Finite-t and Mass Corrections in Deeply Virtual Compton Scattering

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based on: Vladimir Braun, YJ, and Alexander Manashov, to appear

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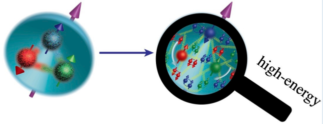
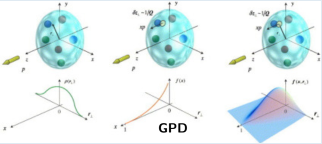
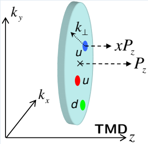
Internal structures of hadrons

- **Hadrons are composite particles with complicated internal structures**

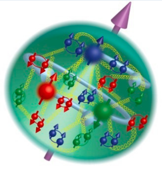


- **PDFs only encode longitudinal information; GPDs and TMDs include transverse information**





- **GPDs are crucial for understanding the spin structure of hadrons**

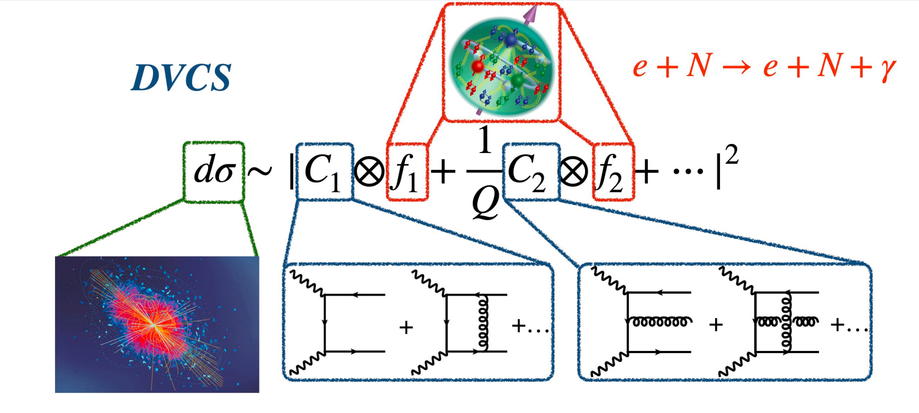
$\sum_{s,j}$  $\stackrel{?}{=} \frac{1}{2}$

Unsolved problem in physics:

? *How do the quarks and gluons carry the spin of protons?*

Extracting structure information of hadrons from high-energy experiment

- **Theoretical foundation for accessing structural information of hadrons: factorization**



Deeply Virtual Compton Scattering (DVCS)

- Golden channel for extracting the **Generalized Parton Distributions (GPDs)**

$$\gamma^*(q) + N(p) \to \gamma(q') + N(p')$$

Kinematics: $q'^2 = 0,\ q^2 = -Q^2,\ t = \Delta^2 = (p' - p)^2,\ p^2 = p'^2 = m^2,\quad P_\mu = (p_\mu + p'_\mu)/2$

- **DVCS** amplitude defined as (hadronic)

$$\delta(p + q - p' - q')\mathcal{A}_{\mu\nu}(q, q', p) = i \int \frac{d^4x d^4y}{(2\pi)^4}\, e^{-iq\cdot x + iq'\cdot y}\, \langle p' |\, T\{j^{\rm em}_\mu(x) j^{\rm em}_\nu(y)\}\, |p\rangle$$

decomposable into helicity amplitudes [V. Braun, A. Manashov, B. Pirnay, (2012)]

$$\mathcal{A}_{\mu\nu}(q, q', p) = \varepsilon^+_\mu \varepsilon^{*+}_\nu \mathcal{A}^{++} + \varepsilon^-_\mu \varepsilon^{*-}_\nu \mathcal{A}^{--} + \varepsilon^0_\mu \varepsilon^{*+}_\nu \mathcal{A}^{0+} + \varepsilon^0_\mu \varepsilon^{*-}_\nu \mathcal{A}^{0-} + \varepsilon^+_\mu \varepsilon^{*-}_\nu \mathcal{A}^{+-} + \varepsilon^-_\mu \varepsilon^{*+}_\nu \mathcal{A}^{-+} + q'_\nu \mathcal{A}^{(3)}_\mu$$

Parity conservation dictates:

$$\mathcal{A}^{++} = \mathcal{A}^{--} \equiv \mathcal{A}^{(0)}\,, \qquad \mathcal{A}^{0\pm} \equiv (\varepsilon^\pm_\mu P^\mu)\mathcal{A}^{(1)}\,, \qquad \mathcal{A}^{\mp\pm} \equiv (\varepsilon^\pm_\mu P^\mu)^2 \mathcal{A}^{(2)}\,,$$

- Other parameterizations available, exact relations between them known

[A. Belitsky, D. Müller, YJ, (2014)], [V. Braun, A. Manashov, D. Müller, B. Pirnay, (2014)]

Helicity amplitudes in DVCS

- **Helicity amplitudes $\mathcal{A}^{ij}$ can be expanded in $1/Q$,**

$$\mathcal{A}^{++} = \mathcal{A}_0^{(0)} + \frac{t}{Q^2}\mathcal{A}_{1,1}^{(0)} + \frac{m^2}{Q^2}\mathcal{A}_{1,2}^{(0)} + \cdots ,$$
$$\mathcal{A}^{0+} = \frac{Q}{q\cdot q'}\left(\mathcal{A}_0^{(1)} + \frac{t}{Q^2}\mathcal{A}_{1,1}^{(1)} + \frac{m^2}{Q^2}\mathcal{A}_{1,2}^{(1)} + \cdots\right),$$
$$\mathcal{A}^{-+} = \frac{\mathcal{O}(1)}{q\cdot q'}\left(\mathcal{A}_0^{(2)} + \frac{t}{Q^2}\mathcal{A}_{1,0}^{(2)} + \frac{m^2}{Q^2}\mathcal{A}_{1,1}^{(2)} + \cdots\right),$$

$$\mathcal{A}_{j,z}^{(i)} \sim \ln(\mu^2/Q^2), \qquad q\cdot q' \sim Q^2$$

- **subleading corrections are generated from two sources:**

$$\mathcal{A}_{j>0,z}^{(i)} = \mathcal{A}_{j,z}^{(i),\text{kin}} + \mathcal{A}_{j,z}^{(i),\text{dyn}}$$

- **dynamical corrections carries genuine higher-twist information of the hadron, while kinematic ones are induced by twist-two GPDs**
- **Each term is a convolution of coefficient function $\mathcal{C}_{j,z}^{(i)}$ and GPD H_k, schematically*:**

$$\mathbb{A}_{j,z}^{(i),\text{kin}} = C_{j,z}^{(i)} \otimes H_2\,, \qquad \mathcal{A}_{j,z}^{(i),\text{dyn}} = C_{j,z}^{(i,k)} \otimes H_k\,, k>2\,,$$

***if factorization holds**

- **DIS case: $t = 0 \mapsto$ Nachtmann correction, (1973)**

Kinematic power corrections

- We study the kinematic corrections induced by **twist-two** GPDs to **NNLP** $\mathcal{A}^{(i)}$ at tree-level
- GPDs are defined in terms of light-ray operators, twist-two vector case:

$$\langle p'| O_2(z_1 n, z_2 n) |p\rangle = 2P_+ \int_{-1}^{1} dx\, e^{-iP_+[z_1(\xi-x)+z_2(\xi+x)]} F_q(x,\xi,t)\,,$$

$$O_2(z_1 n, z_2 n) = \frac{1}{2}\Big(\bar{q}(z_1 n)\gamma_+ q(z_2 n) - \bar{q}(z_2 n)\gamma_+ q(z_1 n)\Big)\,,$$

$$n_\mu = q'_\mu, \qquad \xi = \frac{p_+ - p'_+}{p_+ + p'_+}\,, \qquad \widetilde{n}_\mu = -q_\mu + \frac{Q^2}{(Q^2+t)}\, q'_\mu\,, \qquad a_+ \equiv a\cdot n$$

[V. Braun, A. Manashov, B. Pirnay, (2012)]

actual expansion parameter $1/(n\cdot\widetilde{n}) = 2/(t+Q^2)$

- Two-point nonlocal operator admits local OPE (**x^2 not necessary on the lightcone**):

$$O_2(z_1 x, z_2 x) = \sum_{N=1} \frac{2(2N+1)}{N!} z_{12}^{N-1} \int_0^1 du\, (u\bar{u})^N\, [\mathcal{O}_N(z_{21}^u x)]_{lt}\,,$$

$$[\mathcal{O}_N(y)]_{lt} = x^{\mu_1}\cdots x^{\mu_N}[\mathcal{O}_{\mu_1\cdots\mu_N}(y)]_{lt}\,, \qquad \text{conformal operator}$$

convention: $z_{12} \equiv z_1 - z_2 = 1,\; z_{21}^u \equiv \bar{u}z_2 + u z_1$

- kinematic corrections are induced by: hard to separate from genuine higher-twist in old technique

$$[\partial_y^{\mu_1}\mathcal{O}_{\mu_1\cdots\mu_N}(y)]_{lt}\,, \qquad [\partial_y^2\mathcal{O}_{\mu_1\cdots\mu_N}(y)]_{lt}\,, \;\cdots$$

kinematic correction

- consider local OPE of $\langle p_1|\, \{j_\mu(z_1) j_\nu(z_2)\, |p_2\rangle$, kinematic corrections are induced by:

$$[\partial_y^{\mu_1} \mathcal{O}_{\mu_1\cdots\mu_N}(y)]_{lt}\,, \qquad [\partial_y^2 \mathcal{O}_{\mu_1\cdots\mu_N}(y)]_{lt}\,, \ \cdots \ \Longrightarrow \ \langle p_1|\,|[\cdots]|\,|p_2\rangle \neq 0$$

- they must be included to restore EM gauge invariance for nonforward scattering!
- "kinematic approximation" $\Leftrightarrow$ setting all genuine higher-twist operators to zero at all scales
  - consistent under quantum corrections? evolution?
  - yes! genuine and kinematic higher-twist (all twist-2 descendant) evolve autonomously
    ↰ but not true for each genuine higher-twists operators/GPDs: mixing

[V. Braun, A. Manashov, J. Rohrwild (2009), YJ, A Belitsky, (2014)]

- vanish for onshell quark, subtlety in separating kinematic and genuine higher-twists

[Ferrara, Grillo, Parisi, Gatto, (1971) - (1973)]

$$\partial^\mu O_{\mu\nu} = \boxed{2i\bar{q} g F_{\nu\mu}\gamma^\mu q} \qquad\qquad O_{\mu\nu} = \frac{1}{2}[\bar{q}\gamma_\mu \overleftrightarrow{D}_\nu q + (\mu \leftrightarrow \nu)]$$

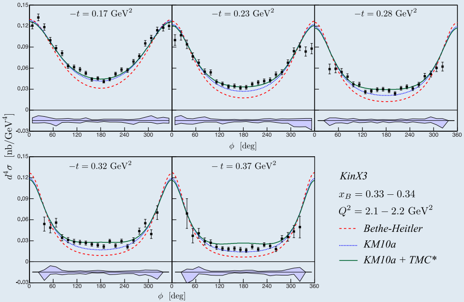
must use RHS, but hard to distinguish RHS from genuine twist-4 operator

Motivation

- **Theory side:**
  - **necessary to remove frame dependence, restore EM gauge and translation symmetry**
  - **a systematic framework for higher-power kinematic contribution**
  - **verify factorization at NNLP order**
  - **application of conformal symmetry to higher-power corrections**
- **Phenomenological side:**
  - **provide theory support for nuclei DVCS measurements (significant m^2 corrections)**

[M. Hattaway et al. [CLAS], Phys. Rev. Lett. 119, no.20, 202004 (2017)]

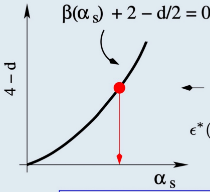
  - **mismatch between theory and experiment, resolvable by higher-power corrections?**



[M. Defurne et al. [Hall A Collaboration], (2015)]

Application of conformal technique in QCD: general idea

QCD is not a conformal theory, but



conformal theory

$$\epsilon^*(a_s) = -\beta_0 a_s + \dots\,, \qquad a_s = \frac{\alpha_s}{4\pi}$$

$$\mathcal{A}_{\rm QCD} = \mathcal{A}^{\rm conf}_{\rm QCD} + \mathcal{O}\left(\frac{\beta(\alpha_s)}{\alpha_s}\right)$$

- "conformal QCD": Wilson-Fisher critical point $\beta(\alpha_s, \epsilon) = 0$ with $d = 4 - 2\epsilon$
- used to compute kernels (-1 loop), RG-resummation, and CFs: two-loop by "hand"

e.g., [V. Braun, YJ, A. Manashov, (2019), V. Braun, YJ, A. Manashov (2017), V. Braun, A. Manashov, S. Moch, J. Schoenleber]

Kinematic corrections: local form

- **OPE expansion of current correlator (scalar current for demonstration):**

vector-vector current: [V. Braun, YJ, A. Manashov (2021)]

$$T\{j(x_1)j(x_2)\} = \sum_N \Big\{ A_N^{\mu_1\cdots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\cdots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\cdots\mu_N}}_{\text{descendants of twist-2}}$$

$$+ C_N^{\mu_1\cdots\mu_N} \partial^2 \underbrace{\mathcal{O}^N_{\mu_1\cdots\mu_N}} + D_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu \partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\cdots\mu_N}}_{\text{descendants}} + \cdots \Big\}$$

$$+ \text{ genuine higher-twist contributions}$$

$$\equiv \sum_N C_N^{\mu_1\cdots\mu_N}(x,\partial)\mathcal{O}^N_{\mu_1\cdots\mu_N} + \text{genuine higher-twist contributions}$$

- **employ conformally covariant OPE to extract $C_N^{\mu_1\cdots\mu_N}(x,\partial)$:** [S. Ferrara, A. F.Grillo, and Gatto, 1971-1973]

CFs of descendants are related to the CFs of the twist-2 operators by conformal symmetry

$$A_N^{\mu_1\cdots\mu_N} \overset{O(4,2)}{\mapsto} C_N^{\mu_1\cdots\mu_N}(x,\partial)$$

conformal triangles

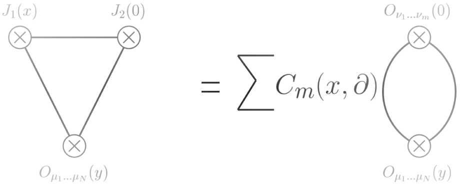
A. M. Polyakov, 1970

$$\langle O_1(x_1)O_2(x_2)\rangle = \frac{\text{const}}{|x_1-x_2|^{2\Delta_1}}\delta_{\Delta_1\Delta_2}\,,$$

$$\langle O_1(x_1)O_2(x_2)O_2(x_3)\rangle = \frac{\text{const}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}}\,,$$

Δ_i are scaling dimensions: canonical + anomalous

- $A_N, B_N, \cdots$ are extracted by exploiting conformal properties of 3-point function



- most efficient to apply shadow operator formalism to extract $C_m(x,\partial)$ to all orders!
  - four(one) independent Lorentz structures for vector(axial-vector) case
  - in general, calculation carried out at d^*-dimensions (critical point), here $d^* = 4$
- normalization (**const**) fixed by DIS coefficient functions
  - agree with previous lower-order result (different approach) [V. Braun and A. Manashov, (2012)]

CFs for kinematic corrections: local OPE of leading twist and descendants

- **A glimpse of local coefficient functions**

[V. Braun, YJ, A. Manashov (2021)]

  - **vector case:**

$$
\begin{aligned}
\Big[j^\mu(x_1)j^\nu(x_2)\Big]_V =& \sum_N \frac{r_{N,\mathrm{V}}}{(-x_{12}^2+i0)^2}\int_0^1 du\,(u\bar u)^N \Big[(N+1)g^{\mu\nu}\Big(1-\frac{1}{4}\frac{u\bar u}{N+1}x_{12}^2\partial^2\Big)\\
&+\frac{1}{2N}x_{12}^2(\partial_1^\mu\partial_2^\nu-\partial_1^\nu\partial_2^\mu)+\Big(1-\frac{1}{4}\frac{u\bar u}{N}x_{12}^2\partial^2\Big)\Big(\frac{\bar u}{u}x_{21}^\mu\partial_1^\nu+\frac{u}{\bar u}x_{12}^\nu\partial_2^\mu\Big)\\
&-\frac{1}{4}\frac{u\bar u}{N(N+1)}x_{12}^2\partial^2\Big(x_{21}^\nu\partial_1^\mu+x_{12}^\mu\partial_2^\nu\Big)\\
&-\frac{x_{12}^\mu x_{12}^\nu}{N+1}u\bar u\partial^2\Big(1-\frac{1}{4}\frac{u\bar u}{N+2}x_{12}^2\partial^2\Big)\Big]\mathcal{O}^{(0)}_{N,V}(x_{21}^u)+\cdots,\qquad x_{21}^u=\bar u x_2+ux_1\,,
\end{aligned}
$$

  - **axial-vector case:**

$$
\begin{aligned}
\Big[j^\mu(x_1)j^\nu(x_2)\Big]_A =& \frac{r_{N,\mathrm{A}}}{(-x_{12}^2+i0)^2}\int_0^1 du\,(u\bar u)^N\Big\{\epsilon^{\mu\nu}{}_{\beta\gamma}x_{12}^\beta\Big[N\Big(\frac{u}{\bar u}\partial_2^\gamma-\frac{\bar u}{u}\partial_1^\gamma\Big)\\
&-\frac{1}{4}\frac{u\bar u x_{12}^2\partial^2}{(N+1)}\Big(\partial_2^\gamma-\partial_1^\gamma+(N+1)\Big(\frac{u}{\bar u}\partial_2^\gamma-\frac{\bar u}{u}\partial_1^\gamma\Big)\Big)\Big]\\
&-\Big(x_{12}^\nu\epsilon^\mu{}_{\alpha\beta\gamma}+x_{12}^\mu\epsilon^\nu{}_{\alpha\beta\gamma}\Big)x_{12}^\alpha\Big(1-\frac{1}{4}\frac{u\bar u x_{12}^2\partial^2}{N+1}\Big)\partial_1^\beta\partial_2^\gamma\Big\}\mathcal{O}^{(0)}_{A,N}(x_{21}^u)+\cdots
\end{aligned}
$$

Kinematic corrections: nonlocal form

- resumming back into nonlocal operator $\mapsto$ **GPD/DD** [V. Braun, YJ, A. Manashov (2022)]

$$\begin{aligned}
\mathbb{A}_{\rm V}^{\mu\nu} &= \mathrm{T}\{j^\mu(x)j^\nu(0)\} \\
&= \frac{1}{i\pi^2}\Bigg\{\frac{1}{(-x^2+i0)^2}\int_0^1 dv\Big[\big[g^{\mu\nu}(x\partial)-x^\mu\partial^\nu\big]O(v,0)-x^\nu(\partial^\mu-i\Delta^\mu)O(1,v)\Big] \\
&\quad -\frac{1}{(-x^2+i0)}\int_0^1 du\int_0^u dv\left[\frac{i}{2}(\Delta^\nu\partial^\mu-\Delta^\mu\partial^\nu)O(u,v)-\frac{\Delta^2\,\bar u}{4}x^\mu\partial^\nu O(u,v)\right] \\
&\quad +\frac{\Delta^2}{2}\frac{x^\mu x^\nu}{(-x^2+i0)^2}\int_0^1 du\,u\int_0^u dv\,O(u,v)+\cdots\Bigg\},
\end{aligned}$$

$O(z_1,z_2)$ **non-local twist-2 light-ray operators** $\mapsto$ **GPD/DD**, also $\Delta\cdot\partial_x O(z_1,z_2)$, $\cdots$

- many highly nontrivial cancellations must happen to resum local $\mapsto$ nonlocal
  - need “intertwining operator” to relate operators of different conformal spin
- similar for axial-vector case

Phenomenology: nucleon DVCS

- **Fourier transform to momentum space: final prediction to NNLP***

$$\mathcal{A}_{\mu\nu}(q,q',p) = \varepsilon_\mu^+ \varepsilon_\nu^{*+} \mathcal{A}^{++} + \varepsilon_\mu^- \varepsilon_\nu^{*-} \mathcal{A}^{++} + \varepsilon_\mu^0 \varepsilon_\nu^{*+} \mathcal{A}^{0+} + \varepsilon_\mu^0 \varepsilon_\nu^{*-} \mathcal{A}^{0+} + \varepsilon_\mu^+ \varepsilon_\nu^{*-} \mathcal{A}^{-+} + \varepsilon_\mu^- \varepsilon_\nu^{*+} \mathcal{A}^{-+}$$

- **Fourier transform to momentum space: final prediction to NNLP***

$$\mathcal{A}^{++} \sim 1 + \frac{1}{Q^2} + \frac{1}{Q^4} + \cdots ,$$
$$\mathcal{A}^{0+} \sim \frac{1}{Q} + \frac{1}{Q^3} + \cdots ,$$
$$\mathcal{A}^{-+} \sim \frac{1}{Q^2} + \frac{1}{Q^4} + \cdots ,$$

- **Further contributions can be calculated if necessary**

A glimpse to the final analytic results

- **structure of $\mathcal{A}^{-+}$, an example:** [V. Braun, YJ, A. Manashov (2024), to appear]

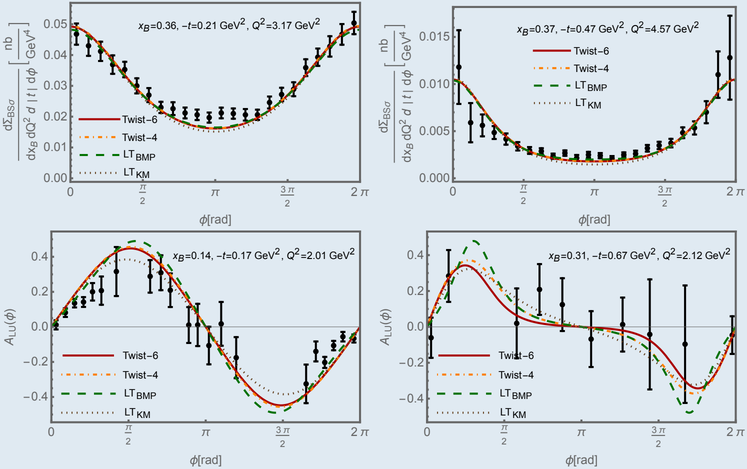
$$\mathcal{A}^{-+}_{1/Q^4} = -\frac{3\varkappa|P_\perp|^2}{2(q\cdot q')^2}\left(t\left(1+\frac{1}{3\xi}D_\xi\right)-\frac{1}{12}|P_\perp|^2 D_\xi^2\right) \times D_\xi^2\int_{-1}^{1}\frac{dx}{\xi}\mathcal{H}(x,\xi,t)\left(\frac{1}{\bar{x}_\xi}\left(\mathrm{Li}_2(x_\xi)-\zeta_2\right)-\ln\bar{x}_\xi\right)+\cdots$$

$$D_\xi = \xi^2\partial_\xi\,,\; x_\xi = \frac{x+\xi}{2\xi}+i0\,,\; \bar{x}_\xi = 1-x_\xi\,,\; |P_\perp|^2 = -m^2-t(1-\xi^2)/(4\xi^2).$$

- **$\mathcal{A}^{-+}$ and $\mathcal{A}^{0+}$ relatively simple, $\mathcal{A}_{++}$ complicated; factorization holds!**
  - **factorization violating $1/q'^2\,,\ln q'^2$ cancel in final result, highly nontrivial for $\mathcal{A}^{++}$**
  - **only Li_2, $\ln^2$ appears in final result**
  - **agree with previous lower-order result** [V. Braun, A. Manashov, B. Pirnay, Phys. Rev. D 86 (2012) 014003]

Numerics

- **Expansion parameter** $1/Q^2 \to 1/(qq') = 2/(Q^2+t)$**!**



GPD model: GK12; lower panel: electron helicity dependent cross section

Conclusions and outlooks

- **Conclusions**
  - NNLP corrections are small with expansion $1/(qq') = -\frac{2}{Q^2+t}$: resummation
  - kinematic corrections to CFFs in BMJ frame are generally larger
  - result respects all symmetries to $1/Q^5$ order
  - tree-level factorization holds to NNLP

- **Outlooks**
  - more extensive numerical analysis for JLAB/EIC data
  - embedding into public codes
  - α_s corrections, kinematic corrections involving gluon GPD: long term
  - apply to other exclusive processes: target mass correction [P. Ball, V. Braun, NPB543 (1999) 201]