$$\equiv \bar{v}_1 \times 3\left(\frac{1+\cos^2\chi}{16}\sigma_T^{{}^3P_0^{[8]}} + \frac{1-\cos^2\chi}{8}\sigma_L^{{}^3P_0^{[8]}}\right).$$

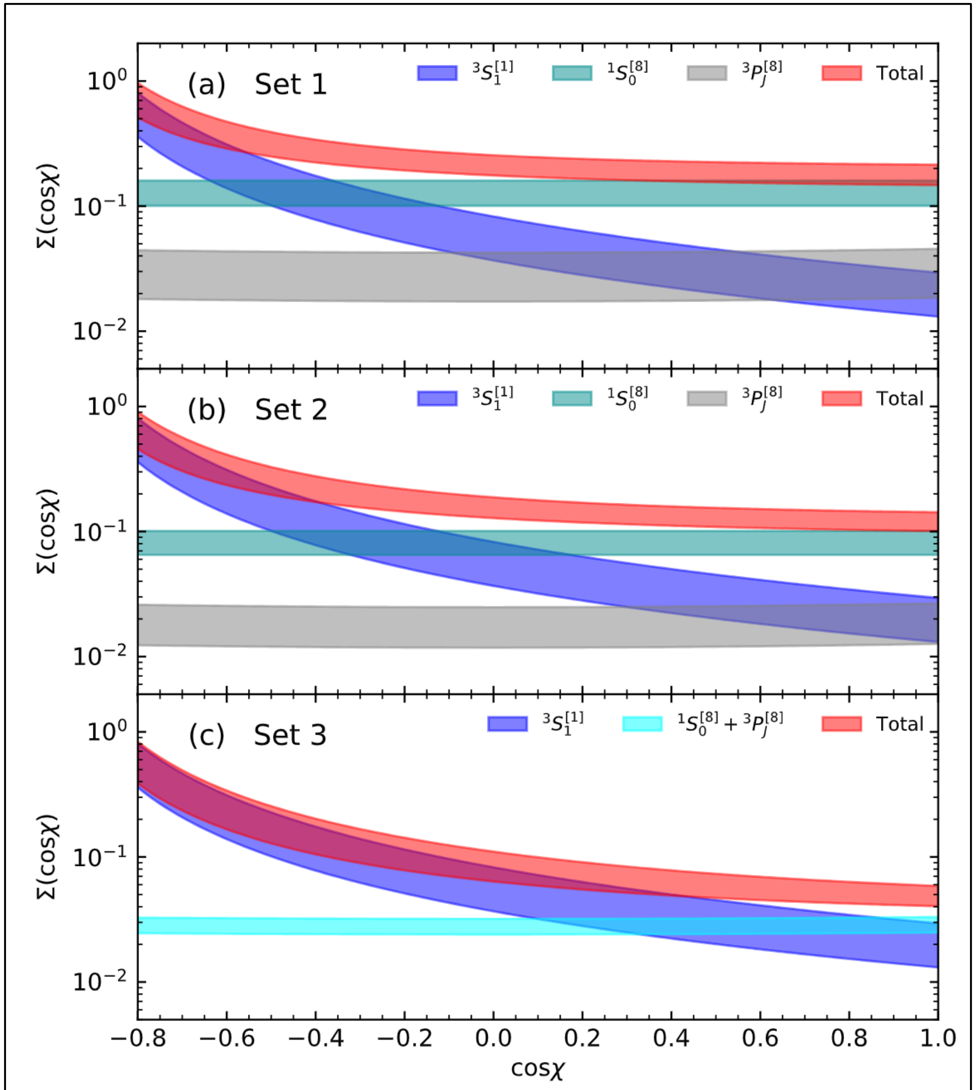
- **with** average energy emitted during hadronization

$$\sigma_T^{{}^3P_0^{[8]}} = -F\frac{2}{(1-r)^4}\left(3r^5 + r^4 + 12r^3 - 16r^2 - 4(r^2+3r+2)r^2\ln r + r - 1\right),$$

$$\sigma_L^{{}^3P_0^{[8]}} = -F\frac{1+r}{(1-r)^4}\left(r^4 - 22r^3 + 16r^2 + 8(r+2)r^2\ln r + 6r - 1\right),$$

$$F = \frac{32\pi^2\alpha^2\alpha_s e_c^2}{3s^2m_c^3}\langle\mathcal{O}^{J/\psi}({}^3P_0^{[8]})\rangle$$

➢ Predictions



$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle = 1.16\,\mathrm{GeV}^3$$

$$\bar{v}_0^2 = \bar{v}_1^2 = 0.25$$

TABLE I. NRQCD LDMEs used in this Letter.

	$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]})\rangle/m_c^2$ 10^{-2} GeV^3
Set 1[41]	8.9 ± 0.98	0.30 ± 0.12	0.56 ± 0.21
Set 2[42]	5.66 ± 0.47	0.177 ± 0.058	0.342 ± 0.102
Set 3	1.0		0.25

Eichten , Quigg, 9503356,
Chao, Ma, Shao, Wang, Zhang, 1201.2675,
Feng, Gong, Chang, Wang, 1810.08989.

- **Theoretical predictions for J/ψ energy correlator in e+e− collision with $\sqrt{s}$=10.6 GeV**

pp collision

➢ The gluon fragmentation mechanism

$$d\sigma_{A+B\to J/\psi+X}(p_\psi) \approx \int_0^1 dz d\hat{\sigma}_{A+B\to g+X}(\hat{p}/z,\mu_F) D_{g\to J/\psi}(z,\mu_F)\,,$$

$d\hat{\sigma}$: partonic production cross section for hadrons A and B to produce parton g

D: gluon fragmentation function

$$D_{g\to J/\psi}(z,\mu_F) = \sum_n d_{g\to c\bar{c}[n]}(z,\mu_F)\langle \mathcal{O}^{J/\psi}(n)\rangle$$

- Leading $\Sigma_{P.T.}$: one hard gluon is emitted in ${}^1S_0^{[8]}$ and ${}^3P_J^{[8]}$ channels

$$\Sigma_{P.T.}(\cos\chi) = \sum_n \int_0^1 dz d\hat{\sigma}_{A+B\to g+X}(\hat{p}/z,\mu_F)\hat{D}_{g\to c\bar{c}[n]}(z,\cos\chi,\mu_F)\langle \mathcal{O}^{J/\psi}(n)\rangle,$$

- **with**

$$\hat{D}_{g\to c\bar{c}[^1S_0^{[8]}]}(z,y,\mu_F) = \frac{5\alpha_s^2}{16}\frac{1}{6m_c^3}\frac{(1-z)^2}{(y+1)((y-1)z+2)^2}\times\Big((y-1)^2z^2+2(y-1)z+2\Big),$$

$$\hat{D}_{g\to c\bar{c}[^3P_J^{[8]}]}(z,y,\mu_F) = \frac{5\alpha_s^2}{16}\frac{1}{6m_c^5}\frac{1}{(y+1)((y-1)z+2)^2}\times\Big(-2\left(y^3-10y+9\right)z^3 + \left(3y^2-14y+25\right)z^2+(y-1)^2(2y+5)z^4+2(5y-7)z+6\Big),$$

$$y = \cos\chi$$

Leading $\Sigma_{N.P.}$: the soft radiation in the $^3S_1^{[8]}$ channel

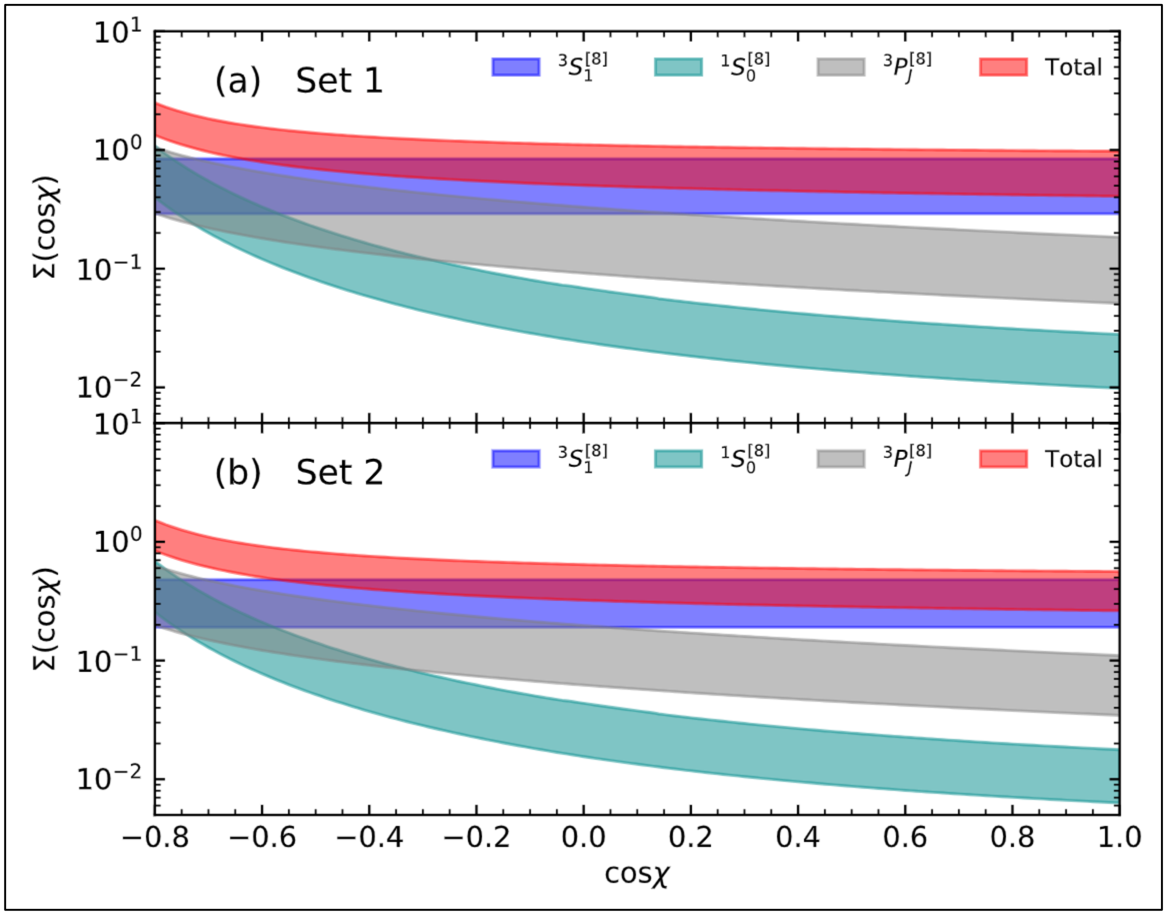
$$\Sigma_{N.P.}(\cos\chi) = \int_0^1 dz d\hat{\sigma}_{A+B\to g+X}(\hat{p}/z,\mu_F)\hat{D}_{g\to c\bar{c}[^3S_1^{[8]}]}(z,\cos\chi,\mu_F)\langle\mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle,$$

- **with**

$$\hat{D}_{g\to c\bar{c}[^3S_1^{[8]}]}(z,\cos\chi,\mu_F) = \int d\Phi_{gg}\frac{G_2}{4\pi}\frac{2k_1^0}{M}\delta(\cos\chi-\cos\theta)d^{\rm LO}_{g\to c\bar{c}[^3S_1^{[8]}]}(z,\mu_F)\frac{1}{\langle\mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle} \equiv \bar{v}_2\times\frac{1}{2}d^{\rm LO}_{g\to c\bar{c}[^3S_1^{[8]}]}(z,\mu_F),$$

$\bar{v}_2$ → average energy emitted during hadronization

➢ Predictions



- **Theoretical predictions for J/ψ energy correlator in pp collision.**

Summary

- The quarkonium energy correlator measures the energy flow at an angle distance χ from the quarkonium in its rest frame.
- The hadronization effect leads to a correction to the energy correlator comparable to that of the perturbative hard radiation in the region where $\cos\chi \gtrsim 0$.
- Quarkonium energy correlator provide a new insight to quarkonium hadronization.

Thank you!