第四届量子场论及其应用研讨会 @ 广州, *Nov. 18, 2024*

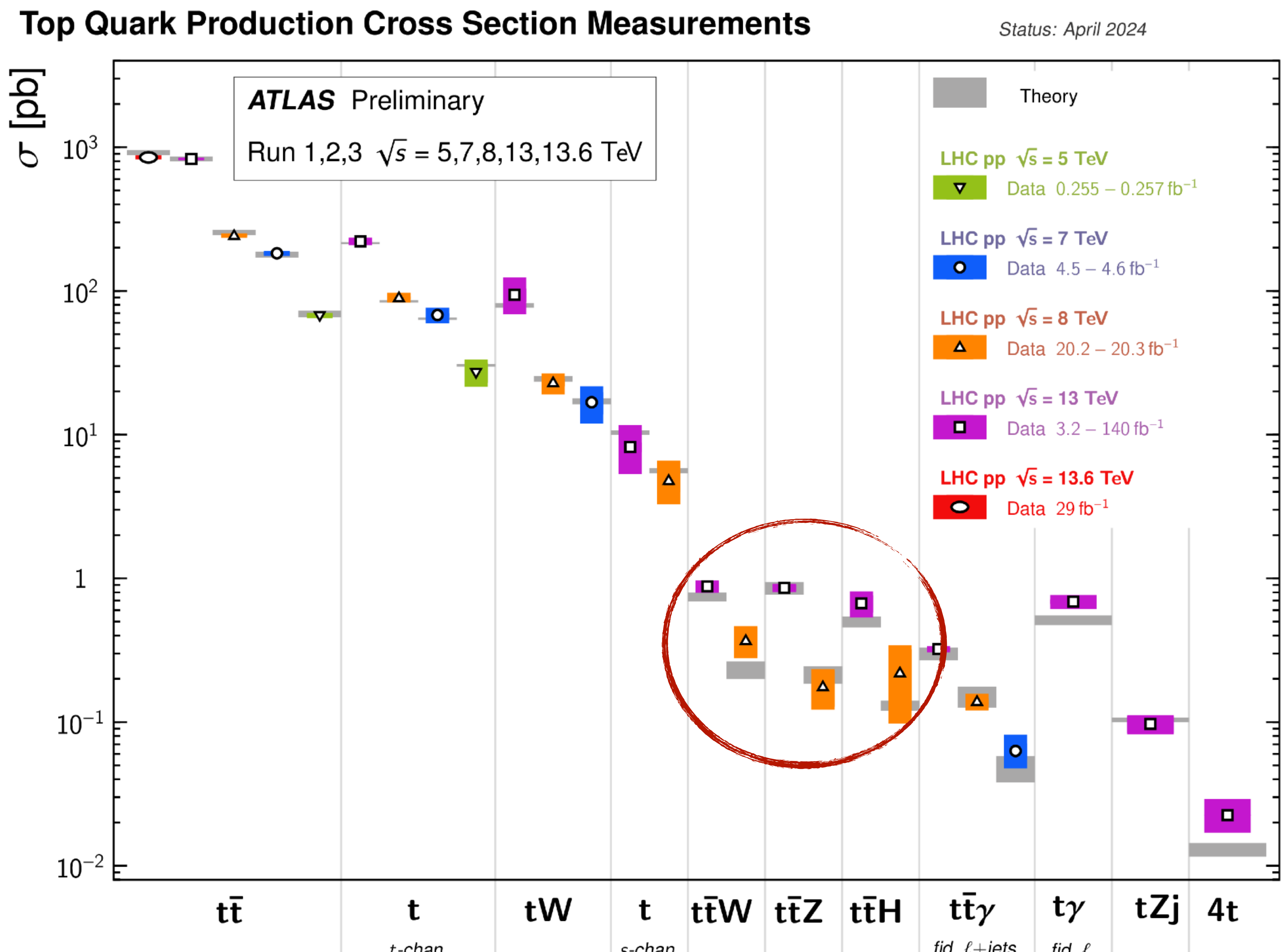
Exclusive Soft Function for Heavy Quark Pair Production

Based on 2411.XXXXX with Pier Monni

Ze Long Liu (刘泽茏)

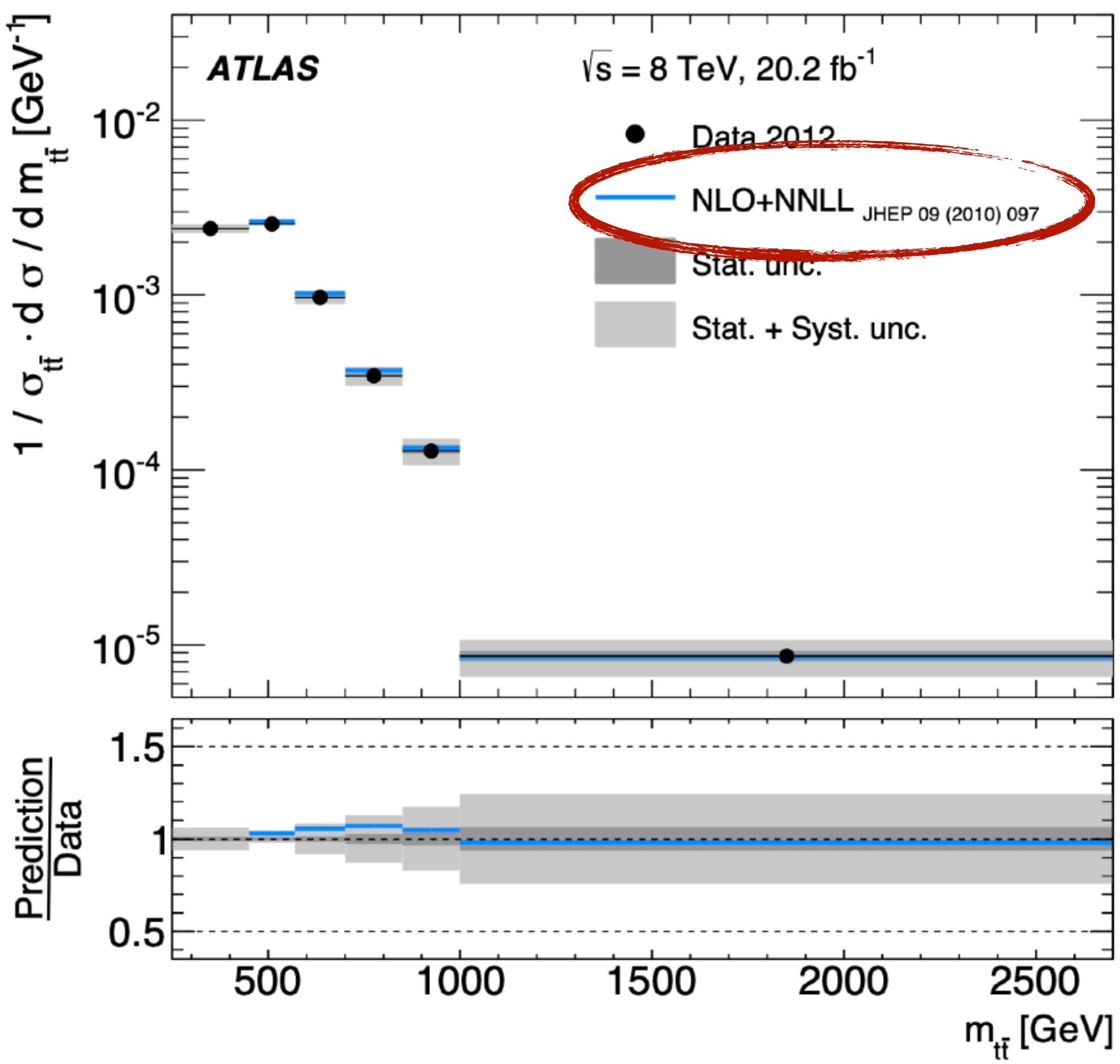
中国科学院高能物理研究所
Institute of High Energy Physics
Chinese Academy of Sciences

Measurements for Top Pair Production

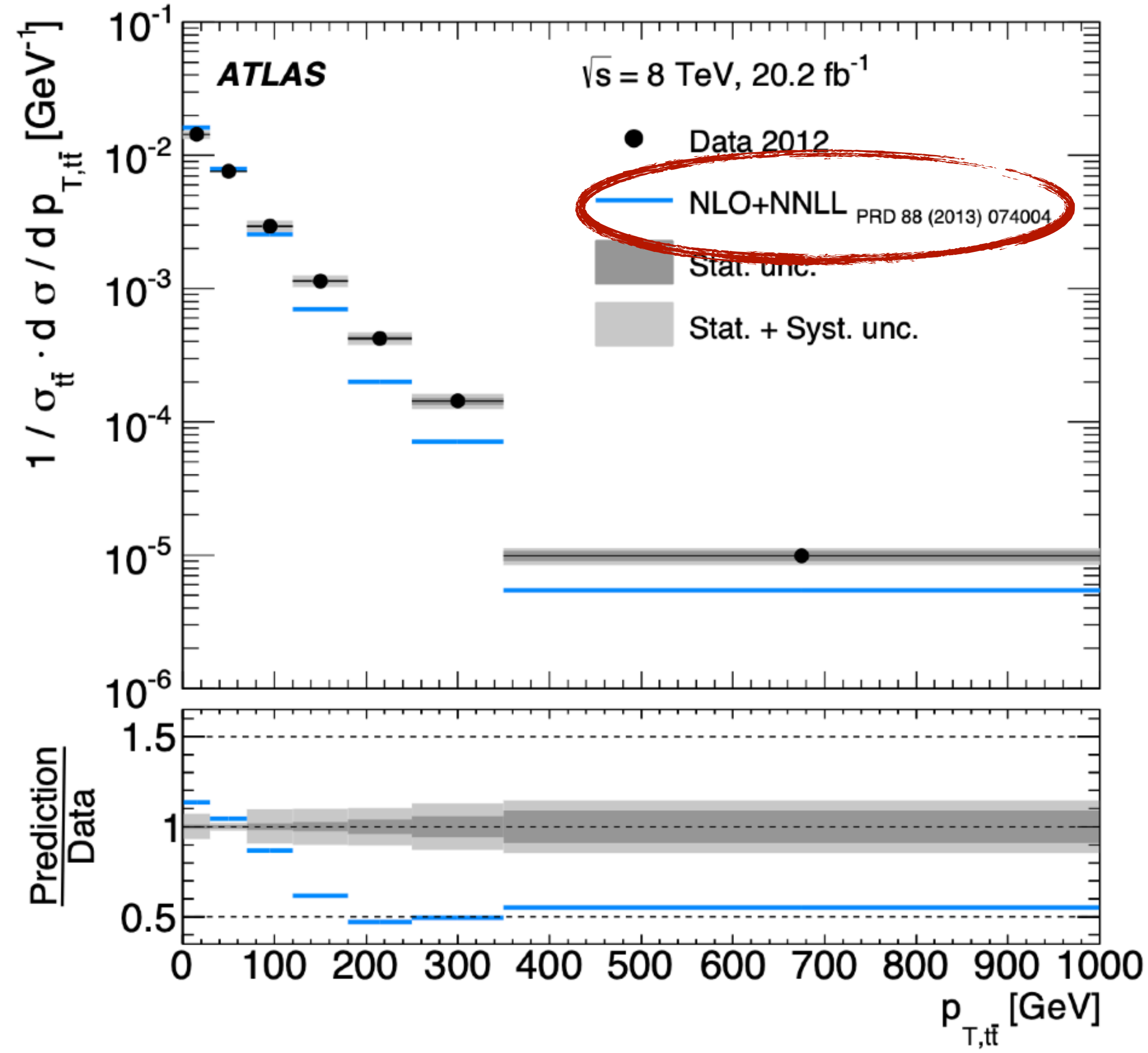


Differential Measurements

- RG improved : NLO QCD + NNLL



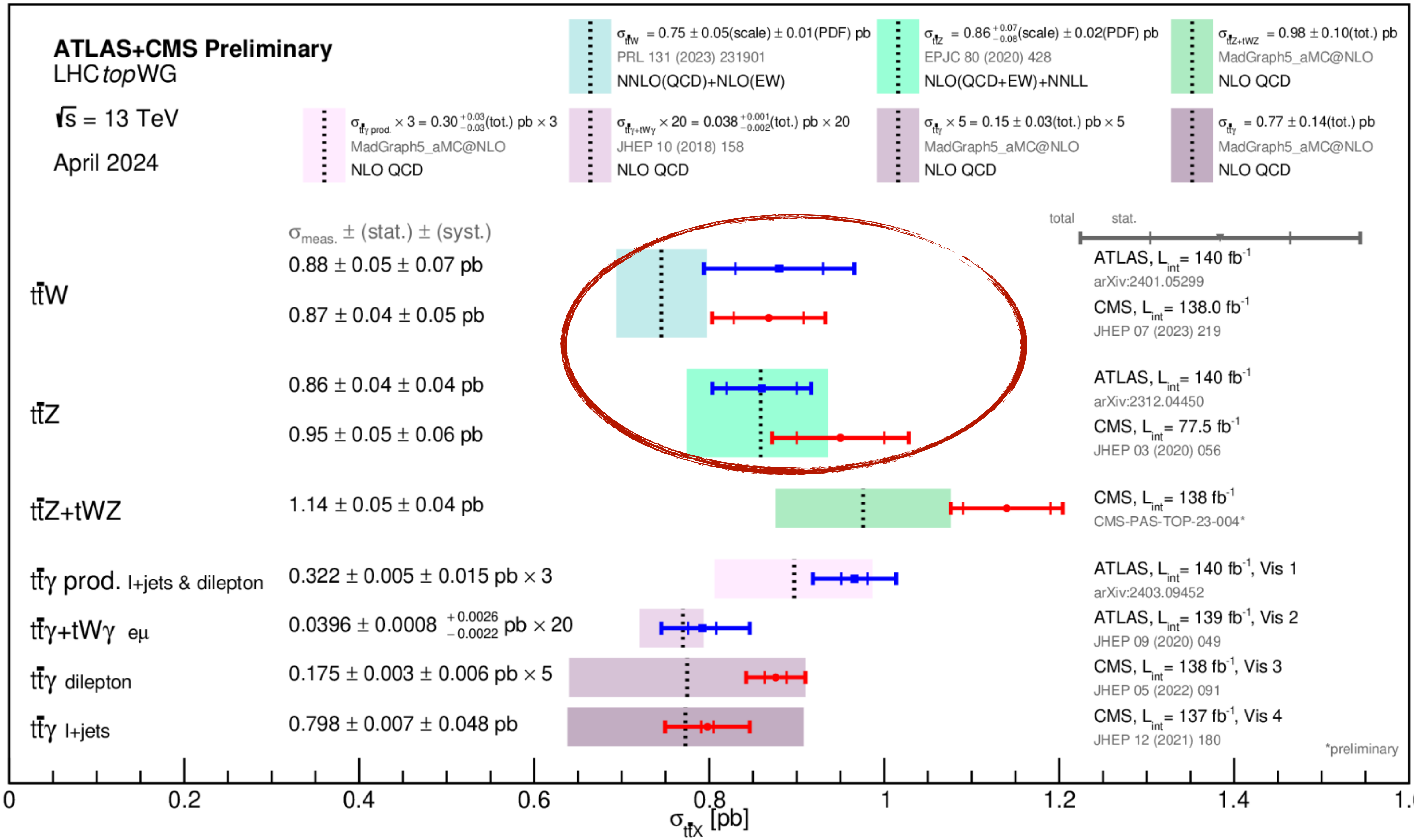
Ahrens, Ferroglia, Neubert, Pecjak, Yang '10



Zhu, Li, Li, Shao, Yang '12

- Fixed-order : NNLO QCD +NLO EW, differential Czakon, Mitov '12

Top Pair +W/Z Boson

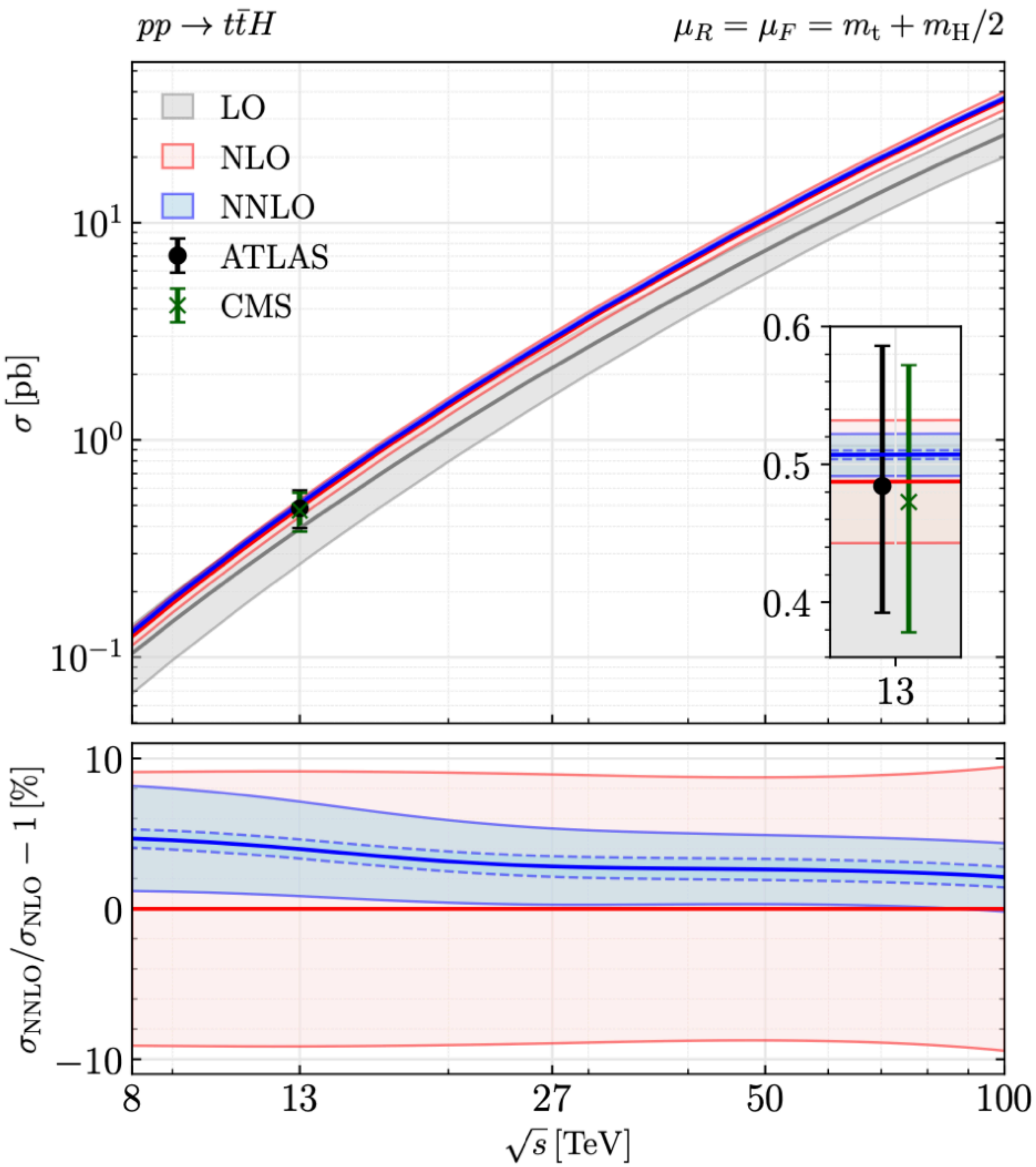


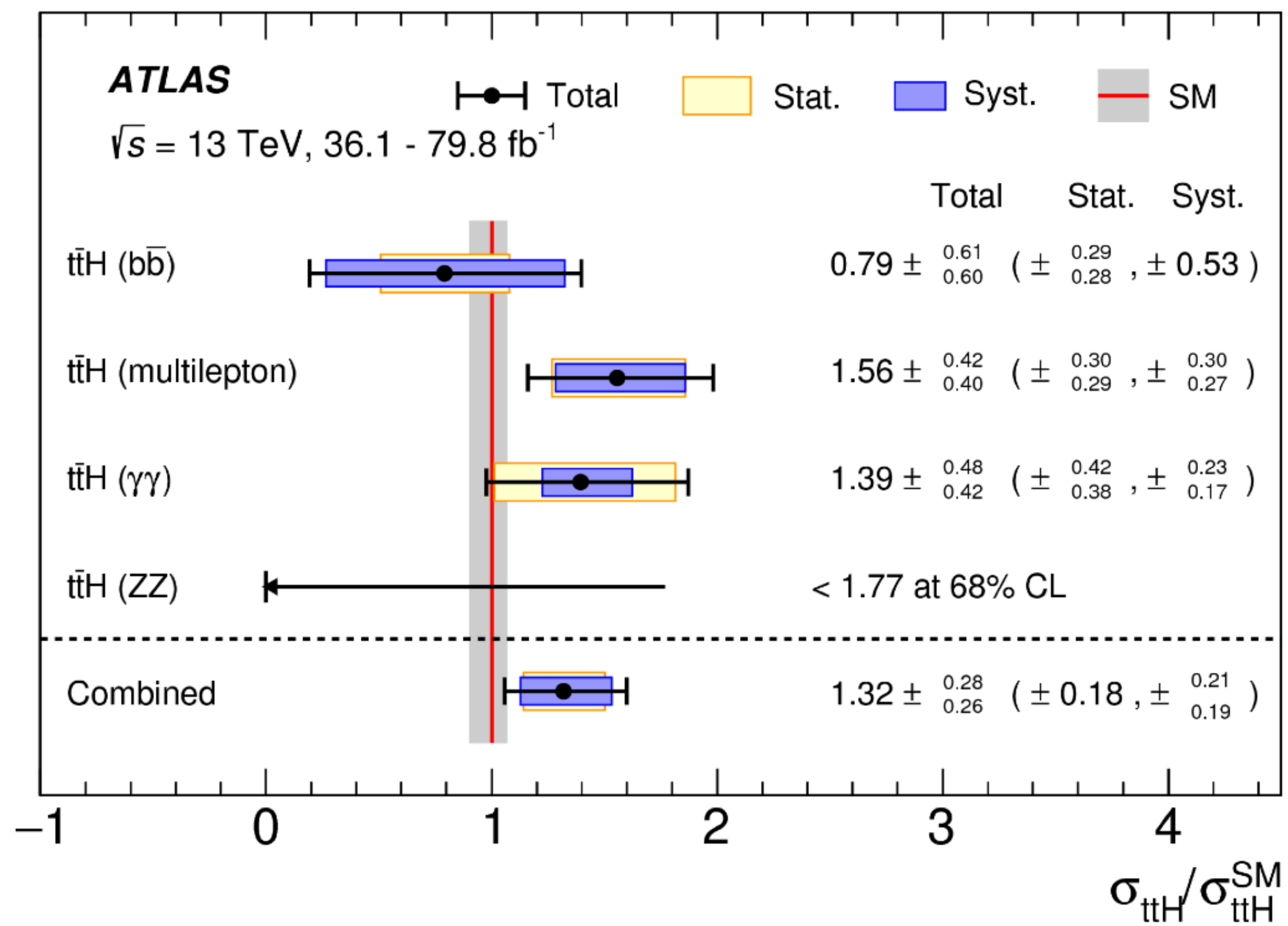
ttbar+W : NNLO QCD + NLO EW — Grazzini et al. '22

ttbar+Z : NLO (QCD + EW) +NNLL — Kulesza et al. '20

Top Pair + Higgs

- Top Pair + Higgs





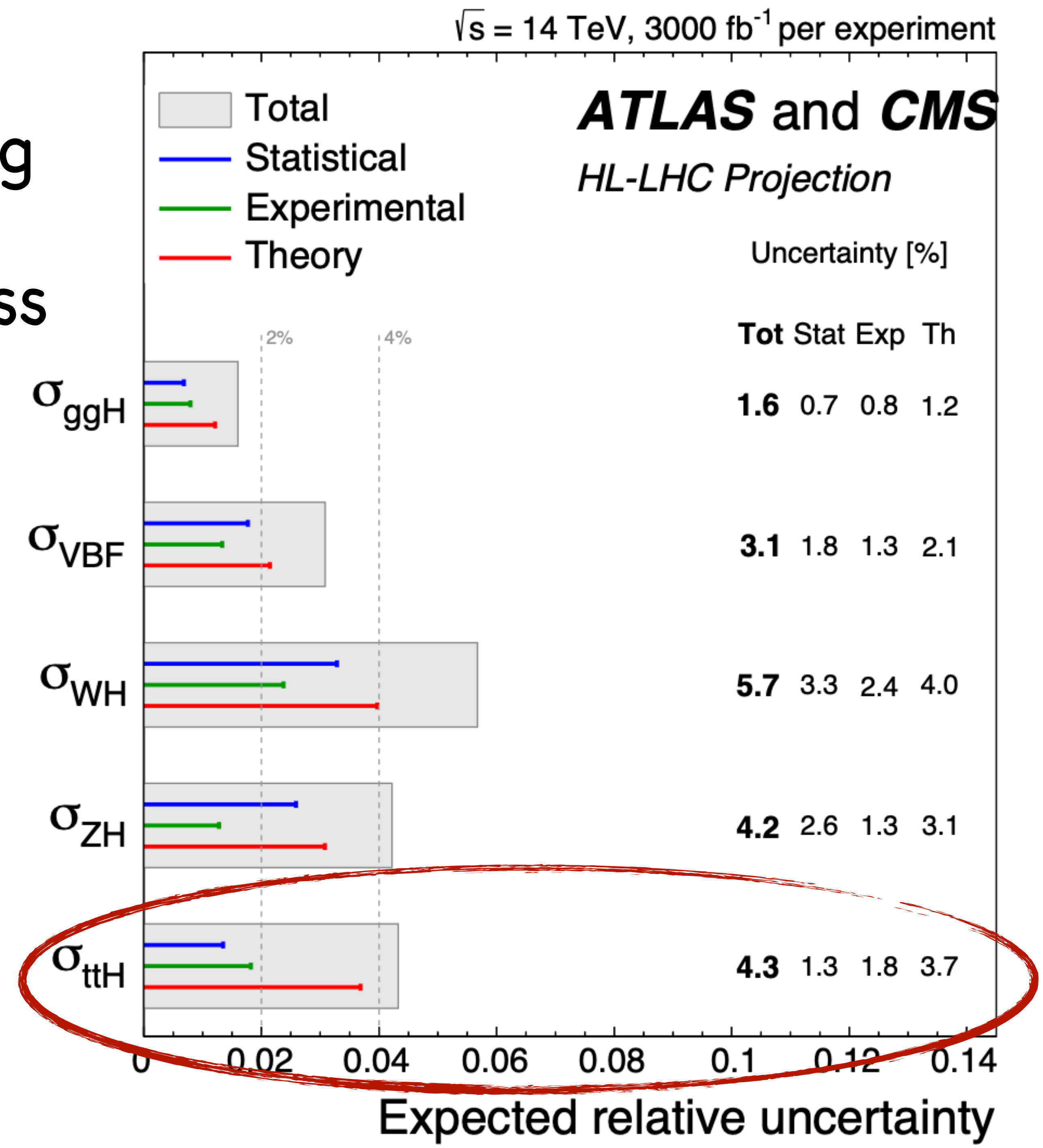
NLO+NNLL QCD

Broggio, Ferroglia, Pecjak, Yang ‘10

NNLO QCD Catani, Grazzini et al. ’22

Why Precision ?

- Determination of strong coupling
- Measurements of top quark mass
- Parton Distribution Functions
- Direct probe for Yukawa Coupling
- Higgs measurements
- Reduce background in BSM searches



Challenges towards NNLO and N3LL

- Two-loop virtual corrections for QQh
  - Soft Higgs/W approximation Catani et al. '22
  - Massification Mitov, Moch. '06 Becher, Melnikov. '07

Full two-loop amplitudes are still unknown !!!

L. Reina et al. '23

G. Heinrich et al. '24

- Subtraction of IR divergences
  - Local subtraction - STRIPPER Czakon '10
  - qT slicing Catani et al. '07 ←

$$\sigma^{\mathrm{NNLO}} = \int_0^{p_T} d\sigma_{t\bar{t}}^{\mathrm{NNLO}} + \int_{p_T} d\sigma_{t\bar{t}+j}^{\mathrm{NLO}}$$

Talk by D.Y. Shao

- More differential - **Large logs !**
  - Small pT
  - Boosted top L.L. Yang et al. '13-'19
  - Threshold enhancement
  - Jet veto - reduce background
- Anomalous Dimensions at 3- and 4-loops

Resummation

- For $p_T \ll Q$, perturbative convergence is violated by $L = \ln(p_T/Q)$
- A good parameter for expansion is $1/L$, not α_s $\qquad \alpha_s L \sim 1$
- Large logs should be resummed to all orders in α_s

$$\sigma(p_T) \sim \left[1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n c_n\right] \exp\left[L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots\right] + \mathcal{O}(p_T/Q)$$

LL $\quad$ NLL $\quad$ NNLL

- Resummation can be achieved by renormalization-group (RG) evolution in SCET

$$\sigma \sim H \otimes S \otimes B \otimes B$$

$$\frac{d}{d\ln\mu} H = \left(\Gamma_{\rm cusp} \ln\frac{Q}{\mu} + \gamma^H\right) H$$

	$\Gamma_{\rm cusp}$	$\gamma^{S/H/J}$	$c^{S/H/J}$
LL	1-loop	—	—
NLL	2-loop	1-loop	—
NNLL	3-loop	2-loop	1-loop

Soft Anomalous Dimensions

General formula of anomalous dimensions

$$\boldsymbol{\Gamma}(\{\underline{p}\},\{\underline{m}\},\mu) = \sum_{(i,j)} \frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\gamma_{\rm cusp}(\alpha_s)\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)\,\mathbf{1}$$

Becher, Neubert, 0901.0722, 0903.1126
Gardi, Magnea, 0901.1091

$$+\sum_{I,j}\boldsymbol{T}_I\cdot\boldsymbol{T}_j\,\gamma_{\rm cusp}(\alpha_s)\ln\frac{m_I\mu}{-s_{Ij}} - \sum_{(I,J)}\frac{\boldsymbol{T}_I\cdot\boldsymbol{T}_J}{2}\gamma_{\rm cusp}(\beta_{IJ},\alpha_s) + \sum_I \gamma^I(\alpha_s)\,\mathbf{1}$$

Becher, Neubert 0904.1021

$$+ f(\alpha_s)\sum_{(i,j,k)}\mathcal{T}_{iijk} + \sum_{(i,j,k,l)}\mathcal{T}_{ijkl}\,F_4(\beta_{ijkl},\beta_{ijkl}-2\beta_{ilkj},\alpha_s)$$

Almelid, Duhr, Gardi 1507.00047 + McLeod, White 1706.10162

← Starting at 3L

$$+\sum_I\sum_{(i,j)}\mathcal{T}_{ijII}\,F_{\rm h2}(r_{ijI},\alpha_s) + \sum_I\sum_{(i,j,k)}\mathcal{T}_{ijkI}\,F_{\rm h3}(r_{ijI},r_{ikI},r_{jkI},\alpha_s)$$

ZLL, Schalch 2207.02864

$$+\ [\text{non-dipole contributions involving two or more massive partons}]$$

Ferroglia, Neubert, Pecjak, Yang, '09 only up to two loops

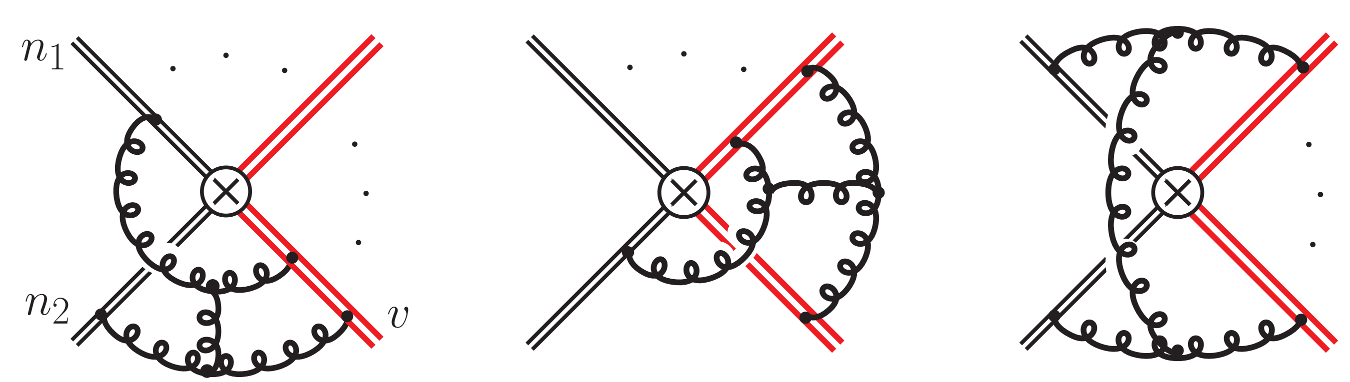
$$+\mathcal{O}(\alpha_s^4)$$

Boels, Huber, Yang, '17

violation of Casimir scaling due to $d_R^{abcd}\boldsymbol{T}_i^a\boldsymbol{T}_j^b\boldsymbol{T}_k^c\boldsymbol{T}_l^d$

- For N3LL resummation, tripole and quadrupole anomalous dimensions are required at 3 loops !!!

ZLL, Schalch, PRL 129 (2022) 23



Factorization

Threshold factorization

$$\frac{d\sigma}{dM_{t\bar{t}}d\Delta y_{t\bar{t}}} \sim \int_\tau^1 dz \int_{\tau/z}^1 \frac{dx}{x} f_{q/N_1}(x)\, f_{\bar{q}/N_2}(\tau/(xz))$$
$$\times \int dx^0\, e^{-i\sqrt{s}(1-z)x^0}\, \mathrm{tr}\left[\mathbf{H}(\{\underline{q}\})\, \mathbf{S}(\{x^0, \vec{x}=0\}, \{\underline{n}, \underline{v}\})\right]$$

in different positions

TMD factorization

$$\frac{d\sigma}{d^2p_\perp\, dM_{t\bar{t}}d\Delta y_{t\bar{t}}} \sim \int \frac{dz}{z} \int d^2x_\perp\, e^{-ip_\perp\cdot x_\perp} B_{q/N_1}(z, x_T^2) B_{\bar{q}/N_2}(\tau/z, x_T^2)\, \mathrm{tr}\left[\mathbf{H}(\{\underline{q}\})\, \mathbf{S}(x_\perp, \{\underline{n}, \underline{v}\})\right]$$

Luo, Yang, Zhu, Zhu '19 available at 3L

The soft function in position space is defined as

$$\mathbf{S}(x, \{\underline{n}, \underline{v}\}) = \langle 0 \left| \overline{\mathbf{T}} \left[Y_{n_1} Y_{n_2}^\dagger Y_{v_3} Y_{v_4}^\dagger \right](x)\ \mathbf{T}\left[Y_{n_1}^\dagger Y_{n_2} Y_{v_3}^\dagger Y_{v_4} \right](0) \right| 0 \rangle$$

With soft Wilson line $$Y_{v_i}(x) = \mathbf{P}\exp\left(-ig_s \int_0^\infty dt\ v_i \cdot A_s^a(x + tv_i)\ \mathbf{T}^a \right)$$

Exclusive Soft Function

Y. Li, S. Mantry, F. Petriello '11

If we keep the full dependence on x^μ Exclusive soft function !

$$\mathbf{S}(x,\{\underline{n},\underline{v}\}) = \langle 0 \left| \overline{\mathbf{T}} \left[Y_{n_1} Y^\dagger_{n_2} Y_{v_3} Y^\dagger_{v_4} \right] (x) \; \mathbf{T} \left[Y^\dagger_{n_1} Y_{n_2} Y^\dagger_{v_3} Y_{v_4} \right] (0) \right| 0 \rangle$$

and transform it into momentum space for convenience

$$\mathbf{S}(x,\{\underline{n},\underline{v}\}) = \int d\omega \; e^{i\omega t} \; \mathbf{S}(\omega,\{\eta,\underline{n},\underline{v}\})$$

with

$$\mathbf{S}(\omega,\{\eta,\underline{n},\underline{v}\}) = \langle 0 \left| \overline{\mathbf{T}} \left[Y_{n_1} Y^\dagger_{n_2} Y_{v_3} Y^\dagger_{v_4} \right] (0) \, \delta(\omega - \eta \cdot \hat{p}) \; \mathbf{T} \left[Y^\dagger_{n_1} Y_{n_2} Y^\dagger_{v_3} Y_{v_4} \right] (0) \right| 0 \rangle$$

Threshold : $\eta^\mu = (1,0,0,0) \,, \quad t = x^0$

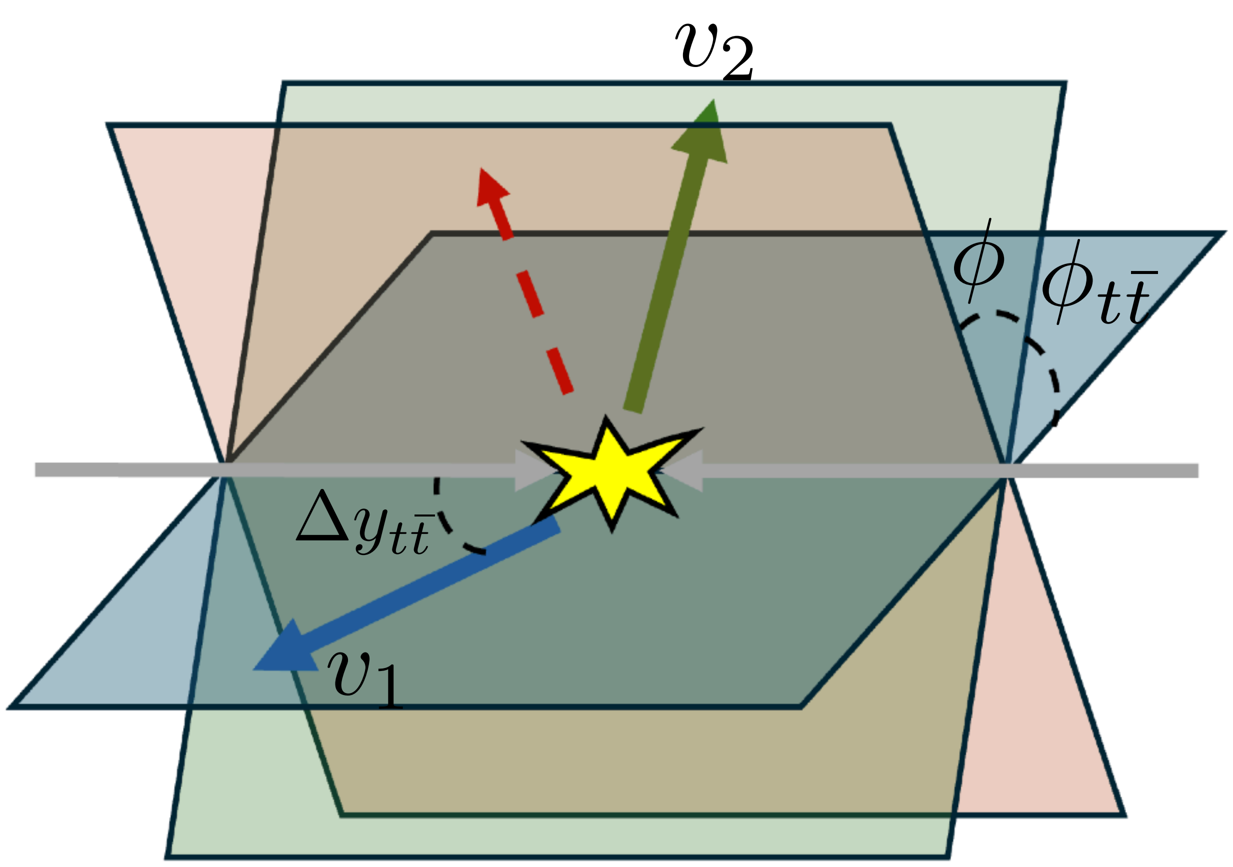
TMD : $\eta^\mu = (\tau/x_T, -i\cos\phi, -i\sin\phi, 0) \,, \qquad t = i\, x_T$

Regulator for rapidity divergences Y. Li, H.X. Zhu '16

Kinematics

- Consider $p + p \to t + \bar{t} + h + X$

1. Azimuthal angle of t-tbar $\phi_{t\bar{t}}$
2. Azimuthal angle of gluon-top ϕ
3. Difference of rapidity $\Delta y_{t\bar{t}}$
4. Velocity of top v_1
5. Velocity of anti-top v_2



The exclusive soft function depends on 5 kinematic variables

$$\{\cos\phi\,, \cos\phi_{t\bar{t}}\,, \Delta y_{t\bar{t}}\,, v_1^2\,, v_2^2\}$$

But they are not suitable for analytical calculation.

What are good variables for analytical calculation?

Kinematics

The soft Wilson line $$Y_{v_i}(x) = \mathbf{P}\exp\left(-ig_s\int_0^\infty dt\ v_i\cdot A_s^a(x+tv_i)\ \mathbf{T}^a\right)$$

is invariant under rescaling $v_i \to \alpha\, v_i$ cross ratios are preferred !!!

- 1 : massless dipole: $\frac{\eta^2(n_1\cdot n_2)}{(\eta\cdot n_1)(\eta\cdot n_2)}$
- 2 : $n_j - v_i$ dipole: $\frac{\eta\cdot v_i}{\sqrt{\eta^2}\sqrt{v_i^2}}$ $\frac{(v_i\cdot n_j)\sqrt{\eta^2}}{(\eta\cdot n_j)\sqrt{v_i^2}}$
- 3 : massive dipole: $\frac{\eta\cdot v_1}{\sqrt{\eta^2}\sqrt{v_1^2}}$ $\frac{\eta\cdot v_2}{\sqrt{\eta^2}\sqrt{v_2^2}}$ $\frac{v_1\cdot v_2}{\sqrt{v_1^2}\sqrt{v_2^2}}$
- 4 : $n_1 - n_2 - v_i$ tripole: $\frac{\eta^2(n_1\cdot n_2)}{(\eta\cdot n_1)(\eta\cdot n_2)}$ $\frac{\eta\cdot v_i}{\sqrt{\eta^2}\sqrt{v_i^2}}$ $\frac{(v_i\cdot n_1)\sqrt{\eta^2}}{(\eta\cdot n_1)\sqrt{v_i^2}}$ $\frac{(v_i\cdot n_2)\sqrt{\eta^2}}{(\eta\cdot n_2)\sqrt{v_i^2}}$
- 5 : $n_i - v_1 - v_2$ tripole:

$$\frac{\eta\cdot v_1}{\sqrt{\eta^2}\sqrt{v_1^2}}\quad \frac{\eta\cdot v_2}{\sqrt{\eta^2}\sqrt{v_2^2}}\quad \frac{v_1\cdot v_2}{\sqrt{v_1^2}\sqrt{v_2^2}}\quad \frac{(v_1\cdot n_i)\sqrt{\eta^2}}{(\eta\cdot n_i)\sqrt{v_1^2}}\quad \frac{(v_2\cdot n_i)\sqrt{\eta^2}}{(\eta\cdot n_i)\sqrt{v_2^2}}$$

Method

- Two contributions: Double real and real virtual
- IBP reduction with reverse unitary
- Differential equations (DEs)
- Canonical form : leading singularities and Magnus series expansion
- Rationalization
- Master integrals (MIs) are expressed by GPLs

Example: DE for MIs at NLO

$$\partial_z \vec{\mathcal{I}} = \epsilon \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\frac{1}{\sqrt{z^2-1}} & \frac{2z}{z^2-1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{4\sqrt{x^2-1}(yz-x)}{\sqrt{z^2-1}(x^2-2xyz+y^2+z^2-1)} & -\frac{4\sqrt{x^2-1}\sqrt{y^2-1}}{x^2-2xyz+y^2+z^2-1} & \frac{2(z-xy)}{x^2-2xyz+y^2+z^2-1} \end{pmatrix} \cdot \vec{\mathcal{I}}\,.$$

Challenge at NNLO

- Canonical form for DEs for all the cases are obtained!
- Mutilple square roots in DEs for tripole correlations , can NOT be rationalized !

Square roots appear in $n_i - v_1 - v_2$ tripole

$$\Big\{\sqrt{x^2-1}, \sqrt{y^2-1}, \sqrt{z^2-1}, \\ \sqrt{u^2z^2-u^2+2uwx-2uwyz-2uxz+2uy+w^2y^2-w^2-2wxy+2wz+x^2-1}\Big\}$$

First, we have to verify how many letters are involved in the amplitudes up to $\mathcal{O}(\epsilon^0)$ at NNLO

Soft function in Lepton Collisions

The renormalized soft function

Z.L. Liu, P. Monni, to appear

$$\tilde{S}(L) = 1 + \frac{\alpha_s}{4\pi}\left(-\Gamma_0^S L + c_1^S\right) + \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\left(\beta_0\Gamma_0^S + \frac{1}{2}\left(\Gamma_0^S\right)^2\right) L^2 - \left[c_1^S\left(\Gamma_0^S + 2\beta_0\right) + \Gamma_1^S\right] L + c_2^S\right]$$

The non-logarithmic term at NLO

$$\begin{aligned} c_1^s =& 8\Bigg(\frac{y}{\sqrt{y^2-1}} G_{1,t} + \frac{z}{\sqrt{z^2-1}} G_{1,u} \\ & - \frac{x}{\sqrt{x^2-1}} \Big[G_{\lambda_1,1,r} + G_{\lambda_2,1,r} + G_{\lambda_3,1,r} + G_{\lambda_4,1,r} - 2G_{0,1,r} - 2G_{2,1,r} \\ & + G_{1,t}\left(G_{\lambda_1,r} - G_{\lambda_2,r} - G_{\lambda_3,r} + G_{\lambda_4,r}\right) + G_{1,u}\left(G_{\lambda_1,r} - G_{\lambda_2,r} + G_{\lambda_3,r} - G_{\lambda_4,r}\right)\Big]\Bigg) \end{aligned}$$

There are 23 letters in GPLs for NNLO. 18 of them are the roots of

$$\begin{aligned} &\lambda^4 + \lambda^3(4yz-4) + 4\lambda^2\left(y^2 - 3yz + z^2 + 1\right) - 8\lambda(y-z)^2 + 4(y-z)^2 = 0\,, \\ &\lambda^4 + \lambda^3(4yz-4) + 4\lambda^2\left(z^2 - 3yz + 2\right) - 8\lambda\left(z^2 - 2yz + 1\right) + 4\left(z^2 - 2yz + 1\right) = 0\,, \\ &\lambda^4 + \lambda^3(4yz-4) + 4\lambda^2\left(y^2 - 3yz + 2\right) - 8\lambda\left(y^2 - 2yz + 1\right) + 4\left(y^2 - 2yz + 1\right) = 0\,, \\ &\lambda^4 + 4\lambda^3(y-1) - 12\lambda^2(y-1) + 16\lambda(y-1) - 8(y-1) = 0\,, \\ &\lambda^2 + 2\lambda(y-1) - 2(y-1) = 0 \end{aligned}$$

The NNLO expression is very long, but already available!!!

Summary

- Factorization and Resummation are required to match the precise experimental measurements
- Exclusive soft function fits to various observables
- Azimuthal angle dependence of the soft function is obtained for the first time
- more differential and more efficient

Thanks for your attention!