Large N-point energy correlator in the collinear limit

代 林

廣西大學
GUANGXI UNIVERSITY

with Chul Kim (SeoulTech) and Adam Leibovich (U. Pittsburgh)

(arXiv:2410.11614 and PRD 95, 074003 (2017))

第四届量子场论及其应用研讨会

广州 2024/11/17

Outline

- Energy Correlators and Jet Substructures
- Fragmentation to a jet (FFJ) in the large z limit
- Energy Correlators and FFJ
- An application to e+e- annihilation

Energy Correlators

$$\text{EEC}(z) = \sum_{a,b} \int \frac{d\sigma_{ee\to a+b+X}}{\sigma} \frac{E_a E_b}{Q^2} \delta\left(z - \sin\frac{\theta_{ab}^2}{2}\right)$$

[Basham, Brown, Ellis, Love, PRL, 1978]

- Properties: IR safe; Friendly to Perturbative Calculation and Experimental Measurements

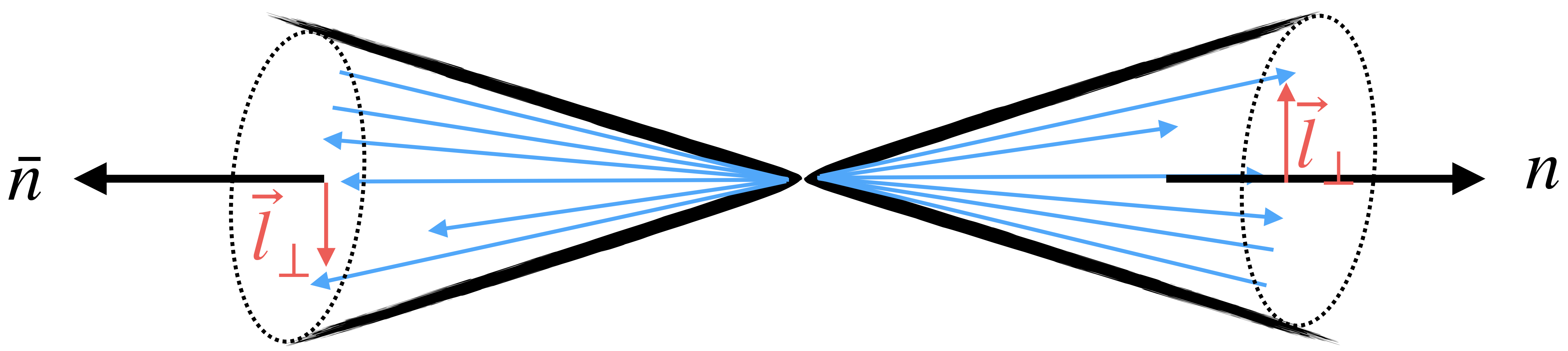
- Recent Surge of Research:

- *L. J. Dixon, M.-X. Luo, V. Shtabovenko, T.-Z. Yang and H. X. Zhu, Phys. Rev. Lett. 120 (2018) 102001, [arXiv:1801.03219]*
- *M.-X. Luo, V. Shtabovenko, T.-Z. Yang and H. X. Zhu, HEP 06 (2019) 037, [arXiv:1903.07277].*
- *J. M. Henn, E. Sokatchev, K. Yan and A. Zhiboedov, Phys. Rev. D 100 (2019) 036010, [arXiv:1903.05314]*
- *M. A. Ebert, B. Mistlberger, and G. Vita, JHEP 08 (2021) 022, [arXiv:2012.07859]*
- *P. T. Komiske, I. Moult, J. Thaler, and H. X. Zhu, Phys. Rev. Lett. 130 (2023), no. 5 051901, [arXiv:2201.07800]*
- *A. Chen, X. Liu, Y. Ma, Phys.Rev.Lett. 133 (2024) 19 [arXiv:2405.10056]*
- *Y. Guo, X. Liu, F. Yuan [arXiv:2408.14693]*
- *X. Liu, W. Vogelsang, F. Yuan and H. X. Zhu, [arXiv:2410.16371]*

⋮

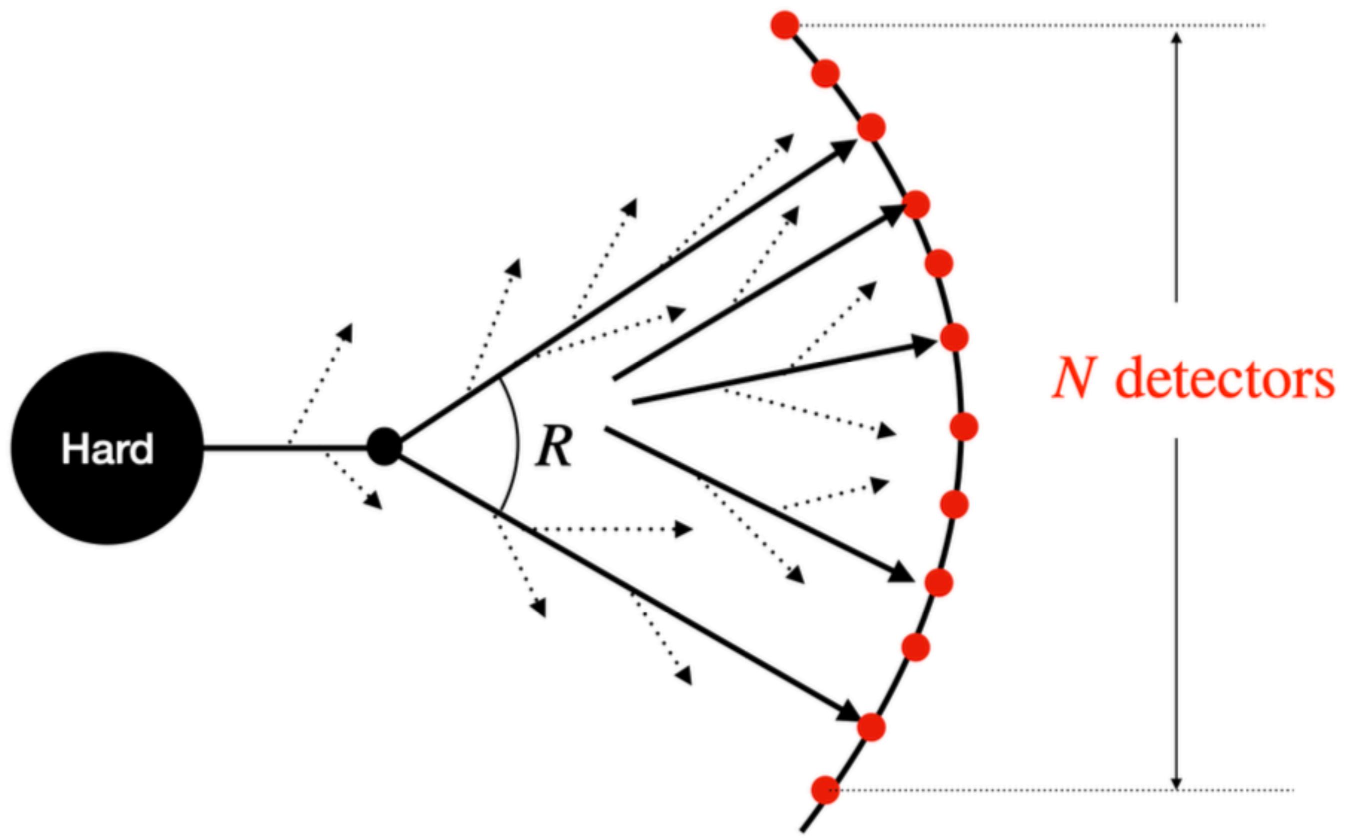
EEC and Jet Substructure

Back-to-Back limit: related to TMD Fragmentation:

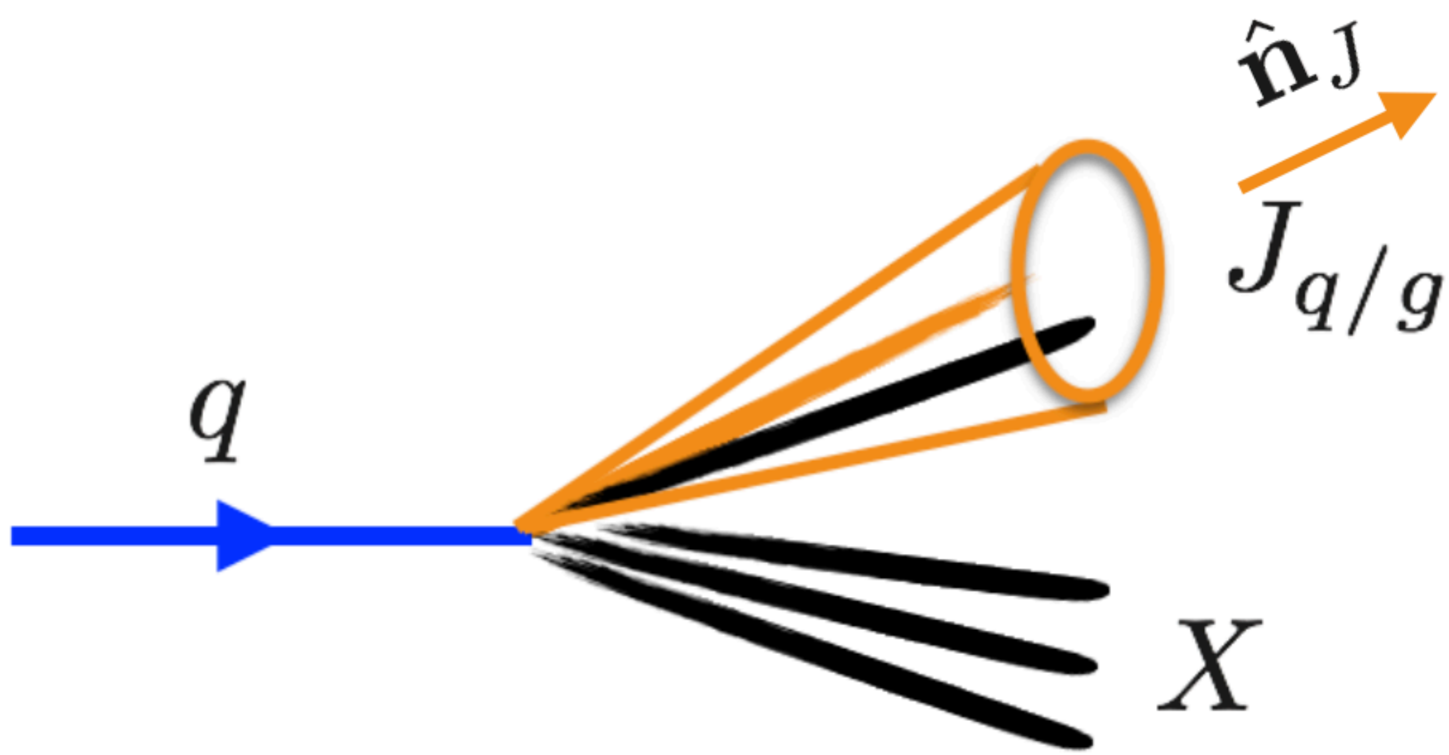


N-Point Energy Correlators （ENC）

Collinear Limit:
Related to Inclusive Fragmentation Function to a Jet (FFJ).



Fragmentation to a Jet (FFJ)



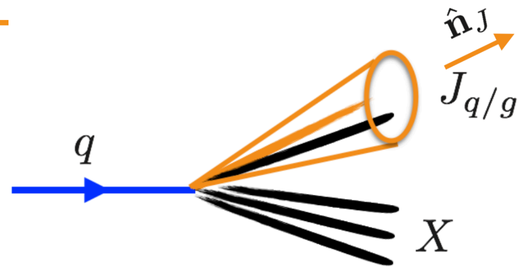
- Quark FFJ:

$$p^+ \equiv \bar{n} \cdot p$$
$$p^- \equiv n \cdot p$$

$$D_{J_k/q}(z,\mu) = \sum_X \frac{z^{D-3}}{2N_c} \mathrm{Tr}\langle 0|\delta\left(\frac{p_J^+}{z} - \mathcal{P}_+\right)\frac{\bar{\not{n}}}{2}\chi_n|J_k X\rangle\langle J_k X|\bar{\chi}_n|0\rangle$$

[LD, Kim, Leibovich, Phys. Rev. D 94 (2016), 114023]
[Z. B. Kang, F. Ringer and I. Vitev, JHEP 10 (2016), 125]

FFJ in the large *z* limit

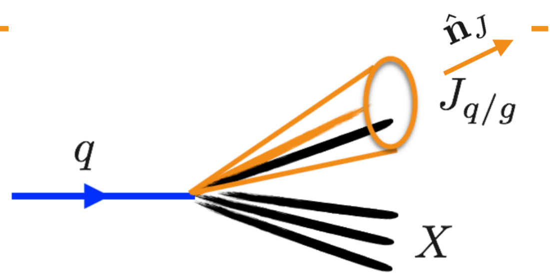


- Quark to quark Jet:

$$D_{J_q/q}(z,\mu;E_J R') = \delta(1-z) + \frac{\alpha_s C_F}{2\pi}\Bigg\{\delta(1-z)\left(\frac{3}{2}\ln\frac{\mu^2}{E_J^2 R'^2} + \frac{13}{2} - \frac{2\pi^2}{3}\right)$$
$$-(1-z) + (1+z^2)\left[\frac{1}{(1-z)_+}\left(\ln\frac{\mu^2}{E_J^2 R'^2} - 2\ln z\right) - 2\left(\frac{\ln(1-z)}{1-z}\right)_+\right]\Bigg\}$$

[LD, Kim, Leibovich, PRD 95, 074003 (2017)]

FFJ in the large *z* limit



- Quark to quark Jet:

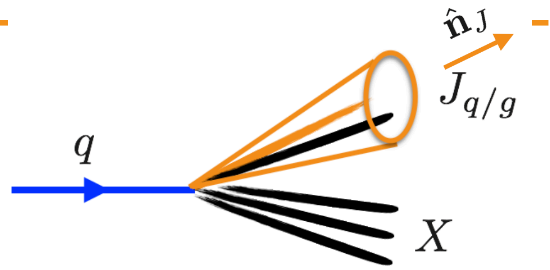
$$D_{J_q/q}(z,\mu;E_JR') = \delta(1-z) + \frac{\alpha_s C_F}{2\pi}\left\{\delta(1-z)\left(\frac{3}{2}\ln\frac{\mu^2}{E_J^2R'^2}+\frac{13}{2}-\frac{2\pi^2}{3}\right)\right.$$
$$\left.-(1-z)+(1+z^2)\left[\frac{1}{(1-z)_+}\left(\ln\frac{\mu^2}{E_J^2R'^2}-2\ln z\right)-2\left(\frac{\ln(1-z)}{1-z}\right)_+\right]\right\}$$

- There are two logs: log[E_JR], log[$E_JR(1-z)$]

[LD, Kim, Leibovich, PRD 95, 074003 (2017)]

FFJ in the large *z* limit

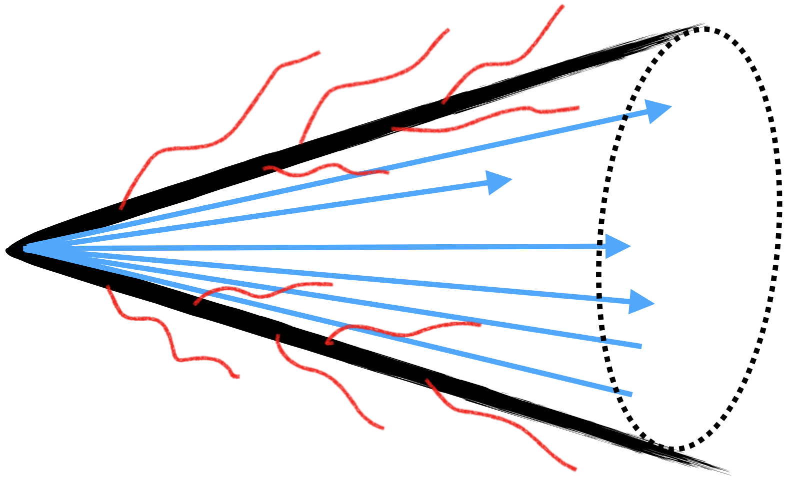
- Quark to quark Jet:



$$D_{J_q/q}(z,\mu;E_JR') = \delta(1-z) + \frac{\alpha_s C_F}{2\pi}\Bigg\{\delta(1-z)\left(\frac{3}{2}\ln\frac{\mu^2}{E_J^2R'^2}+\frac{13}{2}-\frac{2\pi^2}{3}\right)$$
$$-(1-z)+(1+z^2)\left[\frac{1}{(1-z)_+}\left(\ln\frac{\mu^2}{E_J^2R'^2}-2\ln z\right)-2\left(\frac{\ln(1-z)}{1-z}\right)_+\right]\Bigg\}$$

  - There are two logs: log[$E_J R$], log[$E_J R(1\text{-}z)$]

- In the large *z* limit, (1-z) ≪ 1, 1-z ~ O(η)
  - Collinear-Soft (CS) mode $p_{cs} \sim E_J \eta(1, R^2, R)$.
  - CS at the boundary: $p_{cs}^-/p_{cs}^+ \sim R^2$
  - Factorize collinear and cs interactions to resum both logarithms

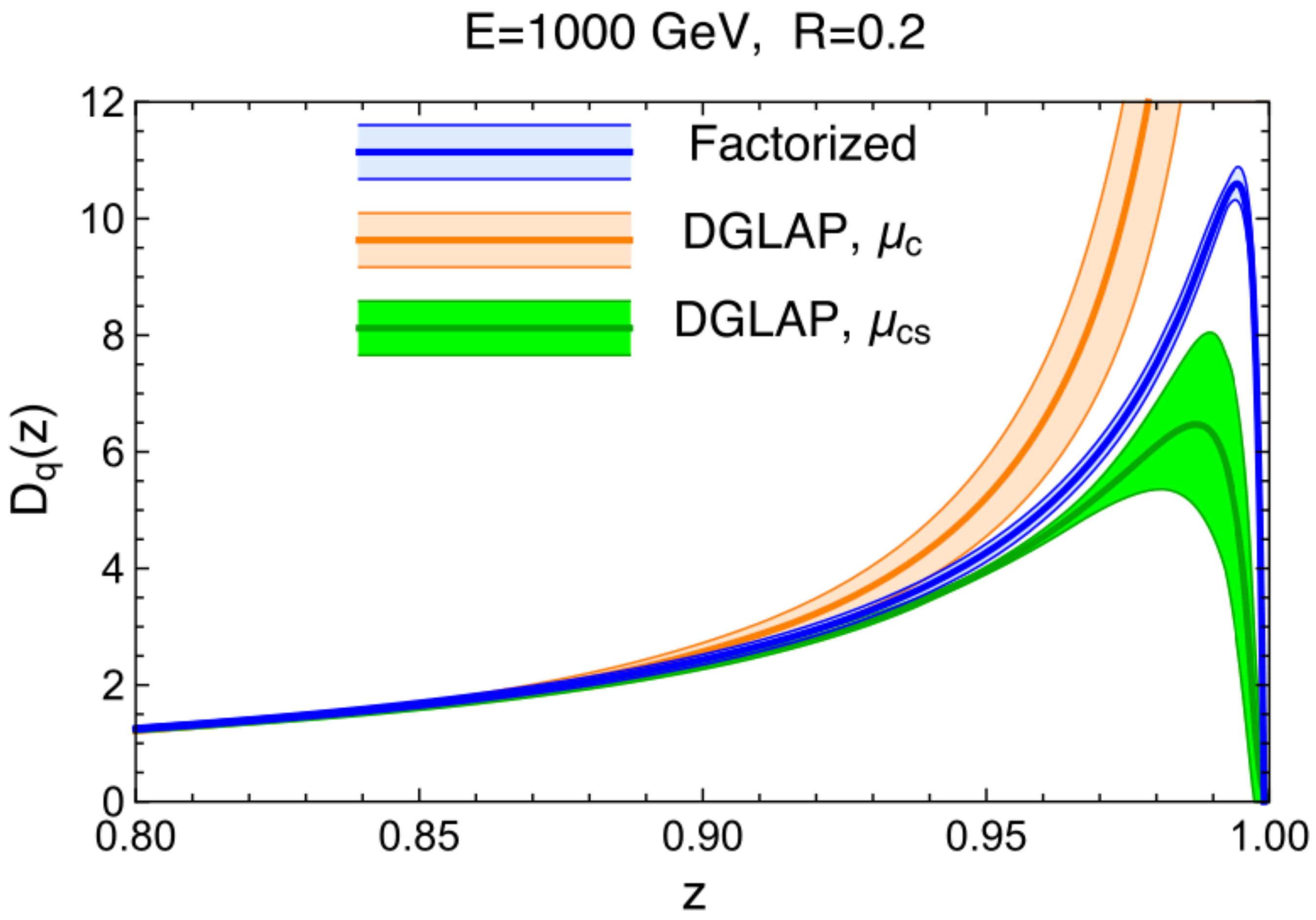


[LD, Kim, Leibovich, PRD 95, 074003 (2017)]

FFJ in the large z limit

- In the large z limit, $(1\text{-}z) \ll 1$, $1\text{-}z \sim O(\eta)$

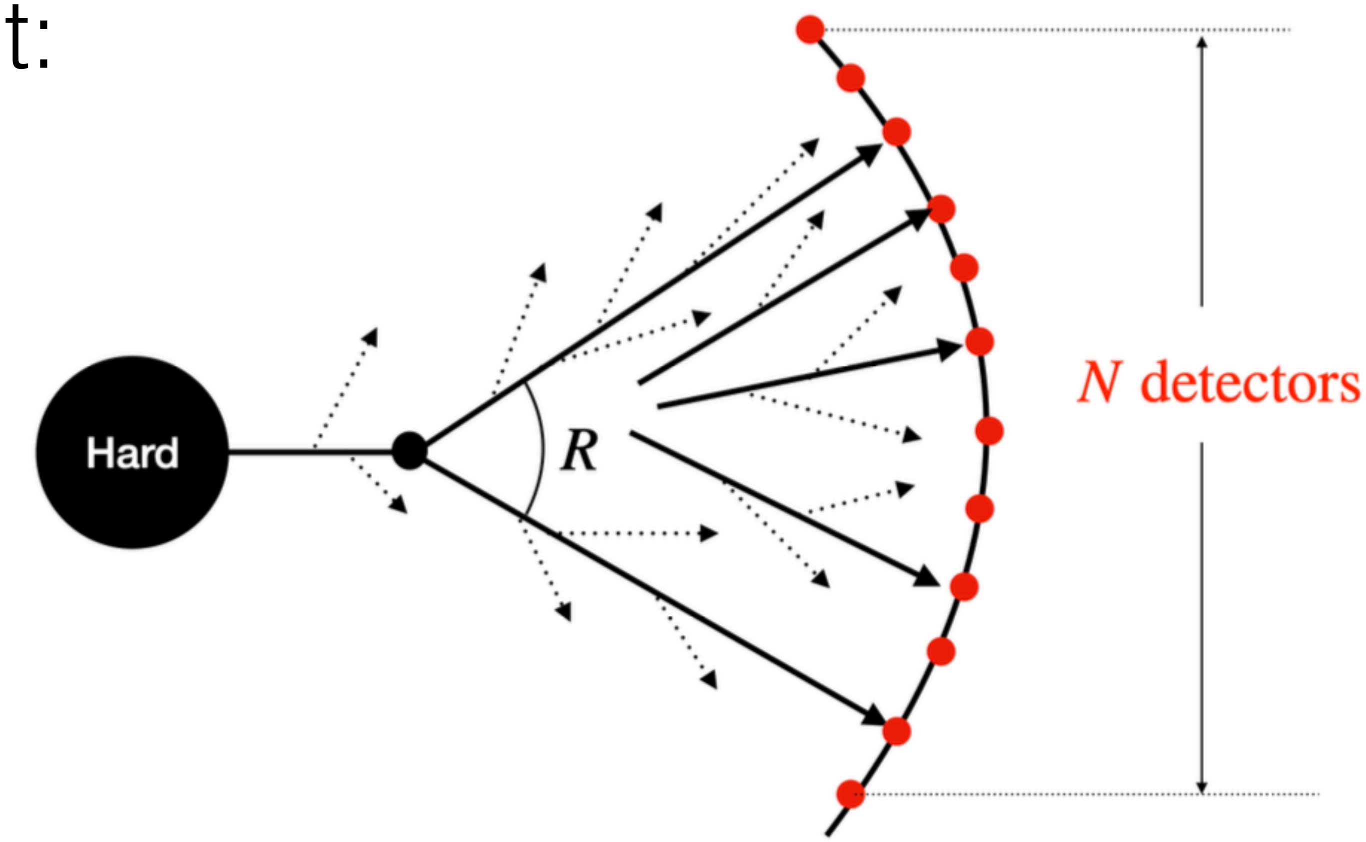
$$D_{J_q/q}(z \to 1, \mu; E_J R', (1-z)E_J R')$$
$$= \mathcal{J}_q(\mu; E_J R', \theta < R') \cdot S_q(z, \mu; (1-z)E_J R')$$



[LD, Kim, Leibovich, PRD 95, 074003 (2017)]

N-Point Energy Correlators （ENC）

Collinear limit:



$$\frac{d\Sigma}{dz_1 \cdots dz_{M_N}} = \sum_{a_1 \cdots a_N} \int d\sigma \frac{E_{a_1} \cdots E_{a_N}}{Q^N}$$
$$\times \delta\left(z_1 - \sin^2 \frac{\theta_{12}}{2}\right) \cdots \delta\left(z_{M_N} - \sin^2 \frac{\theta_{N(N-1)}}{2}\right)$$

$$M_N = \frac{N(N-1)}{2}$$

Related to Inclusive Jet Fragmentation Function.

N-point Energy Correlator (ENC)

- N-point Energy Correlator at z_L:

$$\frac{d\Sigma}{dz_L} = \int dz_1 \cdots dz_{M_N} \delta\left(z_L - z_{\max}\right) \frac{d\Sigma}{dz_1 \cdots dz_{M_N}}$$

- ENC cumulant to jet radius R:

$$\Sigma_N(R^2) = \int_0^{\sin^2(R/2)} dz_L \frac{d\Sigma}{dz_L}$$

[LD, Kim, Leibovich, arXiv:2410.11614]

ENC and FFJ

[LD, Kim, Leibovich, arXiv:2410.11614]

ENC and FFJ

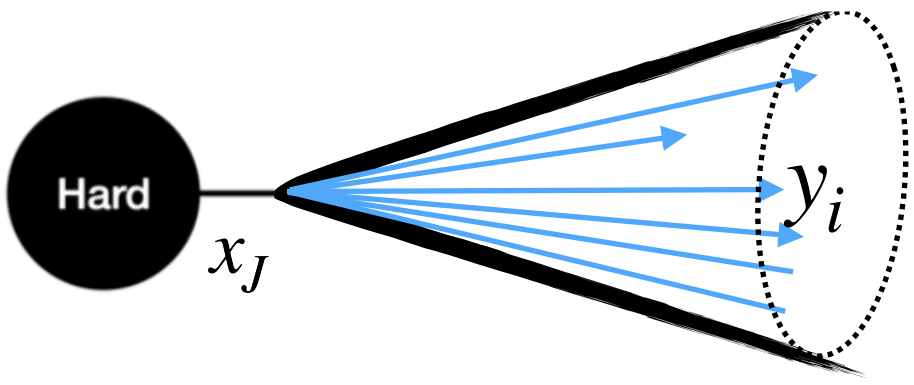
$$\Sigma_N(R^2) = \frac{1}{2^N}\int \{dx\}_N \cdot \{x\}_N \cdot \frac{d\sigma(R)}{\{dx\}_N}$$
$$= \frac{1}{2^N}\int_0^1 dx_J x_J^N \sum_\ell \left(\frac{d\sigma}{dx_J}\right)_\ell \int \{dy\}_N \cdot \{y\}_N \cdot \Phi_l(y_1, \cdots, y_N, R)$$

[LD, Kim, Leibovich, arXiv:2410.11614]

ENC and FFJ

$$\Sigma_N(R^2) = \frac{1}{2^N}\int \{dx\}_N \cdot \{x\}_N \cdot \frac{d\sigma(R)}{\{dx\}_N}$$

$$= \frac{1}{2^N}\int_0^1 dx_J x_J^N \sum_\ell \left(\frac{d\sigma}{dx_J}\right)_\ell \int \{dy\}_N \cdot \{y\}_N \cdot \Phi_l(y_1, \cdots, y_N, R)$$

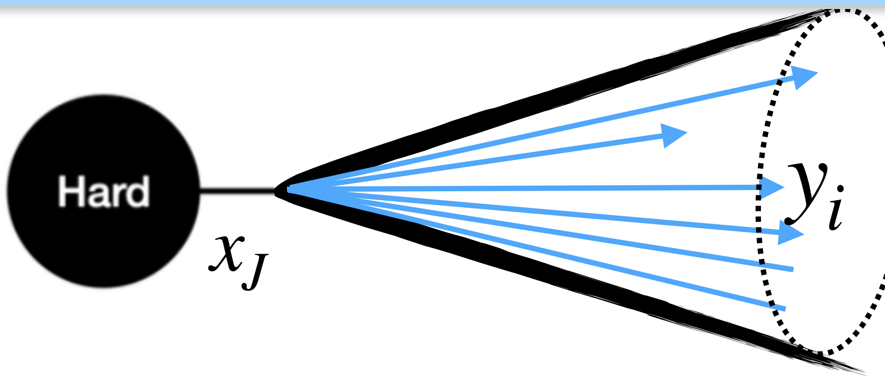


[LD, Kim, Leibovich, arXiv:2410.11614]

ENC and FFJ

$$\Sigma_N(R^2) = \frac{1}{2^N}\int \{dx\}_N \cdot \{x\}_N \cdot \frac{d\sigma(R)}{\{dx\}_N}$$

$$= \frac{1}{2^N}\int_0^1 dx_J x_J^N \sum_\ell \left(\frac{d\sigma}{dx_J}\right)_\ell \int \{dy\}_N \cdot \{y\}_N \cdot \Phi_l(y_1, \cdots, y_N, R)$$



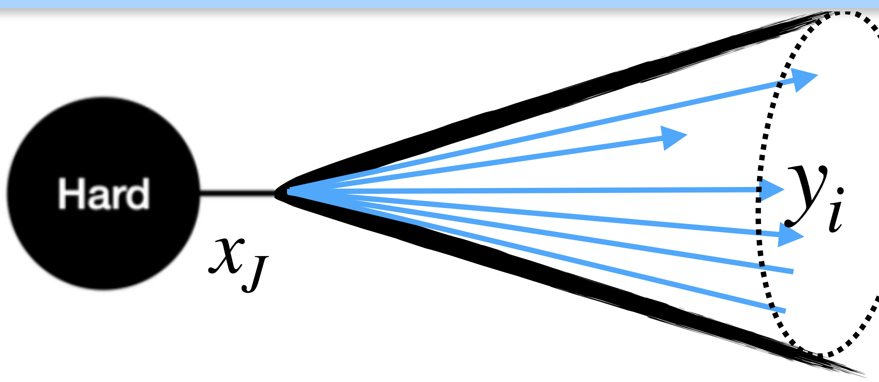
[LD, Kim, Leibovich, arXiv:2410.11614]

ENC and FFJ

$$\Sigma_N(R^2) = \frac{1}{2^N}\int \{dx\}_N \cdot \{x\}_N \cdot \frac{d\sigma(R)}{\{dx\}_N}$$

$$= \frac{1}{2^N}\int_0^1 dx_J x_J^N \sum_\ell \left(\frac{d\sigma}{dx_J}\right)_\ell \int \{dy\}_N \cdot \{y\}_N \cdot \Phi_l(y_1, \cdots, y_N, R)$$

$$= \frac{1}{2^N}\int_0^1 dx_J x_J^N \left(\frac{d\sigma}{dx_J}\right)$$



[LD, Kim, Leibovich, arXiv:2410.11614]

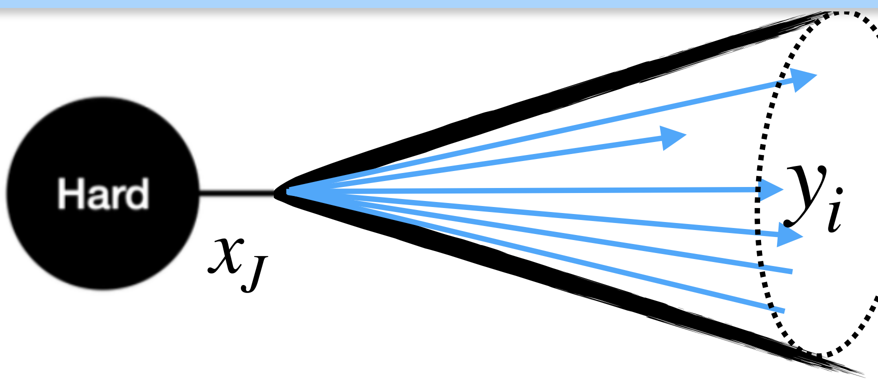
ENC and FFJ

$$\Sigma_N(R^2) = \frac{1}{2^N}\int \{dx\}_N \cdot \{x\}_N \cdot \frac{d\sigma(R)}{\{dx\}_N}$$

$$= \frac{1}{2^N}\int_0^1 dx_J x_J^N \sum_\ell \left(\frac{d\sigma}{dx_J}\right)_\ell$$

$$\int \{dy\}_N \cdot \{y\}_N \cdot \Phi_l(y_1, \cdots, y_N, R)$$

$$= \frac{1}{2^N}\int_0^1 dx_J x_J^N \left(\frac{d\sigma}{dx_J}\right)$$



$$= \frac{\sigma_0}{2^N}\sum_k \int_0^1 d\zeta \zeta^N H_k(\zeta, Q, \mu)\bar{J}_{k,\mathrm{ENC}}(N, \zeta QR/2, \mu)$$

$$\bar{J}_{k,\mathrm{ENC}}(N, ER, \mu) = \int_0^1 dx x^N D_{J/k}(x, ER, \mu)$$

[LD, Kim, Leibovich, arXiv:2410.11614]

Large N-point Limit

- Large N-point limit also means large x limit

$$\bar{J}_{k,\mathrm{ENC}}(N, ER, \mu) = \int_0^1 dx x^N D_{J/k}(x, ER, \mu)$$

$$D_{J/k}(x \to 1, ER, \mu) \approx D_{J_k/k}(x \to 1, E_J R, \mu)$$
$$= \mathcal{J}_k(E_J R, \mu) S_k(x, E_J R, \mu).$$

$$\bar{J}_{k,\mathrm{ENC}}(N, E_J R, \mu) = \mathcal{J}_k(E_J R, \mu) \bar{S}_k(N, E_J R, \mu)$$

[LD, Kim, Leibovich, arXiv:2410.11614]

Large N-point Limit

- Large N-point limit also means large x limit

$$\bar{J}_{k,\mathrm{ENC}}(N, E_J R, \mu) = \mathcal{J}_k(E_J R, \mu)\bar{S}_k(N, E_J R, \mu)$$

$$\mathcal{J}_Q^{(1)}(E_J R, m, \mu) = \frac{\alpha_s C_F}{4\pi}\left[\frac{3+b}{1+b}\ln\frac{\mu^2}{B^2} + \ln^2\frac{\mu^2}{B^2} + 2f(b) + 2g(b) + \frac{4+2\ln(1+b)}{1+b} - \ln^2(1+b) - 2\mathrm{Li}_2(-b) + 4 - \frac{\pi^2}{6}\right]$$

$$\bar{S}_Q^{(1)}(N, E_J R, m, \mu) = \frac{\alpha_s C_F}{4\pi}\left[\frac{2b}{1+b}\ln\frac{\mu^2\bar{N}^2}{B^2} - \ln^2\frac{\mu^2\bar{N}^2}{B^2} - \frac{2\ln(1+b)}{1+b} + \ln^2(1+b) + 2\mathrm{Li}_2(-b) - \frac{\pi^2}{2}\right]$$

$$B = \sqrt{(E_J R)^2 + m^2} \qquad \mu_j = B,\ \mu_s = B/N$$

[LD, Kim, Leibovich, arXiv:2410.11614]

Large N-point Limit

- Resummation small R and large N

$$\frac{d\bar{J}_{k,\mathrm{ENC}}(\mu)}{dR} = e^{\mathcal{M}_N(\mu,\mu_j,\mu_s)}\Big[\frac{dM_N(\mu_j,\mu_s)}{dR}\bar{J}^{\mathrm{NL}}_{k,\mathrm{ENC}}(\mu_j,\mu_s) + \frac{d\bar{J}^{\mathrm{NL}}_{k,\mathrm{ENC}}(\mu_j,\mu_s)}{dR}\Big]$$

$$M_{\bar{N}}^{\mathcal{Q}}(\mu,\mu_j,\mu_s) = -2S_\Gamma(\mu_j,\mu_s) - \ln\frac{\mu_j^2}{B^2}\cdot a_\Gamma(\mu_j,\mu_s) - \ln\bar{N}^2\cdot a_\Gamma(\mu,\mu_s) - \frac{C_F}{\beta_0}\Big(\frac{3+b}{1+b}\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_j)} + \frac{2b}{1+b}\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_s)}\Big)$$

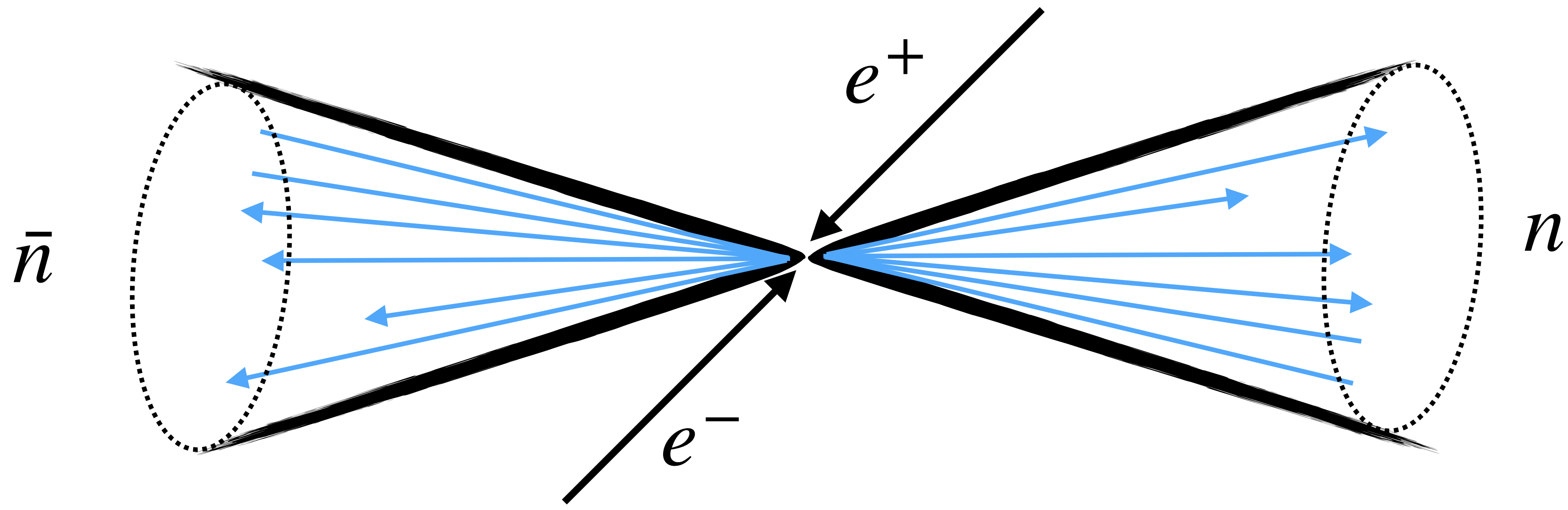
$$\frac{dM_{\bar{N}}^{\mathcal{Q}}(\mu_j,\mu_s)}{dR} = \frac{2E_J^2R}{B^2}\Big[a_\Gamma(B,B/\bar{N}) + \frac{C_F}{\beta_0}\frac{2b}{1+b}\ln\frac{\alpha_s(B)}{\alpha_s(B/\bar{N})} - C_F\Big(\frac{\alpha_s(B)}{4\pi}\frac{3+b}{1+b} + \frac{\alpha_s(B/\bar{N})}{4\pi}\frac{2b}{1+b}\Big)\Big]$$

$$S_\Gamma(\mu_1,\mu_2) = \int_{\mu_2}^{\mu_1}\frac{d\mu}{\mu}\Gamma_C(\alpha_s)\ln\frac{\mu}{\mu_1}\ ,$$

$$a_\Gamma(\mu_1,\mu_2) = \int_{\mu_2}^{\mu_1}\frac{d\mu}{\mu}\Gamma_C(\alpha_s)\ .$$

[LD, Kim, Leibovich, arXiv:2410.11614]

$e^+e^- \to$ 2 jets



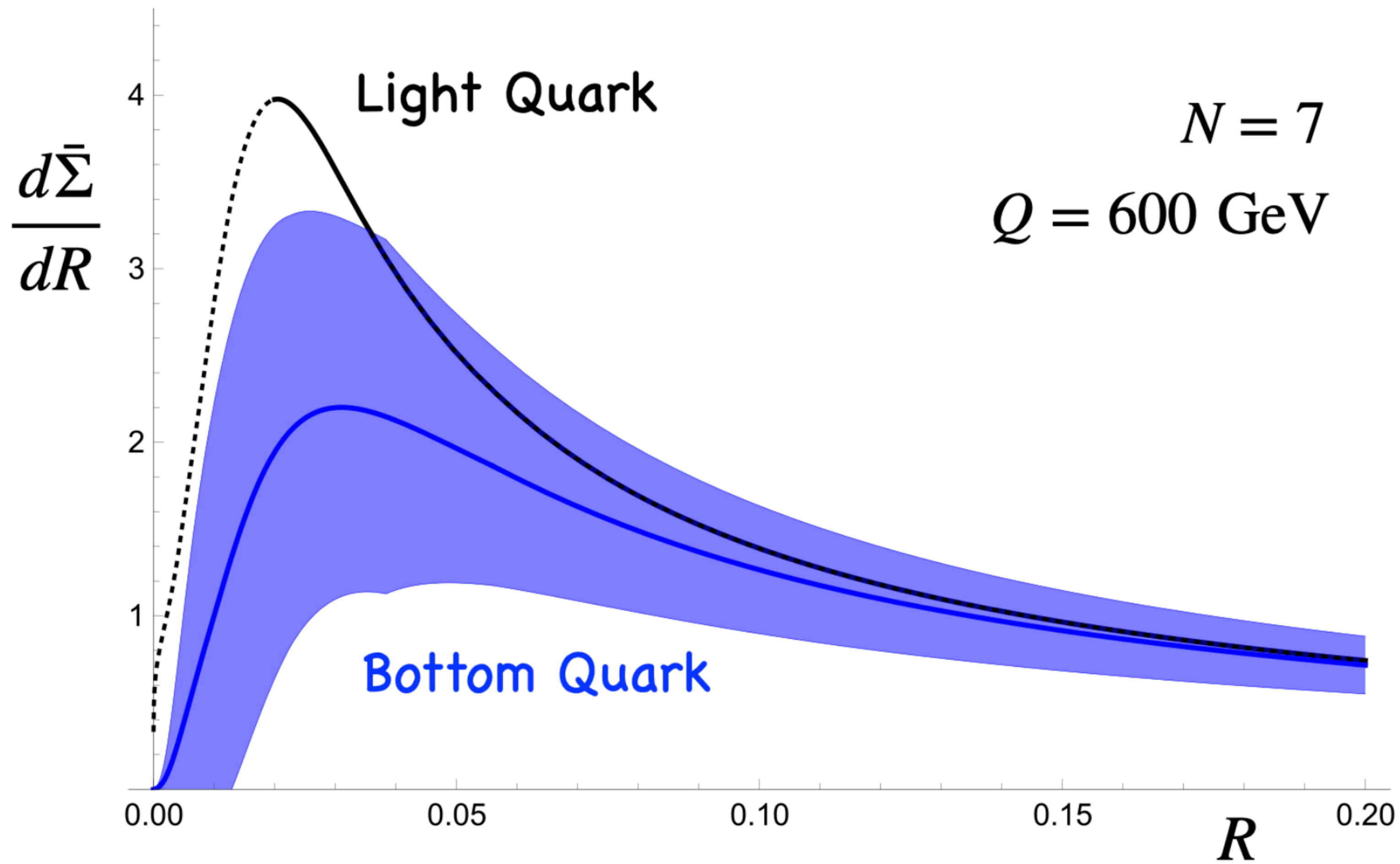
$$\bar{\Sigma}(R^2) \equiv \frac{2^N}{\sigma_0}\Sigma(R^2) = \sum_{k=q,\mathcal{Q}} H_k^v(Q,\mu)\mathcal{J}_k(QR/2,\mu)\,\bar{J}_k^{\bar{n}}(Q,N,\mu)\bar{S}_k(QR/2,N,\mu)$$

$$H_{q,\mathcal{Q}}^{v(1)}(Q,\mu) = \frac{\alpha_s C_F}{2\pi}\left(-3\ln\frac{\mu^2}{Q^2} - \ln^2\frac{\mu^2}{Q^2} - 8 + \frac{7\pi^2}{6}\right), \tag{31}$$

$$\bar{J}_{q,\mathcal{Q}}^{\bar{n}(1)}(Q,N,\mu) = \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\ln\frac{\mu^2\bar{N}}{Q^2} + \ln^2\frac{\mu^2\bar{N}}{Q^2} + \frac{7}{2} - \frac{\pi^2}{3}\right).$$

[LD, Kim, Leibovich, arXiv:2410.11614]

$e^+e^- \to$ 2 jets (NLL)



Scales: $B = \sqrt{(E_J R)^2 + m^2}$ $\quad \mu_j = B,\ \mu_s = B/N$

Differences between light and heavy quark: small R region

[LD, Kim, Leibovich, arXiv:2410.11614]

Summary

- In the Collinear Limit:
  - N-point EEC cumulant is related to moments of FFJ
  - Large-N point EEC corresponds to the large z limit of FFJ
  - Factorization Theorems of FFJ can be applied to EEC

Backup Slides

$$f(b) = \int_0^1 dz \frac{1+z^2}{1-z} \ln \frac{z^2+b}{1+b},$$

$$g(b) = \int_0^1 dz \frac{2z}{1-z} \Big(\frac{1}{1+b} - \frac{z^2}{z^2+b} \Big).$$