DiPion LCDAs and H_{l4} decays

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Overview

I LCDAs in the "hard" processes

II Dipion LCDAs at leading twist

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IV Conclusion and Prospect

LCDAs in the ”hard” processes

LCDAs in the "hard" processes

- The conformal group is the maximal extension of Poincaré group that leaves the light-cone invariant

  - the conformal algebra in four-dimension has 15 generators: translations $\mathbf{P}_\mu$, Lorentz rotation $\mathbf{M}_{\mu\nu}$, dilatation $\mathbf{D}$ and special conformal translation $\mathbf{K}_\mu$

- In the parton model, hadrons are replaced by a bunch of collinear partons, consider only the quantum field "living" on the light-ray $\Phi(x) \to \Phi(\alpha n)$

  - $\mathbf{P}_+$, $\mathbf{M}_{-+}$, $\mathbf{D}$, and $\mathbf{K}_-$, collinear subalgebra of the conformal algebra
  - introduce $\mathbf{L}_\pm = \mathbf{L}_1 \pm i\mathbf{L}_2 = -i\mathbf{P}_+(\mathbf{K}_-/2)$, $\mathbf{L}_0(\mathbf{E}) = i/2(\mathbf{D} \pm \mathbf{M}_{-+})$ satisfying $[\mathbf{L}_0, \mathbf{L}_\mp] = \mp\mathbf{L}_\mp$ and $[\mathbf{L}_-, \mathbf{L}_+] = -2\mathbf{L}_0$
  - $\mathbf{E}$ counts the collinear twist of the field Φ: $[\mathbf{E}, \Phi(\alpha)] = (l - s)/2\Phi(\alpha)$, commutes with all $\mathbf{L}_i$
  - the collinear twist (dimension - spin projection on the plus direction) is different from the geometric twist (dimension - spin) in full conformal algebra respecting full Lorentz symmetry
  - **DAs of definite collinear twist are well understood by the classification in terms of geometric twist**

- LCDAs is a use of conformal symmetry in massless QCD

  - keep in mind that the conformal symmetry of a quantum theory implies a vanished β-function
  - only the case for the free theory $\alpha_s \to 0$ and in the limit it essentially reduces to the parton model (large momentum transfers and large energies)

LCDAs in the "hard" processes

- In the "hard" processes, a certain hadron can be described by LCDAs at different (collinear) twist

- The structure of perturbative prediction for light-cone dominated processes reveals the underlying conformal symmetry of the QCD $\mathcal{L}$
  - pQCD is the calculation of the scale dependence (evolution equations) of physical observables
  - the evolution equations of DA, GPD can be understood as the RGE for the light-cone operators

- The conformal partial expansion of hadron distribution amplitudes
  - similar to partial-wave expansion of wave function in quantum mechanism
  - invariance of massless QCD under conformal trans. *VS* rotation symmetry
  - longitudinal $\otimes$ transversal dofs *VS* angular $\otimes$ radial dofs for spherically symmetry potential
  - transversal-momentum dependence (scale dependence) is governed by the RGE
  - longitudinal-momentum dependence (orthogonal polynomials) is described in terms of irreducible representations of the collinear subgroup of conformal group $SL(2,R) \cong SU(1,1) \cong SO(2,1)$

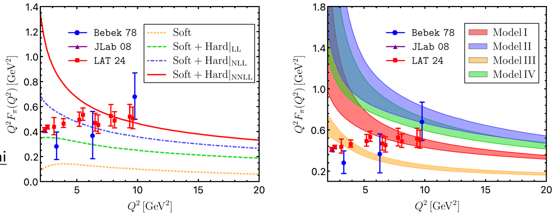
- This expansion was instrumental for the proof of the QCD factorization for the elastic and transition form factors

LCDAs in the "hard" processes

- **Example I Pion electromagnetic form factor**

  - N^2LO factorization at leading power of Λ^2_{QCD}/Q^2 expansion, the light-cone projections on the leading-twist-collinear operators [Chen[2], Feng, Jia 2312.17228]
  - rigorous two-loop computation of leading-twist contribution in the hard-collinear factorization [Ji, Shi, Wang[3], Yu 2411.03658]

see talk from Bo-xuan Shi



Model I [Khodjamirian, et al 2011.11275] Model II [SC, et al 2007.05550]
Model III [Stefanis 2006.10576] Model IV [Cloet, et al 2407.00206]

  - the N^2LO QCD correction to the short-distance coefficient function is enormous
  - large uncertainty from a_2, a_4 in pion meson LCDAs

LCDAs in the "hard" processes

- **Example II Exclusive heavy-to-light decays $B \to \pi l^+ \nu$ and $B \to \rho l^+ \nu$**

◦ proportional to $B \to \pi, \rho$ form factors obtained from LCSRs/LQCD

◦ $|V_{ub}|$ extracted from $B^0 \to \pi^-, \rho^0 l^+ \nu$ has $\sim 3\sigma$ deviation [Belle II 2407.17403]

$$|V_{ub}|_{B\to\pi l\nu} = (3.93 \pm 0.19 \pm 0.13 \pm 0.19(\text{theo})) \times 10^{-3} \quad [\text{LQCD}]$$
$$|V_{ub}|_{B\to\pi l\nu} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16(\text{theo})) \times 10^{-3} \quad [\text{LQCD + LCSRs}]$$
$$|V_{ub}|_{B\to\rho l\nu} = (3.19 \pm 0.12 \pm 0.18 \pm 0.26(\text{theo})) \times 10^{-3} \quad [\text{LCSRs}]$$

◦ LCSRs calculations do not consider the width effect of ρ in the $\pi\pi$ invariant mass spectral or optimistically estimates the uncertainty from B-meson LCDAs

- The introduction of Dipion LCDAs and the 2πDAs LCSRs
  [SC, Khodjamirian, Virto 1709.0173, SC 1901.06071]

◦ high partial waves give few percent contributions to $B \to \pi\pi$ form factors

◦ ρ', ρ'' and NR background contribute $\sim 20\% - 30\%$ to P-wave

◦ qualitatively explains the $|V_{ub}|$ tension obtained from $B \to \pi, \rho l\nu$

Dipion LCDAs at leading twist

Dipion LCDAs at leading twist

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\overline{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

- ◦ $n^2 = 0$, index f, f' respects the (anti-)quark flavor, a, b indicates the electric charge
- ◦ coefficient $\kappa_{+-/00} = 1$ and $\kappa_{+0} = \sqrt{2}$, $k = k_1 + k_2$ is the invariant mass of dipion state
- ◦ $\tau = 1/2, \tau^3/2$ corresponds to the isoscalar and isovector 2πDAs
- ◦ higher twist $\propto 1, \gamma_\mu\gamma_5$ have not been discussed yet, γ_5 vanishes due to P-parity conservation

- Three independent kinematic variables

- ◦ momentum fraction u carried by anti-quark respecting to the total momentum of DiPion state
- ◦ longitudinal momentum fraction carried by one pion $\zeta = k_1^+/k^+$, $2q\cdot\bar{k}\,(\propto 2\zeta - 1)$ and k^2

- Normalization conditions $\int_0^1 du\, \Phi_{\parallel}^{I=1}(u,\zeta,k^2) = (2\zeta - 1)F_\pi(k^2)$

$$\int_0^1 du\,(2u-1)\Phi_{\parallel}^{I=0}(u,\zeta,k^2) = -2M_2^{(\pi)}\zeta(1-\zeta)F_\pi^{\rm EMT}(k^2)$$

- ◦ $F_\pi^{em}(0) = 1$, $F_\pi^{\rm EMT}(0) = 1$, $M_2^{(\pi)}$ is the moments of SPDs

Dipion LCDAs at leading twist

- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=0,\mathrm{even}}^{\infty}\sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=1,\mathrm{odd}}^{\infty}\sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

- $B_{n\ell}(k^2,\mu)$ have similar scale dependence as the a_n of π,ρ,f_0 mesons

- Soft pion theorem relates the chirarlly even coefficients with a_n^π

$$\sum_{\ell=1}^{n+1} B_{n\ell}^{\parallel,I=1}(0) = a_n^\pi\,, \quad \sum_{\ell=0}^{n+1} B_{n\ell}^{\parallel,I=0}(0) = 0$$

- 2πDAs relate to the skewed parton distributions (SPDs) by crossing
  - moments of SPDs M_N^π is expressed in terms of $B_{nl}(k^2=0)$ in the forward limit

- In the vicinity of the resonance, 2πDAs reduce to the DAs of ρ/f_0
  - relation between the a_n^ρ and the coefficients $B_{n\ell}$
  - f_ρ relates to the imaginary part of $B_{nl}(m_\rho^2)$ divided by the strong coupling $g_{\rho\pi\pi}$

Dipion LCDAs at leading twist

- **What's the evolution from $4m_\pi^2$ to large k^2** $\mathcal{O}(m_c^2)$, furtherly to $\mathcal{O}(m_b\lambda_{\rm QCD})$?

◦ Watson theorem of π-π scattering amplitudes implies an intuitive way to express the imaginary part of 2πDAs, leads to the Omnés solution of N-subtracted DR for the coefficients

$$B^I_{n\ell}(k^2) = B^I_{n\ell}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B^I_{n\ell}(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\,\frac{\delta^I_\ell(s)}{s^N(s-k^2-i0)}\right]$$

- 2πDAs in a wide k^2 range is given by δ^I_ℓ and a few subtraction constants

- The subtraction constants of $B_{n\ell}(s)$ at low s (around the threshold)

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

◦ firstly studied in the low-energy EFT based on instanton vacuum [Polyakov '99]

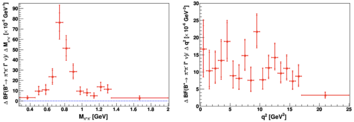
◦ updated with the kinematical constraints and the new a_2^π, a_2^ρ [SC '19,'23]

- We are now at leading twist, subleading twist LCDAs are not known yet

Semileptonic B_{l4} decays

$B \to \pi\pi$ form factors and $B \to [\rho^+ \to]\, \pi^+\pi^0 l\bar{\nu}$ decay

- H_{l4} decays have rich observables, nontrivial tests of SM [Faller '14]
- Different exclusive $b \to u$ processes help in the $|V_{ub}|$ **determination**



[Belle 2005.07766]

- ○ $B \to \rho l \bar{\nu}_l$ $(1.63 \pm 0.20) \times 10^{-4}$
  [BABAR '11, Belle '13, Belle II '24]
- ○ first measurement of $B^+ \to \pi^+\pi^- l^+ \bar{\nu}_l$
  $(2.3 \pm 0.4) \times 10^{-4}$ [Belle '20]
- ○ First measurement of $D^0 \to \pi^+\pi^- e^+ e^-$ [LHCb-PAPER-2024-047, prelim.]
- ○ $(4.53 \pm 1.38) \times 10^{-7}$ in ρ/ω and $(3.84 \pm 0.96) \times 10^{-7}$ in ϕ
- ○ **$c \to u$-typed FCNC** upper limit 0.7×10^{-5} [BES III '18]
- ○ $D^0 \to K^- \pi^0 \mu^+ \nu$ S-wave accounts $\sim 2.06\%$, $(0.729 \pm 0.014 \pm 0.011)\,\%$ [BESIII 2403.10877]

- Dynamics of B_{l4} is governed by the $B \to \pi\pi$ form factors
- A big task for the practitioners of QCD-based methods
- ○ First Lattice QCD study of the $B \to \pi\pi l\bar{\nu}$ transition amplitude in the region of large q^2 and $\pi\pi$ invariant mass near the ρ resonance [Leskovec et.al. 2212.08833[hep-lat]]

$B \to \pi\pi$ form factors and $B \to [\rho^+ \to]\, \pi^+\pi^0 l\bar{\nu}$ decay

- DiPion LCDAs will shine a light on **the width effect encounted in FP** (multibody B meson decays, $B \to [\pi\pi]\, l\nu$, $b \to sll$, $c \to ull$, $D\pi$ system $\cdots$) and **the controversial structure of scalar meson** ?

- How large of ρ contribution in P-wave $B \to \pi\pi$ transition ? How about the contributions from high partial waves ?

- $B \to \pi\pi$ **form factors** [Hambrock, Khodjamirian '15]

$$i\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma$$
$$+F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right)$$
$$+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right)$$

- ○ $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function
- ○ $q\cdot k = (m_B^2 - q^2 - k^2)/2$ and $q\cdot\bar{k} = \sqrt{\lambda}\beta_\pi(k^2)\cos\theta_\pi/2 = \sqrt{\lambda}\,(2\zeta - 1)$
- ○ $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$, θ_π is the angle between the 3-momenta of the neutral pion and the B-meson in the dipion rest frame

$B \to \pi\pi$ form factors

- Starting with the correlation function

$$F_\mu(k_1, k_2, q) = i\int d^4x e^{iq\cdot x}\langle \pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{j_\mu^{V-A}(x), j_5(0)\}|0\rangle$$
$$\equiv \varepsilon_{\mu\nu\rho\sigma} q^\nu k_1^\rho k_1^\sigma F^V + q_\mu F^{(A,q)} + k_\mu F^{(A,k)} + \bar{k}_\mu F^{(A,\bar{k})}$$

- Take $F_\perp^I(q^2, k^2, \zeta)$ as an example

$$\frac{F_\perp^I(q^2, k^2, \zeta)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{m_b}{\sqrt{2} f_B m_B^2 f_{2\pi}^\perp (2\zeta - 1)} \int_{u_0}^1 \frac{du}{u} \Phi_\perp^I(u, \zeta, k^2)\, e^{-\frac{s(u)+m_B^2}{M^2}}$$

- ◦ the Chiral-odd DiPion LCDAs $\Phi_\perp^{ab,ff'}(u, \zeta, k^2)$
- ◦ partial wave expansion $F_{\perp,\parallel}(k^2, q^2, \zeta) = \sum_\ell \sqrt{2\ell+1}\, F_{\perp,\parallel}^{(\ell)}(k^2, q^2)\, P_\ell^{(1)}(\cos\theta_\pi)/\sin\theta_\pi$

- The leading result is obtained by using the orthogonality relation of the Legender polynomials

$$F_\perp^{(\ell)}(k^2, q^2) = \frac{\sqrt{k^2}}{\sqrt{2} f_{2\pi}^\perp} \frac{\sqrt{\lambda_B} m_b}{m_B^2 f_B} e^{\frac{m_B^2}{M^2}} \sum_{n=0,2,\cdots} \sum_{\ell'=1,3}^{n+1} I_{\ell\ell'} B_{n\ell'}^\perp(k^2, \mu) J_n^\perp(q^2, k^2, M^2, s_0^B)$$

$$I_{\ell\ell'} \equiv -\frac{\sqrt{2\ell+1}(\ell-1!)}{2(\ell+1)!} \int_{-1}^1 \frac{dz}{z} \sqrt{1-z^2} P_\ell^{(1)}(z) P_{\ell'}^{(0)}(z)$$

$$J_n^\perp(q^2, k^2, M^2, s_0^B) = \int_{u_0}^1 du e^{\frac{-s}{M^2}} 6(1-u) C_n^{3/2}(2u-1)$$

- ◦ $I_{\ell\ell'} = 0$ when $\ell > \ell'$, $I_{11} = 1/\sqrt{3}$, $I_{13} = -1/\sqrt{3}$, $I_{15} = 4/(5\sqrt{3})$
- ◦ $\ell' = 1$, asymptotic DAs, P-wave term retains in the DiPion LCDAs

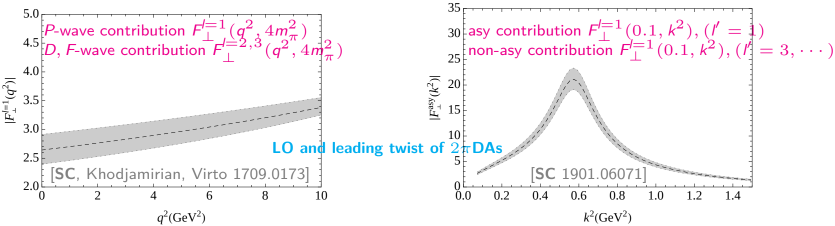
$B \to \pi\pi$ form factors

- How large of $P-$wave contribution to $B \to \pi\pi$ FFs $(\ell = 1)$?

$$R_\ell \equiv F_\perp^{(\ell>1)}(k^2, q^2)/F_\perp^{(\ell=1)}(k^2, q^2)$$

- How much ρ contained in $P-$wave $B \to \pi\pi$ FFs $(\ell = 1, \ell' = 1)$?

  ◦ Short-distance part of the correlation: $\mu = 3$ GeV without NLO correction
  $f_B = 207^{+9}_{-17}$ MeV, $M^2 = 16.0 \pm 4.0$ GeV^2 $\leftrightarrow$ $s_0^B = 37.5 \pm 2.5$ GeV^2 [P. Gelhausen '13]

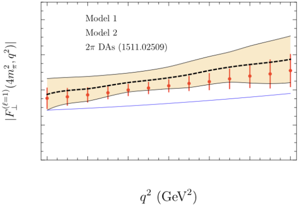


⋆ High partial waves give few percent contributions to $B \to \pi\pi$ form factors

⋆ ρ', ρ'' and NR background contribute $\sim 20\% - 30\%$ to P-wave

$B \to \pi\pi$ form factors

- 30% smaller than the result obtained from B-meson LCSRs
  [SC, Khodjamirian and Virto 1701.01663]
  - high twist contributions ?
  - large uncertainty from B-meson LCDAs ?



- Comparison with the B meson LCSRs

$$F_\mu(k_1, k_2, q) = i\int d^4x e^{iq\cdot x}\langle \pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{j_\mu^{V-A}(x), j_5^B(0)\}|0\rangle$$

$$F_{\mu\nu}(k, q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), \mathrm{T}\{j_\mu^{V-A}(x)\}|\bar{B}^0(q+k)\rangle$$

- At the current accuracy, they give same order plots of $B \to \pi^+\pi^0$ FFs
- For the $P-$wave FFs, they both predict sizable non-ρ contribution ($\sim 10\%$) ρ', ρ'', $\cdots$ and NR background
- B-meson LCSRs can not predict the contributions from higher partial waves, but it indeed exist while is small in the DiPion LCSRs
- B-meson LCSRs rely on the resonance model (an inverse problem), DiPion LCSRs is currently limited by the poor knowledge of DiPion LCDAs

Conclusion and Prospect

- The introduction of DiPion LCDAs provides an opportunity to study the width effects and the structures of nonstable mesons in H_{l4} processes
  - a new booster on the accurate calculation in flavor physics
  - improvement study in the CKM determinations and the flavor anomalies

- DiPion LCDAs study is at leading twist so far QCD definitions and double expansion
  - determine the parameters by low energy effective theory and data constraints
  - evolution of k^2 from the threshold to large scale $\mathcal{O}(m_c^2, m_b\lambda_{QCD})$
  - universal phase shift in $\pi\pi$ scattering and heavy decay ?

- Go further to high twist LCDAs, not only to match the precise measurement
  - $B \to \pi\pi l\nu, B \to [\rho\rho \to] \to 4\pi,\ D_s \to \pi\pi l\nu, D \to K\pi\mu\nu,\ D \to \pi\pi e^+e^-$ et al.

Thank you for your patience.