B工厂中e^+e^-湮灭产生双粲偶素两圈QCD修正

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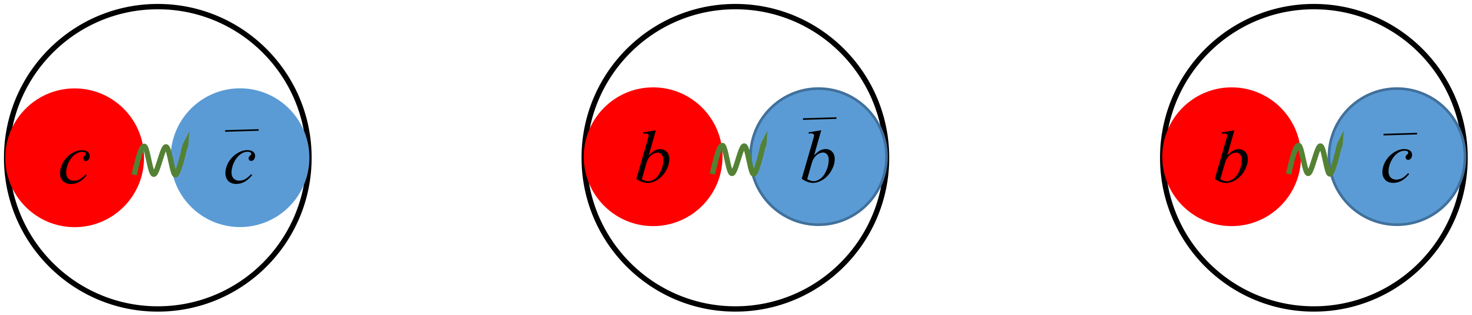
Contents

Background



Charmonium | Bottomonium | $b\bar{c}$ meson

- Since the discovery of J/ψ in 1974, the heavy quarkonium production has been a focus of theoretical and experimental researches.

- Heavy quarkonium is the simplest hadron in QCD, similar to the hydrogen atom in QED.

Experiment

- $\sigma(e^+e^- \to J/\psi + \eta_c) \times B_{\geq 2}$=$25.6 \pm 2.8 \pm 3.4$fb (Belle, PRD 2004)
- $\sigma(e^+e^- \to J/\psi + \eta_c) \times B_{\geq 2}$=$17.6 \pm 2.8^{+1.5}_{-2.1}$fb (BaBar, PRD 2005)
- $\sigma(e^+e^- \to J/\psi + J/\psi) \times \mathcal{B}_{>2} < 9.1$fb (Belle, PRD 2004)

Background

JHEP 02 049 (2023)

Theoretical Calculation

- The LO NRQCD predictions by three groups are smaller than Belle measurements by an order of magnitude!
  E. Braaten, J. Lee, PRD 2003、K. Y. Liu, Z. G. He, K. T. Chao, PLB 2003、
  K. Hagiwara, E. Kou, C. F. Qiao, PLB 2003
- Some other attempts have also been suggested to solve this discrepancy, such as the light-cone approach.
  J.P. Ma and Z.G. Si, PRD 2004、G.T. Bodwin, D. Kang and J. Lee, PRD 2006、
  V.V. Braguta, PRD 2009
- The relativistic corrections have been studied by several groups.
  G.T. Bodwin, D. Kang, T. Kim, J. Lee and C. Yu, AIPCP 2007、
  Z.-G. He, Y. Fan and K.-T. Chao, PRD 2007、G.T. Bodwin, J. Lee and C. Yu, PRD 2008
- The NLO NRQCD predictions is very significant, reduce the large discrepancy.
  Y. J. Zhang, Y. J. Gao and K.-T. Chao, PRL 2006、B. Gong, J. X. Wang, PRD 2008 K factor: 1.8~2.1
- The joint NLO QCD and relativistic correction has been investigated.
  H.-R. Dong, F. Feng and Y. Jia, PRD 2012、X.-H. Li and J.-X. Wang, Chin. Phys. C 2014
- The improved NLO prediction by using PMC shows agreement with the experimental measurements.
  Z. Sun, X.-G. Wu, Y. Ma and S.J. Brodsky, PRD 2018
- The challenging NNLO correction of this process has been calculated.
  F. Feng, Y. Jia, Z. Mo, W.-L. Sang and J.-Y. Zhang, PLB 2024 arXiv:1901.08447
- The light-cone sum rules has also been suggested to solve this discrepancy.
  L. Zeng, H.-B. Fu, D.-D. Hu, L.-L. Chen, W. Cheng and X.-G. Wu, PRD 2021

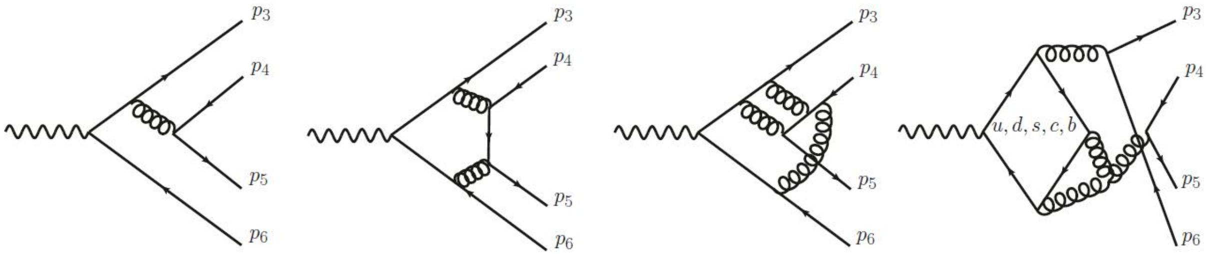
Motivation

1、In 2019, the challenging NNLO correction of this process was calculated in arXiv:1901.08447, however the precision of master integrals is not satisfied.

2、In 2022, a powerful algorithm named Auxiliary Mass Flow has been pioneered by Liu and Ma, which can be used to compute the Feynman integrals with very high precision.

Calculation of the NNLO SDCs

- Nearly 2000 two-loop diagrams for the processes $\gamma^* \to (c\bar{c})[\,^3S_1^{[1]}] + (c\bar{c})[\,^1S_0^{[1]}]$
  (FeynArts) T. Hahn, CPC 2001



- Handle the Lorentz index contraction and Dirac/SU(Nc) traces， Decompose the Feynman amplitudes into 150 Feynman integral families
  (CalcLoop) Yan-Qing Ma, https://gitlab.com/multiloop-pku/calcloop

- Calculate the Feynman integral families
  (Kira, AMFlow) J. Klappert, F. Lange, P. Maierhöfer and J. Usovitsch, CPC 2021
  X. Liu and Y.-Q. Ma, CPC 2023

- Input parameters:

$$\sqrt{s} = 10.58\text{GeV}, \quad m_b = 4.78\text{GeV}, \quad \alpha(\sqrt{s}) = 1/130.9,$$
$$\alpha_s(M_z) = 0.1179, \quad \langle \mathcal{O}^{J/\psi} \rangle = 0.440\text{GeV}^3, \quad \langle \mathcal{O}^{\eta_c} \rangle = 0.437\text{GeV}^3$$

PDG, PTEP 2022
G.T. Bodwin, J. Lee and C. Yu, PRD 2008

NNLO cross section

- The NNLO cross section (in fb) of $e^+e^- \to J/\psi + \eta_c$ with two typical renormalization scales μ_R under two factorization scale μ_Λ choices.

		α_s^2-terms	α_s^3-terms	α_s^4-terms	Total
$\mu_\Lambda = m_c$	$\mu_R = 2m_c$	7.40	7.04 $\boxed{+0.13}$	2.17 $\boxed{+0.28}$	16.61 $\boxed{+0.41}$
	$\mu_R = \sqrt{s}/2$	5.06	5.57 $\boxed{-0.05}$	3.43 $\boxed{-0.08}$	14.06 $\boxed{-0.13}$
$\mu_\Lambda = 1\,\mathrm{GeV}$	$\mu_R = 2m_c$	7.40	7.04 $\boxed{+0.13}$	4.62 $\boxed{+0.27}$	19.06 $\boxed{+0.40}$
	$\mu_R = \sqrt{s}/2$	5.06	5.57 $\boxed{-0.05}$	4.58 $\boxed{-0.09}$	15.21 $\boxed{-0.14}$

1 : 97% : 33% for $\mu_R = 2m_c$, 1 : 109% : 66% for $\mu_R = \sqrt{s}/2$

Contributions from bottom quark : 0.9% ~2.4%

- Results of arxiv: 1901.08447 (last revised 25 Nov. 2022)

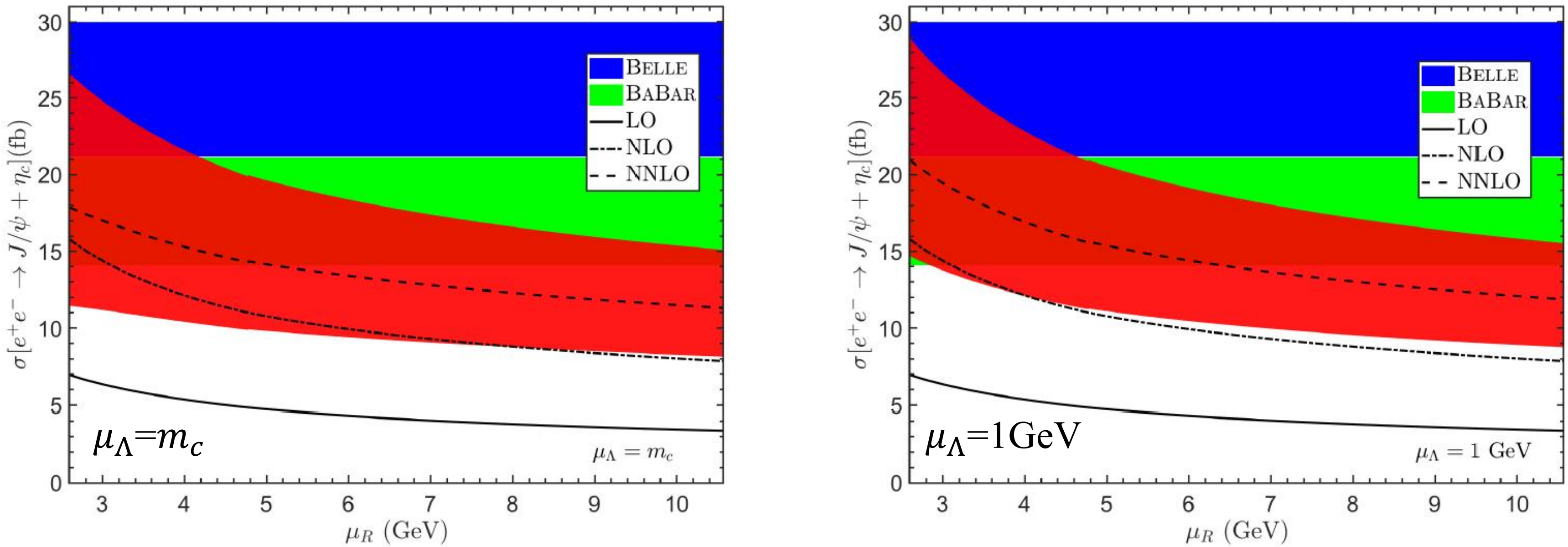
	m(GeV)	μ_{R}	LO	NLO	NNLO
μ_Λ=1GeV	1.5	$\sqrt{s}/2$	$5.05^{+0.92}_{-0.99}{}^{+2.31}_{-1.49}$	$10.54^{+2.86}_{-2.60}{}^{+3.92}_{-2.66}$	$15.00^{+5.03}_{-4.14}{}^{+4.29}_{-3.14}$

m_c $\mu_R \in [2m_c, \sqrt{s}]$

Our results are consistent with table I of arxiv: 1901.08447.

NNLO cross section

- The μ_R dependence of the predicted cross sections at LO, NLO and NNLO levels (central value for m_c=1.5GeV, bound for $m_c \in$ [1.3GeV, 1.7GeV])



1, NNLO has a milder μ_R dependence than NLO in μ_Λ=m_c

2, NNLO is much closer to experimental value in μ_Λ=1GeV

3, Theoretical prediction near $\mu_R = 2m_c$ agree with the experimental results better

Background

JHEP 02 055 (2024)

Theoretical Calculation

- The LO NRQCD predictions is even greater than the LO NRQCD prediction for $e^+e^- \to J/\psi + \eta_c$
  G. T. Bodwin, J. Lee and E. Braaten, PRL 2003 8.7fb
  G. T. Bodwin, J. Lee and E. Braaten, PRD 2003 6.65fb
- Two-photon exchange model, considered the photon fragmentation contribution only
  M. Davier, M. E. Peskin and A. Snyder, arXiv:hep-ph/0606155 2006 2.38fb
- Further took into account the non-fragmentation contribution within the NRQCD factorization framework,
  G. T. Bodwin, E. Braaten, J. Lee and C. Yu, PRD 2006 1.69 ± 0.35fb
- The NLO NRQCD predictions, the combined NLO perturbative and relativistic corrections
  B. Gong and J. X. Wang, PRL 2008 -3.4~2.3fb
  Y. Fan, J. Lee and C. Yu, PRD 2013 -12~-0.43fb
- Following the recipe practised in PRD 74, 074014 (2006), splitting the amplitude into the photon-fragmentation and non-fragmentation parts
  Y. Fan, J. Lee and C. Yu, PRD 2013 1~1.5fb
- Following PRD 74, 074014 (2006), the interference and the non-fragmentation parts are then computed through NNLO within NRQCD
  W. L.Sang, F. Feng, Y. Jia, Z. Mo, J. Pan and J. Y. Zhang, PRL 2023 $2.13^{+0.30}_{-0.06}$fb

Motivation

1、The NLO perturbative correction turns out to be negative and significant, the NNLO correction in the standard NRQCD?

2、How to obtain an positive, physical cross section in the standard NRQCD?

Calculating amplitudes

- The differential cross section can be written as

$$
\begin{aligned}
\frac{d\sigma_{e^+e^-\to(c\bar{c})[^3S_1^{[1]}]+(c\bar{c})[^3S_1^{[1]}]}}{d|\cos\theta|} &= \frac{1}{8s}\frac{\kappa}{16\pi}\left|\mathcal{A}^{0l}+\mathcal{A}^{1l}+\mathcal{A}^{2l}+\mathcal{O}(\alpha_s^3)\right|^2 \\
&= \frac{1}{8s}\frac{\kappa}{16\pi}\left(|\mathcal{A}^{0l}|^2+2\mathrm{Re}(\mathcal{A}^{1l}\mathcal{A}^{0l,*})+2\mathrm{Re}(\mathcal{A}^{2l}\mathcal{A}^{0l,*})+|\mathcal{A}^{1l}|^2\right. \\
&\quad \left.+2\mathrm{Re}(\mathcal{A}^{2l}\mathcal{A}^{1l,*})+|\mathcal{A}^{2l}|^2+\cdots\right),
\end{aligned}
$$

where $\kappa = \sqrt{1-(16m_c^2)/s}$ and θ is the angle between the J/ψ and the beam.

- The square of NNLO-amplitude (S-NNLO) Finite and gauge invariant

$$
\underbrace{\boxed{|\mathcal{A}^{0l}|^2}}_{\text{LO}}+\underbrace{\boxed{2\mathrm{Re}(\mathcal{A}^{1l}\mathcal{A}^{0l,*})}}_{\text{NLO}}+\underbrace{\boxed{2\mathrm{Re}(\mathcal{A}^{2l}\mathcal{A}^{0l,*})+|\mathcal{A}^{1l}|^2}}_{\text{NNLO}}+\boxed{2\mathrm{Re}(\mathcal{A}^{2l}\mathcal{A}^{1l,*})+|\mathcal{A}^{2l}|^2}+\cdots
$$

$$
\mathcal{A}^{nl}|_{n=0,1,2} = \sum_{i=1}^{10} c_i^{nl}|e_i\rangle
$$

$$
\begin{pmatrix} |e_1\rangle \\ |e_2\rangle \\ |e_3\rangle \\ |e_4\rangle \\ |e_5\rangle \\ |e_6\rangle \\ |e_7\rangle \\ |e_8\rangle \\ |e_9\rangle \\ |e_{10}\rangle \end{pmatrix} = \begin{pmatrix}
g^{\rho_1\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) \\
k_1^{\rho_1}k_1^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1)+k_2^{\rho_1}k_2^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) \\
k_1^{\rho_1}k_1^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1)-k_2^{\rho_1}k_2^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) \\
k_1^{\rho_1}k_2^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) \\
k_2^{\rho_1}k_1^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) \\
k_1^{\rho_2}\bar{v}_{m_e}(k_2).\gamma^{\rho_1}.u_{m_e}(k_1)+k_2^{\rho_1}\bar{v}_{m_e}(k_2).\gamma^{\rho_2}.u_{m_e}(k_1) \\
k_1^{\rho_2}\bar{v}_{m_e}(k_2).\gamma^{\rho_1}.u_{m_e}(k_1)-k_2^{\rho_1}\bar{v}_{m_e}(k_2).\gamma^{\rho_2}.u_{m_e}(k_1) \\
k_1^{\rho_1}\bar{v}_{m_e}(k_2).\gamma^{\rho_2}.u_{m_e}(k_1)+k_2^{\rho_2}\bar{v}_{m_e}(k_2).\gamma^{\rho_1}.u_{m_e}(k_1) \\
k_1^{\rho_1}\bar{v}_{m_e}(k_2).\gamma^{\rho_2}.u_{m_e}(k_1)-k_2^{\rho_2}\bar{v}_{m_e}(k_2).\gamma^{\rho_1}.u_{m_e}(k_1) \\
\bar{v}_{m_e}(k_2).\not{p}_2.\gamma^{\rho_1}.\gamma^{\rho_2}.u_{m_e}(k_1)
\end{pmatrix}
$$

$$
c_i^{nl}|_{n=0,1,2} = \sum_{j=1}^{10} G_{i,j}^{-1}d_j^{nl}
$$

$$
d_i^{nl}|_{n=0,1,2} = \langle\mathcal{A}^{nl}|e_i\rangle
$$

$$
G_{i,j} = \langle e_i|e_j\rangle
$$

$$
\mathcal{A}^{ml}\mathcal{A}^{nl,*} = \sum_{i=1}^{10}\sum_{j=1}^{10} c_i^{ml}G_{i,j}c_j^{nl,*}
$$

Phenomenological results

- Input parameters:

PDG, PTEP 2022
G.T. Bodwin, J. Lee and C. Yu, PRD 2008

$\sqrt{s} = 10.58\text{GeV}$, $m_b = 4.8\text{GeV}$, $m_c = 1.5\text{GeV}$, $\alpha(2m_c) = 1/132.6$, $\alpha_s(M_z) = 0.1179$,

$\left|R_s^{J/\psi}(0)\right|^2_{LO}$ =0.492GeV^3, $\left|R_s^{J/\psi}(0)\right|^2_{NLO}$ =0.796GeV^3, $\left|R_s^{J/\psi}(0)\right|^2_{NNLO,\mu_\Lambda=1GeV}$ =1.810GeV^3,

- The leptonic decay widths : $\Gamma_{J/\psi\to e^+e^-} = 5.53\text{keV}$

$$\boxed{\Gamma_{J/\psi\to e^+e^-} = C_0\left|R_s^{J/\psi}(0)\right|^2 (1 + a_1\alpha_s + a_2\alpha_s^2)^2}$$

M. Beneke, A. Signer and V. A. Smirnov, Phys. Rev. Lett. 80, 2535-2538 (1998)
F. Feng, Y. Jia, Z. Mo, J. Pan, W.-L. Sang, and J.-Y. Zhang, arXiv:2207.14259

- Short-distance coefficients f_0, f_1, f_2 : (cosθ=0.999)

$$f_2: \quad 10.2369 n_L + 4.2634 n_H \boxed{-22.6496 n_M - 0.0923 lbl} - 758.7978 - 187.8744\beta_0 L_\mu - 283.8247 L_{\mu_\Lambda}$$

This work:

$f_0\{1, -10.78, -130.51\}$

PRL 131 (2023) 161904:

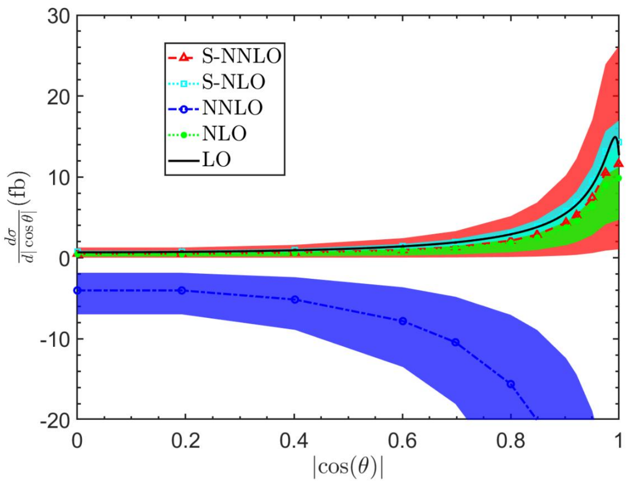
$f_0\{1, -10.78, -130.52\}$

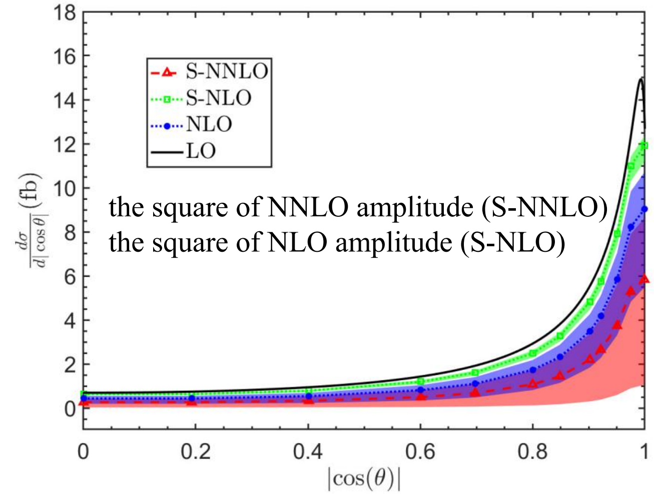
- Cross section

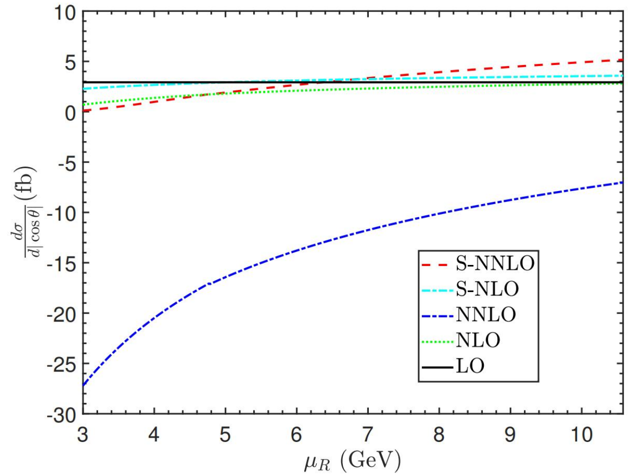
$\sigma(fb)$	LO	NLO	NNLO	S-NLO	S-NNLO
$\mu_R = 2m_c$	2.29	0.61	-21.10	1.83	0.12
$\mu_R = \sqrt{s}/2$	2.29	1.54	-11.97	2.37	1.76
$\mu_R = \sqrt{s}$	2.29	2.25	-5.27	2.84	4.17

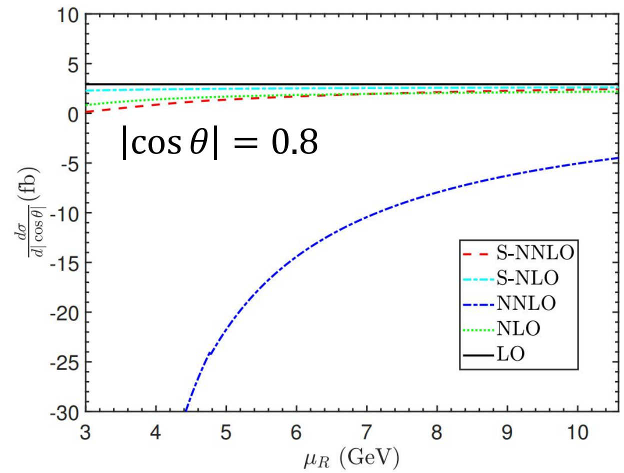
σ (fb)	Fragmentation	LO	NLO	NNLO
Optimized NRQCD	2.52	1.85	$1.93^{+0.05}_{-0.01}$	$2.13^{+0.30}_{-0.06}$
Traditional NRQCD		6.12	$1.56^{+0.73}_{-2.95}$	$-2.38^{+1.27}_{-5.35}$

renormalization scales μ_R dependence









$$\left|R_S^{J/\psi}(0)\right|^2_{\mu_R=2m_c} \qquad \sigma_{\text{S-NNLO}} = 1.76^{+2.41+0.25}_{-1.64-0.25} = 1.76^{+2.42}_{-1.66}\ \text{(fb)},$$

$$\left|R_S^{J/\psi}(0)\right|^2_{\mu_R\ denpendent} \qquad \sigma_{\text{S-NNLO}} = 0.88^{+0.50+0.25}_{-0.77-0.25} = 0.88^{+0.56}_{-0.81}\ \text{(fb)},$$

Summary

- The NNLO corrections to the total cross section for $e^+e^- \to J/\psi + \eta_c$ exhibits reasonable perturbative convergence behavior.
- Our prediction can both agree with the BaBar and Belle measurements.
- The NNLO prediction for $e^+e^- \to J/\psi + J/\psi$ suffers from an unphysical, negative cross section.
- We obtain a physical prediction of the cross section for $e^+e^- \to J/\psi + J/\psi$ in the standard NRQCD method.

Thanks!