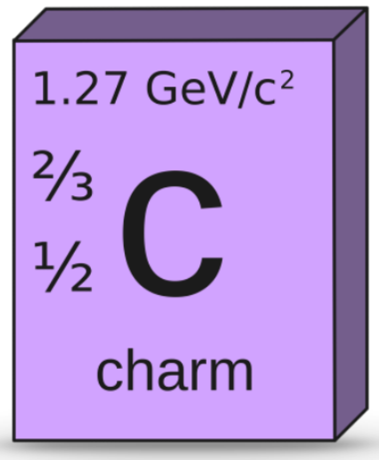


# Inclusive charm decays

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**第四届量子场论及其应用研讨会**

**广州，2024年11月16-20日**

# Semi-inclusive charm decays

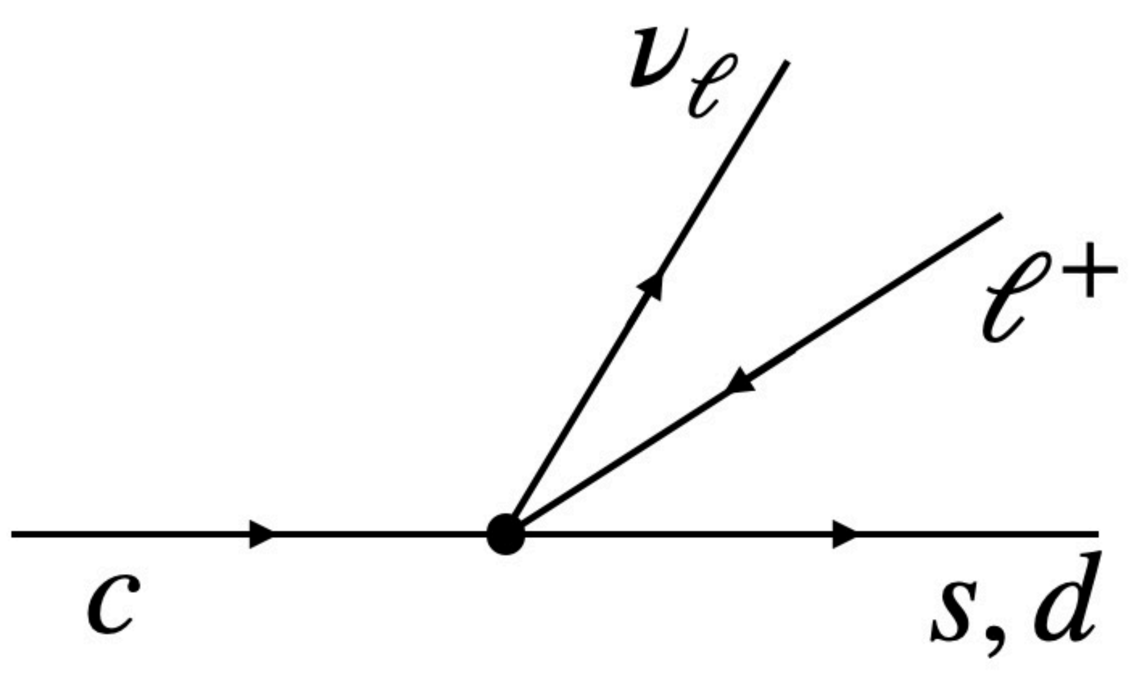
- **Experimental detection of partial final state particles**

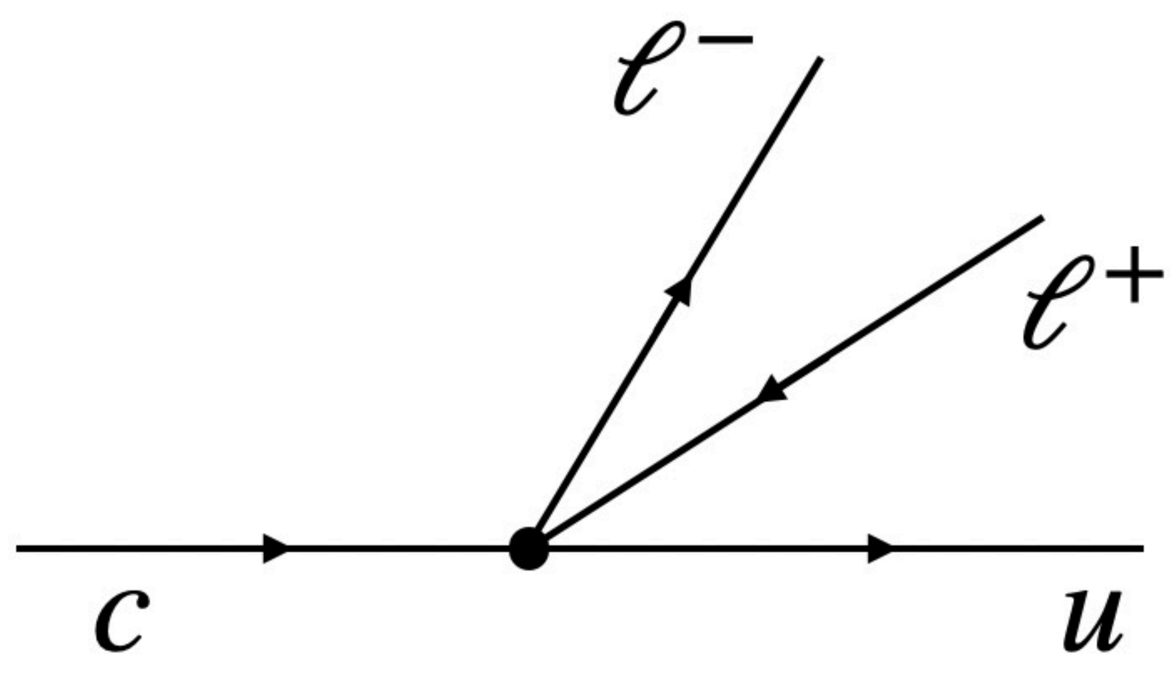
➡ $D \to e^+X$ ($D \to e^+\nu_e X$, only $e^+$ is detected)

- **Sum of a group of exclusive channels**

➡ $D^0 \to e^+X_s = D \to e^+\nu_e K^-, e^+\nu_e K^-\pi^0, e^+\nu_e \bar{K}^0\pi^-, \ldots$

➡ $D^0 \to e^+X_d = D \to e^+\nu_e \pi^-, e^+\nu_e \pi^-\pi^0, e^+\nu_e \pi^-\pi^+\pi^-, \ldots$



# Why inclusive charm decays?

- **As weak decays of heavy hadrons**

  ➡ Probe new physics

  ➡ Understand QCD

- **Compared to exclusive decays**

  ➡ **Better theoretical control**

- **Compared to beauty decays**

  ➡ **Special to new dynamics attached with up-type quarks**

  ➡ **More sensitive to power corrections**

★ Determination by charm, application in beauty.

# Why inclusive charm decays?

- **Resolve (or at least give hints to) current flavor puzzles/anomalies**

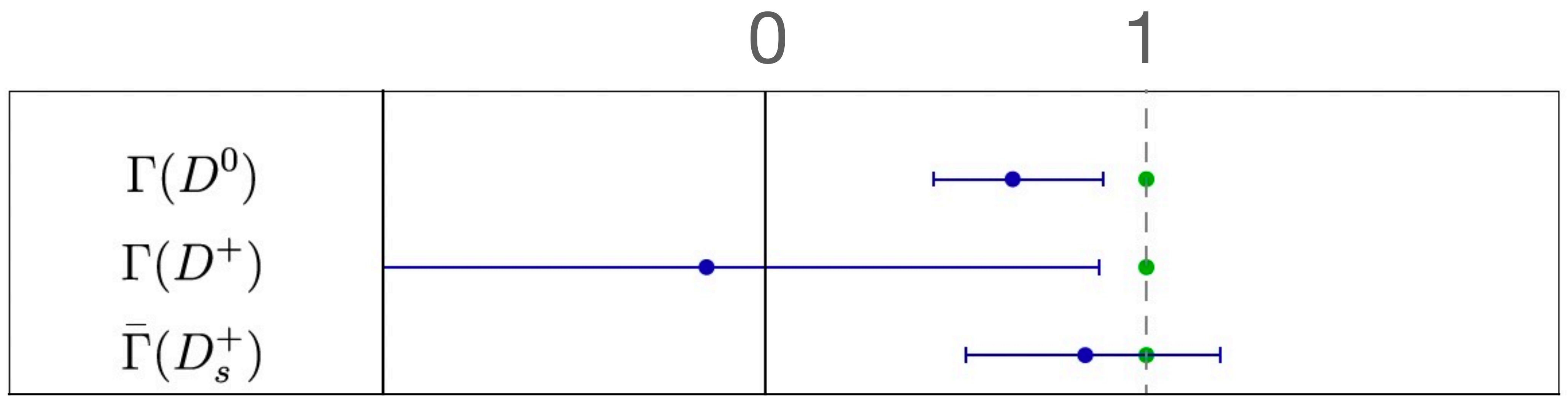
  ➡ Puzzles in charmed hadron lifetimes: theory vs experiment

  ➡ $V_{cb}$, $V_{ub}$ puzzles: inclusive vs exclusive

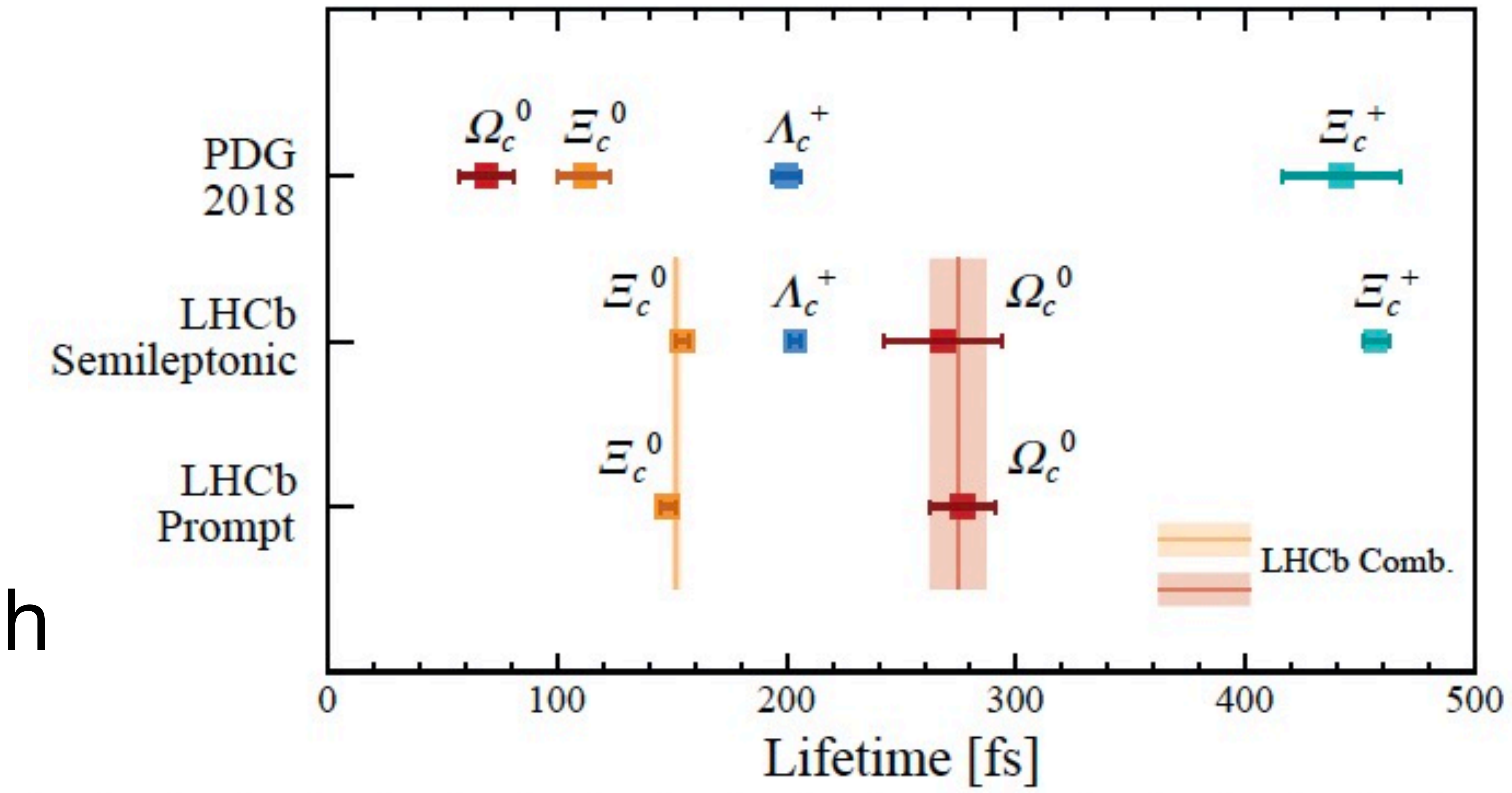
  ➡ $b \to s$ anomalies: $P_5'$ in $B \to K^* \ell\ell$

# Why inclusive charm decays?

- **Flavor puzzle 1.** Charmed hadron lifetimes: theory vs experiment




[Lenz et al, '22]




- **Key issue:** Nonperturbative power corrections with large/unknown uncertainties

  ➡ Dependence on identical hadronic parameters in HQET, $\langle H_c | O_i | H_c \rangle$

$$\mathcal{O}(1/m_c^3) \Rightarrow \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\boldsymbol{\Omega_c^0}),$$
$$\mathcal{O}(1/m_c^4) \Rightarrow \tau(\boldsymbol{\Omega_c^0}) > \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0),$$
$$\mathcal{O}(1/m_c^4) \text{ with } \alpha \Rightarrow \tau(\Xi_c^+) > \tau(\boldsymbol{\Omega_c^0}) > \tau(\Lambda_c^+) > \tau(\Xi_c^0).$$

[Cheng, '21]

- **Solution**: Extraction in the inclusive decay spectrum and application to lifetime

Again a more precise experimental determination of $\mu_\pi^2$ from fits to semileptonic $D^+$, $D^0$ and $D_s^+$ meson decays – as it was done for the $B^+$ and $B^0$ decays – would be very desirable.

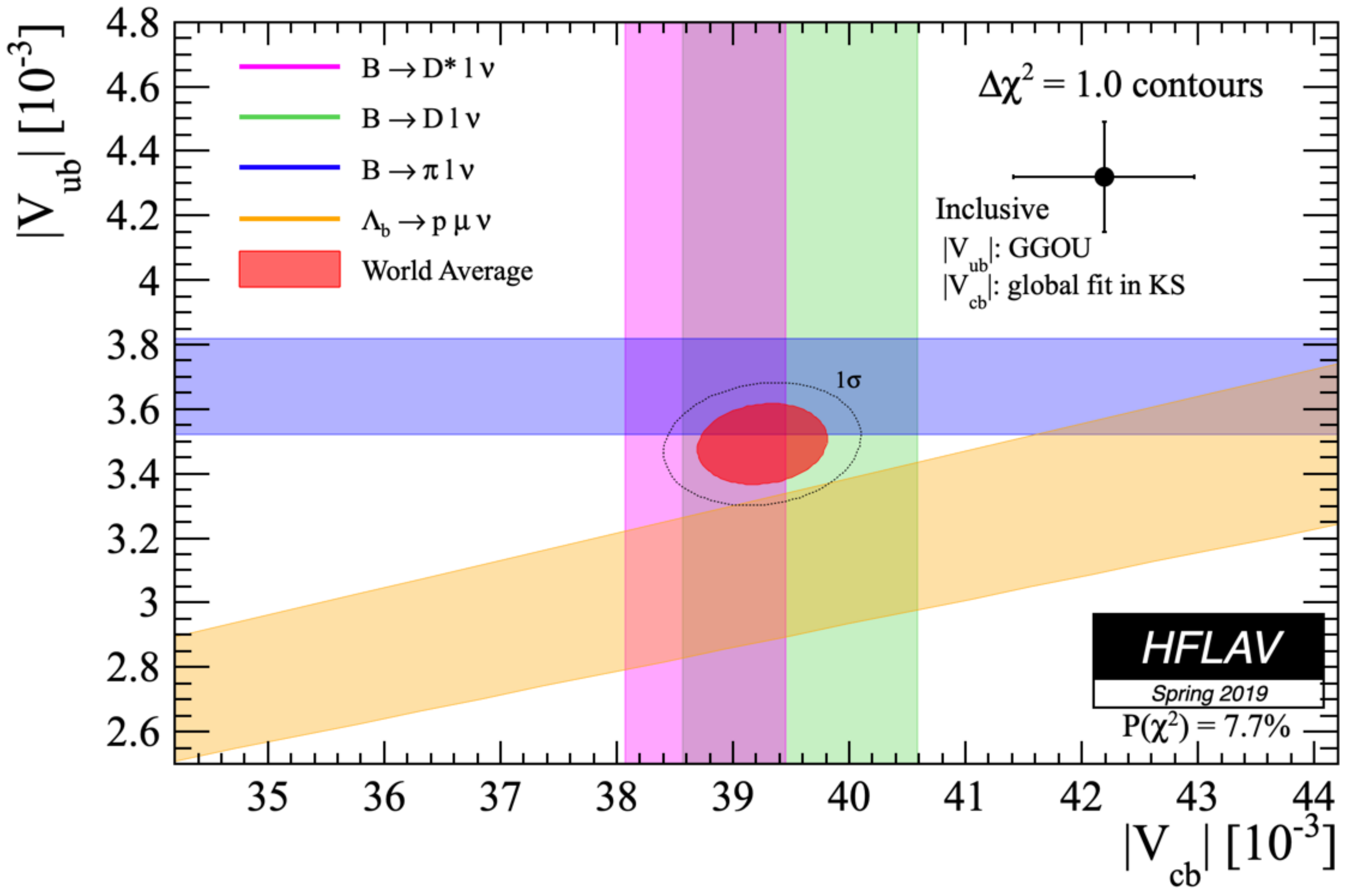
[Lenz et al, '22]

# Why inclusive charm decays?

- **Flavor puzzle 2.** $V_{cb}$, $V_{ub}$: inclusive vs exclusive
- **Key issue:** Systematic uncertainties from theoretical inclusive and exclusive frameworks
- **Give hints:**

  ➡ Test $V_{cd}$, $V_{cs}$: inclusive vs exclusive

  ➡ Inclusive still missing



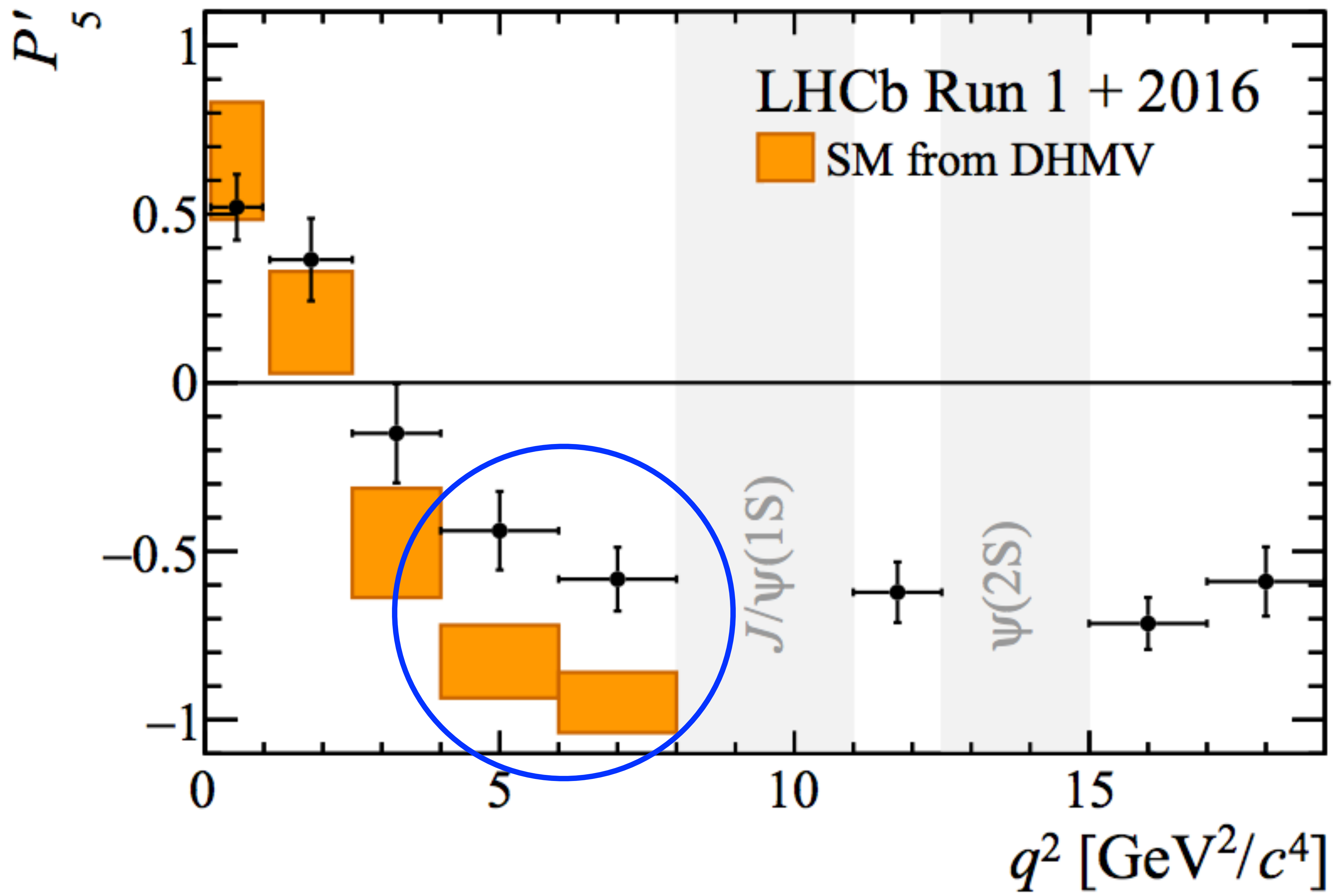
# Why inclusive charm decays?

- **Flavor puzzle 3.** $b \to s$ anomalies: $P_5'$ in $B \to K^*\ell\ell$

- **Key issue:** $B \to K^*$ form factor receive large long-distance quark loop contributions, whose first-principle calculation is still missing

- **Give hints:**

➡ Test the $c \to u$ transition, by angular distribution in inclusive $D \to X_u\ell\ell$



# Theoretical framework

- Optical theorem

$$\sum \langle D|H|X\rangle\langle X|H|D\rangle \propto \mathrm{Im}\int \mathrm{d^4x}\langle \mathrm{D}|\mathrm{T}\{\mathrm{H(x)H(0)}\}|\mathrm{D}\rangle$$

- Operator product expansion (OPE)

➡ Short distance $x \sim 1/m_c$

➡ Dynamical fluctuation in D meson $\sim \Lambda_{\mathrm{QCD}}$

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0) \to 1 + \frac{\Lambda_{\mathrm{QCD}}}{m_c} + \frac{\Lambda^2_{\mathrm{QCD}}}{m_c^2} + \dots$$

<u>**Systematic OPE in HQET.**</u>

# Theoretical framework

- **Heavy quark effective theory**

$$h_v(x) \equiv e^{-im_c v\cdot x}\frac{1+\gamma\cdot v}{2}c(x) \qquad v=(1,0,0,0)$$

**Subtract the big intrinsic momentum,**
**Leave only ~$\Lambda_{QCD}$ degrees of freedom.**

$$L \ni \bar{h}_v iv\cdot Dh_v$$

$$-\bar{h}_v\frac{D_\perp^2}{2m_c}h_v - a(\mu)g\bar{h}_v\frac{\sigma\cdot G}{4m_c}h_v + \dots$$

**Similar to** $\dfrac{m}{\sqrt{1-v^2}} = m + \dfrac{1}{2}mv^2 + \dots$

# Theoretical framework

- **OPE**

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0)$$

➡ Dim-3: $\bar{h}_v h_v$ $(\bar{c}\gamma^\mu c) \to$ partonic decay rate.

➡ Dim-5: $\bar{h}_v D_\perp^2 h_v,\ g\bar{h}_v\sigma \cdot G h_v\,.$

➡ Dim-6: $\bar{h}_v D_\mu (v \cdot D) D^\mu h_v,\ (\bar{h}_v \Gamma_1 q)(\bar{q}\Gamma_2 h_v), \ldots$

- Contribute to inclusive decay rate and also lifetime

➡ Matrix elements of the same operators

➡ Only different short-distance coefficients

$$\lambda_1 \equiv \frac{1}{4m_D}\langle D | \bar{h}_v (iD)^2 h_v | D\rangle = -\mu_\pi^2$$

$$\lambda_2 \equiv \frac{1}{16(s_c \cdot s_q)}\frac{1}{2m_D}\langle D | \bar{h}_v g\sigma \cdot G h_v | D\rangle = \frac{\mu_G^2}{3}$$

# Theoretical results

- Analytical differential decay rate

$$
\begin{aligned}
\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = {} & 2(3-2y)y^2\theta(1-y) \\
& +\frac{2\mu_\pi^2}{m_c^2}\Big[-\frac{5}{3}y^3\theta(1-y)+\frac{1}{6}\delta(1-y)+\frac{1}{6}\delta'(1-y)\Big] \\
& -\frac{2\mu_G^2}{3m_c^2}\Big[-y^2(6+5y)\theta(1-y)+\frac{11}{2}\delta(1-y)\Big]+\mathcal{O}(\alpha_s,\frac{\Lambda^3}{m_c^3})
\end{aligned}
$$

$$y \equiv 2E_e/m_c$$

- Up to finite power, the obtained differential decay rate is **NOT** the experimental spectrum

➡ Observables require integration over final states

➡ $\Gamma = \int \frac{d\Gamma}{dy}dy,\ \langle E_\ell \rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy}E_\ell\, dy,\ \langle E_\ell^2 \rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy}E_\ell^2\, dy, \ldots$

# Theoretical results

- Analytical results for total decay rate and energy moments

$$\Gamma = \frac{G_F^2 m_c^5 |V_{cq}|^2}{192\pi^3}\Big[1 + \frac{\alpha_s C_F}{2\pi}(\frac{25}{4} - \pi^2) - \frac{\mu_\pi^2}{2m_c^2} - \frac{3\mu_G^2}{2m_c^2}\Big]$$

$$\langle E_e \rangle = \frac{G_F^2 m_c^6 |V_{cq}|^2}{192\pi^3 \Gamma}\Big[\frac{7}{20} + \frac{\alpha_s C_F}{2\pi}(\frac{638}{300} - \frac{105\pi^2}{300}) - \frac{\mu_G^2}{m_c^2}\Big]$$

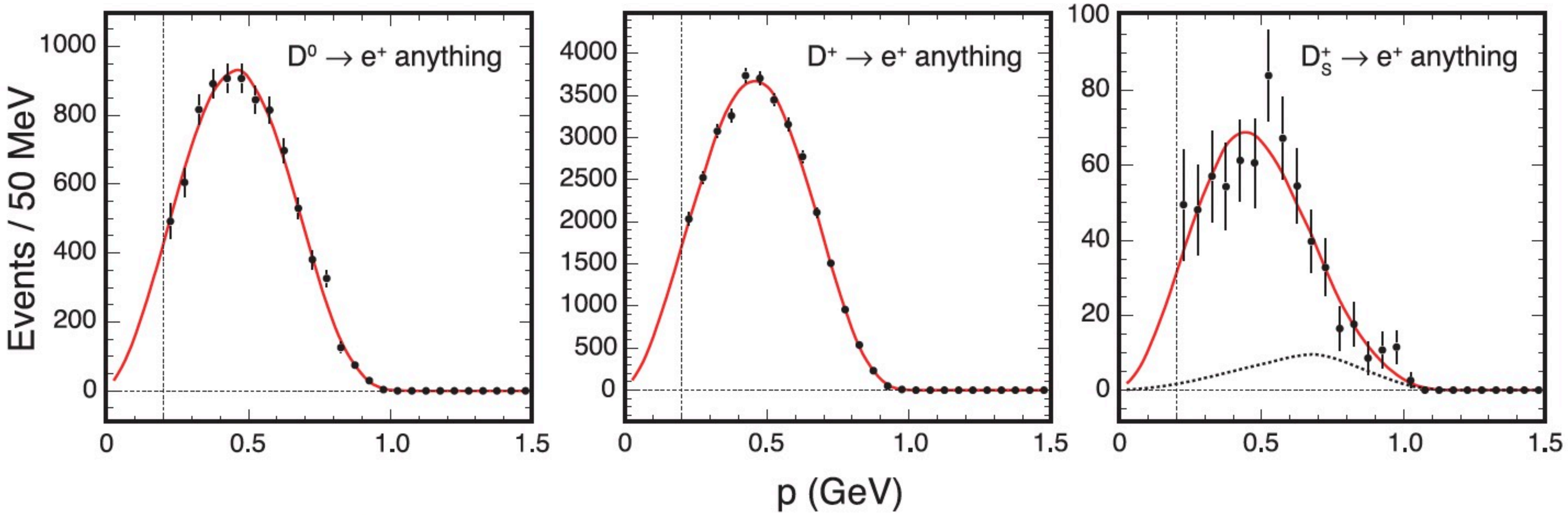
$$\langle E_e^2 \rangle = \frac{G_F^2 m_c^6 |V_{cq}|^2}{192\pi^3 \Gamma}\Big[\frac{2}{15} + \frac{\alpha_s C_F}{2\pi}(\frac{59}{75} - \frac{10\pi^2}{75}) + \frac{\mu_\pi^2}{9m_c^2} - \frac{26\mu_G^2}{45m_c^2}\Big]$$

- Fit them to data

# Experimental status

## CLEO measurements

$D^0 \to e^+X$ $\quad D^+ \to e^+X$ $\quad D_s^+ \to e^+X$

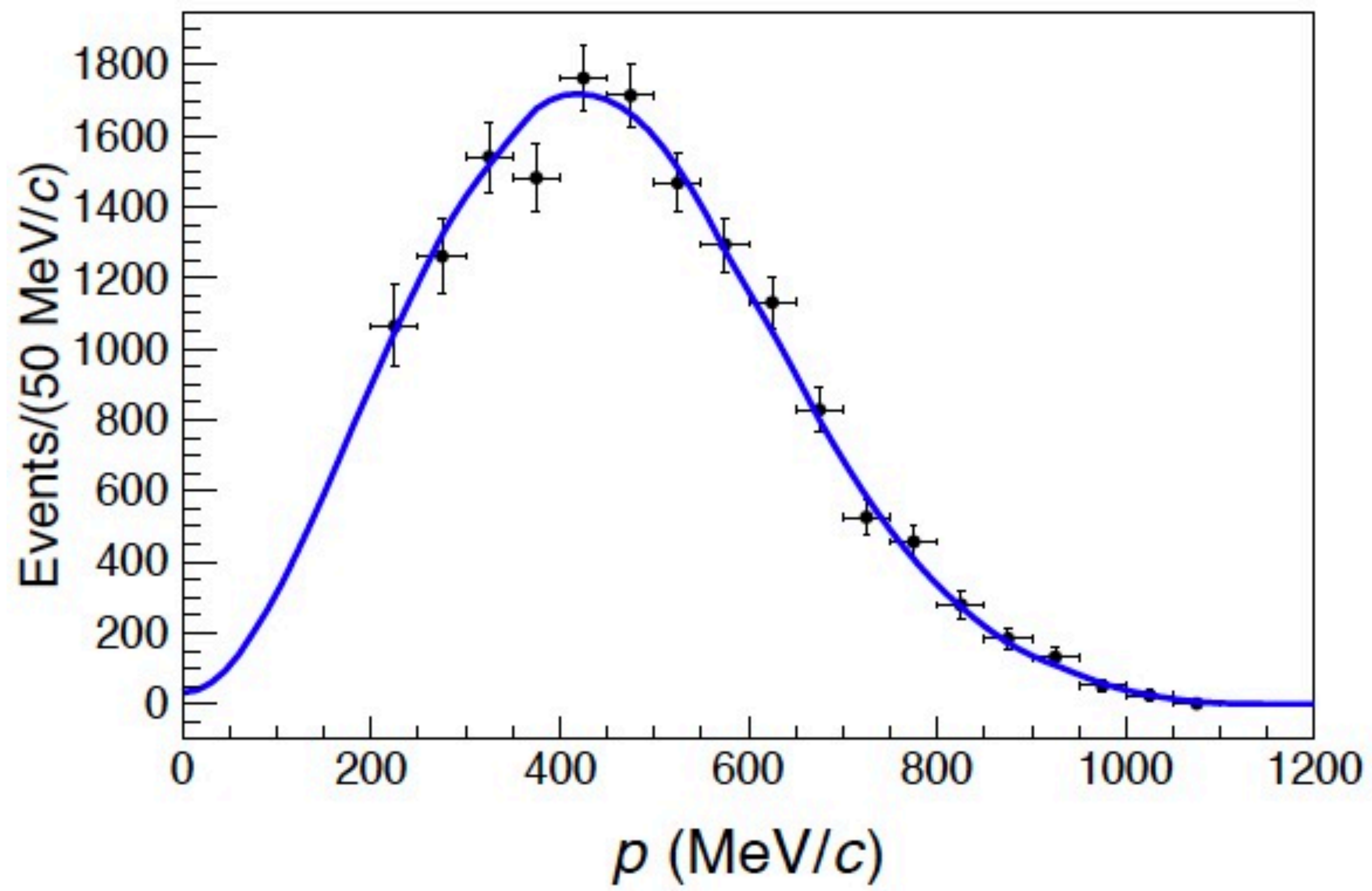



$$\mathcal{B}(D^0 \to Xe^+\nu_e) = (6.46 \pm 0.09 \pm 0.11)\%,$$
$$\mathcal{B}(D^+ \to Xe^+\nu_e) = (16.13 \pm 0.10 \pm 0.29)\%,$$
$$\mathcal{B}(D_s^+ \to Xe^+\nu_e) = (6.52 \pm 0.39 \pm 0.15)\%,$$

[CLEO, '09]

## BESIII measurements

$D_s^+ \to e^+X$



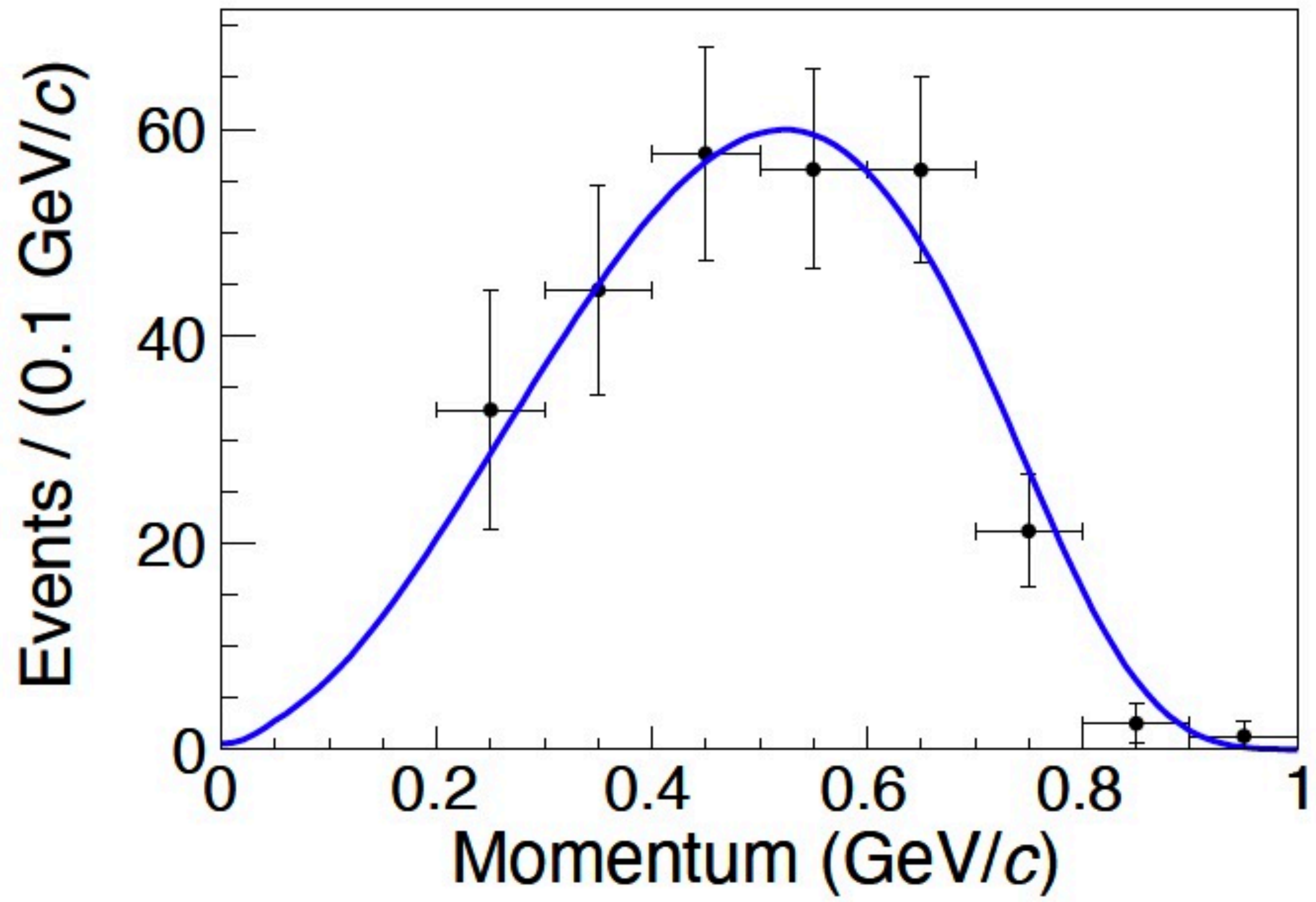

$$B(D_s^+ \to Xe^+\nu_e) = (6.30 \pm 0.13 \pm 0.10)\,\%$$

[BESIII, '21]

# Experimental status

## BESIII measurements

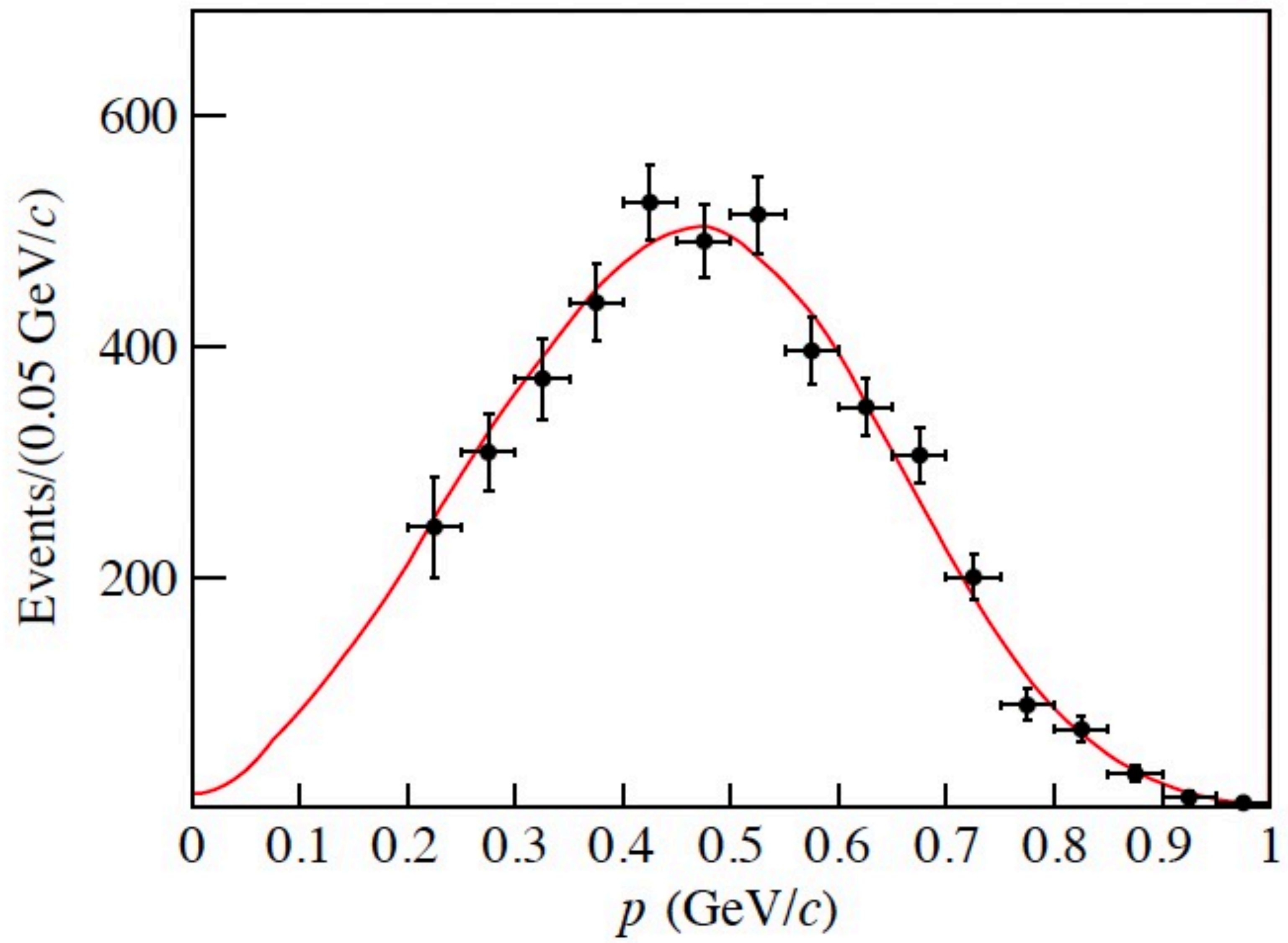
$\Lambda_c \to e^+X$




$$B(\Lambda_c^+ \to Xe^+\nu_e) = (3.95 \pm 0.34 \pm 0.09)\ \%$$

[BESIII (567 pb$^{-1}$), '18]

$\Lambda_c \to e^+X$



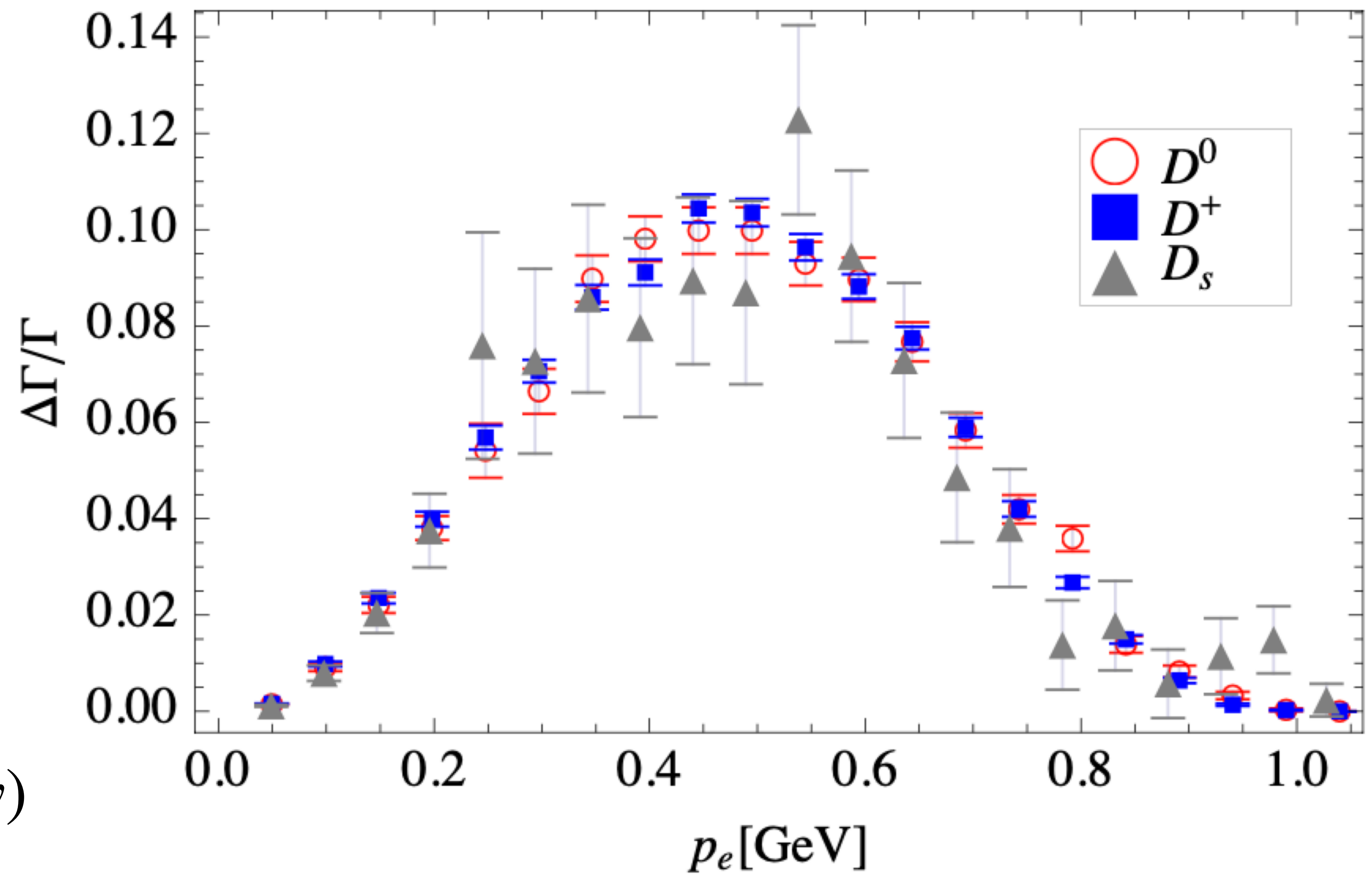

$$\mathcal{B}(\Lambda_c^+ \to Xe^+\nu_e) = (4.06 \pm 0.10_{\text{stat.}} \pm 0.09_{\text{syst}})\%$$

[BESIII (4.5 fb$^{-1}$), '23]

# Experimental status

**CLEO results extrapolation**

$$\frac{d\Gamma}{dy} = ay^2(1 + by)(1 - y)$$




**Lorentz boost**

$$\langle E_\ell \rangle_{exp}^{D^0} = 0.459(3)\text{GeV}\,,$$
$$\langle E_\ell \rangle_{exp}^{D^+} = 0.455(1)\text{GeV}\,,$$
$$\langle E_\ell \rangle_{exp}^{D_s} = 0.456(11)\text{GeV}\,,$$

$$\langle E_\ell^2 \rangle_{exp}^{D^0} = 0.240(2)\text{GeV}^2\,,$$
$$\langle E_\ell^2 \rangle_{exp}^{D^+} = 0.236(1)\text{GeV}^2\,,$$
$$\langle E_\ell^2 \rangle_{exp}^{D_s} = 0.239(12)\text{GeV}^2\,,$$

$$\langle E_e \rangle_{lab}^{D^0} = 0.465(3)\text{GeV}\,, \qquad \langle E_e^2 \rangle_{lab}^{D^0} = 0.248(2)\text{GeV}^2\,,$$
$$\langle E_e \rangle_{lab}^{D^+} = 0.459(1)\text{GeV}\,, \qquad \langle E_e^2 \rangle_{lab}^{D^+} = 0.242(1)\text{GeV}^2\,,$$
$$\langle E_e \rangle_{lab}^{D_s} = 0.466(12)\text{GeV}\,, \qquad \langle E_e^2 \rangle_{lab}^{D_s} = 0.254(13)\text{GeV}^2\,.$$

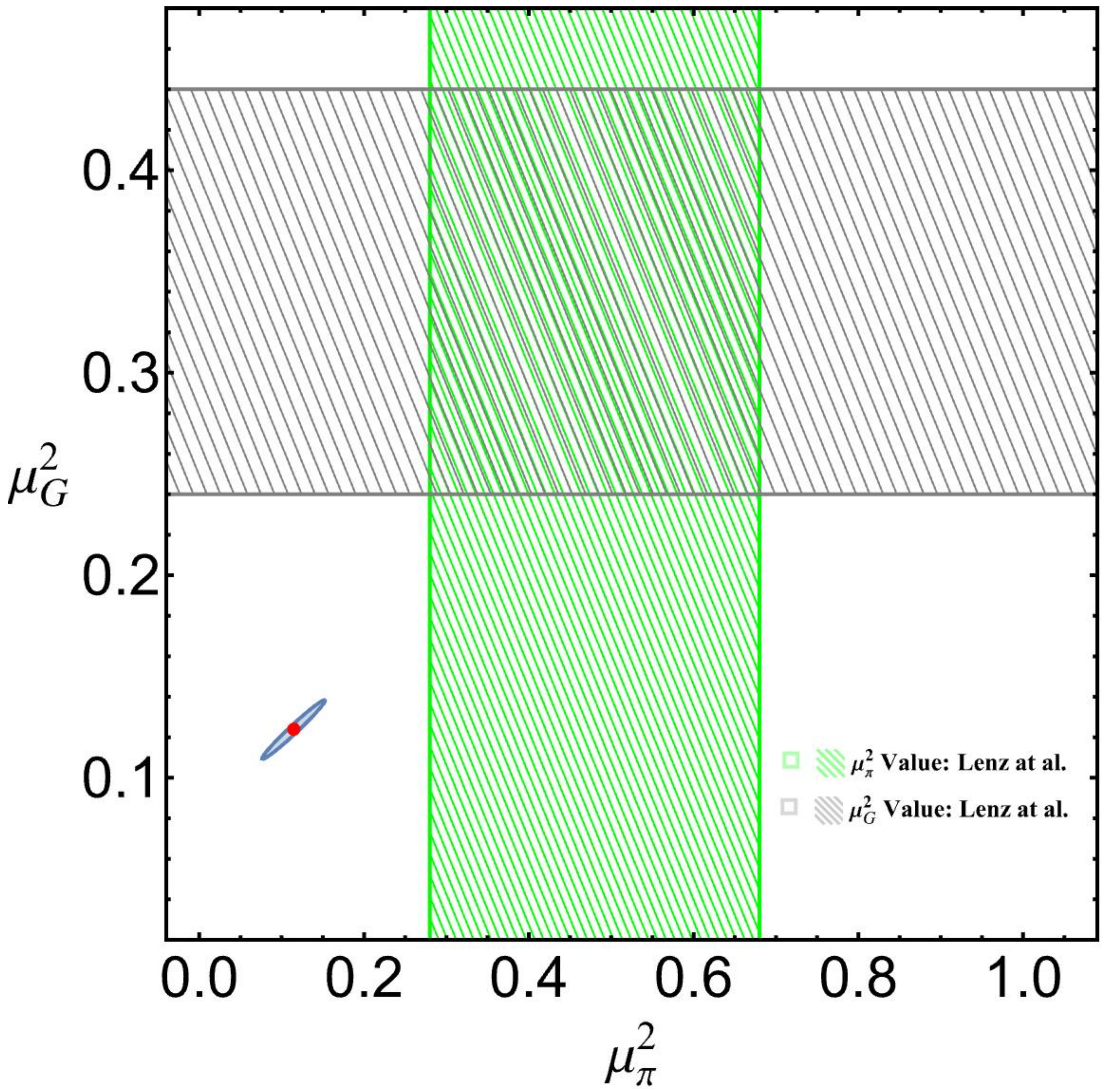
**Lab frame**

**Rest frame**



[Gambino,Kamenik,

# Global fit (preliminary)




$$\mu_\pi^2(D) = (0.48 \pm 0.20)\mathrm{GeV}^2$$

$$\mu_G^2(D) = (0.34 \pm 0.10)\mathrm{GeV}^2$$

[Lenz et al, ’22]

$$\mu_\pi^2(D) = (0.124 \pm 0.014)\mathrm{GeV}^2$$

$$\mu_G^2(D) = (0.115 \pm 0.038)\mathrm{GeV}^2$$

[Our results]

To improve

➡ Modifications from higher power corrections

➡ Direct experimental measurements of moments

# Further Plans

- Include higher-order radiative corrections

- Include dimension-6 operator contributions

- Extract more hadronic parameters (and the charm mass)

# Wishlist

- Precision measurements of $\langle E_e^n \rangle$ in the rest frame of charmed hadrons

- $q^2$ spectrum, good for higher-dimensional operators

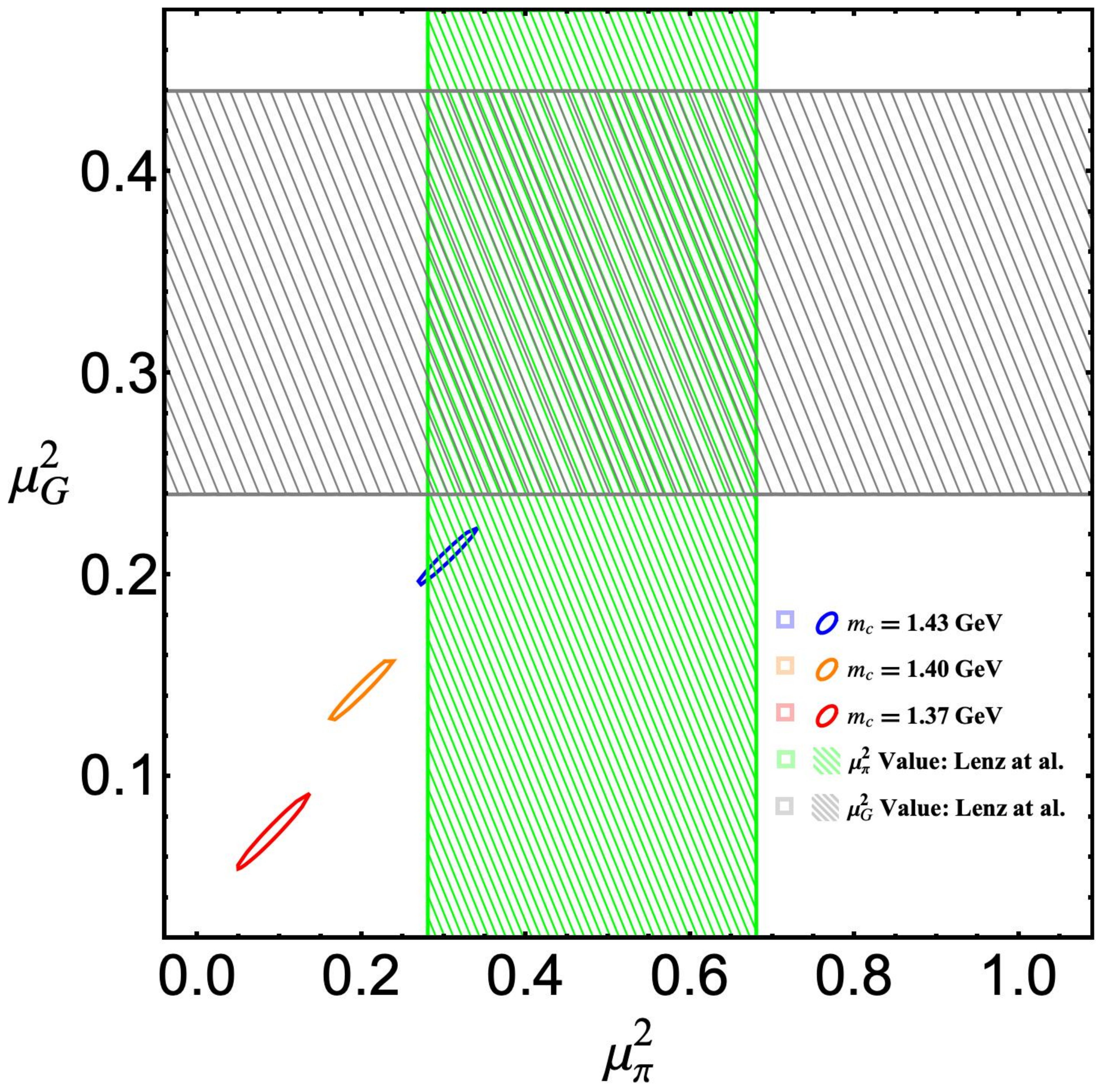
- Separate $X_d$, $X_s$, to give first inclusive measurements of $V_{cd}$, $V_{cs}$

- Rare decays: $D \to X_u \ell \ell$. STCF?

**Thank you!**

# Backup

# Global fit (preliminary)




*Pole mass scheme: mc=1.48 GeV*

*MS bar mass scheme:*

$$m_c^{Pole} = \bar{m}_c\left(\bar{m}_c\right)\left[1 + \frac{4}{3}\frac{\alpha_s(\bar{m}_c)}{\pi}\right]$$

$$\bar{m}_c\left(\bar{m}_c\right) = 1.27\text{GeV}$$

*Kinetic mass scheme*：

$$m_c^{Pole} = m_c^{Kin}\left[1 + \frac{4\alpha_s}{3\pi}\left(\frac{4}{3}\frac{\mu^{\text{cut}}}{m_c^{Kin}} + \frac{1}{2}\left(\frac{\mu^{\text{cut}}}{m_c^{Kin}}\right)^2\right)\right]$$

$$m_c^{kin}(0.5\text{GeV}) = 1.363\text{GeV}$$

*1S mass scheme*：

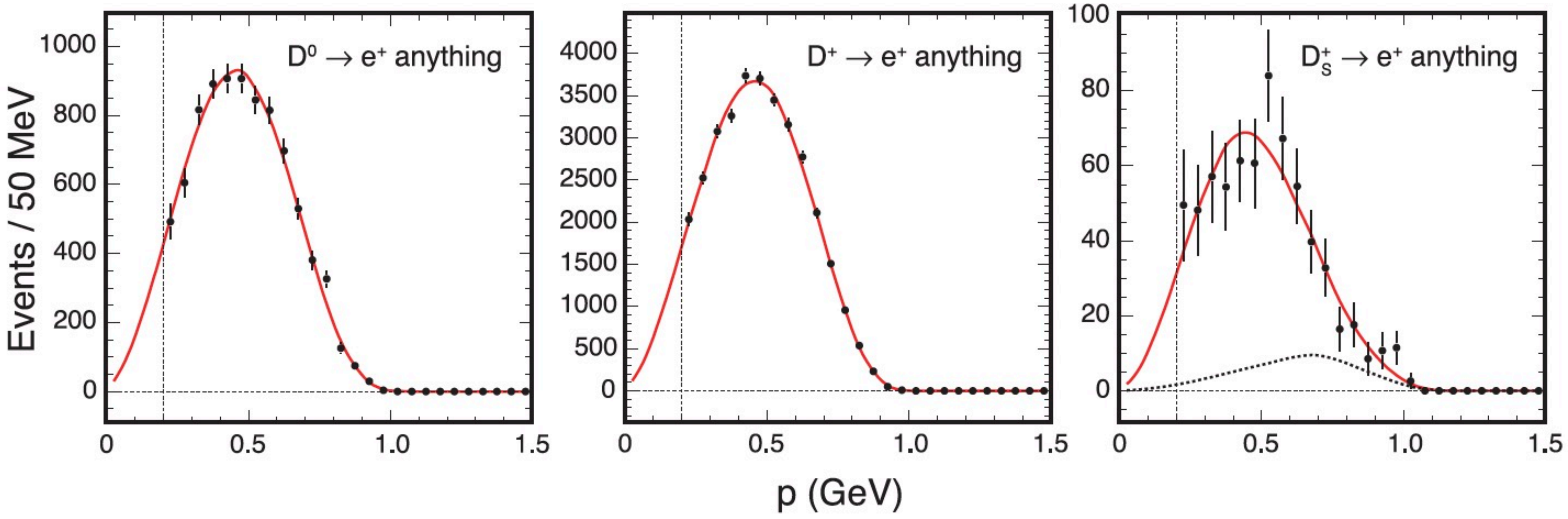
$$m_c^{Pole} = m_c^{1S}\left(1 + \frac{\left(C_F\alpha_s\right)^2}{8}\right)$$

$$m_c^{1S} \approx 1.44\text{GeV}$$

# Experimental status

## CLEO measurements

$D^0 \to e^+ X$ $\qquad$ $D^+ \to e^+ X$ $\qquad$ $D_s^+ \to e^+ X$

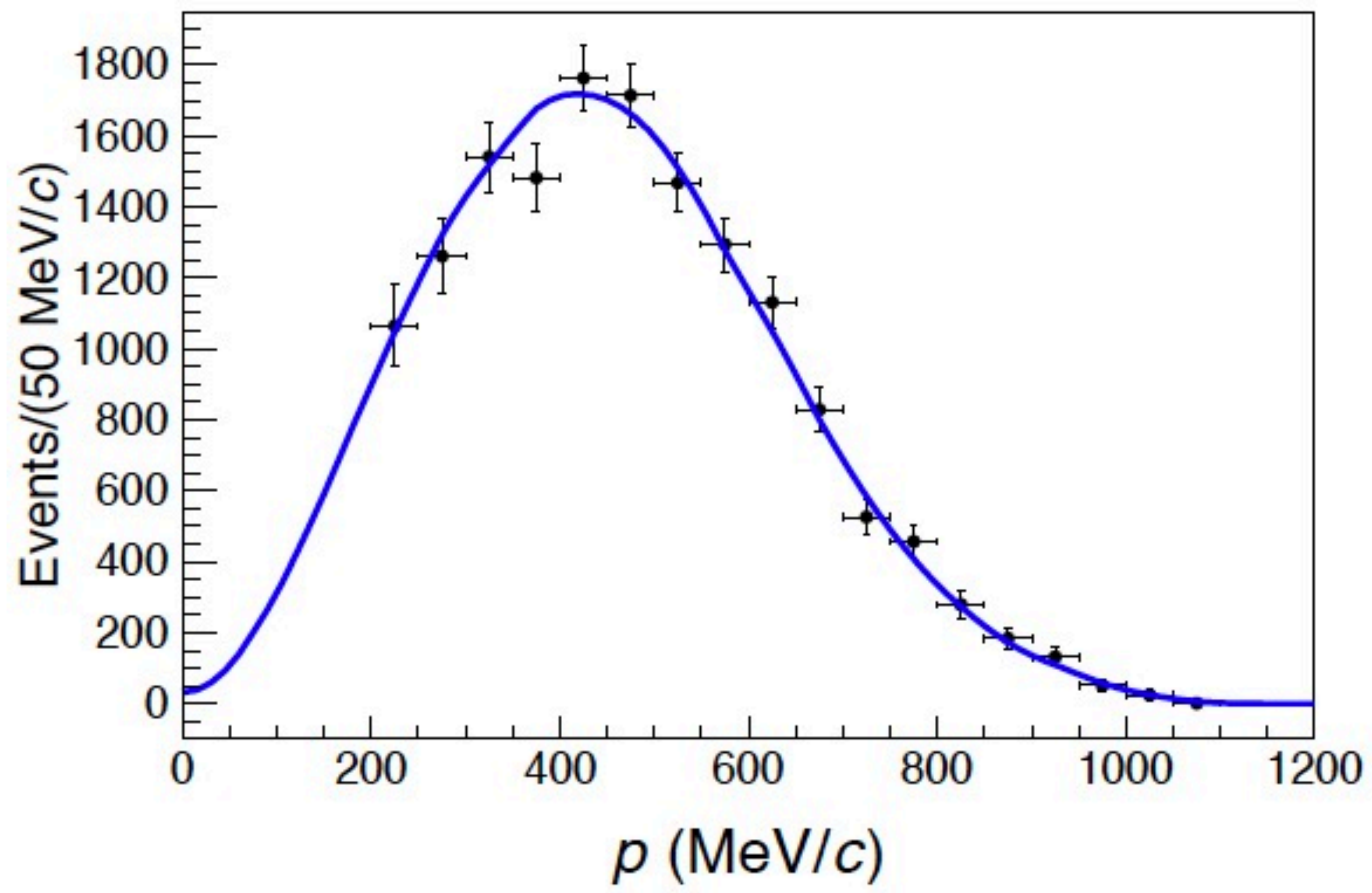



$3.0 \times 10^6$ $D^0\bar{D}^0$ and $2.4\times 10^6$ $D^+D^-$ pairs, and is used to
ays. The latter data set contains $0.6 \times 10^6$ $D_s^{*\pm}D_s^{\mp}$ pairs,

[CLEO (818pb$^{-1}$($D^{0,\pm}$), 602pb$^{-1}$($D_s^{\pm}$)), '09]

## BESIII measurements

$D_s^+ \to e^+ X$




| $E_{\rm cm}$ (MeV) | $\int \mathcal{L}\, dt$ (pb$^{-1}$) | $N_{D_s}(\times 10^6)$ |
|---|---|---|
| 4178 | $3189.0 \pm 0.9 \pm 31.9$ | 6.4 |
| 4189 | $526.7 \pm 0.1 \pm 2.2$ | 1.0 |
| 4199 | $526.0 \pm 0.1 \pm 2.1$ | 1.0 |
| 4209 | $517.1 \pm 0.1 \pm 1.8$ | 0.9 |
| 4219 | $514.6 \pm 0.1 \pm 1.8$ | 0.8 |
| 4225 – 4230 [32] | $1047.3 \pm 0.1 \pm 10.2$ [33] | 1.3 |

[BESIII, '21]