Q_T-slicing for multijet processes

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Refs: arXiv:241X.XXXXX   Rong-Jun Fu, Rudi Rahn, DYS, Wouter Waalewijn, Bin Wu

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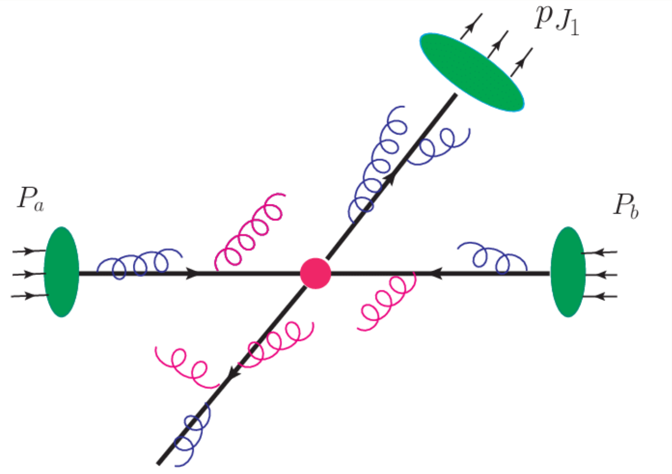
Collinear factorization formula in hadron colliders

$$\mathrm{d}\sigma_{pp\to X} = \sum_{ij} \int \mathrm{d}x_1\, \mathrm{d}x_2 f_i\,(x_1)\, f_j\,(x_2)\, \mathrm{d}\sigma_{ij\to X}\,(x_1 P_1, x_2 P_2)\, F_J \left[1 + \mathcal{O}(\Lambda^2_{\mathrm{QCD}}/Q^2)\right]$$



A key challenge in precision QCD calculations for the LHC is handling infrared (IR) divergences, which must cancel between real and virtual emissions.

Two main approaches for the cancellation of IR divergences

Slicing

$$\int |\mathcal{M}|^2 F_J \, \mathrm{d}\phi_d = \int_0^\delta \left[|\mathcal{M}|^2 F_J \, \mathrm{d}\phi_d\right]_{\text{sing}} + \int_\delta^1 |\mathcal{M}|^2 F_J \, \mathrm{d}\phi_4 + \mathcal{O}(\delta)$$

- Non-local in phase space;
- Large cancellations between singular and regular terms;
- Straightforward to implement;

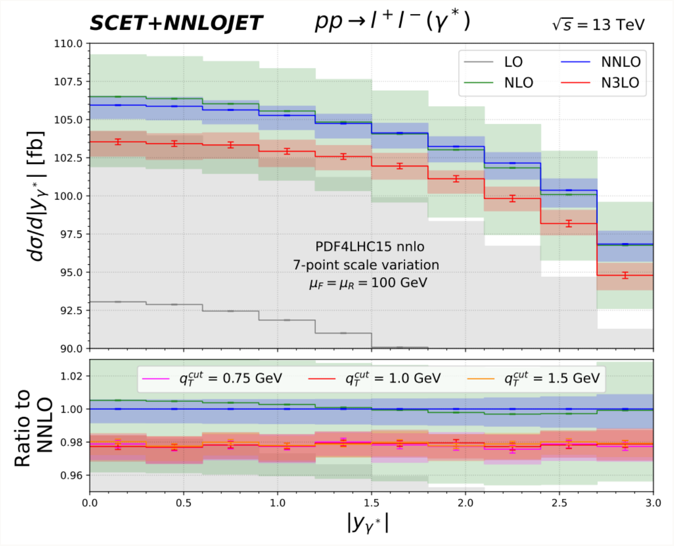
Subtraction [See Xuan Chen's talk]

$$\int |\mathcal{M}|^2 F_J \, \mathrm{d}\phi_d = \int \left[|\mathcal{M}|^2 F_J - S\right] \mathrm{d}\phi_d + \int S \, \mathrm{d}\phi_d$$

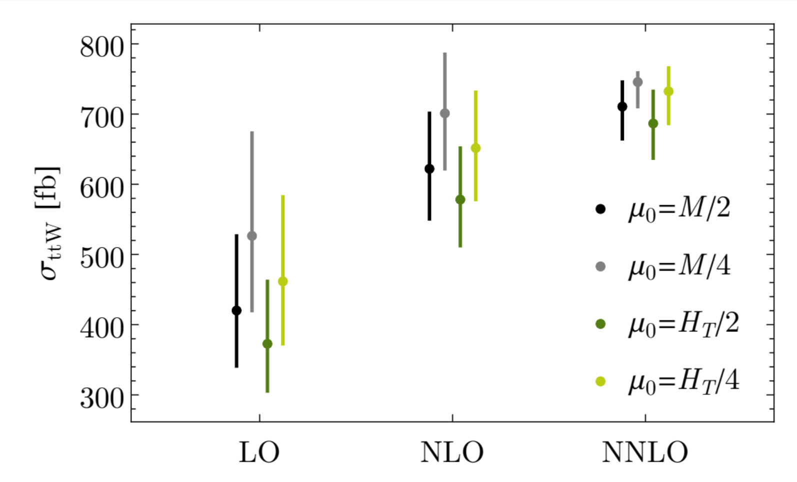
- Local in phase space;
- Potentially, offers better numerical stability;
- More difficult conceptually;

Modern slicing variables

1. Transverse momentum of a colorless or colored (but massive) system; [Catani, Grazzini '07]



[Chen, Gehrmann, Glover, Huss, Yang, Zhu '23]



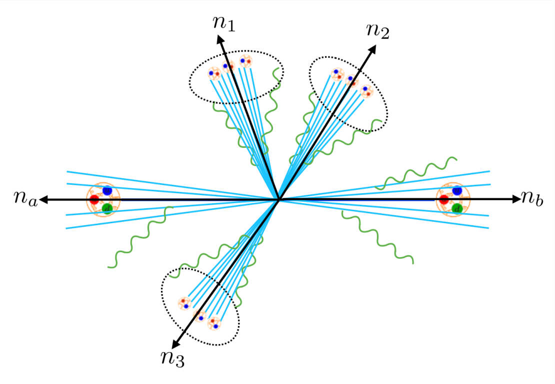
[Buonocorea, Devotob, Grazzinia, Kallweitc, Mazzitellid, Rottolia, Savoini '23]

For processes with jets, such as $pp \to V$+jet or $pp \to 2$ jets, q_T is unsuitable because $q_T = 0$ for radiation emitted inside jets.

2. Jettiness; [Boughezal, Focke, Liu, Petriello '15; Gaunt, Stahlhofen, Tackmann, Walsh '15]

$$\tau_N = \sum_i \min\left\{\frac{2q_a k_i}{Q^2}, \frac{q_b k_i}{Q^2}, \ldots, \frac{2q_N k_i}{Q^2}\right\}$$

- Computation of singular contributions is facilitated by factorization formula; [Stewart, Tackmann, Waalewijn '10]

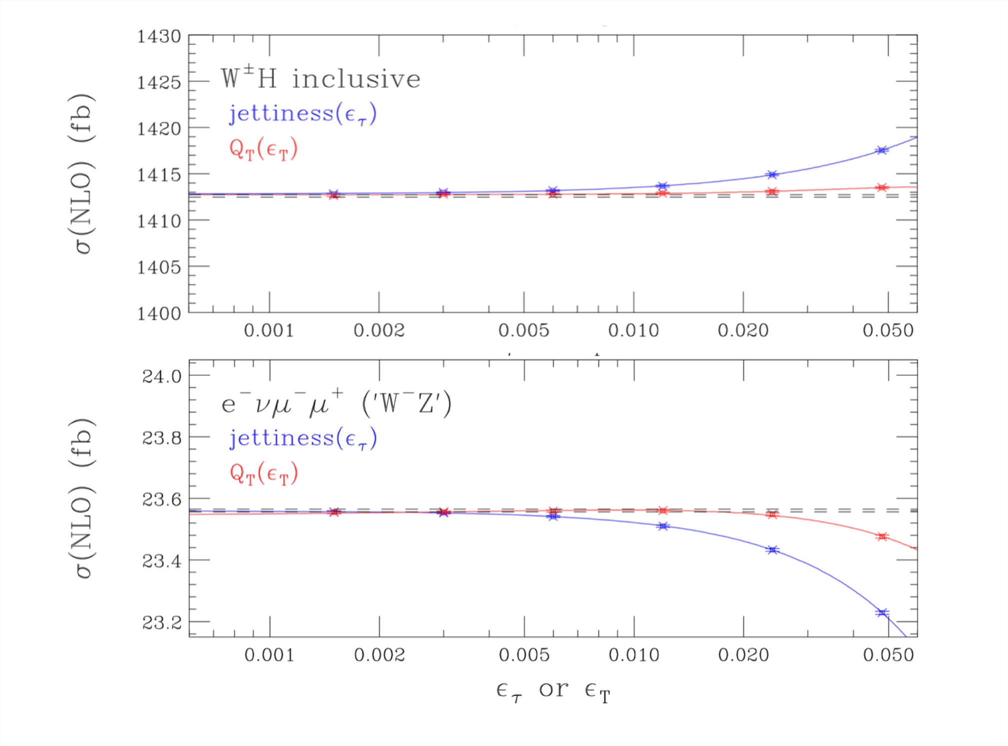


$$\lim_{\tau\to 0}\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = B\otimes B\otimes S\otimes H\otimes J_1\otimes\cdots\otimes J_n$$

- Some N^{3}LO ingredients are known:
  - Jet function: [Bruser, Liu, Stahlhofen '18]
  - Beam function: [Behring, Melnikov, Rietkerk, Tancredi, Wever '19; Ebert, Mistlberger, Vita '20]
  - Zero-jettiness soft function: [Baranowski, Delto, Melnikov,Pikelner, Wang '24]

q_T and jettiness slicing variables

- For most color-singlet processes, q_T performs better than 0-jettiness as slicing variable [Campbell, Ellis, Seth '22]

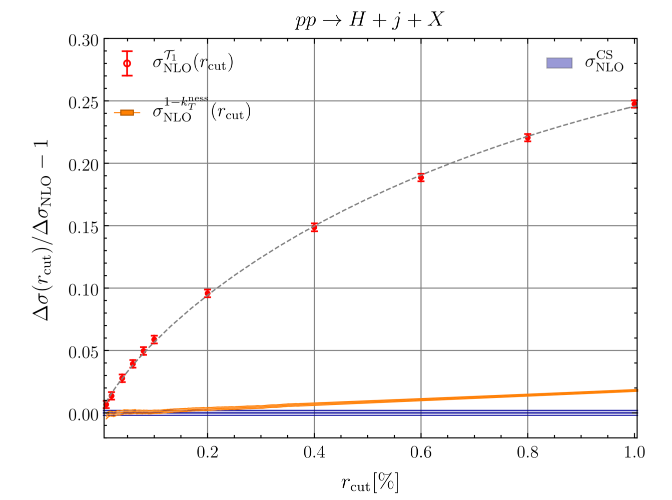


- Motivating the search for an extension of q_T to processes with jets;

- k_T-ness and ΔE_T was recently introduced: [Buonocore, Grazzini, Haag, Rottoli, Savoini '22; '23]
  - One-jet resolution variable ΔE_T: an event in which F is accompanied by m partons with momenta $p_3, \ldots, p_{m+2}$

$$\Delta E_T = \sum_{i=3}^{m+2} |\vec{p}_{i,T}| - |\vec{p}_{F,T}|$$

  - n-jet resolution variable k_T-ness: from the k_T-clustering algorithm

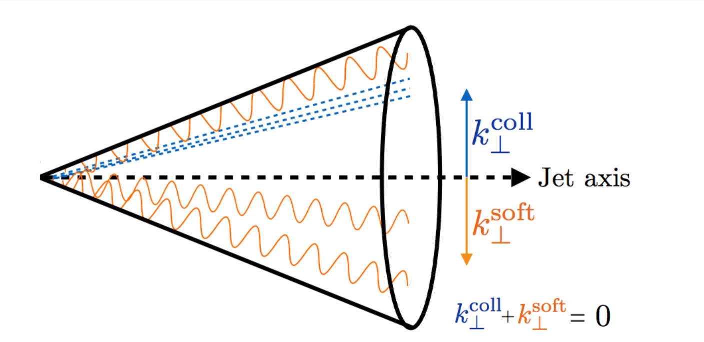


- Both jettiness and k_T-ness results nicely converge
- The resolution variable dependence in the case of jettiness is strong

- However, implementing it beyond NLO will be very challenging in the absence of a factorization formula.

TMD and jet physics

- QCD factorization of jet TMDs:



$$q_T = \left| \sum_{i \notin \text{jets}} \vec{k}_{T,i} \right| + \mathcal{O}(k_T^2) \ll Q$$

  - Sum over all soft and collinear partons not combined with hard jets
  - Deviation from $q_T = 0$ is caused only by particle flow outside jets
  - Non-global observables [Dasgupta & Salam '01]
- Factorization formula ($q_T \ll Q, R \ll 1$) within SCET and Jet effective theory [Becher, Neubert, Rothen, DYS '16 PRL]

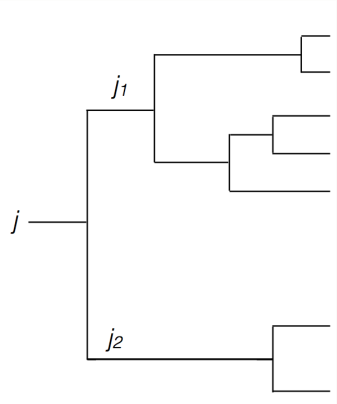
$$\frac{d\sigma}{d^2\vec{q}_T dX} = \sum_{ijk} \int \frac{d^2 b_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T} \mathcal{S}_{ij\to Vk}(\vec{b}_T) \mathcal{B}_{i/N_1}(\xi_1, b_T)\, \mathcal{B}_{j/N_2}(\xi_2, b_T)$$

$$\times \mathcal{H}_{ij\to Vk}(\hat{s}, \hat{t}, m_V) \sum_{m=1}^{\infty} \left\langle \mathcal{J}_m^k(\{\underline{n_J}\}, Rp_J) \otimes \mathcal{U}_m^k(\{\underline{n_J}\}, R\vec{b}_T) \right\rangle$$

Refs: $pp \to V$+jet [Chien, DYS, Wu '19 JHEP]

TMD and jet physics

- Define $\vec{q}_T$ using p_T^n-weighted recombination scheme [Banfi, Dasgupta, Delenda '08]:



$$p_{T,r} = p_{T,i} + p_{T,j},$$
$$\phi_r = \frac{w_i\phi_i + w_j\phi_j}{w_i + w_j}, \quad y_r = \frac{w_i y_i + w_j y_j}{w_i + w_j}, \quad w_i = p_T^n$$

where $n \to \infty$: Winner-take-all scheme [Bertolini, Chan, Thaler '13]

- N^{3}LL resummation for jet q_T @ e^+e^- and ep: [Reyes, Scimemi, Waalewijn, Zoppi '18, '19]
- N^{2}LL resummation for $\delta\phi$ @ LHC: [Chien, Rahn, DYS, Waalewijn & Wu '22 JHEP + Schrinder '21 PLB]
- N^{3}LL resummation for $\delta\phi$ @ ep: [Fang, Ke, DYS, Terry '23 JHEP; Fang, Gao, Li, DYS '24]

q_T-slicing for jets

- Recently, we propose two ways of extending q_T to processes with jets; [Fu, Rahn, DYS, Waalewijn, Wu '24]

The key ingredient is the use of a recoil-free jet axis!

- Azimuthal decorrelation: $\delta\phi$ or q_x

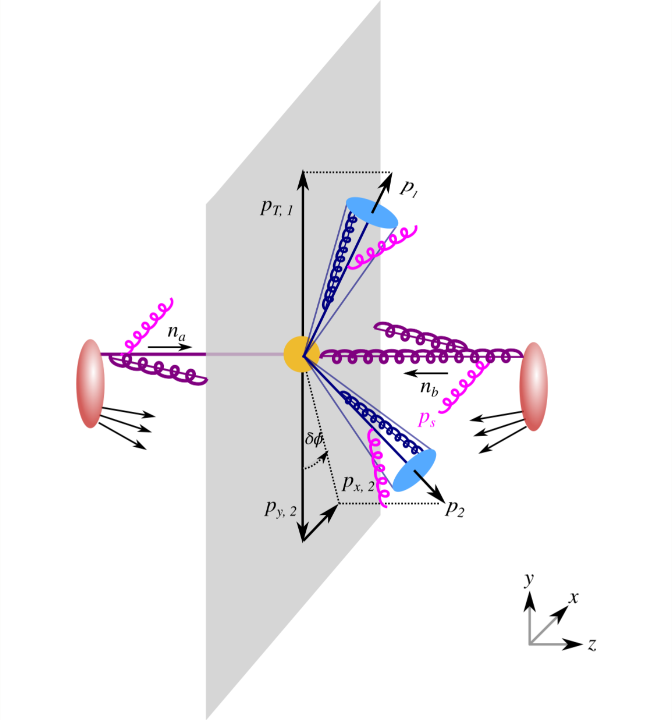
$$q_x = p_{x,1} + p_{x,2}$$

- Total transverse momentum: $\vec{q}_T$:

$$\vec{q}_T = \sum_{i=\text{jets},V,\ldots} \vec{p}_{T,i}$$

Azimuthal decorrelation ($\delta\phi$ or q_x)

- A simple factorization formula; [Chien, Rahn, DYS, Waalewijn & Wu '22 + Schrinder '21]
- Suitable for processes that are planar at LO: $pp \to 2$ jets, $pp \to V$+jet, $e^+e^- \to 3$ jets, etc.;



By using the recoil-free scheme, the transverse momentum perpendicular to the scattering plane q_x or equivalently the azimuthal decorrelation $\delta\phi$, is a suitable slicing variable.

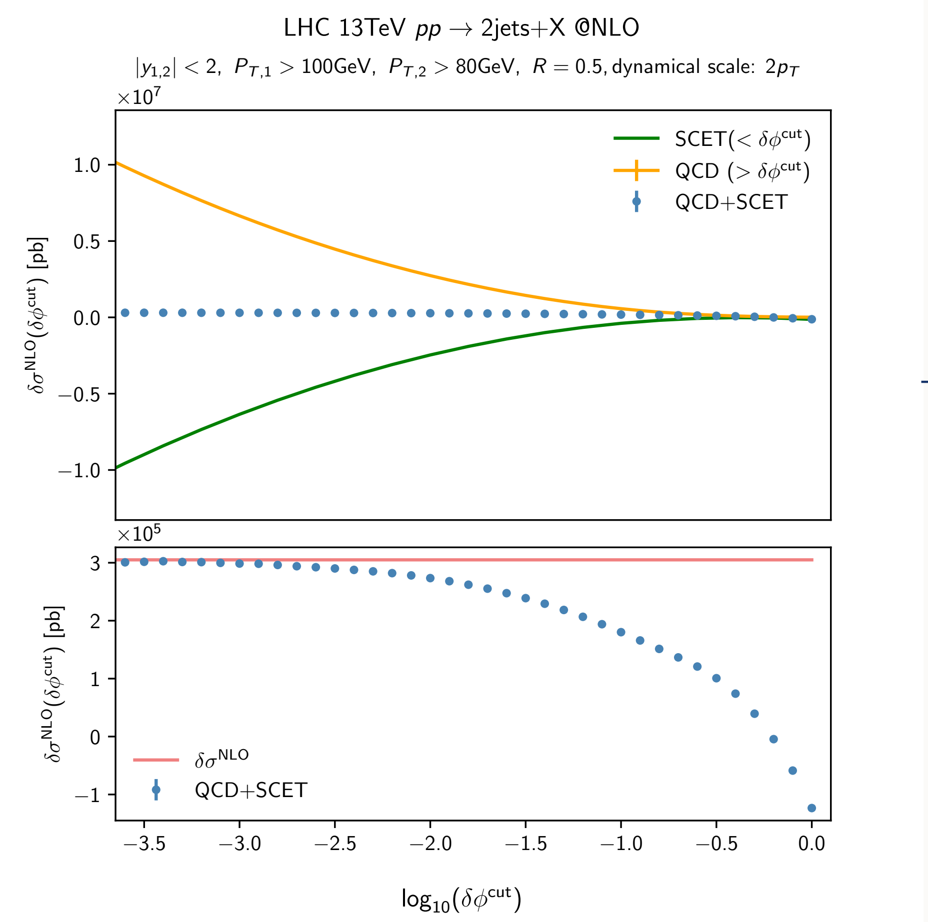
Azimuthal decorrelation ($\delta\phi$ or q_x)

- Factorization formula ($pp \to 2$ jets):

$$\frac{\mathrm{d}\sigma_{\mathrm{fact}}}{\mathrm{d}X\,\mathrm{d}q_x} = \int \frac{\mathrm{d}b_x}{2\pi} e^{\mathrm{i}q_x b_x} \sum_{i,j,k,\ell} B_i\left(x_a, b_x\right) B_j\left(x_b, b_x\right) \mathcal{J}_k\left(b_x\right) \mathcal{J}_\ell\left(b_x\right) \\ \times \operatorname{tr}\left[\hat{\mathcal{H}}_{ij\to k\ell}\left(p_{T,1}, \eta_1 - \eta_2\right) \hat{S}_{ijk\ell}\left(b_x, \eta_1, \eta_2\right)\right].$$

- Factorization ingredients:
  - Standard TMD PDFs $B_{i,j}$: known at N^{3}LO; [Luo, Yang, Zhu, Zhu, '19; Ebert, Mistlberger, Vita '20]
  - Soft function $\hat{S}_{ijk\ell}$: can directly be obtained from the standard TMD soft function at NNLO; [Gao, Li, Moult, Zhu '19]
  - TMD jet functions $\mathcal{J}_{k,\ell}$: partially known at NNLO; [Reyes, Scimemi, Waalewijn, Zoppi '19; Bell, Brune, Das, Wald '23; Fang, Gao, Li, DYS '24]

Azimuthal decorrelation ($\delta\phi$ or q_x)



- NLO correction $\delta\sigma^{\rm NLO}$;

$$\frac{d\sigma}{dX} = \int_0^\delta dq_T \frac{d\sigma_{\rm fact}}{dX\, dq_T} + \int_\delta^\infty dq_T \frac{d\sigma}{dX\, dq_T}$$

- Jets defined using the anti-k_T algorithm with $R = 0.5$.
- Converge nicely to the NLO result

Transverse momentum ($\vec{q}_T$)

- We can still use the total transverse momentum $q_T = |\vec{q}_T|$ of the color-singlet and jets as a slicing variable, when using the WTA scheme.
- Especially, for processes that are non-planar at LO, such as $pp \to Z + 2$ jets, $pp \to 3$ jets, etc.
- Factorization formula:

$$\frac{\mathrm{d}\sigma_{\text{fact}}}{\mathrm{d}X\,\mathrm{d}q_T} = q_T \int \frac{\mathrm{d}^2\vec{b}_T}{2\pi} J_0(q_T b_T) \sum_{i,j,k,\ell} B_i(x_a, b_T) B_j(x_b, b_T) \mathcal{J}_k(b_x) \mathcal{J}_\ell(b_x)$$
$$\times \operatorname{tr}\left[\hat{\mathcal{H}}_{ij\to k\ell}(p_{T,1}, \eta_1 - \eta_2)\, \hat{S}_{ijk\ell}(R, \vec{b}_T, \eta_1, \eta_2)\right].$$

- Only soft function is new: Outside the jet (b_T), Inside the jet (b_x);

Soft function

- In small R limit, soft function refactorizes into simpler global and collinear-soft contributions:

$$\hat{S}_{ijk\ell}(R,\vec{b}_T) = \hat{S}^{\text{global}}_{ijk\ell}(\vec{b}_T)\hat{S}^{\text{out}}_{k}(R\vec{b}_T)\hat{S}^{\text{in}}_{k}(Rb_x)\hat{S}^{\text{out}}_{\ell}(R\vec{b}_T)\hat{S}^{\text{in}}_{\ell}(Rb_x) + \mathcal{O}(R^{2n})$$

The leading term in the small R limit is sufficient to check RG consistency.

- The power corrections of R^{2n} can be obtained by expanding in R. Similar method has been applied in some processes. E.g. Two-loop jet veto p_T^{veto} beam function [Bell, Brune, Das, DYS, Wald '24 JHEP]

Two-loop gluon rapidity anomalous dimension:

$$\Delta d_2^{C_A^2}(R) = \left(-\frac{524}{9} + \frac{16\pi^2}{3} + \frac{176}{3}\ln 2\right)\ln R + \frac{3220}{27} - \frac{44\pi^2}{9} - \frac{560}{9}\ln 2 - \frac{176}{3}\ln^2 2 - 48\zeta_3$$
$$+ 17.9R^2 - 1.28R^4 - 0.0169R^6 + 0.000402R^8 - 2.62\cdot 10^{-5}R^{10}$$

Soft function

$$\hat{\hat{S}}^{(1)}(q_T^{\text{cut}}) = \frac{\alpha_s}{4\pi}\Bigg\{ -4L_\mu^2 \sum_i \boldsymbol{T}_i^2 + L_\mu \Bigg[4L_\nu \sum_i \boldsymbol{T}_i + 8 \sum_{i<j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \ln \frac{n_i \cdot n_j}{2}$$

$$- 8 \ln 2\, (\boldsymbol{T}_1 + \boldsymbol{T}_2) \cdot (\boldsymbol{T}_3 + \boldsymbol{T}_4) - 16 \ln 2\, \boldsymbol{T}_3 \cdot \boldsymbol{T}_4 \Bigg] - \frac{\pi^2}{6} \sum_i \boldsymbol{T}_i^2$$

$$+ \Big[(\boldsymbol{T}_1 + \boldsymbol{T}_2) \cdot (\boldsymbol{T}_3 + \boldsymbol{T}_4) + 2\, \boldsymbol{T}_3 \cdot \boldsymbol{T}_4\Big] \left(4 \ln 2 L_\nu + \frac{\pi^2}{3} + 4 \ln^2 \frac{R}{2}\right)$$

$$+ \sum_{j \in \text{jets}} (\boldsymbol{T}_1 + \boldsymbol{T}_2) \cdot \boldsymbol{T}_j\, 8 \ln 2 \ln (2 \cosh y_j)$$

$$+ \boldsymbol{T}_3 \cdot \boldsymbol{T}_4 \big[8 \ln 2 \ln(4 \cosh \eta_3 \cosh \eta_4) - 2 \ln^2(2 + 2 \cosh(\eta_3 - \eta_4))$$

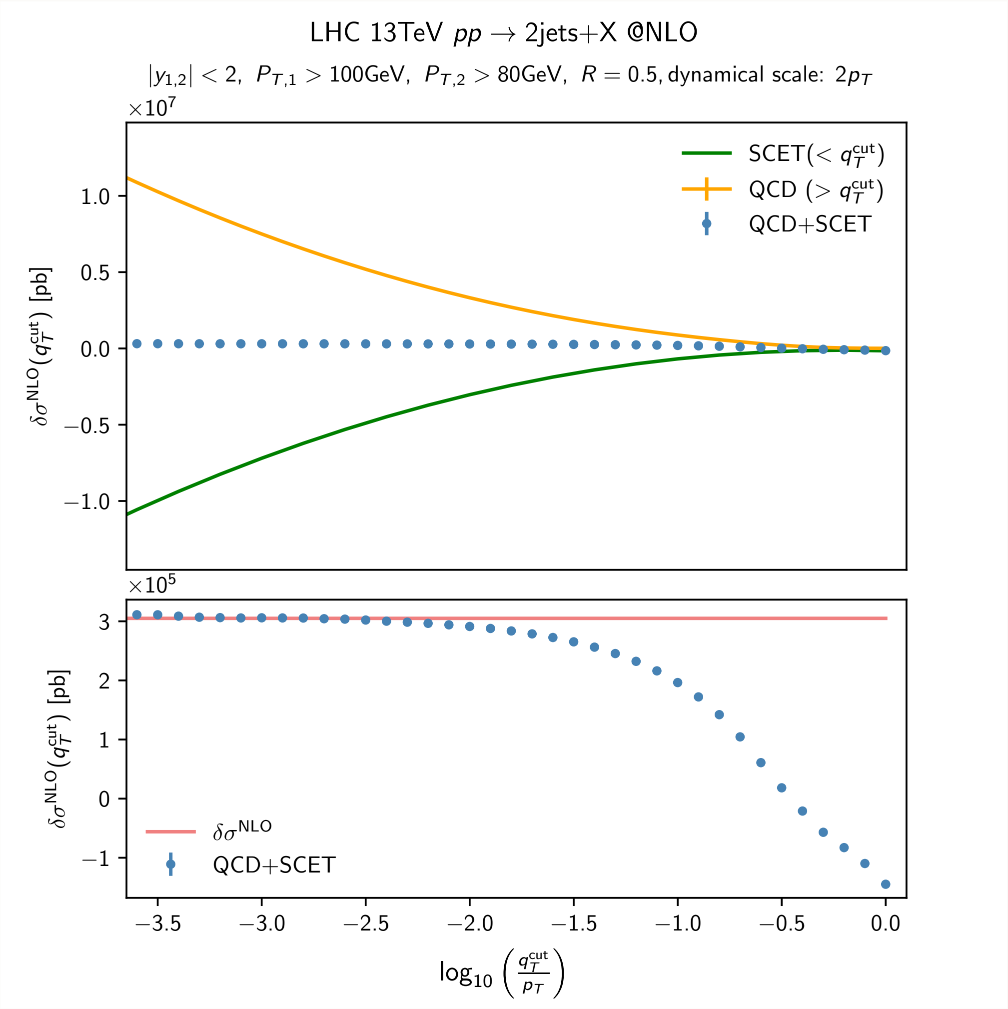
$$+2(\eta_3 - \eta_4)^2\big] - \sum_{i<j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j S_{ij}^{\text{corr}}(R, \eta_3, \eta_4) \Bigg\}$$

Jet-jet dipole:

$$S_{34}^{\text{corr}}(R) = -2R^2 \ln \frac{R}{2} \tanh^2 \left(\frac{\eta_3 - \eta_4}{2}\right) + R^2 \left[\frac{7}{3} - \frac{6}{1 + \cosh(\eta_3 - \eta_4)}\right]$$

$$+ R^4 \left[\frac{49}{720} - \frac{e^{\eta_3+\eta_4}(3e^{2\eta_3} + 3e^{2\eta_4} - 8e^{\eta_3+\eta_4})}{2(e^{\eta_3} + e^{\eta_4})^4} - \ln\left(\frac{R}{2}\right) \frac{(e^{2\eta_3} + e^{2\eta_4} - 10e^{\eta_3+\eta_4})^2}{36(e^{\eta_3} + e^{\eta_4})^4}\right] + \mathcal{O}(R^6)$$

$\vec{q}_T$-slicing for $pp \to 2$ jets



- q_T converges better than q_x slicing
- We add power corrections up to $\mathcal{O}(R^4)$ which reach 1% accuracy as $R = 0.5$.

Summary and outlook

– IR divergences pose a significant challenge in precision QCD calculations, particularly for multi-jet final states.

– Two novel extensions of transverse momentum slicing variables (q_x and q_T) are proposed, specifically designed for jet processes.

- These extensions simplify the treatment of soft functions, especially for planar processes like $pp \to 2$ jets.
- The slicing approach was successfully demonstrated at NLO.
- The q_x slicing variable is also applicable to hadron processes;

– These developments provide a promising framework for addressing the challenges of multi-jet final states at N^{2}LO, paving the way for advancements in precision QCD calculations.

Thank You

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