Unregulated Divergences of Feynman Integrals

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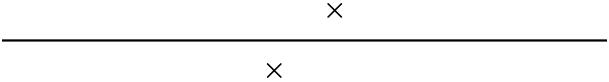
Divergences not regulated by the spacetime dimension

Dimensional regularization

$$\frac{\mathrm{d}^d l}{(l+p)^2 - m^2 + i0^+} = \frac{\mathrm{d}^{d_0} l_\parallel \mathrm{d}^{d-d_0} l_\perp}{(l_\parallel - p)^2 - m^2 - l_\perp^2 + i0^+} = \frac{\Omega(d-d_0)\mathrm{d}^{d_0} l_\parallel \mathrm{d} l_\perp \; l_\perp^{d-d_0-1}}{(l_\parallel - p)^2 - m^2 - l_\perp^2 + i0^+} \tag{1}$$

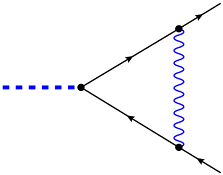
Pinched singularities



Unregulated divergences

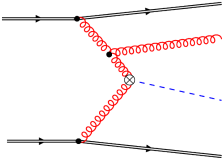
Transverse momentum $l_\perp$ may be absent in some effective theories:

- Heavy quark effective theory (HQET)
- Soft collinear effective theory (SCET)
- ...



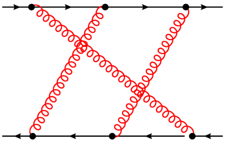
$M_Z \to 0$

(a) Massive Sudakov form factor



$q_T \to 0$

(b) Transverse momentum distribution



$t \to 0$

(c) Regge limit

An example: massive Sudakov form factor

Expansion by regions

Modes in effective theories ⟷ Regions

Massive Sudakov form factor

$$J_0(i_1,\ i_2,\ i_3) \equiv \int \frac{\mathrm{d}^d l}{\pi^{d/2}} \frac{1}{(l+n)^{2i_1} (l-\bar{n})^{2i_2} \left(l^2 - M_Z^2\right)^{i_3}} \ . \tag{2}$$

$$\begin{aligned} n^2 =& \bar{n}^2 = 0 \ , \\ (n+\bar{n})^2 =& Q^2 \gg M_Z \ . \end{aligned}$$

n collinear region: $\quad l^- \sim M_Z^0,\ l^+ \sim l^2 \sim M_Z^2$,

$\bar{n}$ collinear region: $\quad l^+ \sim M_Z^0,\ l^- \sim l^2 \sim M_Z^2$,

hard: $\quad l^+ \sim l^- \sim l^2 \sim M_Z^0$.

Rapidity divergence

n collinear region:

$$
\begin{aligned}
J_n &= \int \frac{\mathrm{d}^d l}{\pi^{d/2}} \frac{1}{(l+n)^2(-2l^-)\,(l^2 - M_Z^2)} \\
&= -\, i\ \Gamma(1+\epsilon) \int_0^\infty \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \left[M_Z^2(1-x_2) - Q^2 x_1 x_2\right]^{-1-\epsilon} \\
&= -\, i\Gamma(1+\epsilon) \int_0^\infty \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \frac{1}{x_2} \left[M_Z^2(1-x_2) - 2x_1\right]^{-1-\epsilon} \ .
\end{aligned} \tag{3}
$$

Expansion by regions breaks down:

$$
\begin{aligned}
J_0(1,\ 1,\ 1) &= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}z}{2\pi i} \Gamma(-z)\Gamma(1+z) \int \frac{\mathrm{d}^d l}{\pi^{d/2}} \frac{(l^2)^z}{(l^2 - M_Z^2)\,(2l^+)^{1+z}(l-p_2)^2} \\
&\equiv \int_{-i\infty}^{i\infty} \frac{\mathrm{d}z}{2\pi i} \Gamma(-z)\Gamma(1+z) J_n(z) \\
&\to \frac{i\Gamma(\epsilon)}{Q^2 M_Z^{2\epsilon}} \left[-\log\left(\frac{M_Z^2}{Q^2}\right) + \psi(\epsilon) - 2\psi(1-\epsilon) - \gamma - i\pi\right] + \mathcal{O}(M_Z) \ .
\end{aligned} \tag{4}
$$

Logarithmic dependence

Differential equations

$$J = \{J_0(0,2,1),\ J_0(2,0,0),\ \epsilon J_0(1,1,1)\}\ . \tag{5}$$

$$\frac{\mathrm{d}J}{\mathrm{d}M_Z} \approx \frac{1}{M_Z} M \cdot J\ , \tag{6}$$

$$M = \epsilon\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & -2 & 0 \\ -2 & -1 & -2 \end{pmatrix}\ . \tag{7}$$

Asymptotic solution

$$J_0(1,1,1) \approx c_1 + c_2 M_Z^{-2\epsilon} + c_3 \log(M_Z) M_Z^{-2\epsilon}\ . \tag{8}$$

Collinear anomaly

analytic regulator

$$J_n \to \int \frac{\mathrm{d}^d l}{\pi^{d/2}} \frac{\nu^{2\eta}}{(l+n)^{2(1+\eta)}(-2l^-)\left(l^2 - M_Z^2\right)}$$

$$= i\Gamma(\epsilon)\frac{M_Z^{-2\epsilon}}{Q^2}\left[\frac{1}{\eta} + \log(\frac{\nu^2}{M_Z^2}) - \psi^{(0)}(1-\epsilon) + \psi^{(0)}(\epsilon) - i\pi\right], \quad (9a)$$

$$J_{\bar{n}} \to \int \frac{\mathrm{d}^d l}{\pi^{d/2}} \frac{\nu^{2\eta}}{(2l^+)^{1+\eta}(l-\bar{n})^2\left(l^2 - M_Z^2\right)}$$

$$= i\Gamma(\epsilon)\frac{M_Z^{-2\epsilon}}{Q^2}\left[-\frac{1}{\eta} + \log(\frac{Q^2}{\nu^2}) - \psi^{(0)}(1-\epsilon) - \gamma\right]. \quad (9b)$$

collinear anomaly

Degeneracies of regions

Unregulated divergences

Parametric representation

$$I(\lambda_0, \lambda_1, \ldots, \lambda_n) = \frac{\Gamma(-\lambda_0)}{\prod_{i=1}^{n+1} \Gamma(\lambda_i + 1)} \int \mathrm{d}\Pi^{(n+1)} \mathcal{F}^{\lambda_0} \prod_{i=1}^{n+1} x_i^{\lambda_i} \ . \tag{10}$$

$$\mathcal{F} \equiv F + U x_{n+1} \equiv \sum_{a=1}^{A} \left(C_{\mathcal{F},a} \prod_{i=1}^{n+1} x_i^{\Lambda_{ai}} \right) \ . \tag{11}$$

region vectors

$$\sum_{i=1}^{n+1} \Lambda_{ai} k_{r,i} = 0, \quad a \in S_r \ , \tag{12a}$$

$$\sum_{i=1}^{n+1} \Lambda_{ai} k_{r,i} > 0, \quad a \notin S_r \ . \tag{12b}$$

Assuming $k_{r,n+1} = 1$, terms in S_r dominates $\mathcal{F}$ when $x_i \sim x_{n+1}^{k_{r,i}}$.

Unregulated divergences

Leading singularity

$$I(\lambda_0, \lambda_1, \ldots, \lambda_n) \to \frac{\Gamma(-\lambda_0)}{\prod_{i=1}^{n+1} \Gamma(\lambda_i + 1)} \int \mathrm{d}\Pi^{(n+1)} \sum_{i \in S_r} \left(C_{\mathcal{F},a} \prod_{i=1}^{n+1} x_i^{\Lambda_{ai}} \right)^{\lambda_0} \prod_{i=1}^{n+1} x_i^{\lambda_i}$$

$$= \frac{\Gamma(-\lambda_0)}{\prod_{i=1}^{n+1} \Gamma(\lambda_i + 1)} \int \mathrm{d}x_{n+1}\, x_{n+1}^{\nu_r - 1} \times \int \mathrm{d}\Pi^{(n)} \sum_{i \in S_r} \left(C_{\mathcal{F},a} \prod_{i=1}^{n} x_i^{\Lambda_{ai}} \right)^{\lambda_0} \prod_{i=1}^{n} x_i^{\lambda_i} \,. \tag{13}$$

$$\nu_r \equiv \sum_{i=1}^{n+1} k_{r,i} (\lambda_i + 1) \tag{14}$$

Criterion for unregulated divergences

$$\nu_r \in \mathbb{Z}^- \cup \{0\} \,, \tag{15}$$

Expansion by regions

$$\mathcal{F} = \sum_{a=1}^{A} \left(C'_{\mathcal{F},a} \prod_{i=0}^{n+1} x_i^{\Lambda'_{ai}} \right) \ . \tag{16}$$

x_0 a small parameter (not a Feynman parameter)

Expansion by regions

$$\sum_{i=0}^{n+1} \Lambda'_{ai} k_{\rho,i} = b_\rho, \quad a \in S_\rho \ , \tag{17a}$$

$$\sum_{i=0}^{n+1} \Lambda'_{ai} k_{\rho,i} > b_\rho, \quad a \notin S_\rho \ . \tag{17b}$$

$$\nu_\rho = b_\rho \lambda_0 + \sum_{i=1}^{n+1} k_{\rho,i}(\lambda_i + 1) \ , \tag{18a}$$

$$I(\lambda_0, \lambda_1, \ldots, \lambda_n) \sim x_0^{\nu_\rho} \ . \tag{18b}$$

Degeneracies of regions

Degenerate

rational number

$$\nu_\rho = \nu_{\rho'} + n \tag{19}$$

$$\text{a singular region: } S_r = S_\rho \cap S_{\rho'} \tag{20}$$

$$k_{\rho',0} = k_{\rho,0} + ck_r \ , \tag{21a}$$

$$\nu_{\rho'} = \nu_\rho + c\nu_r \ . \tag{21b}$$

The presence of unregulated divergences implies the degeneracies of regions.

Summary

Summary

In this talk, I give a brief introduction of unregulated divergences of Feynman integrals through a simple massive Sudakov form factor example. The notion of region degeneracy is introduced. It is shown that the presence of unregulated divergences implies the degeneracies of regions.

Thanks for your attention.