第四届量子场论及其应用研讨会

A try for UV divergences in QFT: The Higgs mass, Einstein gravity and dark energy

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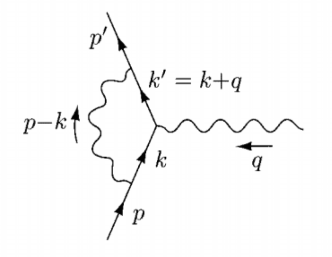
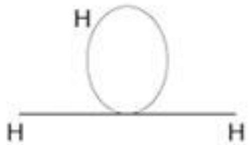
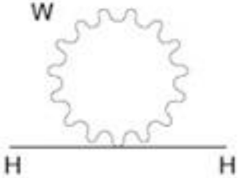
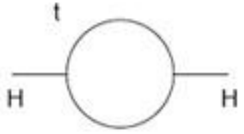
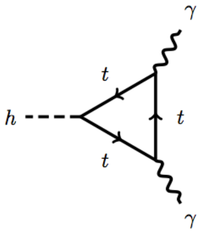
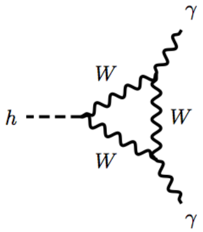








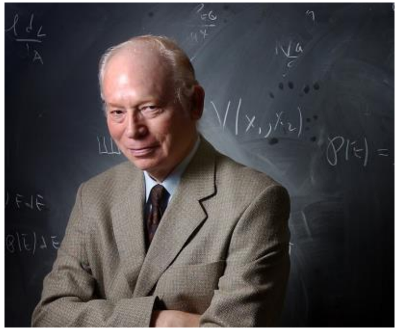




Outline:

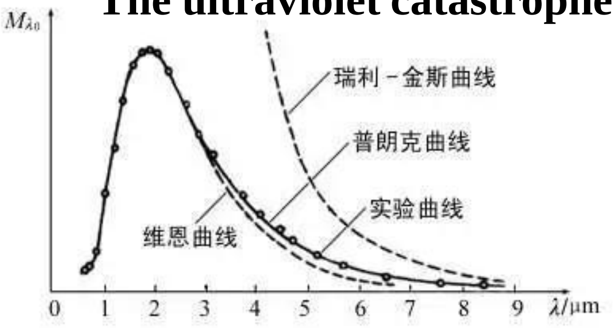
- **I. Background**
- **II. Free flow of ideas**
- **III. Graviton loop in Einstein gravity**
- **IV. Dark energy in QFT**
- **V. Summary and outlook**

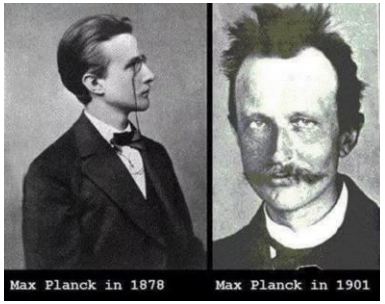
I. Background: UV problems



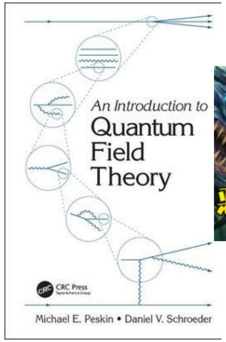
Physics thrives on crisis

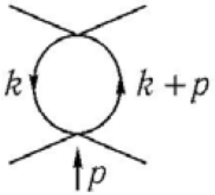
The ultraviolet catastrophe





UV divergences of loops





UV divergence!

$\mathcal{T}_F$

Not well-defined

Paradigm procedure

Devil

∞

−

Devil

∞

Regularization Renormalization

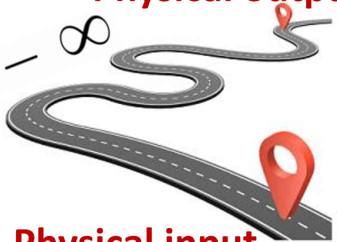
Devil or Angel?

=

Angel

∞ − ∞

Physical output



Physical input

Loop road

I. Background: UV problems

Renormalization

$\infty - \infty$

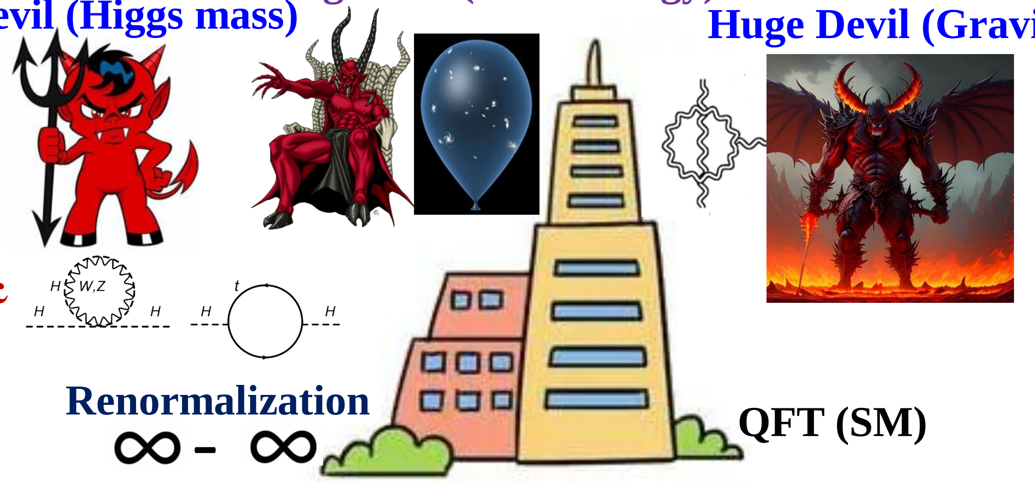
So far so good. **But** …

Log divergence is OK

Step out of the log zone

Power-law divergences are problematic

Three devils over the renormalization building



Three UV problems (power-law divergences):

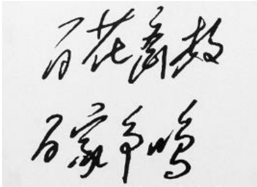
(1) Higgs mass (125GeV)

(2) Einstein gravity (non-renormalizable)

(3) Dark energy (10^{120})

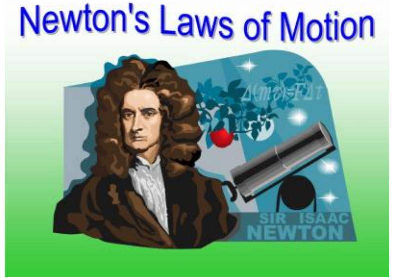
Next

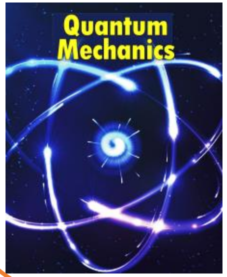
II. Free flow of ideas
--- UV-free scheme



UV-free scheme

arXiv:2305.18104





Negligible

Low-energy corrections

Loop

UV regions (Planck scale)

Loop

Negligible?!

Regularization & renormalization

A presumption:

Local

Physical contributions of loops are finite with contributions from UV regions of momenta being insignificant.

To obtain the physical results of loops, an equation is introduced

$$\mathcal{T}_{\rm F} \Rightarrow \mathcal{T}_{\rm P} = \left[\int d\xi_1 \cdots d\xi_i \frac{\partial \mathcal{T}_{\rm F}(\xi_1, \cdots, \xi_i)}{\partial \xi_1 \cdots \partial \xi_i}\right]_{\{\xi_1,\cdots,\xi_i\}\to 0} + C$$

(primary antiderivative + boundary constant)

or $$\mathcal{T}_{\rm P} = \left[\int (d\xi)^n \frac{\partial^n \mathcal{T}_{\rm F}(\xi)}{\partial \xi^n}\right]_{\xi\to 0} + C,$$

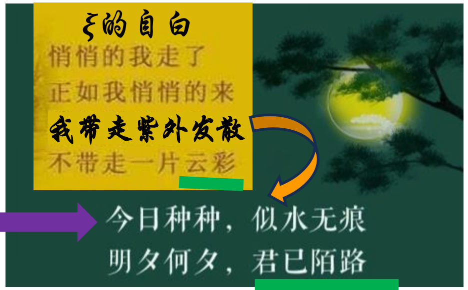
(Similar to $E_p = -\frac{GMm}{r}$+C)

A conceptual breakthrough:

P-V

Derivative method

$\mathcal{T}_{\rm F}$



To a new route: It first reproduces the successful renormalization results, and then gives an explanation to the three UV problems.

$\mathcal{T}_{\rm P}$

$e.g.$ $f(\mathrm{x}) = 1 + x^1 + x^2 + x^3 + x^4 + \ldots$ $\Rightarrow$ $\frac{1}{1-x}$

UV-free scheme:

assume that the physical transition amplitude $\mathcal{T}_\mathrm{P}$ with propagators can be described by an equation of

$$\mathcal{T}_\mathrm{P} = \Big[\int d\xi_1 \cdots d\xi_i \frac{\partial \mathcal{T}_\mathrm{F}(\xi_1, \cdots, \xi_i)}{\partial \xi_1 \cdots \partial \xi_i}\Big]_{\{\xi_1, \cdots, \xi_i\} \to 0} + C \,, (1)$$

a. Tree-level:

the photon propagator $\frac{-ig_{\mu\nu}}{p^2+i\epsilon}$,

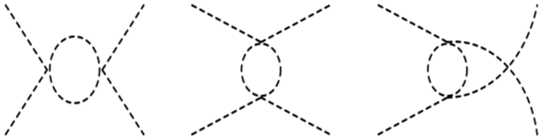
$$\mathcal{T}_\mathrm{F}(\xi) = \frac{-ig_{\mu\nu}}{p^2+\xi+i\epsilon}, \quad \frac{\partial \mathcal{T}_\mathrm{F}(\xi)}{\partial \xi} = \frac{-ig_{\mu\nu}(-1)}{(p^2+\xi+i\epsilon)^2},$$

$$\Big[\int d\xi \frac{\partial \mathcal{T}_\mathrm{F}(\xi)}{\partial \xi}\Big] = \frac{-ig_{\mu\nu}}{p^2+\xi+i\epsilon}, \quad \text{with } \; C = 0$$

$$\mathcal{T}_\mathrm{P} = \Big[\int d\xi \frac{\partial \mathcal{T}_\mathrm{F}(\xi)}{\partial \xi}\Big]_{\xi \to 0} = \frac{-ig_{\mu\nu}}{p^2+i\epsilon}$$

the gauge field propagator restored

b. Loop-level Log:



ϕ^4 theory

The physical scattering amplitude

$$\mathcal{T}_\mathrm{P}(s) = \Big[\int d\xi \frac{\partial \mathcal{T}_\mathrm{F}(\xi)}{\partial \xi}\Big]_{\xi \to 0} + C_1$$

$$= \Big[\frac{-\lambda^2}{2}\int d\xi \int \frac{d^4k}{(2\pi)^4} \frac{-i}{(k^2-m^2+\xi)^2} \frac{i}{(k+q)^2-m^2}\Big]_{\xi \to 0} + C_1 \,,$$

$$\mathcal{T}_\mathrm{P}(s) = \frac{-i\lambda^2}{32\pi^2}\int_0^1 dx \log[m^2 - x(1-x)s] + C_1 \,.$$

A freedom of ξ in propagators

Considering the renormalization conditions, $s = 4m^2$, $t = u = 0$. ➡ $C_1 = \frac{i\lambda^2}{32\pi^2}\int_0^1 dx \log[m^2 - 4m^2 x(1-x)] \,.$

No troublesome UV divergence in loop calculations!

In massless limit $\mathcal{T}_{\mathrm{P}} = \mathcal{T}_{\mathrm{P}}(s) + \mathcal{T}_{\mathrm{P}}(t) + \mathcal{T}_{\mathrm{P}}(u)$

$$s = -t = -u = \mu^2 \quad = \frac{i\lambda^2}{32\pi^2}\Big(\log\frac{\mu^2}{s} + \log\frac{\mu^2}{-t} + \log\frac{\mu^2}{-u}\Big)$$

the n-point physical correlation function $G_{\mathrm{P}}^{(n)}$ can be set by the physical field $\phi_{\mathrm{P}}(x)$ with $\phi_{\mathrm{P}}(x) = Z^{1/2}\phi(x,\mu)$, and the rescaling factor Z is finite here. The local correlation function $G^{(n)}$ (shorthand for a full expression $G^{(n)}(\phi, \lambda, m, \cdots, \mu)$) in the perturbation expansion can be written as $G^{(n)} = Z^{-n/2} G_{\mathrm{P}}^{(n)}$. Considering $\frac{dG_{\mathrm{P}}^{(n)}}{d\mu} = 0$, the variation of μ in the massless limit can be described by a relation

$$(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial\lambda} + n\gamma)G^{(n)} = 0\,.$$

This is the form of the Callan-Symanzik equation [5, 6], and we have another picture about it in UV-free scheme. The μ-dependent term in UV-free scheme is from the boundary constant C. For the ϕ^4 theory in the massless limit, the one-loop result of the parameter γ is zero ($\mathcal{T}_{\mathrm{P}}^{2p} = 0$). The beta function can be derived by Eq. (10), with the result

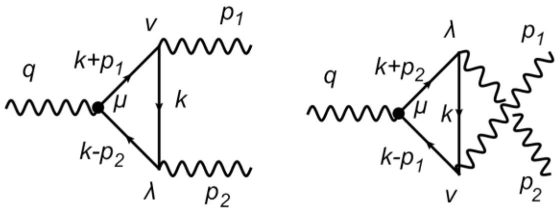
$$\beta = -i\mu\frac{\partial}{\partial\mu}\mathcal{T}_{\mathrm{P}} = \frac{3\lambda^2}{16\pi^2} + \mathcal{O}(\lambda^3)\,.$$

…

UV physics being free

An illustration:

electron physical charge $e = e_0 + \Delta e = e_\mu + \Delta e_\mu$



γ^5 the original form

$$\partial_\mu j^{\mu 5} = i q_\mu \mathcal{T}_{\mathrm{P}}^{\mu\nu\lambda}\epsilon_\nu^*(p_1)\epsilon_\lambda^*(p_2) = -\frac{e^2}{16\pi^2}(\frac{2}{3} - 2\log r)\varepsilon^{\alpha\nu\beta\lambda}F_{\alpha\nu}F_{\beta\lambda}$$

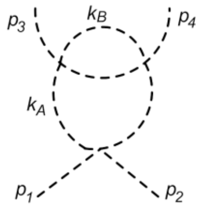
Taking $C_0 = 2\log r$

If $C_0 = -\frac{1}{3}$

SM self-consistent

charge values of quarks coincidence, or correlation?

two-loop transition



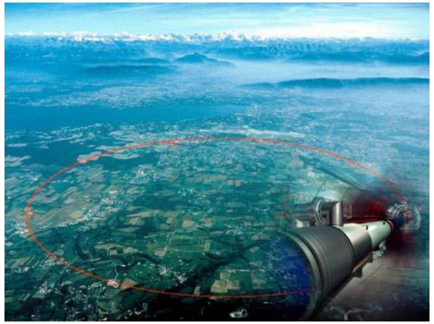
$$\mathcal{T}_{\mathrm{P}} = \Big[\int d\xi \frac{\partial \mathcal{T}_{\mathrm{F}}(\xi)}{\partial\xi}\Big]_{\xi\to 0} + C$$

$$= \Big[\frac{(-i\lambda)^3}{2}\int d\xi \int\frac{d^4k_A}{(2\pi)^4}\frac{d^4k_B}{(2\pi)^4}\frac{i}{k_A^2 - m^2}\frac{i}{(k_A+q)^2 - m^2} \times\frac{-i}{(k_B^2 - m^2 + \xi)^2}\frac{i}{(k_B + k_A + p_3)^2 - m^2}\Big]_{\xi\to 0} + C$$

with $q = p_1 + p_2$

Log divergences are OK

The hierarchy problem (c. Loop-level Λ^2, Λ^4)

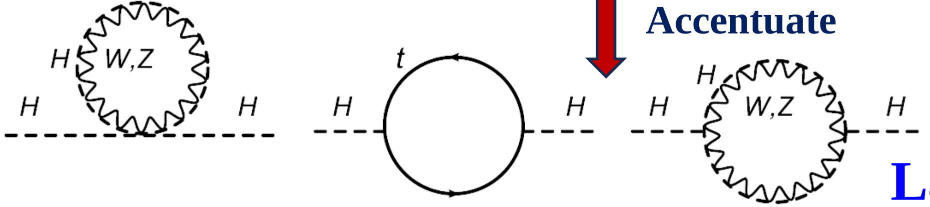


LHC

Higgs boson

125 GeV

Accentuate



The hierarchy problem

$$M_H^2 = (M_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left[M_H^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right]$$

Large Devil (Higgs mass)

Fine-tuning!

A real problem for renormalization!

Higgs in the first diagram

$$\mathcal{T}_{\rm P}^{H1} = \left[\int d\xi_1 d\xi_2 \frac{\partial \mathcal{T}_{\rm F}^{H1}(\xi_1,\xi_2)}{\partial\xi_1\partial\xi_2}\right]_{\{\xi_1,\xi_2\}\to 0} + C$$
$$= \left[(-3i)\frac{m_H^2}{2v^2}\int d\xi_1 d\xi_2 \int \frac{d^4k}{(2\pi)^4} \times \frac{2i}{(k^2 - m_H^2 + \xi_1 + \xi_2)^3}\right]_{\{\xi_1,\xi_2\}\to 0} + C\,.$$

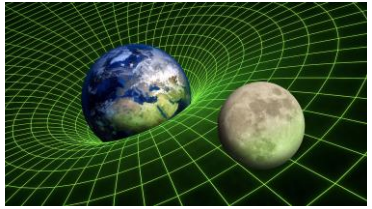
After integral, one has

$$\mathcal{T}_{\rm P}^{H1} = i\frac{3m_H^4}{32\pi^2 v^2}\left(\log\frac{1}{m_H^2} + 1\right) + C$$
$$= i\frac{3m_H^4}{32\pi^2 v^2}\left(\log\frac{\mu^2}{m_H^2} + 1\right).$$

125 GeV Higgs can be obtained without fine-tuning, i.e., an alternative interpretation within SM.

III. Graviton loop in Einstein gravity

$$\mathcal{S}=\int d^4X\sqrt{-g}\,[-\frac{2}{\kappa^2}R+\mathcal{L}_{\rm M}] \qquad g_{\mu\nu}=\eta_{\mu\nu}+\kappa h_{\mu\nu}$$



Huge Devil (Gravity)

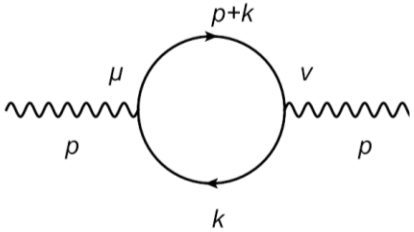
Loops: **Renormalization** → ✗ **Non-renormalizable Einstein Gravity**

? → **Einstein Gravity cannot be quantized!?**

Plan B: **Another alternative method**
UV-free scheme

For the primary antiderivative $\xi-$**dependent choice**

$$\mathcal{T}_{\rm P}^{t2n}=A\Big[\frac{(\xi+\Delta)^n}{n!}(\log|\xi+\Delta|-(\sum_{l=1}^{n}\frac{1}{l}))\Big]_{\xi\to 0}+C_1$$
$$=A\frac{\Delta^n}{n!}\log|\Delta|+C\,.$$

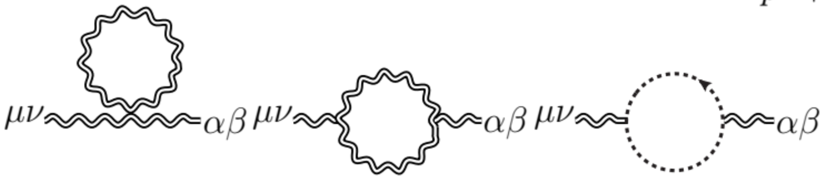


$$\mathcal{T}_{\rm P}^{\mu\nu}=-\frac{ie^2}{2\pi^2}\int_0^1 dx(p^\mu p^\nu-g^{\mu\nu}p^2)x(1-x)$$
$$\times\log(m^2-p^2x(1-x))+C^{\mu\nu}\,,$$

with the Ward identity automatically preserved by the primary antiderivative.

One-loop propagator

$$\frac{i\Pi_{\mu\nu\alpha\beta}/2}{p^2+i\epsilon} \qquad \Pi_{\mu\nu\alpha\beta} = \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}$$



The $\mu\nu \leftrightarrow \alpha\beta$ asymmetry involved at one-loop level in a particle propagation means that time reversal is not invariant in quantum gravity, i.e. an arrow of time at the microscopic level.

$$\mathcal{T}_{\rm P}^a = \Big[i\kappa^2\frac{i\Pi_{\mu_3\nu_3\mu_4\nu_4}}{2}\frac{i}{16\pi^2}\Big(V^{\mu_3\nu_3\mu_4\nu_4|\lambda_1\mu\nu\lambda_2\alpha\beta}p_{\lambda_1}p_{\lambda_2} \times(\xi_1-\xi_1\log\xi_1)+\frac{V^{\mu\nu\alpha\beta|\lambda_3\mu_3\nu_3\lambda_4\mu_4\nu_4}\eta_{\lambda_3\lambda_4}}{4} \times(\xi_1^2\log\xi_1-\frac{3}{2}\xi_1^2)\Big)\Big]_{\xi_1\to 0}+C_a^{\mu\nu\alpha\beta}\,. = 0.$$

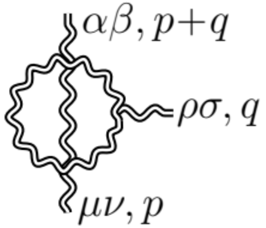
$$\mathcal{T}_{\rm P}^b = \frac{(2i\kappa)^2}{2}\frac{i}{16\pi^2}\int_0^1 dx(-\frac{1}{4})\Big\{\frac{1}{16}[40x^2(1-x)^2p^\mu p^\nu p^\alpha p^\beta +2p^2((1-2x)^2(15x^2-15x-2)(p^\mu p^\nu\eta^{\alpha\beta}+p^\alpha p^\beta\eta^{\mu\nu}) +(10x^4-20x^3+17x^2-7x+2)(p^\nu p^\beta\eta^{\mu\alpha}+p^\mu p^\beta\eta^{\nu\alpha} +p^\nu p^\alpha\eta^{\mu\beta}+p^\mu p^\alpha\eta^{\nu\beta}))+p^4((115x^4-230x^3+103x^2 +12x+1)\eta^{\mu\nu}\eta^{\alpha\beta}+(85x^4-170x^3+139x^2-54x+3) \times(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}))]\log\frac{1}{-p^2x(1-x)}\Big\}+C_b^{\mu\nu\alpha\beta}\,.$$

$$\mathcal{T}_{\rm P}^c = (-1)(i\kappa)^2\frac{4i}{16\pi^2}\int_0^1 dx(-\frac{1}{4})\Big\{\frac{1}{4}[4(4x^4-8x^3+2x^2 +2x+1)p^\mu p^\nu p^\alpha p^\beta + p^2((8x^4-16x^3+4x^2+4x-1) \times(p^\nu p^\beta\eta^{\mu\alpha}+p^\mu p^\beta\eta^{\nu\alpha}+p^\nu p^\alpha\eta^{\mu\beta}+p^\mu p^\alpha\eta^{\nu\beta}) +2x(14x^3-24x^2+13x-4)p^\mu p^\nu\eta^{\alpha\beta}+2p^\alpha p^\beta\eta^{\mu\nu} \times(14x^4-32x^3+25x^2-6x-1))+p^4(2x(11x^3-22x^2 +13x-2)(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha})+(12x^4-24x^3+16x^2 -4x+1)\eta^{\mu\nu}\eta^{\alpha\beta})]\log\frac{1}{-p^2x(1-x)}\Big\}+C_c^{\mu\nu\alpha\beta}\,,$$

$n-$loop with overlapping divergences

superficial degree of divergence $2n+2$

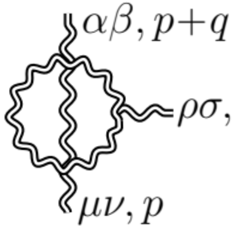
$$\mathcal{T}_{\rm P}^{t2n} = A\frac{\Delta^n}{n!}\log|\Delta|+C \qquad \mathcal{T}_{\rm P}^{\rm total} = \mathcal{T}_{\rm P}^{t2(n+1)}+\mathcal{T}_{\rm P}^{t2n}+\cdots+\mathcal{T}_{\rm P}^{t2} +\mathcal{T}_{\rm P}(\log)+\mathcal{T}_{\rm P}(\text{finite})\,,$$



$$\mathcal{T}_{\rm P}^V = (2i)^3\kappa^5(\frac{i}{16\pi^2})^2\int_0^1 dx\int_0^1 dy\int_0^1 dz\frac{z}{2^4(1-z)^4} \times\Big\{A_3\frac{\Delta_0^3}{3!}+A_2\frac{\Delta_0^2}{2!}+A_1\Delta_0+A_0\Big\}\log\frac{1}{\Delta_0}+C^{\mu\nu\alpha\beta\rho\sigma}$$

Here Δ_0 is $\Delta_0 = b^2-ac$, with $a = z+(1-z)x(x-1)$, $b = yzq+(1-z)x(x-1)p$, $c = yzq^2+(1-z)x(x-1)p^2$. A_3, A_2, A_1, A_0 are coefficients related to sextic, quartic, quadratic, logarithmic divergence inputs respectively.

arXiv: 2403.09487



$$
\begin{aligned}
A_3 = \frac{z-1}{64a^8}\Big(&[440a^2 + a(1564x^2 + 1300x + 23)(z-1) \\
&+4(281x^4 - 562x^3 + 683x^2 - 402x + 273)(z-1)^2] \\
&\times\eta^{\mu\nu}(\eta^{\alpha\rho}\eta^{\beta\sigma} + \eta^{\alpha\sigma}\eta^{\beta\rho}) + [744a^2 + a(1932x^2 + 44x \\
&+1203)(z-1) + 4(297x^4 - 594x^3 + 1563x^2 - 1266x \\
&+673)(z-1)^2]\eta^{\rho\sigma}(\eta^{\alpha\nu}\eta^{\beta\mu} + \eta^{\alpha\mu}\eta^{\beta\nu}) + [440a^2 \\
&+a(1564x^2 - 1100x + 2423)(z-1) + 4(281x^4 \\
&-562x^3 + 683x^2 - 402x + 273)(z-1)^2]\eta^{\alpha\beta}(\eta^{\mu\rho}\eta^{\nu\sigma} \\
&+\eta^{\mu\sigma}\eta^{\nu\rho}) + [1032a^2 + a(3396x^2 - 3020x + 801) \\
&\times(z-1) + 4(591x^4 - 1182x^3 + 1101x^2 - 510x + 215) \\
&\times(z-1)^2](\eta^{\alpha\rho}\eta^{\beta\nu}\eta^{\mu\sigma} + \eta^{\alpha\nu}\eta^{\beta\rho}\eta^{\mu\sigma} + \eta^{\alpha\nu}\eta^{\beta\sigma}\eta^{\mu\rho} \\
&+\eta^{\alpha\sigma}\eta^{\beta\nu}\eta^{\mu\rho} + \eta^{\alpha\rho}\eta^{\beta\mu}\eta^{\nu\sigma} + \eta^{\alpha\mu}\eta^{\beta\rho}\eta^{\nu\sigma} + \eta^{\alpha\mu}\eta^{\beta\sigma}\eta^{\nu\rho} \\
&+\eta^{\alpha\sigma}\eta^{\beta\mu}\eta^{\nu\rho}) + [1696a^2 + a(4844x^2 + 848x + 4147) \\
&\times(z-1) + 4(787x^4 - 1574x^3 + 2521x^2 - 1734x \\
&+795)(z-1)^2]\eta^{\alpha\beta}\eta^{\mu\nu}\eta^{\rho\sigma}\Big)\,.
\end{aligned}
$$

Parameter A_0 $(l = p + q)$

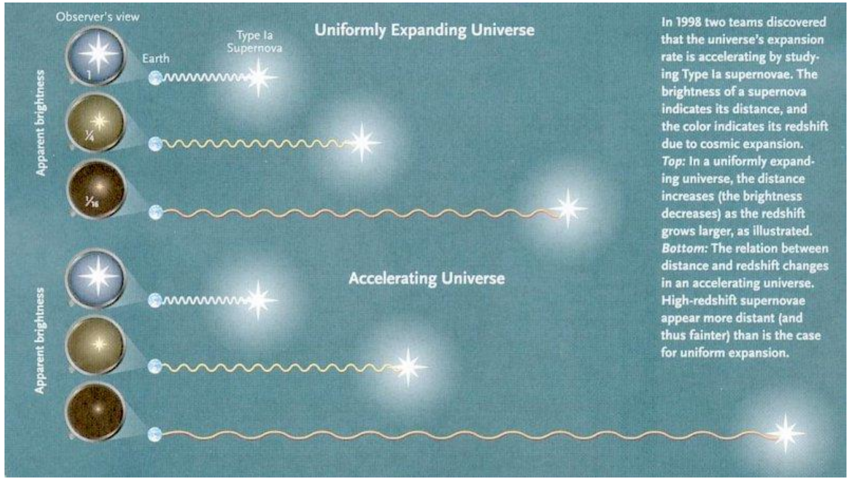
In the case of $p^2 = l^2 = 0$, the result is

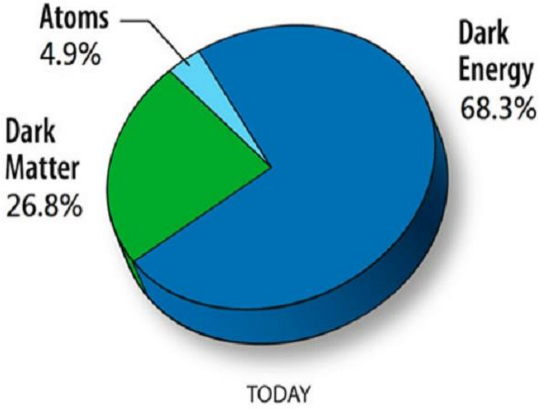
$$
\begin{aligned}
A_0 = -\frac{(z-1)^3}{64a^8}\Big\{&16y^3z^3[a^3(8x^2-8x+7)-2a^2(4x^4-8x^3+16x^2-12x+11)yz+a(14x^4-28x^3+53x^2-39x+28)y^2z^2 \\
&-14(x^2-x+1)^2y^3z^3]q^\alpha q^\beta q^\mu q^\nu q^\rho q^\sigma - 8y^2z^2[a^4(6-9x+9x^2)+a^2(x-1)x(47-75x+83x^2-16x^3+8x^4)y(1-z)z \\
&-2a(x-1)x(21-25x+32x^2-14x^3+7x^4)y^2(1-z)z^2+28(x-1)x(1-x+x^2)^2y^3(1-z)z^3+a^3((x-1)x(-12+41x \\
&-41x^2)(1-z)+(-7+14x-6x^2-16x^3+8x^4)yz)](p^\rho q^\alpha q^\beta q^\mu q^\nu q^\sigma + p^\sigma q^\alpha q^\beta q^\mu q^\nu q^\rho)+8y^2z^2[a^4(7-9x+9x^2)-28 \\
&\times(x-1)x(1-x+x^2)^2y^3(1-z)z^3+ay^2z^2((x-1)x(49-57x+59x^2-4x^3+2x^4)(1-z)+(-14+45x-73x^2+56x^3 \\
&-28x^4)yz)+a^3((x-1)x(-3+29x-29x^2)(1-z)+(-7-9x+x^2+16x^3-8x^4)yz)+a^2yz((x-1)x(-9-21x+13x^2 \\
&+16x^3-8x^4)(1-z)+(12-17x+37x^2-40x^3+20x^4)yz)](p^\beta q^\alpha q^\mu q^\nu q^\rho q^\sigma + p^\alpha q^\beta q^\mu q^\nu q^\rho q^\sigma)-4yz[2a^5+56(x-1)^2 \\
&\times x^2(1-x+x^2)^2y^3(1-z)^2z^3-a^4(9(x-1)x(1-z)+2(3-7x+7x^2)yz)+2a(x-1)xy^2(1-z)z^2((x-1)x(-35+29x \\
&-17x^2-24x^3+12x^4)(1-z)+(14-45x+73x^2-56x^3+28x^4)yz)+a^2(x-1)xy(1-z)z(10(x-1)x(5-6x+6x^2) \\
&\times(1-z)+(-38+31x+9x^2-80x^3+40x^4)yz)+2a^3(-3(x-1)^2x^2(1-z)^2+(x-1)x(17-18x+18x^2)y(1-z)z \\
&+(2-3x+11x^2-16x^3+8x^4)y^2z^2)](p^\beta p^\rho q^\alpha q^\mu q^\nu q^\sigma+p^\alpha p^\rho q^\beta q^\mu q^\nu q^\sigma+p^\beta p^\sigma q^\alpha q^\mu q^\nu q^\rho+p^\alpha p^\sigma q^\beta q^\mu q^\nu q^\rho)-8yz[a^5(5-6x \\
&+6x^2)+28(x-1)^2x^2(1-x+x^2)^2y^3(1-z)^2z^3+2a^4((x-1)x(7-x+x^2)(1-z)+(-21+34x-30x^2-8x^3+4x^4) \\
&\times yz)+2a(x-1)xy^2(1-z)z^2(3(x-1)x(-7+6x-2x^2-8x^3+4x^4)(1-z)+(14-45x+73x^2-56x^3+28x^4)yz) \\
&+a^3(2(x-1)^2x^2(3+2x-2x^2)(1-z)^2+(x-1)x(-47+107x-91x^2-32x^3+16x^4)y(1-z)z+(53-50x+30x^2 \\
&+40x^3-20x^4)y^2z^2)+a^2yz((x-1)^2x^2(-30+87x-79x^2-16x^3+8x^4)(1-z)^2+(x-1)x(11-30x+34x^2-8x^3 \\
&+4x^4)y(1-z)z+2(-8-11x+25x^2-28x^3+14x^4)y^2z^2)]p^\alpha p^\beta q^\mu q^\nu q^\rho q^\sigma-8[a^6+28(x-1)^3x^3(1-x+x^2)^2y^3(1-z)^3 \\
&\times z^3+a^5(2(x-1)x(1-x+x^2)(1-z)+(-2-5x+5x^2)yz)+2a(x-1)^2x^2y^2(1-z)^2z^2((x-1)x(-14+4x+15x^2 \\
&-38x^3+19x^4)(1-z)+(14-45x+73x^2-56x^3+28x^4)yz)+a^4((1-2x)^2(x-1)^2x^2(1-z)^2+(x-1)x(7-8x+8x^2) \\
&\times y(1-z)z+(1+8x-16x^3+8x^4)y^2z^2)+a^2(x-1)xy(1-z)z((x-1)^2x^2(-1+12x-12x^2)(1-z)^2+3(x-1)x \\
&\times(-1-11x+39x^2-56x^3+28x^4)y(1-z)z+2(-8-11x+25x^2-28x^3+14x^4)y^2z^2)+a^3(x-1)x(1-z)(2(x-1)^2 \\
&\times x^2(1-x+x^2)(1-z)^2+(x-1)x(10+9x-9x^2)y(1-z)z+(18-25x+79x^2-108x^3+54x^4)y^2z^2)](p^\alpha p^\beta p^\rho q^\mu q^\nu q^\sigma \\
&+p^\alpha p^\beta p^\sigma q^\mu q^\nu q^\rho)+8y^2z^2[8a^4(1-2x+2x^2)-28(x-1)x(1-x+x^2)^2y^3(1-z)z^3+ay^2z^2((x-1)x(49-88x+142x^2 \\
&-108x^3+54x^4)(1-z)+(-14+45x-73x^2+56x^3-28x^4)yz)+a^3(12(x-1)x(1-x+x^2)(1-z)+(-31+88x-104x^2 \\
&+32x^3-16x^4)yz)+2a^2yz((x-1)x(-16+21x-29x^2+16x^3-8x^4)(1-z)+(17-51x+69x^2-36x^3+18x^4)yz)] \\
&\times(p^\nu q^\alpha q^\beta q^\mu q^\rho q^\sigma+p^\mu q^\alpha q^\beta q^\nu q^\rho q^\sigma)-4yz[2a^5(6-11x+11x^2)+56(x-1)^2x^2(1-x+x^2)^2y^3(1-z)^2z^3+a^2(x-1)x \\
&\times y(1-z)z((x-1)x(68-113x+129x^2-32x^3+16x^4)(1-z)+(-26+117x-213x^2+192x^3-96x^4)yz)+a^4((x-1) \\
&\times x(-11+52x-52x^2)(1-z)+2(-4+7x+x^2-16x^3+8x^4)yz)+2a(x-1)xy^2(1-z)z^2(-5(-1+x)x(7-12x+20x^2 \\
&-16x^3+8x^4)(1-z)+(14-45x+73x^2-56x^3+28x^4)yz)+a^3(-2(x-1)^2x^2(12-37x+37x^2)(1-z)^2+(x-1)x \\
&\times(27-31x+63x^2-64x^3+32x^4)y(1-z)z-2(2-3x+11x^2-16x^3+8x^4)y^2z^2)](p^\nu p^\rho q^\alpha q^\beta q^\mu q^\sigma+p^\nu p^\sigma q^\alpha q^\beta q^\mu q^\rho \\
&+p^\mu p^\rho q^\alpha q^\beta q^\nu q^\sigma+p^\mu p^\sigma q^\alpha q^\beta q^\nu q^\rho)-4yz[-6a^5(3-5x+5x^2)+56(x-1)^2x^2(1-x+x^2)^2y^3(1-z)^2z^3+a^4((x-1)x \\
&\times(-11-32x+32x^2)(1-z)+2(27-44x+52x^2-16x^3+8x^4)yz)+2a(x-1)xy^2(1-z)z^2((x-1)x(-42+67x-95x^2 \\
&+56x^3-28x^4)(1-z)+2(14-45x+73x^2-56x^3+28x^4)yz)+a^2yz((x-1)^2x^2(7+36x-20x^2-32x^3+16x^4)(1-z)^2 \\
&-8(x-1)x(13-29x+43x^2-28x^3+14x^4)y(1-z)z+4(10-21x+35x^2-28x^3+14x^4)y^2z^2)-2a^3((x-1)^2x^2(-3 \\
&+31x-31x^2)(1-z)^2+(x-1)x(-21+17x-33x^2+32x^3-16x^4)y(1-z)z+(35-57x+85x^2-56x^3+28x^4)y^2z^2)] \\
&\times(p^\beta p^\nu q^\alpha q^\mu q^\rho q^\sigma+p^\alpha p^\nu q^\beta q^\mu q^\rho q^\sigma+p^\beta p^\mu q^\alpha q^\nu q^\rho q^\sigma+p^\alpha p^\mu q^\beta q^\nu q^\rho q^\sigma)-4[56(x-1)^3x^3(1-x+x^2)^2y^3(1-z)^3z^3+2a^5 \\
&\times((x-1)^2x^2(1-z)+(1-x+x^2)yz)-a^4(x-1)x(1-z)((x-1)x(1-z)+4(x-1)x^2(1-z)-4(x-1)x^3(1-z) \\
&+(-10-31x+31x^2)yz)+2a(x-1)^2x^2y^2(1-z)^2z^2((x-1)x(-28+39x-53x^2+28x^3-14x^4)(1-z)+2(14-45x \\
&+73x^2-56x^3+28x^4)yz)+a^2(x-1)xy(1-z)z((x-1)^2x^2(37-41x+41x^2)(1-z)^2-2(x-1)x(38-71x+99x^2 \\
&-56x^3+28x^4)y(1-z)z+4(10-21x+35x^2-28x^3+14x^4)y^2z^2)+a^3(x-1)x(1-z)((x-1)^2x^2(-5-2x+2x^2) \\
&\times(1-z)^2+8(x-1)x(6-x+x^2)y(1-z)z-2(25-36x+50x^2-28x^3+14x^4)y^2z^2)](p^\beta p^\nu p^\rho q^\alpha q^\mu q^\sigma+p^\alpha p^\nu p^\rho q^\beta q^\mu q^\sigma \\
&+p^\beta p^\nu p^\sigma q^\alpha q^\mu q^\rho+p^\alpha p^\nu p^\sigma q^\beta q^\mu q^\rho+p^\beta p^\mu p^\rho q^\alpha q^\nu q^\sigma+p^\alpha p^\mu p^\rho q^\beta q^\nu q^\sigma+p^\beta p^\mu p^\sigma q^\alpha q^\nu q^\rho+p^\alpha p^\mu p^\sigma q^\beta q^\nu q^\rho)+4[4a^6(1-x \\
&+x^2)-56(x-1)^3x^3(1-x+x^2)^2y^3(1-z)^3z^3+2a^5((x-1)x(3-8x+8x^2)(1-z)+(2-21x+13x^2+16x^3-8x^4) \\
&\times yz)+2a(x-1)^2x^2y^2(1-z)^2z^2((x-1)x(35-46x+48x^2-4x^3+2x^4)(1-z)-3(14-45x+73x^2-56x^3+28x^4) \\
&\times yz)+a^4(4(x-1)^2x^2(1-5x+5x^2)(1-z)^2+(x-1)x(61-104x+56x^2+96x^3-48x^4)y(1-z)z+2(1+9x+19x^2 \\
&-56x^3+28x^4)y^2z^2)-2a^2(x-1)xy(1-z)z((x-1)^2x^2(-29+75x-67x^2-16x^3+8x^4)(1-z)^2+(x-1)x(-35+66x
\end{aligned}
$$

21 pages

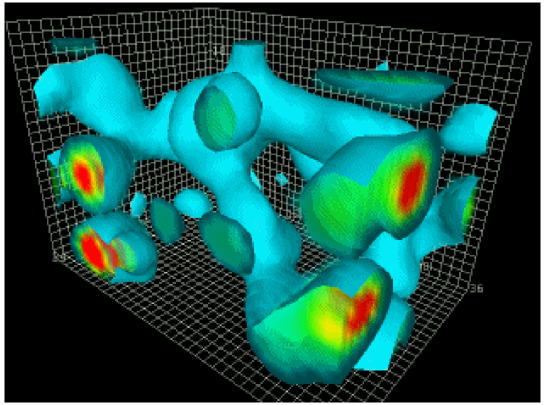
IV. Dark energy in QFT

Big Devil (Dark energy)





$\Lambda + \text{CDM}$



$$\langle\rho\rangle = \int_0^\Lambda \frac{4\pi k^2 dk}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

$$(2.3\ \text{meV})^4$$

10^{120}

Vacuum energy

New ultraviolet catastrophe

The vacuum energy density

$$\rho_0^i = (-1)^{2j} g_i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m_i^2}$$

$$\mathcal{T}_{\rm P}^i = \left[\int (d\xi)^3 \frac{\partial^3 \mathcal{T}_{\rm F}^i(\xi)}{\partial \xi^3} \right]_{\xi\to 0} + C$$

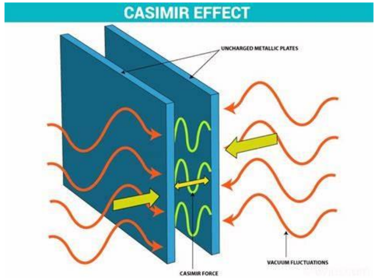
$$= \left[(-1)^{2j} g_i \int (d\xi)^3 \int \frac{d^3k}{(2\pi)^3} \frac{3}{16} \frac{1}{\left(\sqrt{k^2+m_i^2+\xi}\right)^5} \right]_{\xi\to 0} + C$$

$$= (-1)^{2j} g_i \frac{1}{64\pi^2} m_i^4 \log(m_i^2) + C \ .$$

$$\mathcal{T}_{\rm P}^i = (-1)^{2j} g_i \frac{1}{64\pi^2} m_i^4 \log \frac{m_i^2}{\mu_\Lambda^2}$$

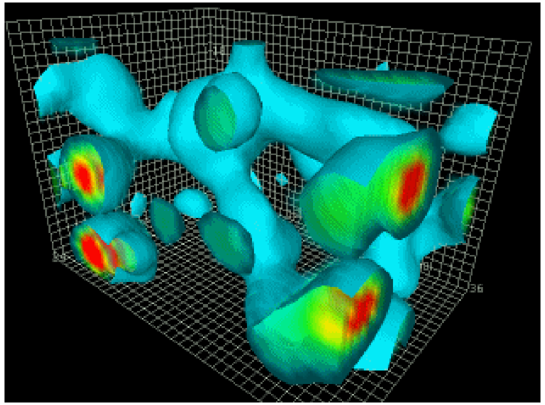
μ_Λ likes $\Lambda_{\rm QCD}$ and the electroweak scale v

arXiv:2410.06604

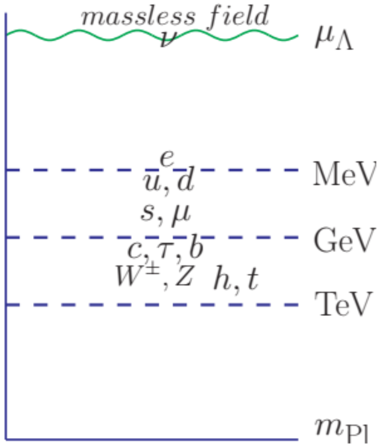


$$\zeta(-1) = -\frac{1}{12}$$

No longer a problem caused by the UV region



If dark energy comes from the vacuum energy, then the contributions of heavy fields should be suppressed by some unknown mechanisms.



The ripple model

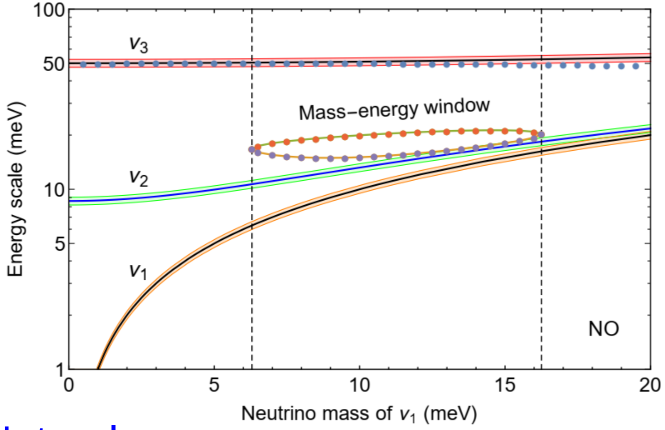
An effective active degrees of freedom $g_i^* = g_i e^{-m_i^2/\mu_\Lambda^2}$

Naturalness

Gaussian distribution

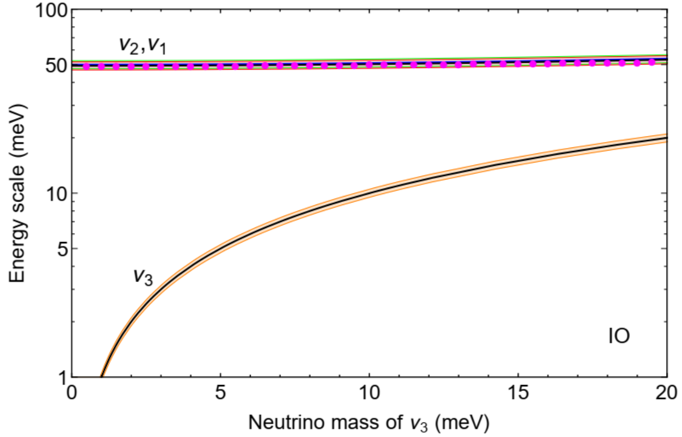
Neutrino fields play a major role

Neutrino mass and dark energy density $(2.3\ \text{meV})^4$ $H_0 \approx 70$ km /(s· Mpc)

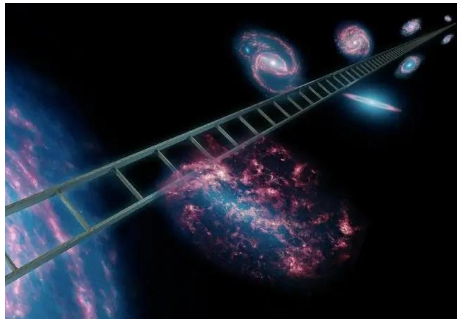


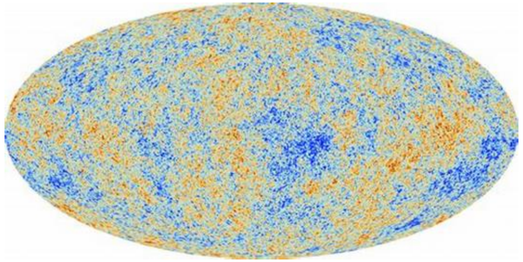
Likewise, the neutrino mass window set by dark energy is 6.3 meV $\lesssim m_1 \lesssim$ 16.3 meV, 10.7 meV $\lesssim m_2 \lesssim$ 18.4 meV, 50.5 meV $\lesssim m_3 \lesssim$ 52.7 meV, and the total neutrino mass is 67.5 meV $\lesssim m_1 + m_2 + m_3 \lesssim$ 87.4 meV.

Naturalness



Hubble tension





The nearby universe

$H_0 = (73.04 \pm 1.04)$ km /(s·Mpc)

$$(2.37\ \text{meV})^4$$

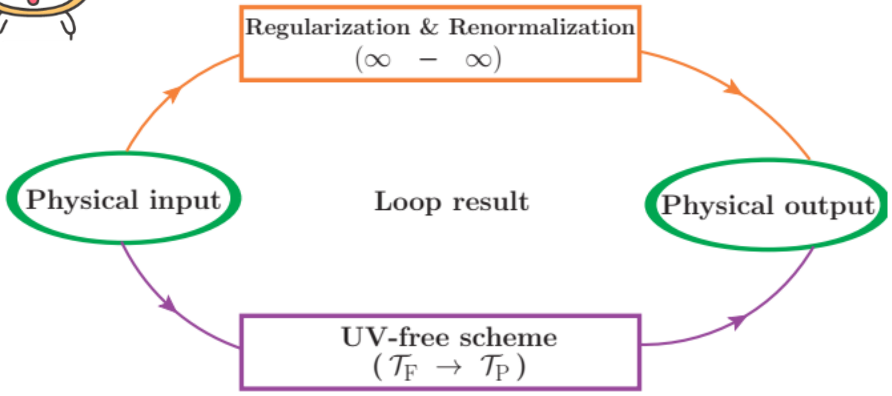
The early universe

$H_0 = (67.4 \pm 0.5)$ km /(s·Mpc)

$$(2.24\ \text{meV})^4$$

μ_Λ has a slow-running behavior

Why does the UV-free scheme still hold for power-law divergences?



Two alternative routes of concern

(a) *Equivalent transformation* of the loop integral from UV divergence to UV divergence mathematically expressed form (regularization), with renormalization required to remove the UV divergence.
(b) *Analytic continuation* of the transition amplitude from UV divergent $\mathcal{T}_F$ to UV converged $\mathcal{T}_P$ (the UV-free scheme here), without UV divergences in calculations.

A test

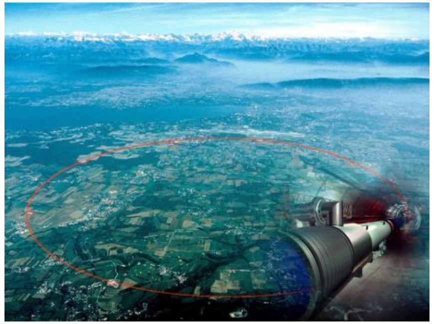
The hierarchy problem of Higgs mass

(a) New particles (TeV) needed to cancel out UV contributions of loops to the Higgs mass

(b) An interpretation within SM

SELECTED BY NATURE

LHC

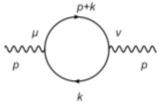


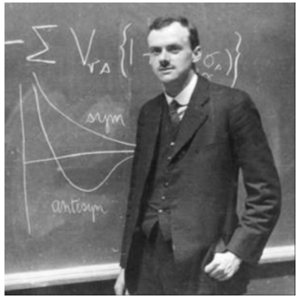
A conservative way

UV-free scheme *Analytic continuation*

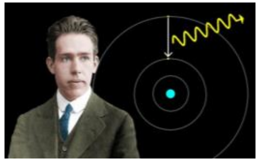
$$\mathcal{T}_F \Longrightarrow \mathcal{T}_P = \left[\int (d\xi)^n \frac{\partial^n \mathcal{T}_F(\xi)}{\partial \xi^n}\right]_{\xi \to 0} + C,$$

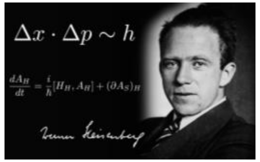
Finite input		UV divergence input (continuation)	
Tree level	Loop finite	Loop Log	Loop $\Lambda^2, \Lambda^4, \Lambda^6, \ldots$
Originally well-defined		Verified	To be verified

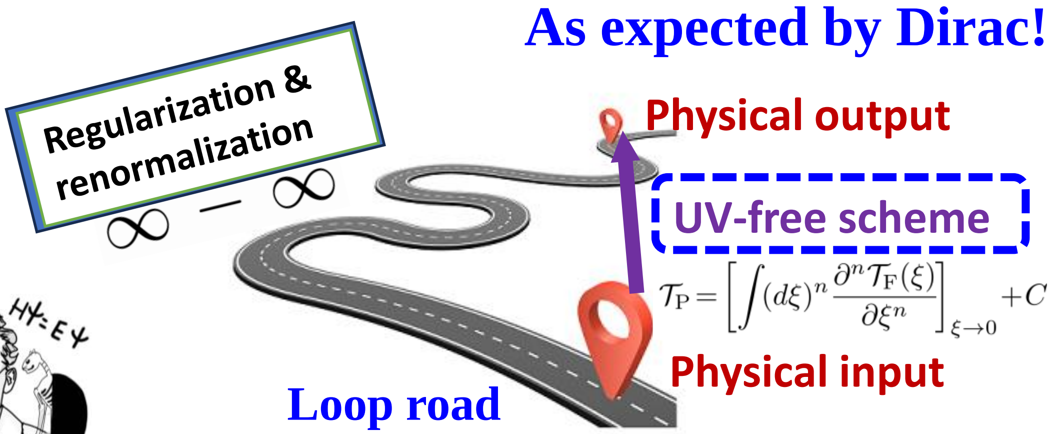




P. A. M. Dirac I believe the successes of the renormalization theory will be on the same footing as the successes of the Bohr orbit theory applied to one-electron problems.

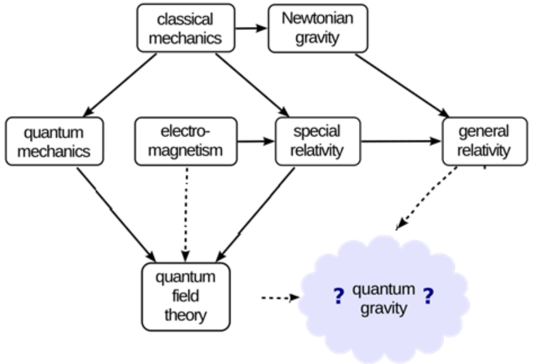




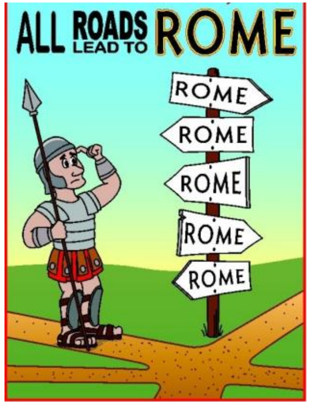


Schemes	Tree level	Loop finite	Loop Log	Loop $\Lambda^2, \Lambda^4, \Lambda^6, \ldots$
Regularization & renormalization (∞-∞)			OK	Problematic
UV-free scheme ($T_F \to T_P$)	OK	OK	OK	OK

Both loops of the renormalizable Standard Model and non-renormalizable Einstein gravity being OK!



V. Summary and outlook



New method reproduces the successful renormalization results for log divergences, and gives an explanation to the three UV problems:

a) To the hierarchy problem of the 125 GeV Higgs, an alternative interpretation without fine-tuning within SM.

b) It is possible to incorporate Einstein gravity into the framework of QFT.

c) An alternative interpretation of dark energy in QFT.

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Outlook:

It is the beginning of a new alternative method.

Thank you!