PRECISION PHENOMENOLOGY TOWARDS N3LO QCD

第四届量子场论及其应用研讨会

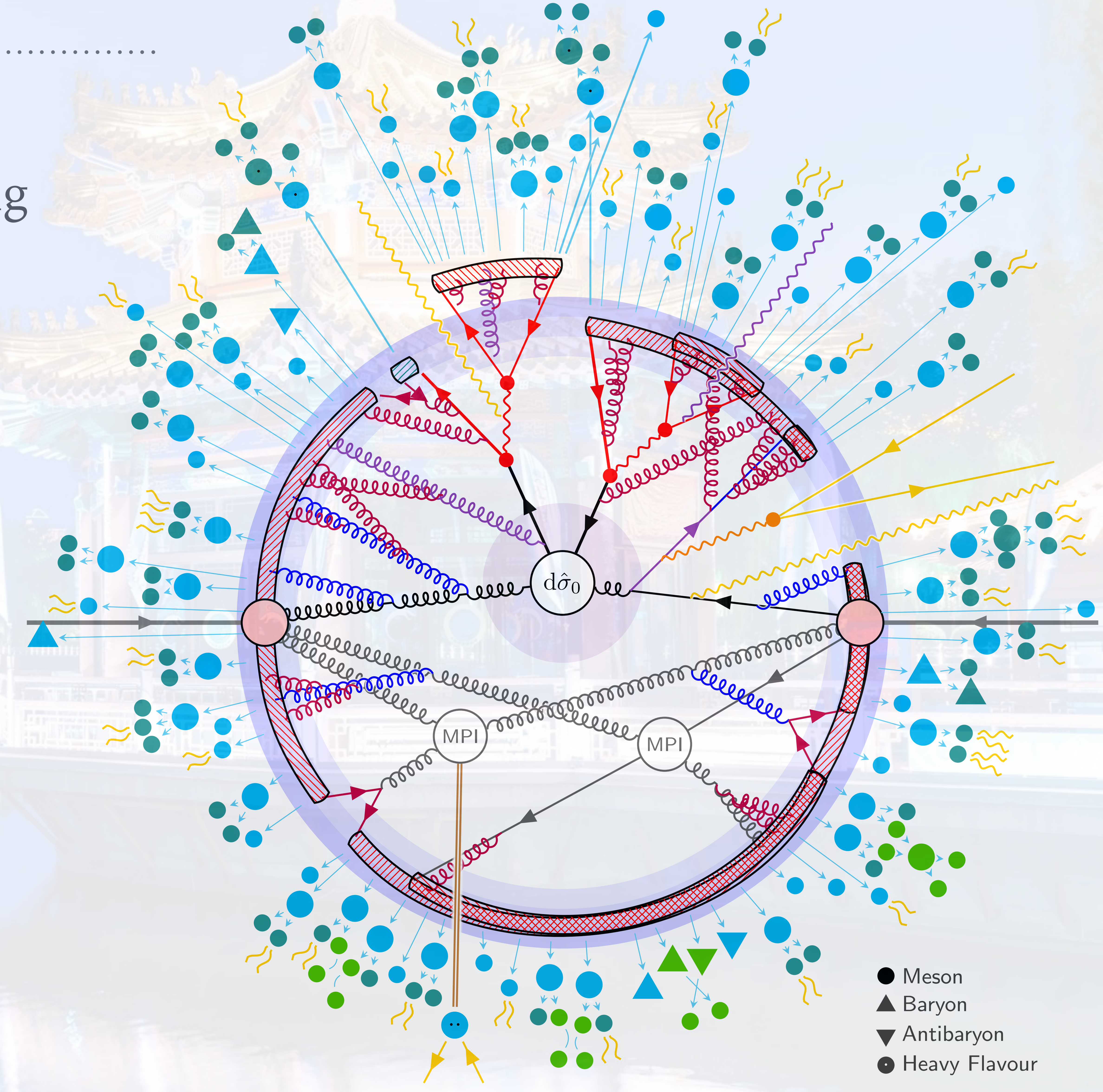
陈暄
山东大学

广州, *18 November, 2024*

Collider Event in Theorist's Eye

➤The idea of factorisation in Quantum Field Theory plays important role to help theorists understanding complex high energy processes:

Time ordering



PYTHIA 8.3

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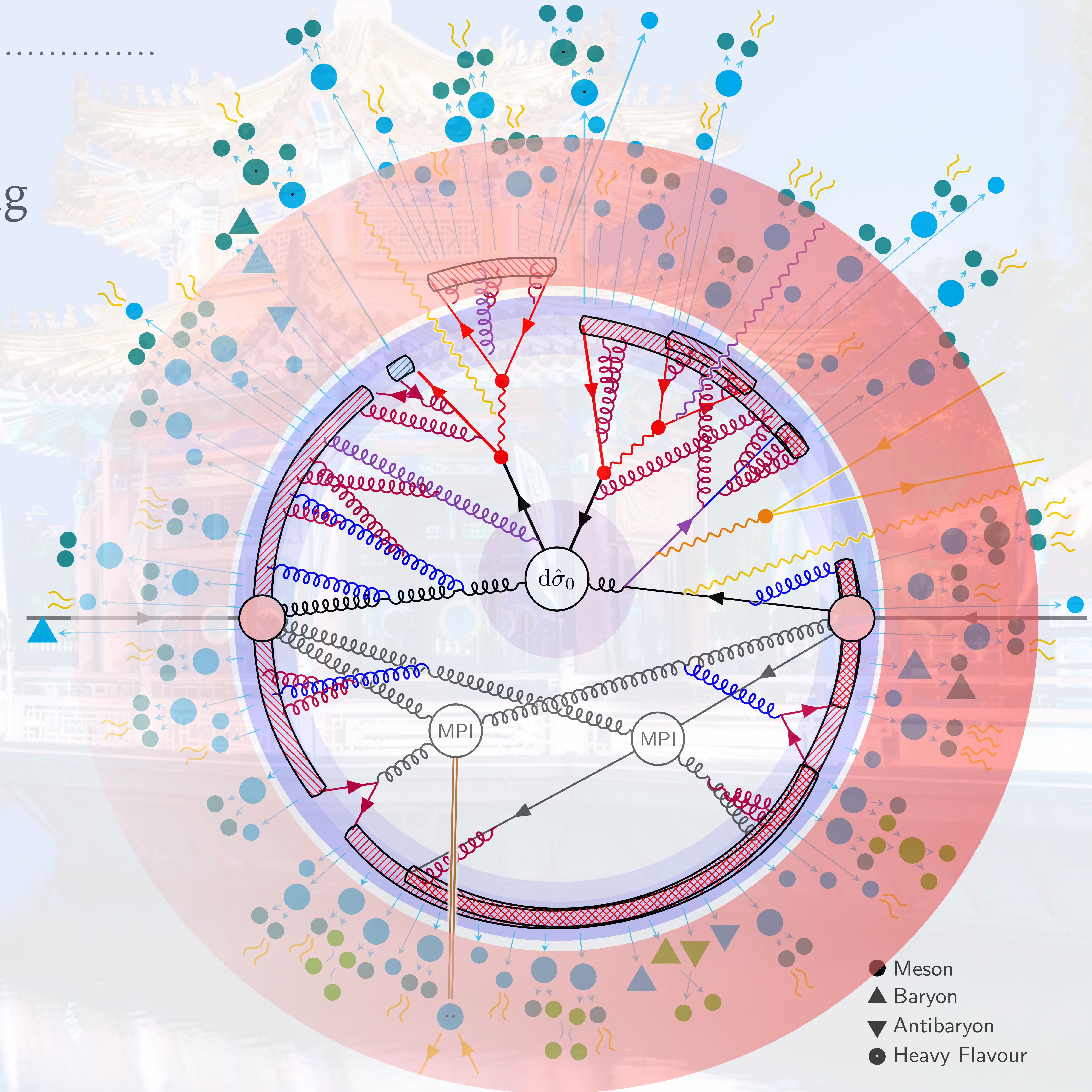
Hadronisation



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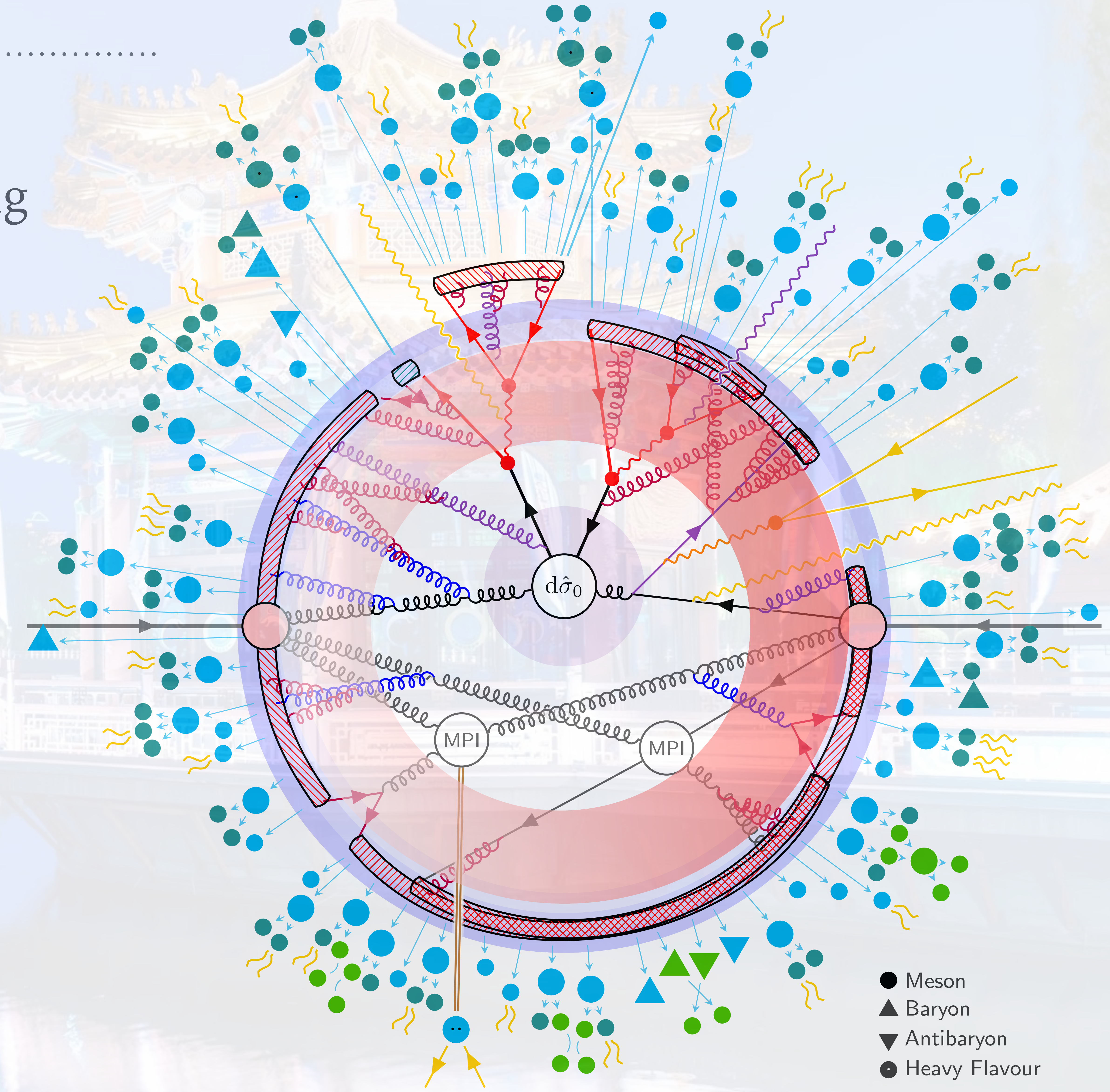

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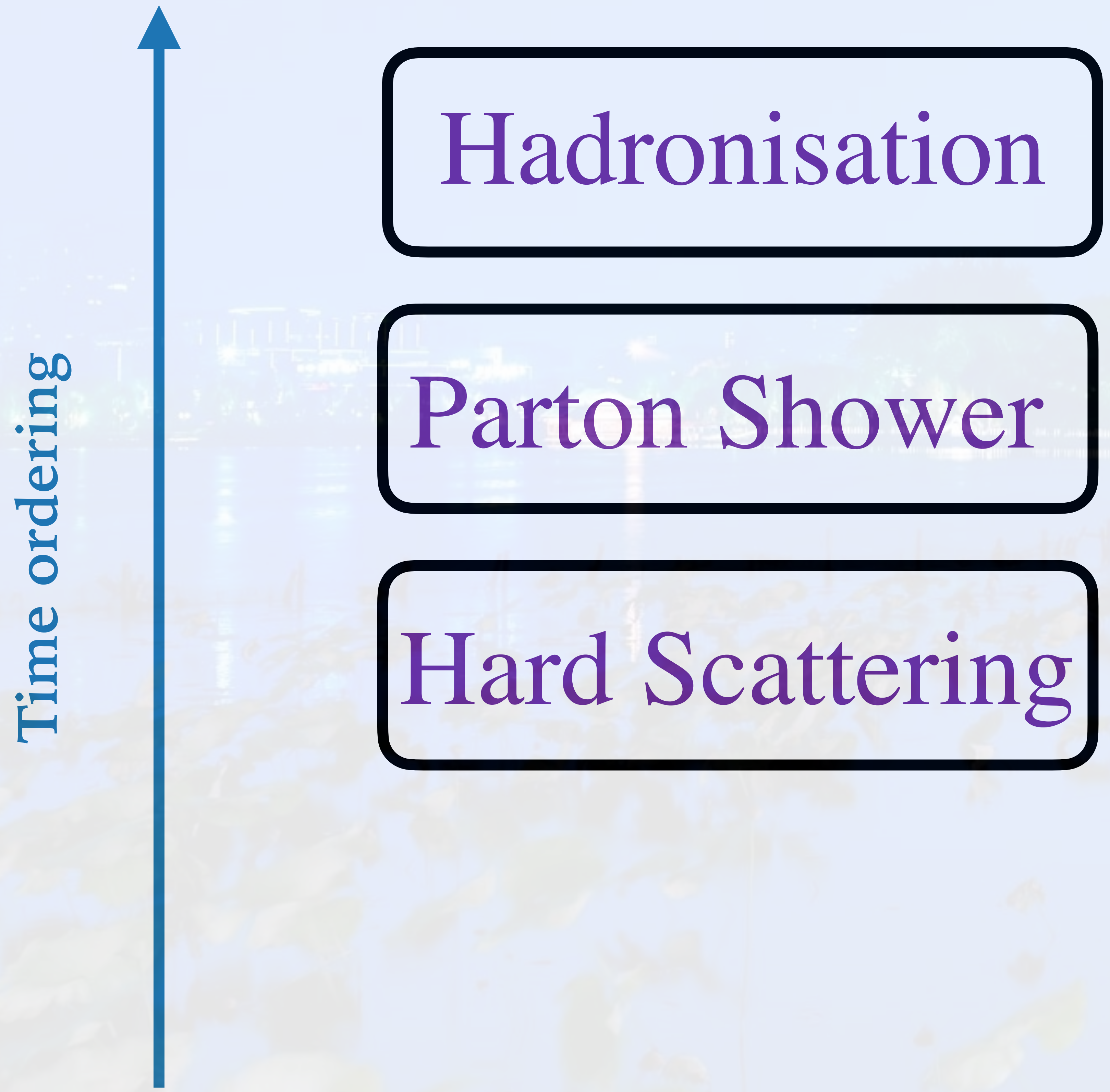
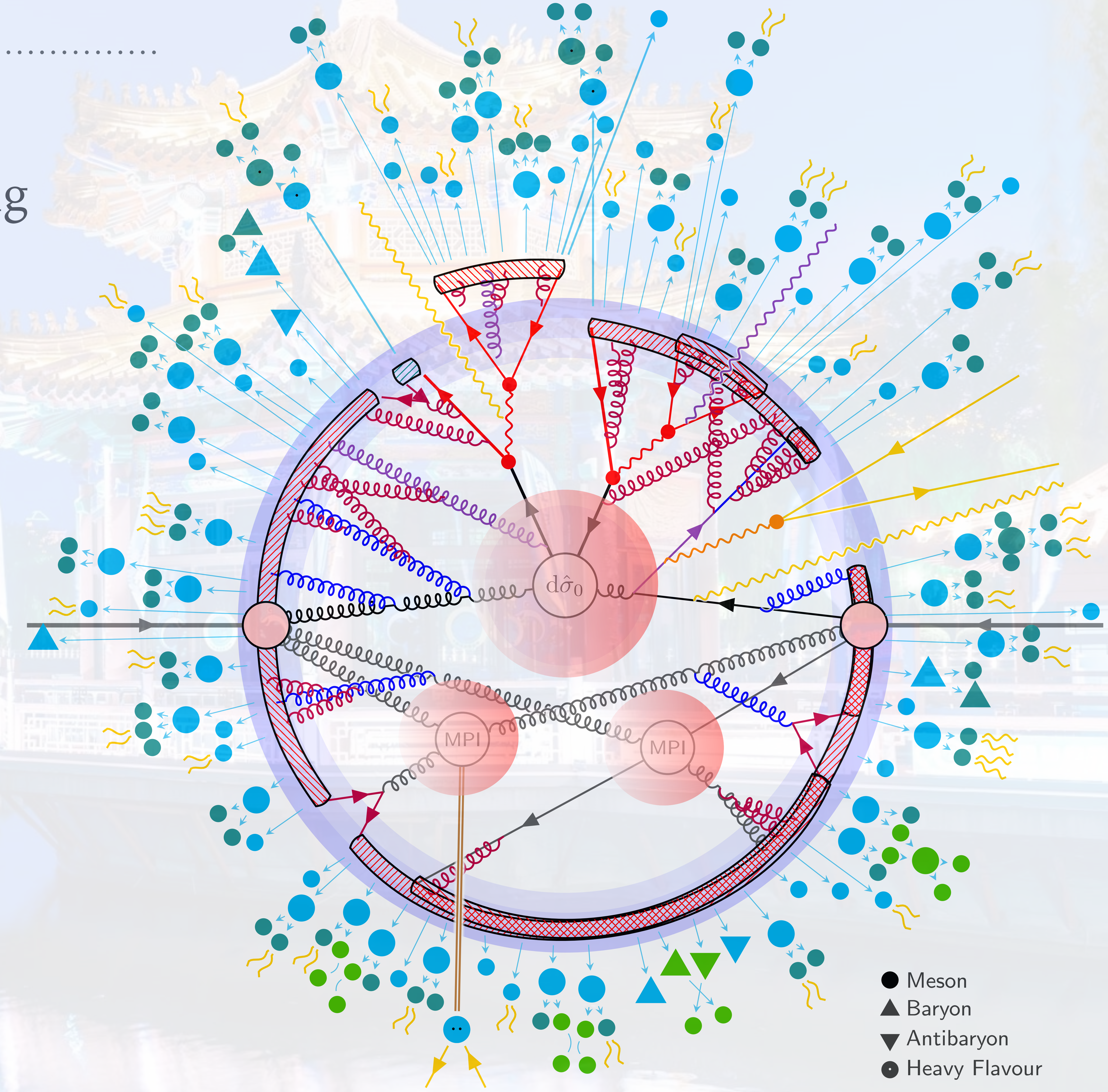




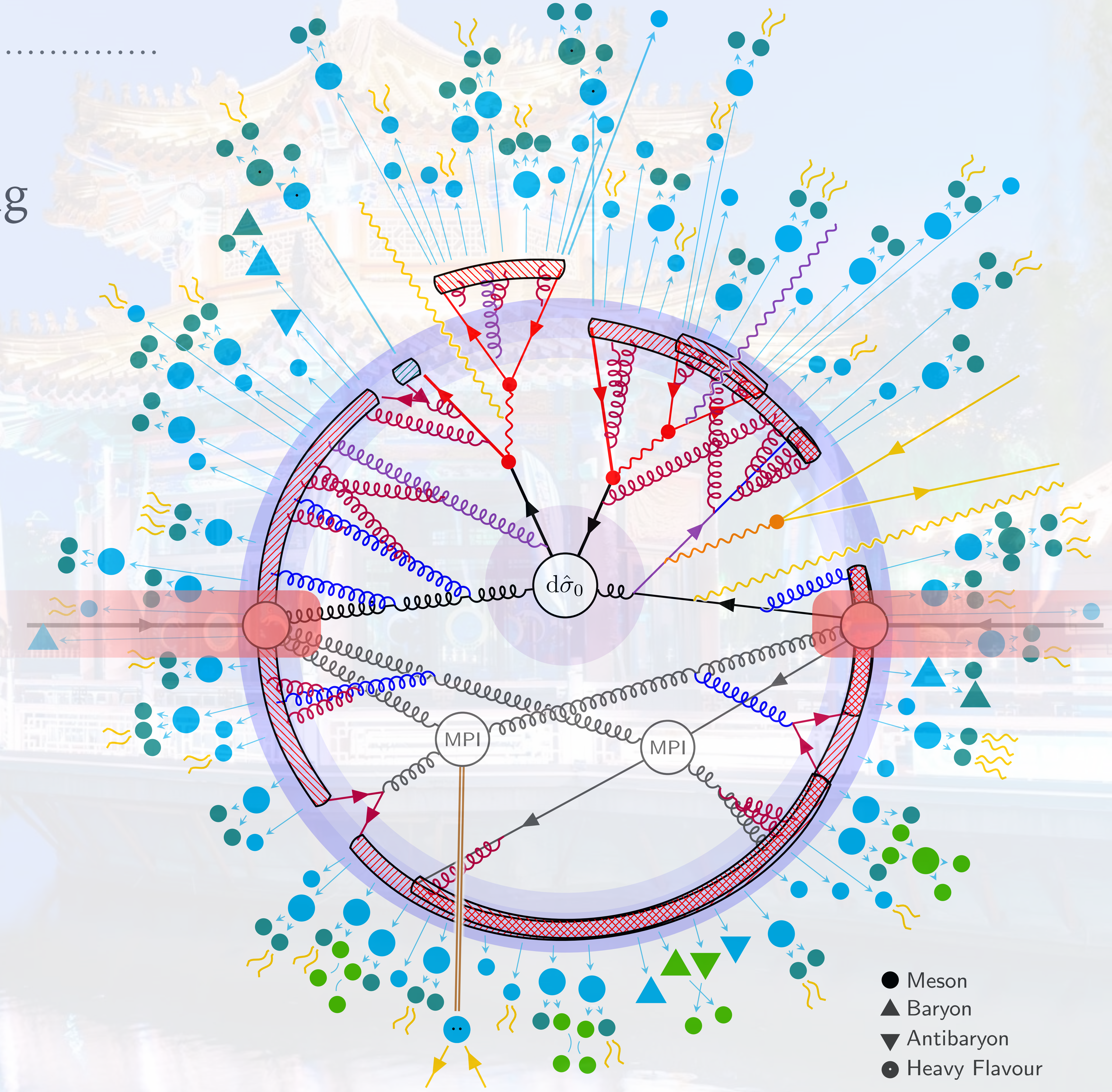
PYTHIA 8.3

Collider Event in Theorist's Eye

➤ The idea of factorisation in Quantum Field Theory plays important role to help theorists understanding complex high energy processes:

Time ordering ↑

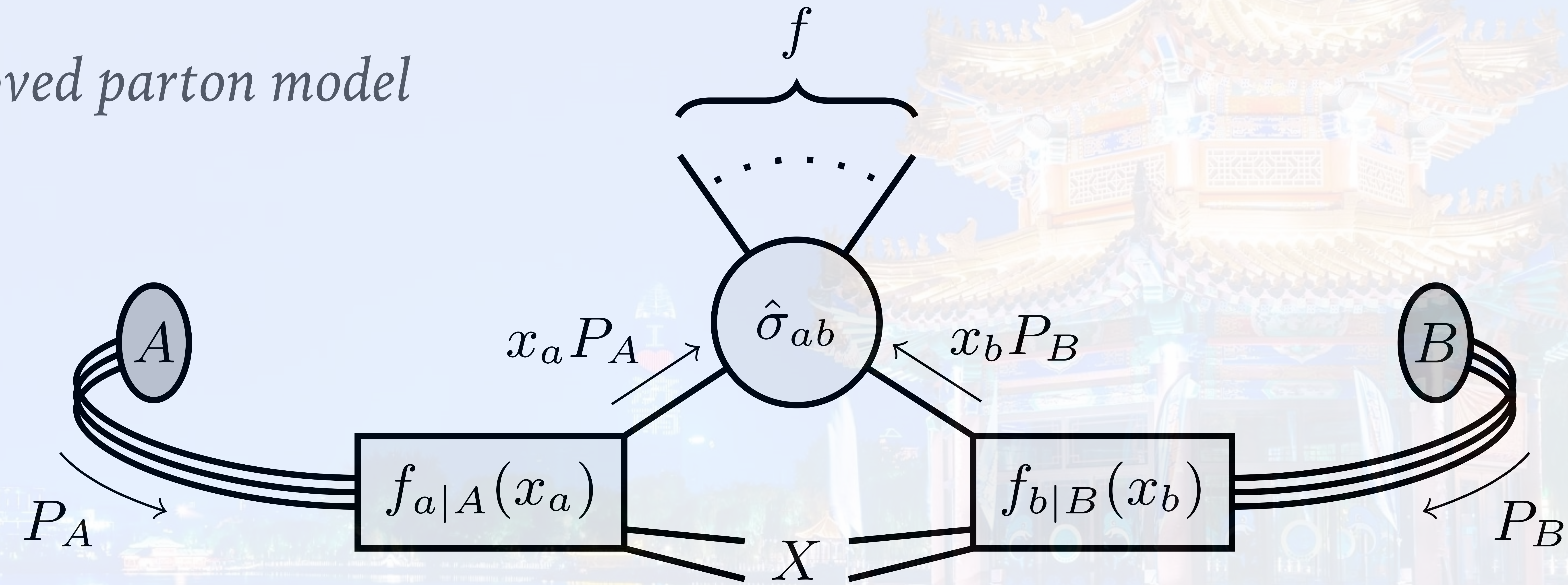
- Hadronisation
- Parton Shower
- Hard Scattering
- Proton ⟶ Parton



PYTHIA 8.3

COLLINEAR FACTORISATION

QCD improved parton model



$$\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \; f_{a|A}(x_a) \; f_{b|B}(x_b) \; \hat{\sigma}_{ab}(x_a, x_b) \; \left(1+\mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)\right)$$

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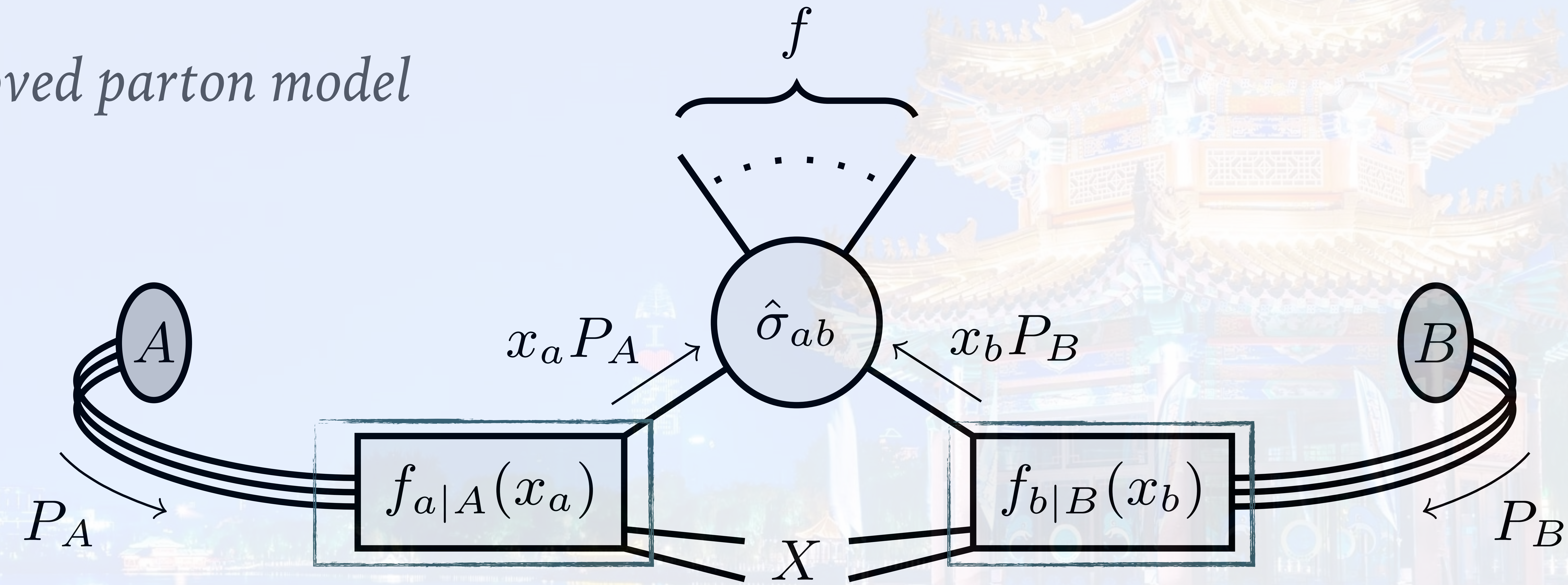

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Parton distribution functions
(Energy evolution from all exp.)
± 3-5 % at LHC energy

COLLINEAR FACTORISATION

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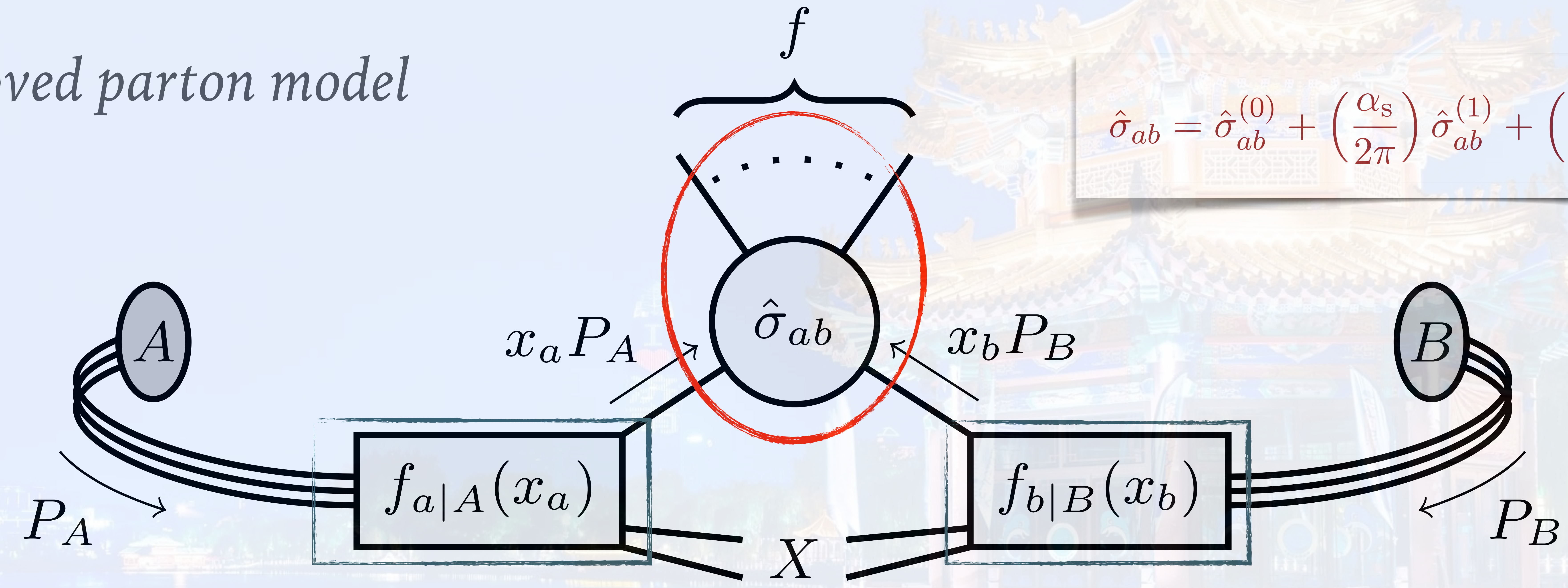
$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}^{(1)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}^{(2)}_{ab} + \ldots$$



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Hard scattering
(Perturbative quantum field theory)
± 1~3 % level!

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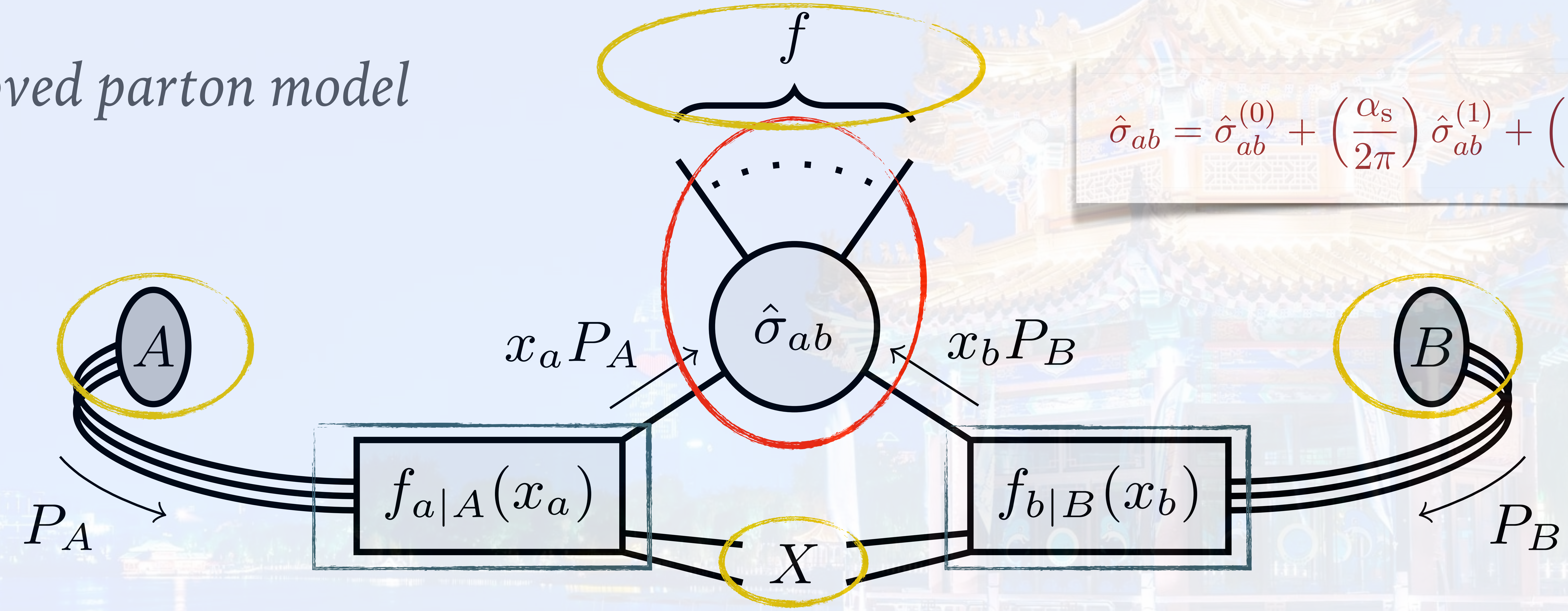

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Parton distribution functions
(Energy evolution from all exp.)
± 3-5 % at LHC energy

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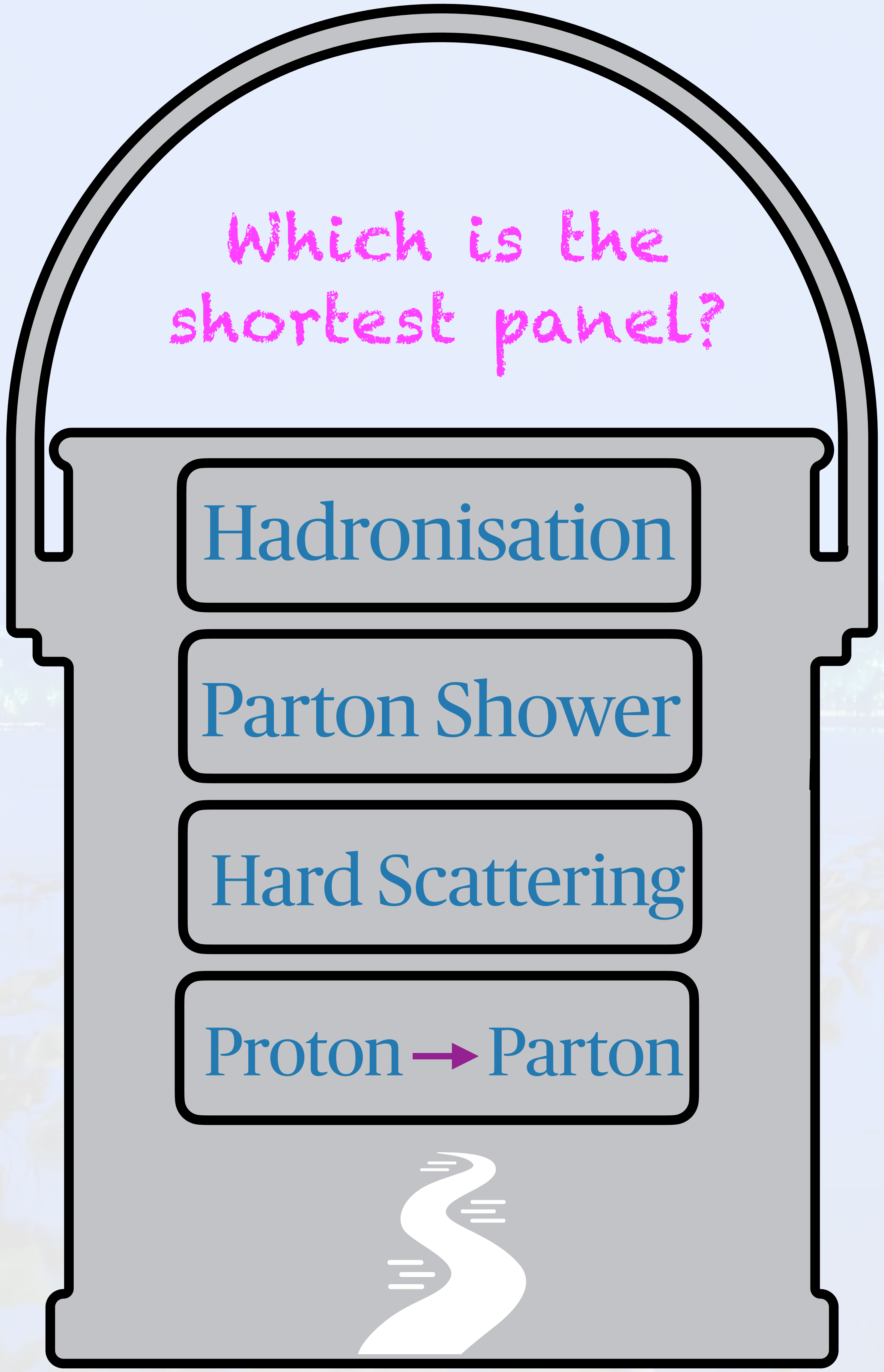
(Perturbative quantum field theory)
± 1~3 % level!

non-perturbative effects
(Fragmentation, hadronisation)
± Λ QCD/ $\sqrt{\hat{s}}$

The Bucket Effect in $pp \to H + X$



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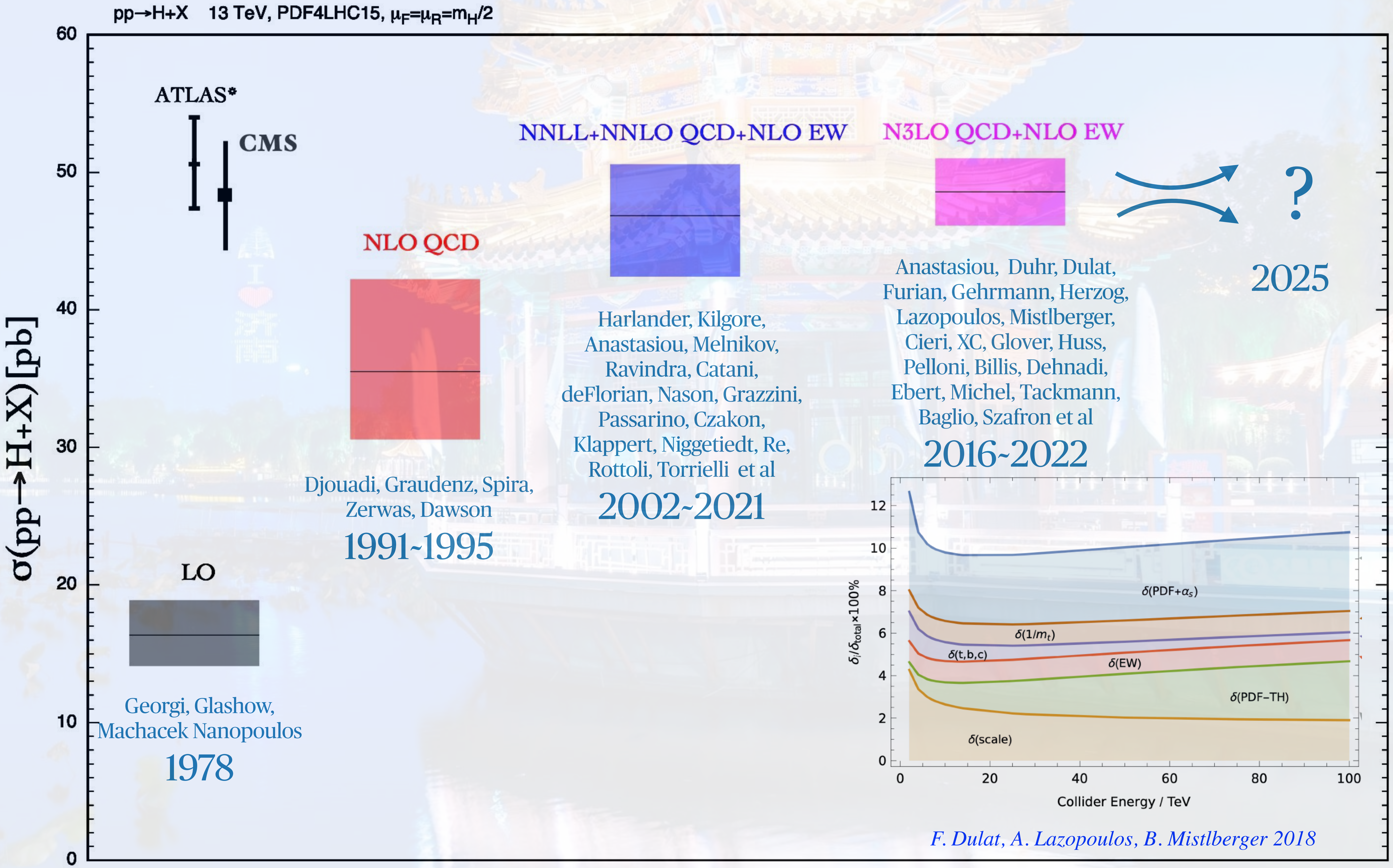

Slide based on M. Grazzini's talk @ Higgs10

ANATOMY OF HARD SCATTERING $\hat{\sigma}_{ab}$

➤Building blocks from perturbative QFT

$$\hat{\sigma} = \alpha_s(\hat{\sigma}^B) + \alpha_s^2(\hat{\sigma}^R + \hat{\sigma}^V) + \alpha_s^3(\hat{\sigma}^{RR} + \hat{\sigma}^{RV} + \hat{\sigma}^{VV}) + \alpha_s^4(\hat{\sigma}^{RRR} + \hat{\sigma}^{RRV} + \hat{\sigma}^{RVV} + \hat{\sigma}^{VVV}) + \mathcal{O}(\alpha_s^5)$$

$\hat{\sigma}_{LO}$ $\quad$ $\hat{\sigma}_{NLO}$ $\quad$ $\hat{\sigma}_{NNLO}$ $\quad$ $\hat{\sigma}_{N^3LO}$ $\quad$ …

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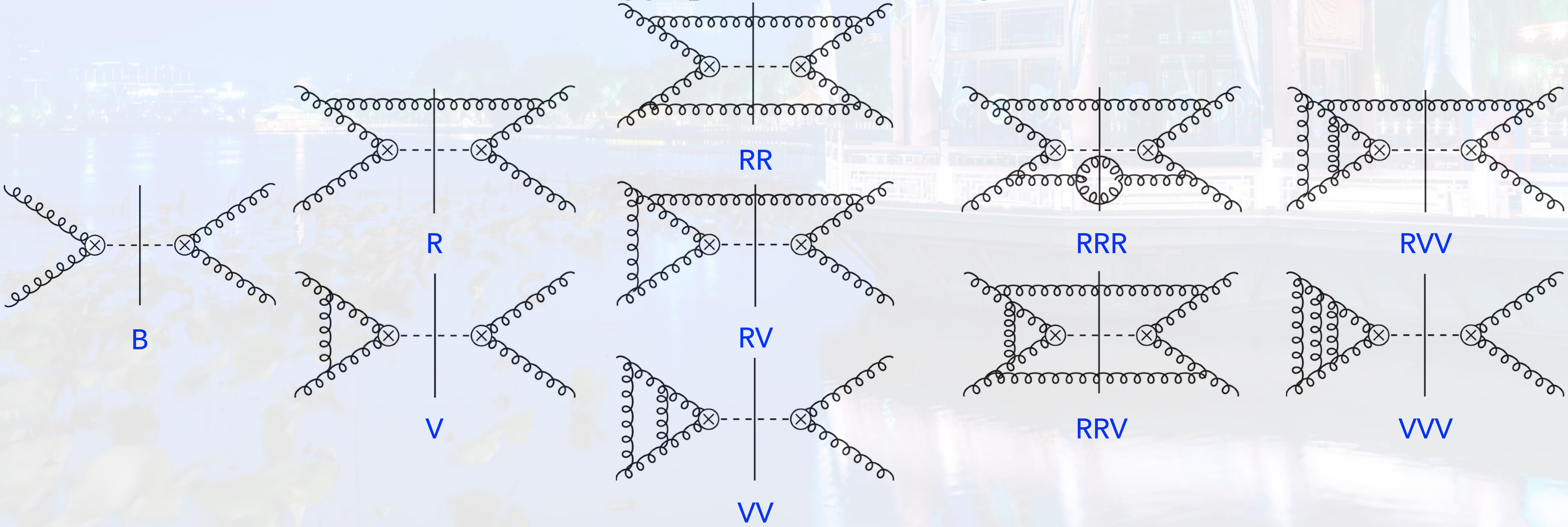
Matrix elements for Higgs production from gluon fusion (in HTL)



ANATOMY OF DIFFERENTIAL CROSS SECTIONS $d\hat{\sigma}_{ab}$

$$d\hat{\sigma}_{NLO} = \int_{d\Phi_{N+1}} (d\hat{\sigma}^{R}_{NLO} - d\hat{\sigma}^{S}_{NLO}) + \int_{d\Phi_{N}} (d\hat{\sigma}^{V}_{NLO} - d\hat{\sigma}^{T}_{NLO})$$

$$0 = \int_{d\Phi_{N+1}} d\hat{\sigma}^{S}_{NLO} + \int_{d\Phi_{N}} d\hat{\sigma}^{T}_{NLO}$$

- Method to control IR divergence?
  - Subtraction of IR divergence

    NLO: ***di-pole*** (S. Catani, M.H. Seymour 1996), ***FKS*** (S. Frixione, Z. Kunszt, A. Signer 1995), ***Nagy-Soper*** (2003)

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  - Slice IR sensitive phase space region (applicable to all orders)

    qT slicing (G. Bozzi, S. Catani, D. De Florian, M. Grazzini et al. 2006-07), ***N-jettiness slicing*** (R. Boughezal, X. Liu, F. Petriello et al. 2015)
  - ***Projection to Born***

    (M. Cacciari, F. A. Dreyer, A. Karlberg, G. P. Salam, G. Zanderighi 2015)
  - Amplitude level removal

    Loop-Tree Duality (I. Bierenbaum, S. Catani, P. Draggiotis et al. 2008-10, R. Runkel, Z. Szor et al. 2019, Z. Capatti, V. Hirschi et al. 2019-20, J. J. Aguilera-Verdugo et al. 2020) ***Universal factorisation*** (C. Anastasiou, R. Haindl, G. Sterman, Z. Yang, M. Zeng 2018-20)

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Perturbative QCD @ NNLO

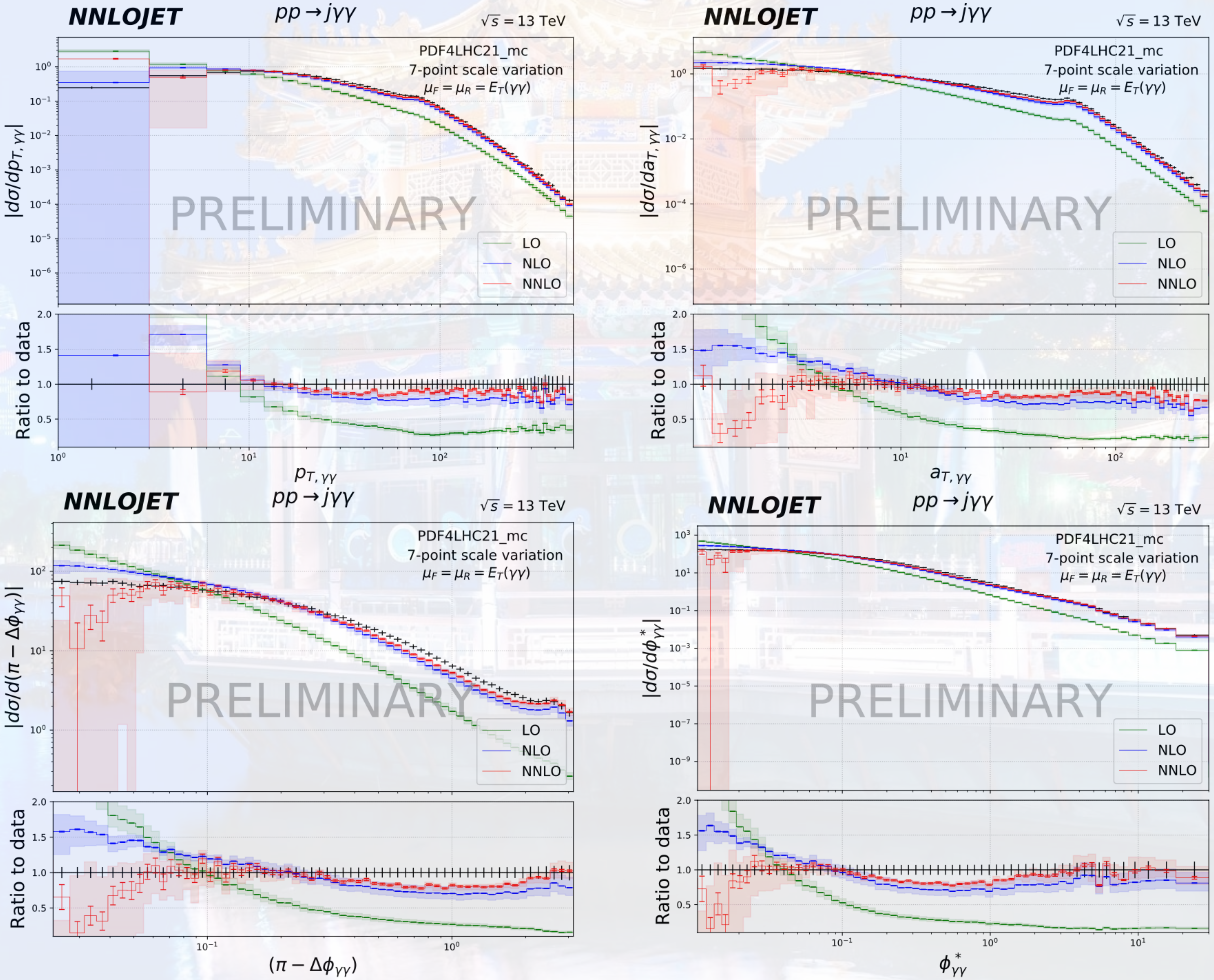
$pp \to \gamma\gamma + \text{jet}$ @NNLO QCD

- Fully differential predictions (α_S^3)
  - $2 \to 3$ ME @ 2-loop.
    B. Agarwal, F. Buccioni et. al. *PRL. 127, 262001 (2021)*
  - $2 \to 4$ ME @ 1-loop from OpenLoops2
    F. Buccioni, J.-N. Lang, J. M. Lindert et. al. *EPJC 79, 866 (2019)*
  - Loop induced contribution @ NLO (α_S^4)
    S. Badger, T. Gehrmann et. al. *Phys.Lett.B 824 (2022)*
  - Antenna subtraction for IR divergence cancellation within each event
  - Reliable fixed order prediction for $p_{T,\gamma\gamma} > 5$ GeV
  - Scale uncertainty reduced by > 50%
  - Improved agreement with ATLAS 13 TeV data.
    ATLAS collaboration *JHEP 11 (2021) 169*

$$a_T = \frac{p_{x,\gamma_1} p_{y,\gamma_2} - p_{y,\gamma_1} p_{x,\gamma_2}}{|\vec{p}_{T,\gamma_1} - \vec{p}_{T,\gamma_2}|},$$

$$\phi^*_\eta = \tan\frac{\pi - \Delta\phi_{\gamma\gamma}}{2}\sqrt{1 - \tanh^2(\Delta\eta_{\gamma\gamma}/2)},$$

$$\phi_{\rm acop} = \pi - \Delta\phi_{\gamma\gamma}.$$



F. Buccioni, XC, W. Feng, T. Gehrmann, A. Huss, M. Marcoli in preparation
Also in H. A. Chawdhry, M. Czakon, A. Mitov, R. Poncelet JHEP 09 (2021) 093

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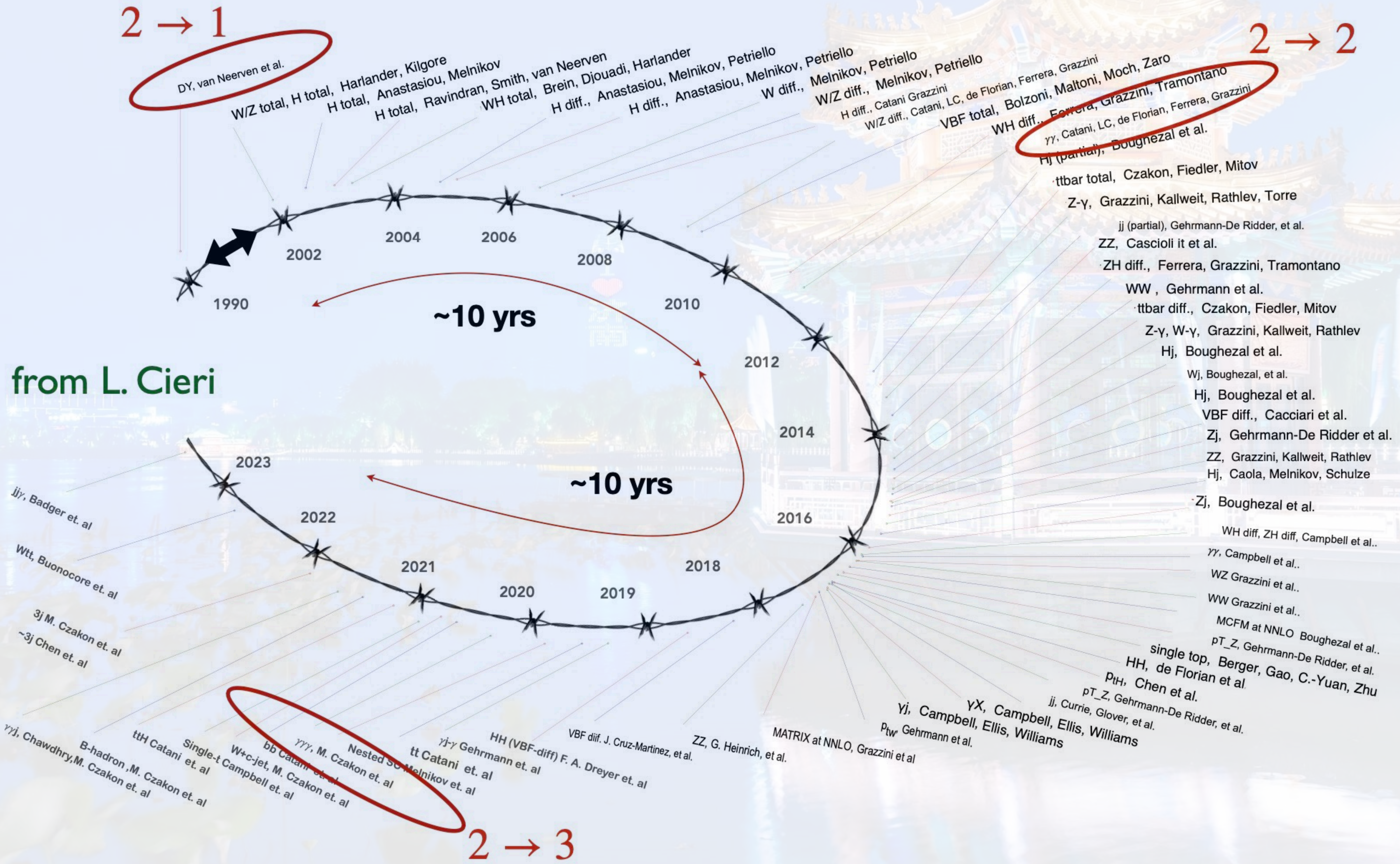

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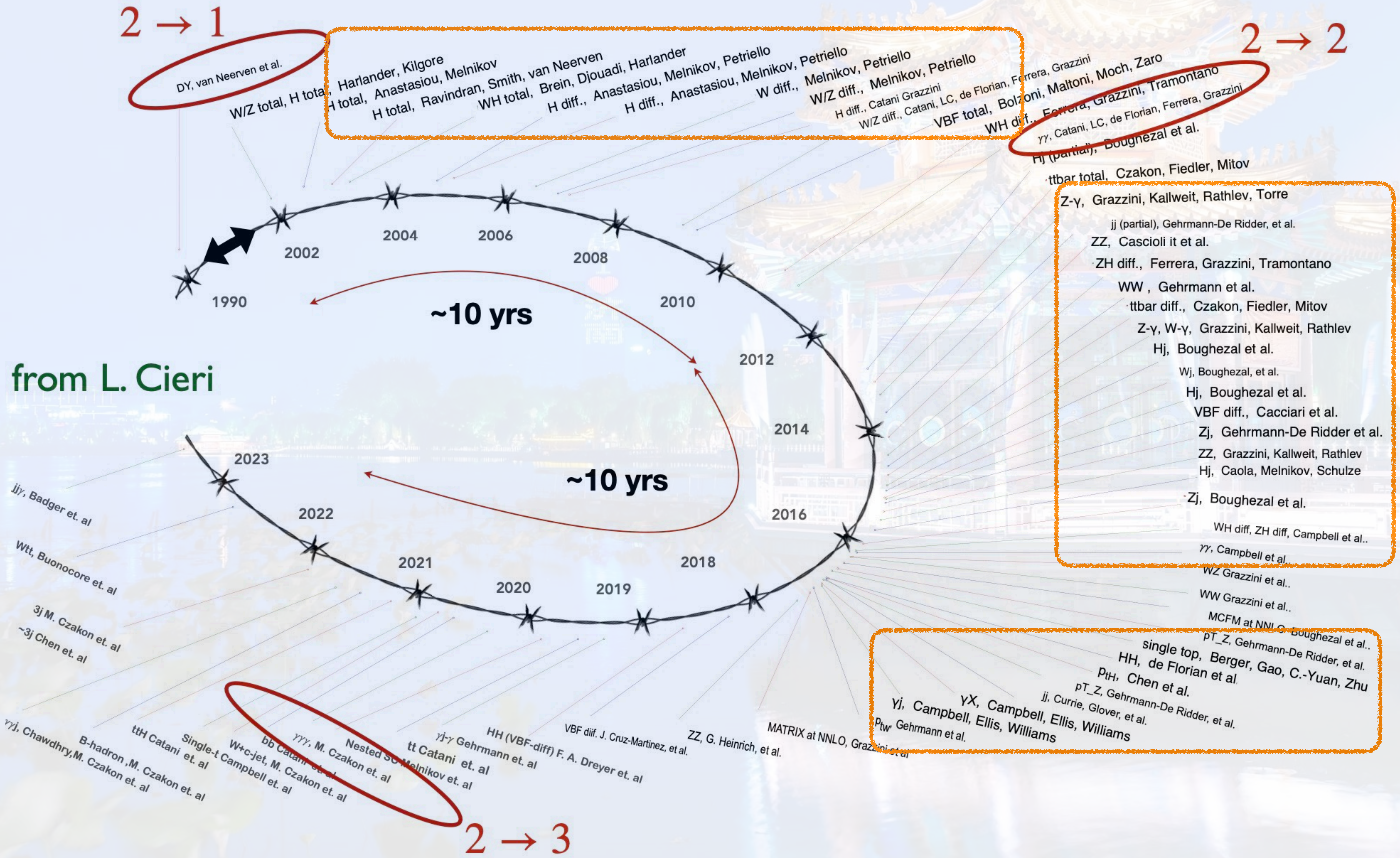


from L. Cieri

Perturbative QCD @ NNLO

SciPost Physics Codebases — Submission

NNLOJET: a parton-level event generator for jet cross sections at NNLO QCD accuracy

NNLOJET Collaboration

A. Huss[1,⋆], L. Bonino[2], O. Braun-White[3], S. Caletti[4], X. Chen[5], J. Cruz-Martinez[1], J. Currie[3], R. Gauld[6], W. Feng[2], E. Fox[3], G. Fontana[2], A. Gehrmann-De Ridder[2,4], T. Gehrmann[2], E.W.N. Glover[3], M. Höfer[7], P. Jakubčik[2], M. Jaquier[8], M. Löchner[2], F. Lorkowski[2], I. Majer[4], M. Marcoli[3], J. Mo[2], T. Morgan[3], J. Niehues[3,9], J. Pires[10], C. Preuss[11], A. Rodriguez Garcia[4], K. Schönwald[2], V. Sotnikov[2], R. Schürmann[2], G. Stagnitto[12], D. Walker[3], S. Wells[3], J. Whitehead[13], T.Z. Yang[2] and H. Zhang[8]

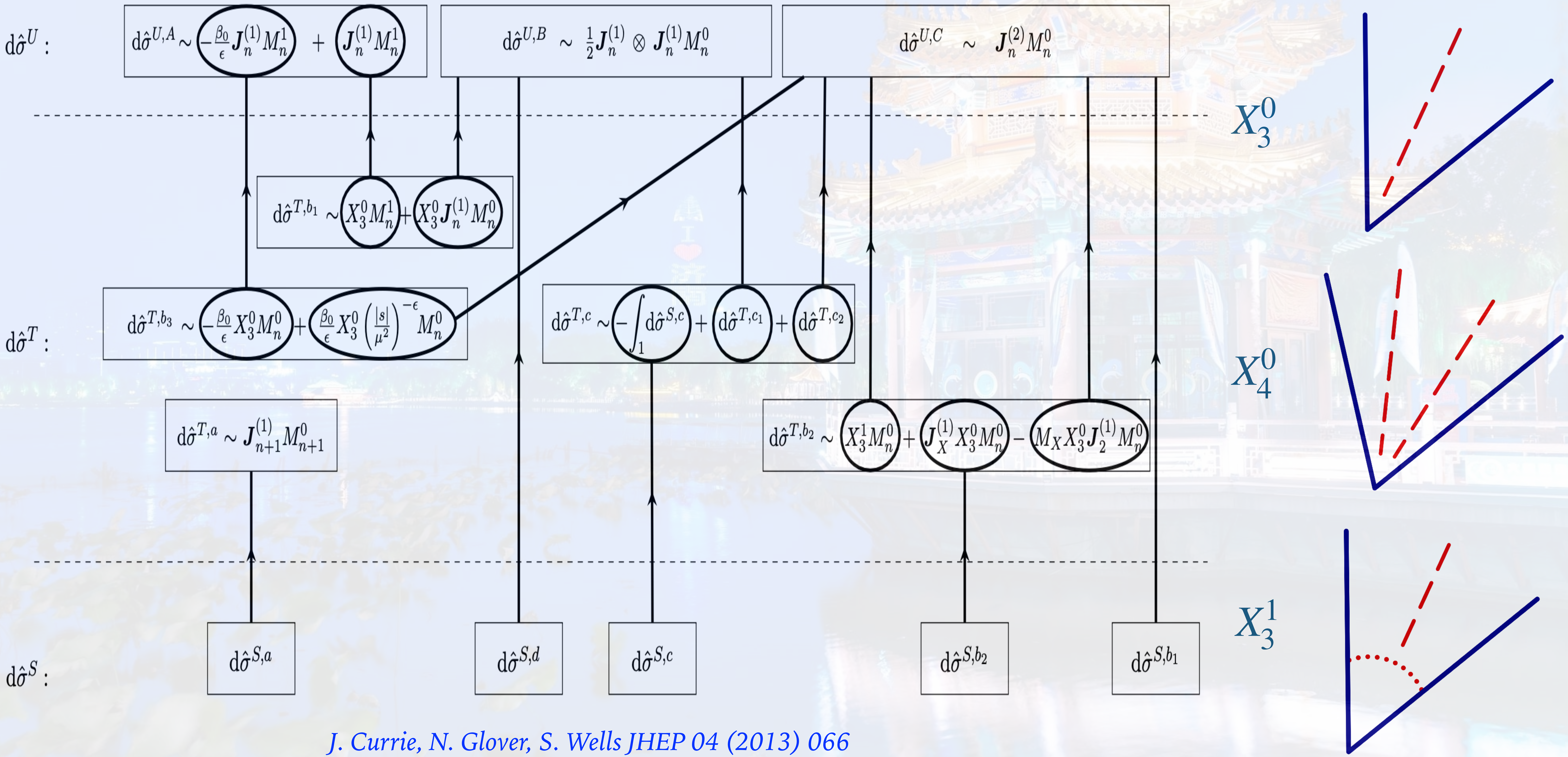
Abstract

The antenna subtraction method for NNLO QCD calculations is implemented in the NNLOJET parton-level event generator code to compute jet cross sections and related observables in electron-positron, electron-proton and proton-proton collisions. We describe the publicly available NNLOJET code and its usage and document several reference applications.

ANTENNA SUBTRACTION @ NNLO

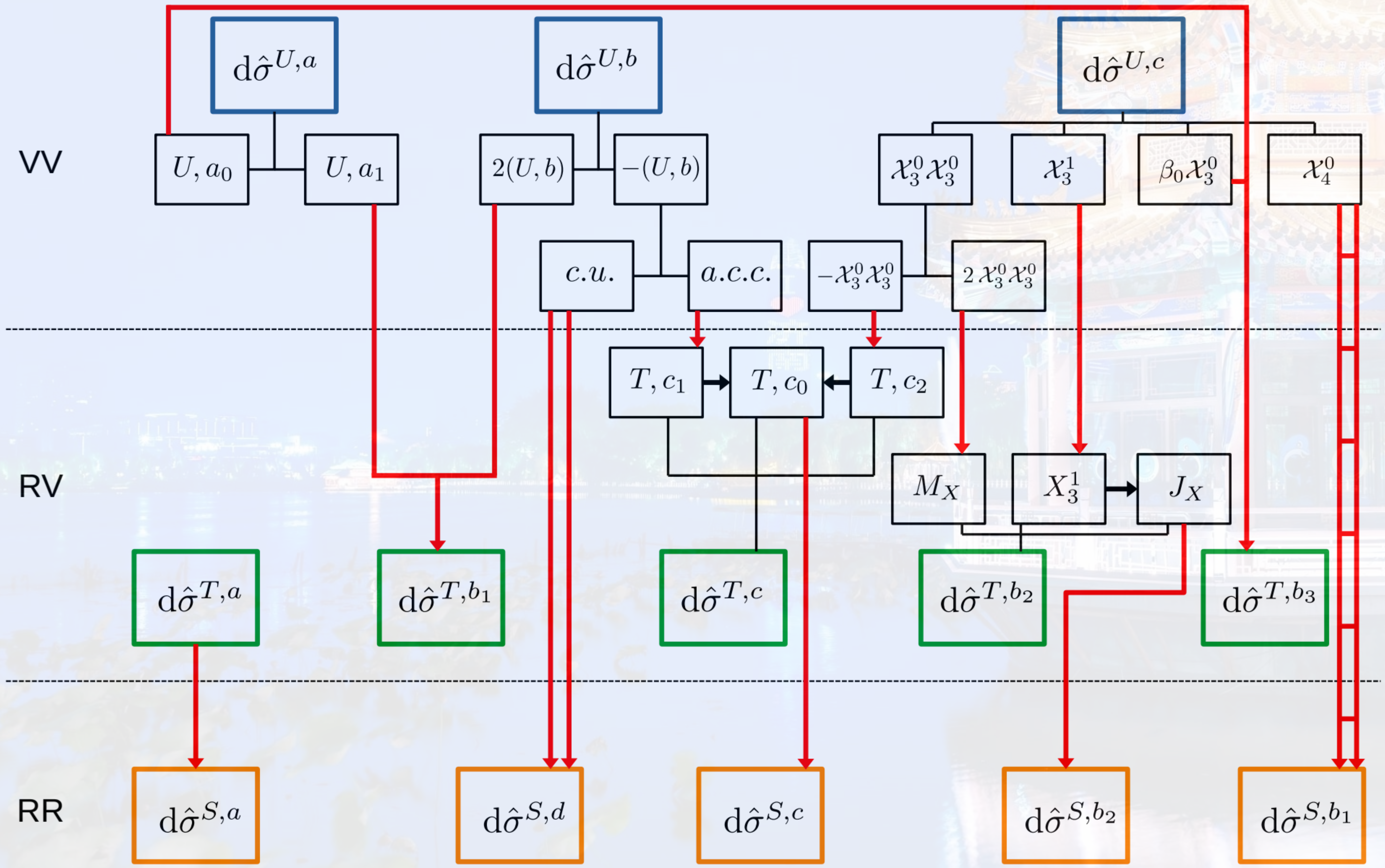
	Feature	Reference
Traditional (NNLOJET v.1.0)	• Built from scattering matrix element (di-pole) • All collider types available: LHC, LEP, DIS etc. • Most 2 to 2 scattering and crossing available • Branch to Trunk and Trunk to Branch algorithms ✳ Partially automated (LHC 3 jets production) ✳ (Massive/heavy quark) ✳ (Identified hadron, photon) $X_3^0(i,j,k) = \frac{\lvert \mathcal{M}_3^0(i,j,k)\rvert^2}{\lvert \mathcal{M}_2^0(\tilde{I},\tilde{K})\rvert^2}$ $X_4^0(i,j,k,l)\lvert = \frac{\lvert \mathcal{M}_4^0(i,j,k,l)\rvert^2}{\lvert \mathcal{M}_2^0(\widetilde{ijk},\widetilde{jkl})\rvert^2}$	• Original: hep-ph/0505111 hep-ph/0612257 0710.0346 0912.0374 1006.1849 1112.3613 1211.2710 1301.4693 • Massive: 0904.3297 1102.2443 1105.0530 1112.4736 1207.6546 1309.6887 1409.3124 • Identified: 2201.06982 2208.02650 2406.09925 • Automation: 2203.13531 2310.19757
Idealised	• Built from desired IR limits • Based on two hard partons (di-pole) • Currently only final state partons for NNLO	2302.12787 2307.14999 2308.10829
Generalised	• Built from desired IR limits • Based on three hard partons (tri-pole) • Currently only final state partons for NNLO • Branch to Trunk algorithms and easy to automate • Efficiency boost at x5 ~ x10	2410.12904

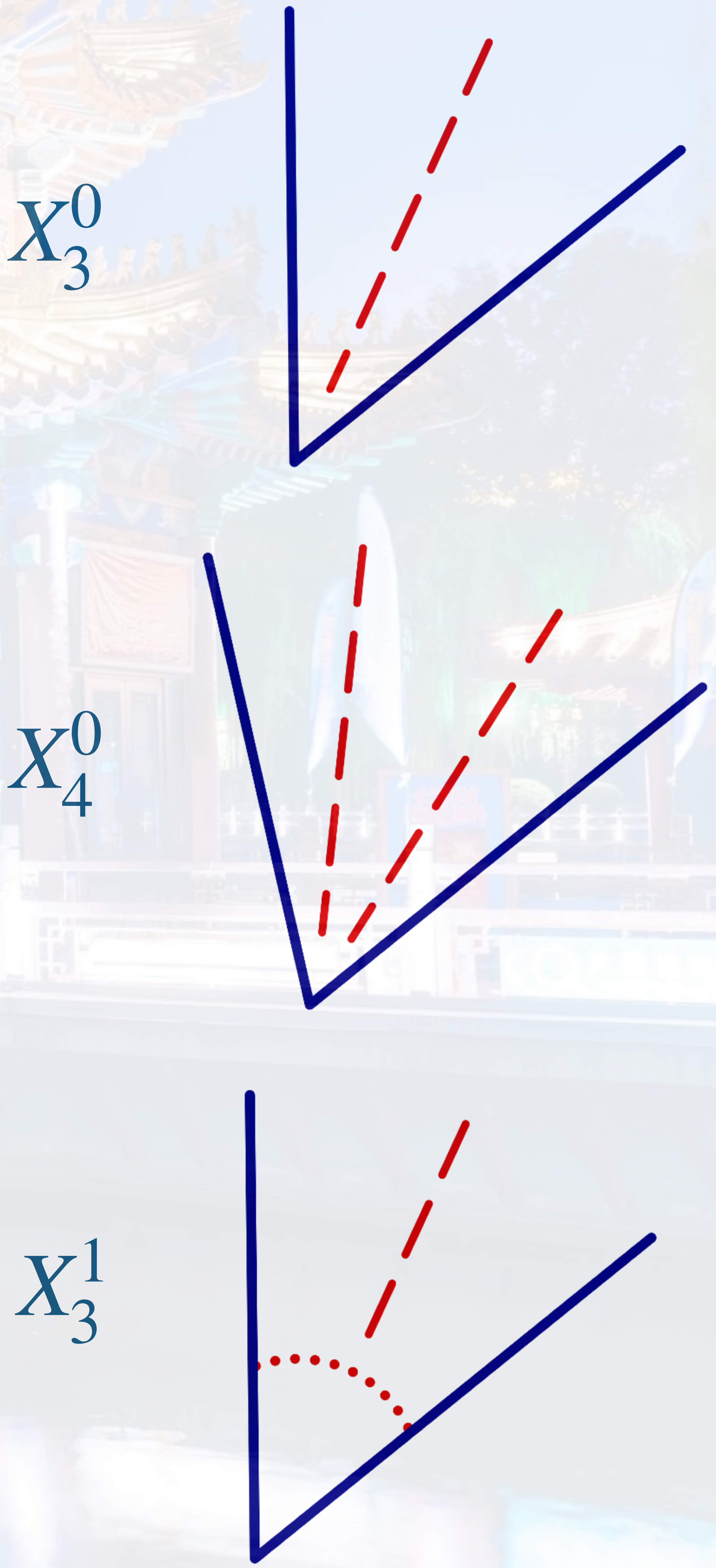
ANTENNA SUBTRACTION @ NNLO



J. Currie, N. Glover, S. Wells JHEP 04 (2013) 066

ANTENNA SUBTRACTION @ NNLO

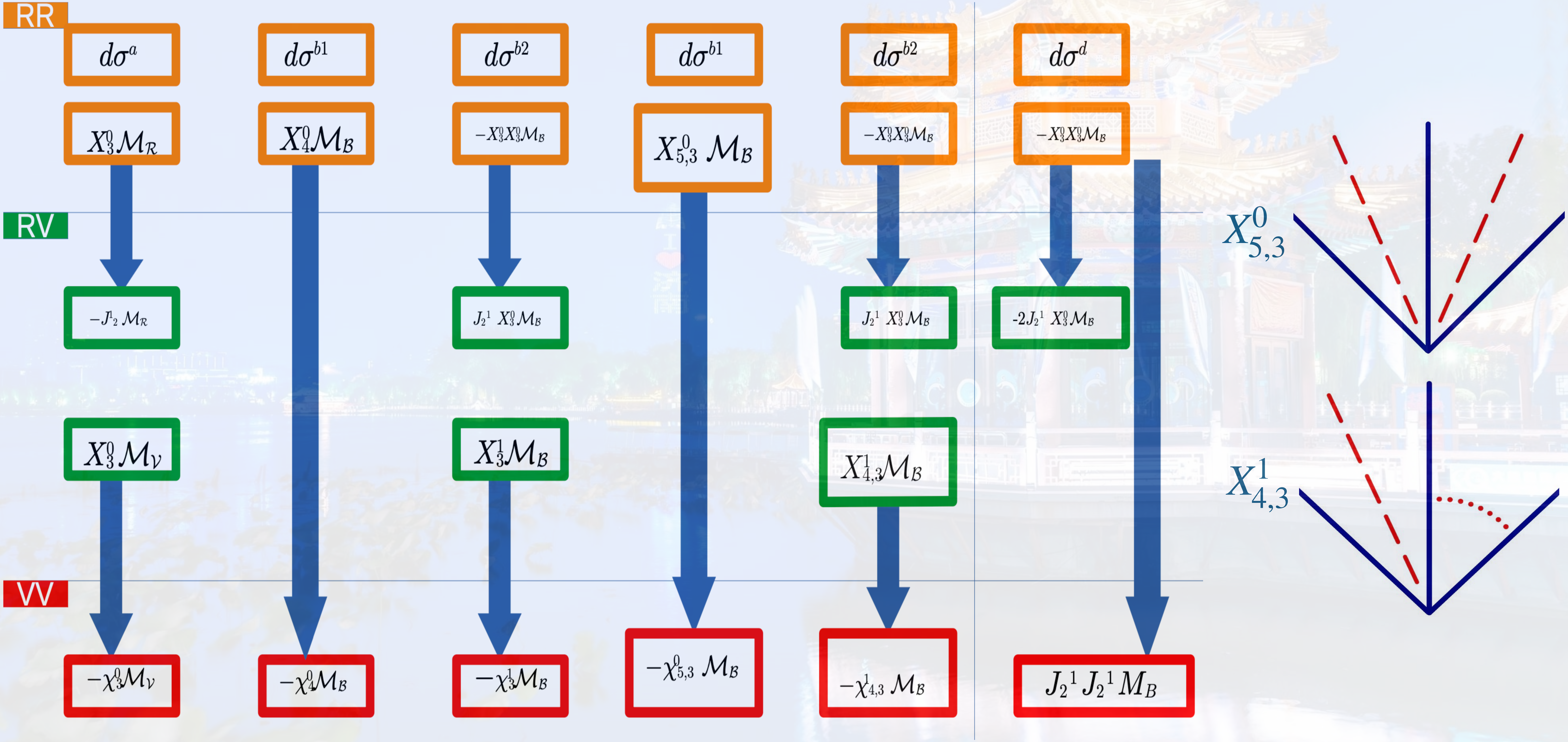




XC, T. Gehrmann, N. Glover, A. Huss, M. Marcoli JHEP 10 (2022) 099

ANTENNA SUBTRACTION @ NNLO

Slide from Elliot @ HP2



E. Fox, N. Glover, M. Marcoli 2410.12904

ANTENNA SUBTRACTION @ NNLO

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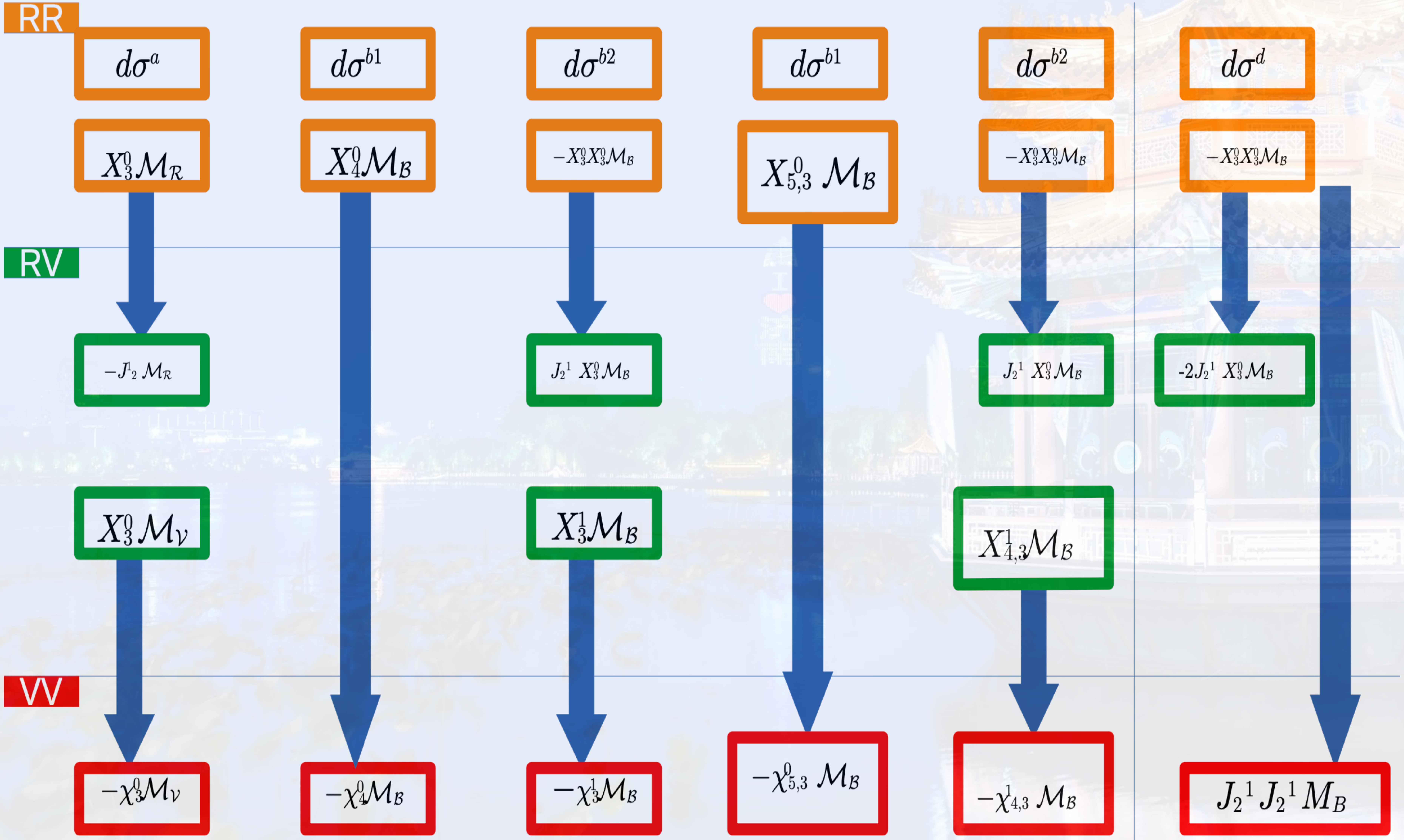

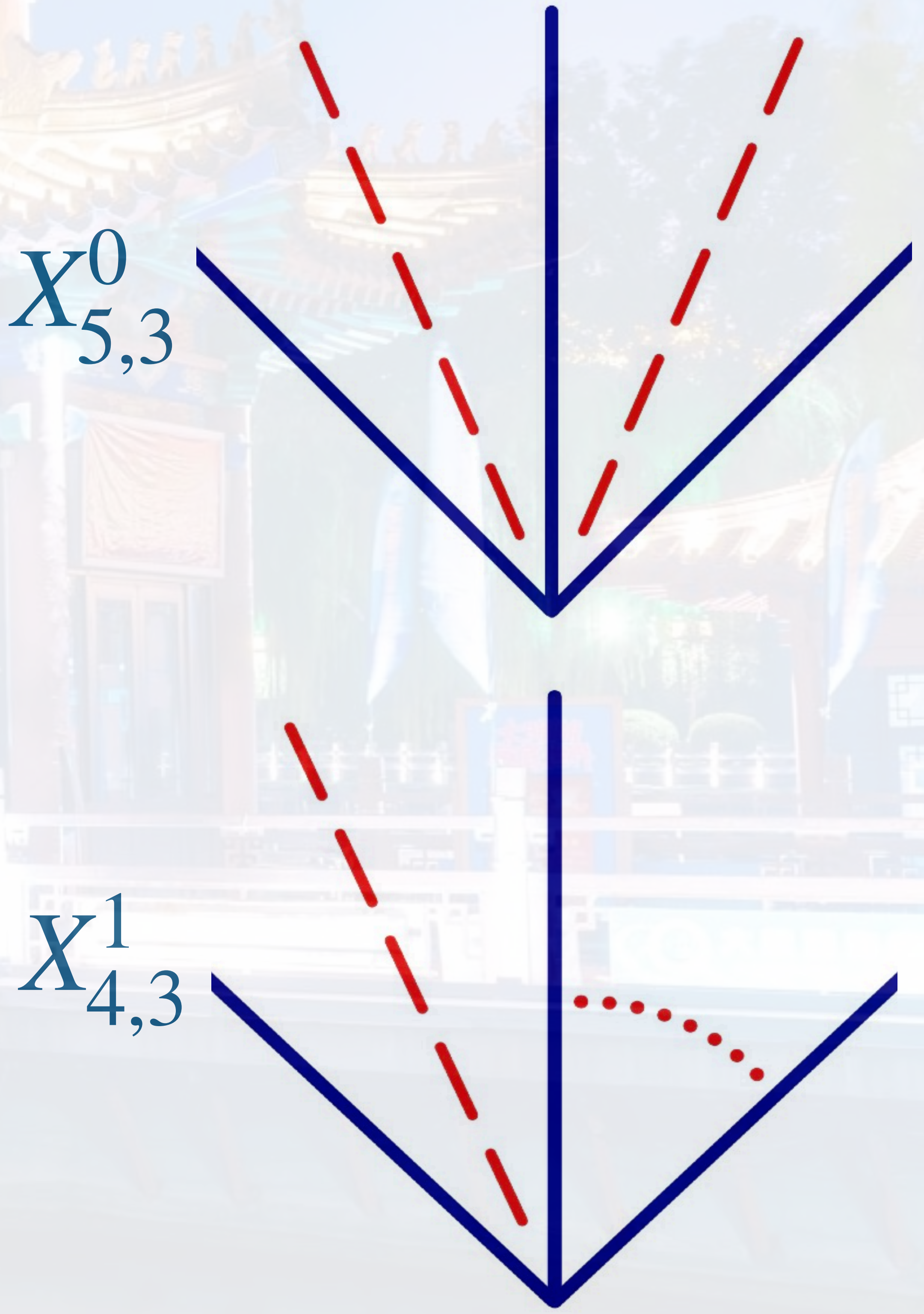

$e^+e^- \to$ 3 jets @ NNLO

Efficiency boost by a factor of 10

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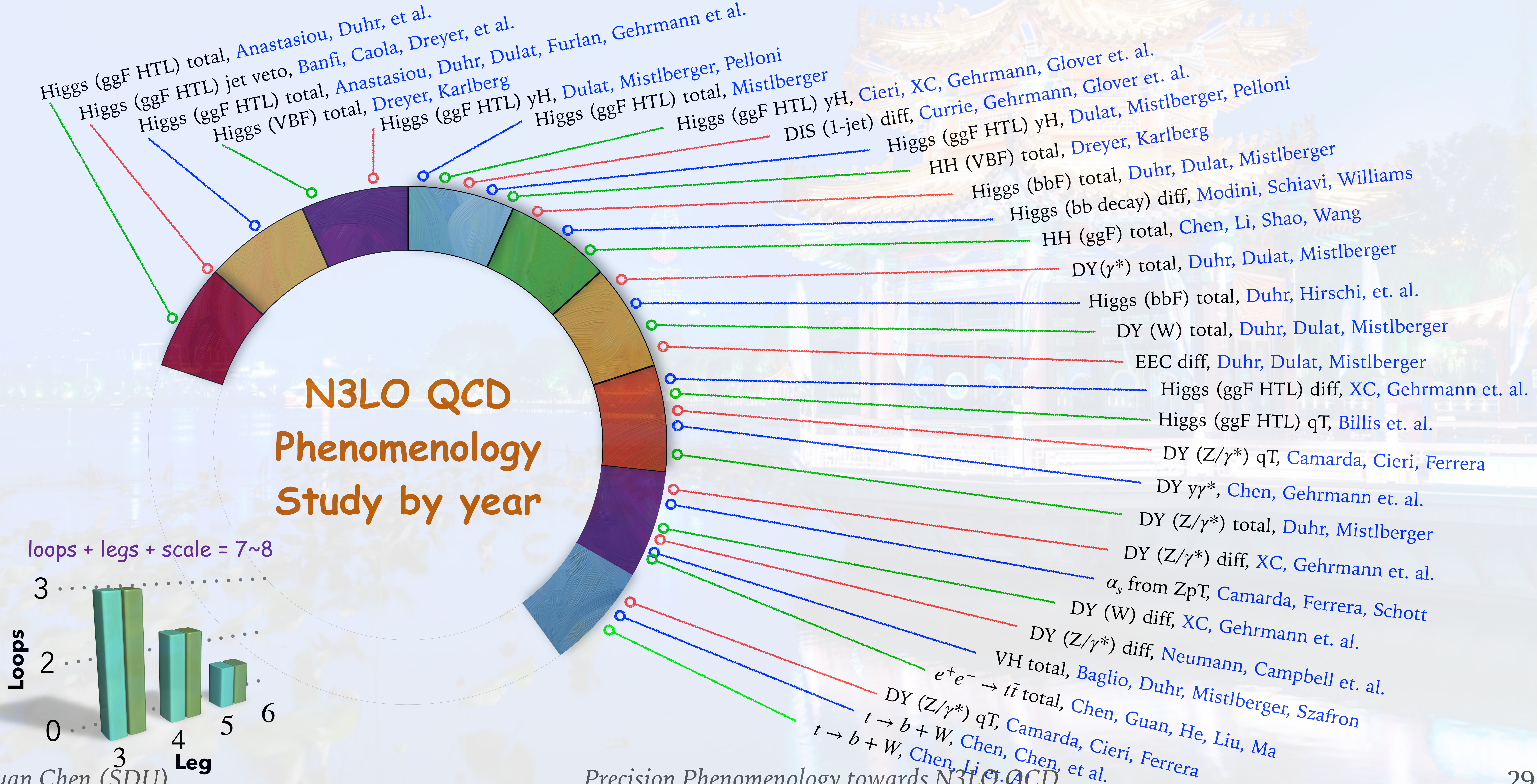
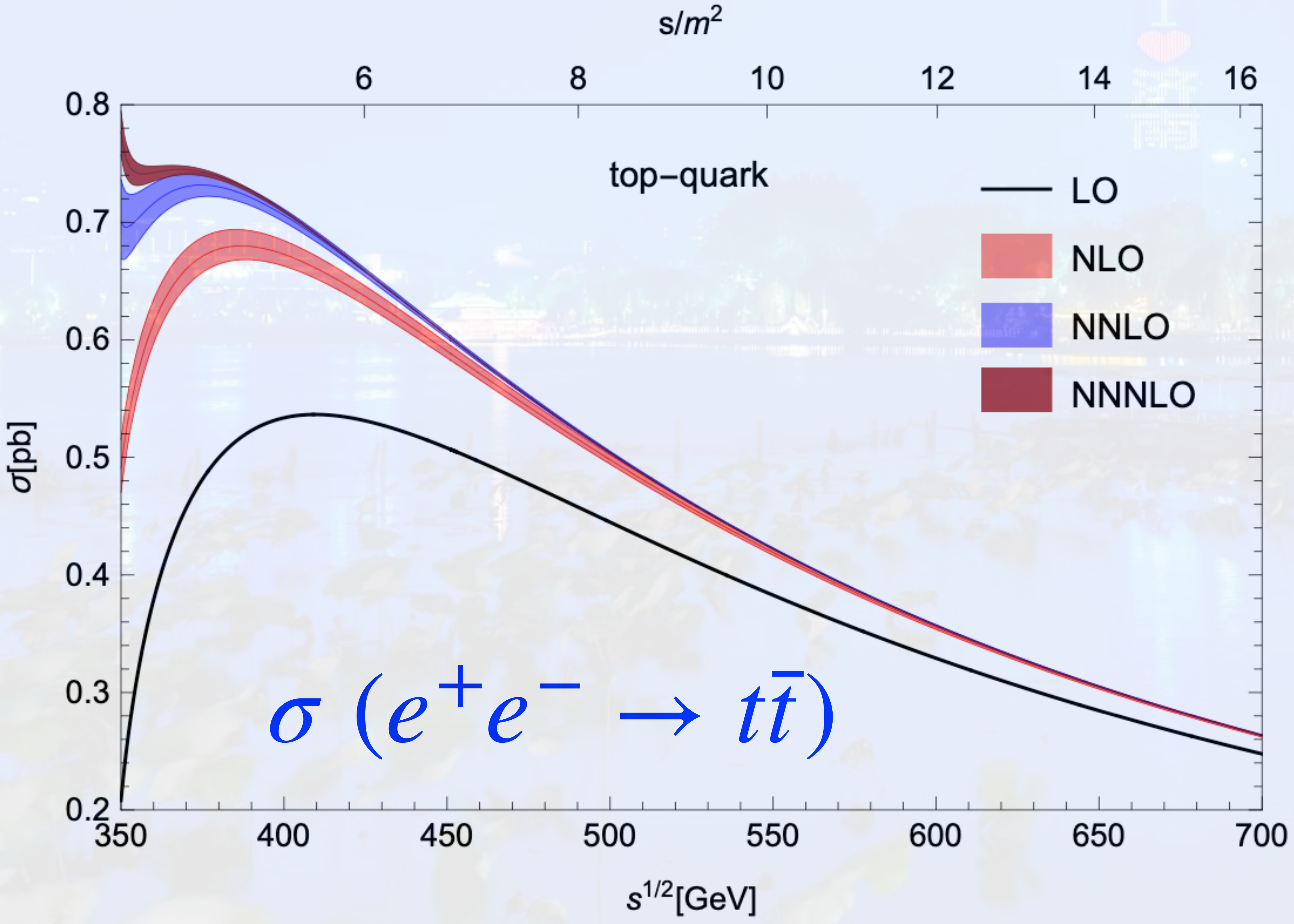
Perturbative QCD @ N3LO



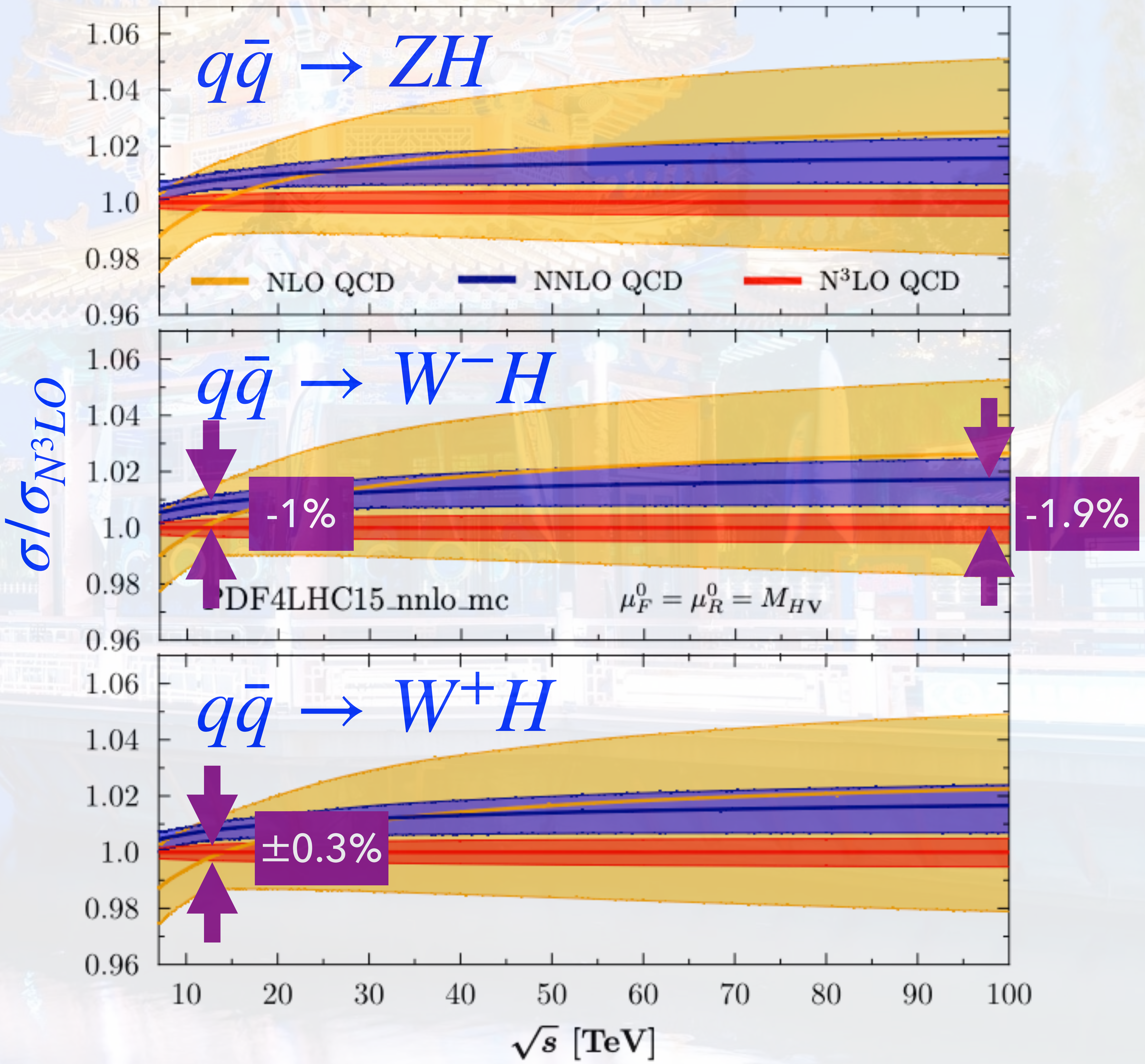
Perturbative QFT for Precision Predictions

$2 \to 2$ @N3LO QCD

- Total cross section for pp and epem collider
  - ME from $2 \to 3$ @ NNLO + ME @ 3-loop.
  - Use reverse unitarity for IR pole cancellation.
  - Different perturbative-series convergent behaviour



X. Chen, X. Guan, C.-Q. He, X. Liu, Y.-Q. Ma PRL. 132, 101901 (2024)



J. Baglio, C. Duhr, B. Mistlberger, R. Szafron JHEP 12 (2022) 066

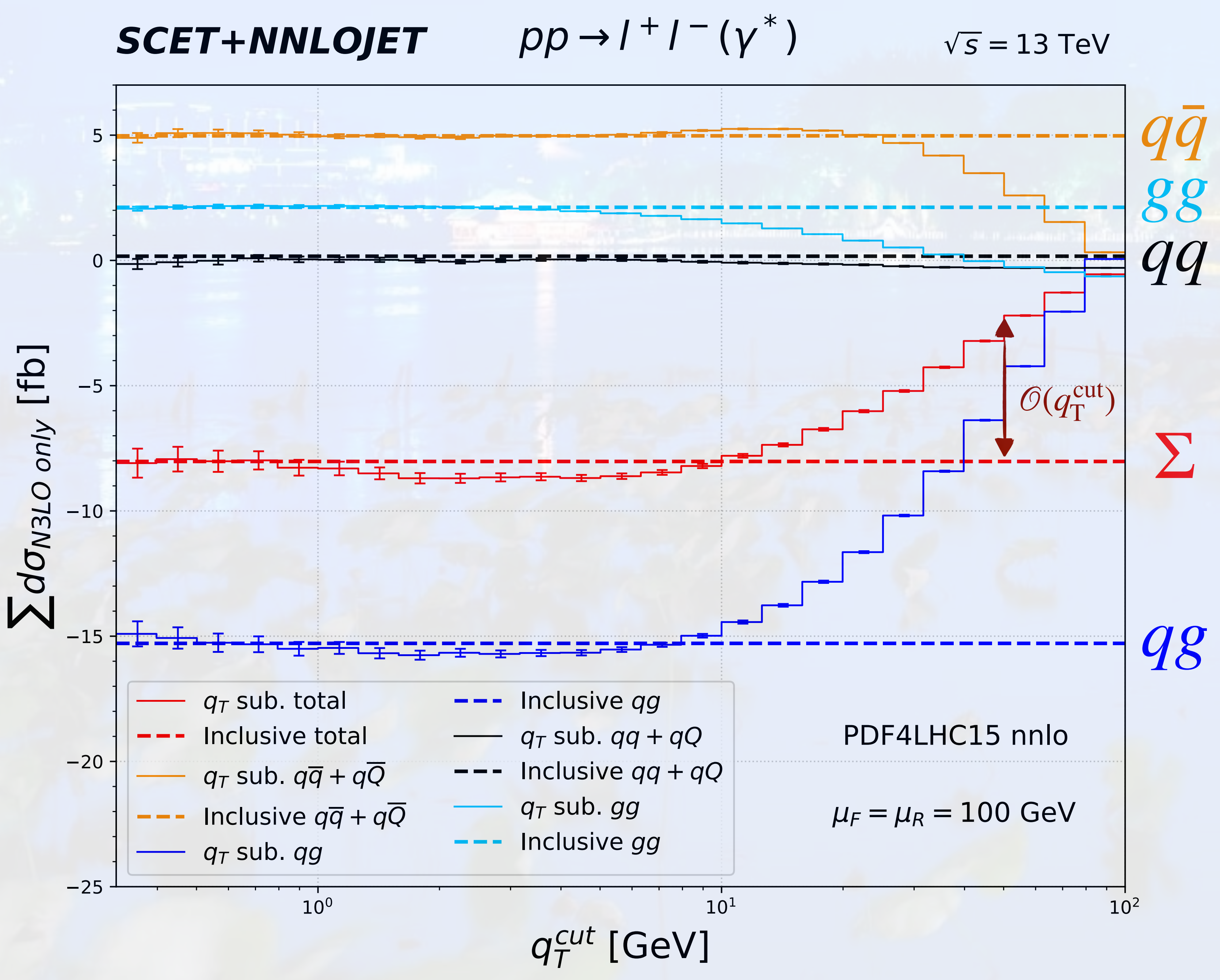
Perturbative QFT for Precision Predictions

$2 \to 1$ @ N3LO (+ N3LL) QCD

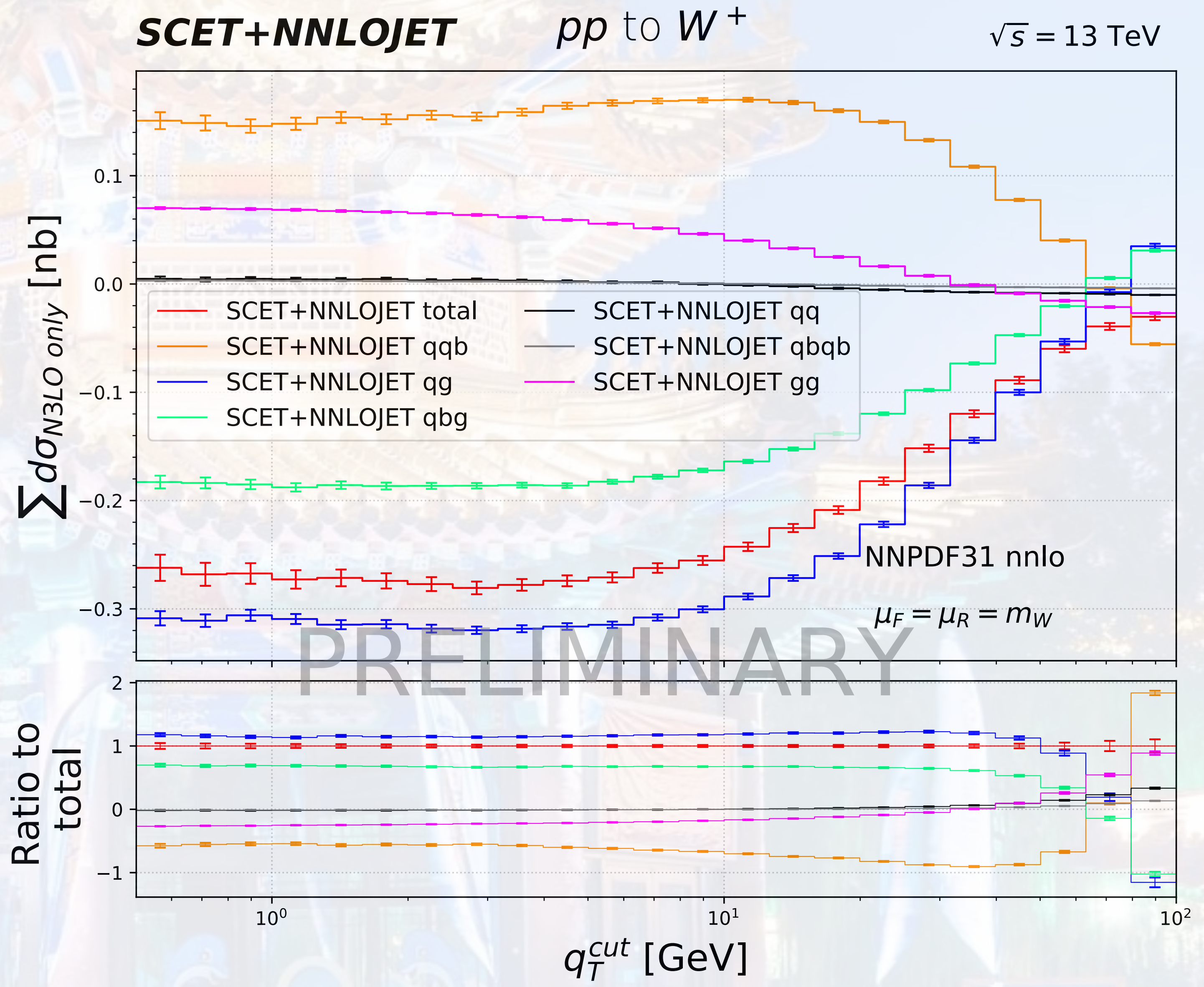
- Fully differential N3LO correction in event generator
  - Recycle $pp \to V + J$ @ NNLO with $\tau_{\rm cut}$ slicing

$$d\sigma^F_{N^kLO} = \mathcal{H}^F_{N^kLO} \otimes d\sigma^F_{LO}\Big|_{\delta(\tau)} + \left[d\sigma^{F+jet}_{N^{k-1}LO} - d\sigma^{F\ CT}_{N^kLO}\right]_{\tau>\tau_{cut}} + \mathcal{O}(\tau^2_{cut}/Q^2)$$

  - Fiducial power correction removed via MC recoil technique.
- Small p_T resummation at N3LL and partial N4LL



XC, T. Gehrmann, et. al. PRL. 128, 052001 (2022)

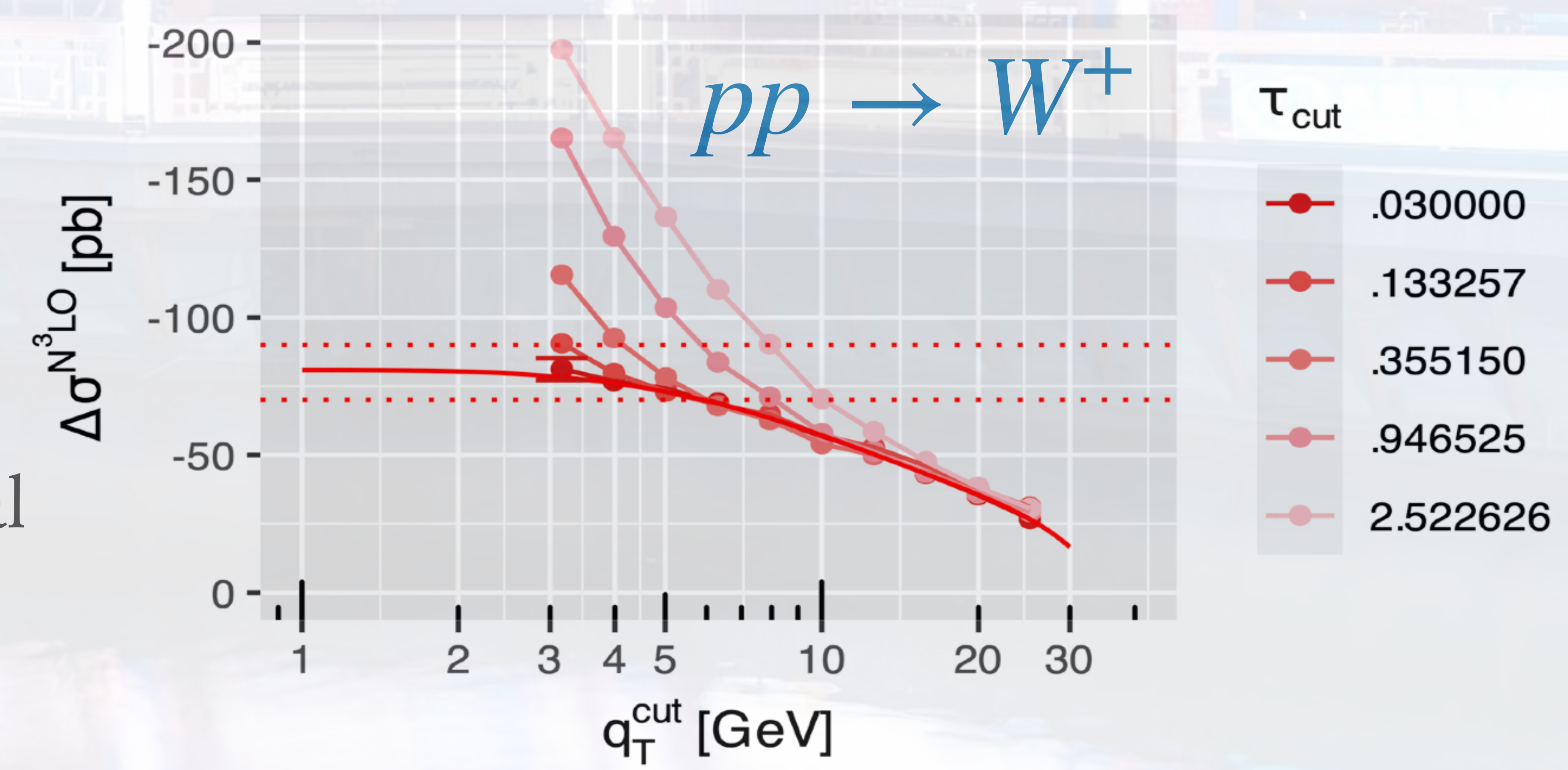


XC, T. Gehrmann, et. al. *Phys.Lett.B 840 (2023)*

- Validation of inclusive total cross section for $q_T^{cut} < 1$ GeV.

  *C. Duhr, F. Dulat, B. Mistlberger. PRL. **125**, 172001 (2020)*

- Separated in parton channels
- Foundation of numerical Monte Carlo setup for differential predictions.



Neumann and Campbell *JHEP 11 (2023) 127*

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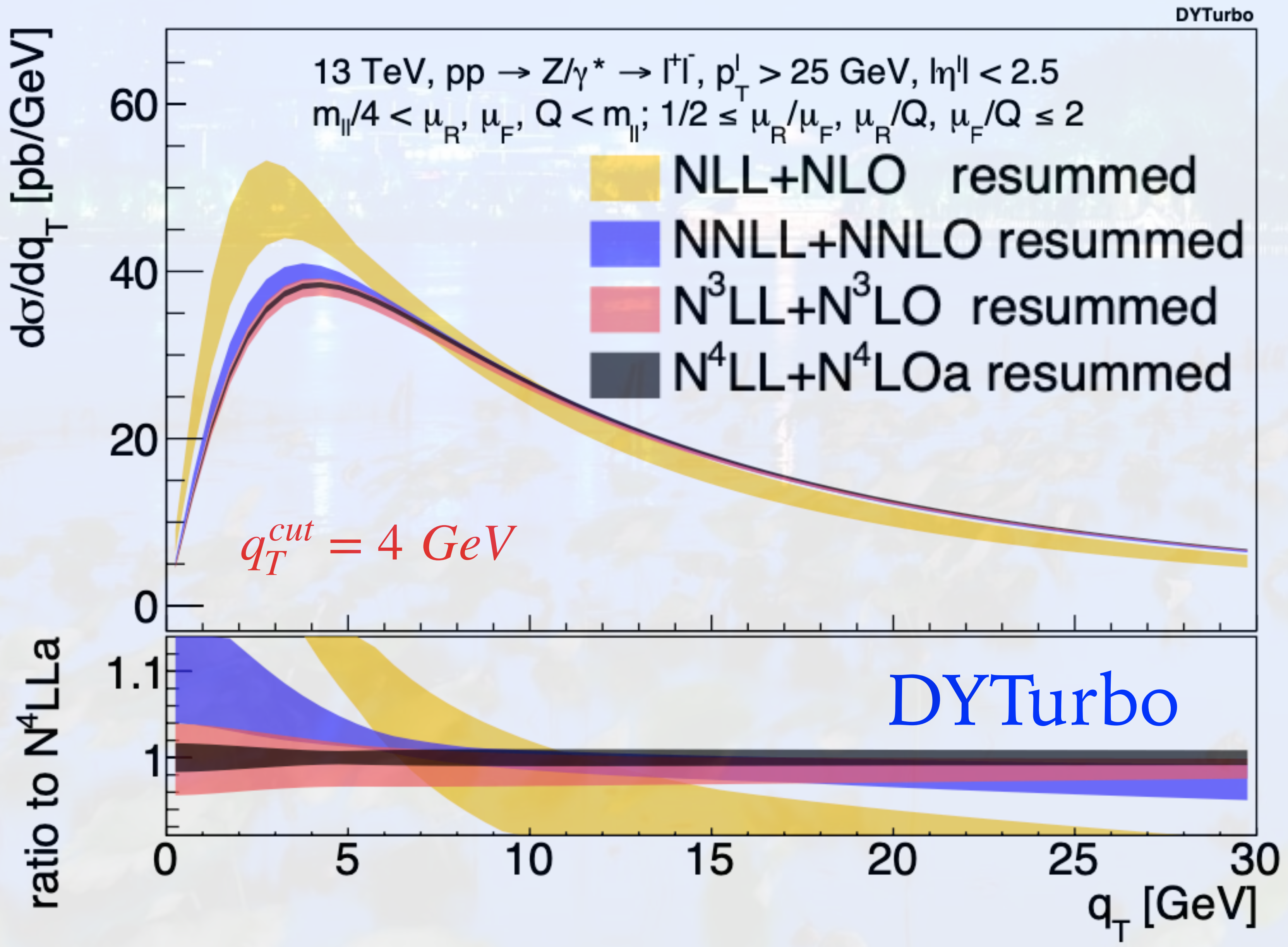
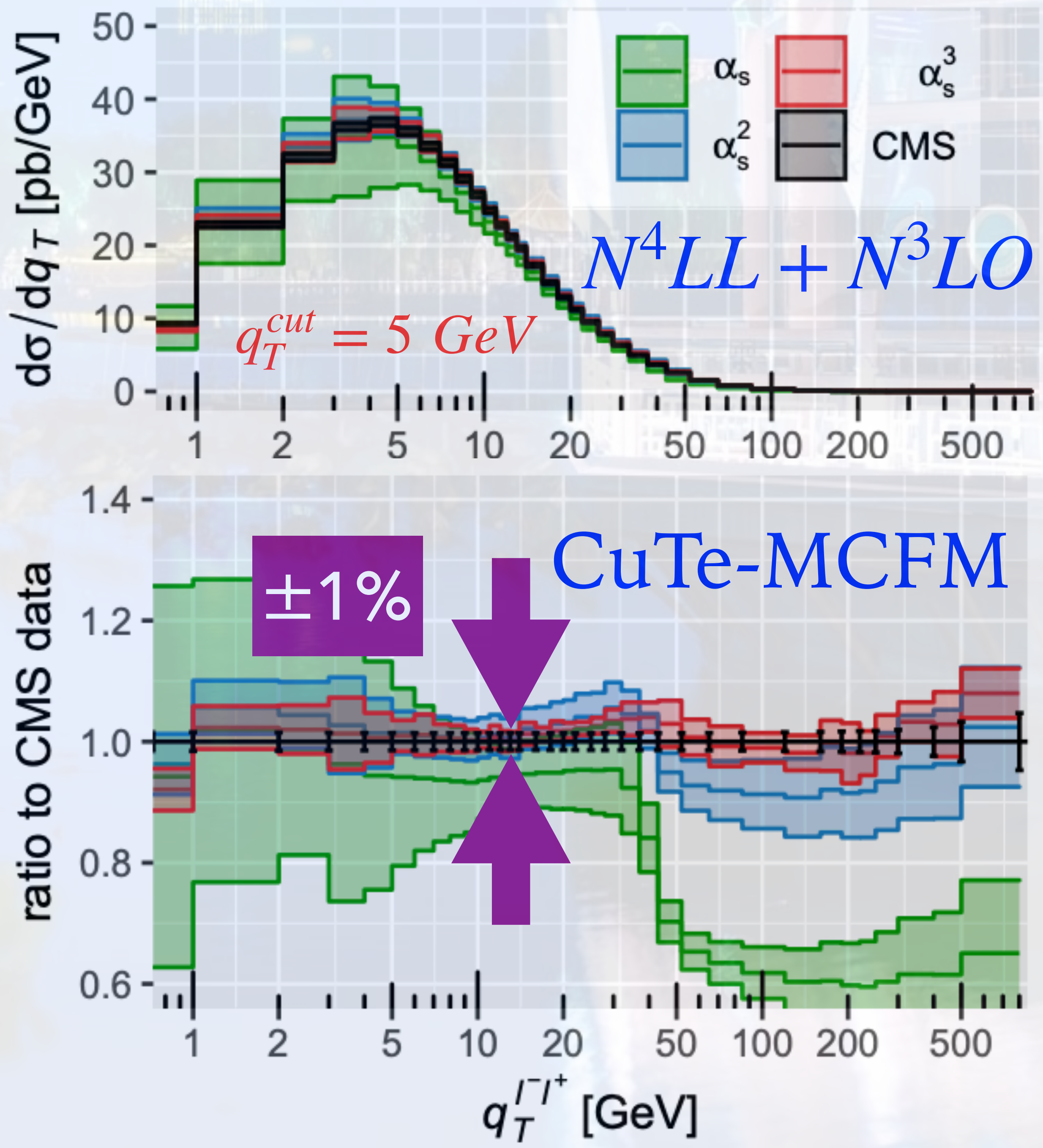
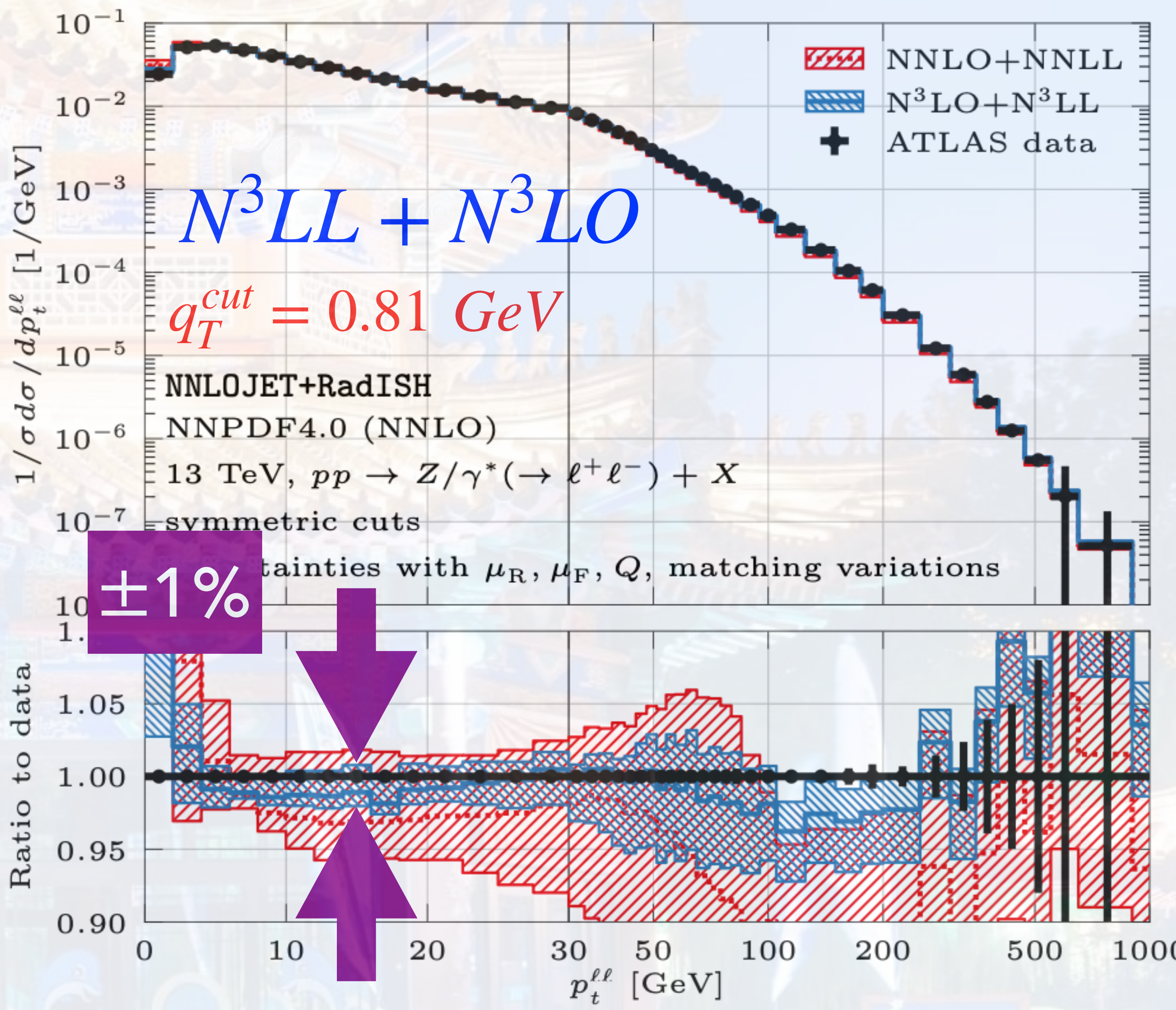
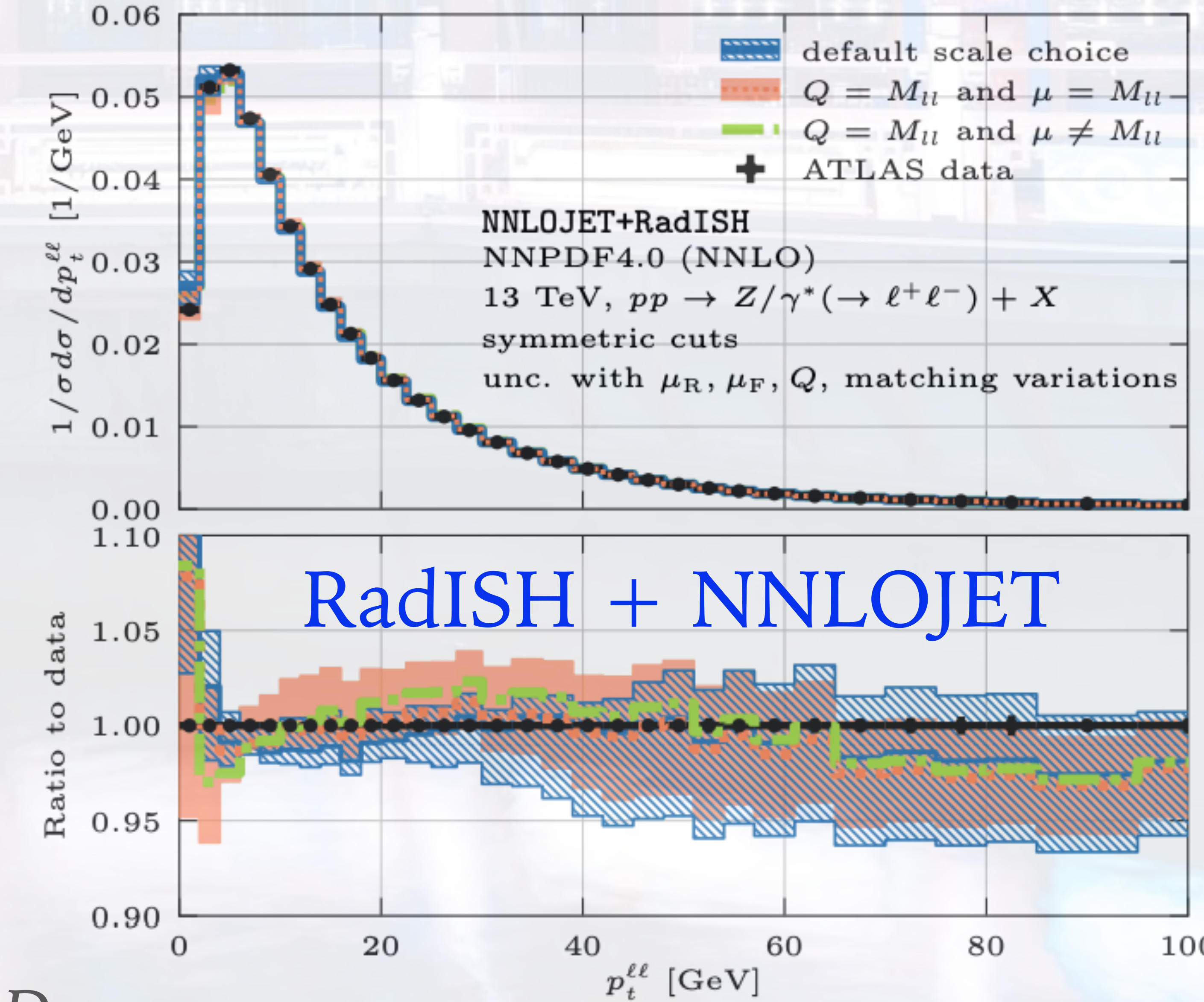








XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)

S. Camarda, L. Cieri, G. Ferrera Phys.Lett.B 845 (2023)

T. Neumann, J. Campbell PRD 107, L011506 (2023)

ANTENNA SUBTRACTION @ N3LO

Slide from Petr @ LL2024

	NLO	NNLO
RRR	$X_3^0 \tilde{M}^0_{n+2}$	$(X_4^0 - X_3^0 X_3^0)\, \tilde{M}^0_{n+1}$
VRR	$X_3^0 \tilde{M}^0_{n+2}$	$(X_3^1 + X_3^0 X_3^0)\, \tilde{M}^0_{n+1} + X_3^0 \tilde{M}^1_{n+1}$
VVR		$(X_4^0 + X_3^1 + X_3^0 X_3^0)\, \tilde{M}^0_{n+1} + X_3^0 \tilde{M}^1_{n+1}$
VVV		

- reduced matrix elem. $\tilde{M}^{\text{loops}}_{\text{hard partons}}$
- unintegrated antenna $X^{\text{loops}}_{\text{emissions}+2}$
- integrated antenna X

ANTENNA SUBTRACTION @ N3LO

Slide from Petr @ LL2024

	NLO	NNLO	N³LO (a sketch)
RRR	$X_3^0 \tilde{M}^0_{n+2}$	$(X_4^0 - X_3^0 X_3^0)\tilde{M}^0_{n+1}$	$(X_5^0 - X_4^0 X_3^0 - X_3^0 X_4^0 + X_3^0 X_3^0 X_3^0)\tilde{M}^0_n$
VRR	$X_3^0 \tilde{M}^0_{n+2}$	$(X_3^1 + X_3^0 X_3^0)\tilde{M}^0_{n+1}$ $+ X_3^0 \tilde{M}^1_{n+1}$	$(X_4^1 - X_3^1 X_3^0 + X_3^0 X_3^0 X_3^0 + X_3^0 X_4^0)\tilde{M}^0_n$ $+(X_4^0 + X_3^0 X_3^0)\tilde{M}^1_n$
VVR		$(X_4^0 + X_3^1 + X_3^0 X_3^0)\tilde{M}^0_{n+1}$ $+ X_3^0 \tilde{M}^1_{n+1}$	$(X_3^2 + X_3^0 X_3^1 + X_3^1 X_3^0 + X_4^0 X_3^0 + X_3^0 X_3^0 X_3^0)\tilde{M}^0_n$ $+(-X_3^1 + X_3^0 X_3^0)\tilde{M}^1_n$ $+ X_3^0 \tilde{M}^2_n$
VVV	reduced matrix elem. (red); unintegrated antenna (blue); integrated antenna (green)	$\tilde{M}$: loops, hard partons; X: loops, emissions+2	$(X_5^0 + X_4^1 + X_3^2 + X_4^0 X_3^0 + X_3^1 X_3^0 + X_3^0 X_3^0 X_3^0)\tilde{M}^0_n$ $+(X_4^0 + X_3^1 + X_3^0 X_3^0)\tilde{M}^1_n$ $+ X_3^0 \tilde{M}^2_n$

ANTENNA SUBTRACTION @ N3LO

Slide from Petr @ LL2024

	NLO	NNLO
RRR	$X_3^0\, \tilde{M}^0_{n+2}$	$(X_4^0 - X_3^0 X_3^0)\, \tilde{M}^0_{n+1}$
VRR	$X_3^0\, \tilde{M}^0_{n+2}$	$(X_3^1 + X_3^0 X_3^0)\, \tilde{M}^0_{n+1} + X_3^0\, \tilde{M}^1_{n+1}$
VVR		$(X_4^0 + X_3^1 + X_3^0 X_3^0)\, \tilde{M}^0_{n+1} + X_3^0\, \tilde{M}^1_{n+1}$
VVV	reduced matrix elem. (red); unintegrated antenna (blue); integrated antenna (green)	$\tilde{M}^{\text{loops}}_{\text{hard partons}}$; $X^{\text{loops}}_{\text{emissions}+2}$; X

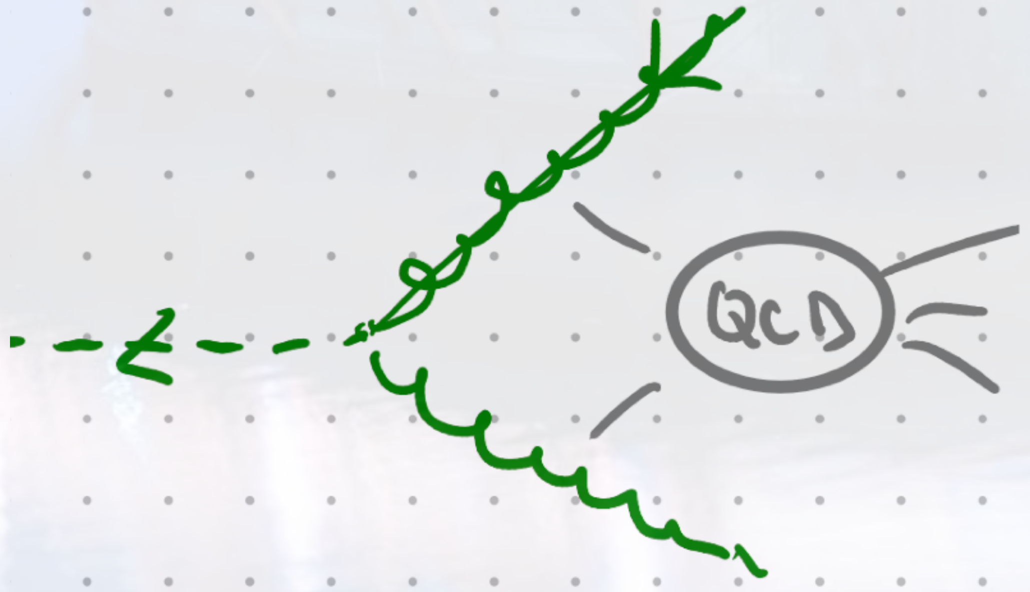
Integration of X_5^0, X_4^1, X_3^2 finished for all final states

$\gamma^* \to q\bar{q}$

P. Jakubcik, M. Marcoli, G. Stagnitto JHEP 01 (2023) 168

$H \to gg$

XC, P. Jakubcik, M. Marcoli, G. Stagnitto JHEP 06 (2023) 192



$\chi \to \tilde{g}g$

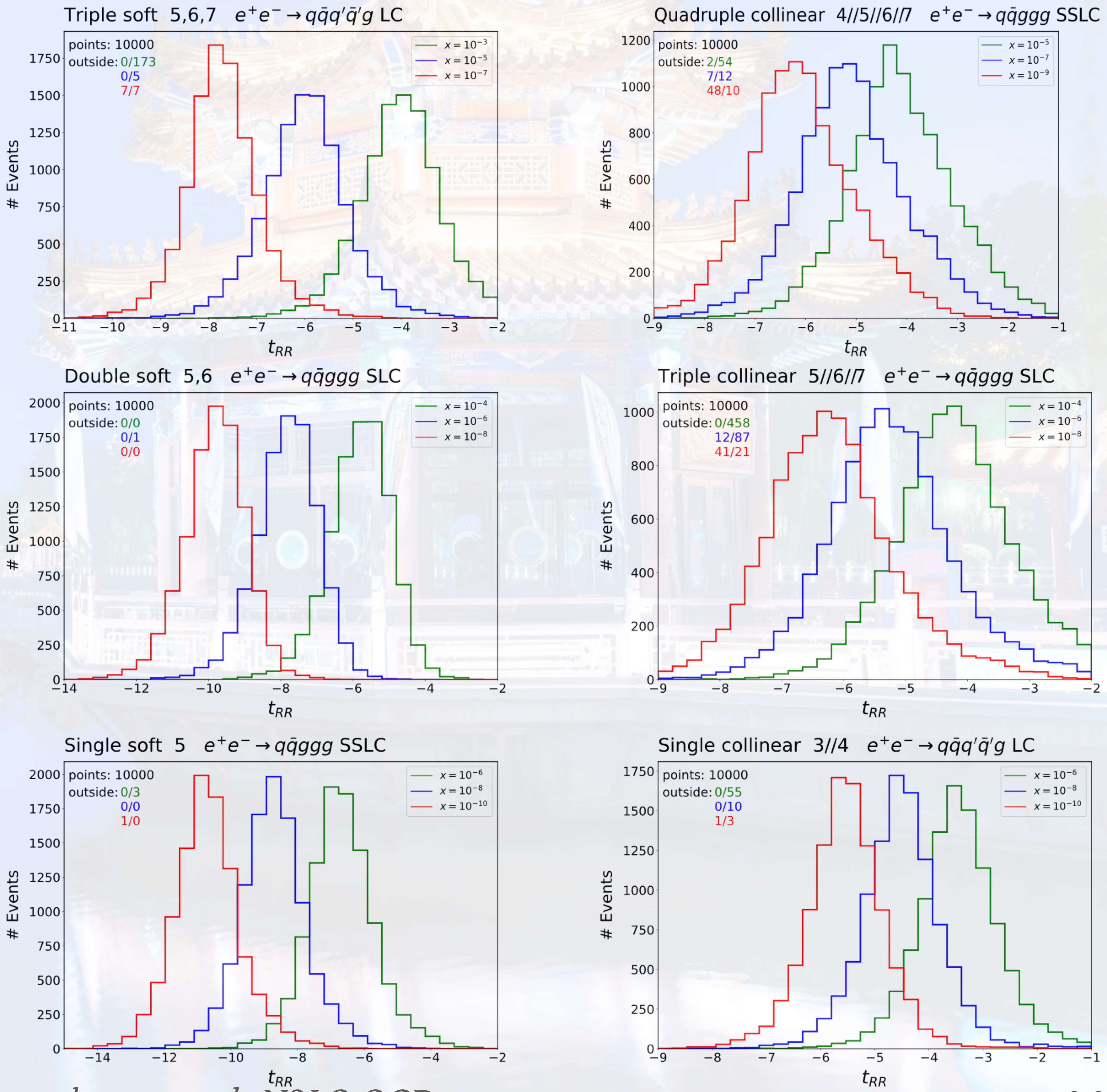
XC, P. Jakubcik, M. Marcoli, G. Stagnitto JHEP 12 (2023) 198

ANTENNA SUBTRACTION @ N3LO

Slide from Matteo @ HP2

Fully working subtraction terms for all RRR partonic channels:

- two quarks: $e^+e^- \to 2$ jets @ N3LO
  - $e^+e^- \to q\,\bar{q}\,g\,g\,g$ LC
  - $e^+e^- \to q\,\bar{q}\,g\,g\,g$ SLC
  - $e^+e^- \to q\,\bar{q}\,g\,g\,g$ SSLC
- four quarks, different flavour:
  - $e^+e^- \to q\,\bar{q}\,q'\,\bar{q}'\,g$ LC
  - $e^+e^- \to q\,\bar{q}\,q'\,\bar{q}'\,g$ SLC
- four quarks, same flavour:
  - $e^+e^- \to q\,\bar{q}\,q\,\bar{q}\,g$ LC
  - $e^+e^- \to q\,\bar{q}\,q\,\bar{q}\,g$ SLC



FUTURE PROSPECTS

- Precision is not the ultimate goal → identify anomaly then understand
- The most famous **failed experiment**: Michelson–Morley in 1887,  foundation of special relativity. → 1907 Nobel Prize to Albert A. Michelson .
- "… it seems probable that **most of the grand underlying principles have been firmly established** and that further advances are to be sought chiefly in the rigorous application of these principles to all the phenomena which come under our notice. … An eminent physicist remarked that **the future truths of physical science are to be looked for in the sixth place of decimals.**" —— Albert A. Michelson, 1894 University of Chicago
- Mass origin, Higgs potential, EW vacuum metastability etc.

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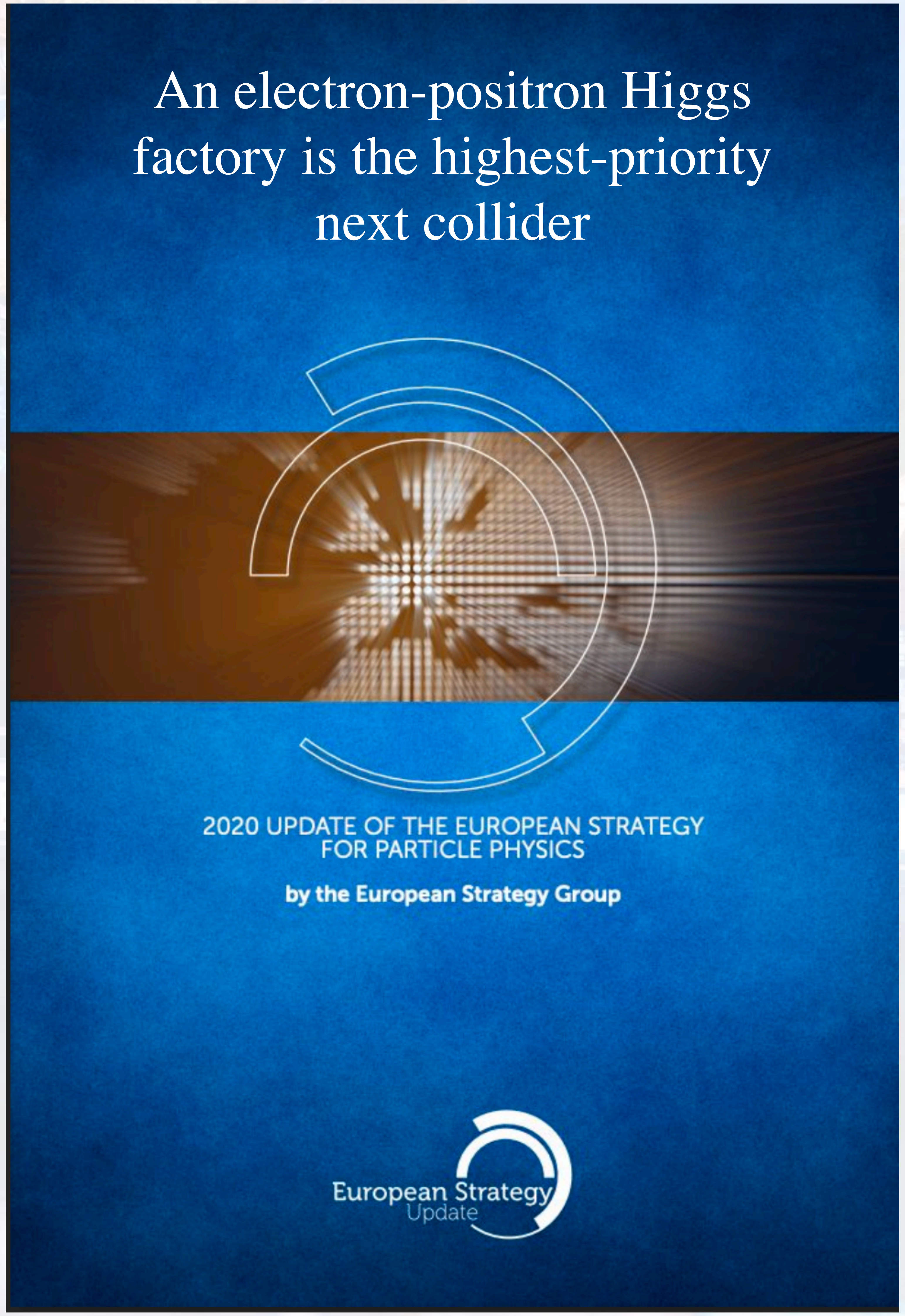
- Europe: HL-LHC, EPPSU due in June 2026, FCC feasibility study etc.
- United States: P5, muon collider, Snowmass conference etc.



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- United States: P5, muon collider, Snowmass conference etc.
- Higgs factory in the making: CEPC, FCC-ee

42nd ICHEP in Prague 2024

There will be Higgs factory without any doubt.

—— Yifang Wang @ ICHEP 2024


Thank You for Your Attention

BACK UP SLIDES

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^{3(4)}LL}$

FO	α_s^n	$H(m_V,\mu)$	$I_{i/j}^{(n)}(x,b)$	$\ln W(x_a,x_b,m_V,\vec{b},\mu=b_0/b) \sim \int_{\mu_h}^{\mu} d\bar{\mu}/\bar{\mu}\left(A(\alpha_s(\bar{\mu}))\ln\frac{m_V^2}{\bar{\mu}^2}+B(\alpha_s(\bar{\mu}))\right)$						
$\frac{d\hat{\sigma}^V_{NLO}}{dq_T}$	NLO	✓	✓	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1				
$\frac{d\hat{\sigma}^V_{NNLO}}{dq_T}$	N2LO	✓	✓	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1			
$\frac{d\hat{\sigma}^V_{N^3LO}}{dq_T}$	N3LO	✓	✓	$\ln^4(b^2m_V^2)$	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1		
$\frac{d\hat{\sigma}^V_{N^4LO}}{dq_T}$	N4LO	✓	✗	$\ln^5(b^2m_V^2)$	$\ln^4(b^2m_V^2)$	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1	
…	…			…	…	…	…		…	…
$\frac{d\hat{\sigma}^V_{N^kLO}}{dq_T}$	NKLO			$\ln^{k+1}(b^2m_V^2)$	$\ln^k(b^2m_V^2)$	$\ln^{k-1}(b^2m_V^2)$	$\ln^{k-2}(b^2m_V^2)$	$\ln^{k-3}(b^2m_V^2)$	…	…
…	…			…	…	…	…		…	…
Resum				LL	NLL	NNLL	N3LL	N4LL	…	$N^{k+1}LL$
A				A1 ✓	A2 ✓	A3 ✓	A4 ✓	A5 ✗	…	A_{k+2}
B					B1 ✓	B2 ✓	B3 ✓	B4 ✓	…	B_{k+1}

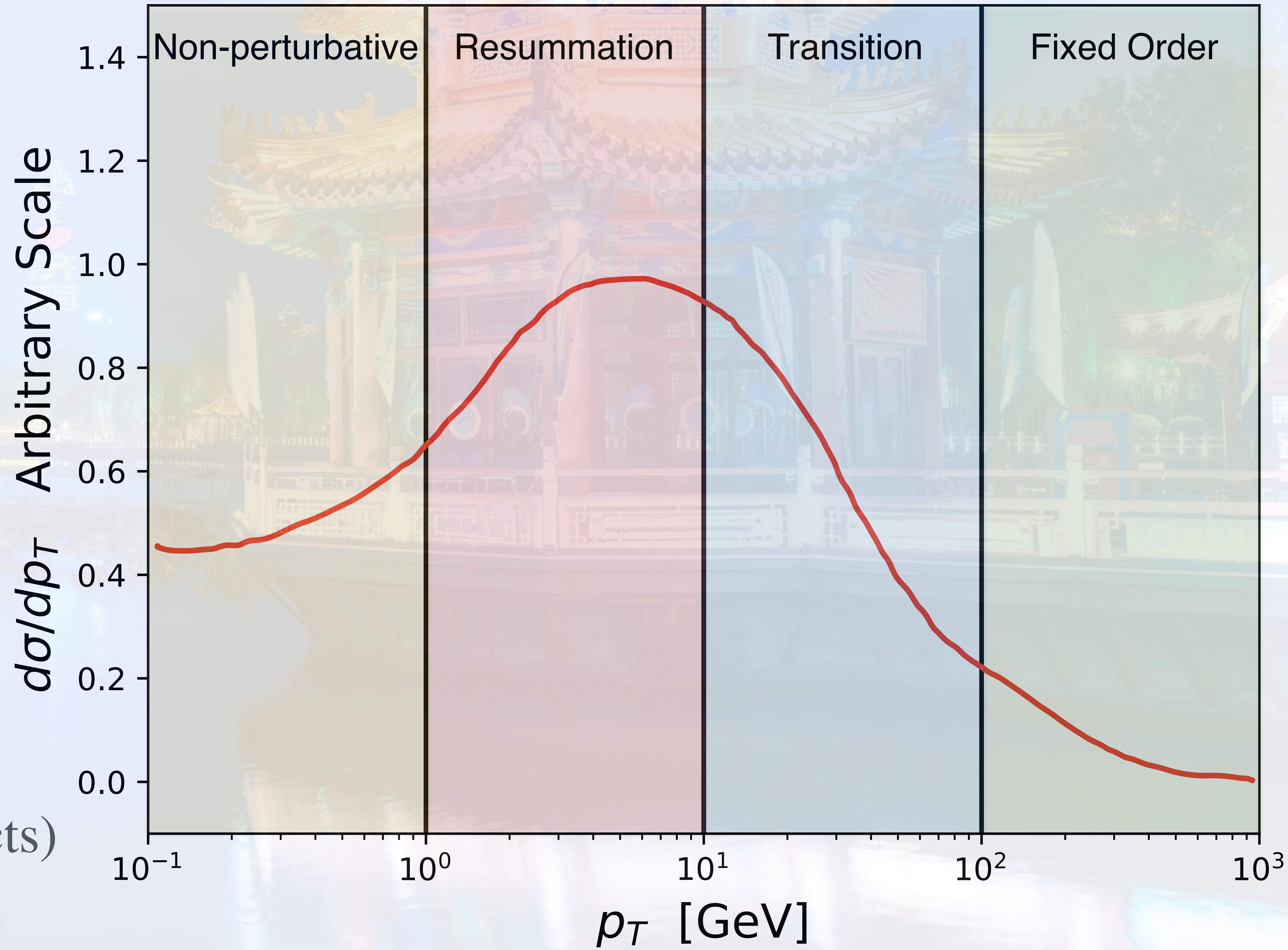
Predictions of Colourless pT at Hadron Collider

- ➤Beyond QCD improved parton model
  - ➤pQCD describes the tail of spectrum
  - ➤Large logarithmic divergence

$$\ln\frac{p_T}{Q} \text{ as } p_T \to 1 \text{ GeV}$$

  - ➤Various LP resummation schemes
  - ➤Multiple solutions in transition region
  - ➤Non-perturbative effects ~ 1 GeV (Short distance and long distance effects)

p_T Spectrum = multi-scale problem



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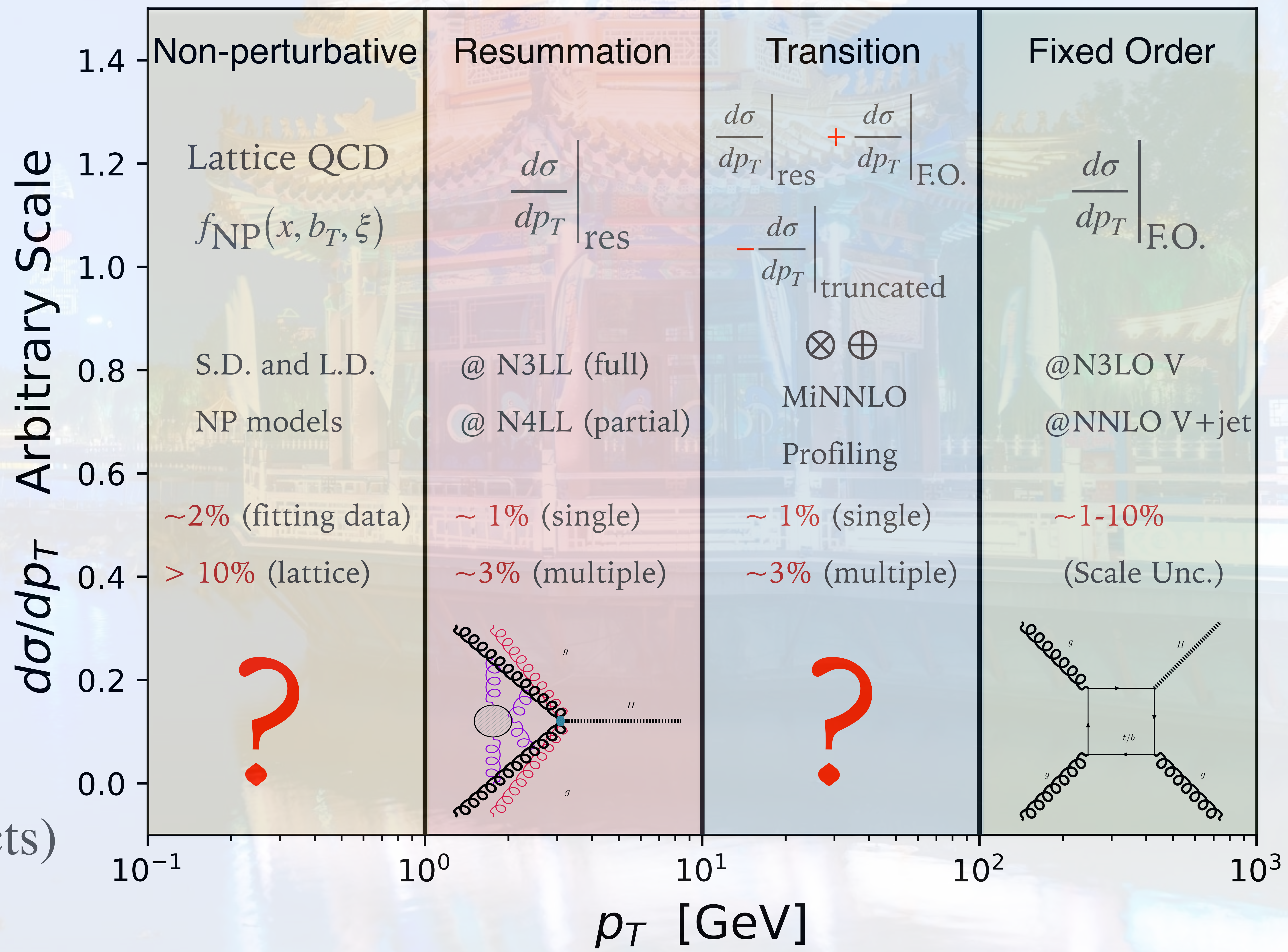


Perturbative QFT for Precision Predictions

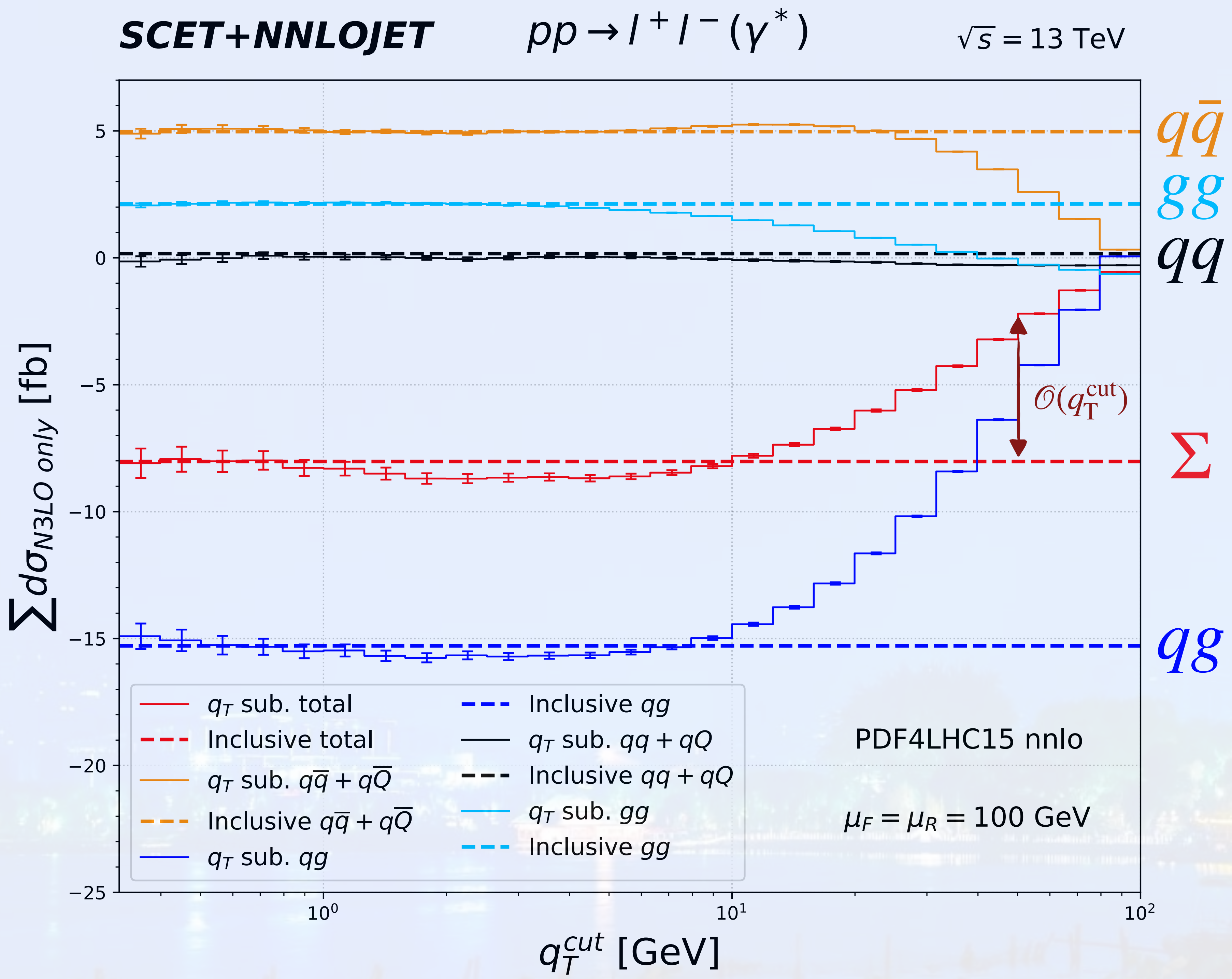
- State-of-the-art differential N3LO predictions ($2 \to 1$)
  - Fully differential N3LO Drell-Yan production (via γ^*) (XC, T. Gehrmann, N. Glover, A. Huss, T.-Z. Yang, H. X. Zhu 2021)
  - Apply qt-slicing at N3LO with SCET factorisation and expand to N3LO:

$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} \sum_q \sigma_{\rm LO}^{\gamma^*} H_{q\bar{q}} \Big[\sum_k \int_{x_1}^1 \frac{dz_1}{z_1} \mathcal{I}_{qk}\left(z_1, b_T^2, \mu\right) f_{k/h_1}(x_1/z_1, \mu) \times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} \mathcal{I}_{\bar{q}j}\left(z_2, b_T^2, \mu\right) f_{j/h_2}(x_2/z_2, \mu) \mathcal{S}\left(b_\perp, \mu\right) + (q \leftrightarrow \bar{q}) \Big] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

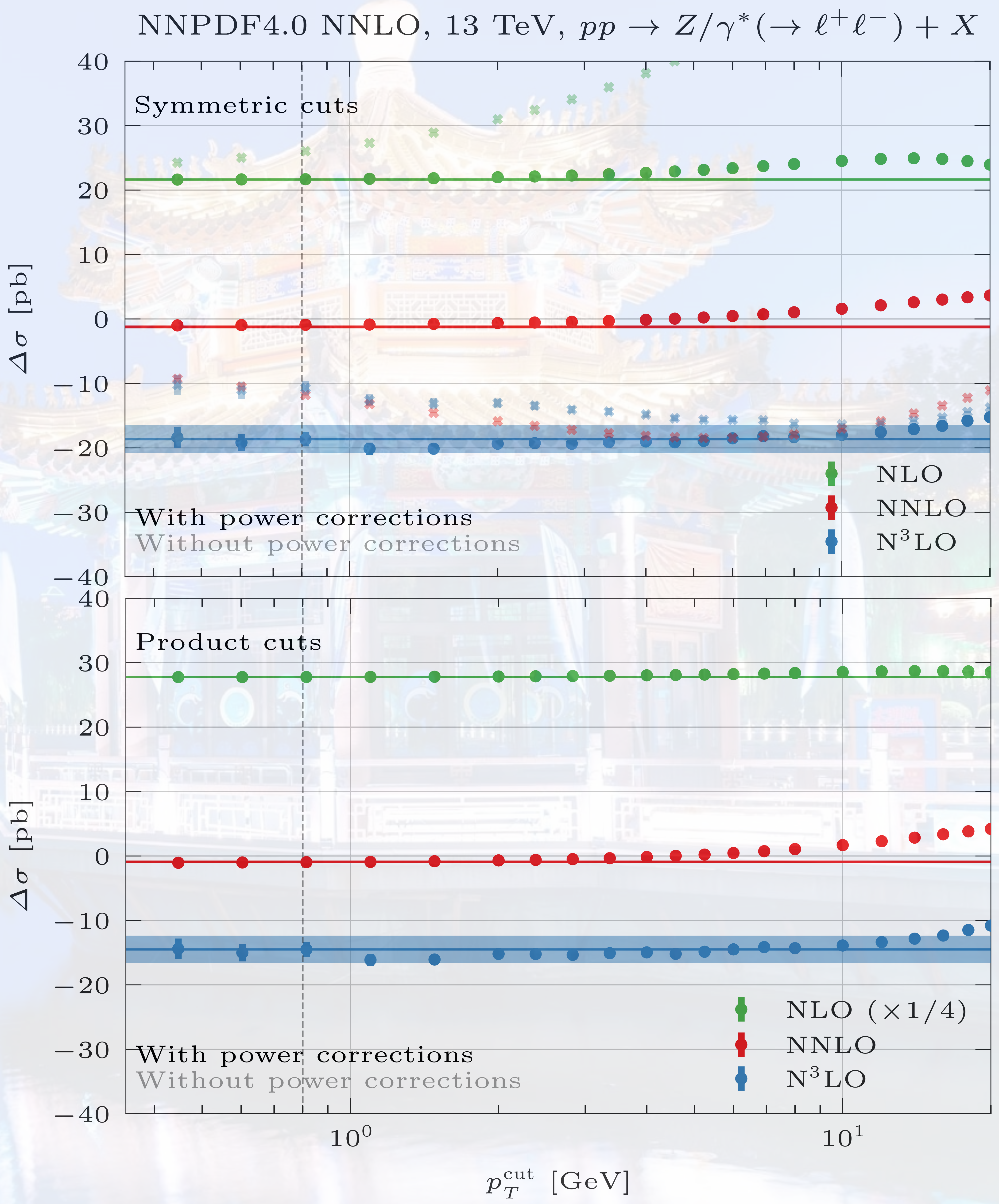
  - All factorised functions are recently known up to N3LO:
    1) 3-loop hard function $H_{q\bar{q}}^{(3)}$ (T. Gehrmann, E.W.N. Glover, T. Huber, N. Ikizlerli, C. Studerus 2010)
    2) Transverse-momentum-dependent (TMD) soft function $S(b_\perp, \mu)$ at α_s^3 (Y. Li, H.X. Zhu 2016)
    3) Matching kernel of TMD beam function I_{qk} at α_s^3 (M.-X. Luo, T.-Z. Yang, H. X. Zhu, Y. J. Zhu 2019, M. A. Ebert, B. Mistlberger, G. Vita 2020)
  - Apply qt cut to factorise N3LO contribution into two parts:

$$d\sigma_{N^3LO}^{\gamma^*} = [\mathscr{H}^{\gamma^*} \otimes d\sigma^{\gamma^*}]_{N^3LO}\Big|_{\delta(p_{T,\gamma^*})} + \left[d\sigma_{NNLO}^{\gamma^*+jet} - d\sigma_{N^3LO}^{\gamma^*\ CT}\right]_{p_{T,\gamma^*} > qt_{cut}} + \mathcal{O}(qt_{cut}^2/Q^2)$$

$pp \to \gamma^*/Z$ @ N^{3}LO



XC, T. Gehrmann, et. al. PRL. 128, 052001 (2022)



Fixed order	$\sigma_{pp\to\gamma^*}$ (fb)		
LO	$339.62^{+34.06}_{-37.48}$		
NLO	$391.25^{+10.84}_{-16.62}$		
NNLO	$390.09^{+3.06}_{-4.11}$		
N^{3}LO	$382.08^{+2.64}_{-3.09}$ [14]		
N^{3}LO only	$q_T^{cut} = 0.63$ GeV	$q_T^{cut} \to 0$ fit	[14]
qg	−15.32(32)	−15.34(54)	−15.29
$q\bar{q}+q\bar{Q}$	+5.06(12)	+5.05(12)	+4.97
gg	+2.17(6)	+2.19(6)	+2.12
$qq+qQ$	+0.09(13)	+0.09(17)	+0.17
Total	−7.98(36)	−8.01(58)	−8.03

C. Duhr, F. Dulat, B. Mistlberger. PRL. **125**, 172001 (2020)

XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

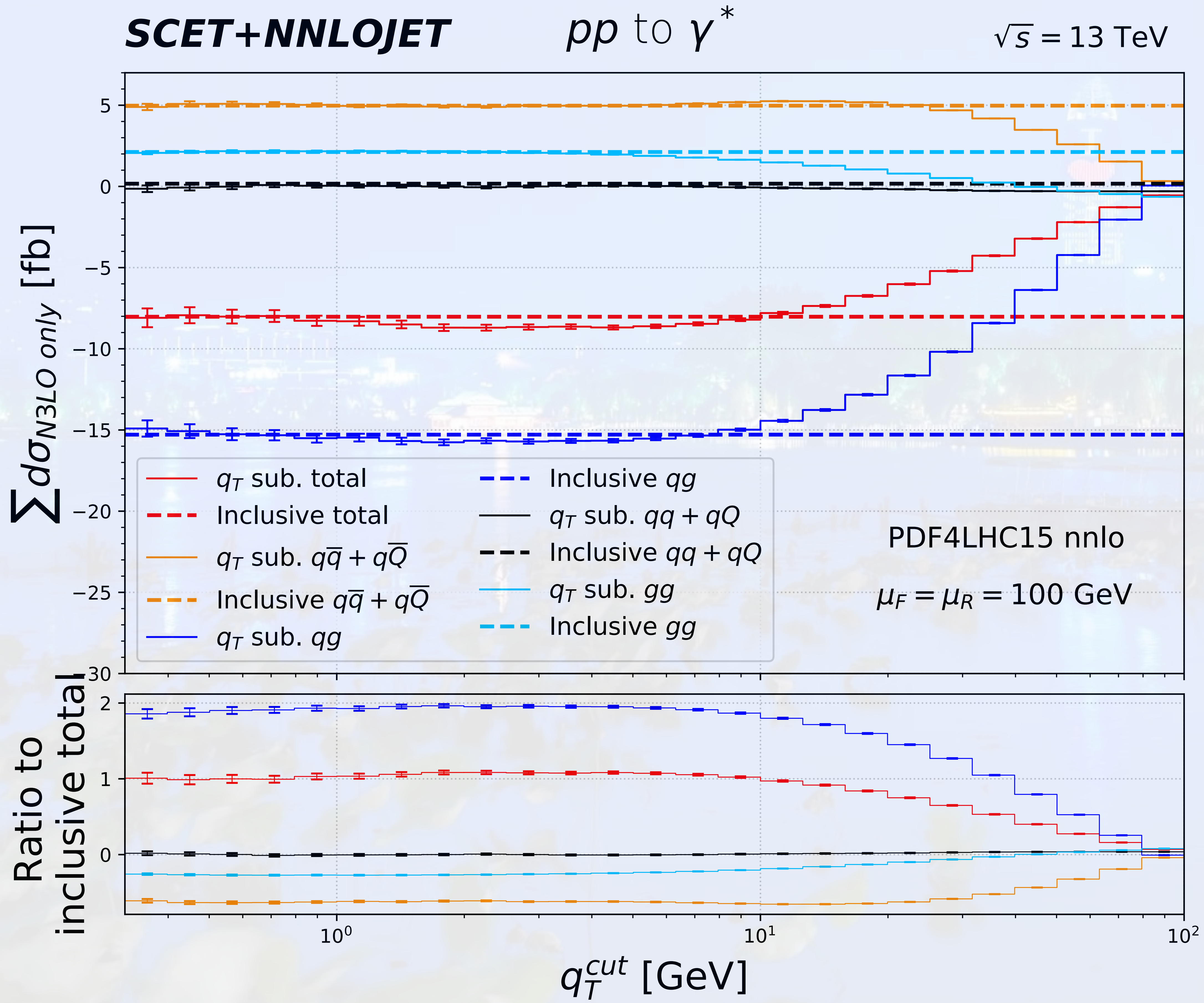
➤qT slicing at N3LO for neutral and charged current production (NNLOJET)

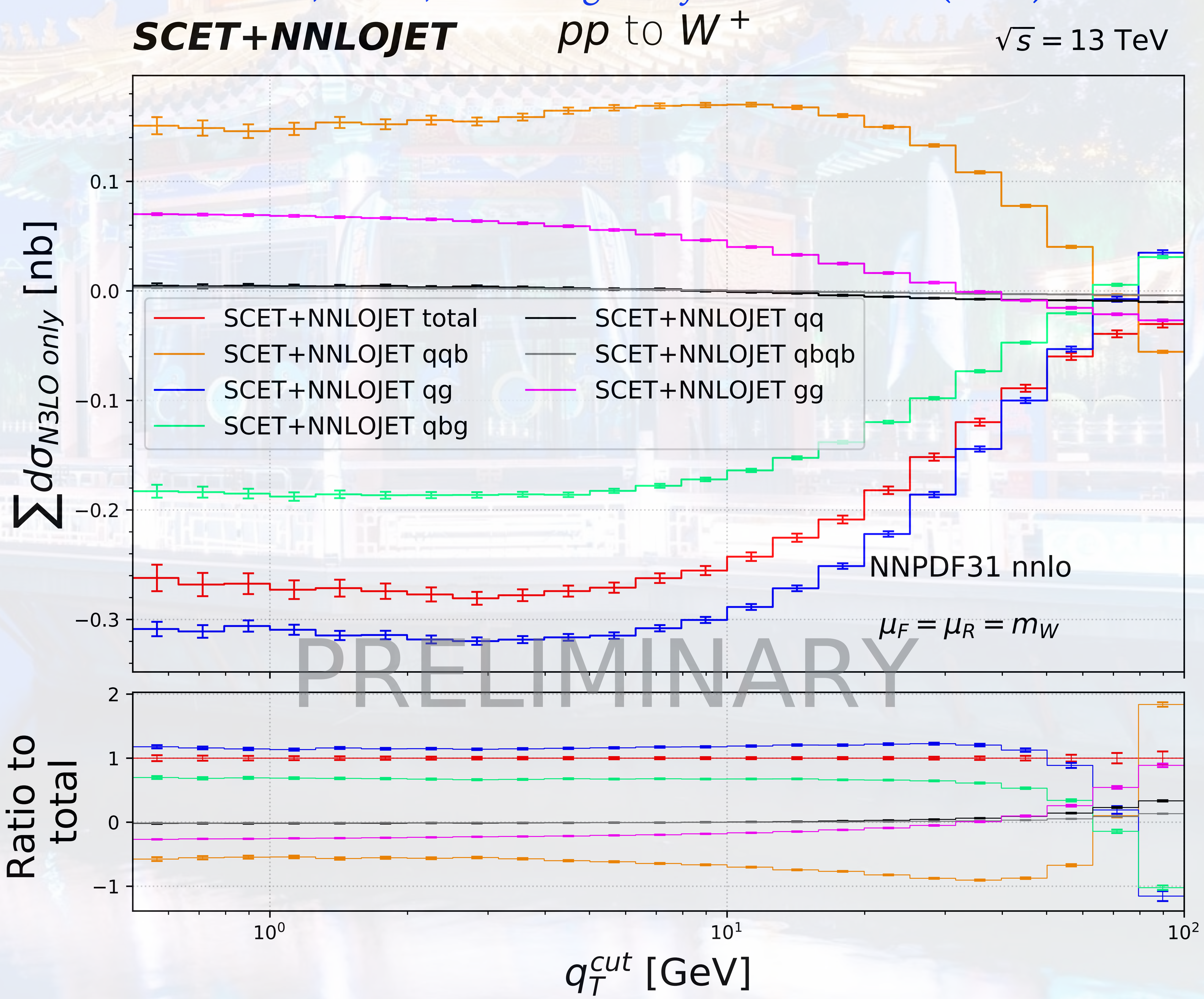
$$\sum \mathrm{d}\sigma^{V}_{N3LO} \equiv \sum_{\mathrm{d}p_{T,V}} \mathrm{d}\sigma^{V+jet}_{NNLO}/\mathrm{d}p_{T,V}|_{p_{T,V}>q_T^{cut}} + \sum_{\mathrm{d}p_{T,V}} \mathrm{d}\sigma^{V\,SCET}_{N^3LO}/\mathrm{d}p_{T,V}|_{p_{T,V}\in[0,q_T^{cut}]}$$

NC and CC Validated against inclusive XS within ± 5% uncertainty

$$\Delta\sigma^{\gamma^*}_{N^3LO} = -7.98 \pm 0.36\,fb \quad vs. \quad -8.03\,fb$$

Duhr, Dulat, Mistlberger *Phys.Rev.Lett.* 125 (2020)





XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Rev.Lett.* 128 (2022) 5

XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Lett.B 840 (2023)*

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

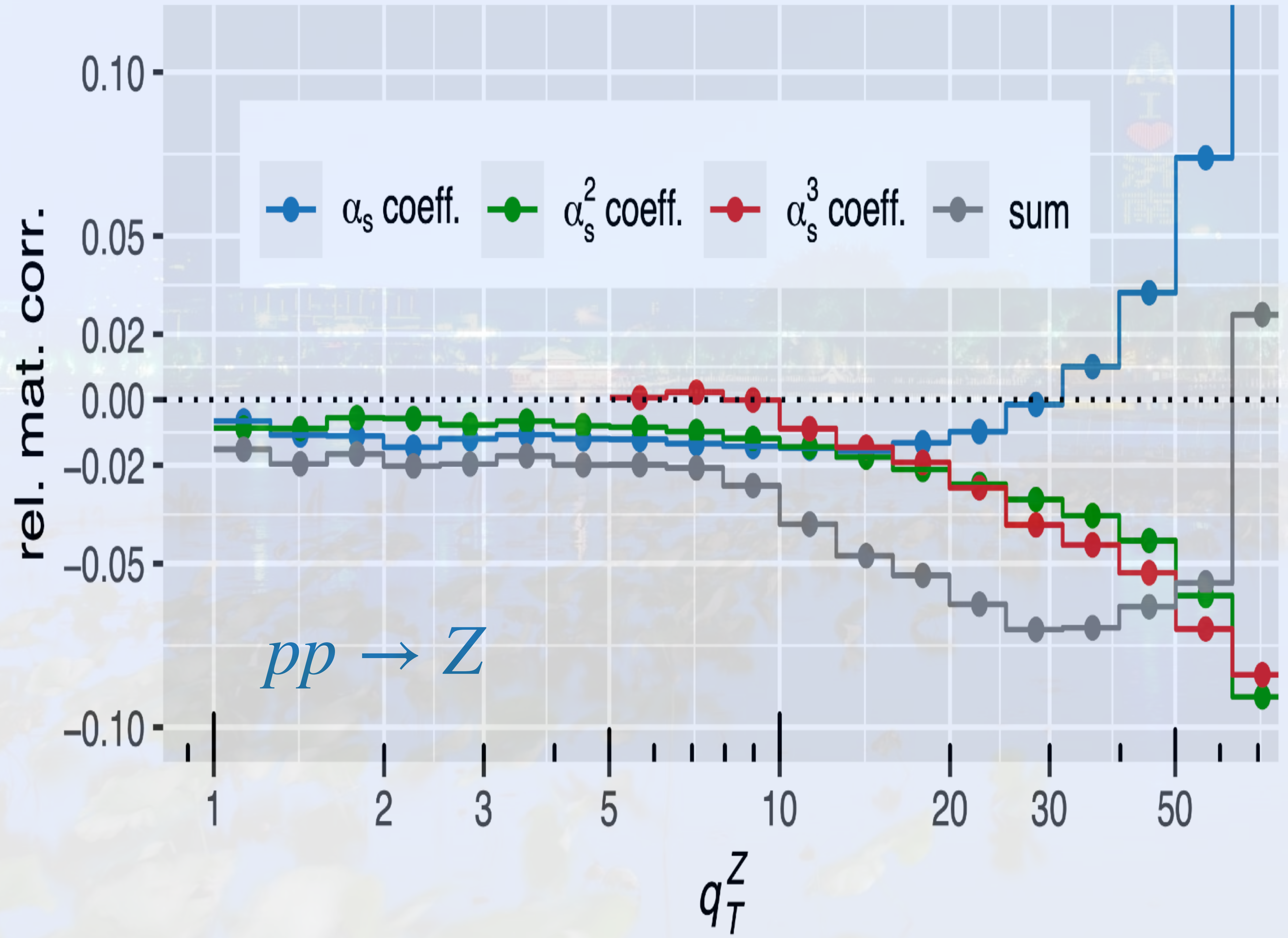
➤qT slicing at N3LO for neutral and charged current production (MCFM)

$$\sum_{dp_{T,V}} d\sigma^V_{N3LO} \equiv \sum_{dp_{T,V}} d\sigma^{V+jet}_{NNLO}/dp_{T,V}|_{p_{T,V}>q_T^{cut}} + \sum_{dp_{T,V}} d\sigma^{V\,SCET}_{N^3LO}/dp_{T,V}|_{p_{T,V}\in[0,q_T^{cut}]}$$

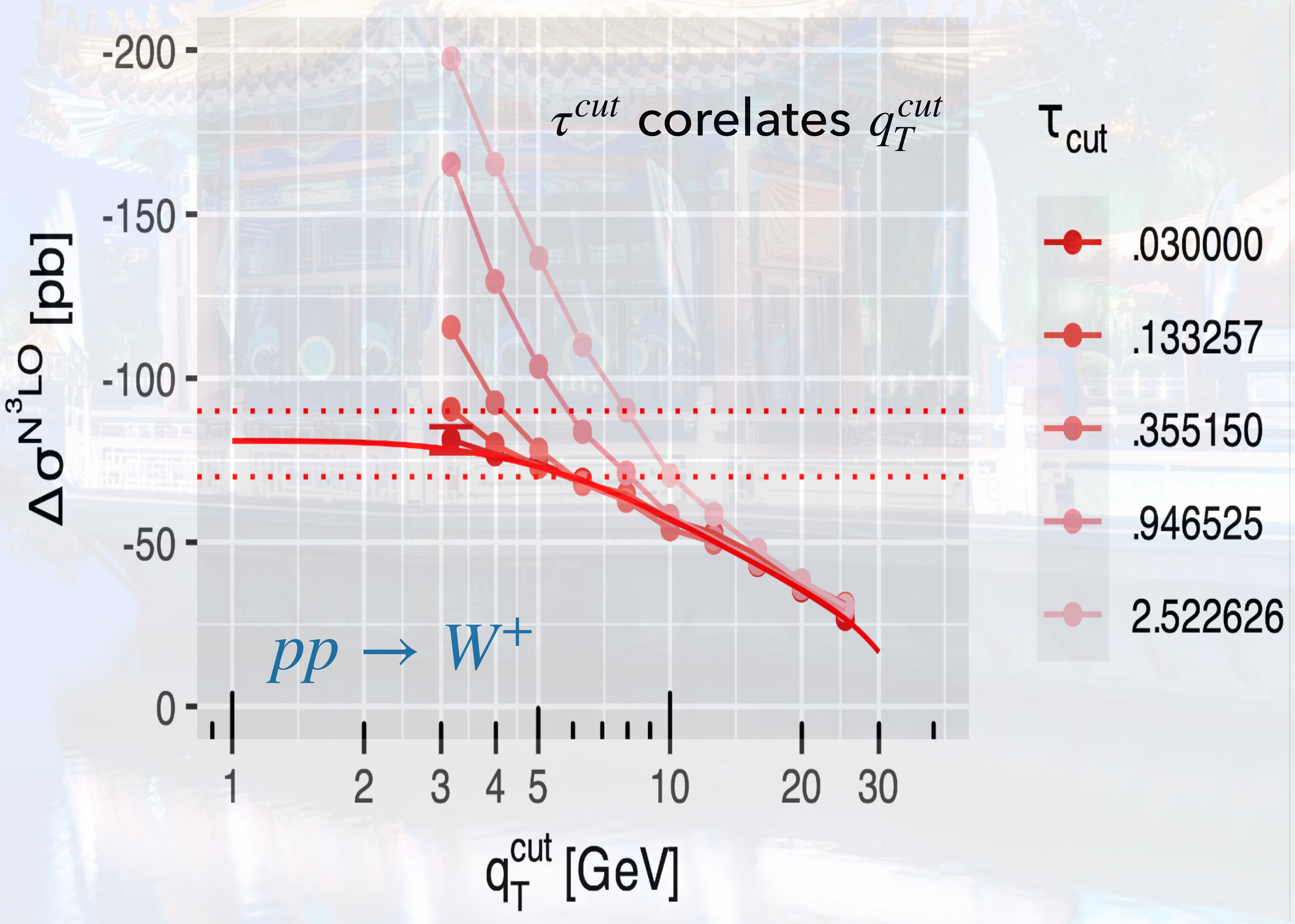
NC MCFM: $-22.6\,\text{pb} \pm 1.4\,\text{pb}\,(\text{num.}) \pm 1\,\text{pb}\,(\text{slicing})$

NC NNLOJET: $-18.7\,\text{pb} \pm 1.1\,\text{pb}\,(\text{num.}) \pm 0.9\,\text{pb}\,(\text{slicing})$

CC agree to inclusive XS within ± 60% uncertainty of $\Delta(\alpha_s^3)$



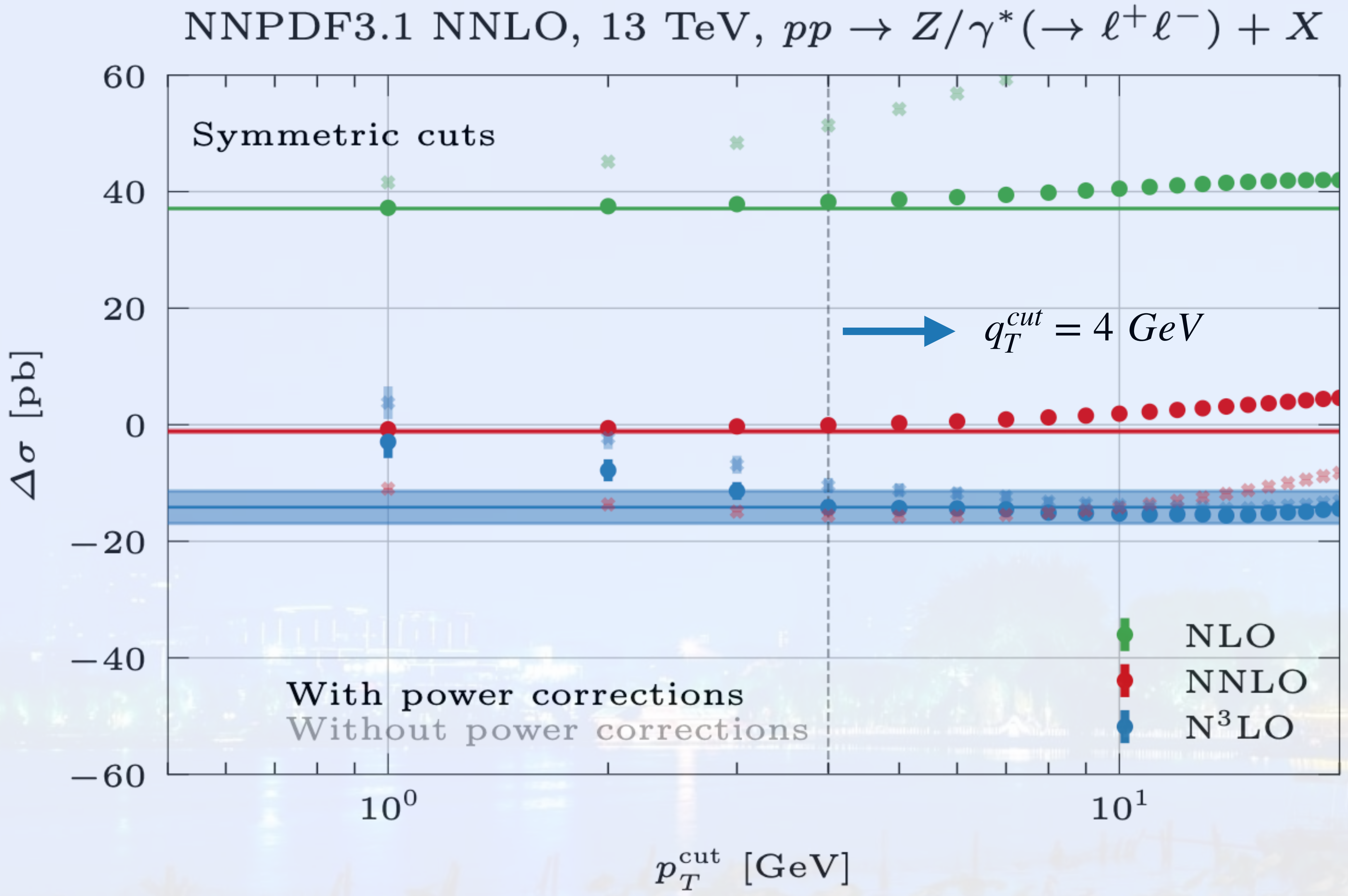
Neumann and Campbell *Phys.Rev.D* 107 (2023) 1



Neumann and Campbell *JHEP 11 (2023) 127*

Precision Predictions at Hadron Collider

$2 \to 1$ @ N3LO (+ N3LL) QCD



XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)

- Solid horizontal lines: NLO, NNLO at 1 GeV, N3LO at 4 GeV with MC error.
  - N3LO shows no plateau in 1905.05171
- Pale dots are values used by DYTurbo in 2103.04974 and 2303.12781 (taken from 1905.05171).
  - Fiducial power corrections are not included.
  - Leads to 30% difference of N3LO coefficients at $q_T^{cut} = 4\ GeV$.
- Solid dots are corrected values with fiducial power correction.
  - Central value shifts 2 pb starting from NLO (the dominant error).
  - ± 2.1 pb uncertainty from MC and q_T^{cut} (estimated from [3,5] GeV region).
  - Not consistent with DYTurbo update result of ± 0.7 pb uncertainty.

DYTurbo result with fiducial power correction

Order	N^{3}LO
q_T subtr. ($q_T^{cut} = 4\,\text{GeV}$)	$747.1 \pm 0.7\,\text{pb}$
recoil q_T subtr.	$745.7 \pm 0.7\,\text{pb}$

S. Camarda, L. Cieri, G. Ferrera Eur.Phys.J.C 82 (2022) 6

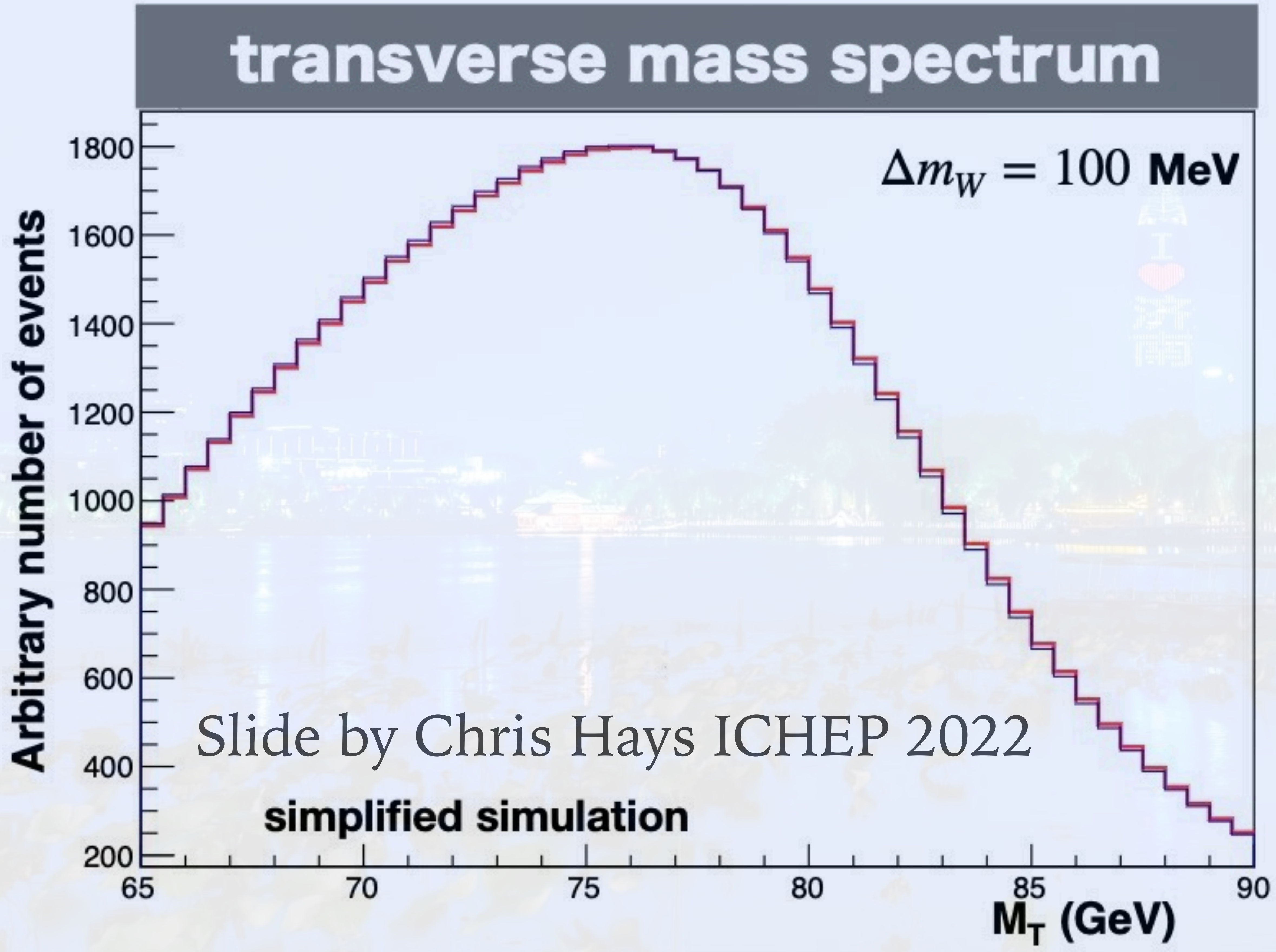
DYTurbo result without fiducial power correction cited in ATLAS α_s fitting

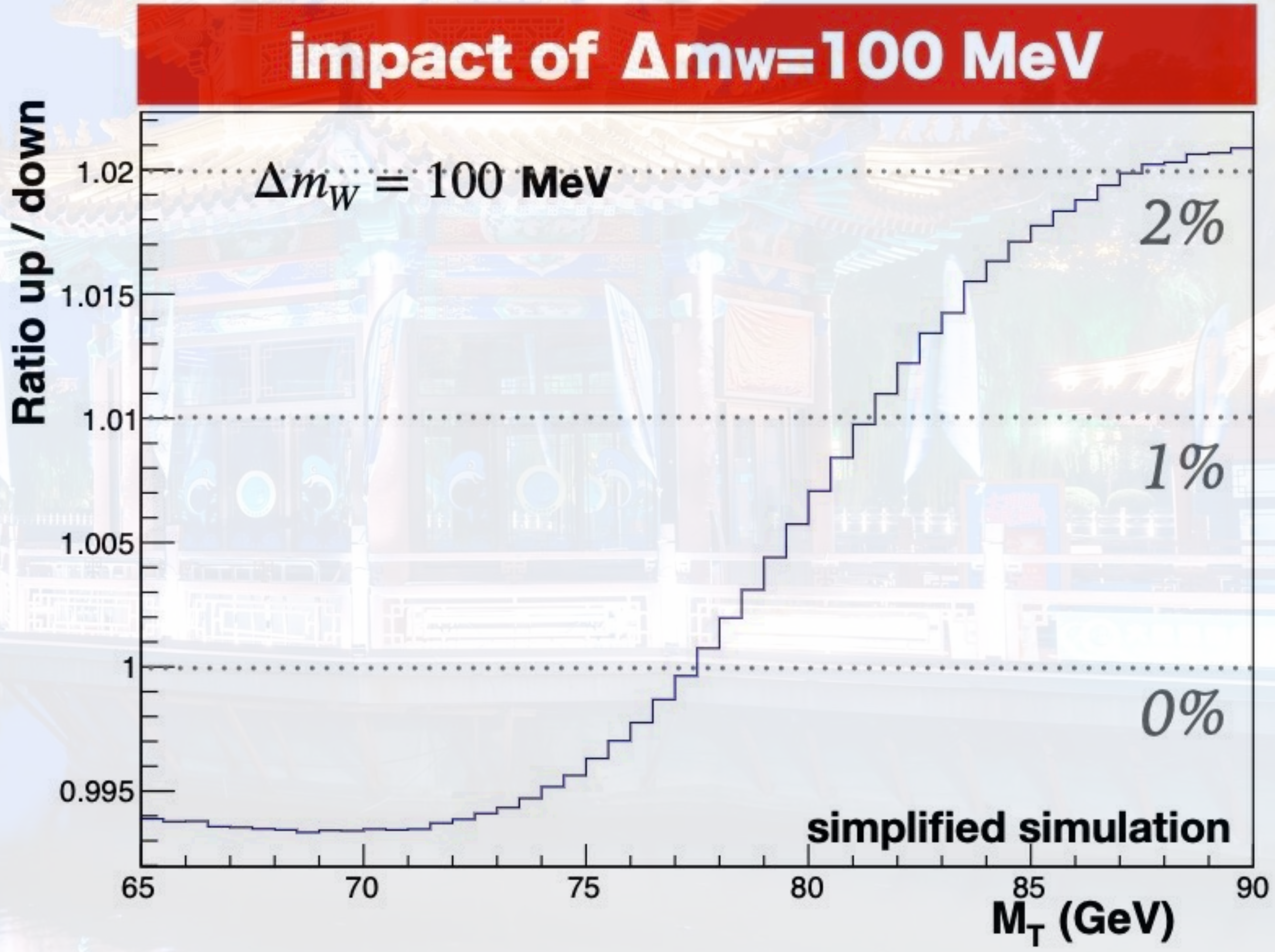
Order	NLO	NNLO	N^{3}LO
$\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb]	766.3 ± 1	757.4 ± 2	746.1 ± 2.5
Order	NLL+NLO	NNLL+NNLO	N^{3}LL+N^{3}LO
$\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb]	773.7 ± 1	759.8 ± 2	749.6 ± 2.5

*S. Camarda, L. Cieri, G. Ferrera Phys. Rev. D **104**, L111503 (2021)*

W mass in CDFII measurement

➤ $d\sigma/dm_T^W$ two templates with $\Delta m_W = 100$ MeV





$\Delta m_W =$ 100 MeV ~ 0.5-2% change in $d\sigma/dm_T^W$ → $\Delta m_W =$ 10 MeV ~ 0.1% precision in $d\sigma/dm_T^W$

Precision predictions in CDF II

➤CDF II use ResBos to generate theory templates

➤NLO+NNLL accuracy for W/Z production

Balazs, Brock, Landry, Nadolsky and Yuan`97 to`03

➤CSS factorisation and resummation of p_T in b space:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}^2\vec{p}_T\,\mathrm{d}y\,\mathrm{d}\cos\theta\,\mathrm{d}\phi} = \sigma_0 \int \frac{\mathrm{d}^2 b}{(2\pi)^2} e^{i\vec{p}_T\cdot\vec{b}} e^{-S(b)}$$

$$\times\, C\otimes f(x_1,\mu)\, C\otimes f(x_2,\mu) + Y(Q,\vec{p}_T,x_1,x_2,\mu_R,\mu_F)$$

Collins, Soper and Sterman`85

➤Non-perturbative effects at $\alpha_s(\Lambda)$ and large b:

$$S(b) = S_{\mathrm{NP}} S_{\mathrm{Pert}}\,,$$

Collins and Soper `77

$$S_{\mathrm{Pert}}(b) = \int_{C_1^2/(b^*)^2}^{C_2^2 Q^2} \frac{\mathrm{d}\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2 Q^2}{\bar{\mu}^2}\right) A(\bar{\mu},C_1) + B(\bar{\mu},C_1,C_2)\right]$$

$$S_{\mathrm{NP}} = \left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \ln(100 x_1 x_2)\right] b^2$$

S_{NP} assumes the BLNY functional form

Brock, Landry, Nadolsky and Yuan `02

➤Use data driven method:

Fix	g1	g2	g3	α_s
p_T^Z	Global fit `03	CDFII fit	Global fit `03	CDFII fit
p_T^Z/p_T^W			Global fit `03	

Global fit by Brock, Landry, Nadolsky and Yuan `03

$m_T^W \sim 0.7$ MeV, $p_T^l \sim 2.3$ MeV, $p_T^\nu \sim 0.9$ MeV

CDF supplementary materials `22

➤Scale uncertainty of p_T^Z/p_T^W by DYQT

Bozzi, Catani, Ferrera, de Florian, Grazzini `09 `11

$m_T^W \sim 3.5$ MeV, $p_T^l \sim 10.1$ MeV, $p_T^\nu \sim 3.9$ MeV

Not included in final result CDF sm`22

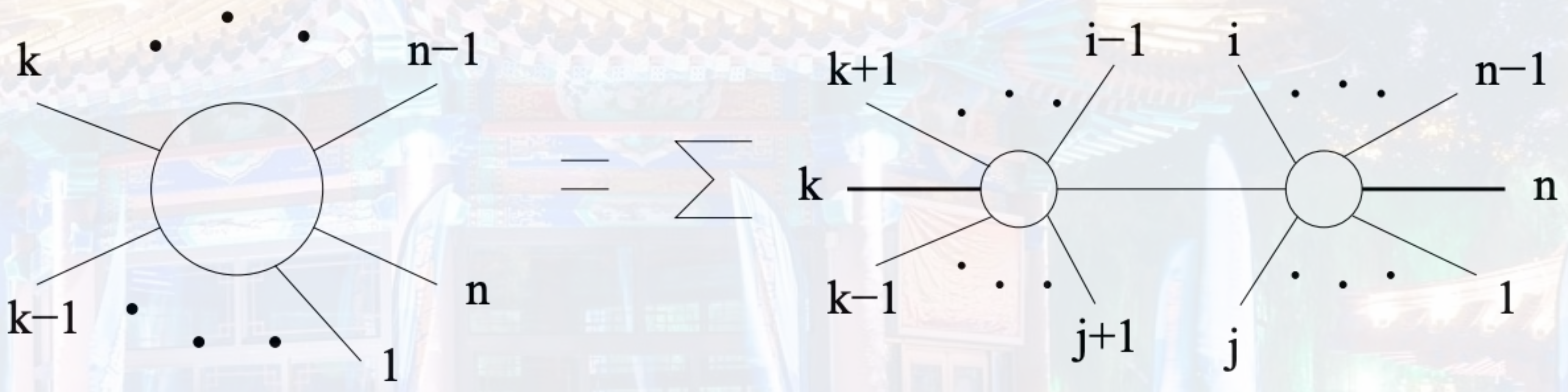
ANATOMY OF HARD SCATTERING $\hat{\sigma}_{ab}$

➤Tree level matrix element (consider solved)

$$\mathcal{M}_4^{(0)}(1^-2^-3^+4^+) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \qquad \mathcal{M}_n^{(0)}(1^+\cdots i^-\cdots j^-\cdots n^+) = i\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle} \quad \text{with} \quad s_{ij} = (p_i+p_j)^2 = 2p_i\cdot p_j = \langle ij\rangle[ji]$$

MHV amplitudes, S. J. Parke and T. R. Taylor (1986)

Recursive relations like CSW, BCFW and CHY methods scale up analytical results to any # of legs

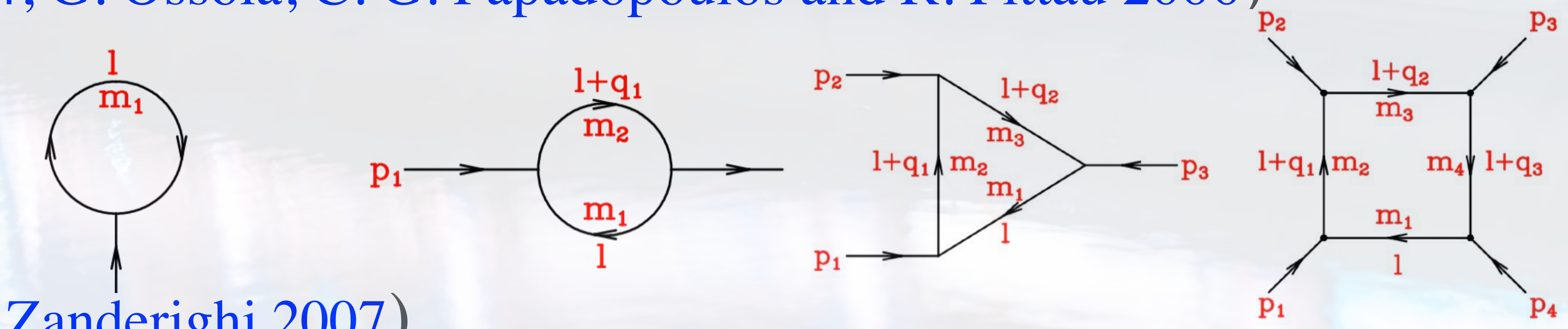


R. Britto, F. Cachazo, B. Feng and E. Witten (2005)

➤1-loop matrix element (consider solved)

- ➤Recursively reduce tensorial 1-loop integrand into scalar integrand (G. 't Hooft, G. Passarino, M. J. G. Veltman 1979, F. del Aguila and R. Pittau 2004)
- ➤Recursively reduce integrand with $n \geq 5$ external legs to $n = 1,2,3,4$ external legs (D.B. Melrose 1965, W. L. van Neerven and J. A. M. Vermaseren 1984, Z. Bern, L. J. Dixon and D. A. Kosower 1993, T. Binoth, J. P. Guillet and G. Heinrich 2000, G. Duplancic and B. Nizic 2004, G. Ossola, C. G. Papadopoulos and R. Pittau 2006)

$$I_n^D = \sum_{i=1}^{4} d_i I_i^D + R_n + \mathcal{O}(\epsilon) \text{ with } D = 4 - 2\epsilon$$



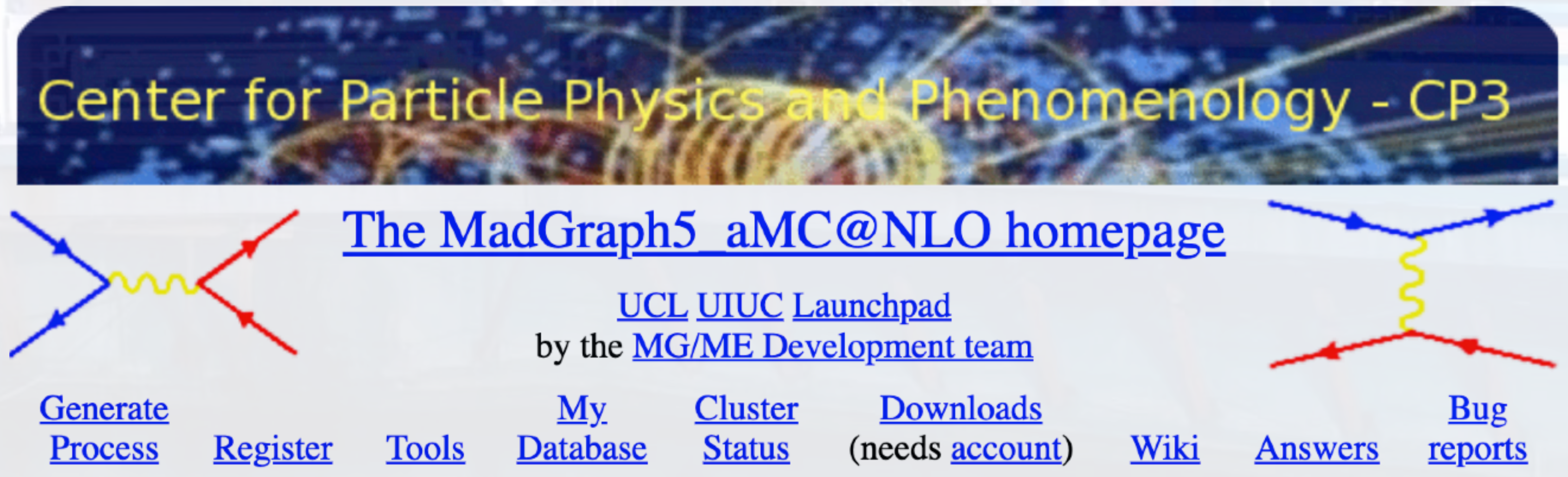
- ➤All master integrals available analytically (R. K. Ellis, G. Zanderighi 2007)

ANATOMY OF HARD SCATTERING $\hat{\sigma}_{ab}$

- 1-loop automation packages on the market
  - BlackHat (Z. Bern, L. Dixon, F. Febres Cordero, S. Höche, H. Ita, D. Kosower, D. Maitre)
  - GoSam (G. Heinrich, S. Jones, M. Kerner, V. Magerya, P. Mastrolia, G. Ossola, T. Peraro, J. Schlenk et al.)
  - RECOLA (S. Actis, A. Denner, L. Hofer, J.-N. Lang, A. Scharf, S. Uccirati)
  - OpenLoops (F. Cascioli, P. Maierhöfer, S. Pozzorini, F. Buccioni, J.-N. Lang, J. M. Lindert, H. Zhang, M. F. Zoller)
  - NJet (S. Badger, B. Biedermann, R. Moodie, P. Uwer, V. Yundin)
  - MadLoop/aMC@NLO (R. Frederix, S. Frixione, V. Hirschi et al.)
  - CutTools (G. Ossola, C. Papadopoulos, R. Pittau)

One-Loop Amplitudes with BlackHat

OpenLoops 2



GoSam

CutTools

Computing 1-loop amplitudes at the integrand level

njet / NJet

njet

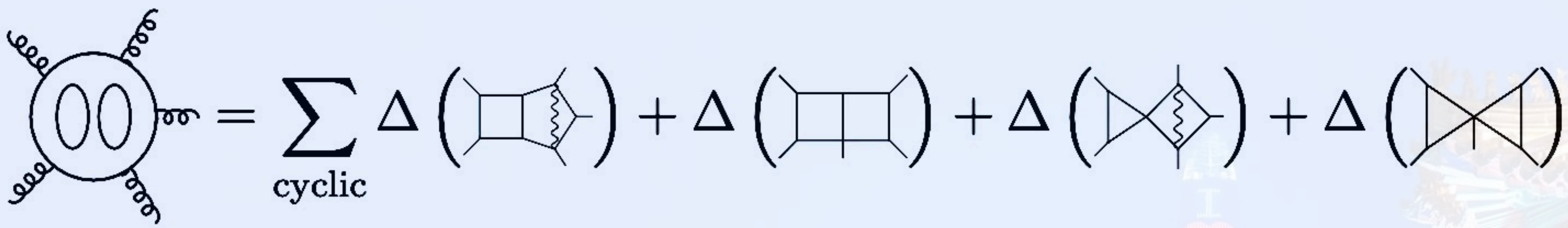
A C++ library for multi-parton one-loop matrix elements via a generalized unitarity construction.

ANATOMY OF HARD SCATTERING $\hat{\sigma}_{ab}$

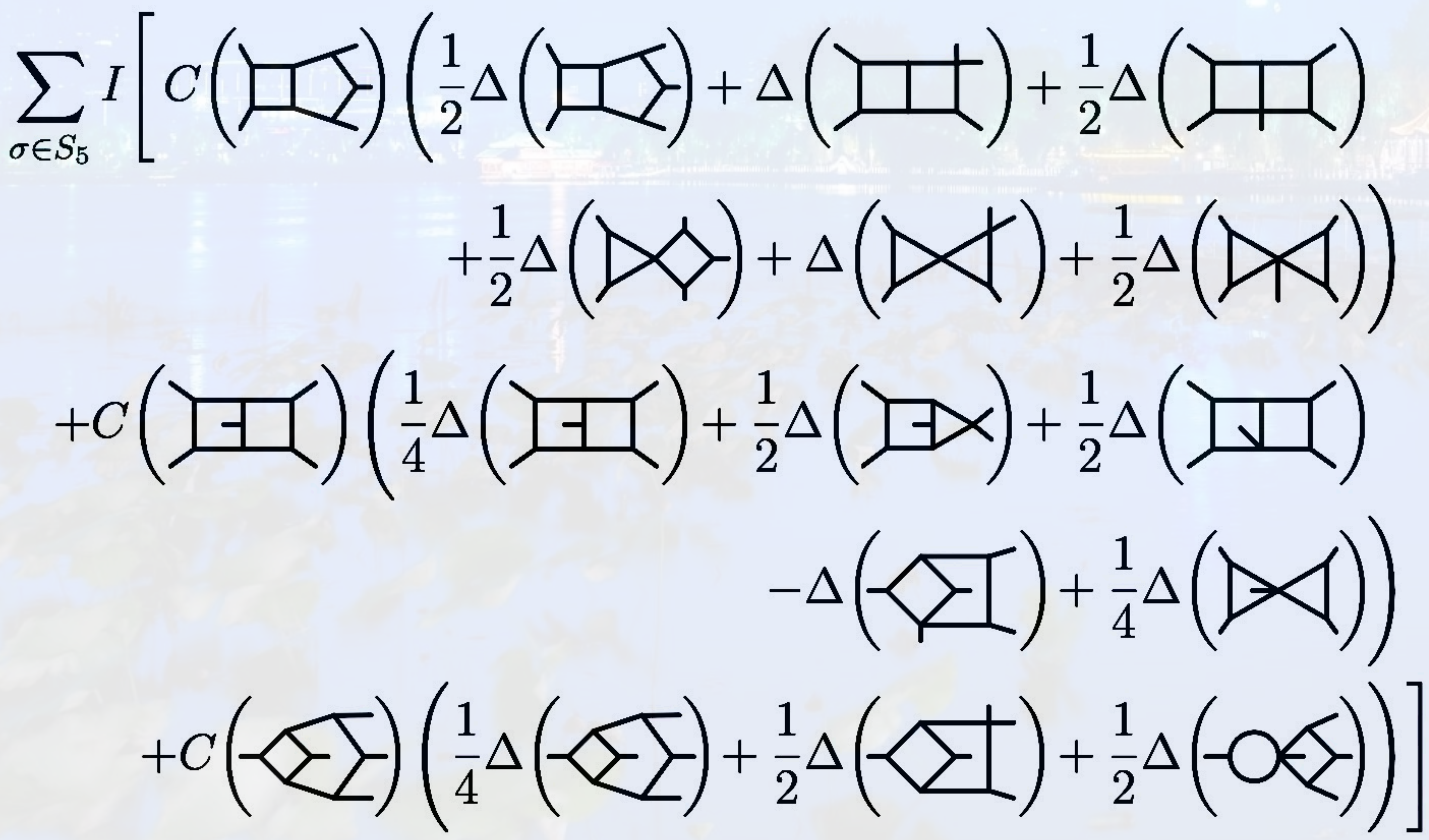
➤ 2-loop matrix elements

➤ Due to the complexity and emerge of non-planar diagrams, we haven't find a finite set of master integrals .

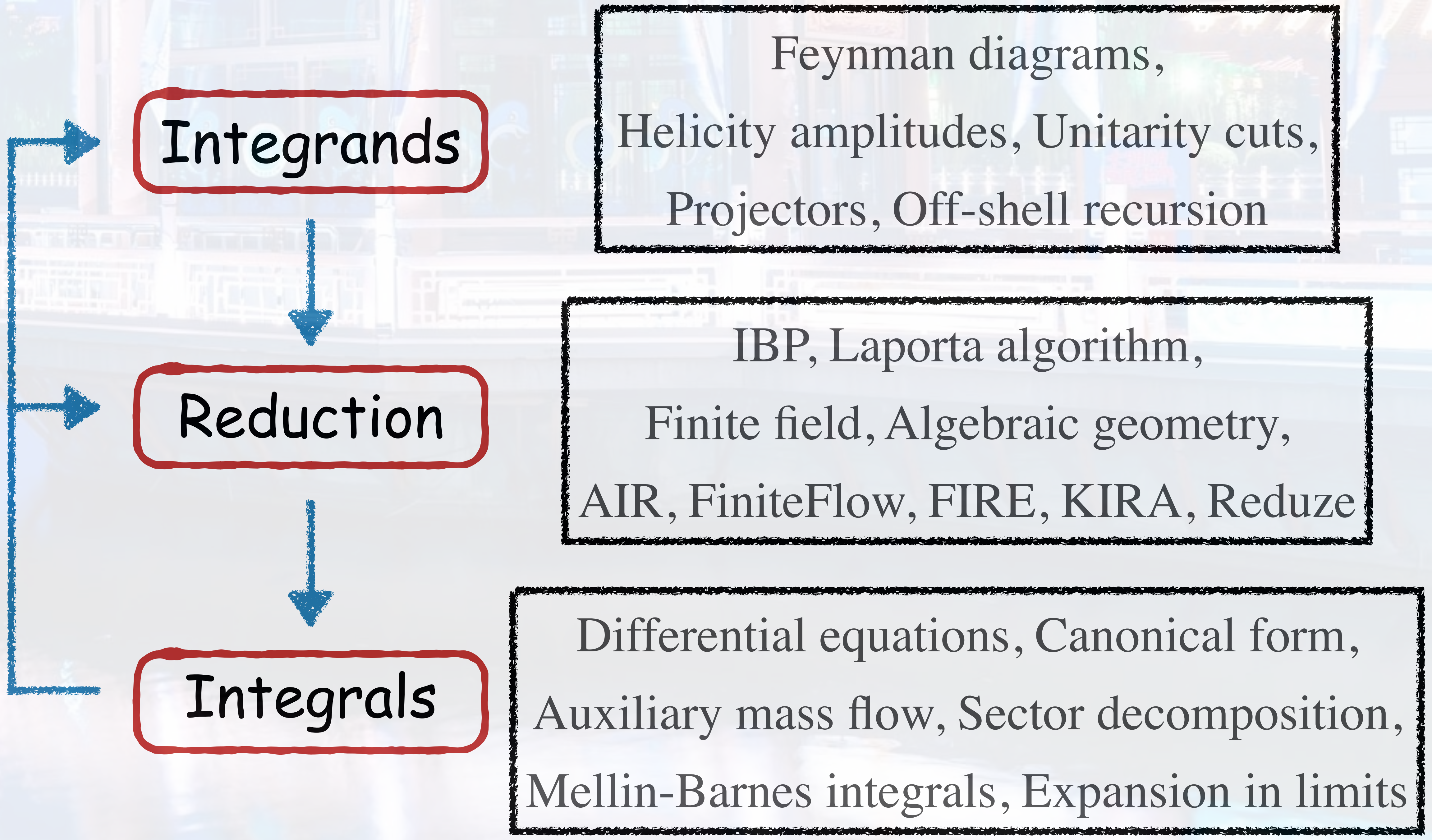
S. Badger, C. Brønnum-Hansen, H. Bayu Hartanto
G. Mogull, A. Ochirov, D. O'Connel, T. Peraro,
H. Frellesvig, Y. Zhang (2013-2016)



$\mathcal{M}_5^{(2)}(1^+,2^+,3^+,4^+,5^+) =$



➤ Calculation is on process-by-process basis



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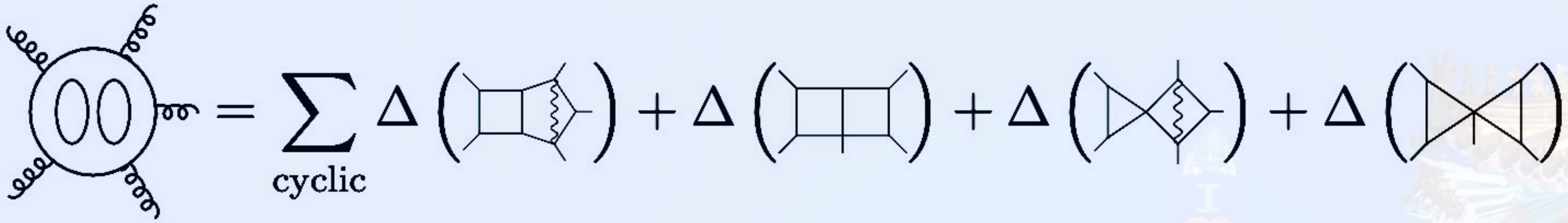
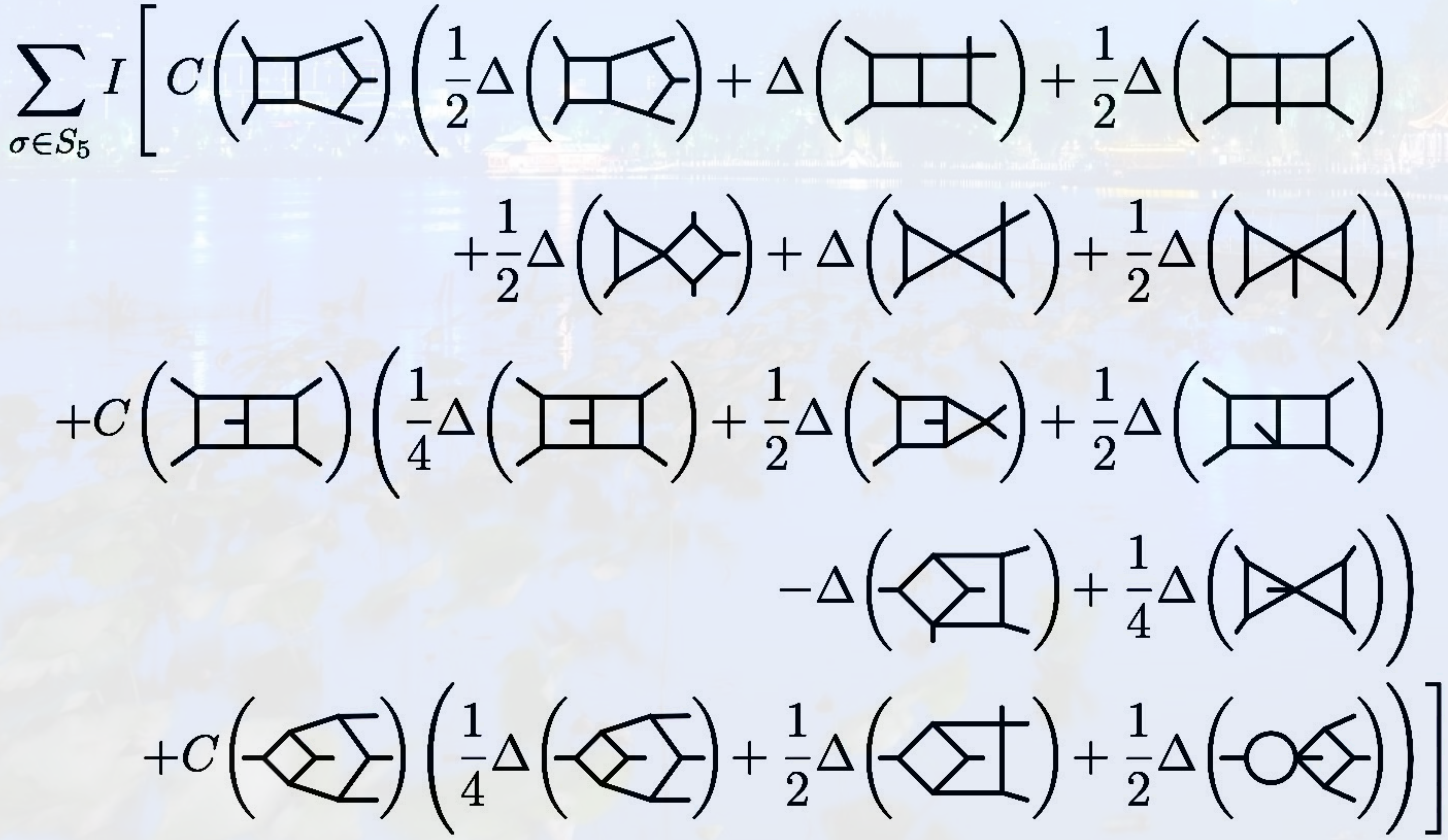
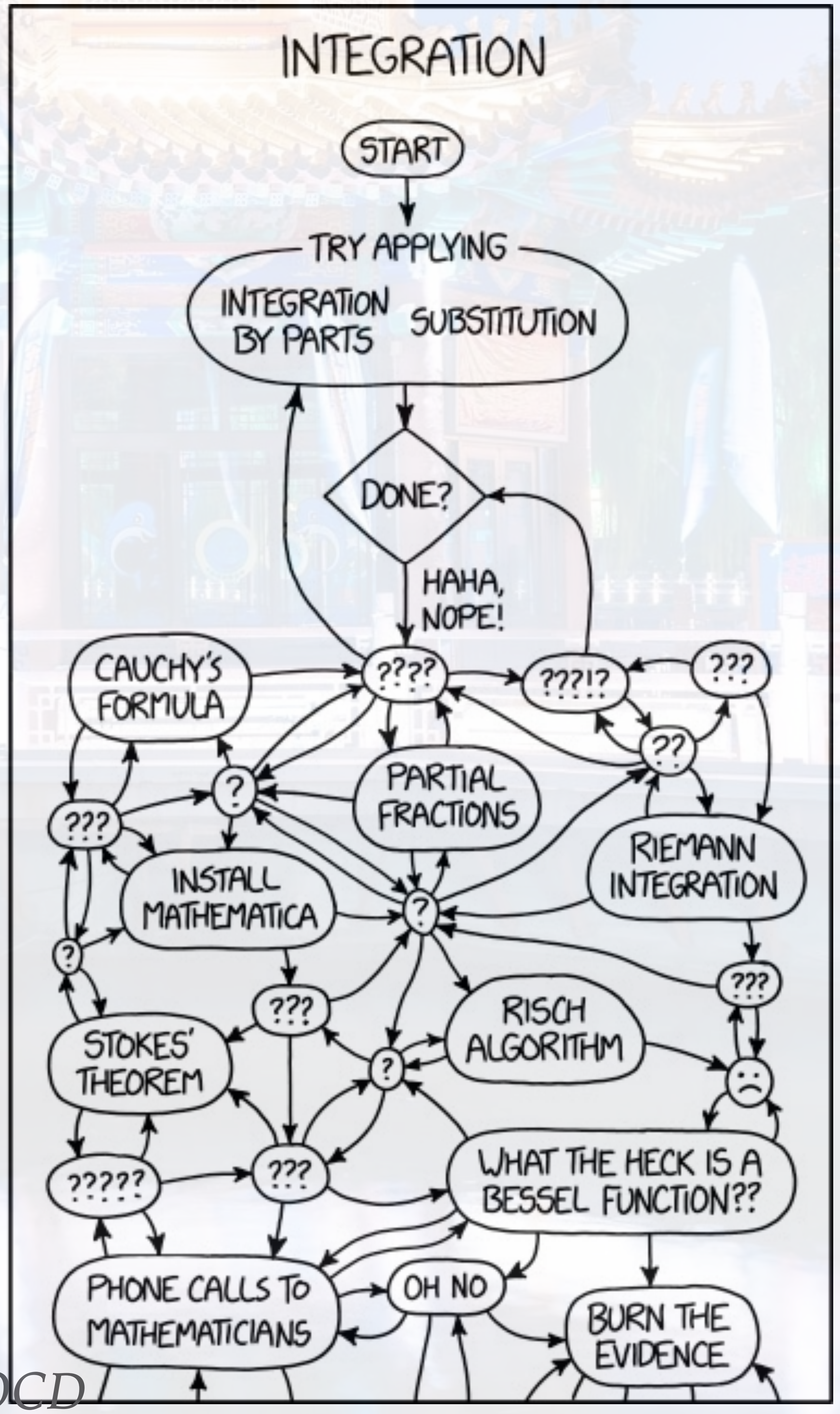
ANATOMY OF HARD SCATTERING $\hat{\sigma}_{ab}$

- 2-loop matrix elements
  - Due to the complexity and emerge of non-planar diagrams, we haven't find a finite set of master integrals .



$\mathscr{M}_5^{(2)}(1^+,2^+,3^+,4^+,5^+) =$





www.xkcd.com

ANATOMY OF HARD SCATTERING $\hat{\sigma}_{ab}$

- 2-loop matrix elements (State-of-the-art progress)
  - $2 \to 2$ processes in general known for massless loop (all master integral known analytically)

$pp \to p + p$ (C. Anastasiou, E.W.N. Glover, C. Oleari, M. Tejeida-Yeomans, Z. Bern, L. Dixon, A. De Freitas)

$pp \to V + p$ (L. Garland, E.W.N. Glover, T. Gehrmann, A. Koukoutsakis, E. Remiddi, L. Tancredi, E. Weihs)

$pp \to H + p$ (E.W.N. Glover, T. Gehrmann, M. Jaquier, A. Koukoutsakis)

$pp \to V + \gamma$ (T. Gehrmann, L. Tancredi)

$pp \to V + V\ (W^+W^-, ZZ, W^\pm Z)$ (F. Caola, T. Gehrmann, J. Henn, A. Von Manteuffel, K. Melnikov, A&V. Smirnov, L. Tancredi)

  - $2 \to 2$ processes cutting edge calculation with massive loop (numerical + analytical)

$pp \to t + \bar{t}$ (R. Bonciani, A. Ferroglia et al. 2011, M. Czakon, P. Fiedler and A. Mitov 2012, S. Di Vita, T. Gehrmann, S. Laporta et al. 2019)

$gg \to H + H$ via top-loop (S. Borowka, N. Greiner, G. Heinrich, S.P. Jones et al. 2016; J. Davies, G. Mishima, M. Steinhauser et al. 2018)

$pp \to H + p$ via top-loop (S.P. Jones, M. Kerner, G. Luisoni 2018, J. M. Lindert, K. Kudashkin et al. 2018, XC, A. Huss et al. 2021)

$pp \to H + p$ via bottom-loop (F. Caola, J. M. Lindert, K. Melnikov, P. F. Monni, L. Tancredi, C. Wever 2018)

$gg \to \gamma + \gamma$ via top-loop (F. Maltoni, M. K. Mandal, X. Zhao 2018, L. Chen, G. Heinrich, S. Jahn, S.P. Jones M. Kerner et al. 2019)

$gg \to Z + Z$ via top-loop (J. Davies, G. Mishima, M. Steinhauser, D. Wellmann 2020)

$gg \to Z + H$ via top-loop (L. Chen, G. Heinrich, S.P. Jones, M. Kerner. J. Klappert, J. Schlenk 2020, G. Wang, X. Xu, Y. Zu, L. Yang 2021)

ANATOMY OF HARD SCATTERING $\hat{\sigma}_{ab}$

- 2-loop matrix elements (State-of-the-art progress)
  - $2 \rightarrow 3$ processes cutting edge calculation with massless loop (all involves some numerical evaluation)

$pp \rightarrow p + p + p$ planar and some non-planar contribution (Abreu, Agarwal, S. Badger, Brønnum-Hansen, Buccioni, Chawdhry, Chicherin, Czakon, Dixon, Dormans, Febres Cordero, Gehrmann, Hartanto, Heinrich, Henn, Herrmann, Ita, Kraus, Kryś, Lo Presti, Mitev, Mitov, Mogull, Ochirov, O'Connell, Page, Papadopoulos, Pascual, Peraro, Poncelet, Ruf, Sotnikov,Tancredi, Tommasini, von Manteuffel, Wasser, Wever, Zeng, Zhang, Zoia et al … starting from 2014)

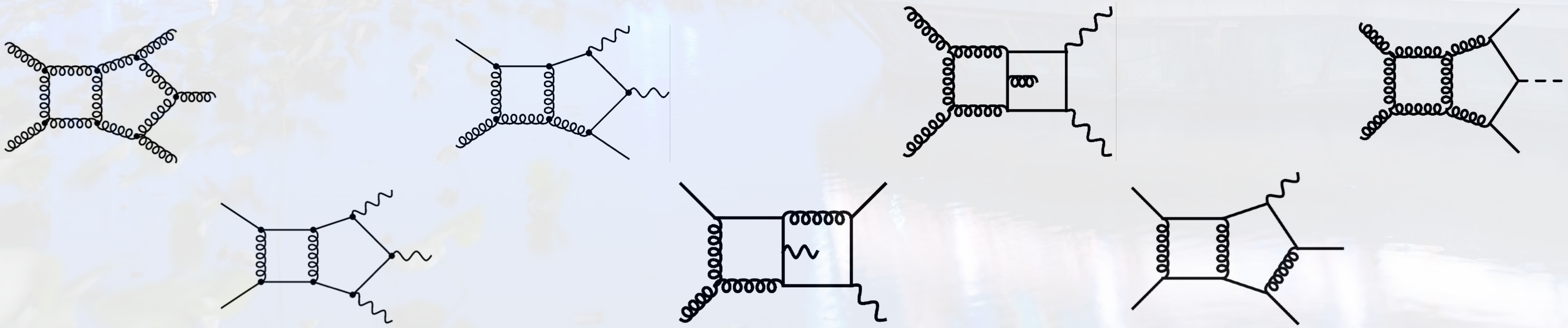
$pp \rightarrow \gamma + \gamma + \gamma$ (H. A. Chawdhry, M. Czakon, A. Mitov, R. Poncelet 2020-21, S. Abreu, B. Page, E. Pascual, V. Sotnikov 2020)

$pp \rightarrow \gamma + \gamma + p$ (B. Agarwal, F. Buccioni et al. 2021 H. A. Chawdhry, M. Czakon et al. 2021, S. Badger, C. Brønnum-Hansen et al. 2021)

$pp \rightarrow W + p + p$ planar (H. B. Hartanto, S. Badger, C. Brønnum-Hansen, S. Zoia 2019-21)

$pp \rightarrow H + p + p$ planar (S. Badger, H. B. Hartanto, J. Kryś, S. Zoia 2021)

$pp \rightarrow V + p + p$ planar and some non-planar (S. Abreu, F. Febres Cordero, H. Ita et al. 2021, D. Chicherin, V. Sotnikov, S. Zoia 2021)



ANATOMY OF HARD SCATTERING $\hat{\sigma}_{ab}$

- 3-loop matrix elements
  - Calculation is on process-by-process basis with similar technology as 2-loop calculations
  - $2 \to 1$ in general known for massless and some massive loop

    $gg \to H,\ qq \to V$ (P. A. Baikov, K.G. Chetyrkin, A&V Smirnov, M. Steinhauser 2009, T. Gehrmann, E.W.N. Glover, T. Huber et al. 2010, R. V. Harlander, M. Prausa, J. Usovitsch 2019, L. Chen, M. Czakon, M. Niggetiedt 2021)

    $gg \to H$ via top-loop (J. Davies, R. Grober, A. Maier, T. Rauh, M. Steinhauser 2019, M. Czakon, M. Niggetiedt 2020)
  - $2 \to 2$ at cutting edge calculation with massless loop

    $pp \to p + p$ all master integrals (V. A. Smirnov 2003, J. M. Henn, V&A. Smirnov 2013, J. M. Henn, B. Mistlberger et al. 2020)

    $pp \to V + p$ planar master integrals (S. Di Vita, P. Mastrolia, U. Schubert, V. Yundin 2014, D. D. Canko, N. Syrrakos et al. 2021)

    $qq \to q + q$ (F. Caola, A. Chakraborty, G. Gambuti, A. Von Manteuffel, L. Tancredi 2021)

    $gg \to g + g$ (F. Caola, A. Chakraborty, G. Gambuti, A. Von Manteuffel, L. Tancredi 2021)

    $q\bar{q} \to \gamma + \gamma$ (P. Bargieta, F. Caola, A. Von Manteuffel, L. Tancredi 2021)

F. Caola, A. Chakraborty, G. Gambuti, A. Von Manteuffel, L. Tancredi 2021