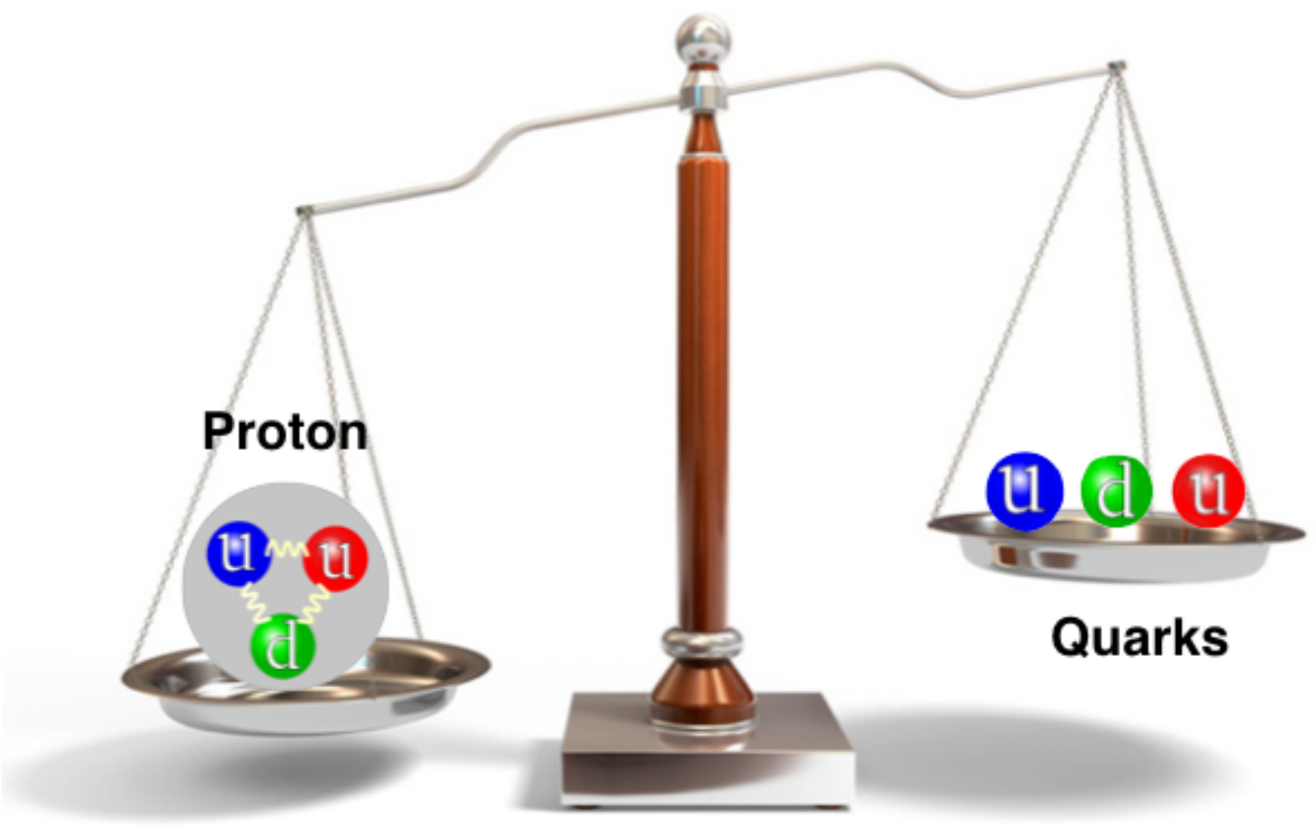
Strong interaction origin of hadron mass

Yi-Bo Yang

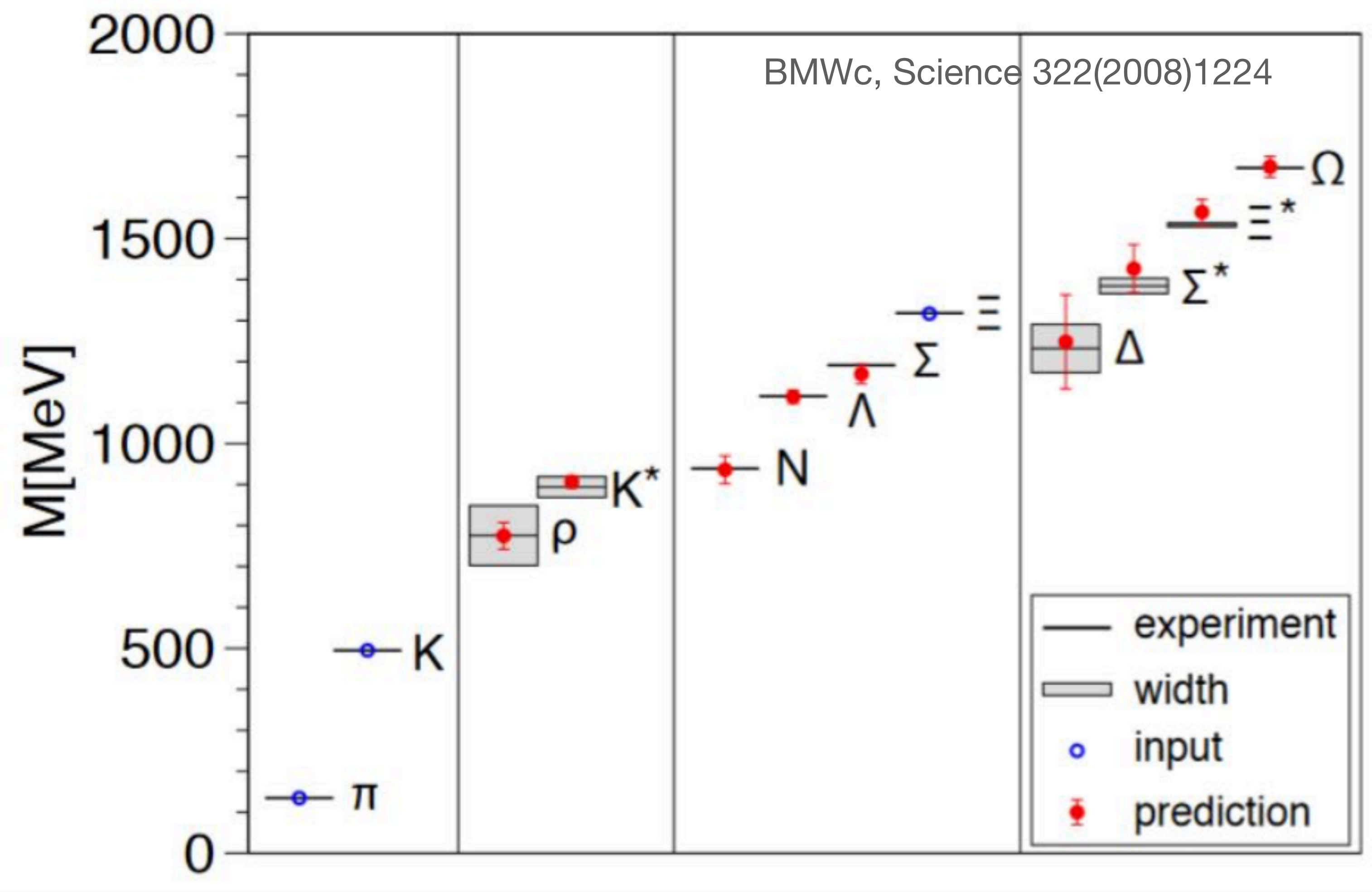
For CLQCD collaboration

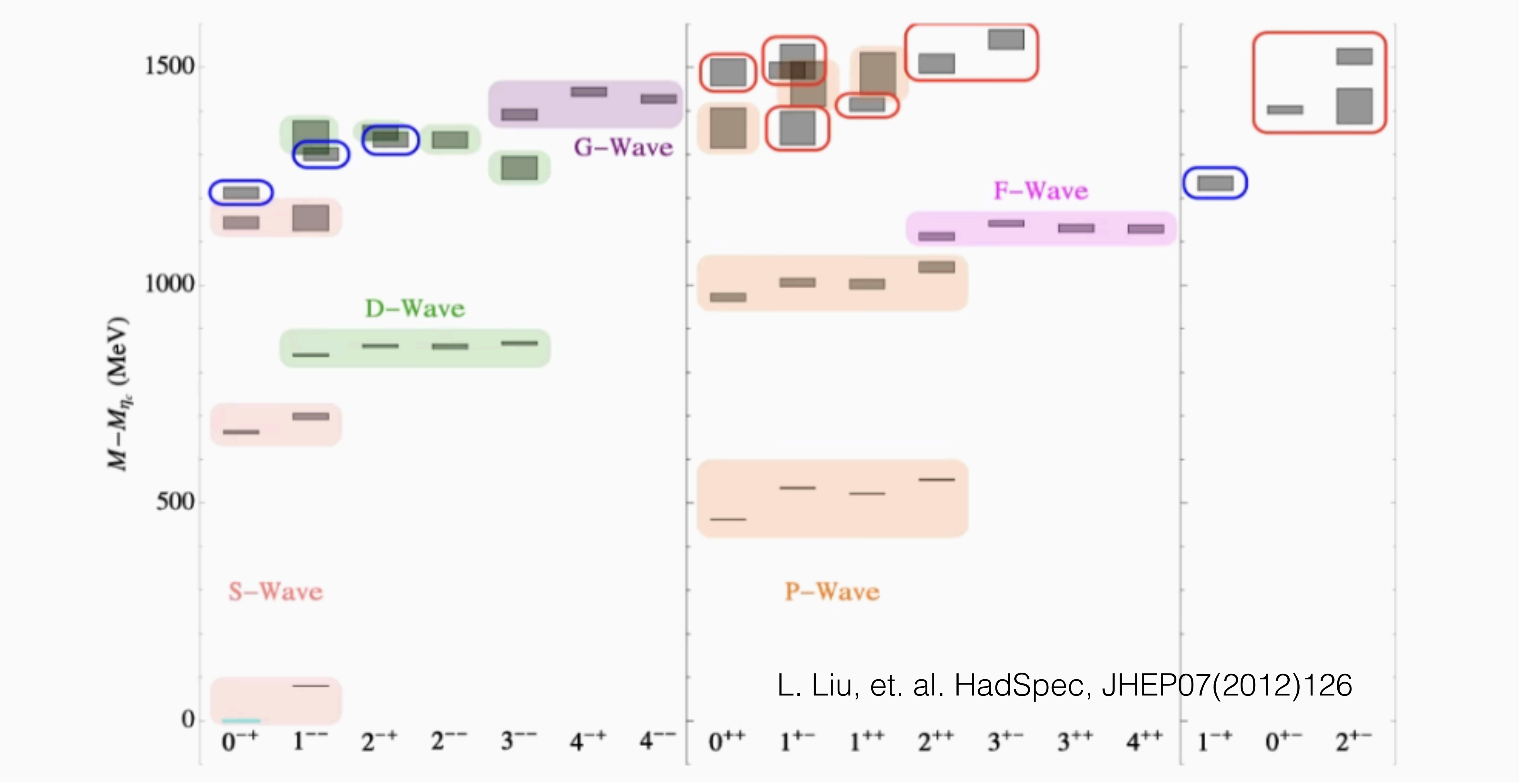
Hadron mass

and its origin



- Higgs boson provides very different coupling to six quark flavors;
- Quarks (and gluons) combine to form hundreds of hadrons with varying masses;
- What would be the origin of the mass gap between different hadrons?





Hadron mass

$$m_H = \langle T^\mu_\mu \rangle_{H,\vec{p}=0}$$

$$T_{\mu\nu} = \sum_q \frac{i}{2}\bar{q}\overleftrightarrow{D}_{(\mu}\gamma_{\nu)}q + \frac{1}{4}g_{\mu\nu}F^2 - F_{\mu\rho}F_{\rho\nu}$$

- When $D = 4$, the UV regularization will introduce an UV scale and break the conformal symmetry, and change T^μ_μ into:

$$T^\mu_\mu = \left[1 + \frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)\right]m\bar{\psi}\psi + \left[(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)\right]F^2,$$

where the terms proportional to α_s are the QCD quantum corrections.

At quantum level

- Under the dimension regularization, we have

$$T^\mu_\mu = m\bar{\psi}\psi - 2\epsilon\frac{F^2_{\mu\nu}}{4} + \mathcal{O}(\epsilon^2),\ \text{and}$$

$$F^2_{\mu\nu} = -\frac{1}{\epsilon}\left(\frac{\beta}{g}F^2_{\mu\nu,R} + 2\gamma_m m\bar{\psi}\psi\right) + \mathcal{O}(\epsilon^0);$$

See Y. Hatta, et.al., JHEP12(2018)008 as an example

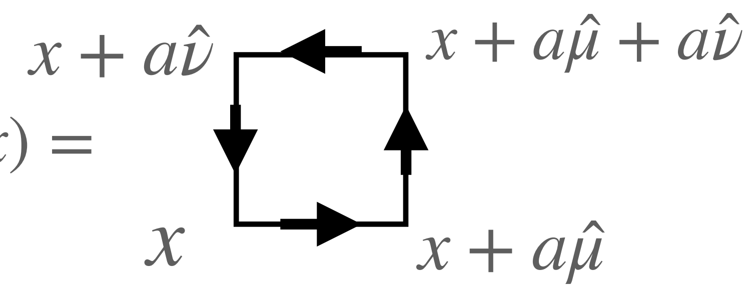
- On the lattice, the exact form of $T_{\mu\nu}$ is unknown, while we can guess how the trace anomaly works:

$$\frac{2}{a^4g^2}\mathrm{Tr}[1 - U^{m\times n}_{P,\mu\nu}] = m^2n^2F^2_{\mu\nu} + \mathcal{O}(a^2, \alpha_s/a^4)$$

$$\frac{2}{a^4g^2}\mathrm{Tr}[U^{1\times1}_{P,\mu\nu} - \frac{1}{16}U^{2\times2}_{P,\mu\nu}] = \mathcal{O}(a^2, \alpha_s/a^4)$$

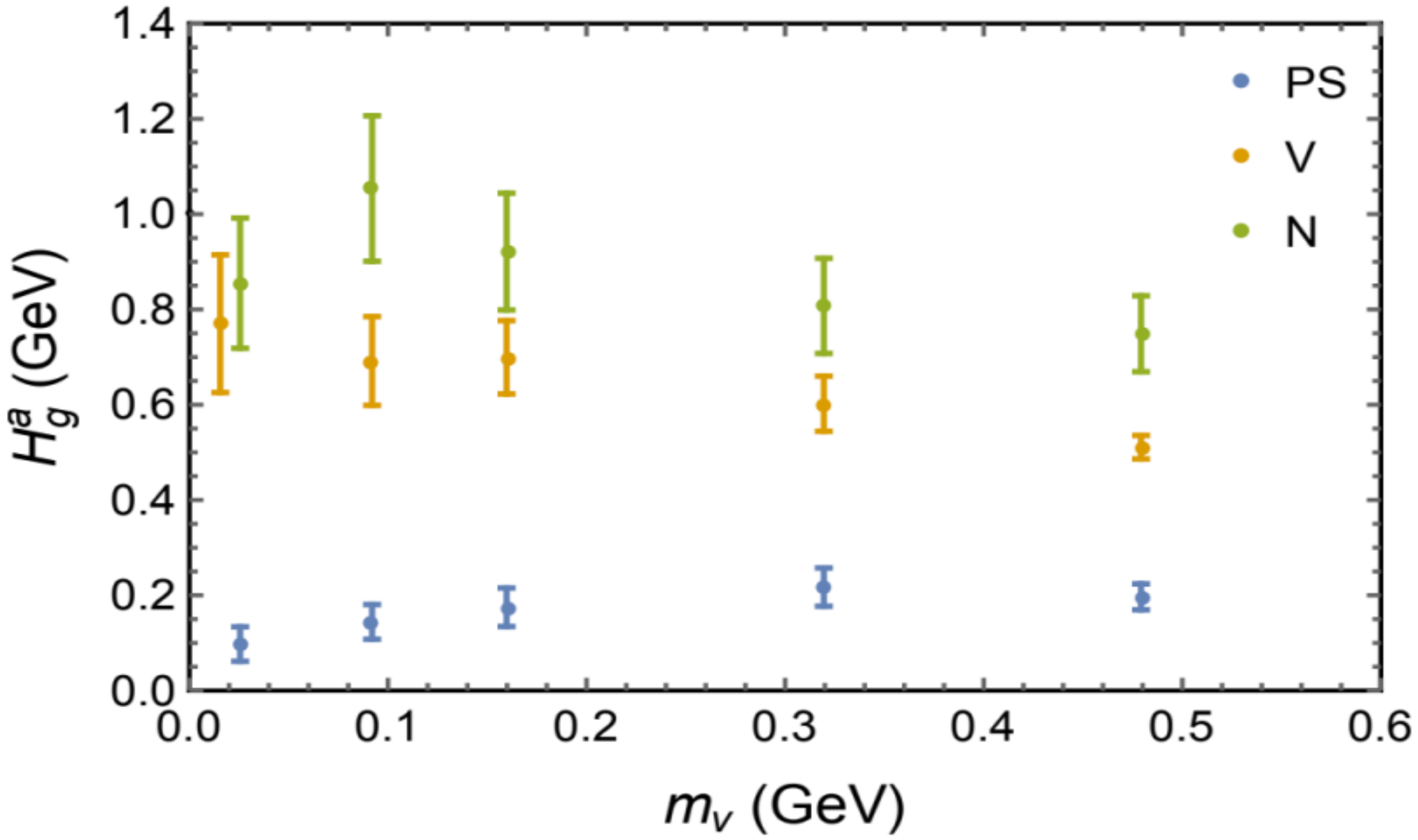
$$\mathcal{L}^{\mathrm{lat}}_{\mathrm{QCD}} = \sum_{q=u,d,s,c,b,t}\bar{q}^i(\mathbb{D}^{\mathrm{lat}}_{ij} - \delta_{ij}m_q)q^j + \frac{1}{2N_cg^2a^4}\sum_{\mu\nu}\mathrm{Tr}[1 - U^{1\times1}_{P,\mu\nu}],$$

$U^{1\times1}_{P,\mu\nu}(x) =$

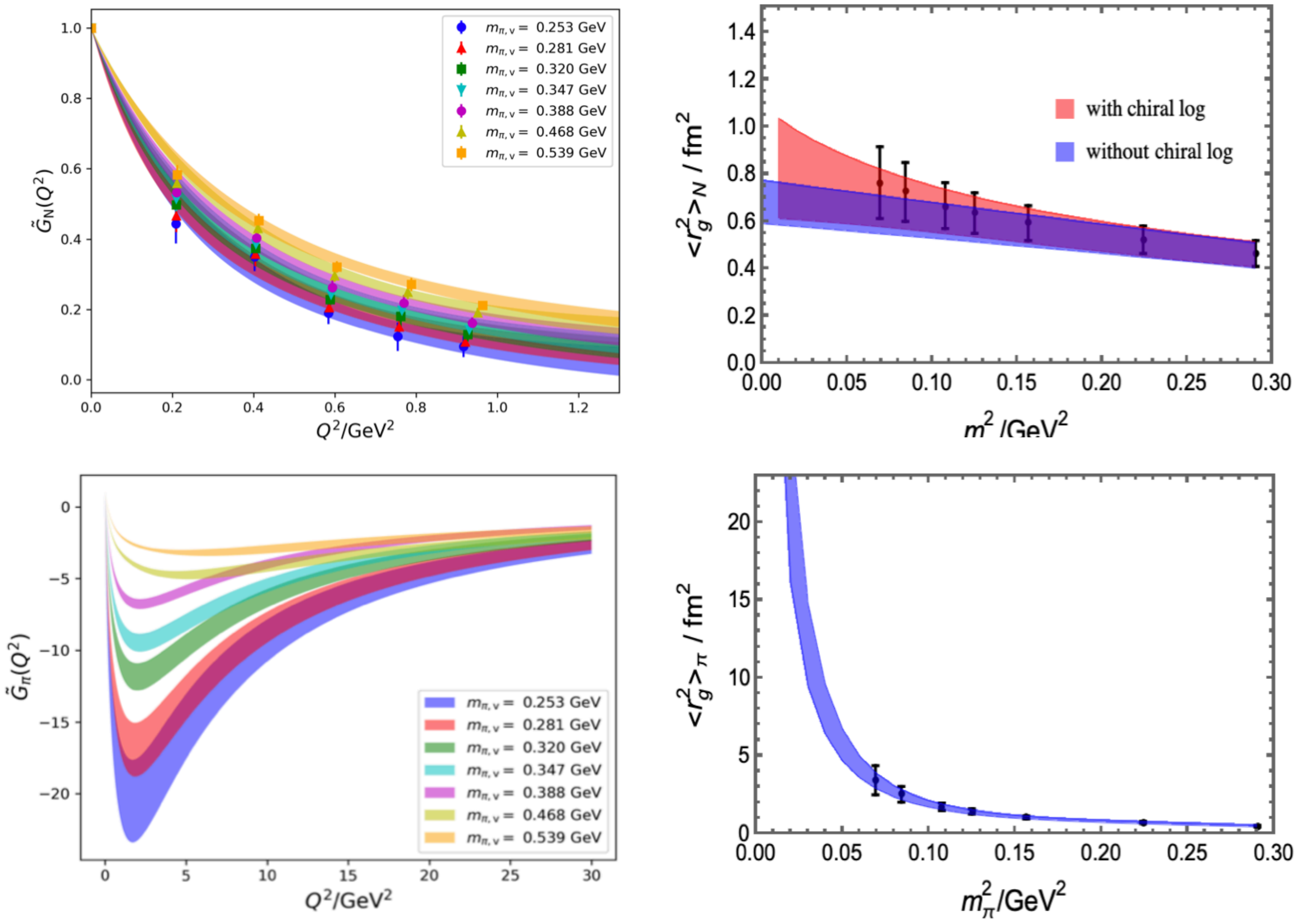


Hadron mass

Gluon trace anomaly contribution



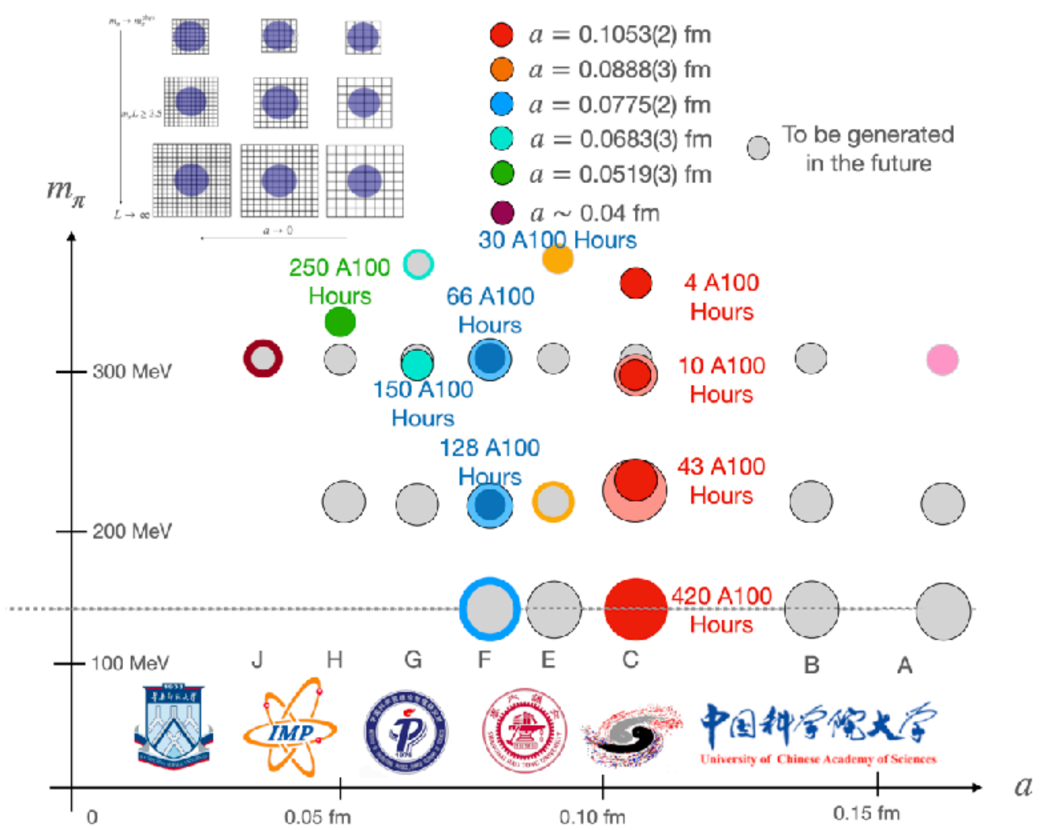
F. He, P. Sun, **YBY**, χQCD, Phys.Rev.D 104(2021)074507

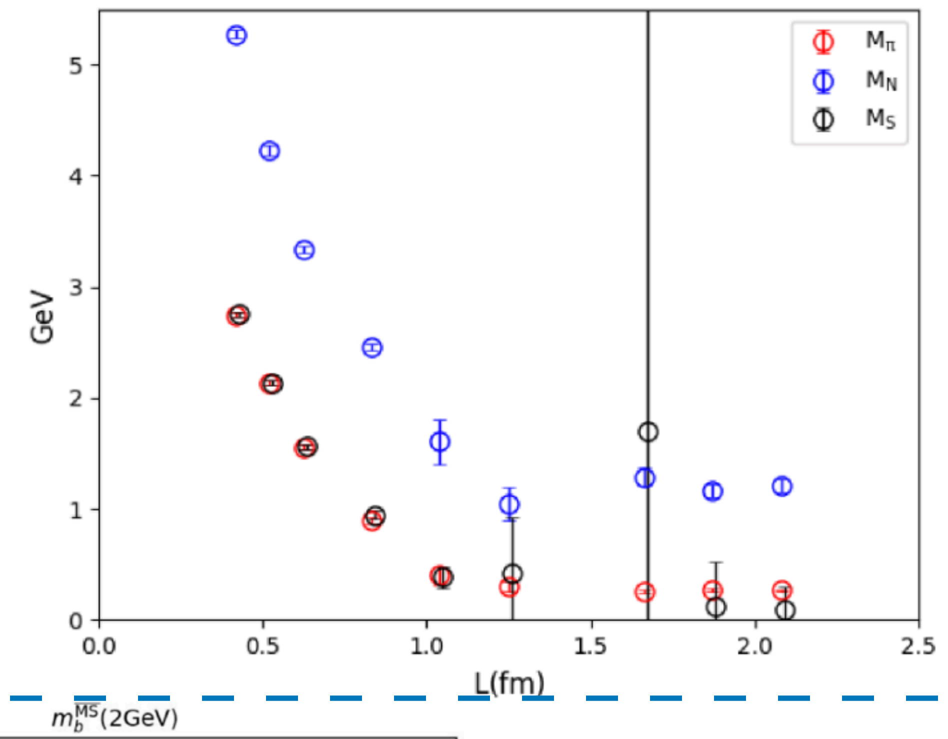


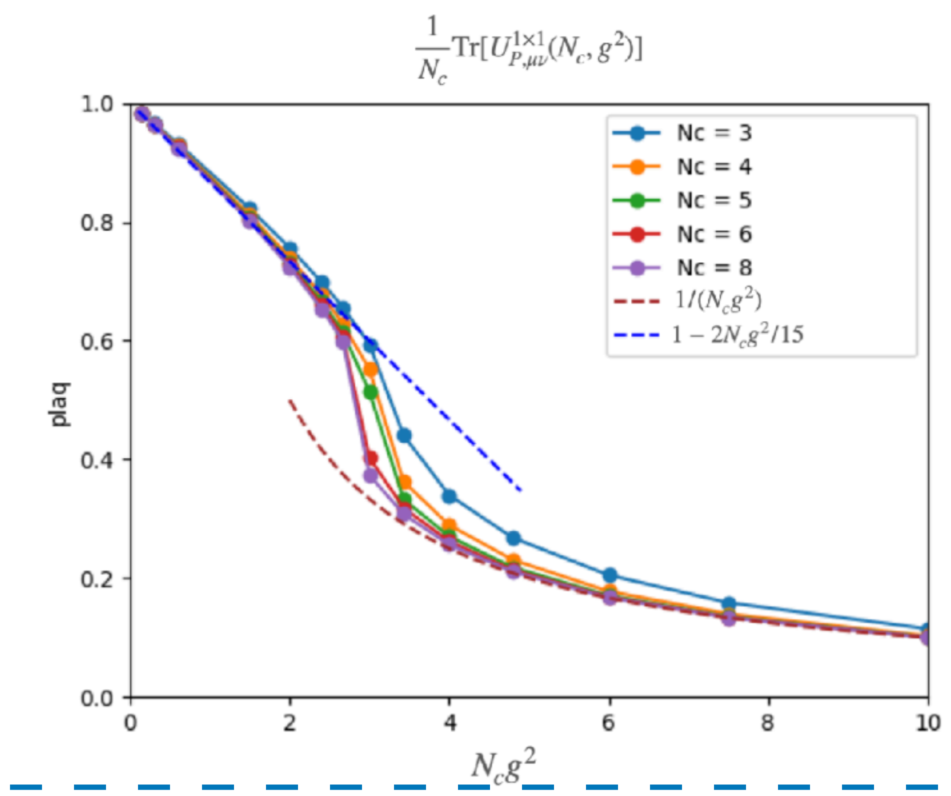
B. Wang, et.al., χQCD, Phys.Rev.D 109(2024)094504

- The gluon contribution to the hadron mass, form factor and mass radius;
- **Overlap fermion at 0.11 fm, too expensive to control the systematics.**

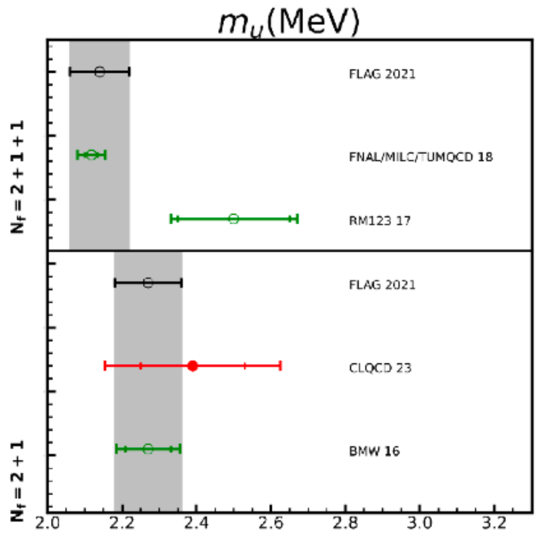
Outline

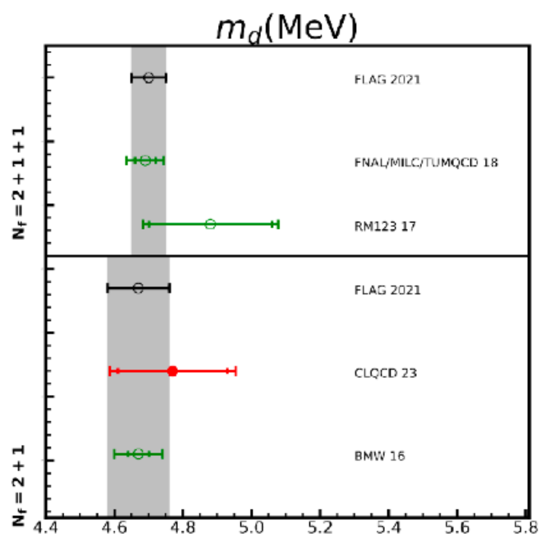


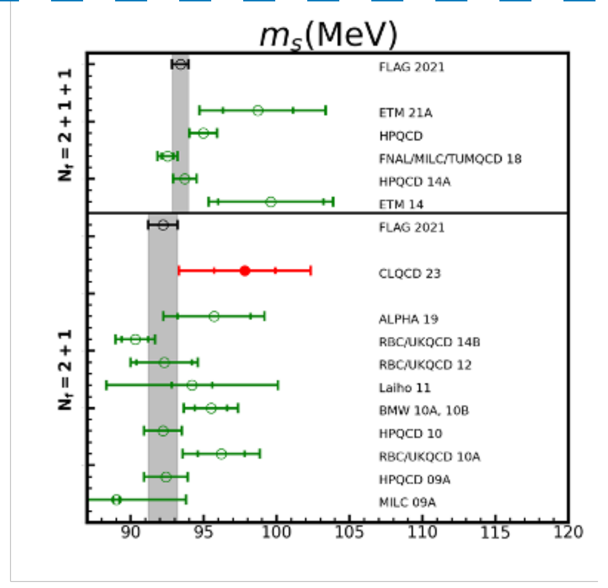


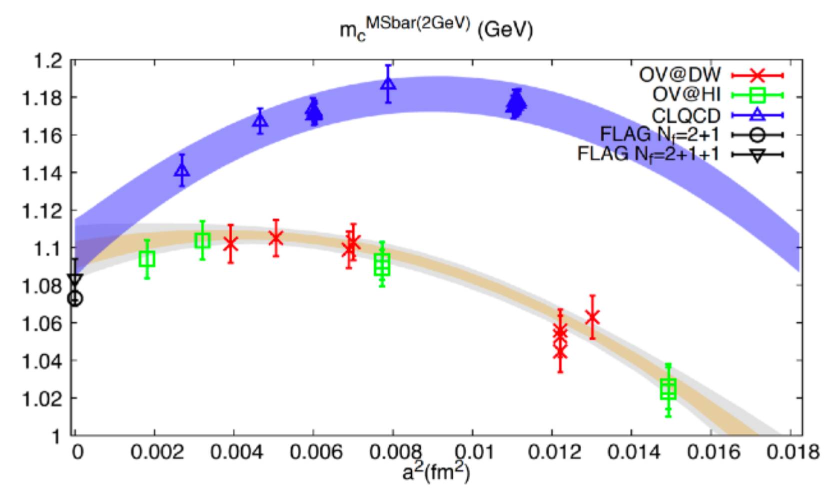


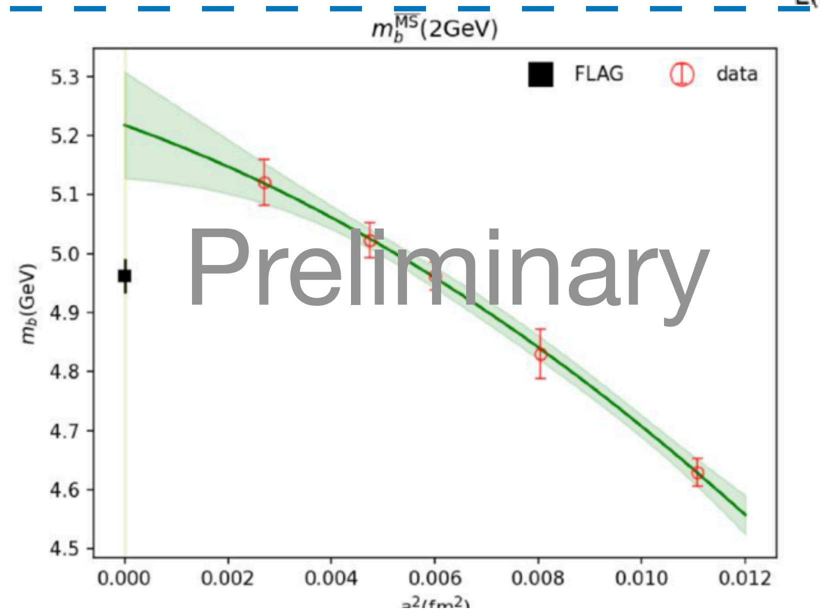
- **Gauge ensembles**





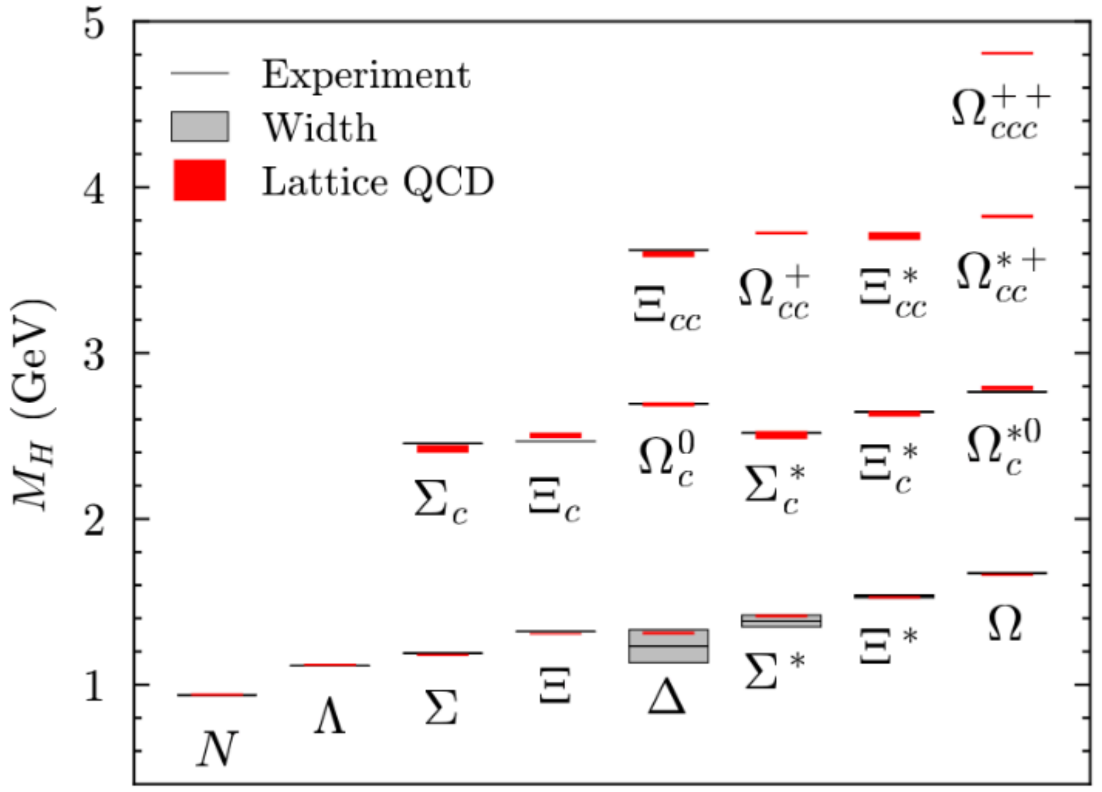


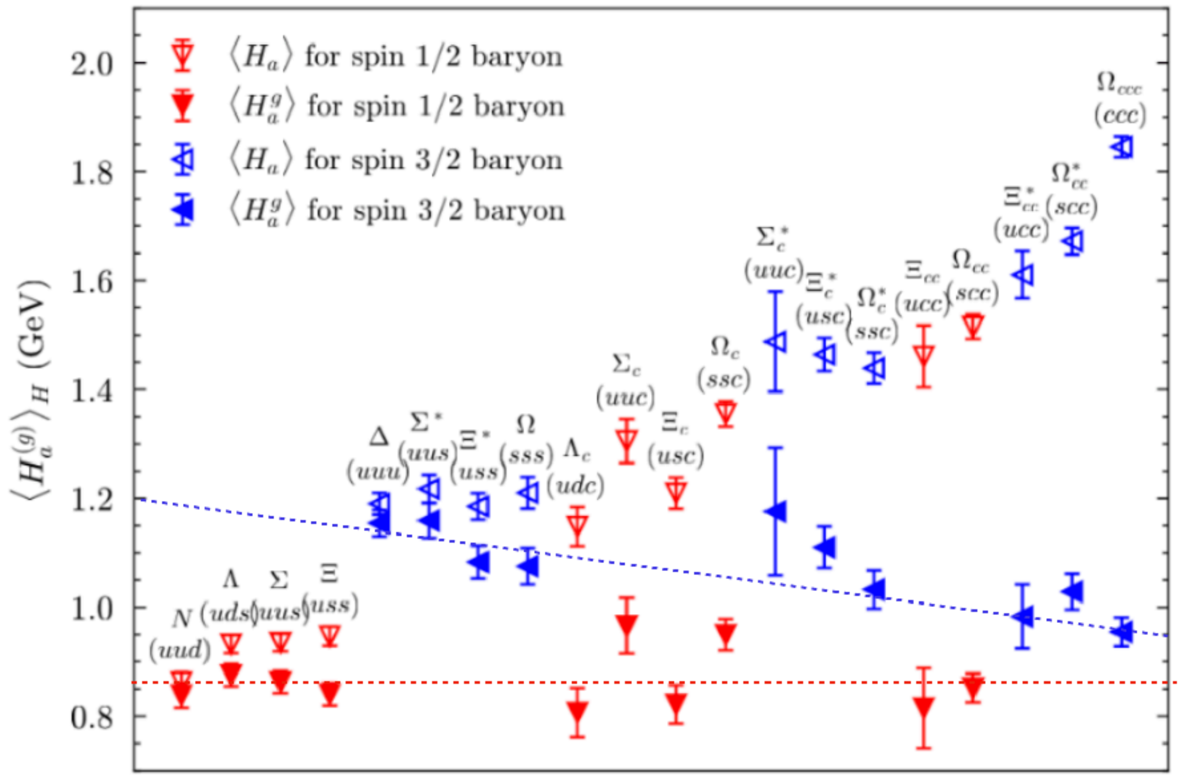




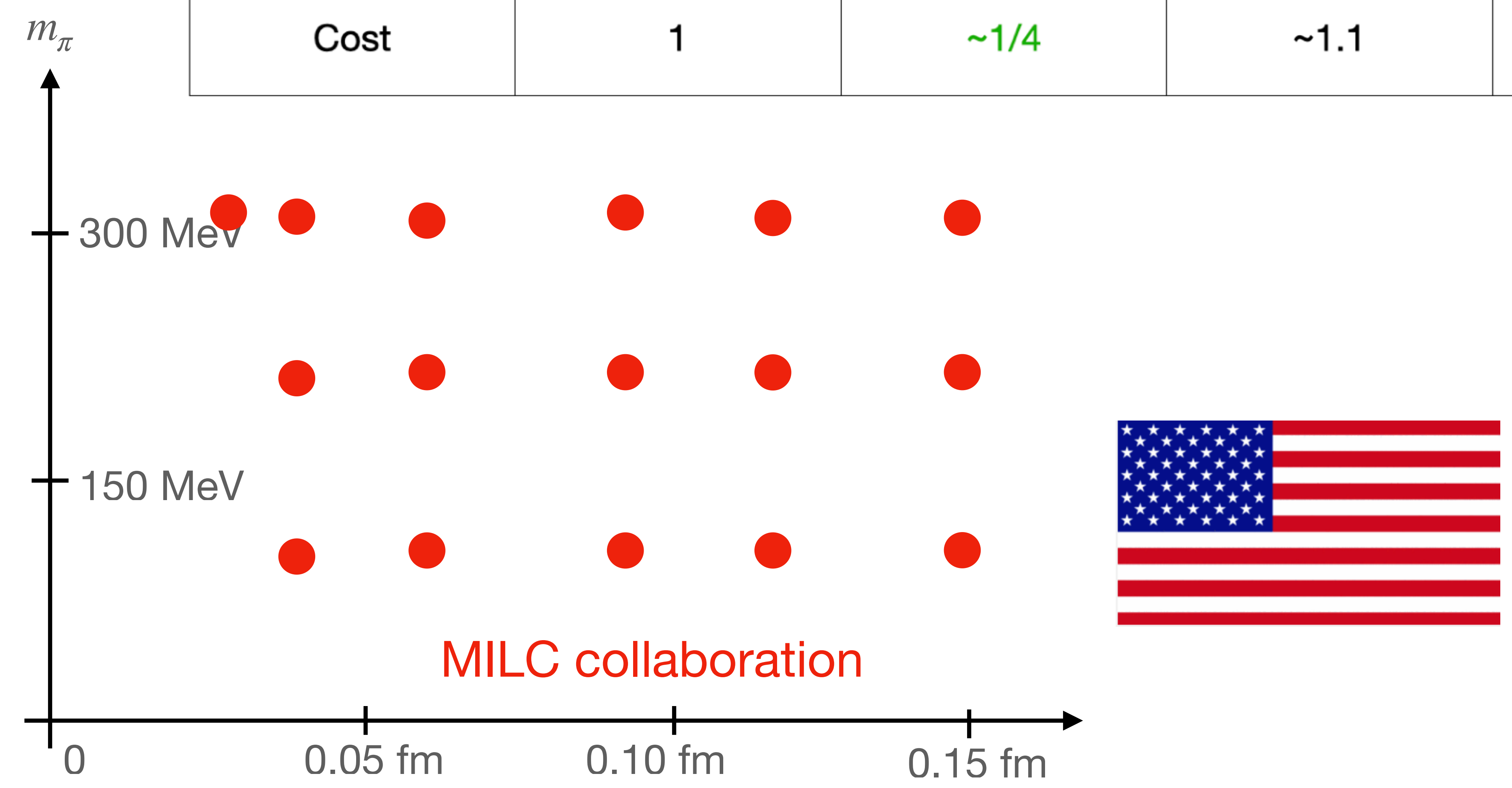
- Benchmark of CLQCD ensembles

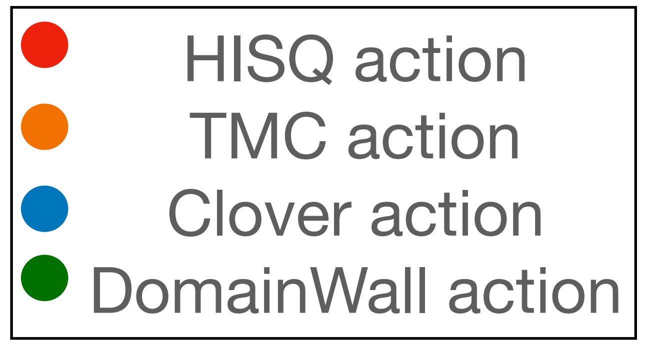
- Strong interaction origin of hadron mass

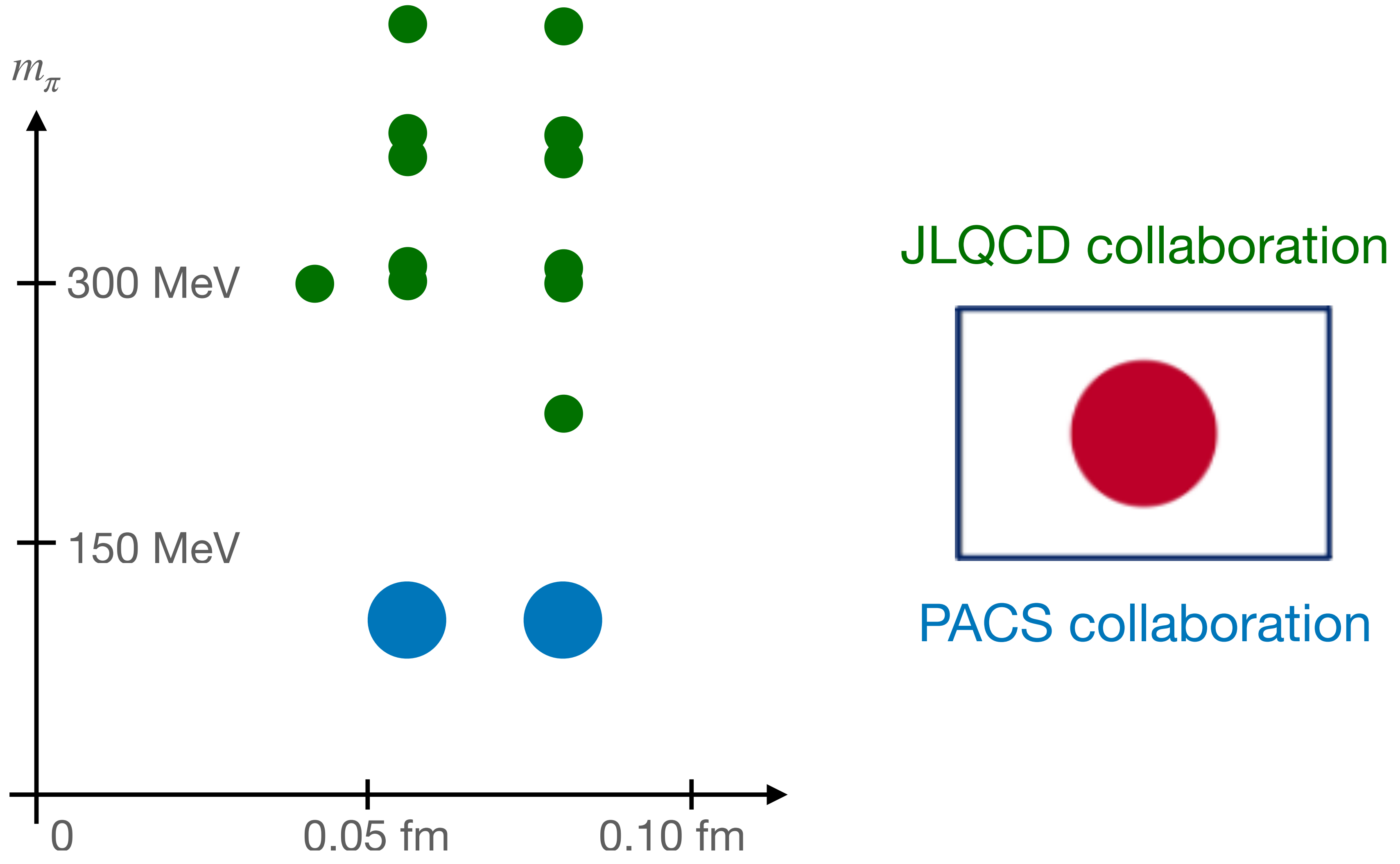


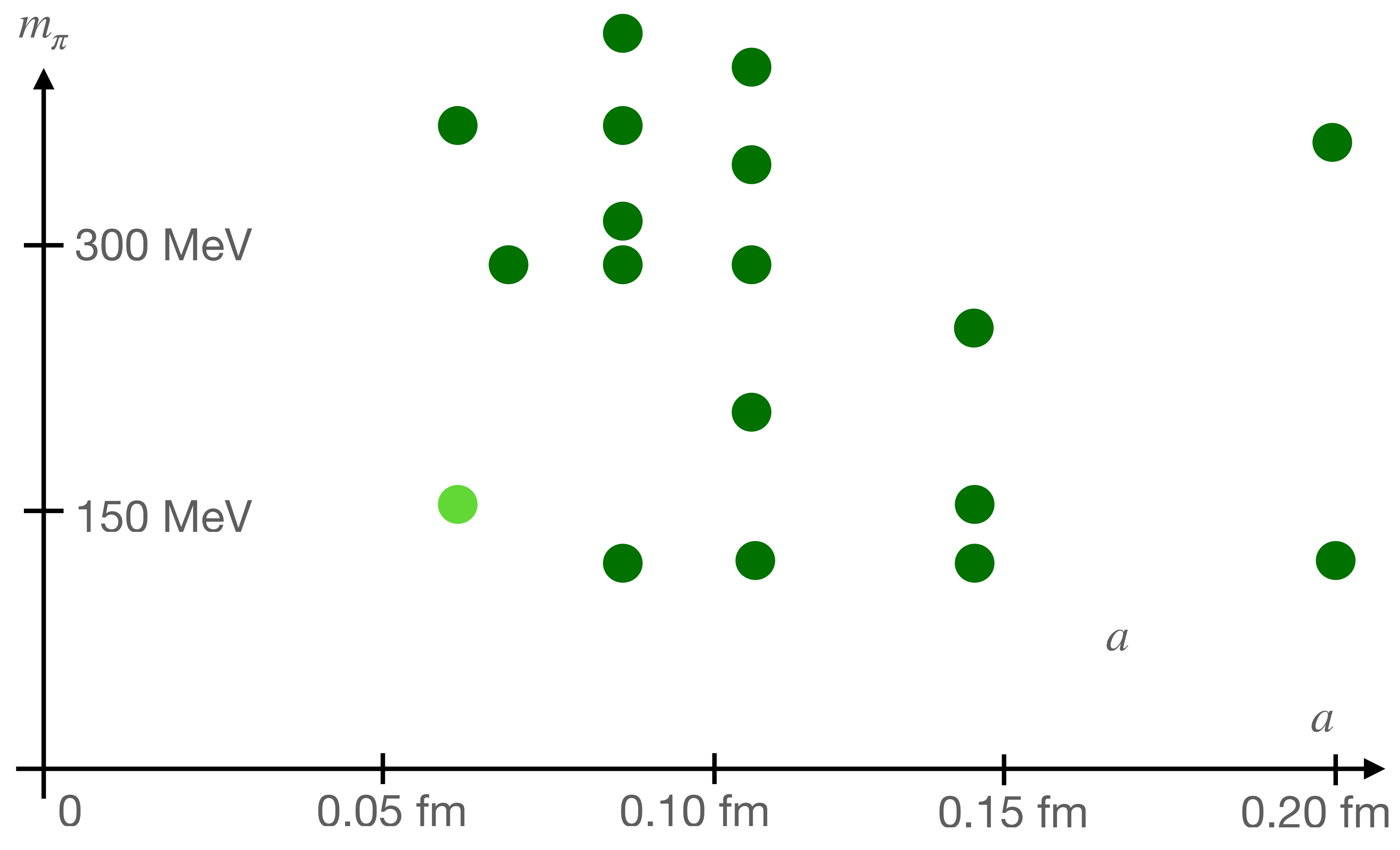


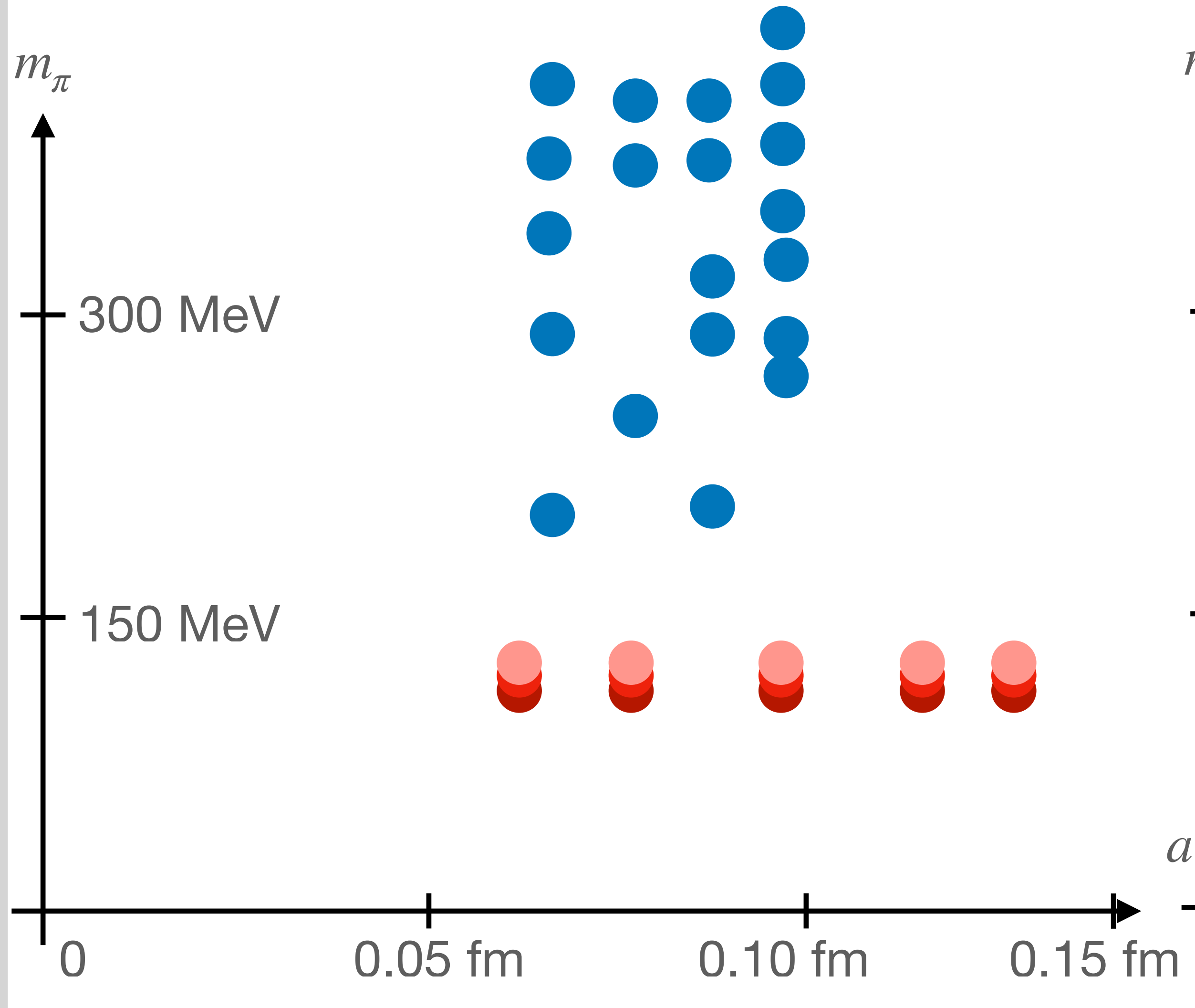
	Naive	Staggered/HISQ	Wilson/Clover	Twisted-mass	Overlap/Domain wall
Form	$D^{\text{naive}} = \gamma_\mu(\delta_{x,x+\mu} - \delta_{x,x-\mu})$	$D^{\text{st}} = \gamma_\mu^{\text{st}}(x)(\delta_{x,x+\mu} - \delta_{x,x-\mu})$	$D^{\text{clv}} = D + aD^2 + ac_{sw}F_{\mu\nu}\sigma^{\mu\nu}$	$D^{\text{tm}} = D^{\text{clv}} + \mathrm{i}\tau_3 m$	$D^{\text{ov}} = [1+\gamma_5 D(-\rho)/\sqrt{D^\dagger(-\rho)D(-\rho)}]/\rho$
Fermion copies	16	4	1	1	1
Chiral symmetry breaking	N/A	$\mathcal{O}(a^4)$	$\mathcal{O}(\alpha_s/a)$	$\mathcal{O}(\alpha_s)$	N/A
Cost	1	~1/4	~1.1	~1.1	~10-100

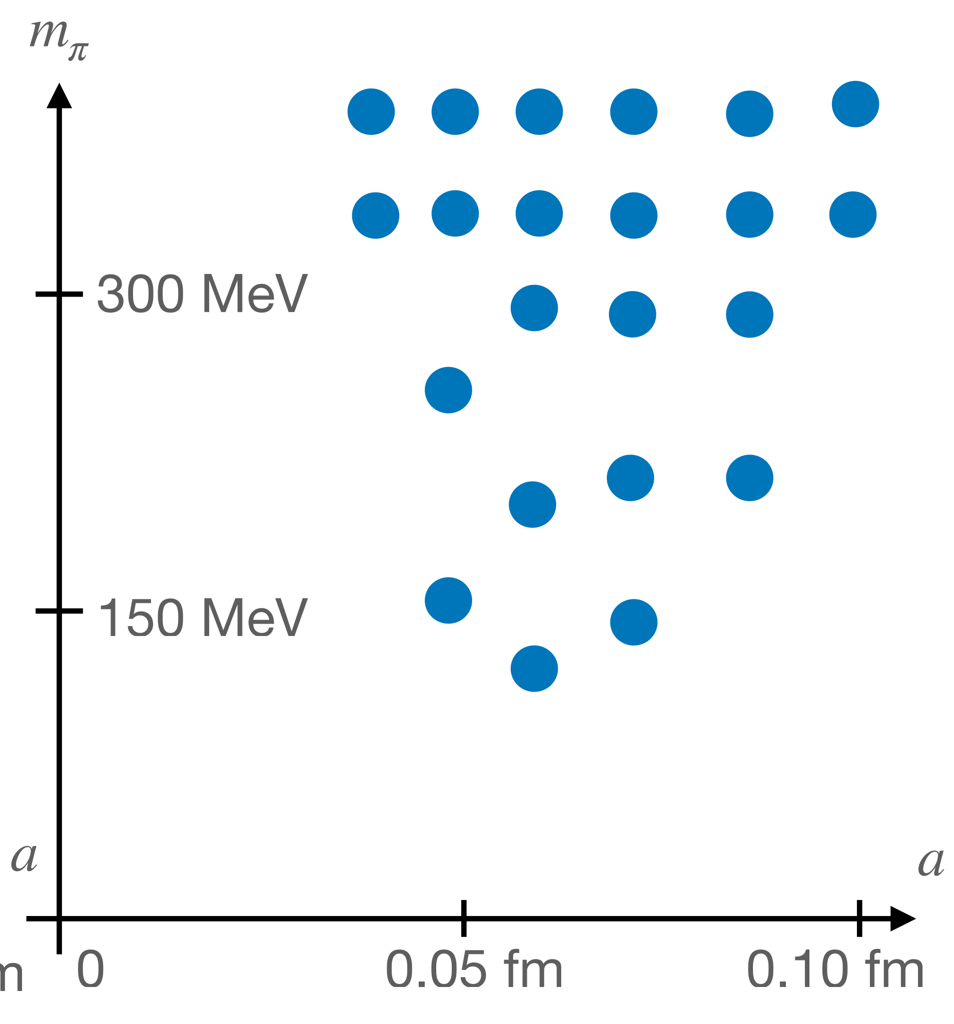


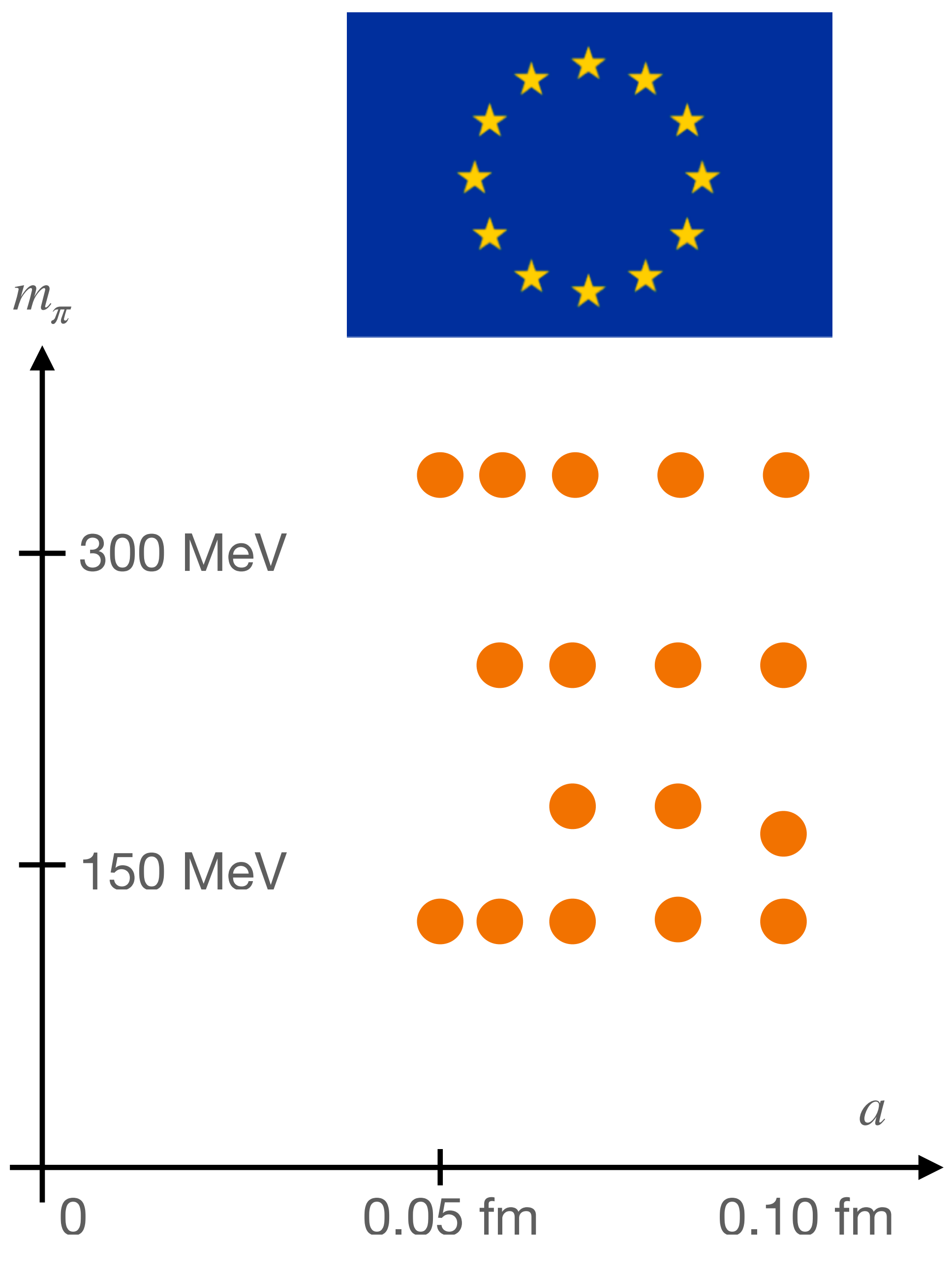




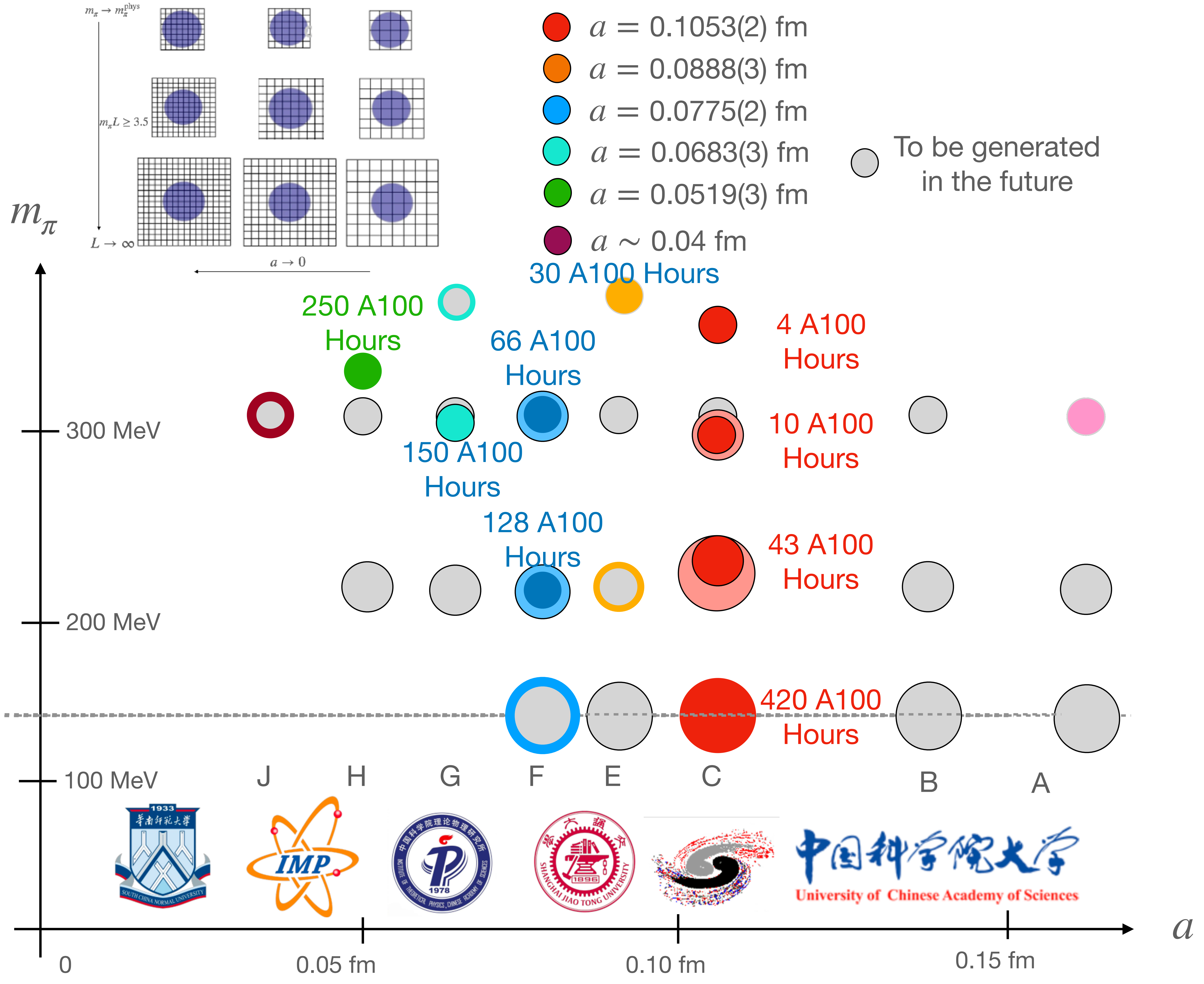








Gauge ensembles



Z.-H. Hu, B.-L. Hu, J.-H. Wang, et. al., CLQCD, PRD109(2024) 054507

CLQCD ensembles

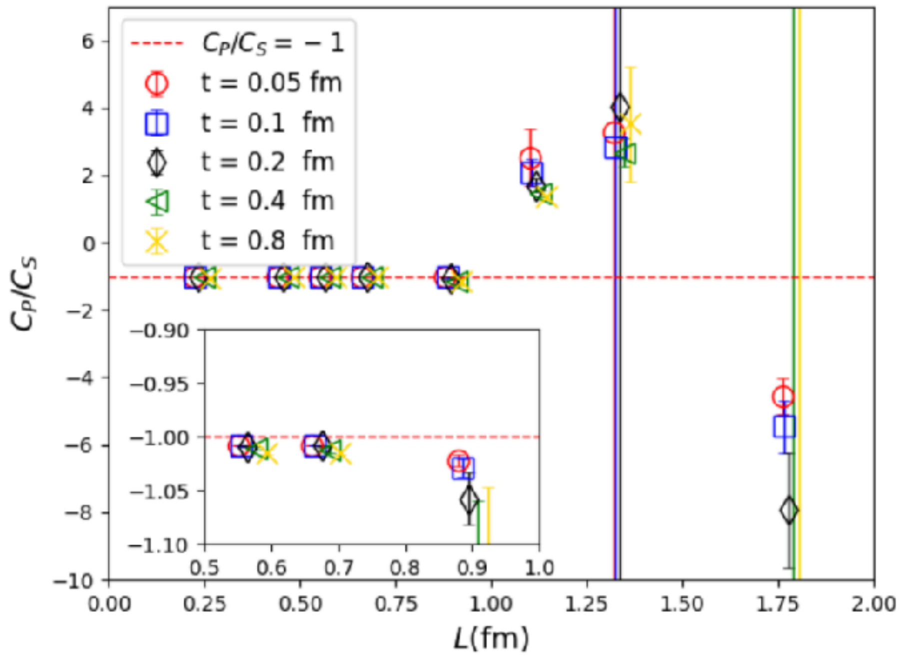
14 ensembles with more than 5,000 configurations in total:

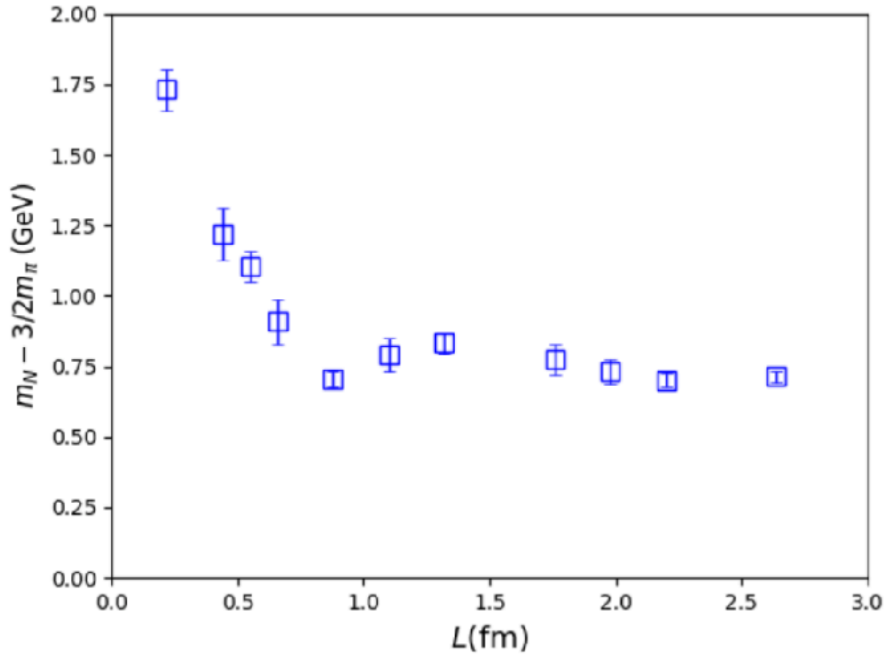
- 5 lattice spacings from 0.05 fm to 0.11 fm;
- 7 pion masses from 130 MeV to 350 MeV;
- 4 Volumes from 2.5 fm to 5.0fm.
- More ensembles are in production;

Ensembles with another setup are preparing for better control on systematic uncertainties.

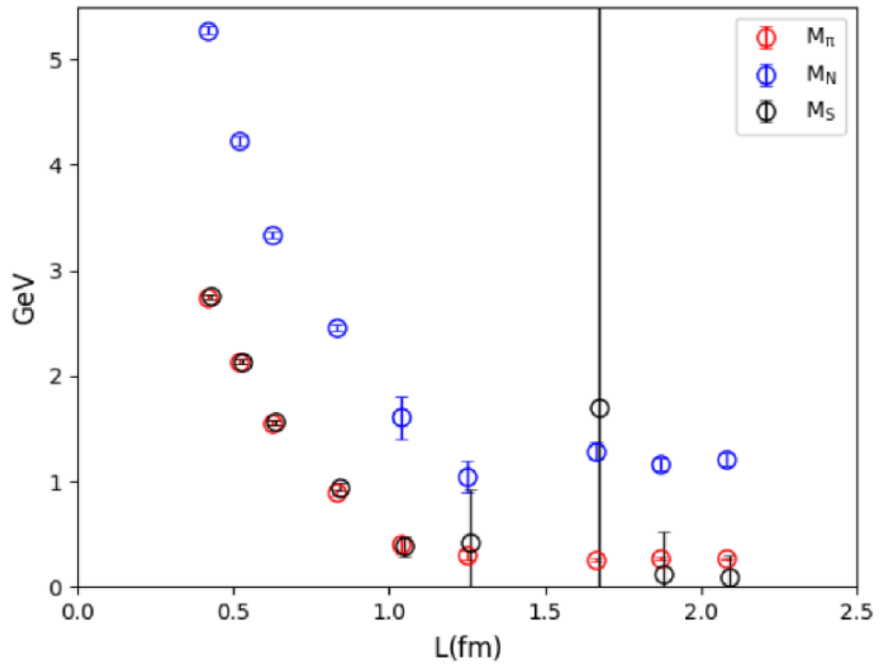
Gauge ensembles

Finite volume

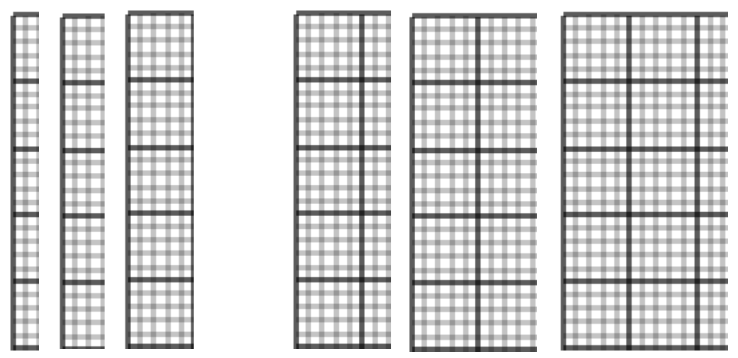




$D = 4,\ N_c = 3,\ L \to 0$



$L > 1$ fm

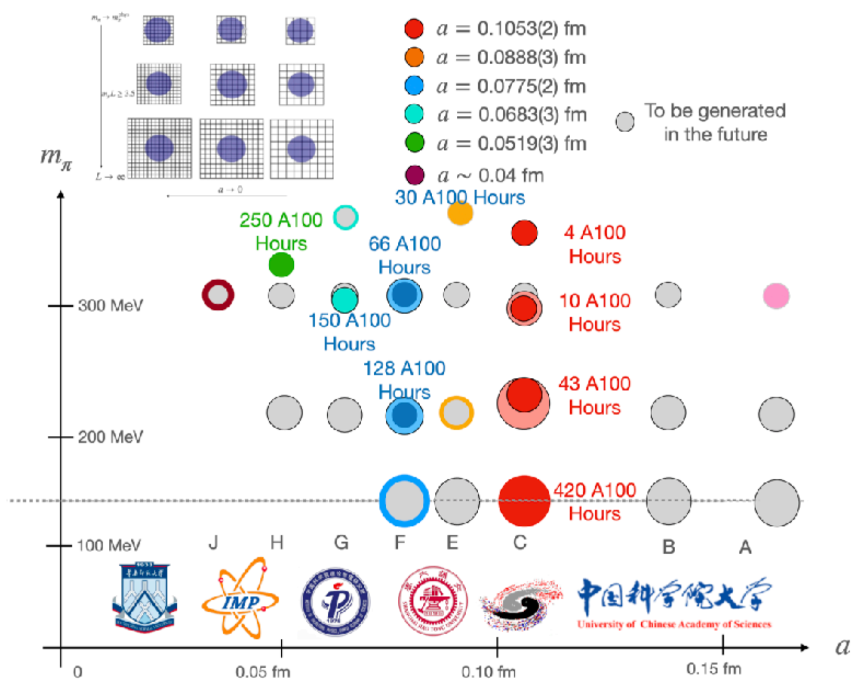


$L \leq 1$ fm

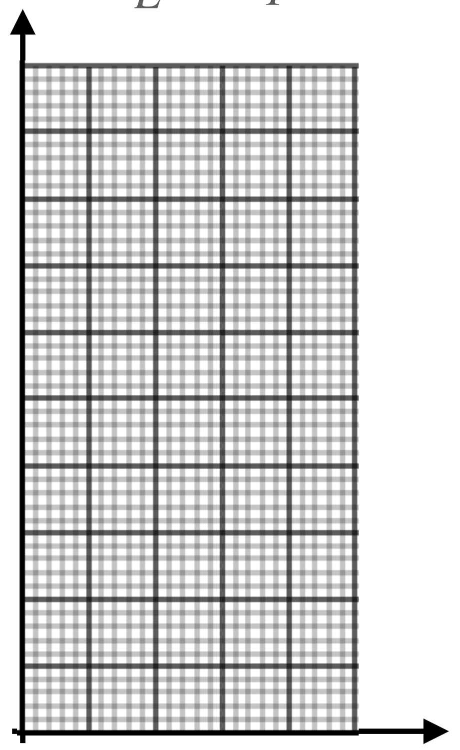
X. Meng, et.al, CLQCD, CTP 76(2024)095203

$D = 4,\ N_c = 3,\ L \to \infty, T \to 0$

$a \to 0,\ m_{l,s,c,b} \to m^{\mathrm{phys}}_{l,s,c,b}$



$n_T = \dfrac{1}{Ta}$ $\quad n_L^3 \times n_T$

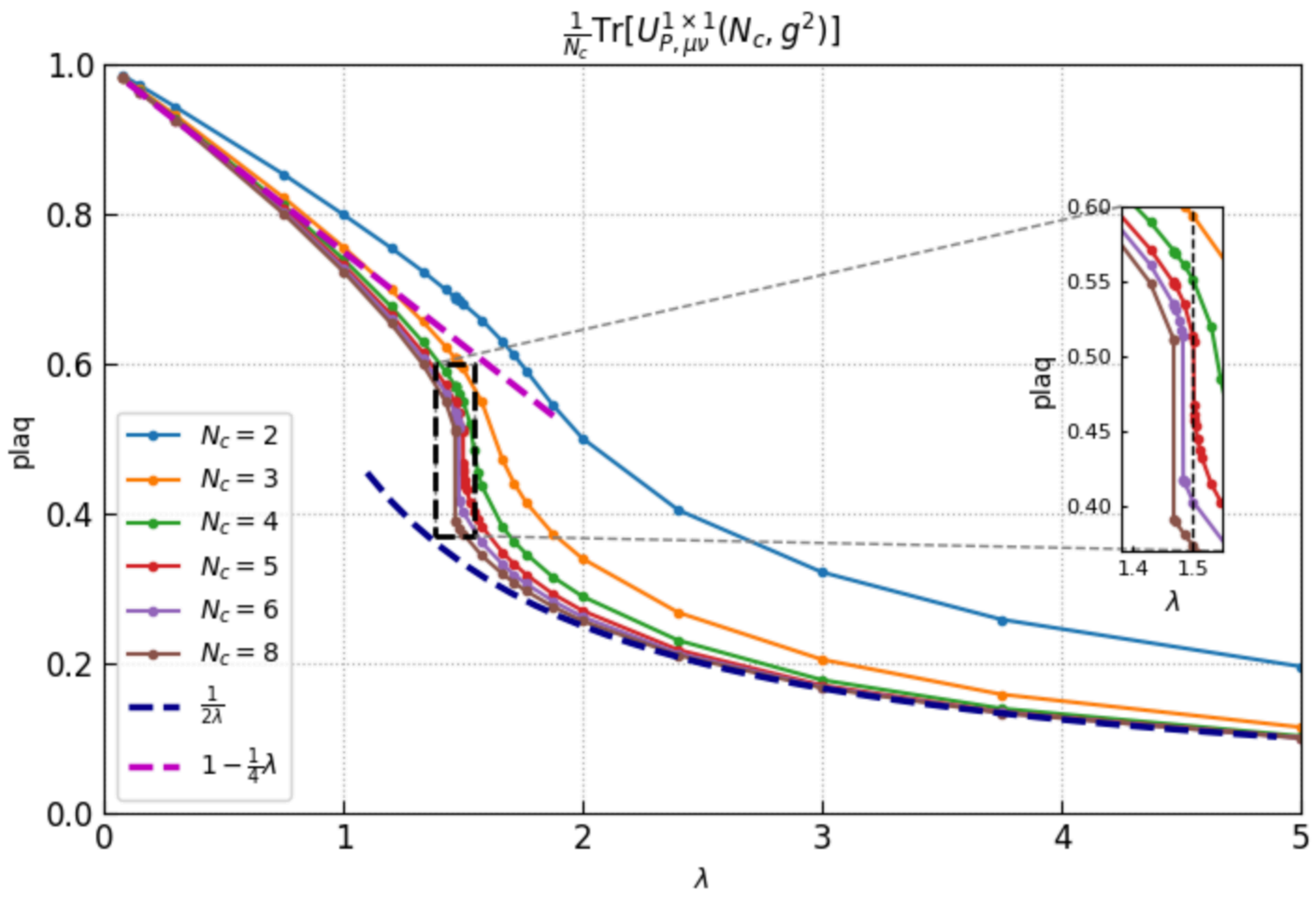


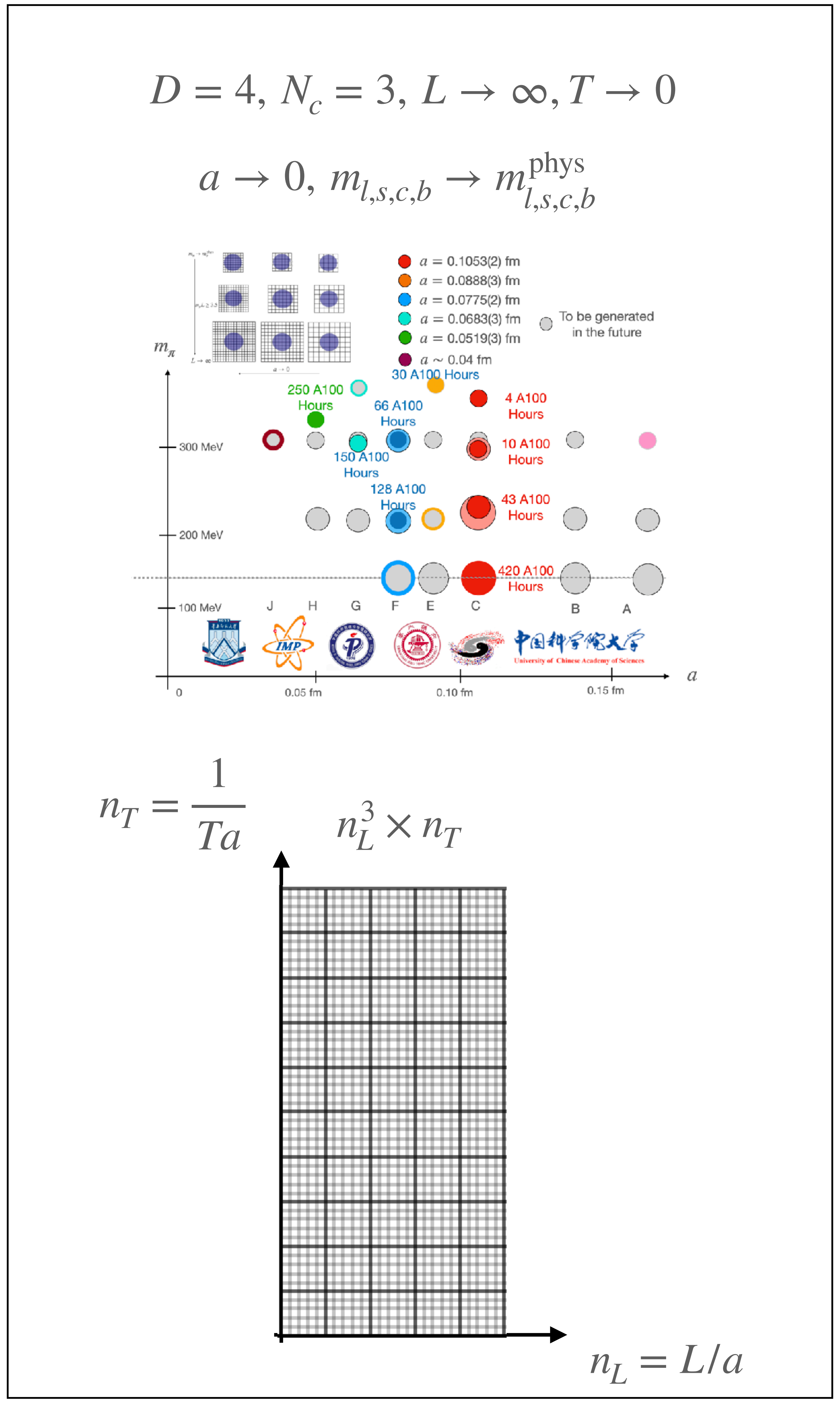
$n_L = L/a$

- Based on our simulation with different spacial size at $a = 0.055$ fm:
- Spontaneous chiral symmetry breaking restored at $L \sim 0.8$ fm which corresponds to $1/L \sim 246$ MeV;
- But the mass gap $m_N - 3/2m_\pi$ remain large even at very small L.
- It suggests that mass gap and spontaneous chiral symmetry breaking can be independent features of QCD.

Gauge ensembles

SU(N)



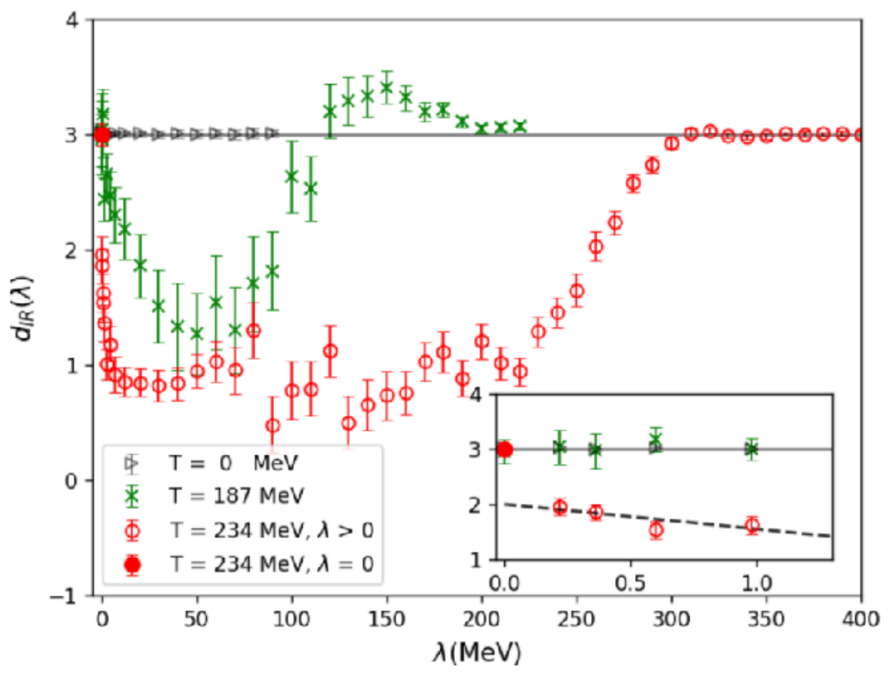


- Based on our pure gauge simulation with different $N_c = 2,3,4,5,6,8$:
- The plaquatte $\frac{1}{N_c}\mathrm{Tr}[U^{1\times1}_{P,\mu\nu}]$ have similar behavior as a function of $N_c g^2$ but not g^2;
- The plaquatte saturates at $N_c \sim 8$ and can be described by analytic behavior at weak/strong coupling limits.
- Studies of the other quantities, dynamical flavors, dimensions are in progress.

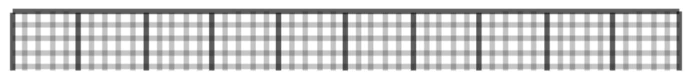
Gauge ensembles

roadmap

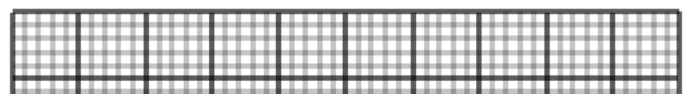
$D=4,\ N_c=3,\ T\to\infty$



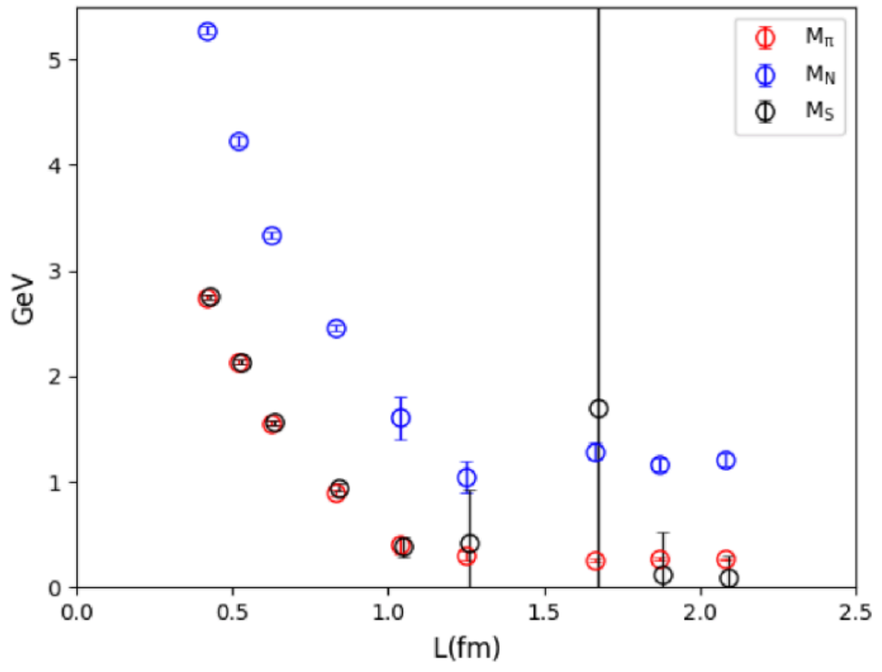
$T=234\ \text{MeV},\ 1/T\sim0.84\ \text{fm}$



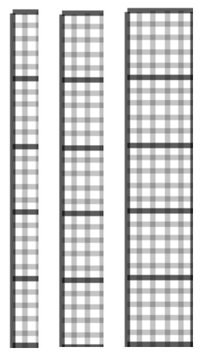
$T=187\ \text{MeV},\ 1/T\sim1.05\ \text{fm}$



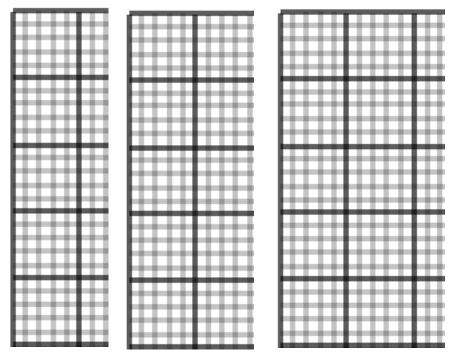
$D=4,\ N_c=3,\ L\to0$



$L\le1\ \text{fm}$

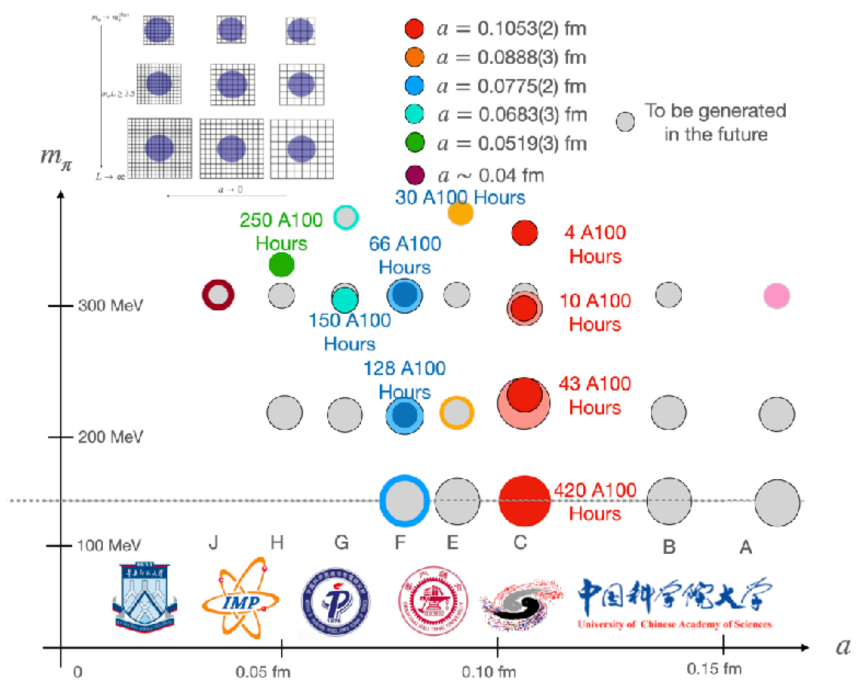


$L>1\ \text{fm}$



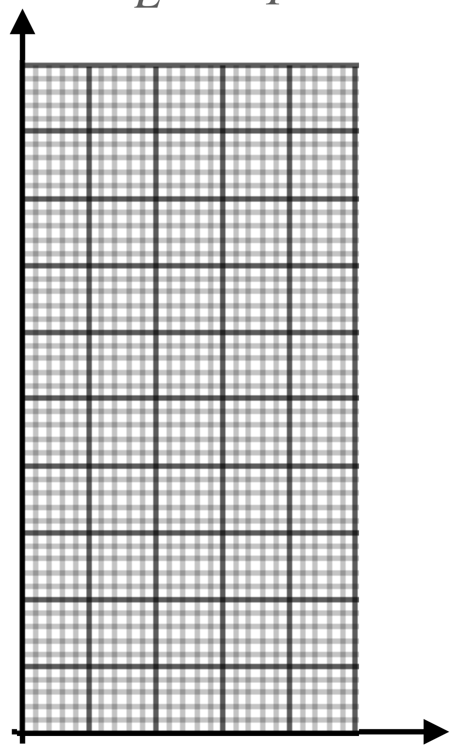
$D=4,\ N_c=3,\ L\to\infty, T\to0$

$a\to0,\ m_{l,s,c,b}\to m_{l,s,c,b}^{\text{phys}}$



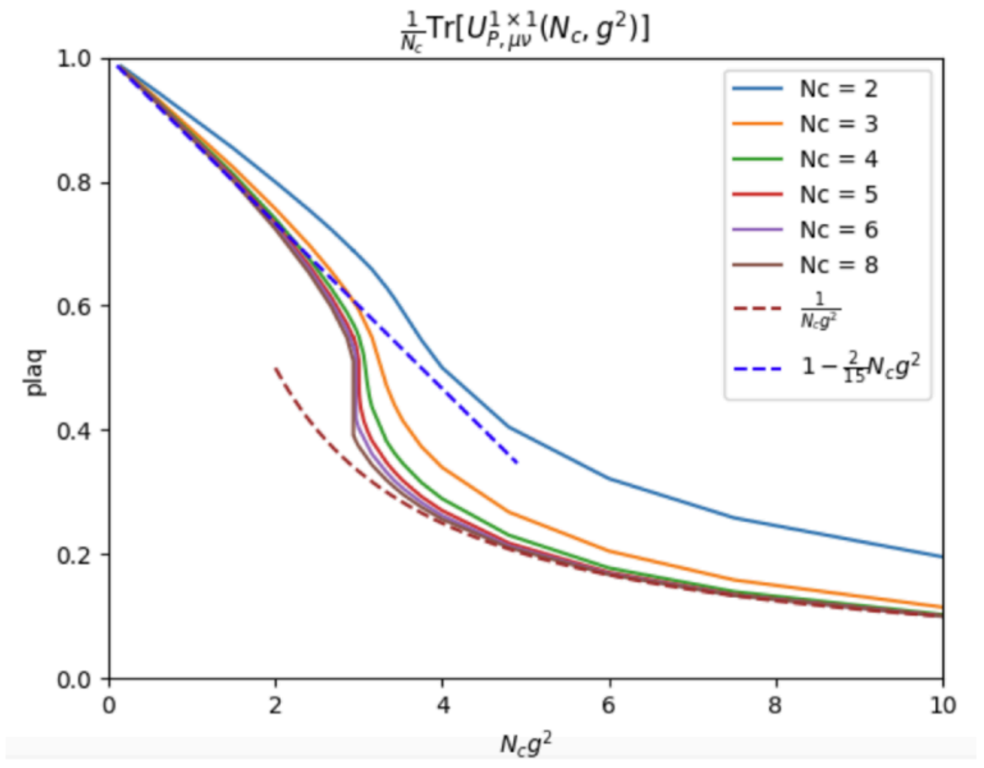
$n_T=\dfrac{1}{Ta}$

$n_L^3\times n_T$



$n_L=L/a$

$D=4,\ N_c=3,4,5,6,8...,\ N_f=0$



$D=4,\ N_c=3,4,5,6,8...,\ N_f=2,2+1...$

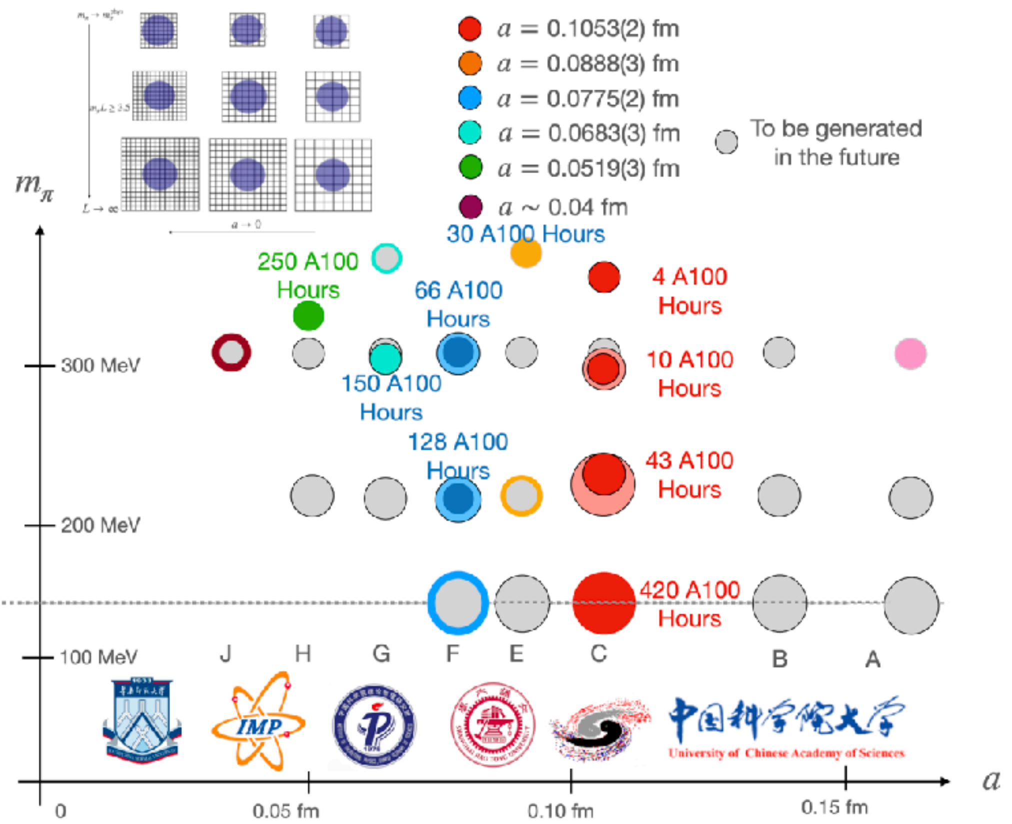
$D=2,3,\ N_c=3,4,5,6,8...,$

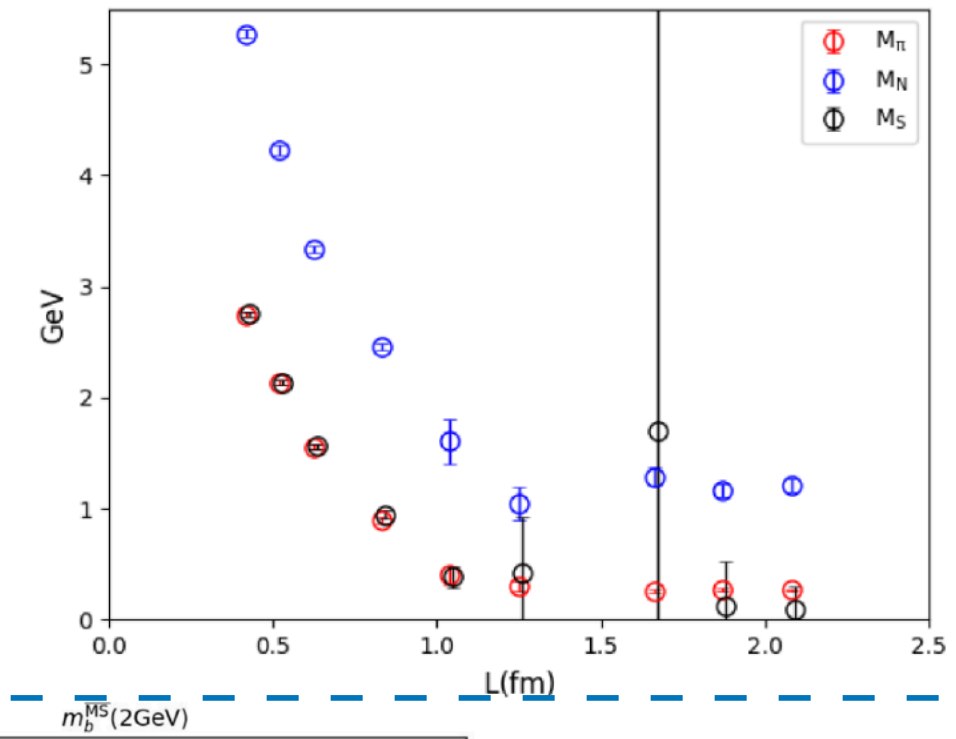
$D=4,\ N_c=3,\ L\to\infty, T\to0,\ m_\pi L\ll1$

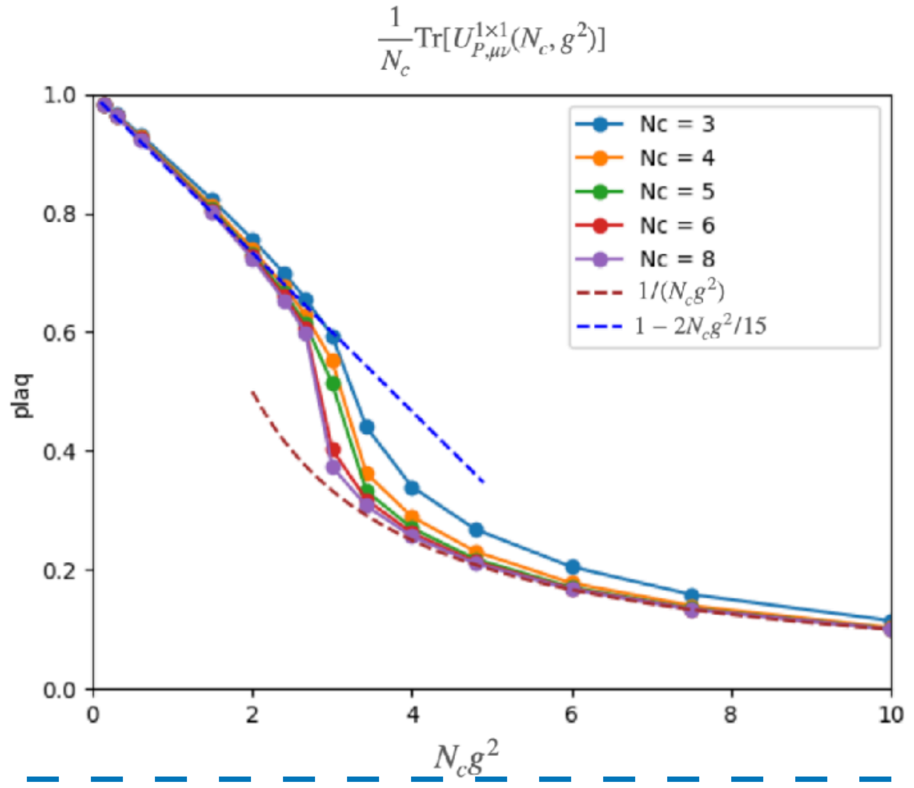
In progress

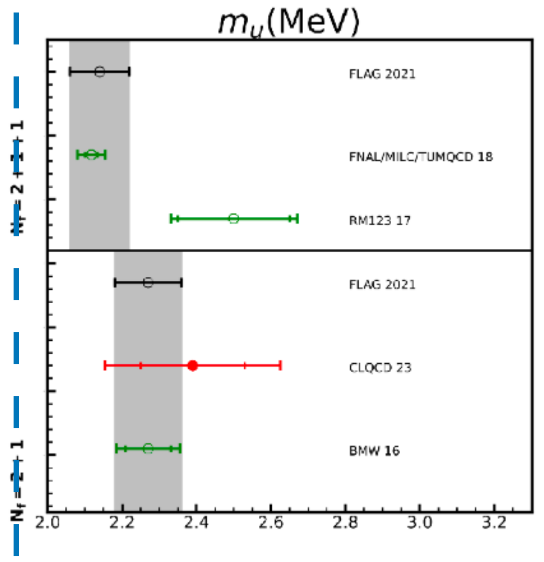
Outline

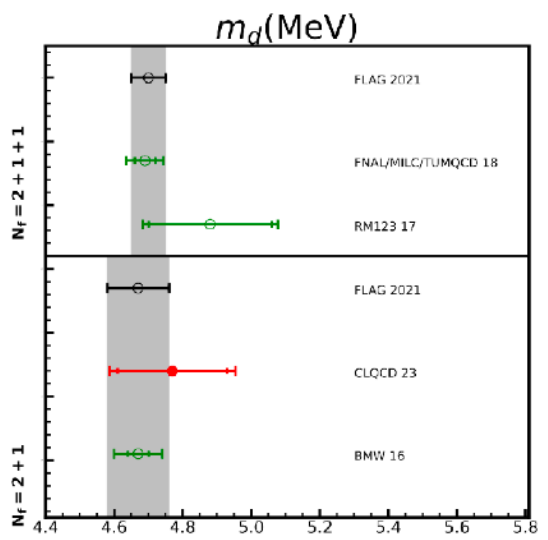
- Gauge ensembles

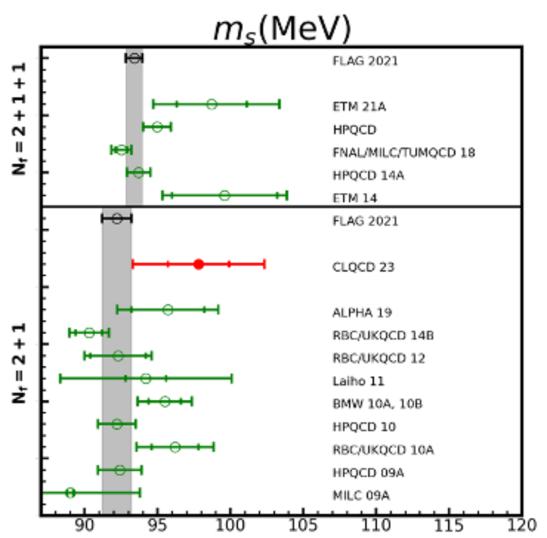


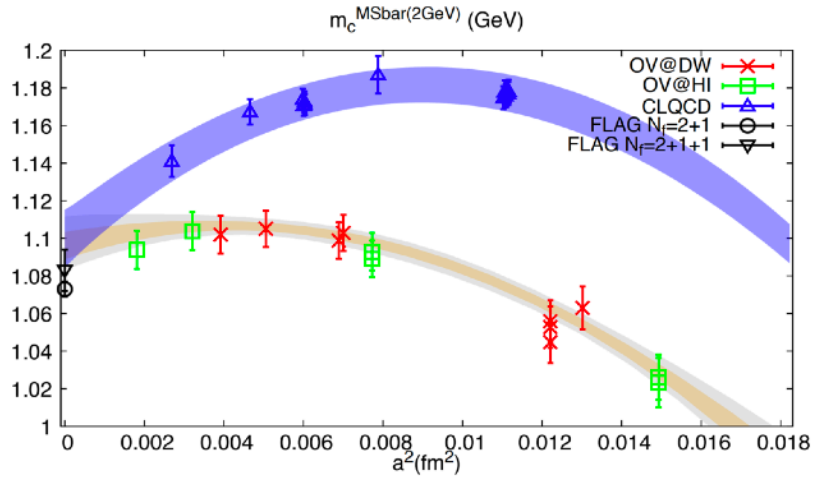


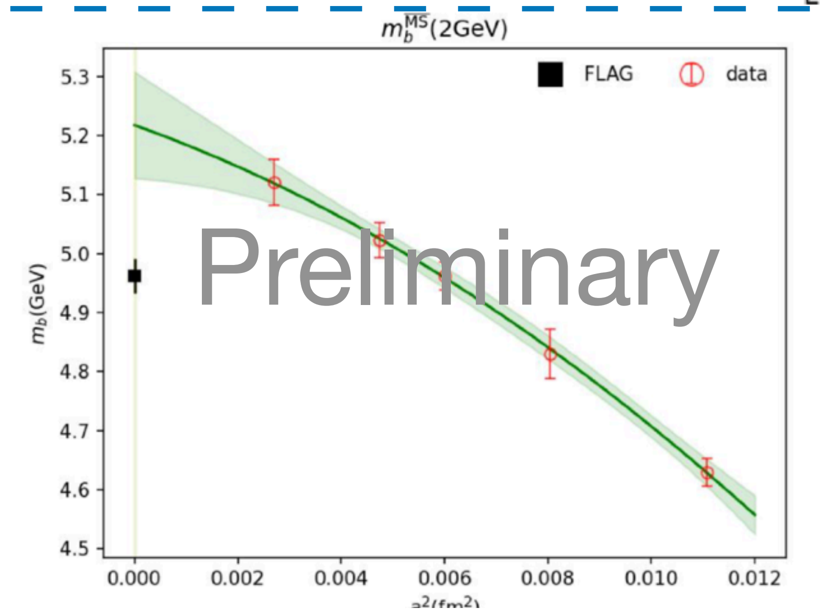






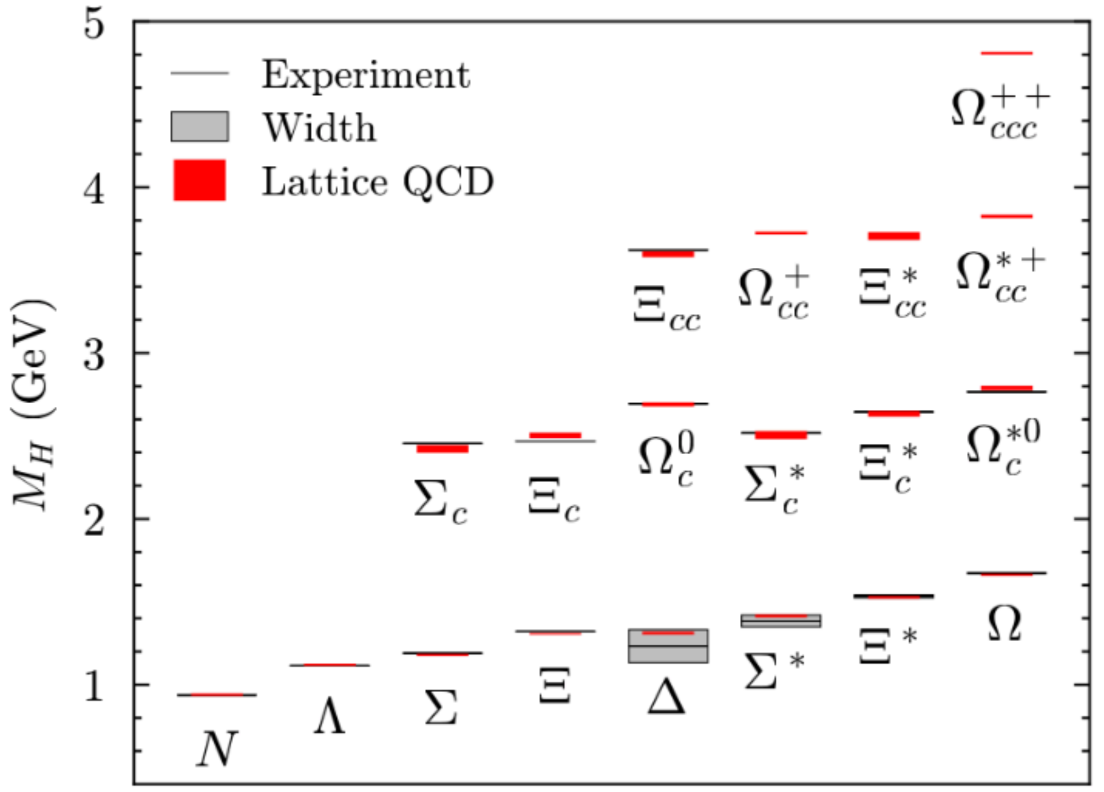


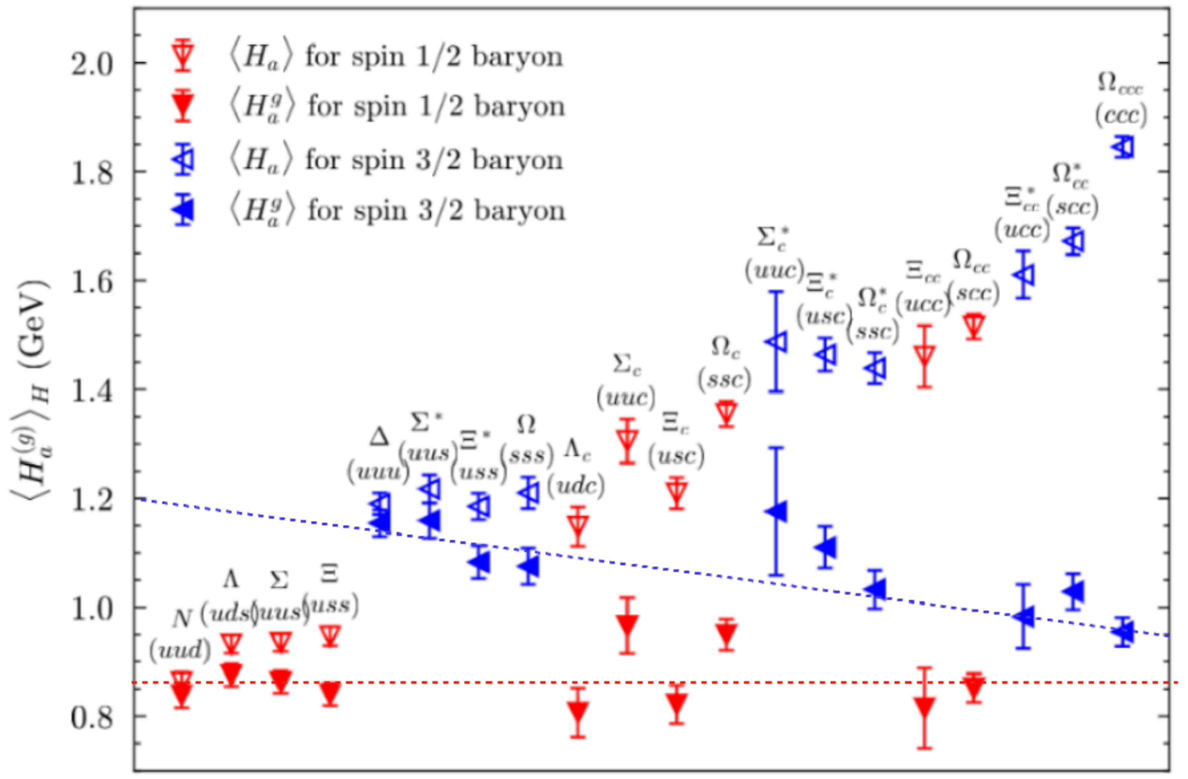




- **Benchmark of CLQCD ensembles**

- Strong interaction origin of hadron mass





Quark mass

Light and strange quark masses

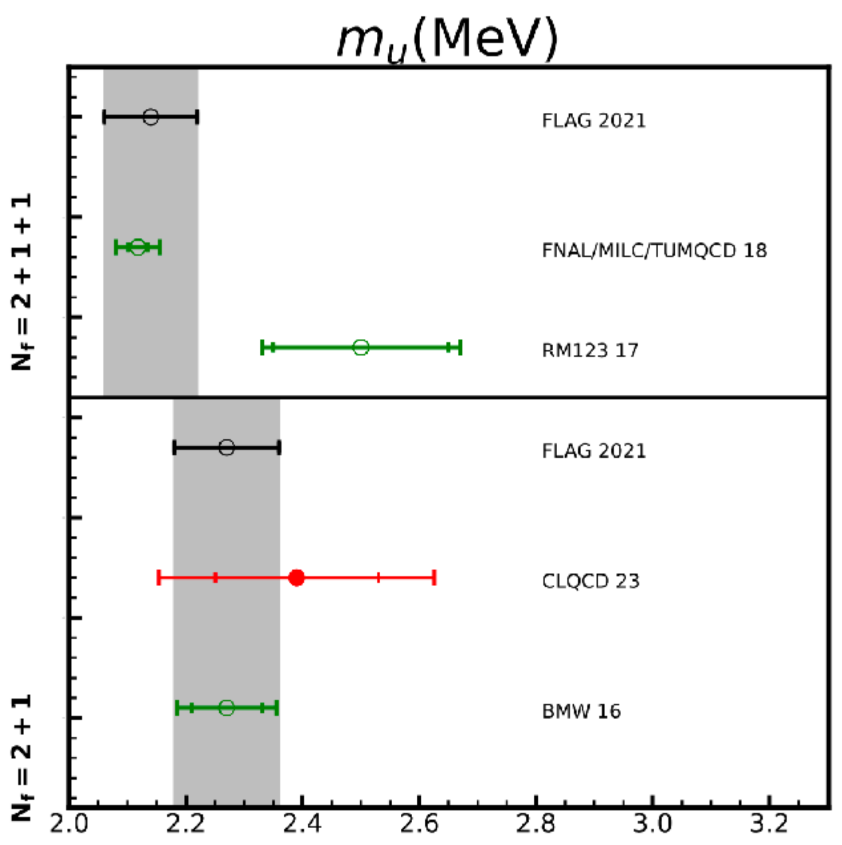
P.Zyla et,al, PTEP(2020)083C01 (PDG2020):

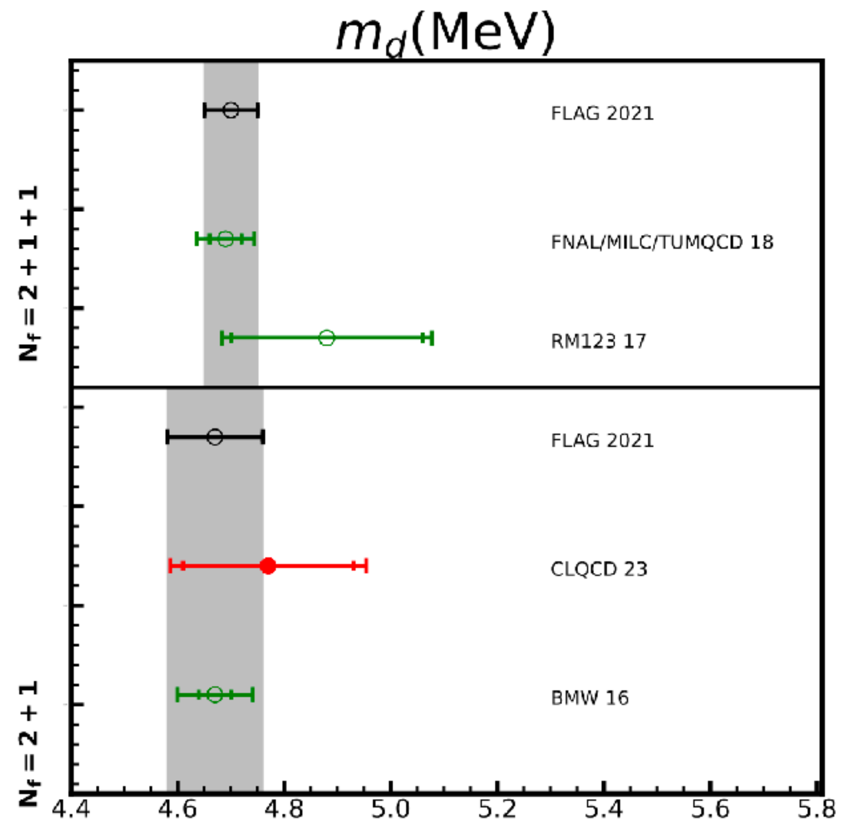
- $m_p = 938.27 \text{ MeV} = m_{p,\mathrm{QCD}} + 1.00(16) \text{ MeV} + \ldots;$
- $m_n = 939.57 \text{ MeV};$
- $\boxed{m_\pi^0 = 134.98 \text{ MeV};}$
- $m_\pi^+ = 139.57 \text{ MeV} = m_\pi^0 + 4.53(6) \text{ MeV} + \ldots;$

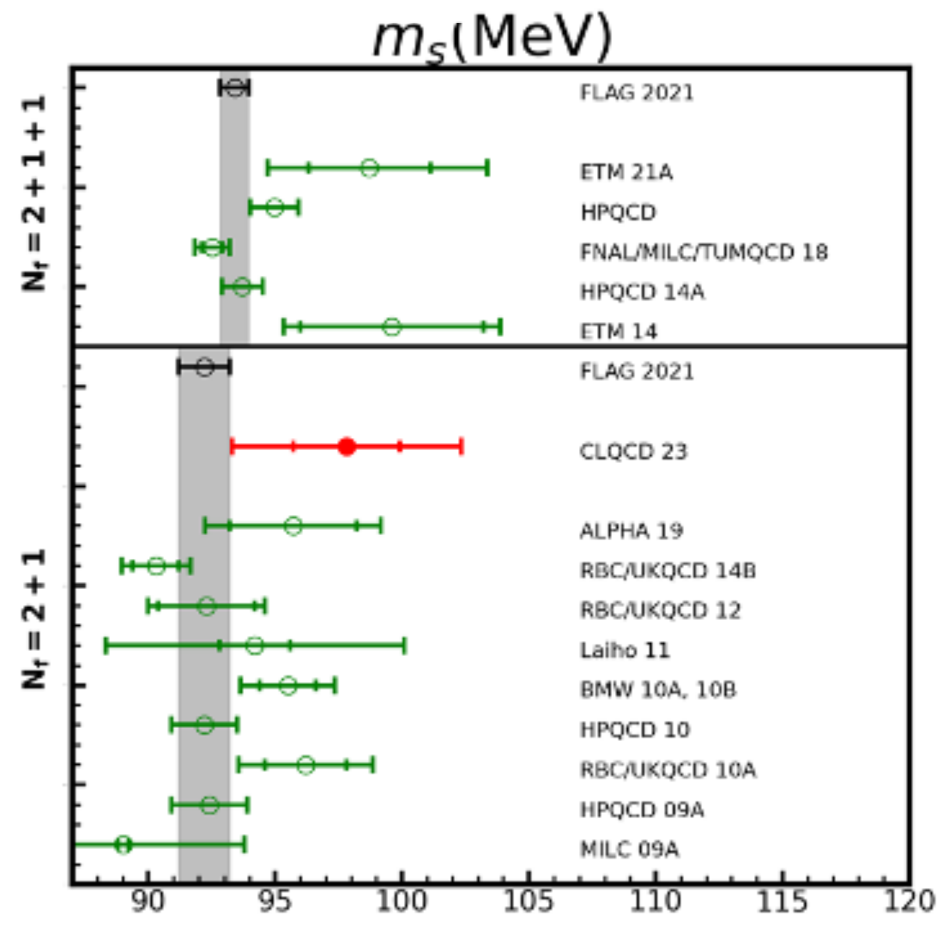
X. Feng, et,al. Phys.Rev.Lett.128(2022)062003

- $m_K^0 = 497.61(1) \text{ MeV} = m_{K,\mathrm{QCD}}^0 + 0.17(02) \text{ MeV} + \ldots;$
- $m_K^+ = 493.68(2) \text{ MeV} = m_{K,\mathrm{QCD}}^+ + 2.24(15) \text{ MeV} + \ldots$

D. Giusti, et,al. PRD95(2017)114504





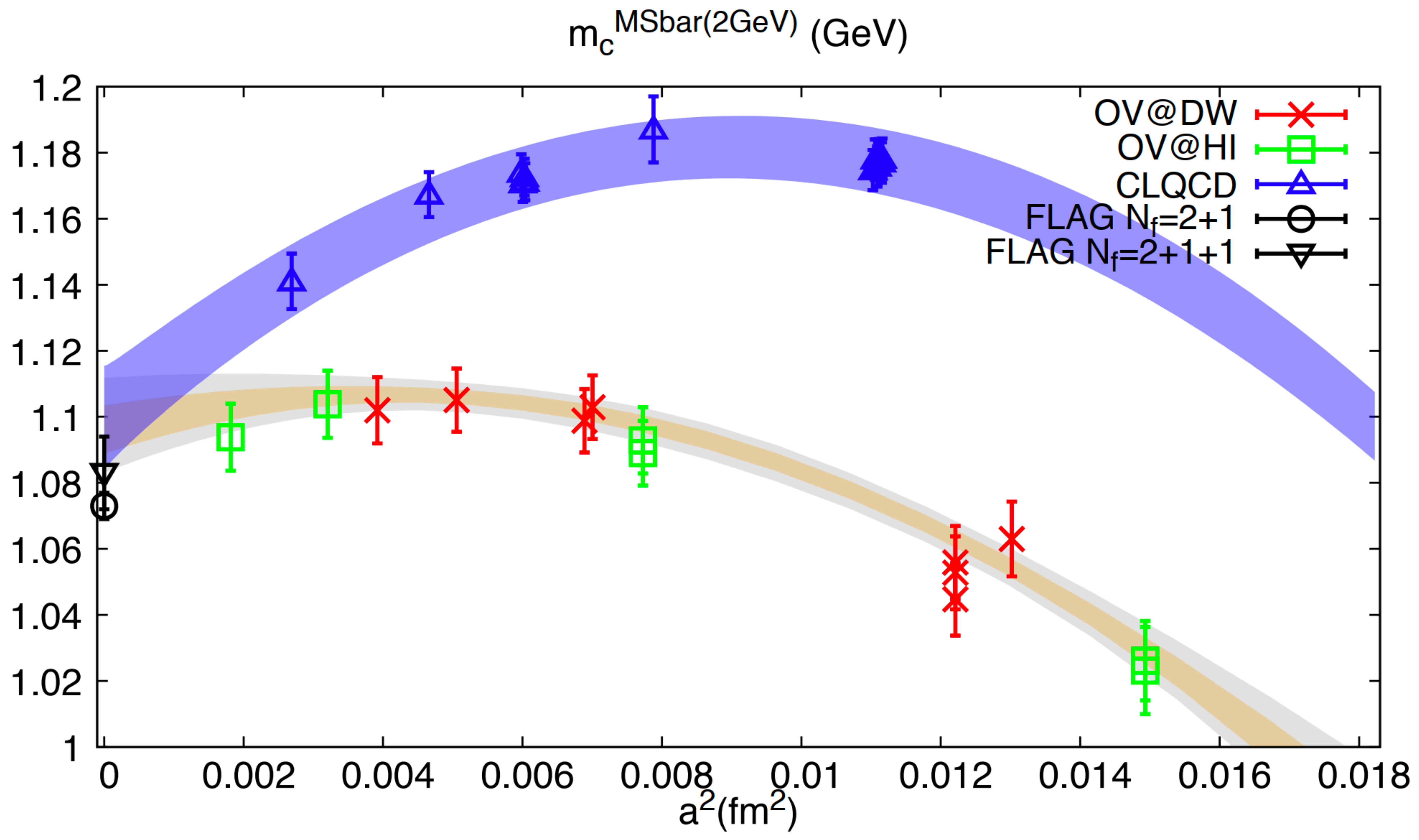


First Lattice QCD determination of up and down quark masses from China

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, PRD109(2024) 054507

Renormalized quark masses

Charm quark mass



Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

D.J. Zhao, et. al., χQCD, in preparation

Based on the $a^2 + a^4$ extrapolation:

- The impact of unphysical light and strange quark masses have been corrected based on the global fit.
- Such a value is similar to the current lattice averages within ~2%.

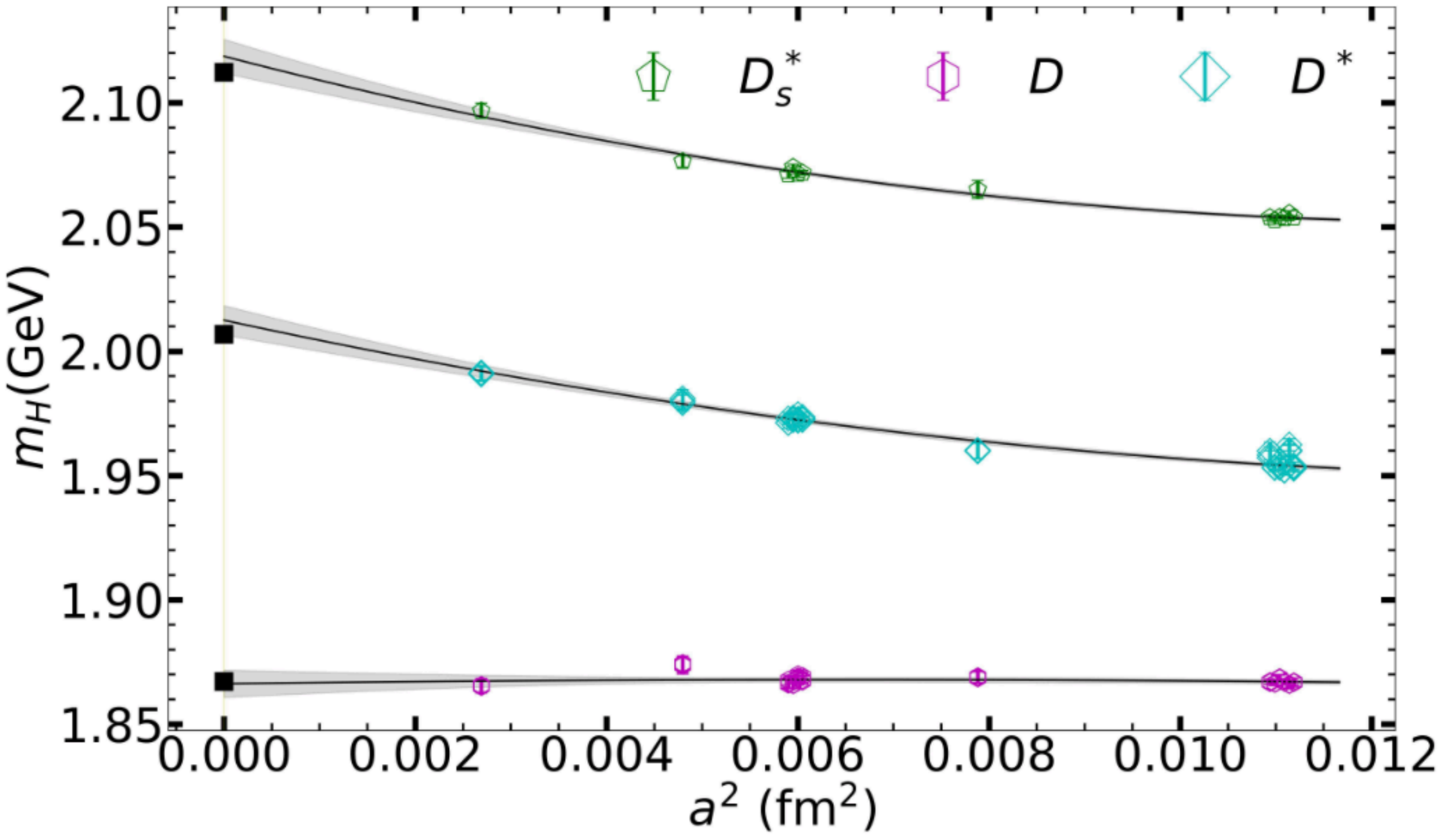
Charmed meson spectrum

Open charm cases

$$m_{D_s}^{\mathrm{QCD}} = m_{D_s}^{\mathrm{phys}} - \Delta^{\mathrm{QED}} m_{D_s} = 1966.7(1.5)\ \mathrm{MeV}\,.$$

RM123, Phys.Rev.D100 (2019) 034514

Input to determine the charm quark mass



- m_D is almost constant at different lattice spacing, with
  $m_D^{\pm} - m_D^0 = 2.9(2)_{\mathrm{QCD}} + 2.4(5)_{\mathrm{QED}} = 5.3(2)(5)\ \mathrm{MeV}$;
  RM123, Phys.Rev.D95(2017) 114504
- Agree with the PDG value 4.8(1) MeV well.
- Both m_D^* and $m_{D_s}^*$ have obvious lattice spacing dependence and the continuum extrapolated values agree with PDG well.

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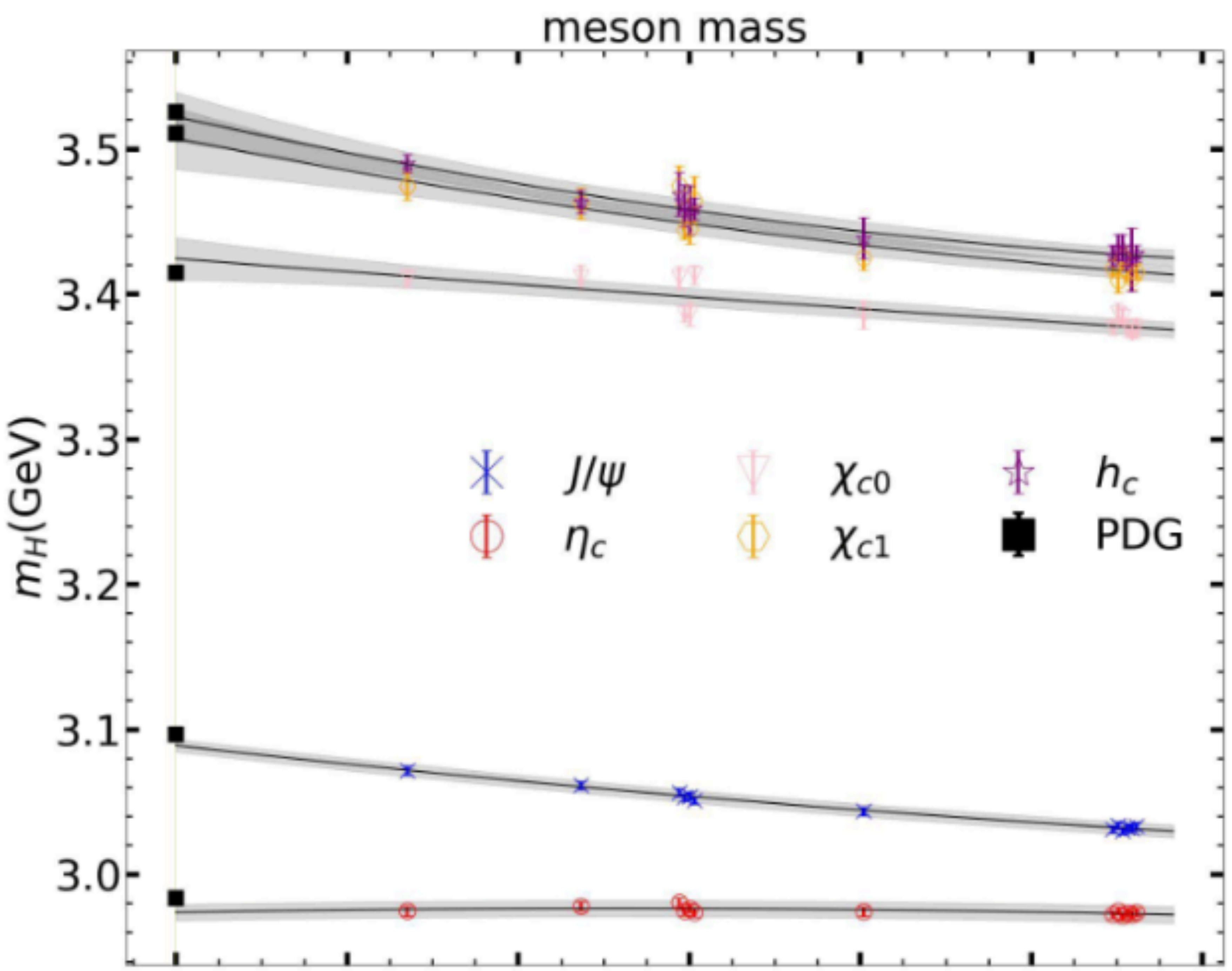
Charmed meson spectrum

charmonium cases

$$m_{D_s}^{\mathrm{QCD}} = m_{D_s}^{\mathrm{phys}} - \Delta^{\mathrm{QED}} m_{D_s} = 1966.7(1.5)\ \mathrm{MeV}\,.$$

RM123, Phys.Rev.D100 (2019) 034514

Input to determine the charm quark mass

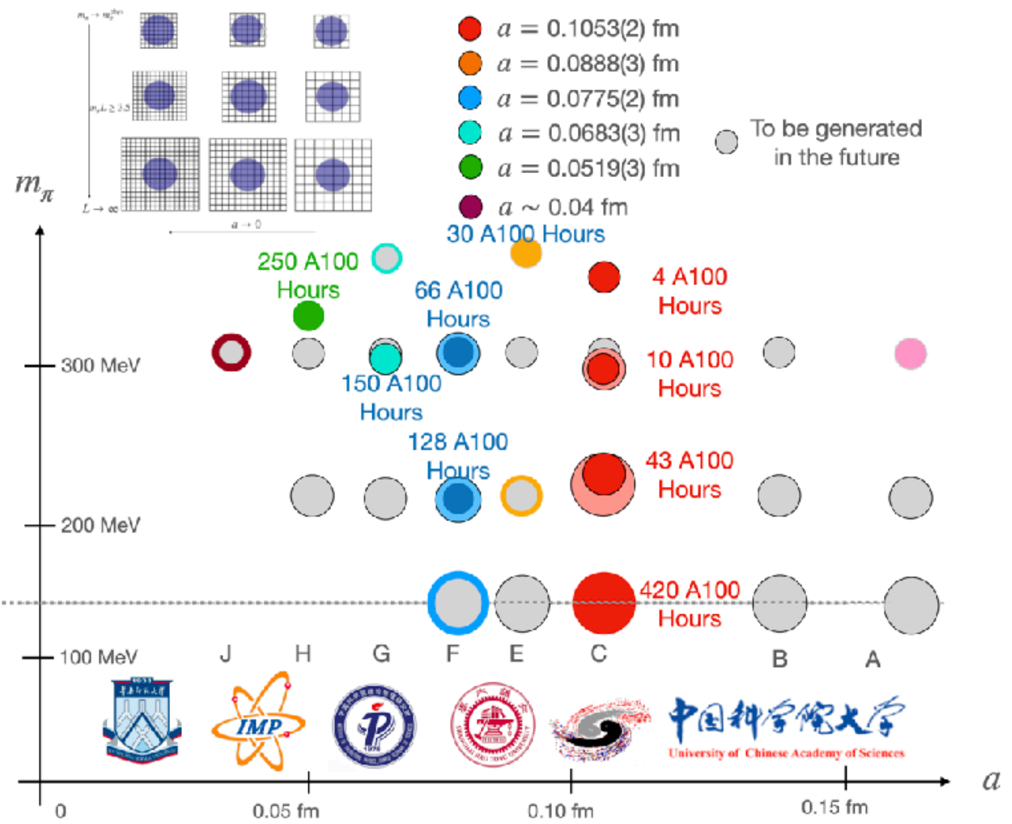


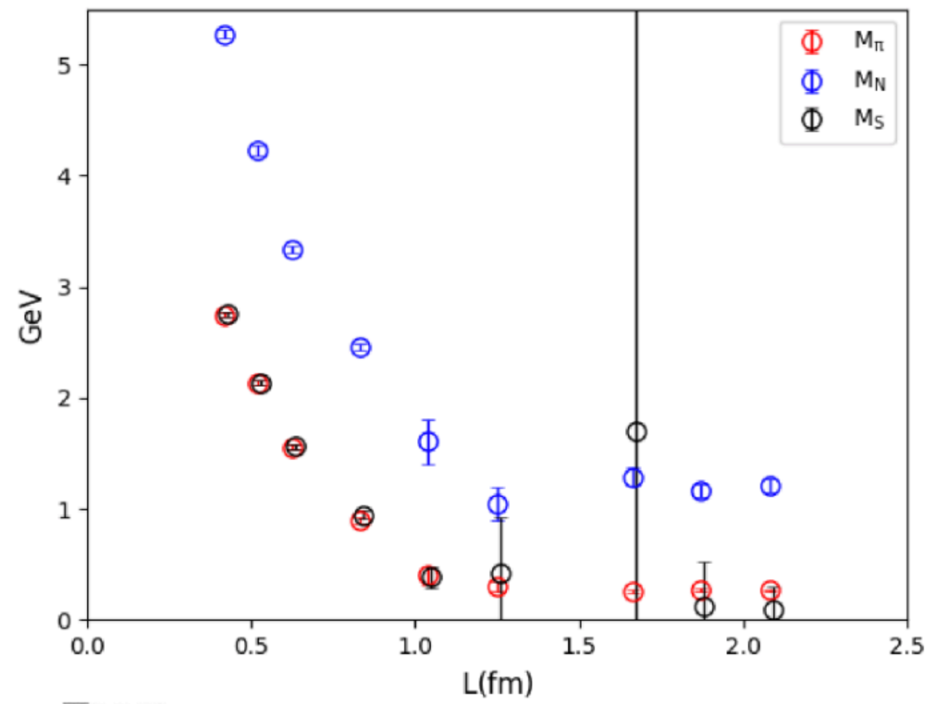
- $m_{J/\psi}$ agrees with PDG well but m_{η_c} is a few MeV lower;
- $m_{J/\psi} - m_{\eta_c} = 116(3)\ \mathrm{MeV}$ agree with previous HPQCD pure QCD prediction 119(1) MeV.
- P-wave charmonium masses also agree with PDG well, with $m_{1P} - m_{1S} = 461(19)\ \mathrm{MeV}.$

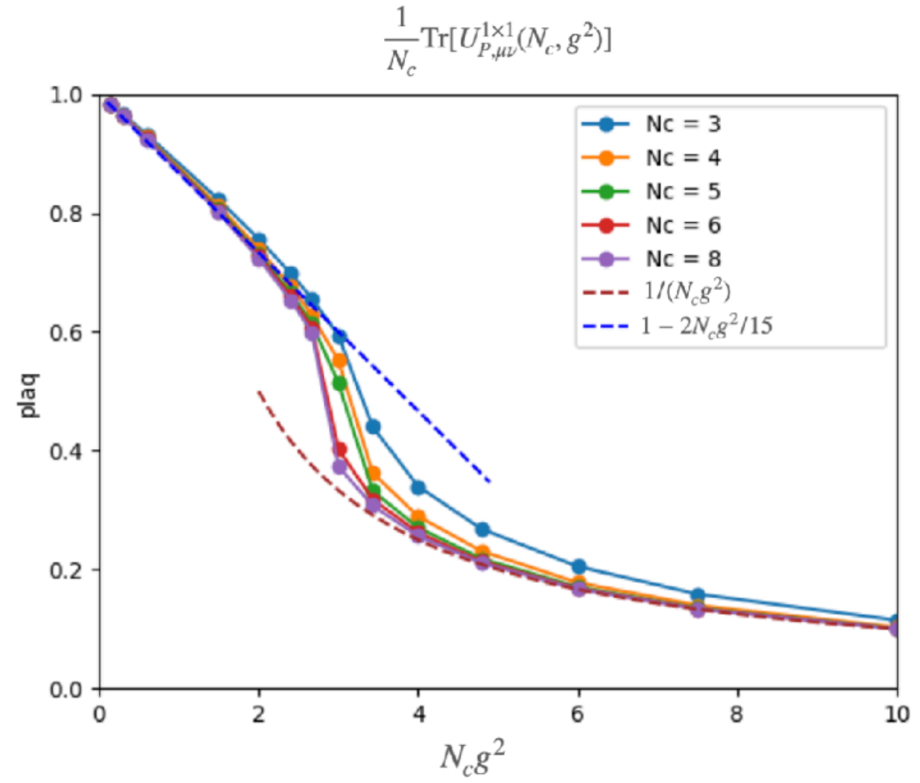
Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

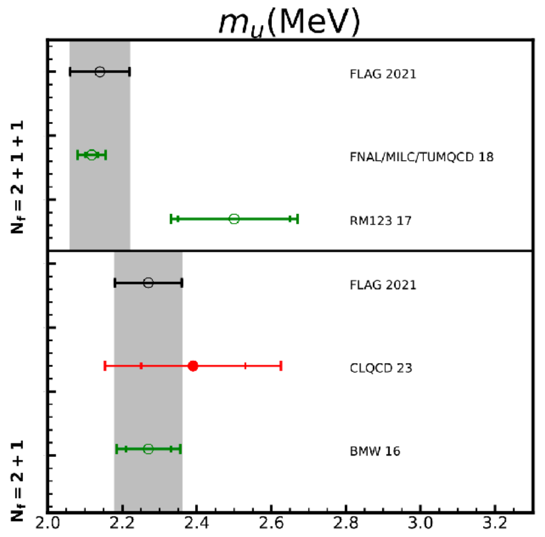
Outline

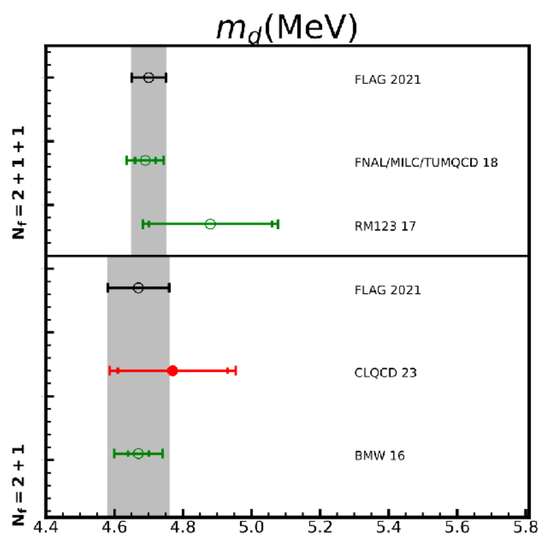
- Gauge ensembles

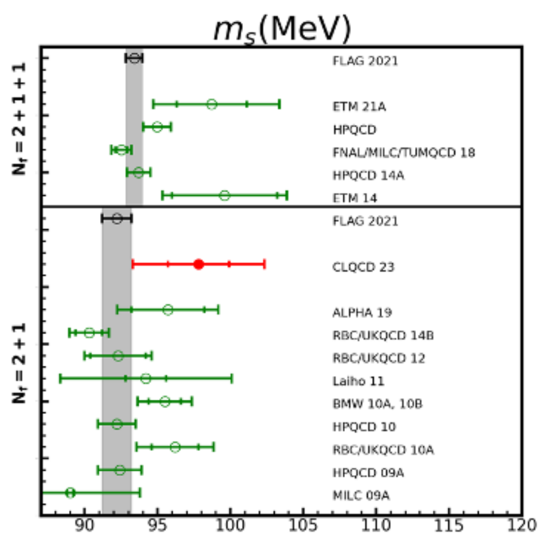


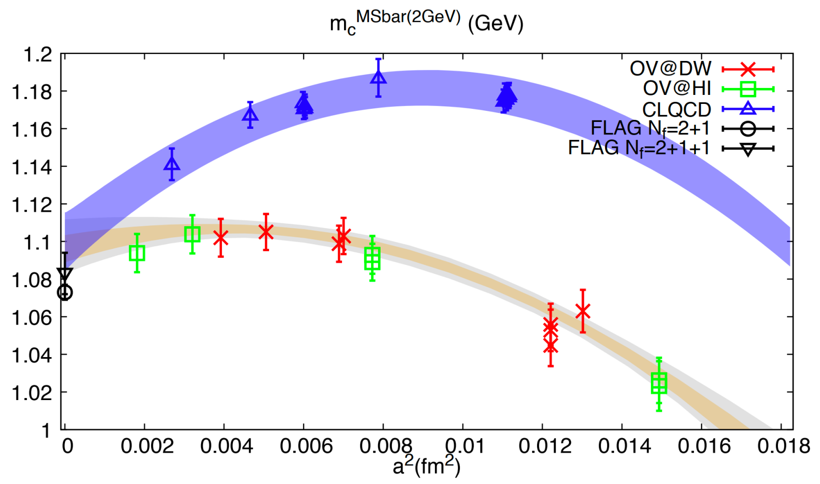


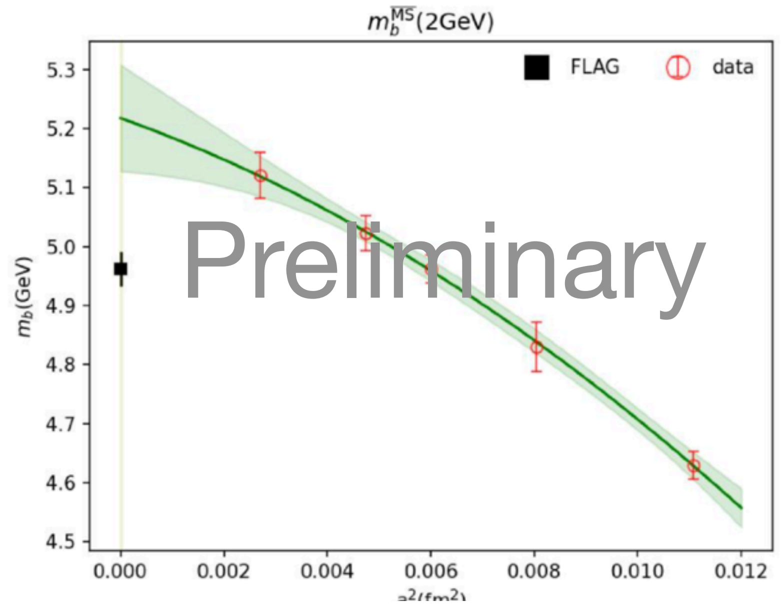






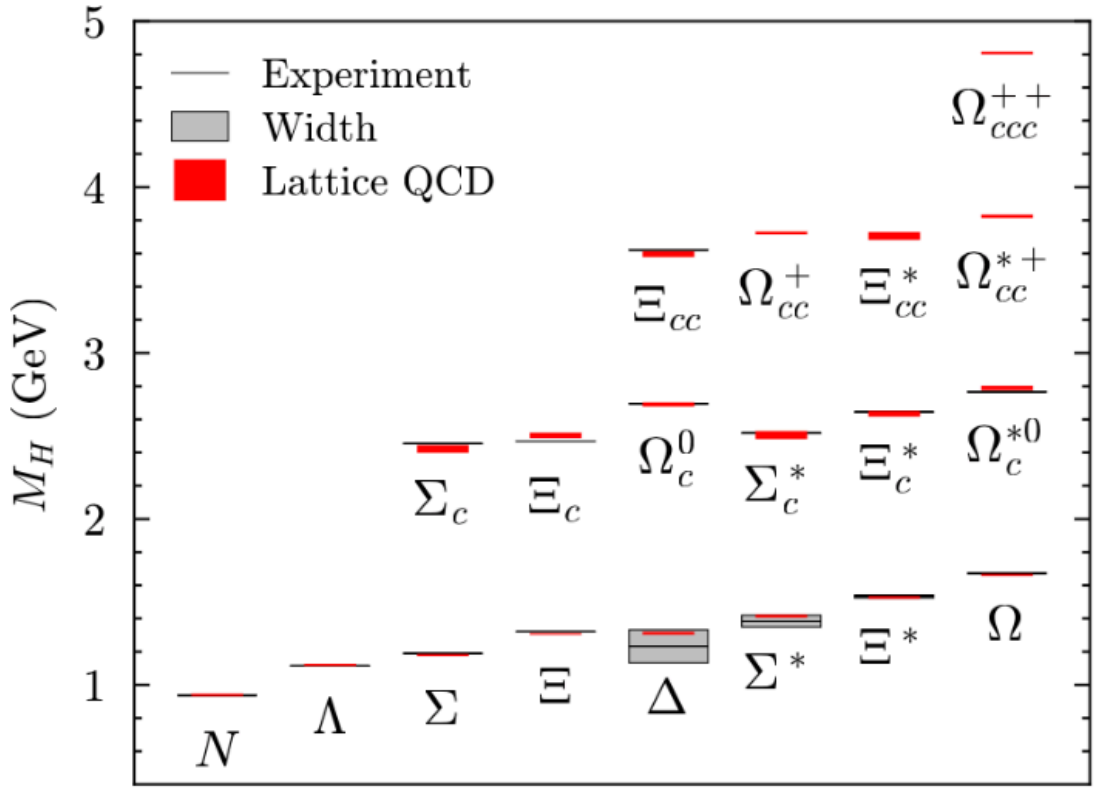


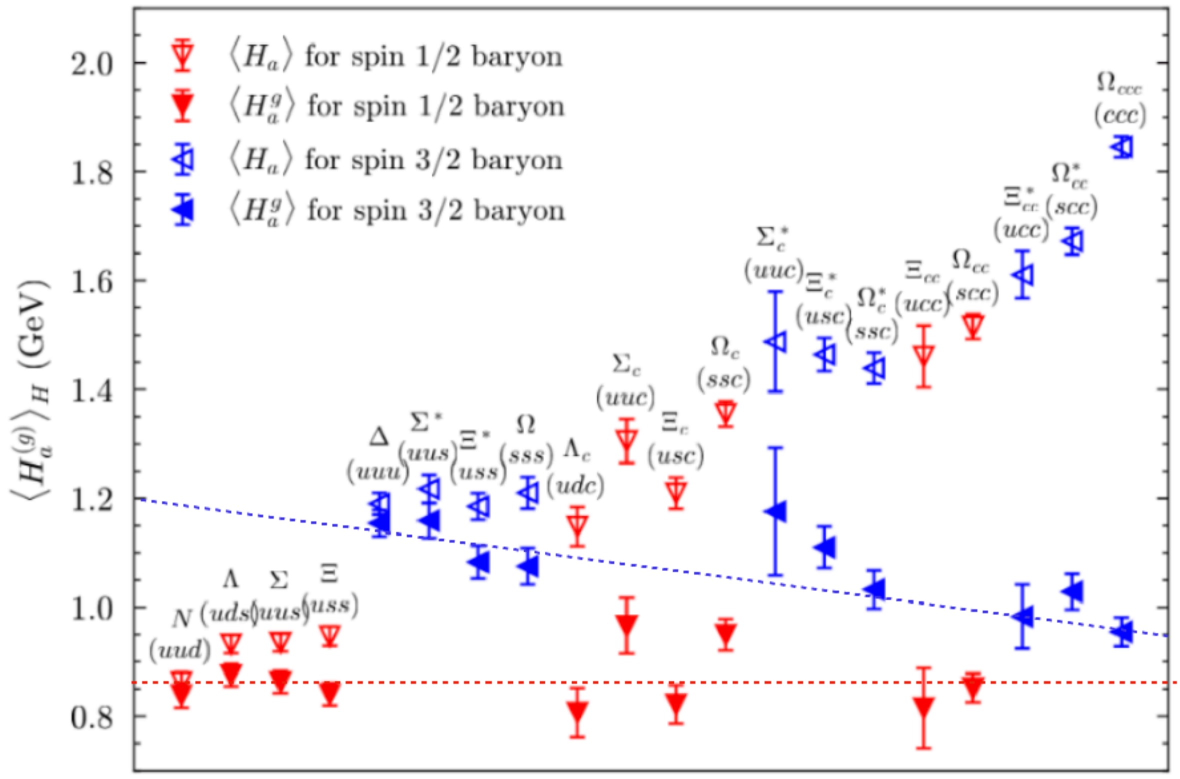




- Benchmark of CLQCD ensembles

- **Strong interaction origin of hadron mass**

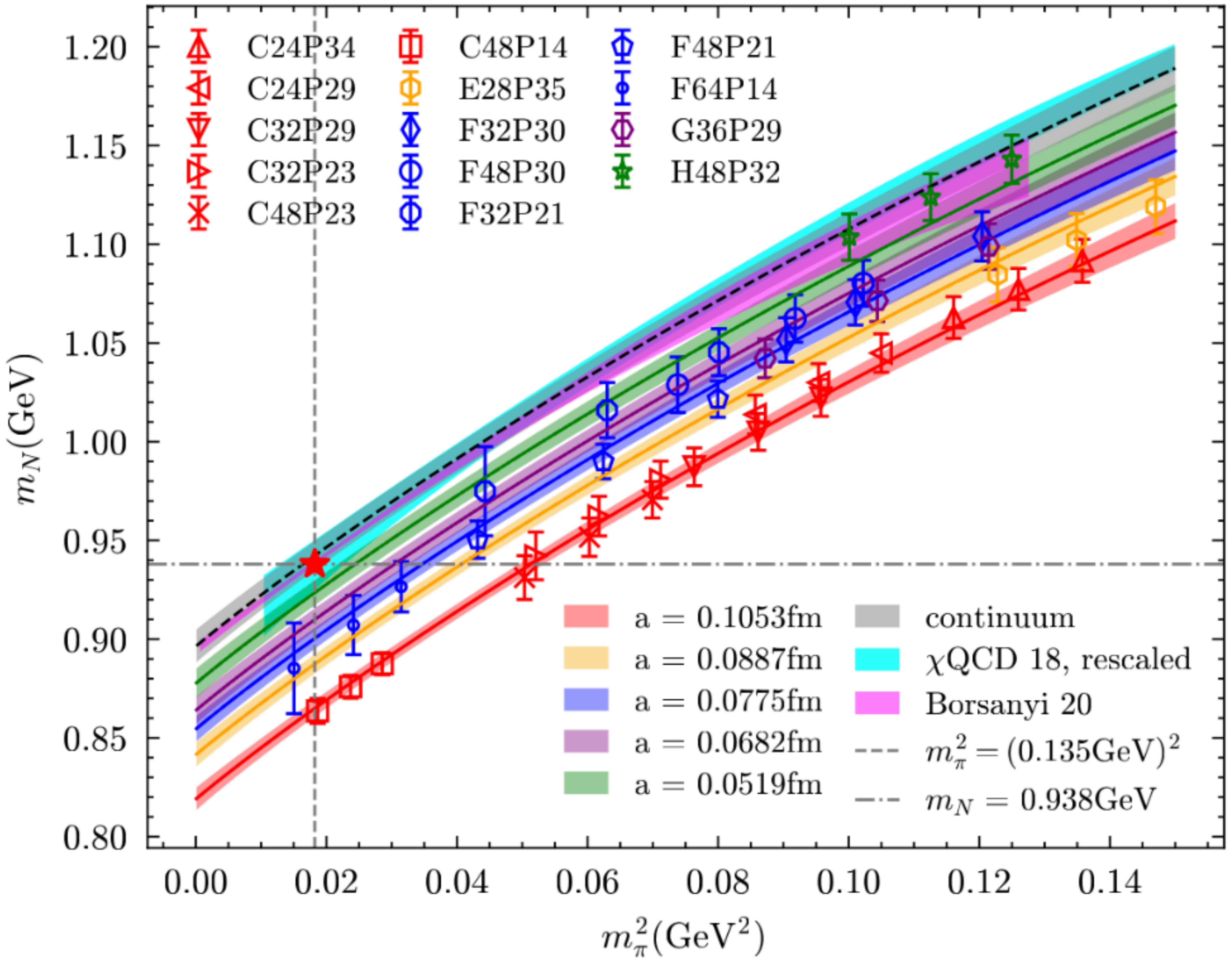




Baryon spectrum

Proton case

Symbol	$\tilde{L}^3 \times \tilde{T}$	$\hat{\beta}$	a (fm)	$\tilde{m}_l^{\mathrm{b}}$	$\tilde{m}_s^{\mathrm{b}}$	m_π(MeV)	m_{η_s}(MeV)	$m_\pi L$	n_{cfg}	n_{src}^{l}	$n_{\mathrm{src}}^{s,c}$
C24P34	$24^3 \times 64$	6.20	0.10521(11)(62)	-0.2770	-0.2310	340.2(1.7)	748.61(75)	4.38	200	32	32
C24P29	$24^3 \times 72$	6.20	0.10521(11)(62)	-0.2770	-0.2400	292.3(1.0)	657.83(64)	3.75	760	3	3
C32P29	$32^3 \times 64$			-0.2770	-0.2400	293.1(0.8)	658.80(43)	5.01	489	3	3
C32P23	$32^3 \times 64$			-0.2790	-0.2400	227.9(1.2)	643.93(45)	3.91	400	3	3
C48P23	$48^3 \times 96$			-0.2790	-0.2400	224.1(1.2)	644.08(62)	5.79	62	3	3
C48P14	$48^3 \times 96$			-0.2825	-0.2310	136.4(1.7)	706.55(39)	3.56	188	48	3
E28P35	$28^3 \times 64$	6.308	0.08970(26)(53)	-0.2490	-0.2170	351.4(1.4)	717.94(93)	4.43	147	4	4
F32P30	$32^3 \times 96$	6.41	0.07751(14)(45)	-0.2295	-0.2050	300.4(1.2)	675.98(97)	3.81	250	3	3
F48P30	$48^3 \times 96$			-0.2295	-0.2050	302.7(0.9)	674.76(58)	5.72	99	3	3
F32P21	$32^3 \times 64$			-0.2320	-0.2050	210.3(2.3)	658.79(94)	2.67	194	3	3
F48P21	$48^3 \times 96$			-0.2320	-0.2050	207.5(1.1)	661.94(64)	3.91	82	12	12
F64P14	$64^3 \times 128$			-0.2336	-0.2030	122.8(0.9)	679.9(0.3)	3.09	109	4	4
G36P29	$36^3 \times 108$	6.498	0.06884(18)(41)	-0.2150	-0.1926	297.2(0.9)	693.05(46)	3.68	270	4	4
H48P32	$48^3 \times 144$	6.72	0.05198(20)(31)	-0.1850	-0.1700	316.6(1.0)	691.88(65)	4.06	157	12	12



- Used 2,636 configurations with 27,158 measurements in total;
- After the continuum extrapolation, agree with the previous lattice results well.

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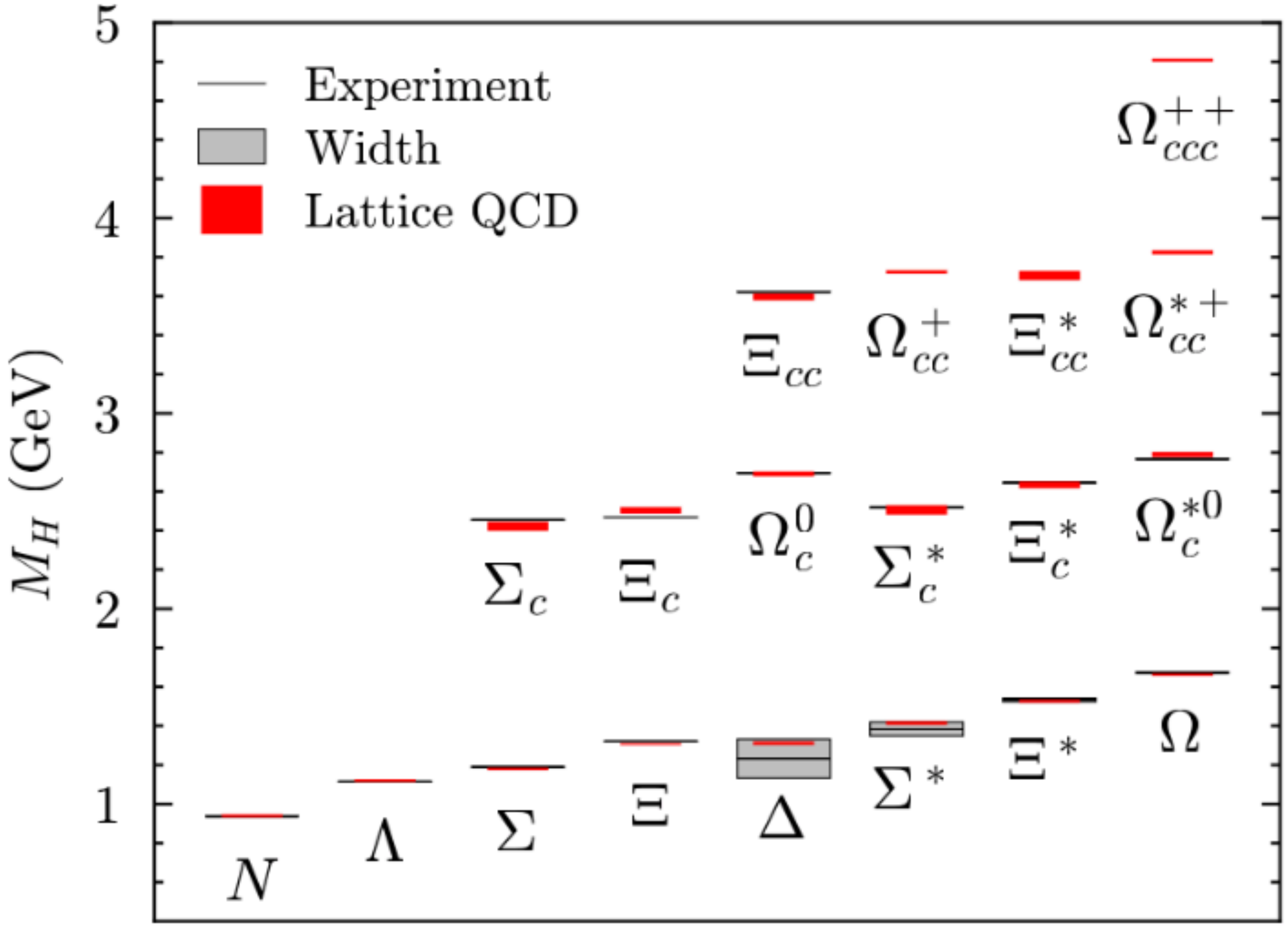
Baryon spectrum

of four light flavors

- Generally agree with the PDG values at 1% level;
- Decay width is added to the experimental values as an uncertainty of the pole position;
- The mass of baryon with 1 charm quark is around 2.5 GeV;
- The mass of baryon with 2 charm quarks is around 3.8 GeV;
- The mass of baryon with 3 charm quarks is around 5 GeV;
- The missing QED effect will be investigated in the near future.



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Baryon spectrum

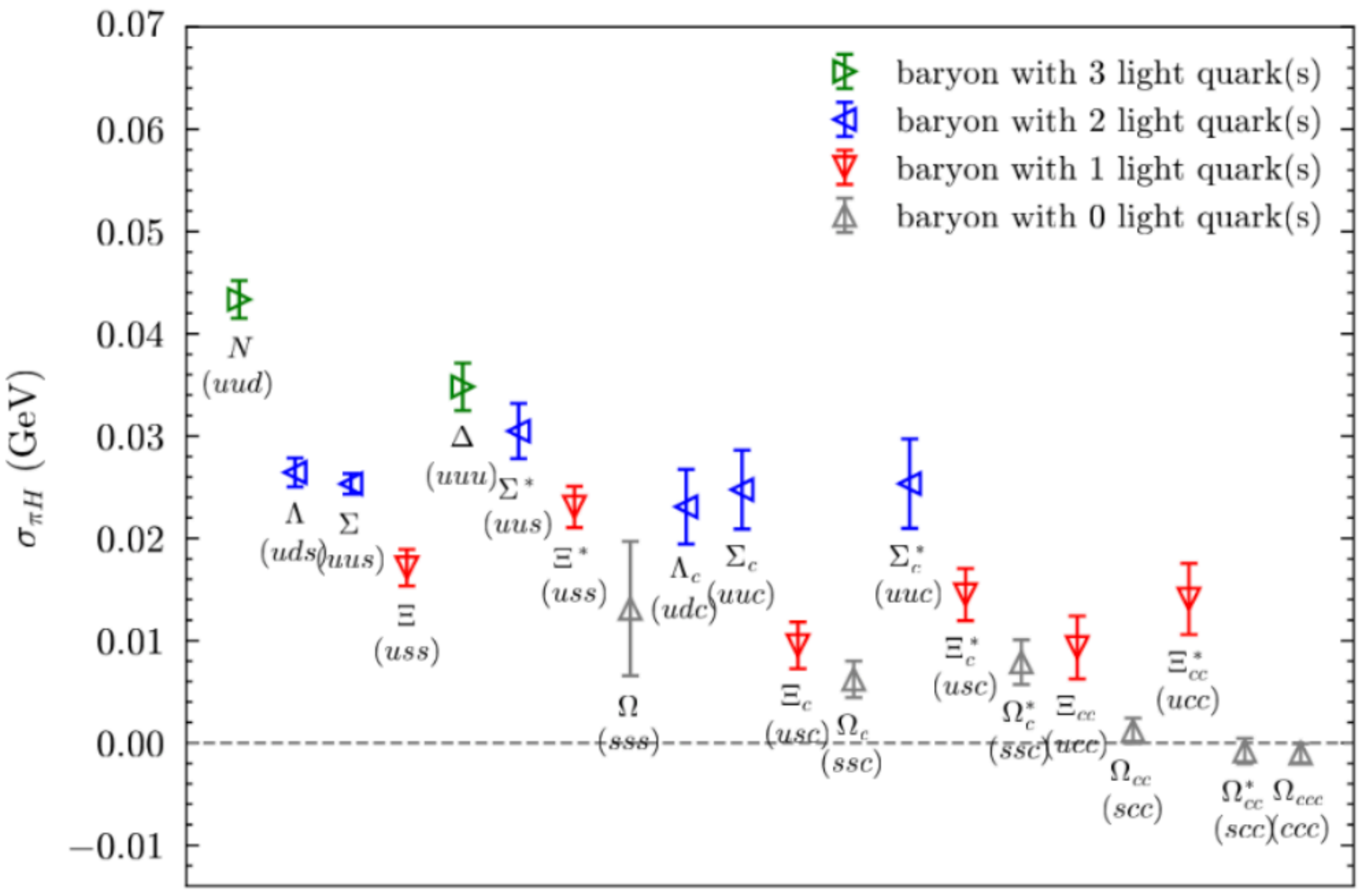
Light quark mass contribution

- After interpolation of valence strange and charm quark masses, the hadron mass on different ensembles are fitted using the following ansatz:
$X(m_\pi, m_{\eta_s}, a) = X(m_\pi^{\text{phys}}, m_{\eta_s}^{\text{phys}}, 0)$
$$+d_1^X(m_\pi^2 - (m_\pi^{\text{phys}})^2) + d_2^X(m_{\eta_s,\text{sea}}^2 - (m_{\eta_s}^{\text{phys}})^2) + d_3^X a^2 + d_4^X a^4;$$

- The light quark mass contribution is defined as

$$\sigma_{\pi X} = d_1^X (m_\pi^{\text{phys}})^2;$$

- That in $N^{1/2}$ or $\Delta^{3/2}$ is **~40 MeV**;
- Those in the baryons with two valence light quarks are **~25 MeV**;
- Those in the baryons with only one valence light quark are **~15 MeV**;
- Contribution to the baryons without valence light quark **decreases with more charm valence quarks.**



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Baryon spectrum

Strange quark mass contribution

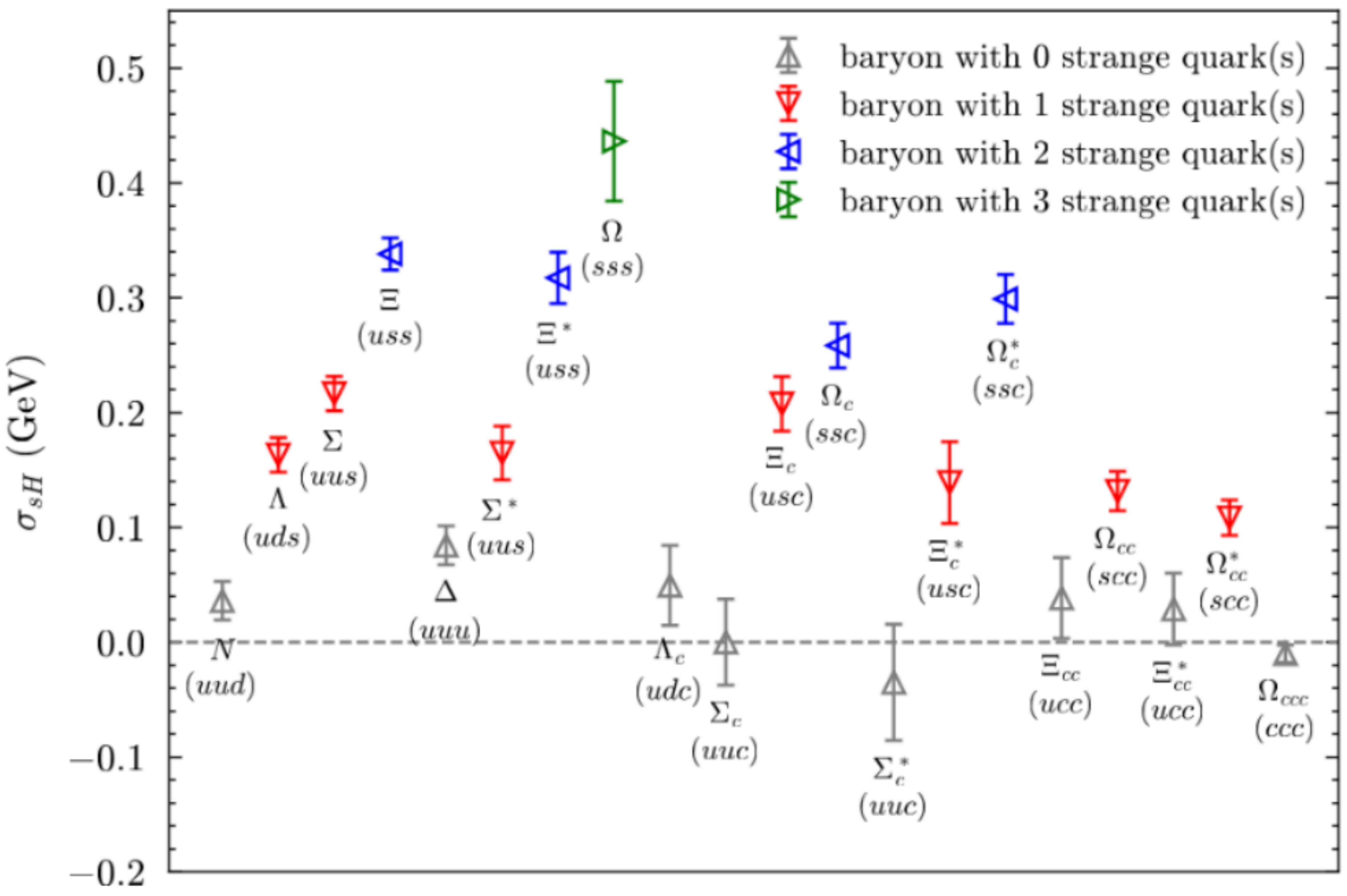
- After interpolation of valence strange and charm quark masses, the hadron mass on different ensembles are fitted using the following ansatz:

$$X(m_\pi, m_{\eta_s}, a) = X(m_\pi^{\rm phys}, m_{\eta_s}^{\rm phys}, 0)$$
$$+ d_1^X(m_\pi^2 - (m_\pi^{\rm phys})^2) + d_2^X(m_{\eta_s,\rm sea}^2 - (m_{\eta_s}^{\rm phys})^2) + d_3^X a^2 + d_4^X a^4;$$

- The strange quark mass contribution is defined as

$$\sigma_{sX} = m_s^{\rm PC} \frac{\partial m_H}{\partial m_s^{\rm PC}}\Big|_{m_{\eta_s}(m_s^{\rm PC}) = m_{\eta_s}^{\rm phys}} + d_2^X (m_{\eta_s}^{\rm phys})^2;$$

- That in $\Omega^{3/2}$ is **~450 MeV**;
- Those in the baryons with two valence strange quarks are **~300 MeV**;
- Those in the baryons with only one valence strange quark are **~150 MeV**;
- Contribution to the baryons without valence strange quark **are relatively small.**



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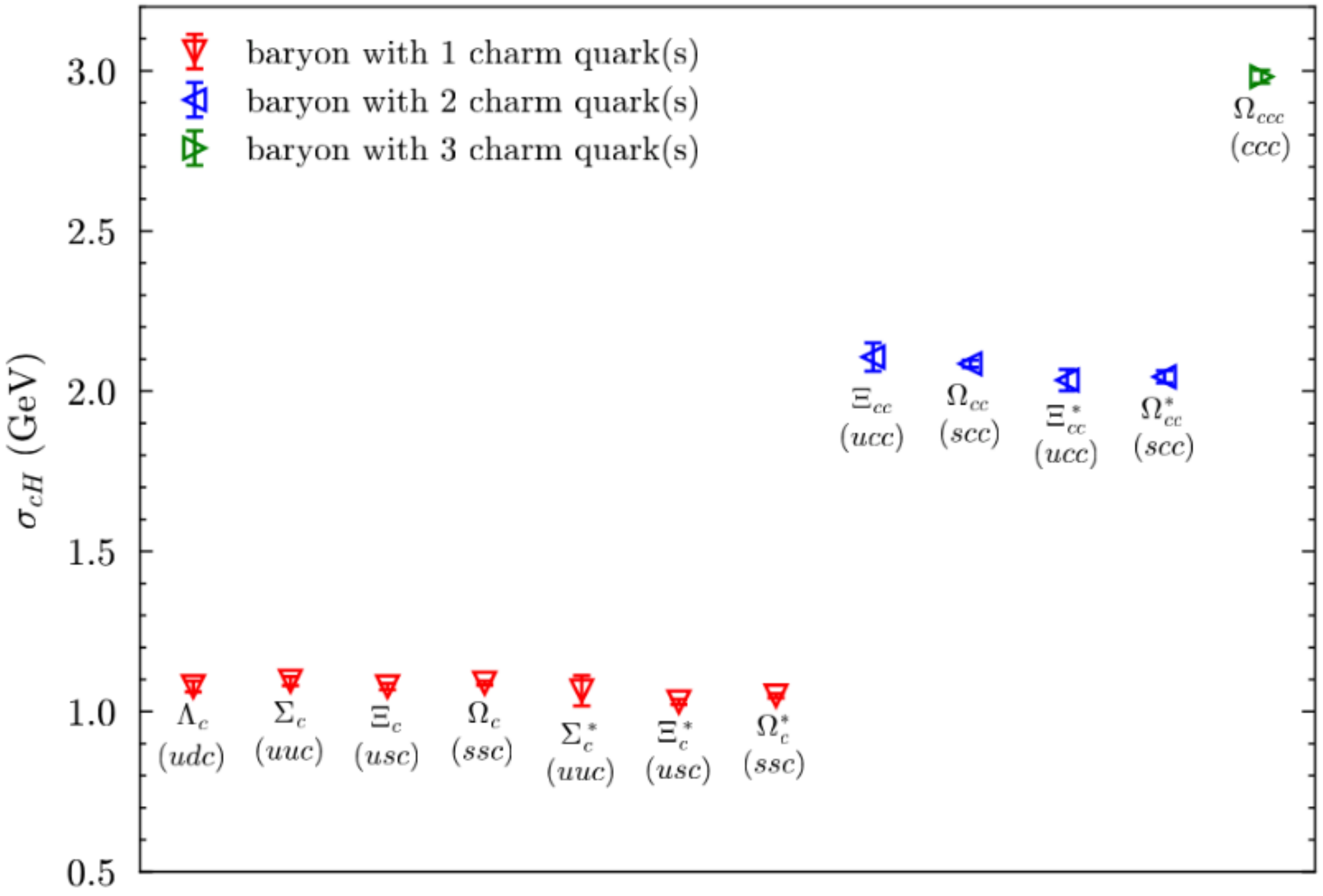
Baryon spectrum

Charm quark mass contribution

- After interpolation of valence strange and charm quark masses, the hadron mass on different ensembles are fitted using the following ansatz:
$X(m_\pi, m_{\eta_s}, a) = X(m_\pi^{\text{phys}}, m_{\eta_s}^{\text{phys}}, 0)$
$$+d_1^X(m_\pi^2 - (m_\pi^{\text{phys}})^2) + d_2^X(m_{\eta_s,\text{sea}}^2 - (m_{\eta_s}^{\text{phys}})^2) + d_3^X a^2 + d_4^X a^4;$$
- The strange quark mass contribution is defined as

$$\sigma_{cX} = m_c^{\text{PC}} \frac{\partial m_H}{\partial m_c^{\text{PC}}}\Bigg|_{\substack{m_{\eta_s}(m_s^{\text{PC}}) = m_{\eta_s}^{\text{phys}}, \\ m_{D_s}(m_{s,c}^{\text{PC}}) = m_{D_s}^{\text{phys}}}};$$

- That in $\Omega_{ccc}^{3/2}$ is **~3 GeV**;
- Those in the baryons with two valence charm quarks are **~2.1 GeV**;
- Those in the baryons with only one valence charm quark are **~1.1 GeV**;
- Contribution to the baryons without valence charm quark **are neglected (would be ~70 MeV based on the heavy quark expansion).**



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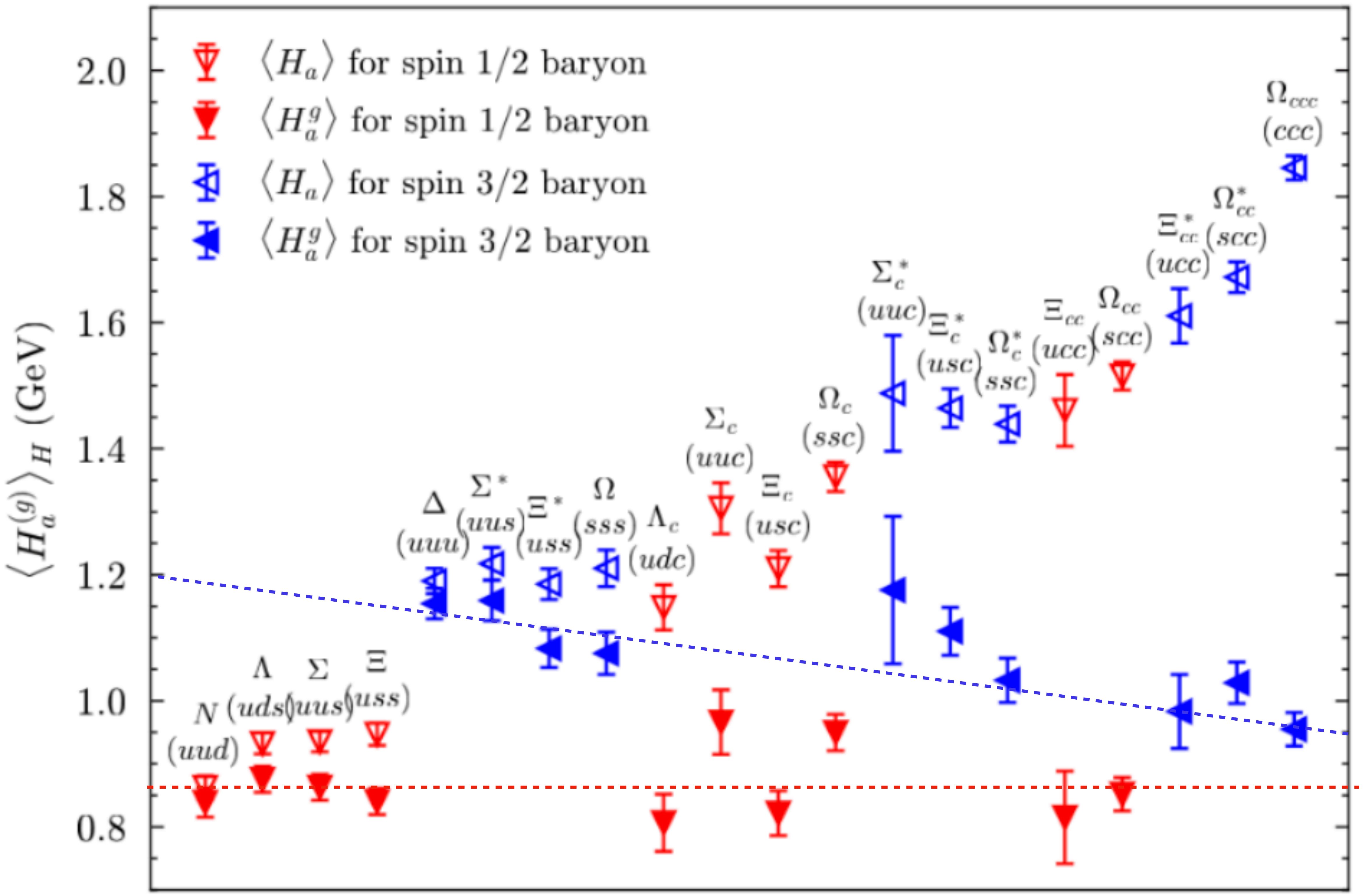
Baryon spectrum

Trace anomaly contribution

$$m_H = m\langle\bar{\psi}\psi\rangle_H + [\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)]m\langle\bar{\psi}\psi\rangle_H + [(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)]\langle F^2\rangle_H .$$



- Total trace anomaly can be obtained through the difference between baryon mass and its quark mass contributions;
- Gluon trace anomaly $\langle H_a^g\rangle_H = \langle H_a\rangle_H - \gamma_m\sigma_H$ can be more insensitive to flavor if we use $\gamma_m \sim 0.3$;
- That in the J=3/2 baryon is larger than that of J=1/2 baryon, while the difference becomes smaller with more heavy flavors.

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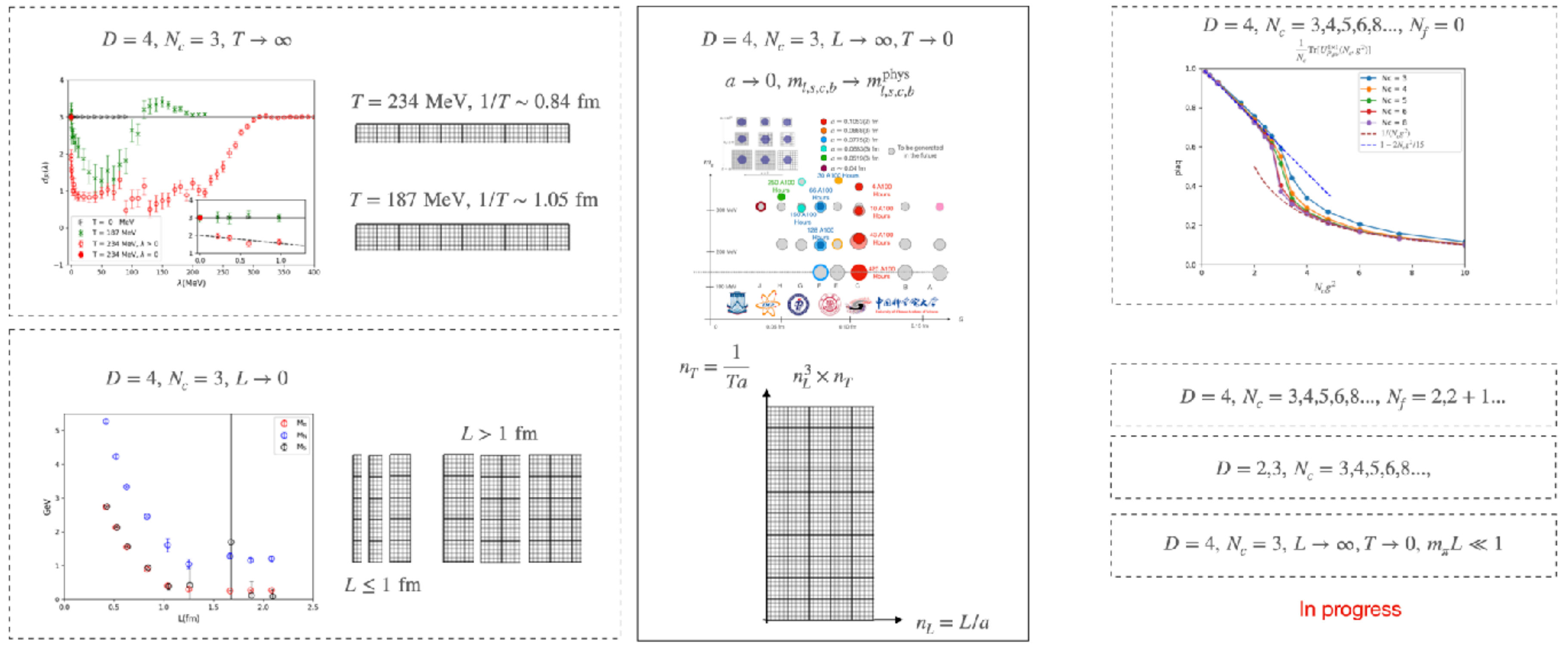
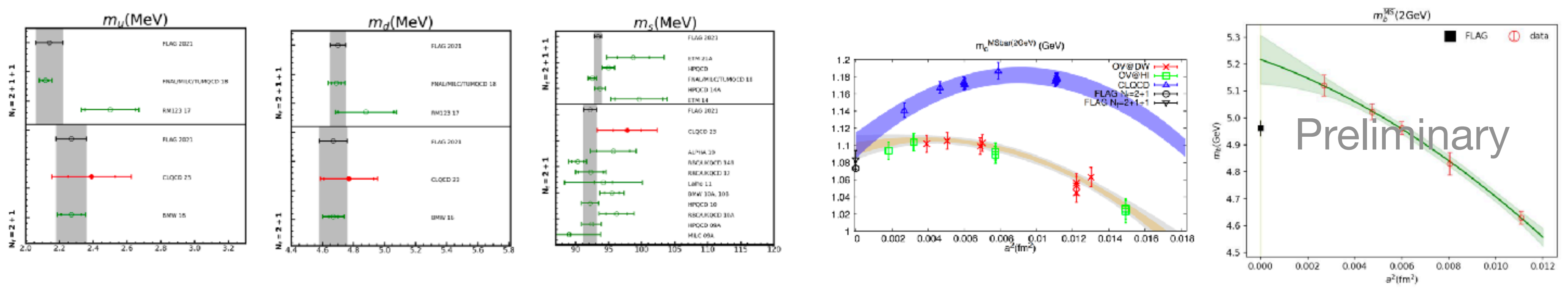
Summary

- Kinds of gauge ensembles in different conditions have been generated to study the mass gap and related feature of the Yang-Mills field;



- Up, down, strange and charm quark masses have been determined at a few percent level, and bottom quark mass is under investigation;



- Ground state baryon masses with the lightest four flavors and contributions from Higgs and strong interaction are obtained, and the latter one is universal within 10%.