Are parton showers inside a quark-gluon plasma strongly coupled?

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Reporting on recent work with Peter Arnold and Shahin Iqbal

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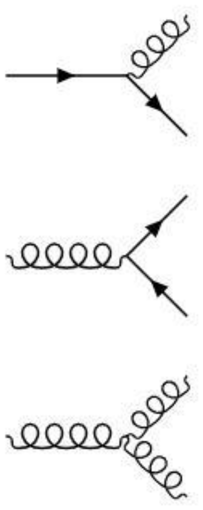
Energy loss in the quark gluon plasma

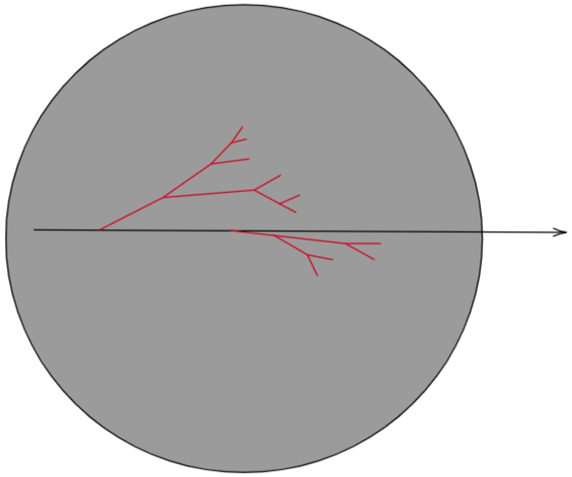
- A high energy parton in the QGP loses energy primarily through splitting process: hard bremsstrahlung or pair production. When repeated, a shower of lower energy particles is produced.



$q \to qg$

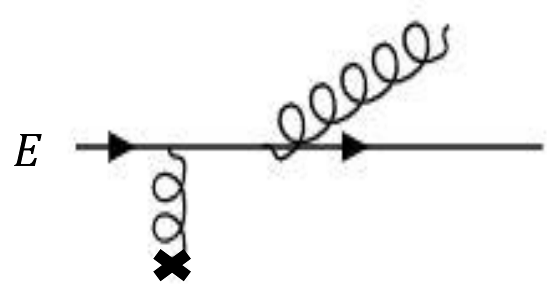
$g \to q\bar{q}$

$g \to gg$



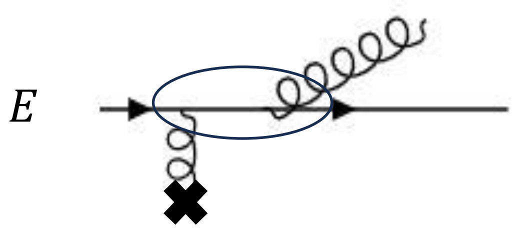
Medium-induced emissions

- Each collision with the medium offers a chance for emission.
- Naively, prob of splitting $\sim \alpha$ per *collision*



Medium-induced emissions

- However, the quantum duration of such splitting (known as the formation time) grows with energy E.

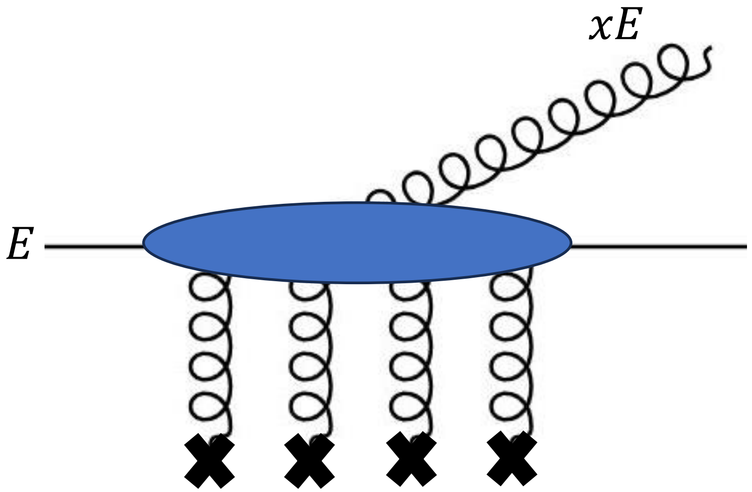


- When the formation time $\gg$ the mean free time between collision with the medium, many scatterings take place within one formation time. This is known as the LPM effect.

prob of splitting $\sim \alpha$ per *formation time*.

This reduction in the rate was figured out in the 1950's for QED (Landau-Pomeranchuk & Migdal) and for QCD (1990s) by authors known collectively as BDMPS-Z.

$$\left[\frac{d\Gamma}{dx}\right]_{LO}$$

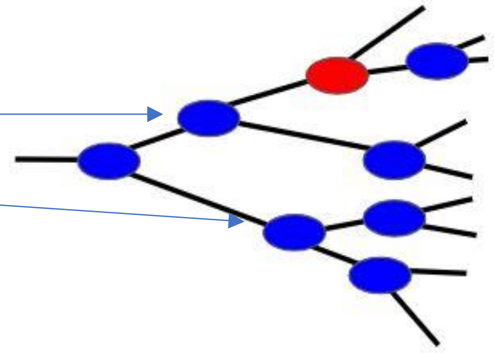


In-Medium Shower development

- Accounting for the LPM effect, we can use the splitting rates to statistically model shower development by treating high-energy particles classically between splittings, and rolling dice based on the splitting rate to decide when each particle splits.

- We call this weakly coupled in-medium shower.

LPM splitting rate $\left[\frac{d\Gamma}{dx}\right]_{LO}$



In-Medium Shower development

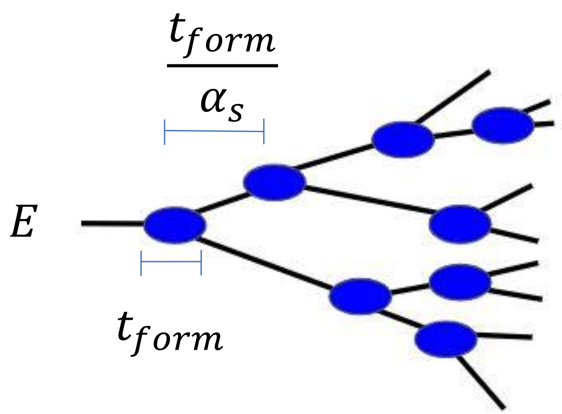
- Or can splittings overlap?
- This could happen if the formations times are large compared to times between splittings. In this case, one may not treat different splittings as quantum mechanically independent, and any classical picture of shower development breaks down.

We call this strongly coupled in-medium shower.

Hierarchy of scale

- For small α_s, the distinction between weakly and strongly-coupled pictures of the shower is controlled by the size of the running coupling constant $\alpha_s(\mu)$ at the transverse momentum scale μ associated with high energy splitting.

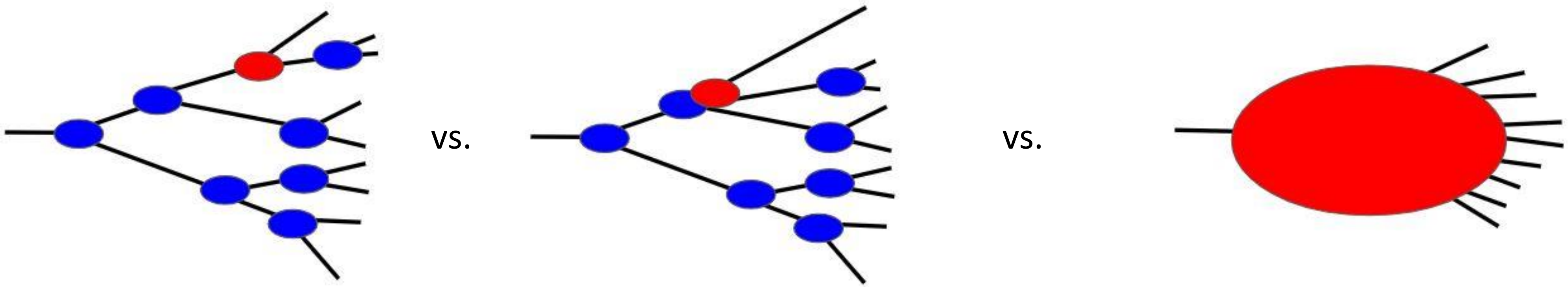
$$\mu \sim (\hat{q}E)^{\frac{1}{4}}$$



- Formally, the contribution of overlapping formation times is suppressed by one factor of $\alpha_s(\mu)$. This factor of $\alpha_s(\mu)$(which is moderately small) is accompanied by a potentially double log.

Our approach

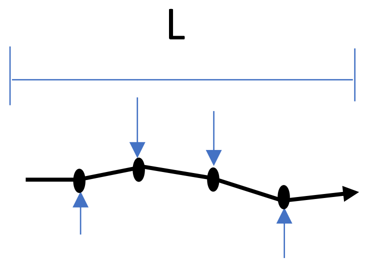
- Treat $\alpha_s(\mu)$ as small but calculate the correction to the weakly-coupled picture by computing the correction from overlapping formation times of two consecutive splittings. For reasonable values of $\alpha_s(\mu)$, how large are these corrections?



Our simplest theoretical situation

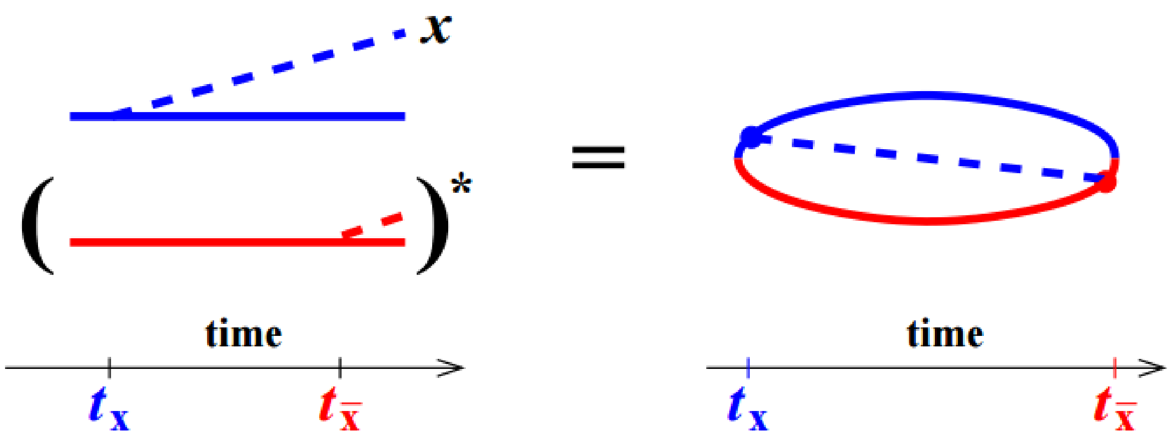
- The QGP is static, homogonous and large enough to stop the shower completely.
- We start with a single high energy parton that is very close to on-shell.
- The Harmonic approximation, also known as the $\hat{q}$ or multiple scattering approximation.

$$\hat{q} = \langle p_\perp^2 \rangle / L$$

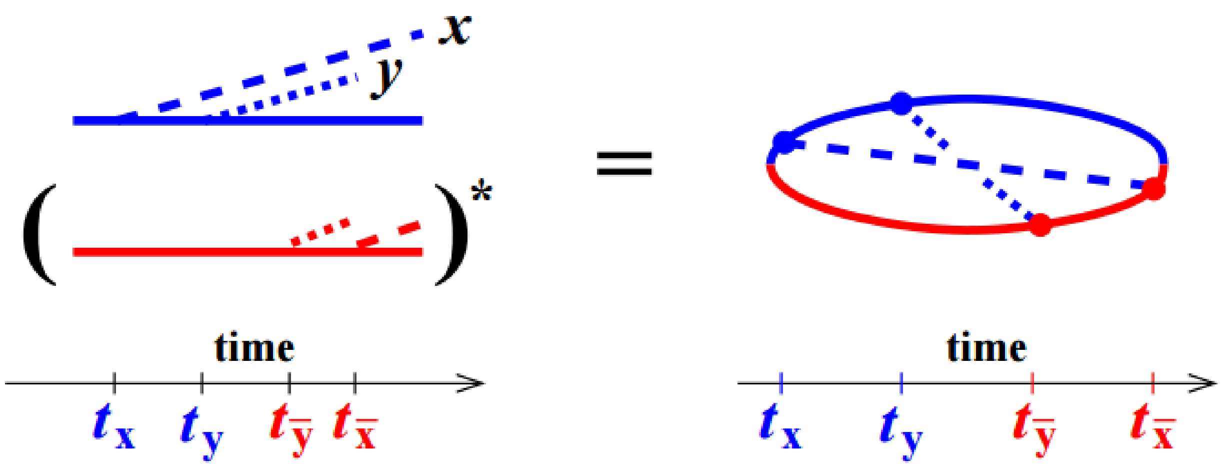


How to draw diagrams? [see Zakharov's work]

- For single splitting:



- For double-splitting:



High-Energy approximation

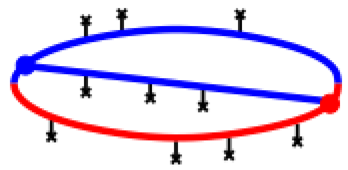
- In the high energy limit, the particle's energy become:

$$E_p = \sqrt{p_z^2 + p_\perp^2} \simeq \frac{p_\perp^2}{2p_z} + p_z \simeq \frac{p_\perp^2}{2p_z} + \text{constant}.$$

- The RHS looks like a 2-D non-relativistic Kinetic energy $\frac{p_\perp^2}{2m}$.

Medium interactions (1)

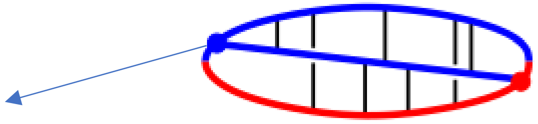
- As the high energy particles propagate through the medium, they interact with the background fields in the plasma.
- For a thermal field these background field are random with well-defined correlations.
- When computing a rate in a random background field, one should average over that randomness.



Medium interactions (2)

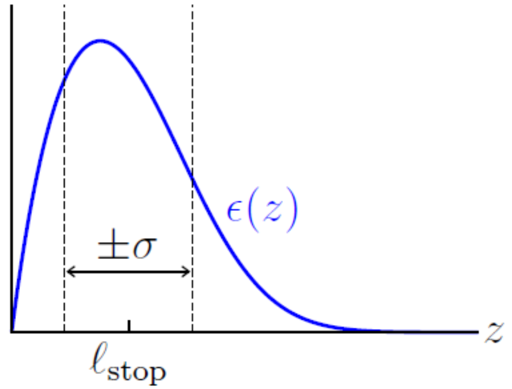
- After this average, the interactions of the particles with the background fields, and their correlations, may be replaced with a "potential energy" term in the QM problem.
- However, unlike a normal quantum mechanics problem, this potential energy is not real-valued.
- The effective QM problem that reproduces the medium averaged splitting –rate has a non-Hermitian Hamiltonian.

The splitting vertices for nearly-collinear bremsstrahlung or pair production (used in discussion of DGLAP evolution).



A theorist's observable

- Imagine measuring the distribution $\epsilon(z)$ in z of where that energy is deposited into the medium, statistically averaged over many showers.

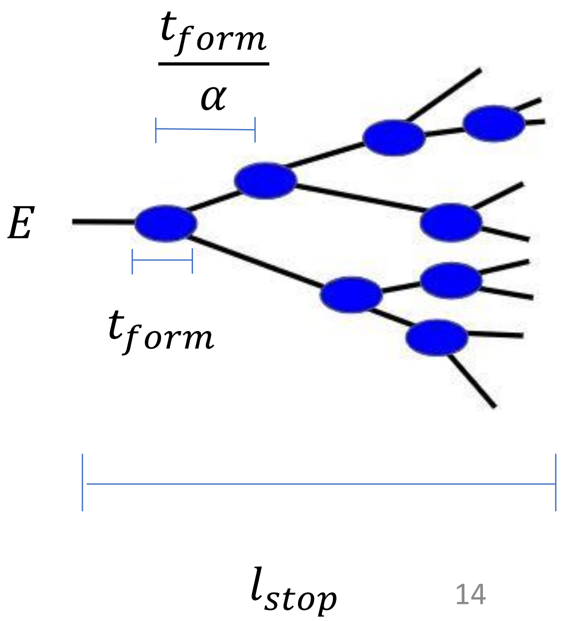


$$t_{form} \sim \sqrt{\frac{E}{\hat{q}}}$$

$$l_{stop} \sim \frac{1}{\alpha_S}\sqrt{\frac{E}{\hat{q}}}$$

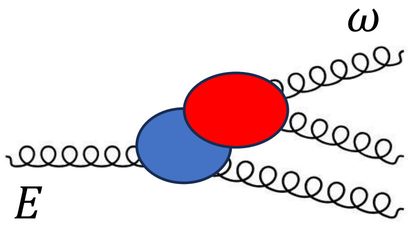
l_{stop} is the first moment of energy deposition distribution $\epsilon(z)$ and σ is the width of the distribution. Note that both depend on $\hat{q}$.

$$\sigma \sim \frac{1}{\alpha_S}\sqrt{\frac{E}{\hat{q}}}$$



The overlapping soft emission

- The previous calculation by (Blaizot,Mehtar-Tani/Iancu/Wu) showed that the probability of a hard splitting overlapping with soft bremsstrahlung ω $(T \ll \omega \ll E)$ is enhanced by a large double logarithm in QCD.
- So the probability of overlap is suppressed not just by a factor of α_s, but by $\alpha_s \ln^2 \frac{E}{T}$ (a large effect for large E).
- However, they found that these soft radiation effects can be absorbed into an effective value of the medium parameter $\hat{q}$:



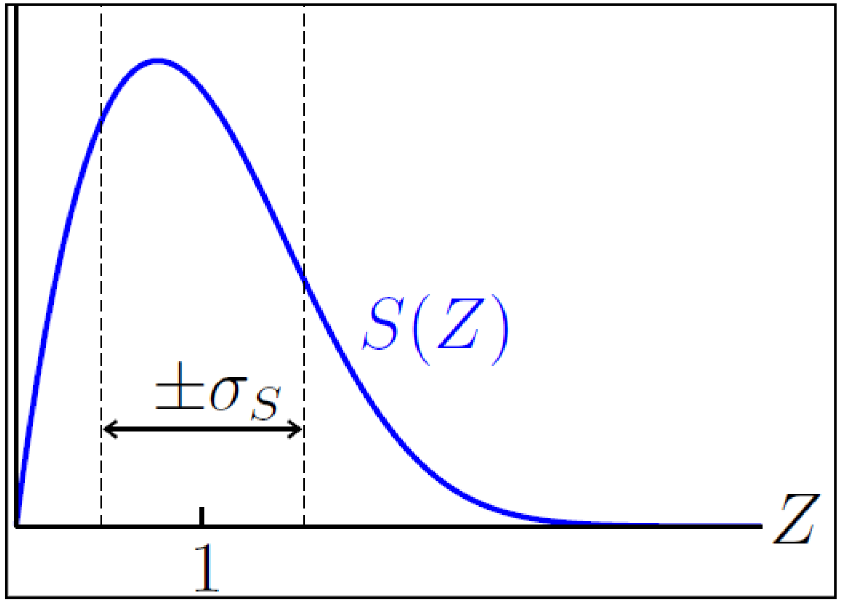
$$\hat{q} \to \hat{q}_{eff} \sim \hat{q}\left(1 + \alpha_s \ln^2 \frac{E}{T}\right)$$

⇒ They also showed how to re-sum these corrections at leading log to all orders in α_s.

A refined question

How large are overlap corrections that cannot be absorbed into an effective value of $\hat{q}$?

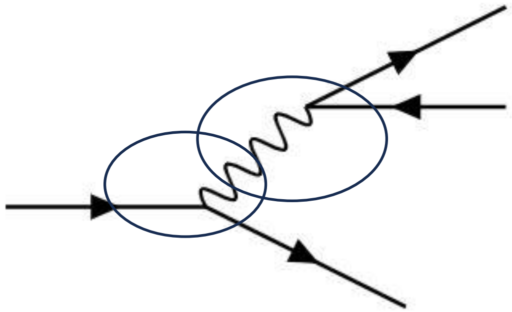
- One can define the shape $S(Z)$ of the energy deposition distribution as $S(Z) = \frac{l_{stop}}{E_0}\epsilon(l_{stop}Z)$ where $Z = \frac{z}{l_{stop}}$.
- The shape function and its moments are insensitive to constant shifts $\delta\hat{q}$ to $\hat{q}$.
- For example: $\sigma_S = \frac{\sigma}{l_{stop}}$, is independent of $\hat{q}$.



Previous Results

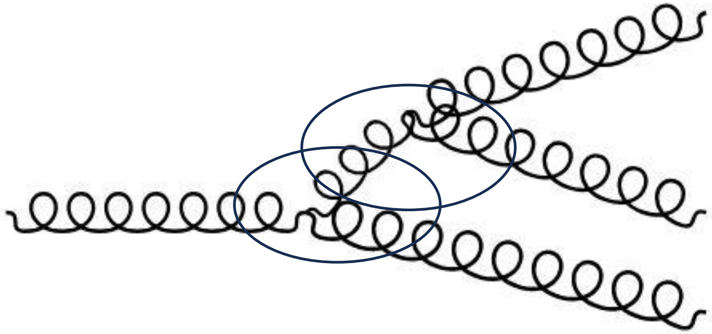
- Large-N_f QED (charge deposition) [2018 P.Arnold& S.Iqbal]:

$$\sigma_S = \frac{\sigma}{l_{stop}} = \left(\frac{\sigma}{l_{stop}}\right)_{LO} [1 - 0.85\, N_f \alpha(\mu)]$$



- Large-N_c QCD (gluons only and energy deposition) [PRL 131 &PRD 108]:

$$\sigma_S = \frac{\sigma}{l_{stop}} = \left(\frac{\sigma}{l_{stop}}\right)_{LO} [1 - 0.01\, N_c \alpha_s(\mu)]$$



Large-N_c QCD (pure gluons) is very different from QED (Large-N_f): speculations

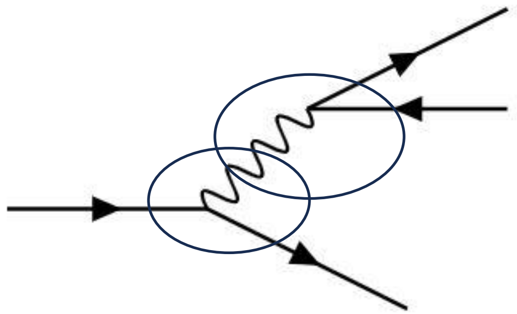
- It could be the difference between calculation energy deposition (QCD) vs. charge deposition (QED).

- Maybe showers involving fermions (QED) behave differently from those that don't.

Energy vs. Charge deposition

- We found that the overlap correction for electron-initiated showers in Large-N_f QED (energy deposition distributions) [JHEP 09 (2024) 131]:

$$\sigma_S = \frac{\sigma}{l_{stop}} = \left(\frac{\sigma}{l_{stop}}\right)_{LO} \left[1 + 1.13\, N_f \alpha(\mu)\right]$$

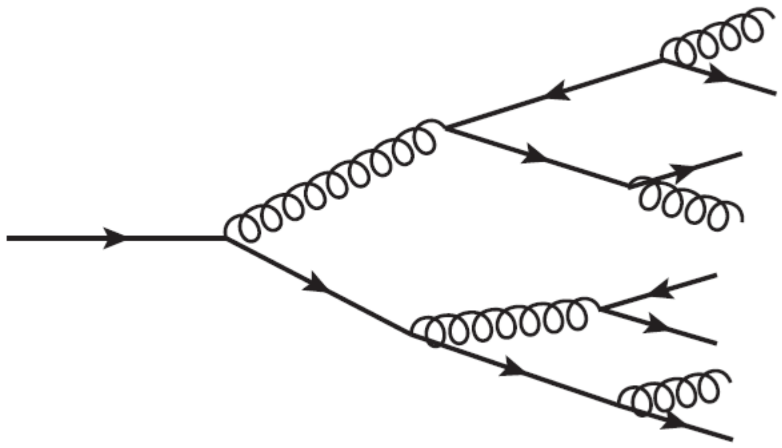


- There is no qualitative difference between charge deposition and energy deposition in the QED case.

Adding quarks to our QCD showers

- We take the large N_f limit, where N_f is the number of quark flavors. Specially, $N_f \gg N_c \gg 1$.

- A simpler calculation. One can adapt existing formulas for large- N_f QED overlap rates, and $\hat{q}$-insensitive quantities to large- N_f QCD.

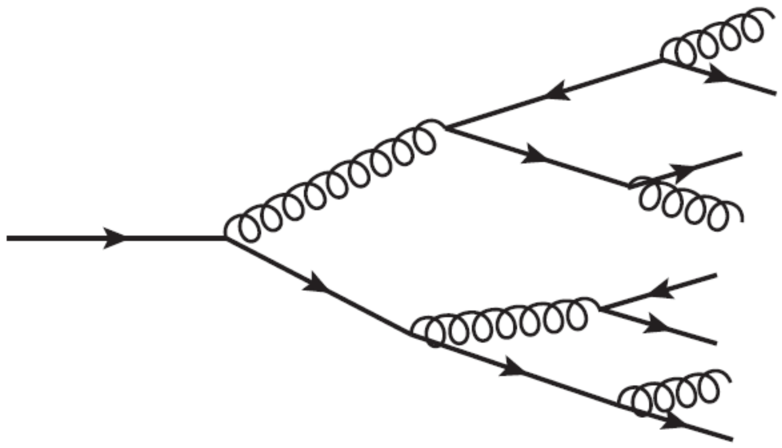


$q \to qg$ & $g \to q\bar{q}$

Results in large-N_f QCD

- The result for quark-initiated shower (energy-deposition) [arxiv:2408.07129]:

$$\sigma_S = \frac{\sigma}{l_{stop}} = \left(\frac{\sigma}{l_{stop}}\right)_{LO} [1 - 0.005\, N_f \alpha_s(\mu)]$$



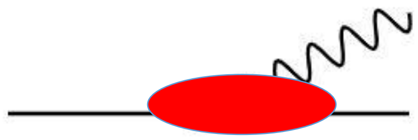
- Adding quarks (many flavors) does not change the qualitative answer that overlap effects in QCD are small (at least in the large-N_f we investigated).

QED vs. QCD
LO review

- Gluons interact much more directly with a QCD medium than photons do with a QED medium:

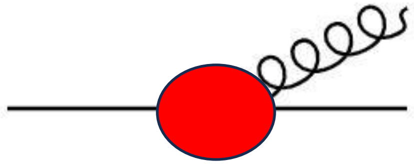
a) For QED, in the soft limit $x_\gamma \to 0$:

$$t_{form}(x_\gamma) \sim \sqrt{\frac{E}{x_\gamma \hat{q}}}$$



b) For QCD, in the soft limit $x_g \to 0$:

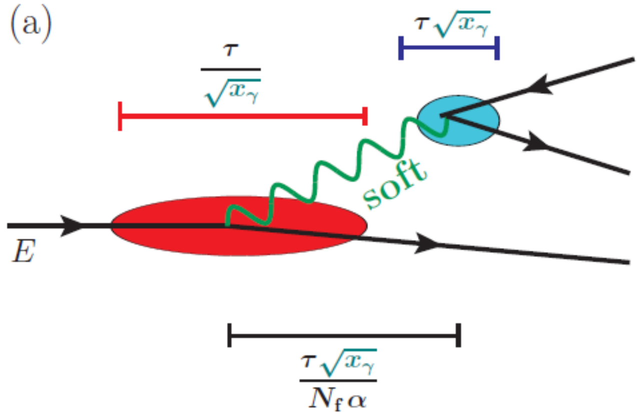
$$t_{form}(x_g) \sim \sqrt{\frac{x_g E}{\hat{q}}}$$

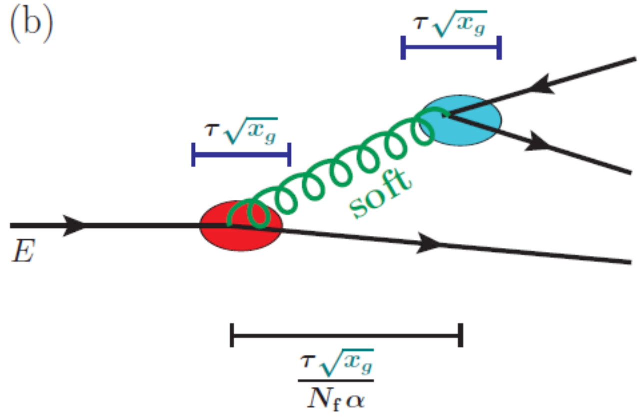


QED vs. QCD

- Overlap allows soft bremsstrahlung photons to convert to a soft electron/positron pair which is easily scattered by the QED medium.
- A significant modification to the LPM effect for the original soft bremsstrahlung.

- In contrast, soft gluons already interact easily with the QCD medium and so overlap does not make the same qualitative change.

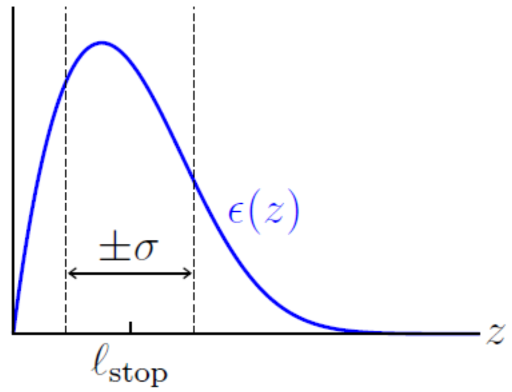




$$\tau \equiv \sqrt{E/\hat{q}}$$

Are the effects that can be absorbed into $\hat{q}$ process-independent?

- Imagine measuring the LO energy deposition distribution of a gluon initiated shower.
- Extract l_{stop} and equate it to its formula to get an effective value of $\hat{q}$.



-What if we had instead extracted the value of $\hat{q}$ from the first moment of energy deposition of a quark-initiated shower?

-Would we get almost the same value of effective $\hat{q}$, or do we need a different value to accurately describe energy deposition in the quark-initiated case?

Are the effects that can be absorbed into $\hat{q}$ process independent?

- Extracting the value of $\hat{q}$ from one measurement to predict other measurements would only work if there is little sensitivity to what type of measurement one uses to extract $\hat{q}$.
- Proposed test: calculate the overlap correction to a LO calculation of the ratio:

$$\frac{l_{stop}(q, energy, \hat{q}, E_0)}{l_{stop}(g, energy, \hat{q}, E_0)}$$

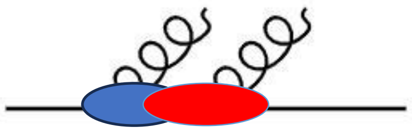
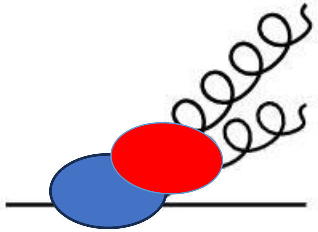
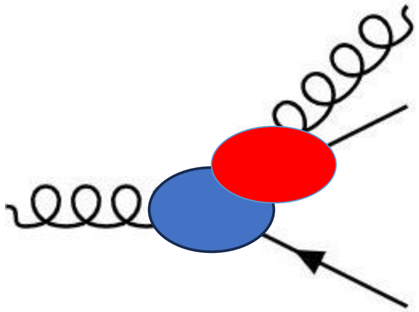
We found the corrections are very small ($0.2\% \times N_f \alpha_s$) for any reasonable value of $N_f \alpha_s$.
This suggest that extracting $\hat{q}$ from one type of measurement would make LO analysis work very well for all types of measurements considered here.

Conclusion

- Overlap effects that cannot be absorbed into an effective value of $\hat{q}$ are small for both $N_f \gg N_c \gg 1$ QCD & Large-N_c QCD (gluons only).

- This makes clear that the earlier small result for $\hat{q}$-insensitive overlap effects of a purely gluonic shower is not a peculiar accident of purely gluonic showers. Adding quarks did not make a difference.

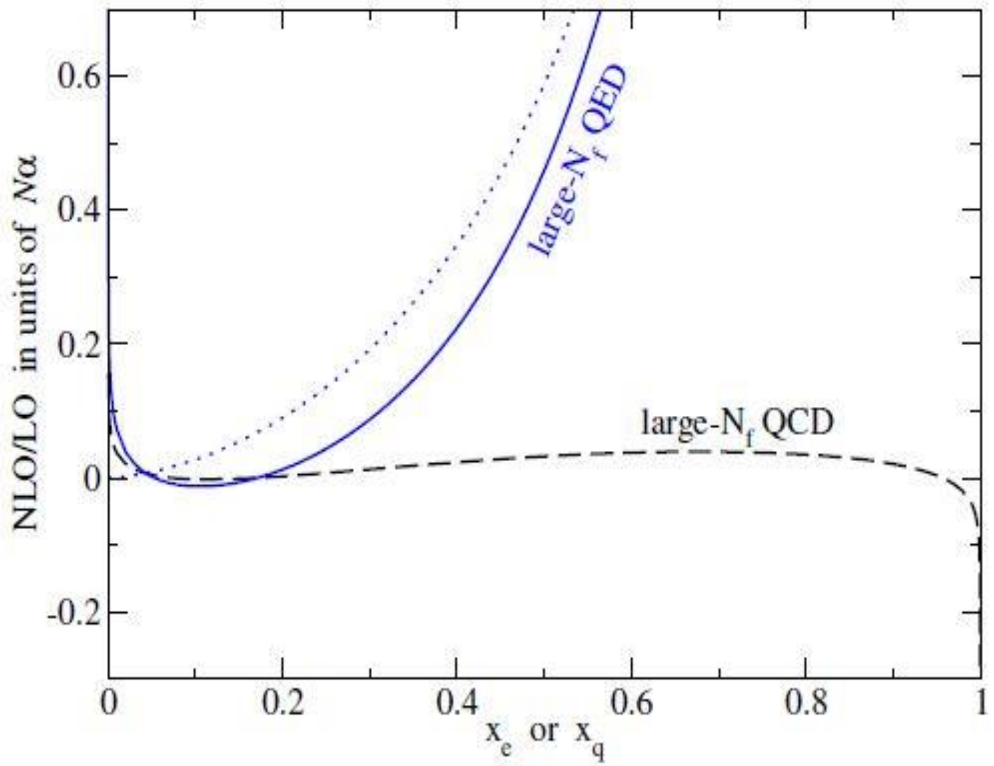
- It would be interesting to check our qualitative conclusions for $N_c \sim N_f \gg 1$ as well.

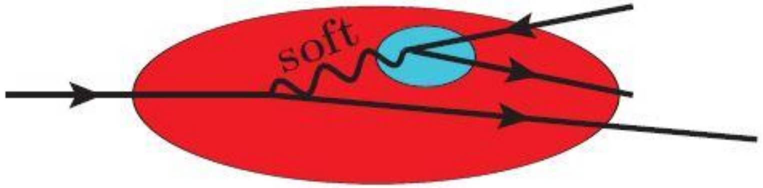
Thanks
Questions?

QCD vs. QED

- We have shown before that overlap corrections are big, but they can be absorbed into the $\hat{q}$ parameter.
- Our $\hat{q}$ insensitive measure of overlap correction is independent of constant shifts to $\hat{q}$.
- The size of overlap effects that cannot be absorbed into $\hat{q}$, depends on the x-dependence of the ratio of NLO to LO contributions to the rate.



QED vs. QCD



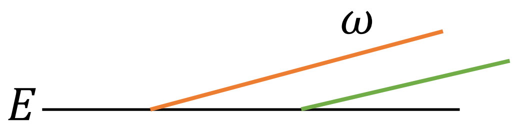
- The overlap correction to leading-log order: treat the splitting with the longer formation time as a vacuum-like DGLAP initial radiation (or final-state fragmentation correction) to the LO LPM formula for the other splitting.
- It captures a large part of the overlap effects in QED that cannot be absorbed into $\hat{q}$.

$$\left[\frac{d\Gamma}{dx_e}\right]^{\text{NLO}}_{\underline{e\to e}} \approx -\frac{3N_{\text{f}}\alpha^2}{8\pi}\,\frac{\ln(1-x_e)}{(1-x_e)^{3/2}}\sqrt{\frac{\hat{q}}{E}} \qquad \text{(QED, } 1-x_e \ll 1\text{)}$$

- In fact, it produces $-78\% \times N_f\,\alpha$ of the $-85\% \times N_f\,\alpha$ that we found for the overlap corrections to $\frac{\sigma}{l_{stop}}$.

A problem

- The potentially large double log correction, arising from a soft bremsstrahlung overlapping a hard splitting, is not constant: it depends logarithmically on the energy scale E of the underlying hard splitting.



$$\delta\hat{q} \sim \hat{q}\ \alpha_S \ln^2 \frac{E}{T}$$

$$g \to ggg$$

$$T \ll \omega \ll E$$

- So $\delta\hat{q}$ is different for different splittings in the shower, and those differences do not exactly cancel in $S(Z)$.

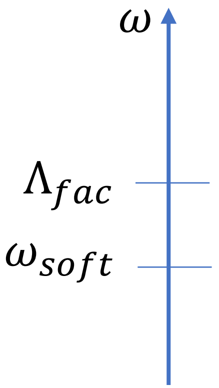
- The naive calculation of overlap corrections to the shape function $S(Z)$ will not be completely independent of soft radiation physics !

How to proceed?

- We introduce a factorization scale, where all overlapping soft radiation that has $\omega_{soft} \leq \Lambda_{fac}$, has been absorbed into $\hat{q}_{eff}(\Lambda_{fac})$ and so into an effective value $[LO\ rate]_{eff}$ of the $g \rightarrow gg$ rate.

- Now, the $[NLO\ rate]_{fac}$ is finite (after we subtract the IR divergences associated with one of the gluons being soft) and can be used to calculate the energy deposition distribution $\epsilon(z)$ and the shape function $S(Z)$.

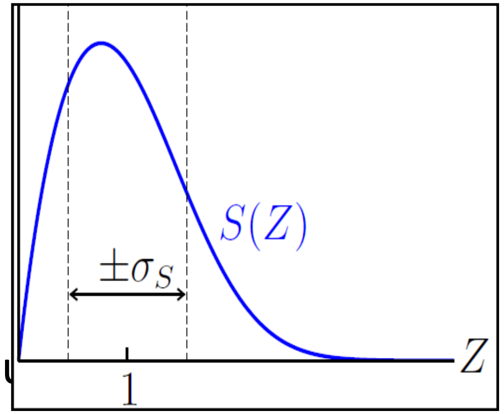
- The large double and single IR logs in $[LO\ rate]_{eff}$ would then have to be tamed by a next to leading –log (NLLO) resummation of IR logs to all orders in $\alpha_s(\mu)$.



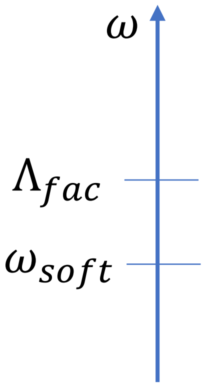
Solution

- It turns out …

$$\frac{\sigma_S^{NLO,fac}}{\sigma_S^{LO,eff}} = \frac{\sigma_S^{NLO,fac}}{\sigma_S^{LO} \times [1 + O(\sqrt{\alpha_s}\,)]} = \frac{\sigma_S^{NLO,fac}}{\sigma_S^{LO}}[1 + O(\sqrt{\alpha_s}\,)]$$



- The ratio $\sigma_S^{NLO,fac}/\sigma_S^{LO,eff}$ is itself an $O(\alpha_s)$ correction, and so the difference between σ_S^{LO} and $\sigma_S^{LO,eff}$ in the denominator is a yet-higher order correction to the ratio and so can be ignored.



- A technical note: we didn't do a full resummation of large logs, we only resummed the energy dependent part. Fortunately, we do not need a full resummation, because the shape function $S(Z)$ and its moments are insensitive to any *constant* (i.e. energy independent) shift in $\hat{q}$.