Scale-invariant total decay width $\Gamma(H \to b\bar{b})$ using the novel method of characteristic operator

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Outline

QCD properties

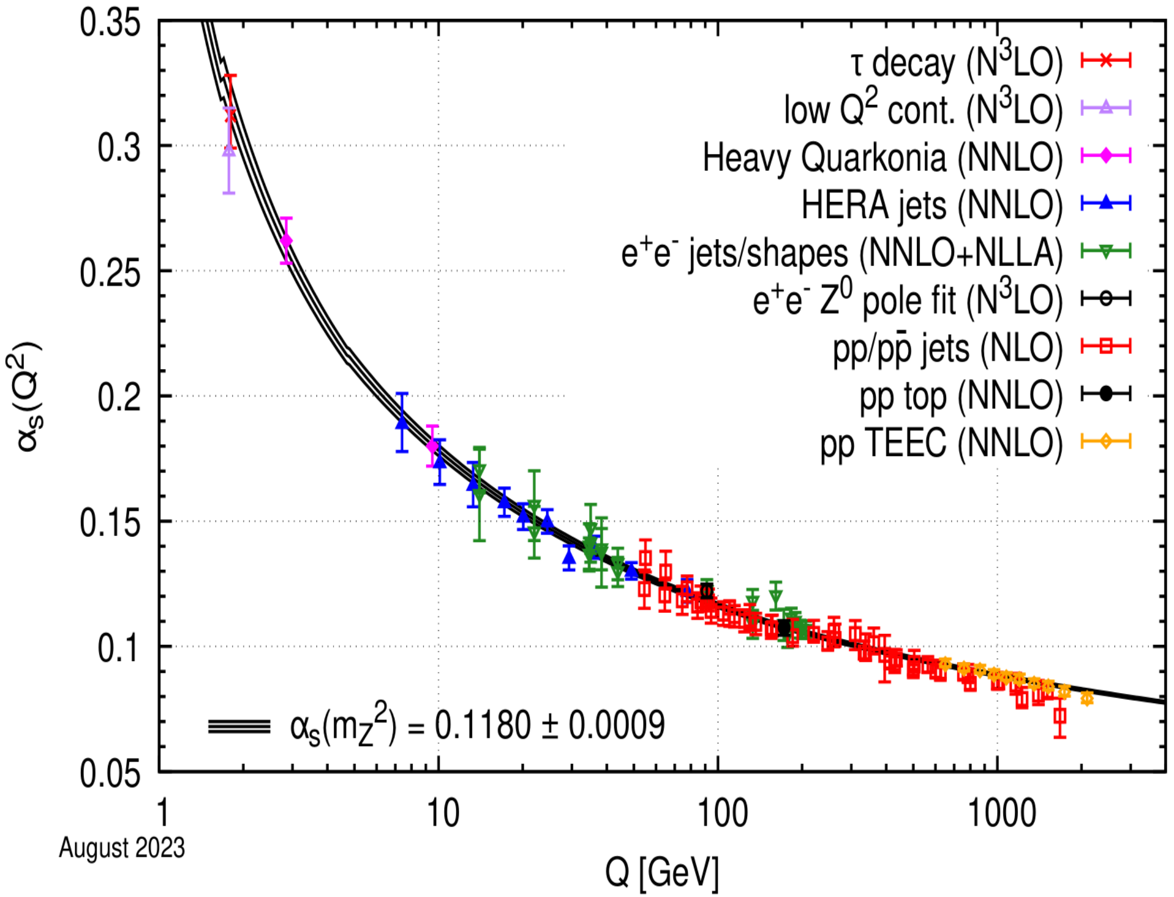
- **quark confinement** $\Lambda_{QCD} \sim 0.2$ GeV
- **asymptotic freedom**

$\alpha_s(Q \gg \Lambda_{QCD}) < 1$

perturbative calculation

$$\rho(Q) = r_1 \alpha_s^n + r_2 \alpha_s^{n+1} + r_3 \alpha_s^{n+2} + \cdots$$

LO **NLO** **N²LO** …



S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024)

Renormalization scale-setting problem

In perturbative calculations of observables → **UV Divergence**
→ **regularization & renormalization**
→ **cancel the divergences**

$$\rho_p = \sum_{i=1}^{p} r_i(\mu_r, Q)\alpha_s^{n+i-1}(\mu_r)$$

↔ Certain renormalization scheme and renormalization scale μ_r have to be introduced !

A valid prediction for a physical observable must be independent to any choices of renormalization scheme and renormalization scale.

---Renormalization group invariance (RGI)

$$\frac{\partial \rho}{\partial \mu_r} = 0, \qquad \frac{\partial \rho}{\partial R} = 0$$

How to achieve such scale invariance for a fixed-order series ? → $\frac{\partial \rho_p}{\partial \mu_r} = 0$? ? ✗ Breaking standard RGI !!

Conventional scale-setting approach

A fixed-order pQCD prediction ρ_p

$$\rho_p = \sum_{i=1}^{p} r_i(\mu_r, Q)\alpha_s^{n+i-1}(\mu_r)$$

Scale- and scheme-dependence due to mismatching of α_s with its coefficients for an arbitrary choice of scale!

$\mu_r = \mu_0$ (initial scale) → central value of the prediction

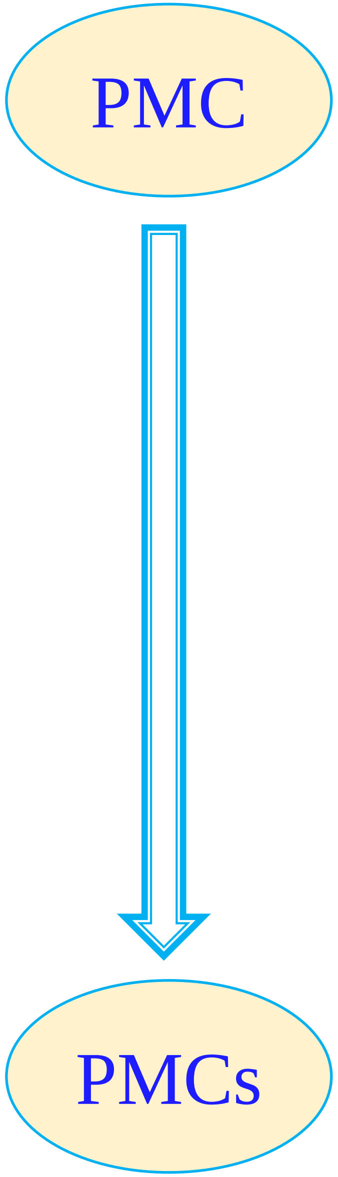
"Choosing" μ_0 to be the typical momentum flow Q of process or to eliminate large/dangerous logarithms or to get more convergent series or to agree with data.

$\mu_r \in [\mu_0/t, t\mu_0]$ → theoretical uncertainties of the prediction

Disadvantages: μ_0-dependent, t-dependent

no decent theoretical basis

Principle of Maximum Conformality



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- ***The first kind of residual scale dependence***

  the unknown terms in the determined PMC scales, such as $Q_{1,\ldots,n-1}$, due to their perturbative nature

- ***The second kind of residual scale dependence***

  the last perturbative terms of the pQCD approximant are not fixed since its PMC scale cannot be fixed

Although these two residual scale dependence are suppressed at high orders in α_s and/or from exponential suppression, if the convergence of the perturbative series of either the PMC scale or the pQCD approximant is weak, such residual scale dependence could be significant.

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Characteristic Operator (CO)

A fixed-order pQCD prediction ρ

Kernel function

$$\rho\big(\mu_r, \alpha_s(\mu_r), \overline{m}_q(\mu_r); Q\big) = \overline{m}_q^{\,n_\gamma}(\mu_r)\,\alpha_s^{n_\beta}(\mu_r)\,R\big(\mu_r, \alpha_s(\mu_r), \overline{m}_q(\mu_r); Q\big)$$

$$R\big(\mu_r, \alpha_s(\mu_r), \overline{m}_q(\mu_r); Q\big) = \sum_{i\geq 1} r_i\big(\mu_r/Q, \overline{m}_q\big)\,\alpha_s^{i-1}(\mu_r)$$

RGI $\quad \dfrac{\mathrm{d}\rho}{\mathrm{d}\ln\mu_r^2} = 0 \quad \Rightarrow \quad \left(n_\gamma \gamma_m + n_\beta \dfrac{\beta(\alpha_s)}{\alpha_s} + \beta(\alpha_s)\dfrac{\partial}{\partial \alpha_s}\right) R = -\left(\dfrac{\partial}{\partial \ln\mu_r^2} + \gamma_m \overline{m}_q \dfrac{\partial}{\partial \overline{m}_q}\right) R$

operates on α_s → process-independent

operates on coefficients → process-dependent

$$\hat{\mathcal{D}}_{n_\gamma, n_\beta} := n_\gamma \gamma_m + n_\beta \frac{\beta(\alpha_s)}{\alpha_s} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}$$

Characteristic Operator (CO)

Characteristic Operator (CO)

$$\widehat{\mathcal{D}}_{n_\gamma,n_\beta} \coloneqq n_\gamma \gamma_m(\alpha_s) + n_\beta \frac{\beta(\alpha_s)}{\alpha_s} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}$$

$$\frac{\mathrm{d}\alpha_s(\mu)}{\mathrm{d}\ln\mu^2} = \beta(\alpha_s)$$

$$\frac{\mathrm{d}\overline{m}_q(\mu)}{\mathrm{d}\ln\mu^2} = \overline{m}_q(\mu)\gamma_m(\alpha_s)$$

$$\frac{\mathrm{d}^k \alpha_s^{n_\beta}(\mu)}{(\mathrm{d}\ln\mu^2)^k} = \alpha_s^{n_\beta}(\mu)\widehat{\mathcal{D}}^k_{0,n_\beta}[1]$$

$$\frac{\mathrm{d}^k \overline{m}_q^{n_\gamma}(\mu)}{(\mathrm{d}\ln\mu^2)^k} = \overline{m}_q^{n_\gamma}(\mu)\widehat{\mathcal{D}}^k_{n_\gamma,0}[1]$$

$$\frac{\mathrm{d}^k [\overline{m}_q^{n_\gamma}(\mu)\alpha_s^{n_\beta}(\mu)]}{(\mathrm{d}\ln\mu^2)^k} = \overline{m}_q^{n_\gamma}(\mu)\alpha_s^{n_\beta}(\mu)\widehat{\mathcal{D}}^k_{n_\gamma,n_\beta}[1]$$

Properties

$$\widehat{\mathcal{D}}^k_{n_\gamma,n_\beta}[1] = \sum_{i=0}^{k} C_k^i\, \widehat{\mathcal{D}}^i_{n_\gamma,0}[1]\widehat{\mathcal{D}}^{k-i}_{0,n_\beta}[1]$$

This relationship allows us to differentiate the contributions from various running quantities within the pQCD series.

$$\widehat{\mathcal{D}}^k_{n_\gamma,n_\beta}[\alpha_s^\ell] = (-1)^k \sum_{i\geq 0} d_i^{[n_\gamma,n_\beta;k,\ell]}\alpha_s^{\ell+k+i}$$

$$\alpha_s^\lambda \widehat{\mathcal{D}}^k_{n_\gamma,n_\beta+\lambda}[\alpha_s^\ell] = \widehat{\mathcal{D}}^k_{n_\gamma,n_\beta}[\alpha_s^{\ell+\lambda}]$$

$$d_i^{[n_\gamma,n_\beta+\lambda;k,\ell]} = d_i^{[n_\gamma,n_\beta;k,\ell+\lambda]}$$

Principle of Maximum Conformality

$$R = r_1 + [c_{2,0} + c_{2,1} n_f]\alpha_s(\mu_r) + [c_{3,0} + c_{3,1} n_f + c_{3,2} n_f^2]\alpha_s^2(\mu_r) + [c_{4,0} + c_{4,1} n_f + c_{4,2} n_f^2 + c_{4,3} n_f^3]\alpha_s^3(\mu_r) + \cdots$$

transform RGE-involved n_f-series into $\{\beta_i, \gamma_i\}$-series

$$R = r_{1,0} + \left(r_{2,0} + d_0^{[n_\gamma,n_\beta;1,0]} r_{2,1}\right)\alpha_s(\mu_r) + \left(r_{3,0} + d_1^{[n_\gamma,n_\beta;1,0]} r_{2,1} + d_0^{[n_\gamma,n_\beta;1,1]} r_{3,1} + \frac{1}{2!} d_0^{[n_\gamma,n_\beta;2,0]} r_{3,2}\right)\alpha_s^2(\mu_r)$$

$$+ \left(r_{4,0} + d_2^{[n_\gamma,n_\beta;1,0]} r_{2,1} + d_1^{[n_\gamma,n_\beta;1,1]} r_{3,1} + d_0^{[n_\gamma,n_\beta;1,2]} r_{4,1} + \frac{1}{2!}\left(d_1^{[n_\gamma,n_\beta;2,0]} r_{3,2} + d_0^{[n_\gamma,n_\beta;2,1]} r_{4,2}\right) + \frac{1}{3!} d_0^{[n_\gamma,n_\beta;3,0]} r_{4,3}\right)\alpha_s^3(\mu_r) + \cdots$$

$$= \sum_{i\geq 1} r_{i,0}\alpha_s^{i-1}(\mu_r) + \sum_{i\geq 2}\sum_{j=1}^{i-1} \frac{(-1)^j}{j!} r_{i,j} \hat{\mathcal{D}}^j_{n_\gamma,n_\beta}\left[\alpha_s^{i-j-1}(\mu_r)\right]$$

conformal terms — non-conformal terms

eliminate all $\{\beta_i, \gamma_i\}$-terms

$$R\Big|_{\rm PMC} = \sum_{i\geq 1} r_{i,0}\alpha_s^{i-1}(Q_*)$$

massless

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$H \to b\bar{b}$

massive *b*-quark

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PMC analysis

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$H \to b\bar{b}$

The decay width of the Higgs boson into a $b\bar{b}$ pair is given by

$$\Gamma_{H\to b\bar{b}} = \frac{G_F M_H}{4\sqrt{2}\pi}\overline{m}_b^2(M_H)\tilde{R}(s=M_H^2) \quad \Rightarrow \quad \tilde{R} = \frac{1}{2\pi s}\mathrm{Im}\,\tilde{\Pi}(-s-\mathrm{i}\epsilon)$$

scalar two-point correlator

$\tilde{\Pi}(\mu;Q)$ satisfies the following RGE when $\tilde{\Pi}(\mu;Q)$ is massless

$$\frac{\mathrm{d}\tilde{\Pi}(\mu;Q)}{\mathrm{d}\ln\mu^2} = \left(\frac{\partial}{\partial\ln\mu^2} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right)\tilde{\Pi}(\mu;Q) = 2\gamma_m(\alpha_s)\tilde{\Pi}(\mu;Q) + Q^2\gamma^{SS}(\alpha_s)$$

The Adler function $\tilde{D}(\mu,Q)$, defined in the Euclidean region, has been introduce to associate the observable defined in the Minkowskian space as

ensure $\tilde{D}(\mu,Q)$ is scale-invariant

$$\tilde{D}(\mu,Q) := \frac{1}{6}\frac{\mathrm{d}}{\mathrm{d}\ln Q^2}\frac{\tilde{\Pi}(\mu;Q)}{Q^2} = \frac{1}{6}\left(2\gamma_m(\alpha_s) + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right)\frac{\tilde{\Pi}(\mu;Q)}{Q^2} - \frac{1}{6}\gamma^{SS}(\alpha_s) = \int_0^\infty \frac{Q^2\tilde{R}(\mu;s)}{(s+Q^2)^2}\mathrm{d}s$$

$H \to b\bar{b}$

$$\frac{1}{6}\left(2\gamma_m(\alpha_s) + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right)\frac{\widetilde{\Pi}(\mu;Q)}{Q^2} - \frac{1}{6}\gamma^{\mathrm{SS}}(\alpha_s) = \int_0^\infty \frac{Q^2\tilde{R}(\mu;s)}{(s+Q^2)^2}\mathrm{d}s$$

$$\int_0^\infty \mathrm{d}s\,\frac{Q^2}{(s+Q^2)^2}\left\{1;\ln\frac{\mu^2}{s};\ln^2\frac{\mu^2}{s};\ln^3\frac{\mu^2}{s};\ln^4\frac{\mu^2}{s}\right\}$$

$$=\left\{1;\ln\frac{\mu^2}{Q^2};\ln^2\frac{\mu^2}{Q^2}+\frac{\pi^2}{3};\ln^3\frac{\mu^2}{Q^2}+\pi^2\ln\frac{\mu^2}{Q^2};\ln^4\frac{\mu^2}{Q^2}+2\pi^2\ln^2\frac{\mu^2}{Q^2}+\frac{7\pi^4}{15}\right\}$$

$$r_{i,0} = -\frac{\gamma^{\mathrm{SS}}_{i-1}}{2}$$

$$r_{i+1,1} = -\frac{\Pi_{i-1}}{2} \qquad r_{i+2,2} = \frac{\pi^2}{6}\gamma^{\mathrm{SS}}_{i-1} \qquad r_{i+3,3} = \frac{\pi^2}{2}\Pi_{i-1} \qquad r_{i+4,4} = -\frac{\pi^4}{10}\gamma^{\mathrm{SS}}_{i-1}$$

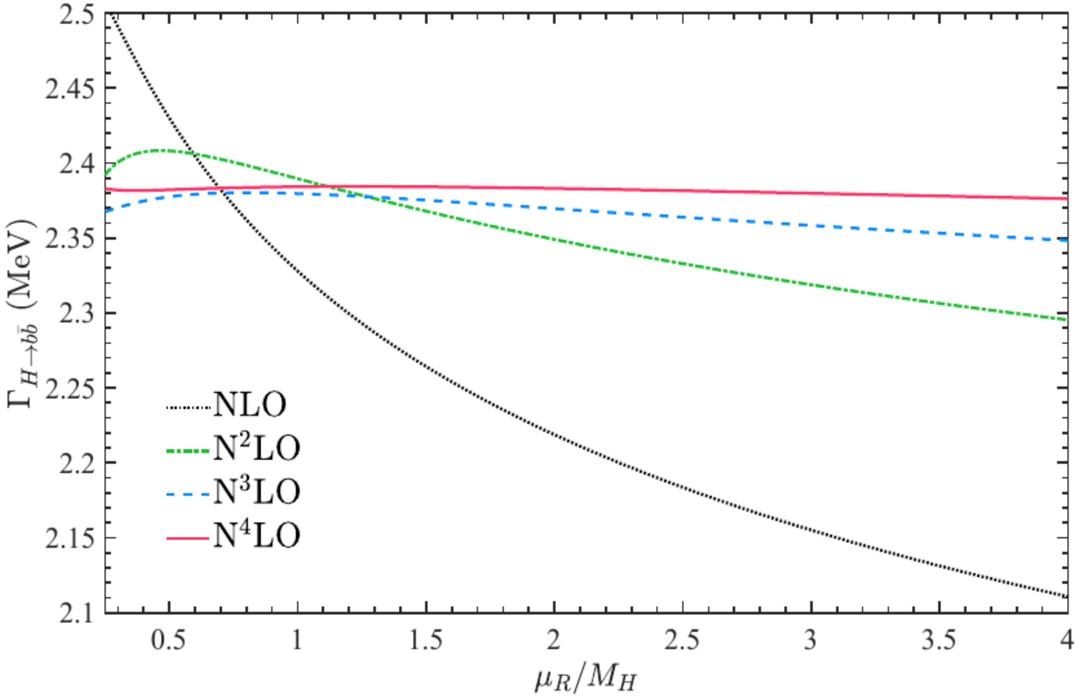
conformal coefficients

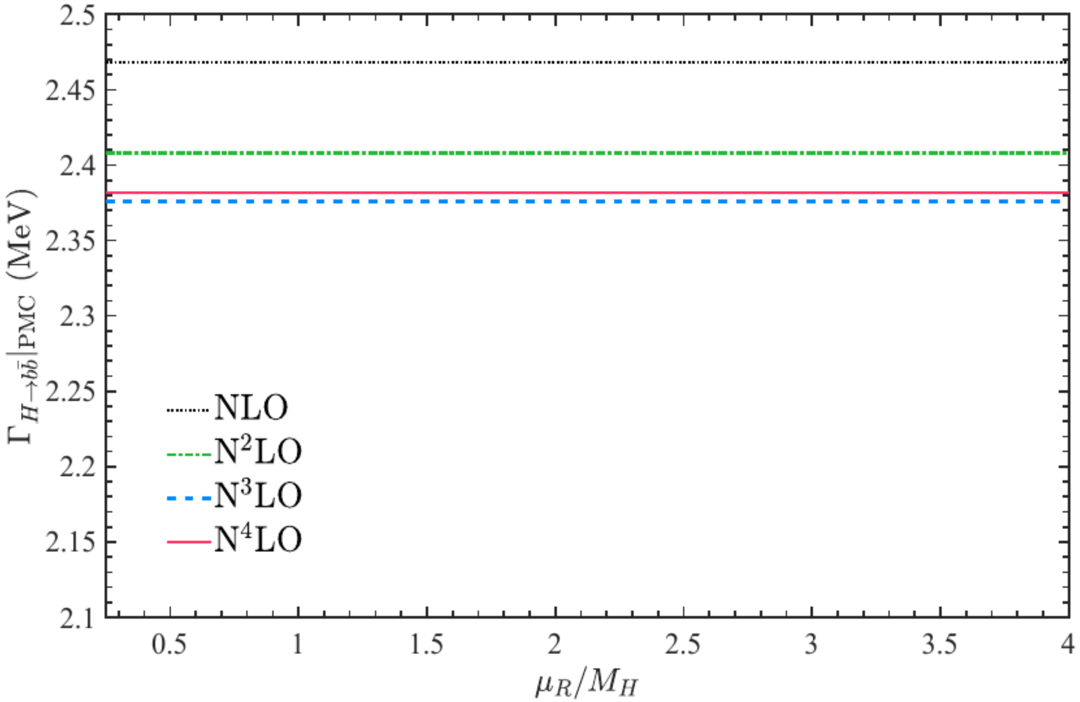
non-conformal coefficients

$$\sum_{i\geq 2}\sum_{j=1}^{i-1}\frac{(-1)^j}{j!}\sum_{k=0}^{j} C_j^k \hat{r}_{i-k,j-k}\ln^k\frac{Q_*^2}{Q^2}\widehat{\mathcal{D}}^j_{n_\gamma,n_\beta}\left[\alpha_s^{i-j-1}(Q_*)\right] = 0$$

$$Q_*^{(\mathrm{LL,NLL,N^2LL,N^3LL})} = \{46.0585, 52.9381, 55.1600, 55.2916\}\ \mathrm{GeV}$$

Numerical results





$$\Gamma_{H\to b\bar{b}}\Big|_{\rm Conv.} = 2.3842^{+0.0002+0}_{-0.0020-0.0060}\ \text{MeV}$$

$\mu_r/M_H \in \left[\frac{1}{2}, 2\right]$ $\mu_r/M_H \in \left[\frac{1}{4}, 4\right]$

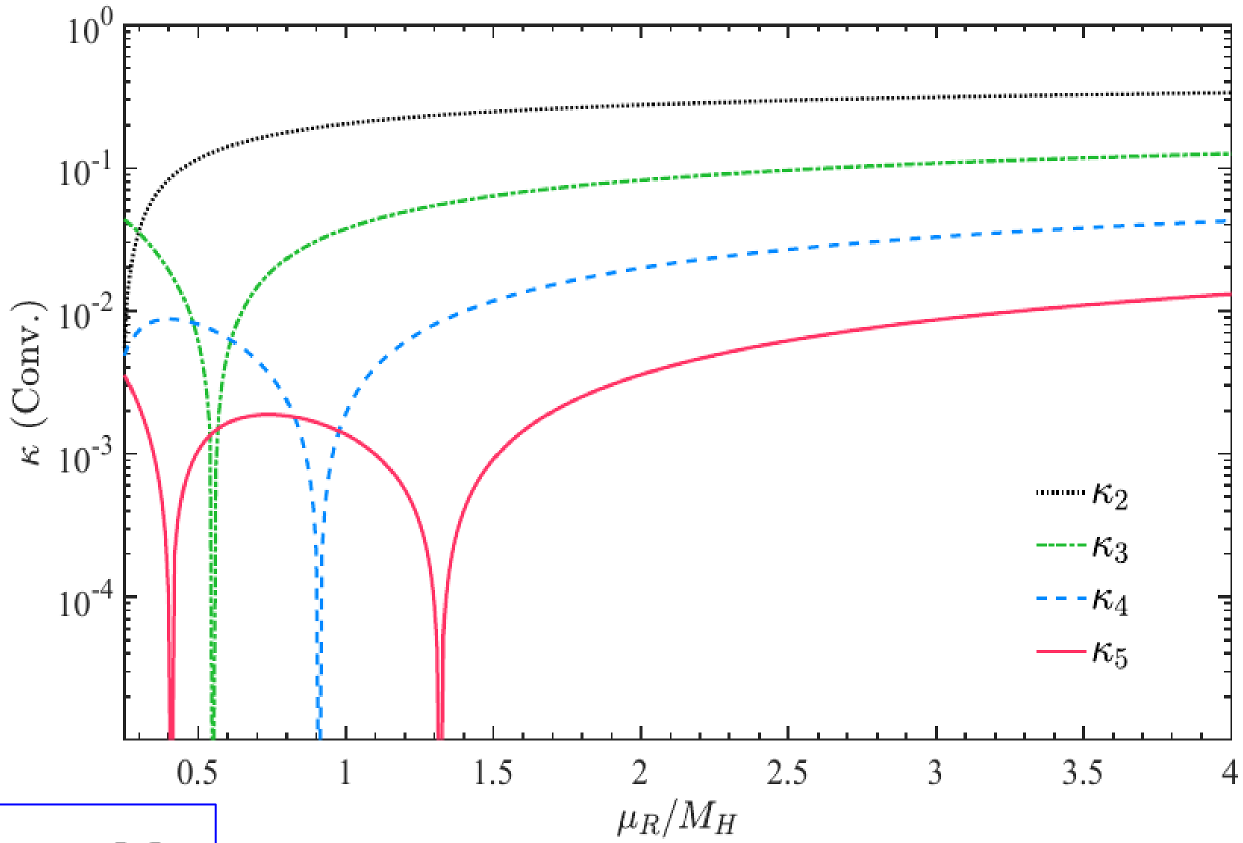
$$\Gamma_{H\to b\bar{b}}\Big|_{\rm PMC} = 2.3819\ \text{MeV}$$

scale-independent

The κ-factor is defined for N^{4}LO series to characterize the convergence of the series

$$\kappa(\text{Conv.}) = \left|\frac{r_i(\mu_r)\alpha_s^{i-1}(\mu_r)}{r_1}\right|,$$

$$\kappa(\text{PMC}) = \left|\frac{r_{i,0}\alpha_s^{i-1}(Q_*)}{r_{1,0}}\right|$$



$$\kappa(\text{Conv.}) = \{1, 0.2031, 0.0374, 0.0019, 0.0014\}, \quad \mu_r = M_H$$
$$\kappa(\text{PMC}) = \{1, 0.0677, 0.0067, 0.0004, 0.0001\}$$

Convergence is strongly dependent on the choice of the renormalization scale!

The convergence of PMC series is **much better than** that of conventional series!

- The unknown higher-order (UHO) terms

Bayesian analysis (BA)

Conventional method

Typically, errors due to variations in unphysical scales are widely considered UHO errors.

However, this approach is **incomplete** since the errors from variations in unphysical scales can only be regarded as the UHO errors for the non-conformal terms associated with scale running, while **the UHO errors for the scale-independent conformal terms are not captured, thus tends to underestimate the contributions from UHO-terms**.

PMC method

Errors associated with variations in the renormalization scale have been eliminated, just the UHO errors for the scale-independent conformal terms need be estimated.

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J. M. Shen, X. G. Wu, S. J. Brodsky et al, Eur. Phys. J. C 83, 326 (2023)

Theoretical uncertainties

- The unknown higher-order (UHO) terms

$$\Delta\Gamma_{H\to b\bar{b}}\Big|^{\rm UHO}_{\rm Conv.} = \left({}^{+0.0004}_{-0.0024}{}^{+0}_{-0.0081}\right)\ \mathrm{MeV}$$

$\mu_r/M_H \in \left[\frac{1}{2}, 2\right]$ $\mu_r/M_H \in \left[\frac{1}{4}, 4\right]$

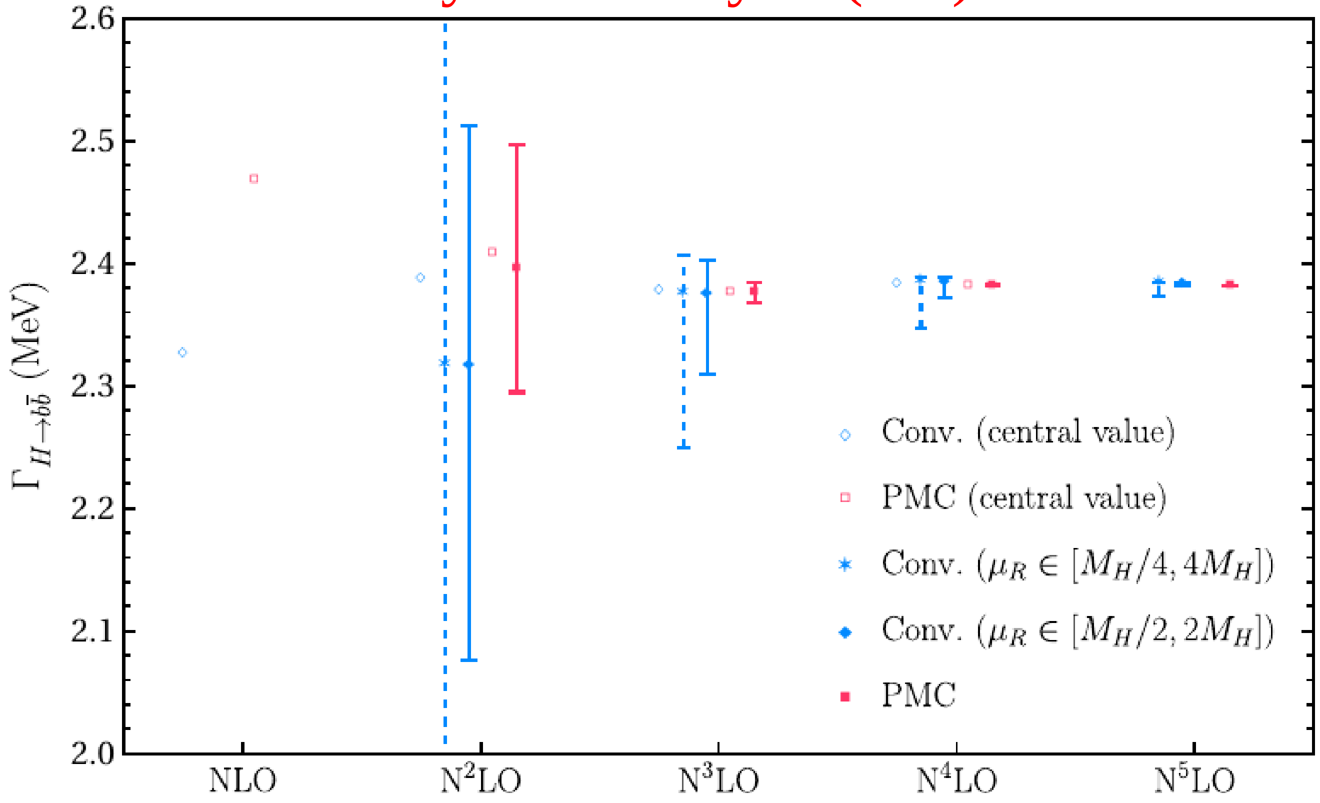
$$\Delta\Gamma_{H\to b\bar{b}}\Big|^{\rm UHO}_{\rm PMC} = \pm 0.0001\ \mathrm{MeV}$$

conventional coefficients ⟺ scale-dependent

Introducing extra uncertainties for BA!

PMC coefficients ⟺ scale-independent ⟹

Bayesian analysis (BA)



Providing a more reliable foundation for constraining predictions regarding the UHO contributions!

Theoretical uncertainties

- Errors caused by input parameters

	$\Delta\Gamma_{H\to b\bar{b}}\Big\vert^{\Delta\alpha_s(M_Z)}$	$\Delta\Gamma_{H\to b\bar{b}}\Big\vert^{\Delta\overline{m}_b(\overline{m}_b)}$	$\Delta\Gamma_{H\to b\bar{b}}\Big\vert^{\Delta M_H}$
Conv.	$\left(^{+0.0208}_{-0.0210}\right)$	$\left(^{+0.0095}_{-0.0094}\right)$	± 0.0021
PMC	$\left(^{+0.0209}_{-0.0210}\right)$	$\left(^{+0.0095}_{-0.0094}\right)$	± 0.0017

Table 1. Additional uncertainties (in unit: MeV) arising from $\Delta\alpha_s(M_Z) = \pm 0.0009$, $\Delta\overline{m}_b(\overline{m}_b) = \pm 0.007$ GeV, $\Delta M_H = \pm 0.11$ GeV under conventional and PMC scale-setting approached, respectively.

Errors from $\Delta\alpha_s(M_Z)$ are so **dominant** that other sorts of errors are heavily diluted!

$$\Gamma_{H\to b\bar{b}}\Big\vert_{\text{Conv.}} = 2.3842^{+0.0230}_{-0.0232}\ \text{MeV} \qquad \Gamma_{H\to b\bar{b}}\Big\vert_{\text{PMC}} = 2.3819^{+0.0230}_{-0.0231}\ \text{MeV}$$

Summary

- Defining the characteristic operator to rederive the scale-displacement relation, QCD degeneracy relations, and PMC formulas involving the running of $\overline{\rm MS}$ mass $\overline{m}_q$

- Obtaining scale-invariant and more convergent pQCD series of $\Gamma_{H\to b\bar{b}}$ using PMC

- Estimating the magnitude of the unknown higher-order terms using the Bayesian analysis

- The scale-independent convergent behavior of the PMC series can be regarded as the inherent perturbative nature of pQCD prediction

Thanks for your attention!