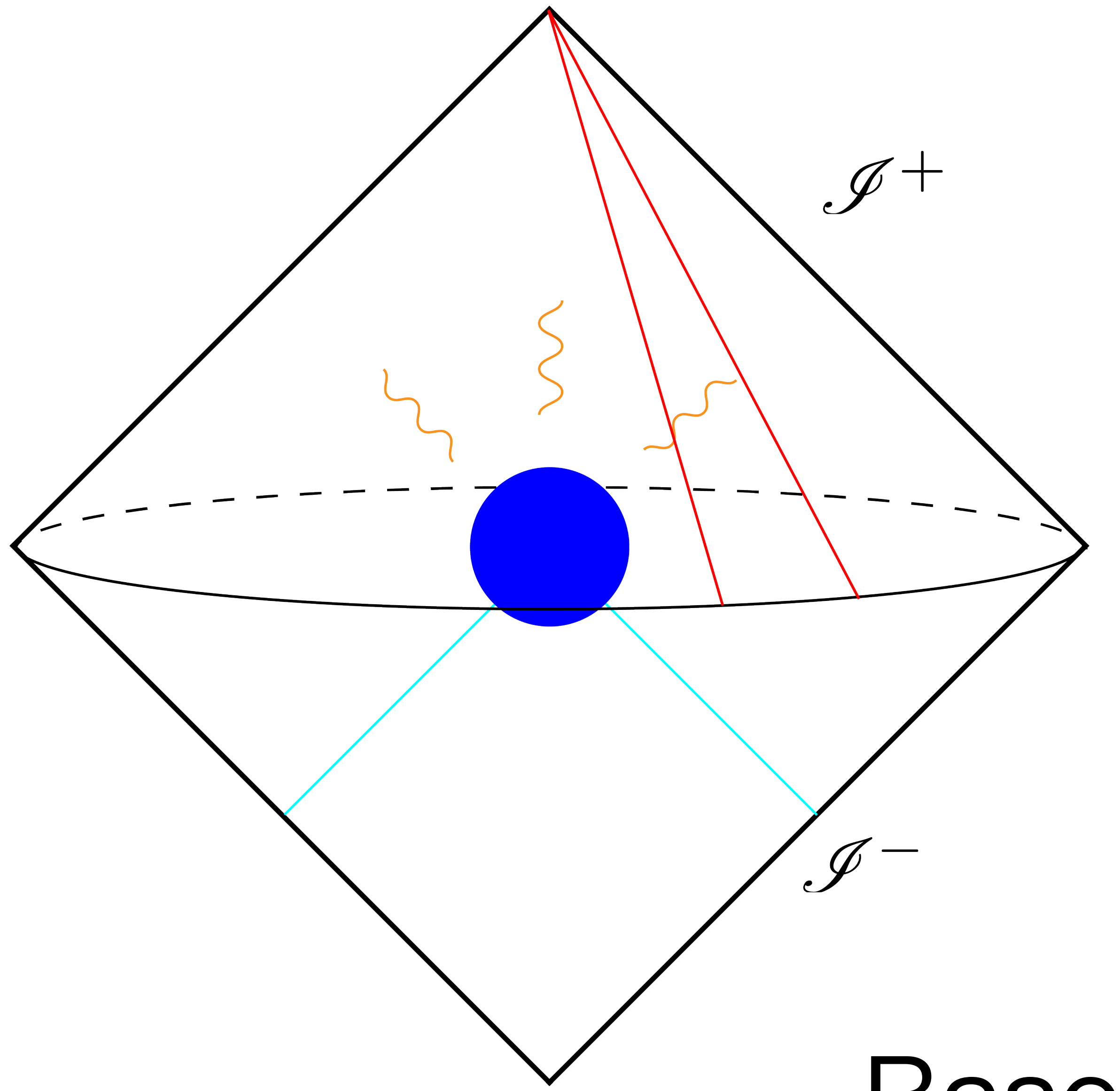
Celestial Block for Hadron Colliders



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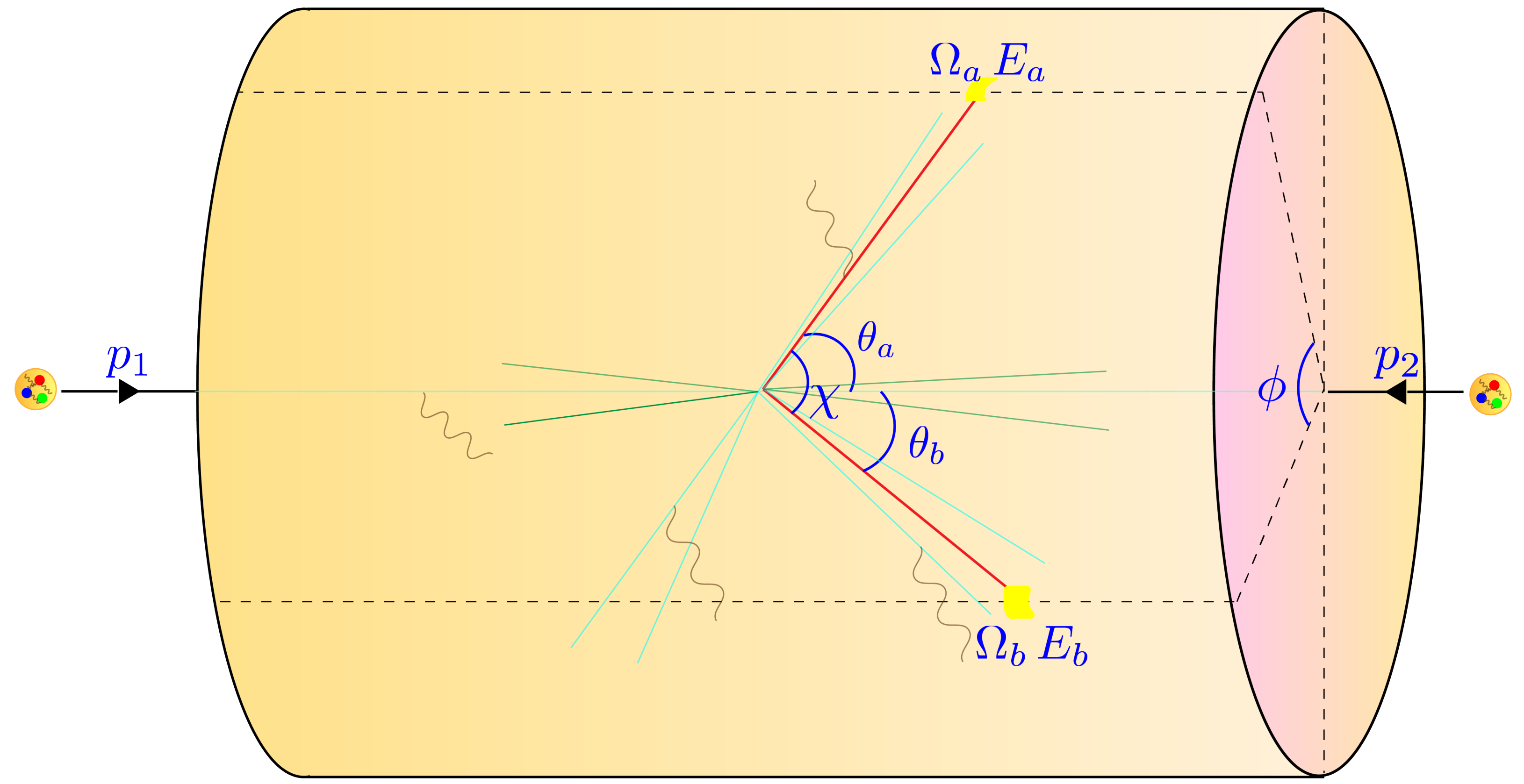


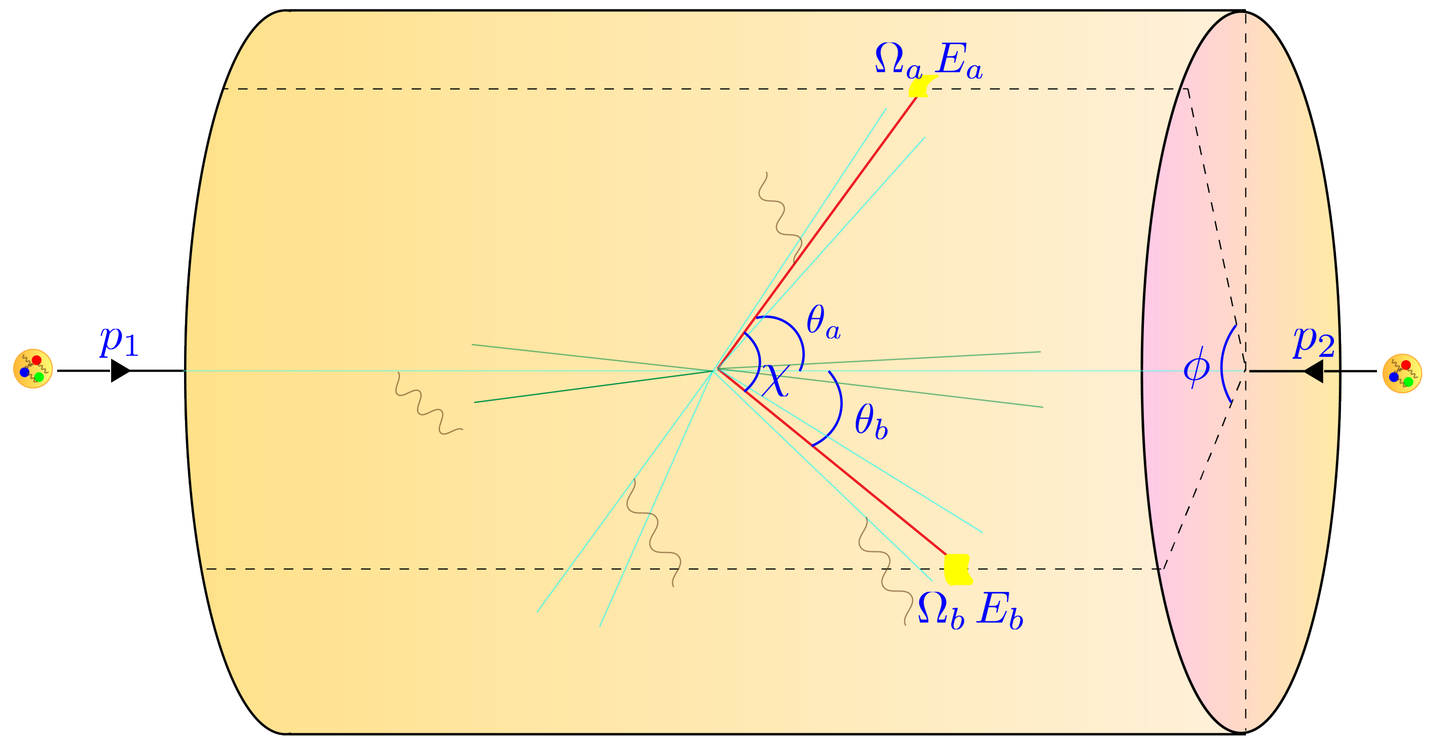
Based on works with Hao Chen and Hua Xing Zhu

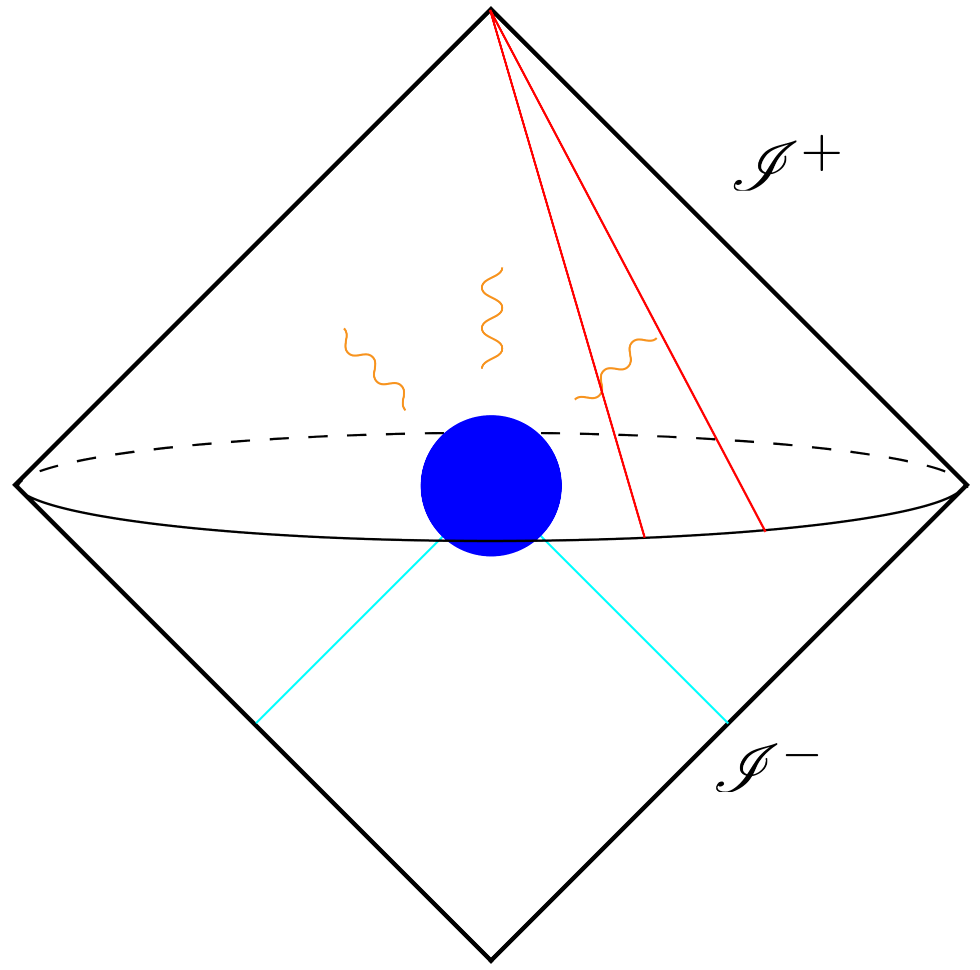
第四届量子场论及其应用研讨会　2024.11.19

Outline

Hadronic Energy–Energy Correlator

- Collider Physics Aspects
  - Definition and Motivation
  - Results and Factorizations
- Formal Theory Aspects
  - Celestial Conformal Symmetry
  - Celestial Block Decomposition on the Celestial Sphere along with the Collider Axis





Energy Flow Operators and EEC

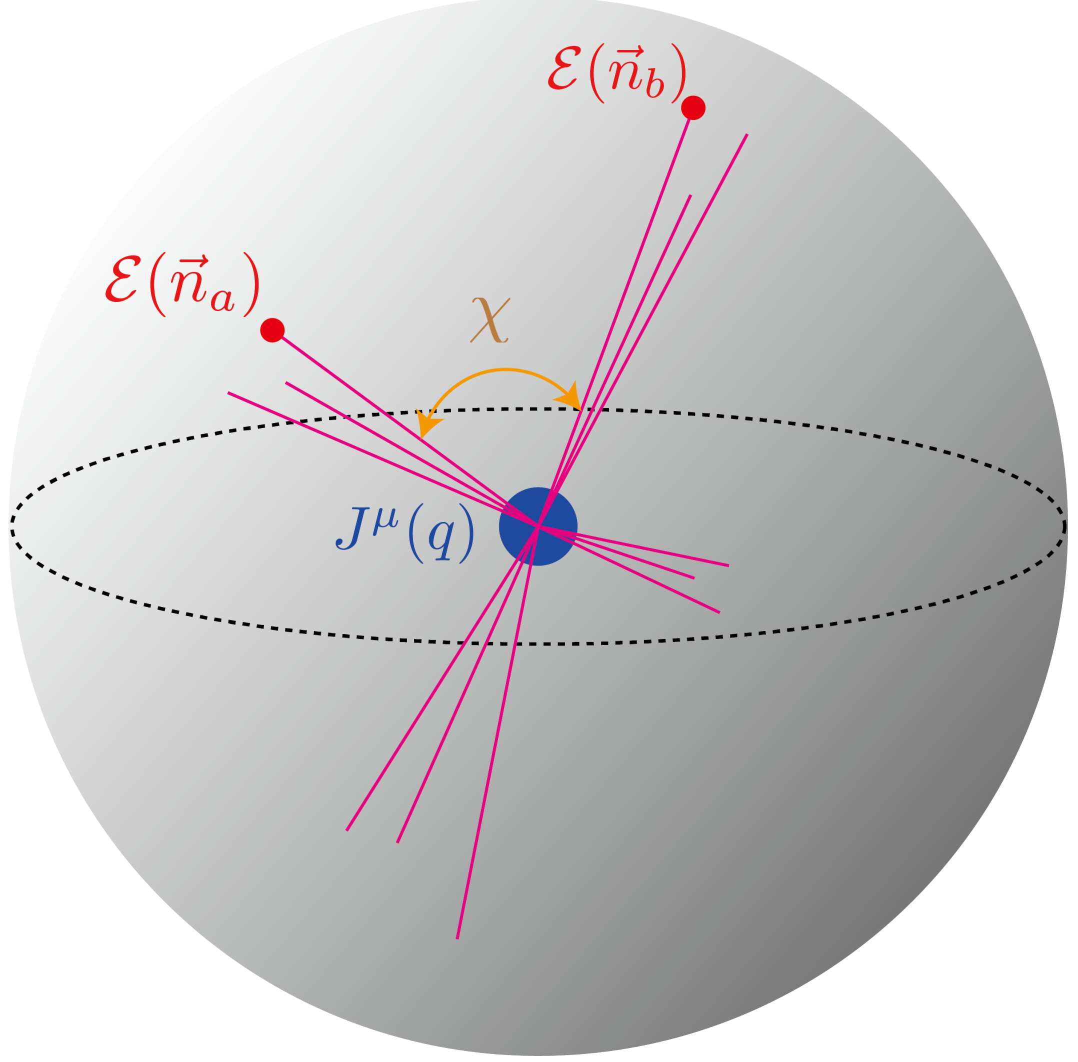
electron–positron colliders

[Basham, Brown, Ellis and Love, 1978]

introduced energy–energy correlation

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} = \sum_{i,j}\int d\sigma\, \frac{E_i E_j}{Q^2}\delta^{(2)}(\Omega_a - \Omega_{p_i})\delta^{(2)}(\Omega_b - \Omega_{p_j})$$

which characterizes the correlation of two energy detectors at spatial infinity (celestial sphere).

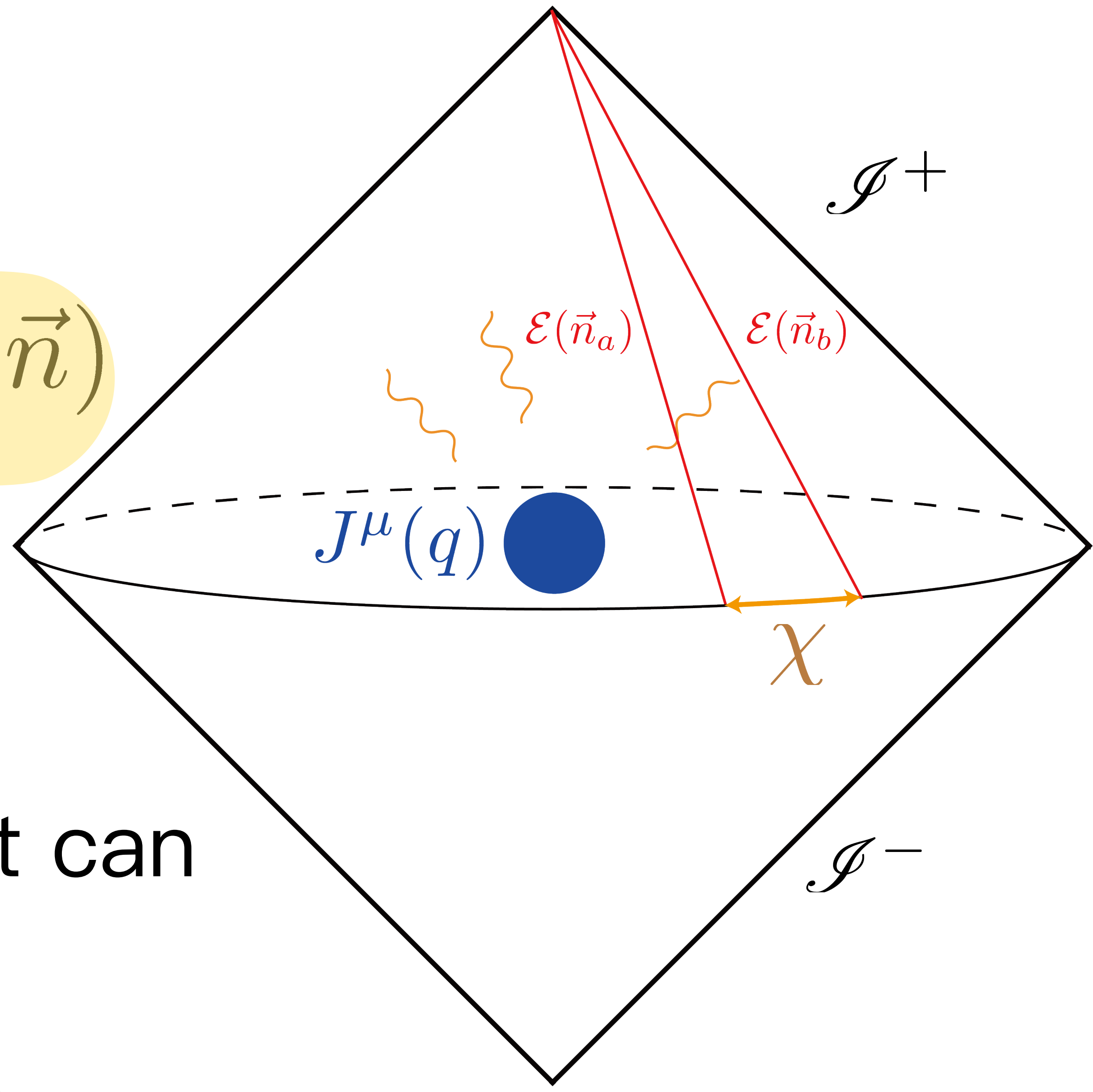


$$\langle\Omega|J_\nu(-q)\boxed{\sum\omega(q)|X\rangle\langle X|}J_\mu(q)|\Omega\rangle$$

$\hat{\omega}$: measurement operator

[Korchemsky, Sterman, 1999;
Hofman, Maldacena, 2008;
Bauer, Fleming Lee, Sterman, 2008; ...]

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty}\int_0^\infty dt\, r^2 n^i T_{0i}(t, r\vec{n})$$



The energy flow operator is defined as an integral along a light ray in the future null infinity. In the free theory, it can be expressed as

$$\mathcal{E}(\vec{n}) = \int \widetilde{d^3p}\, E_p\, a_p^\dagger a_p\, \delta^{(2)}\left(\vec{n} - \vec{n}_p\right)$$

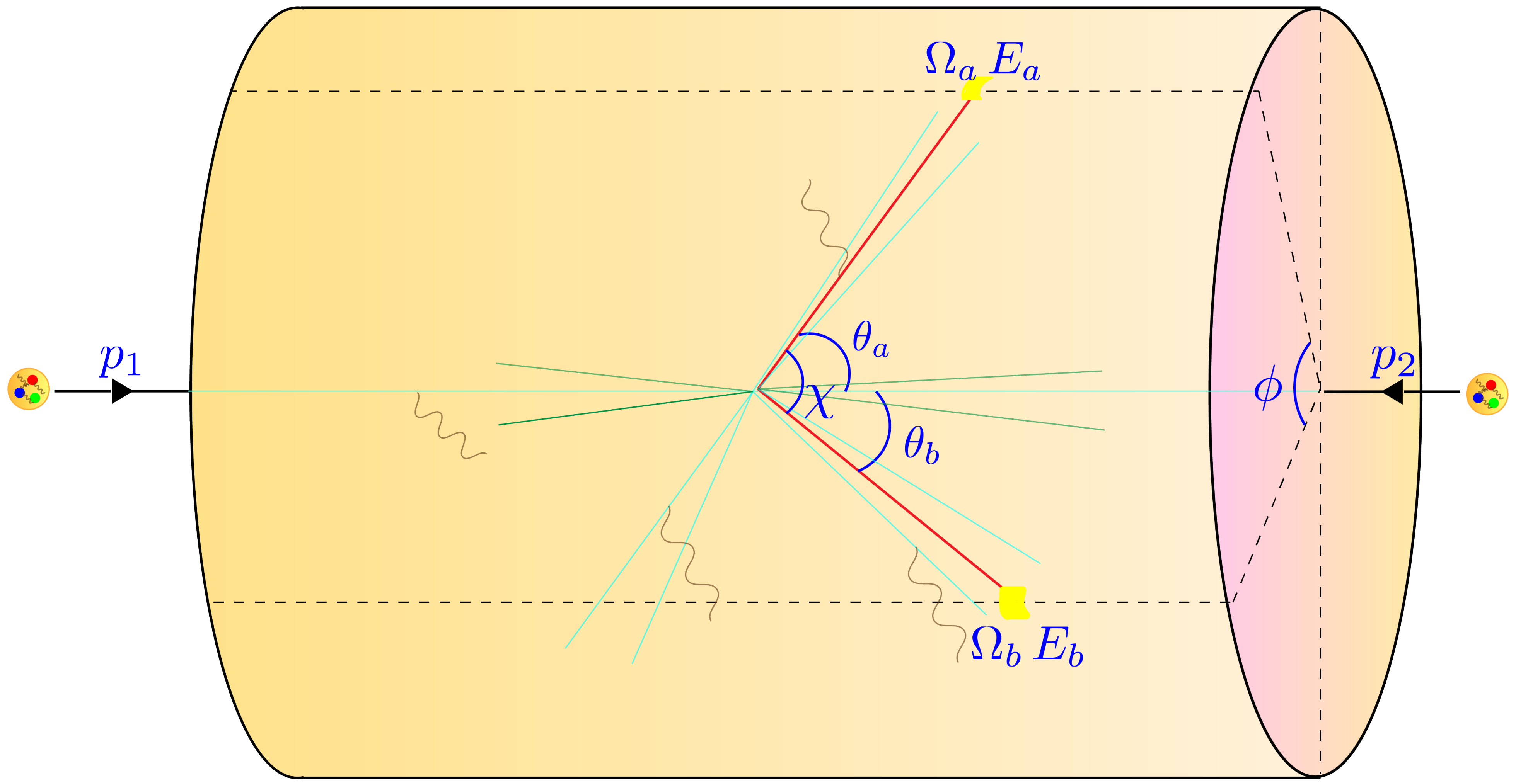
This nice operator definition of the energy flow allows an alternative definition of EEC and its generalization:

$$\langle\Omega|O_{\rho\sigma\ldots}(-q)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\cdots O_{\mu\nu\ldots}(q)|\Omega\rangle$$

Source with total momentum q=(Q, 0, 0, 0)

Hadronic EEC

increased information via symmetry breaking



Hadronic EEC is defined as:

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} = \sum_{i,j} \int d\sigma_{pp\to i+j+X}\, E_i E_j\, \delta^{(2)}(\Omega_a - \Omega_{p_i})\delta^{(2)}(\Omega_b - \Omega_{p_j})$$

with dependence on three angles: θ_a, θ_b, χ.

The collision axis breaks spherical symmetry down to axial symmetry, rendering the final result considerably more complex, yet providing additional insights.

- $\mathcal{N}=4$ SYM: "Hydrogen Atom"

  EEC: NLO [Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2014]

  NNLO [Henn, Sokatchev, Yan, Zhiboedov, 2019]

  E3C: LO, Collinear [Chen, Luo, Moult, Yang, Zhang, Zhu, 2019]

  LO [Yan, Zhang, 2022]

  E4C: LO, Collinear [Chicherin, Moult, Sokatchev, Yan, Yunyue, 2024]

- QCD in electron–positron colliders: "Metal Atom"

  EEC: LO [Basham, Brown, Ellis, Love, 1978]

  NLO [Dixon, Luo, Shtabovenko, Yang, Zhu, 2018]

  E3C: LO, Collinear [Chen, Luo, Moult, Yang, Zhang, Zhu, 2019]

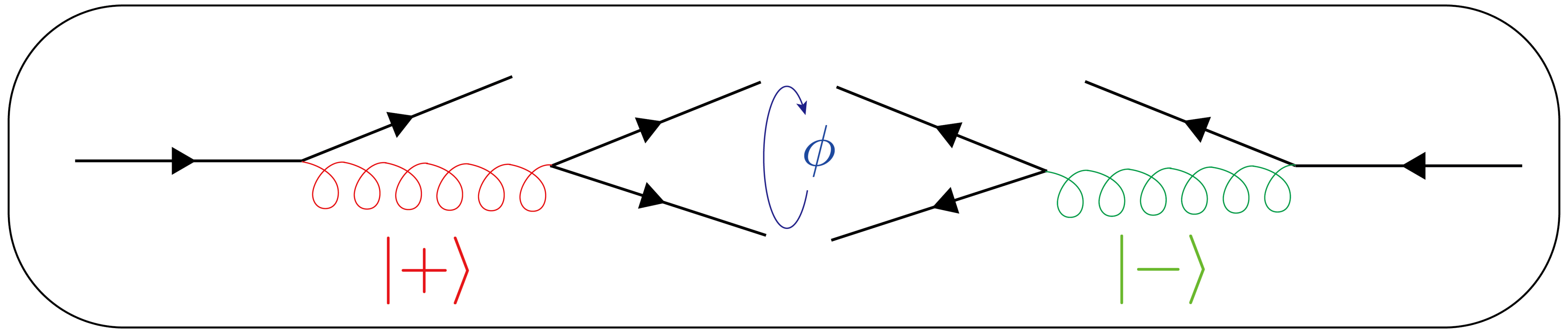
- QCD in hadron colliders: "Zeeman effect"

  TEEC: Back–to–Back [Gao, Li, Moult, Zhu, 2019; 2023]

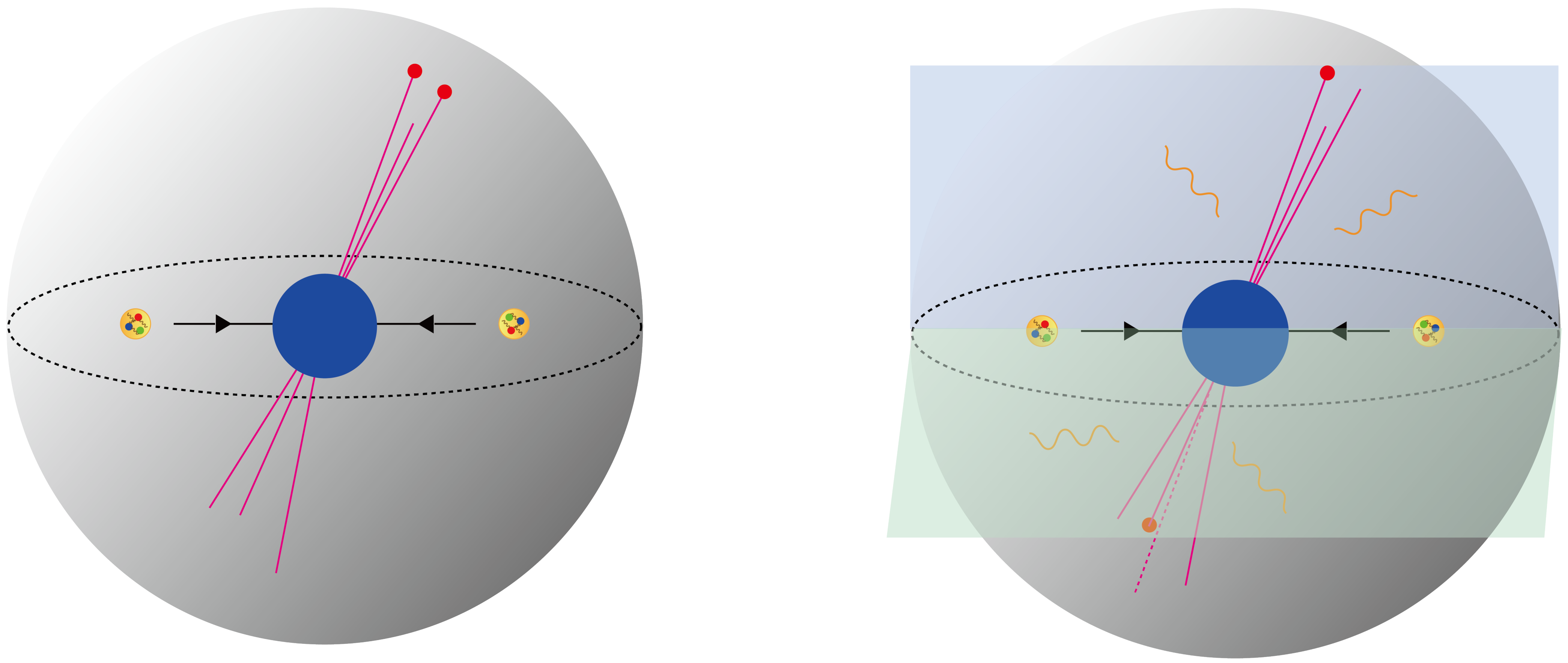
  HEEC: LO, Full angle [Chen, Ruan, Zhu, forthcoming]

What remains to be uncovered?

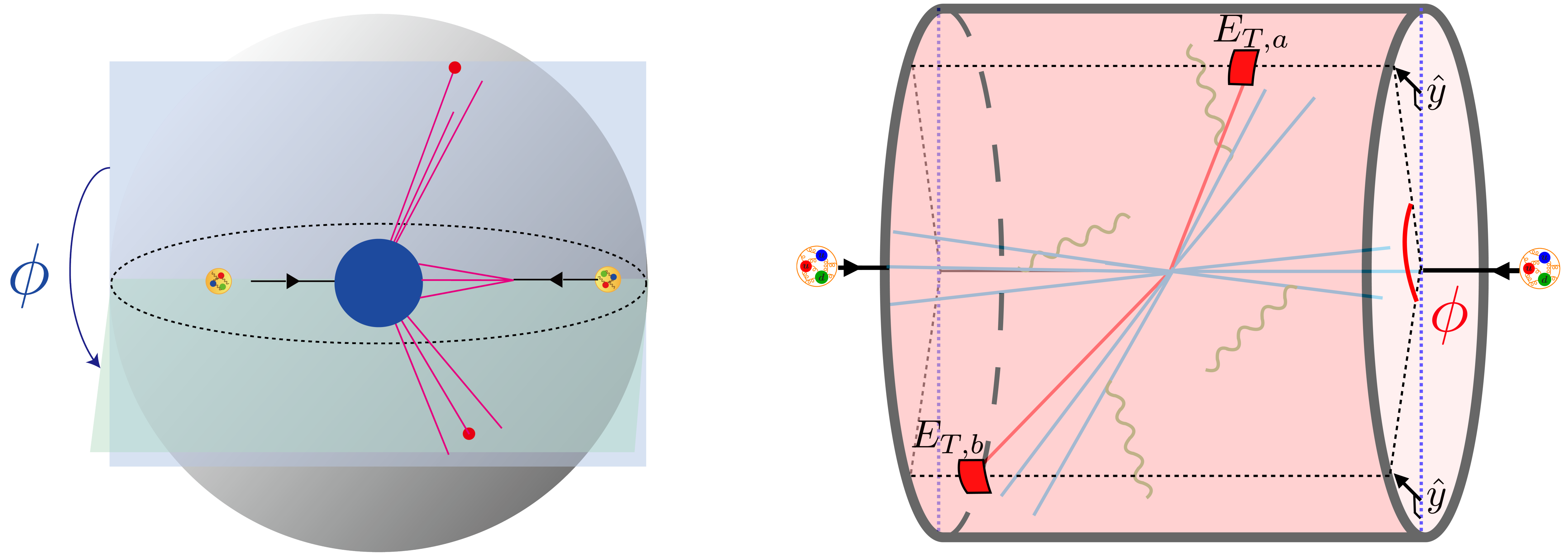
Interference Effect



An interference effect arises between inter–mediate gluons in the squeezed limit, a pheno–menon observed in QCD but absent in $\mathcal{N}=4$ SYM. [Chen, Luo, Moult, Yang, Zhang, Zhu, 2019]

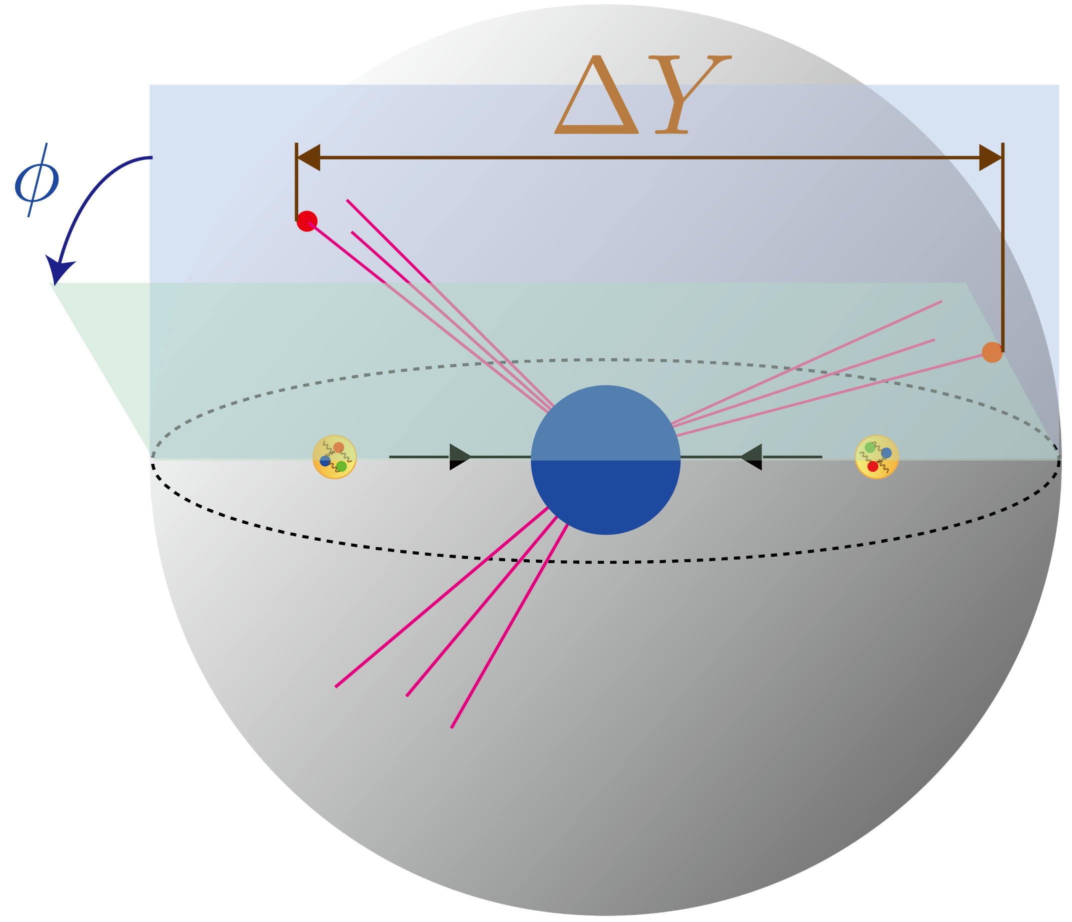


Collinear limit and back–to–back limit, well understood in the EEC of e^+e^- colliders



art from [Gao, Li, Moult, Zhu, 2023]

opposite coplanar limit, well understood as the back–to–back limit in the TEEC of pp colliders



The Regge limit, characterized by a large rapidity difference (ΔY) between the two detectors, remains unexplored in the context of energy correlators.

Hadronic EEC Results

for pure gluon scattering

- Color–ordered amplitude from spinor–helicity formalism: [Parke, Taylor 1986]; [Elvang, Huang, 2014]

$$A_5[1,2,3,4,5] = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

- Using the color decomposition formula, the full 5–gluon tree amplitude can be expressed as:

$$\mathcal{A}_5^{\text{full,tree}} = g^3 \sum_{\sigma\in S_4} \text{Tr}(T^{a_1}T^{\sigma(a_2)}T^{a_3}T^{a_4}T^{a_5}) A_5[1,\sigma(2,3,4,5)]$$

- The averaged squared amplitude can also be expressed in a compact form

$$\overline{\sum_{\text{h,c}} \left|\mathcal{A}_5^{\text{full,tree}}\right|^2} = \frac{27 g^6}{16} \frac{\sum\limits_{1\le i<j\le 5} s_{ij}^4}{\prod\limits_{1\le i<j\le 5} s_{ij}} \sum_{\sigma\in S_4} \left(s_{1\sigma(2)}s_{\sigma(2)\sigma(3)}s_{\sigma(3)\sigma(4)}s_{\sigma(4)\sigma(5)}s_{\sigma(5)1}\right)$$

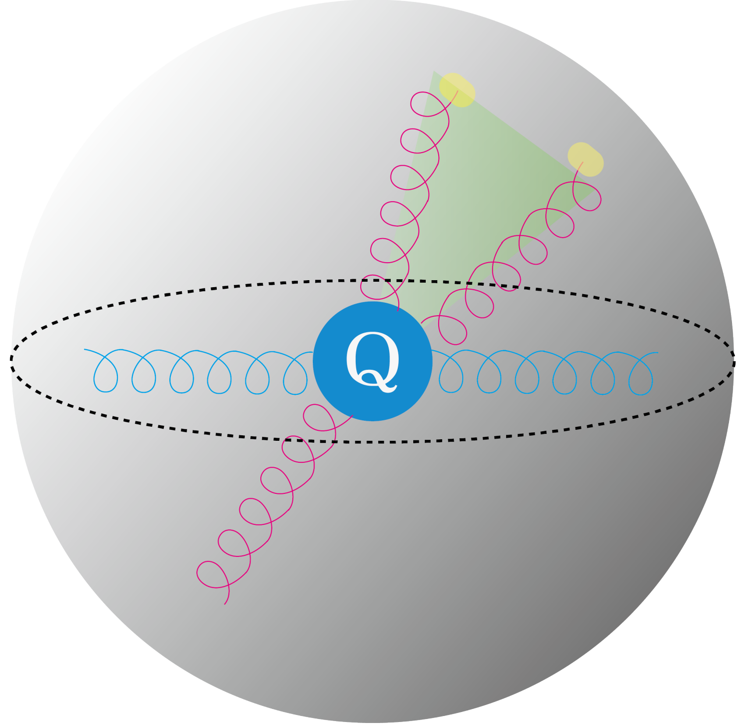
- The LO HEEC for gluon scattering can then be expressed as

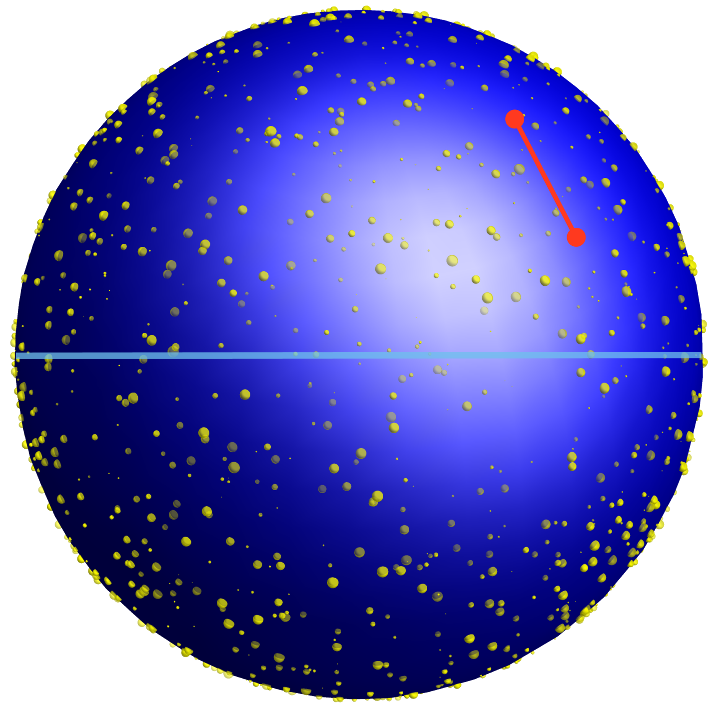
$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} = \frac{Q^2}{16384\pi^5}\int_0^1 dx\, \frac{(1-x)^2 x^2}{(1-x\xi)^3} \overline{\sum_{\text{h,c}} \left|\mathcal{A}_5^{\text{full,tree}}\right|^2}$$

The final result spans over three pages; thus, we only present its main structure:

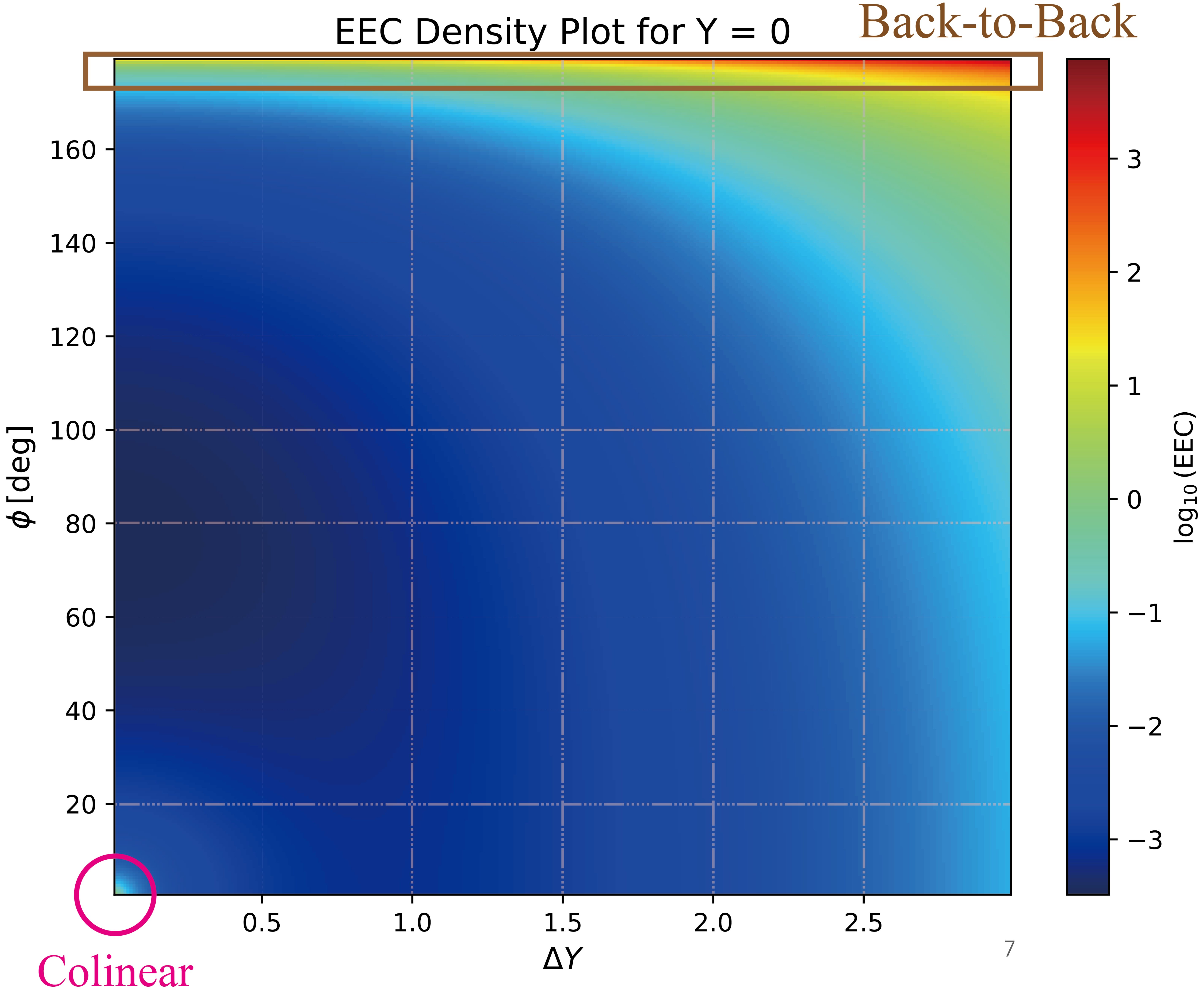
$$\begin{aligned}\frac{d^2\Sigma}{d\Omega_a d\Omega_b} =& f_1(Y,\Delta Y,\phi) + f_2(Y,\Delta Y,\phi)\Delta Y + f_3(Y,\Delta Y,\phi)\phi \\ &+ f_4(Y,\Delta Y,\phi)\arccos\frac{\cosh Y\cos\phi + 1}{\cosh Y + \cos\phi} \\ &+ f_5(Y,\Delta Y,\phi)\ln\frac{(\cosh Y+\sinh Y)(\cosh\Delta Y+\sinh\Delta Y)+1}{\cosh Y+\sinh Y\cosh\Delta Y+\sinh\Delta Y} \\ &+ f_6(Y,\Delta Y,\phi)\ln\frac{\cosh Y+\cos\phi}{\cosh Y+\cosh\Delta Y}\end{aligned}$$

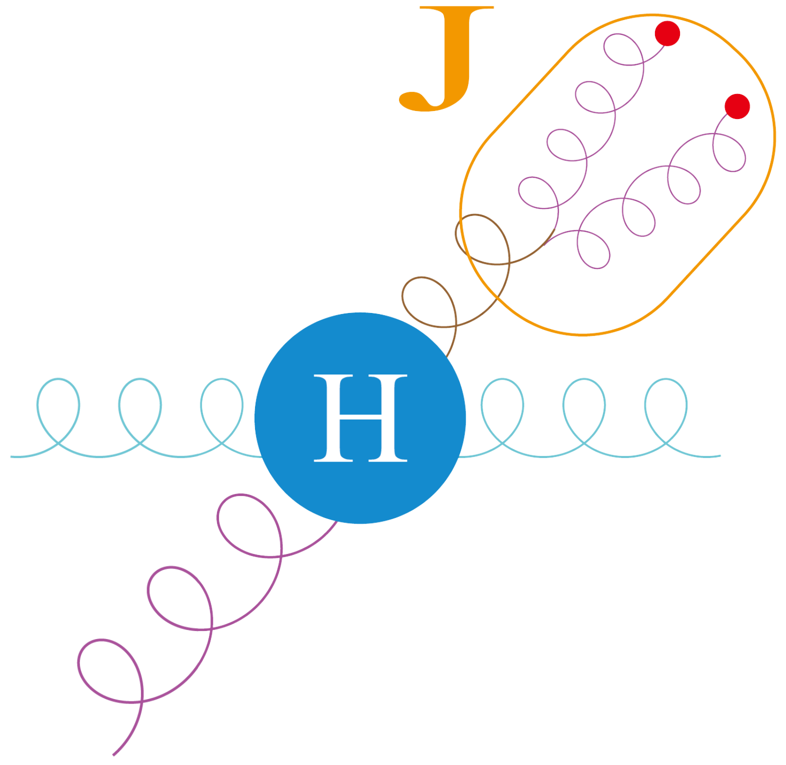
We utilize variables that facilitate comparison with experimental data: the rapidity sum Y and difference ΔY of the two detectors, along with their azimuthal angle separation ϕ. The functions $f_i(Y,\Delta Y,\phi)$ represent distinct and highly intricate rational expressions.





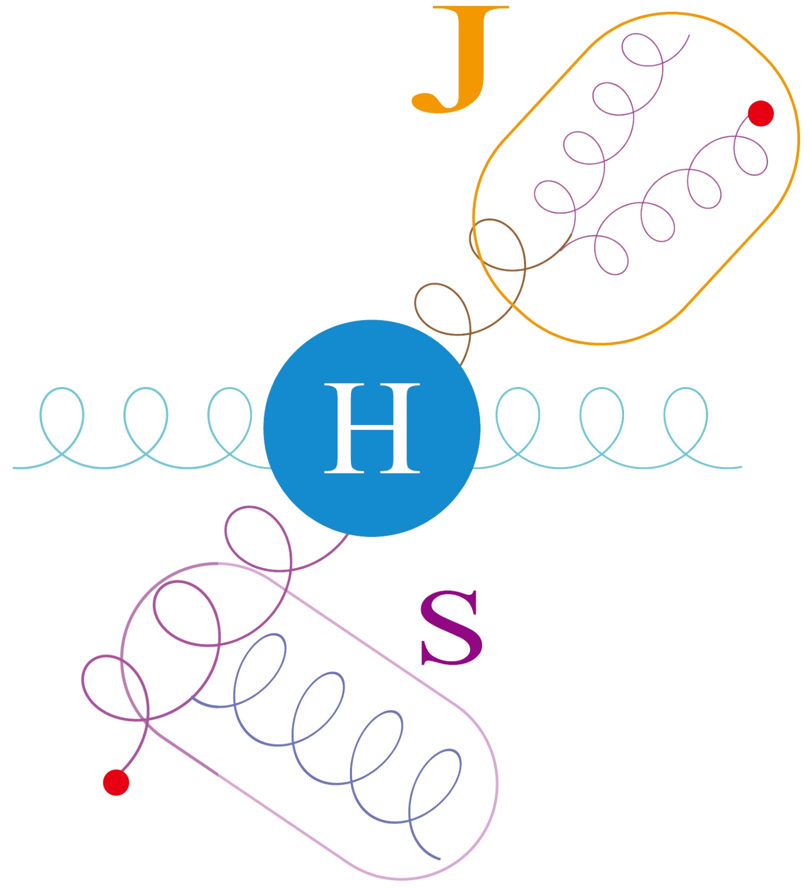
Plots & Limit Factorizations





$H \otimes J \sim \frac{1}{\phi^2+\Delta Y^2}$

Colinear (LL)

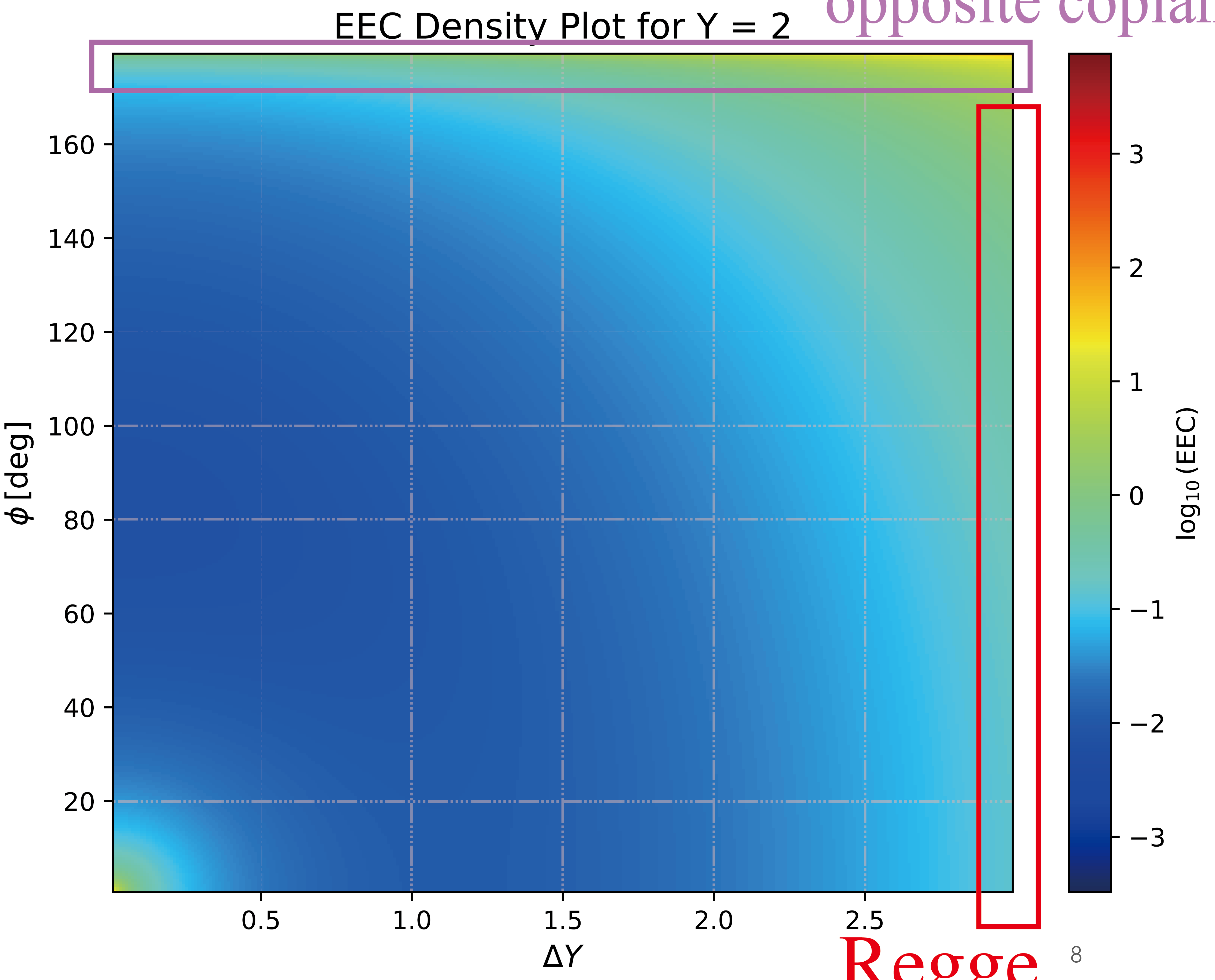


$$H \otimes J \otimes S \sim \frac{\#}{(\pi-\phi)^2+Y^2} + \frac{\#\ \ln\left((\pi-\phi)^2+Y^2\right)}{(\pi-\phi)^2+Y^2}$$

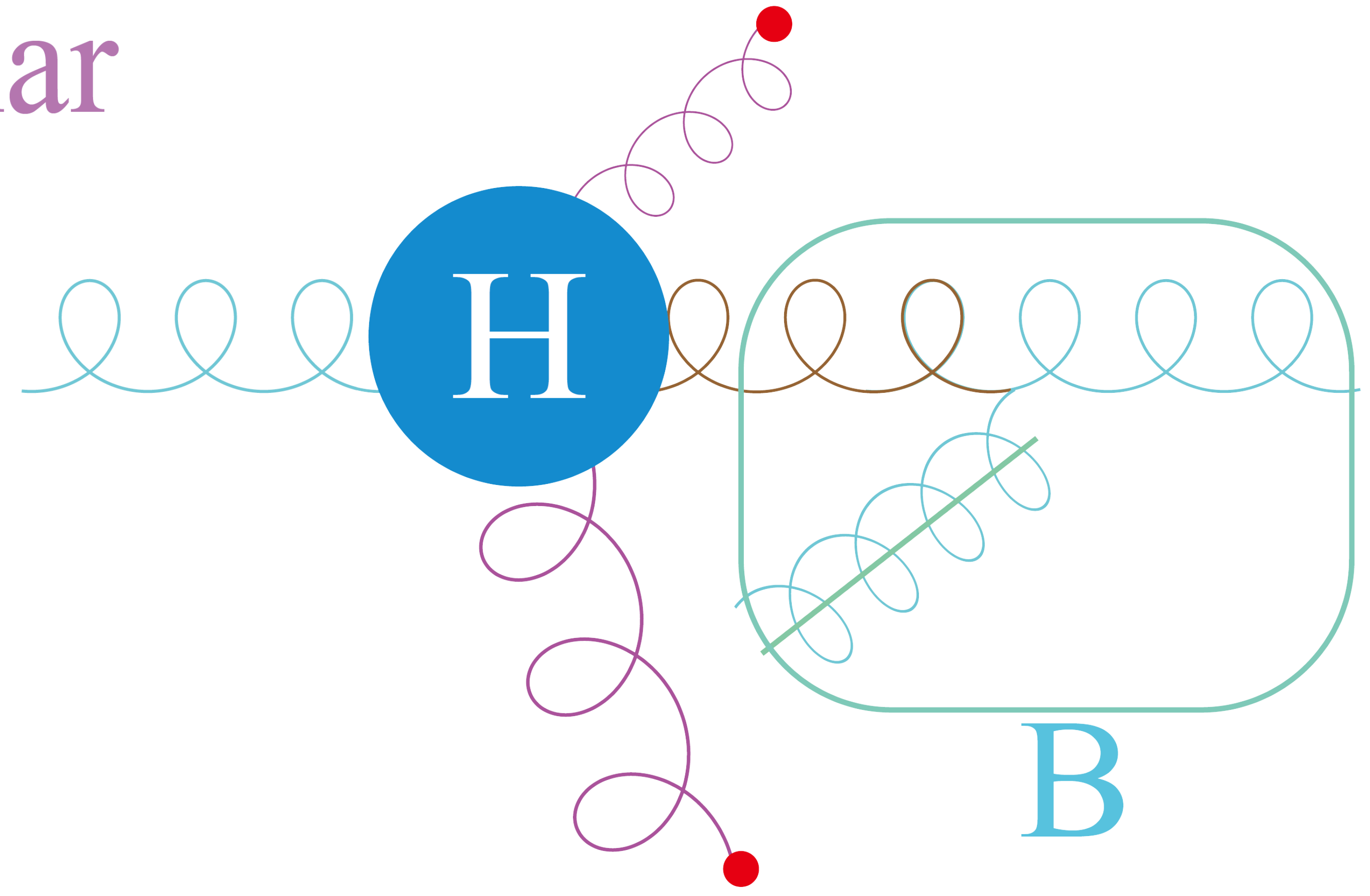
Back-to-Back (NLL)

Plots & Limit Factorizations

opposite coplanar

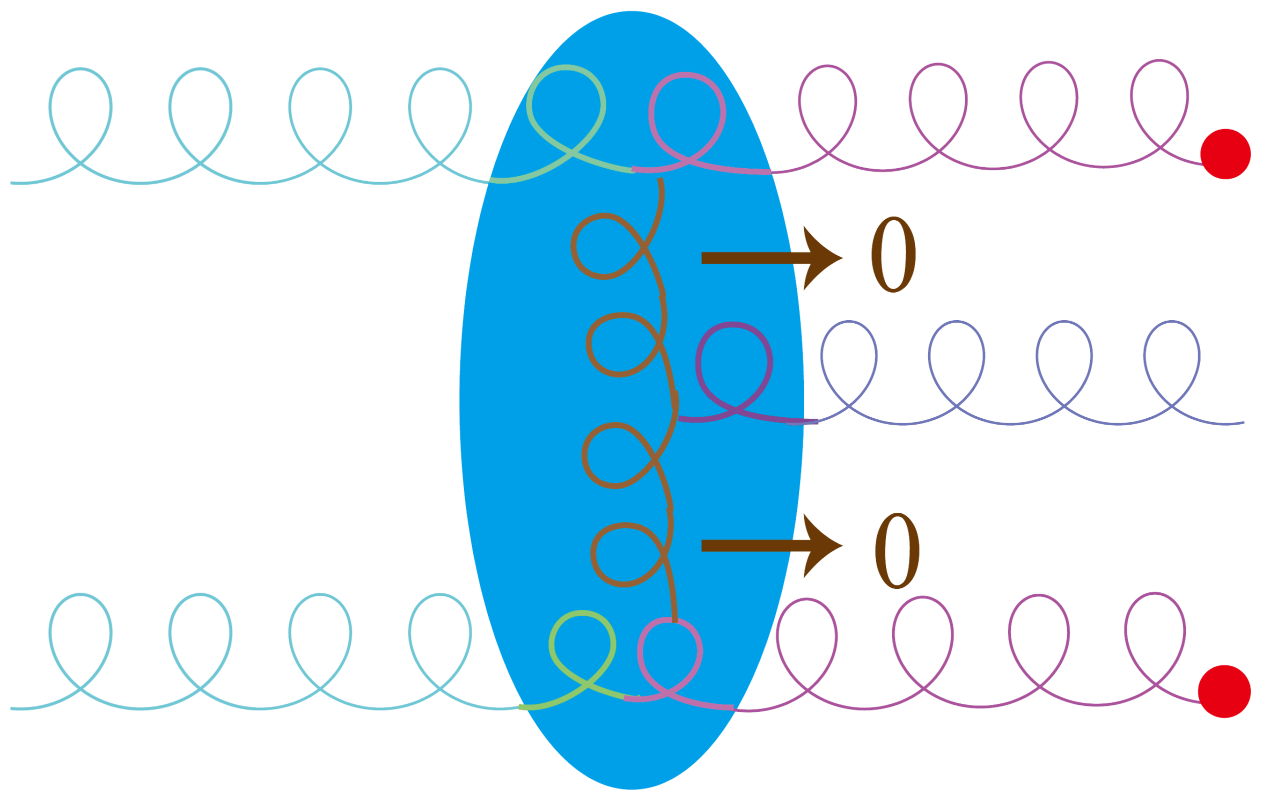


Regge



$$H \otimes B \sim \frac{1}{\pi - \phi}$$

opposite coplanar (LL)



$$??? \sim \frac{\Delta Y\, e^{3\Delta Y + Y}}{1 + 2\, e^{Y} \cos\phi + e^{2Y}}$$

Regge (LL)

Conformal and Lorentz Symmetry

From 4D Minkowski to Celestial Sphere

First noticed by Dirac:

$$\mathfrak{so}(1,3)_{\mathbb{C}} \cong \mathfrak{sl}(2,\mathbb{C}) \oplus \mathfrak{sl}(2,\mathbb{C})$$

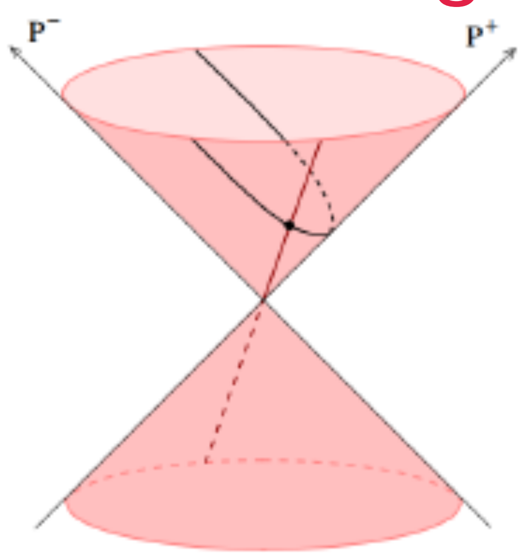
that is, Lorentz transformations in 4d Minkowski spacetime can be described as global transformations (Möbius transformations) in a 2d complex plane.

More generally, we have the following correspondence:

$$\text{(Global) Conformal Symmetry in } \mathbb{R}^d$$
$$\cong \text{ Lorentz Symmetry in } \mathbb{R}^{d+1,1}$$

$$L_{ab} \text{ in } \mathbb{R}^{d+1,1} \begin{cases} L_{\mu\nu} = M_{\mu\nu} \longrightarrow \text{Lorentz Transformation} \\ L_{-1,0} = D \longrightarrow \text{Dilation} \\ L_{0,\mu} = \frac{1}{2}(P_\mu + K_\mu) \quad \text{Translation} \\ L_{-1,\mu} = \frac{1}{2}(P_\mu - K_\mu) \quad \text{SCT (Inv+Trans+Inv)} \end{cases}$$

We consider Lorentz transformations on the celestial sphere via embedding formalism [Chen, Moult, Sandor, Zhu, 2022] :



$$x^\alpha \to \left(\delta^\alpha_\beta - i\epsilon\,(M^{\mu\nu})^\alpha_\beta\right) x^\beta = x^\alpha + \epsilon\,(g^{\mu\alpha}x^\nu - g^{\nu\alpha}x^\mu)$$

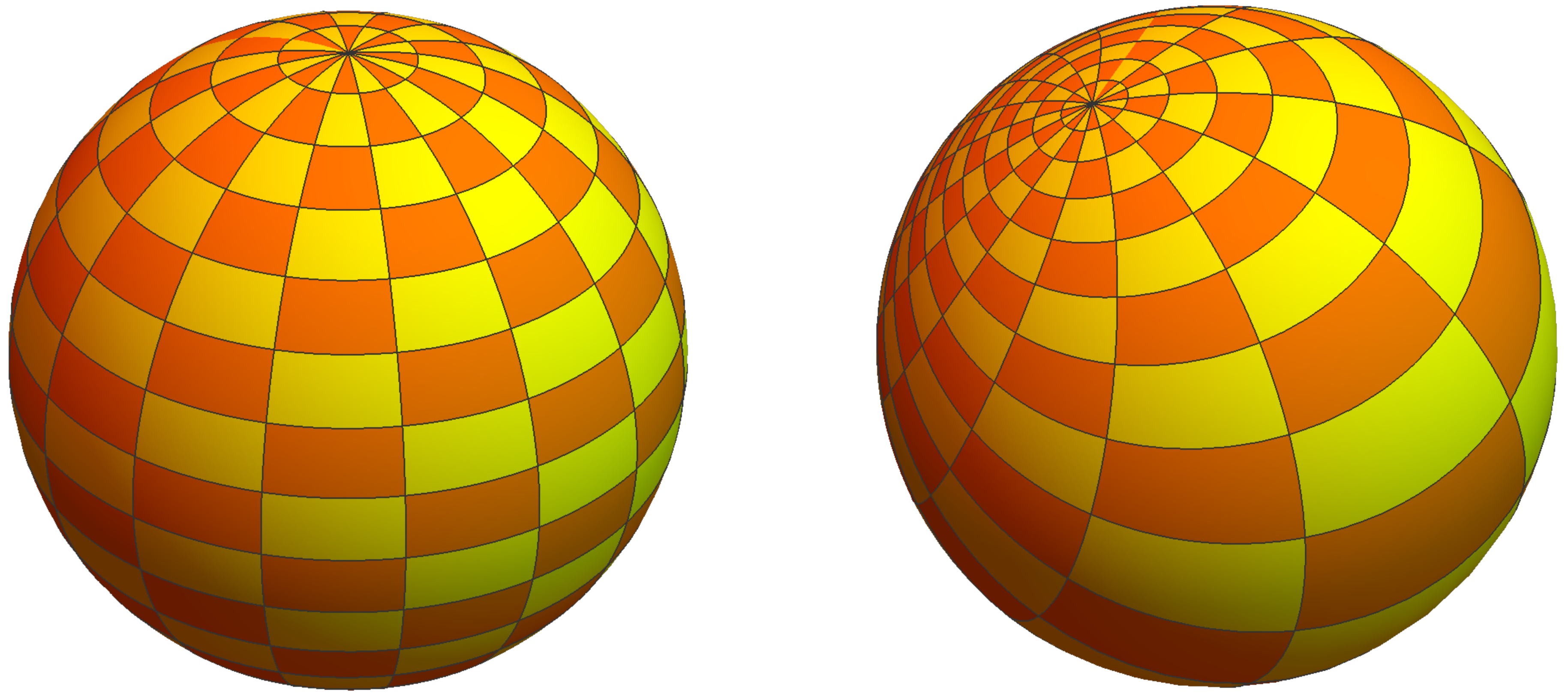
$$M^{0a} : (1, n^i) \to (1 + \epsilon n^a, n^i + \epsilon\delta^{ia}) \to n^i + \epsilon(\delta^{ia} - n^i n^a)$$

$$M^{ab} : (1, n^i) \to (1, n^i + \epsilon(\delta^{ib}n^a - \delta^{ia}n^b)) \to n^i + \epsilon(\delta^{ib}n^a - \delta^{ia}n^b)$$

we have used $n^2 = 0$ and $n \sim \lambda n$

$$M^{03} = -\sin\theta\partial_\theta, \quad M^{01} = \cos\theta\cos\phi\partial_\theta - \frac{\sin\phi}{\sin\theta}\partial_\phi, \quad M^{02} = \cos\theta\sin\phi\partial_\theta + \frac{\cos\phi}{\sin\theta}\partial_\phi,$$

$$M^{12} = \partial_\phi, \quad M^{31} = \cos\phi\partial_\theta - \frac{\cos\theta}{\sin\theta}\sin\phi\partial_\phi, \quad M^{32} = \sin\phi\partial_\theta + \frac{\cos\theta}{\sin\theta}\cos\phi\partial_\phi.$$

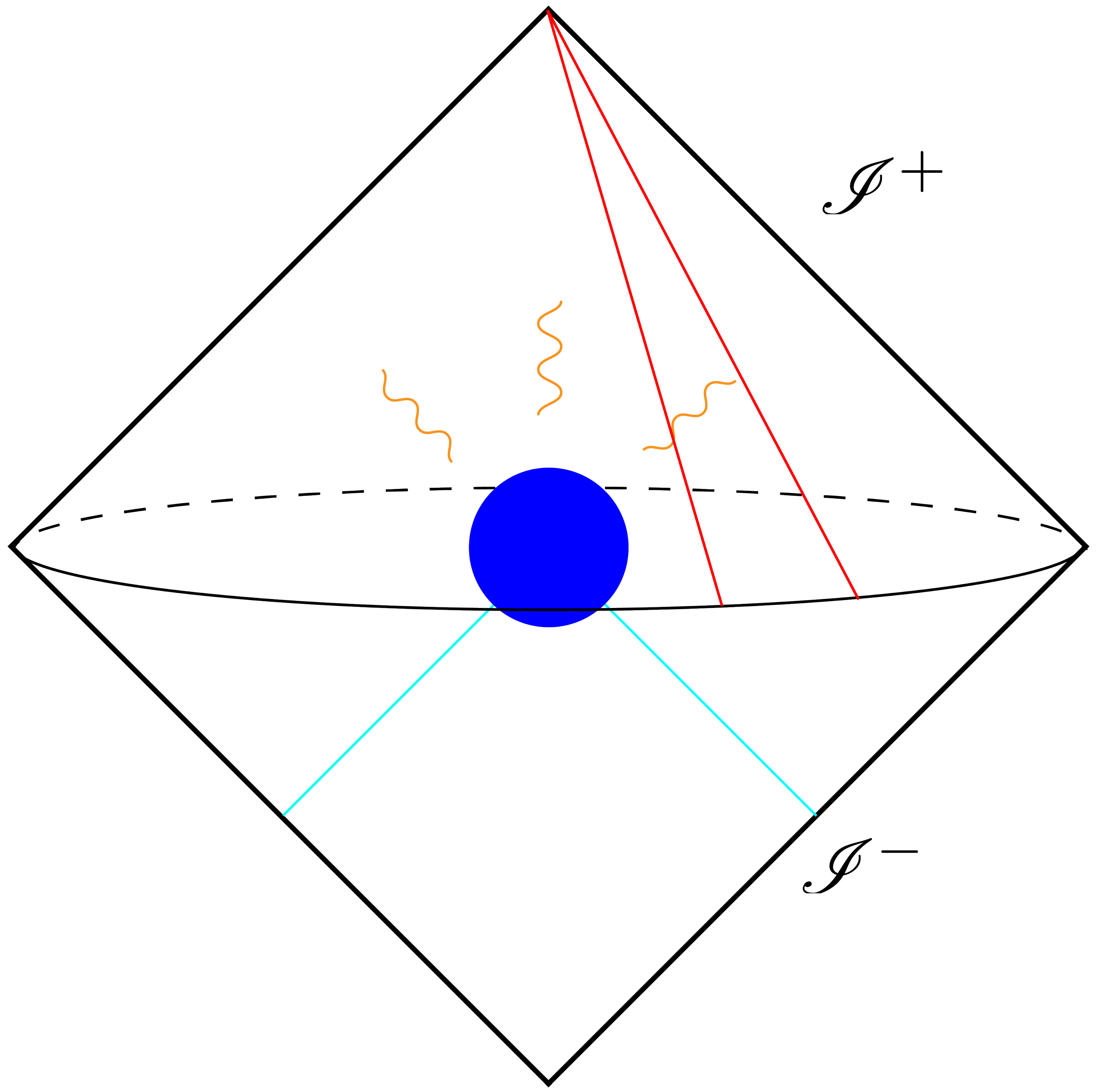


art from [Chen, Moult, Sandor, Zhu, 2022]

Poincaré Symmetry:
Open for Further Discussion During Q&A

Celestial Block

Solving the Casimir Equation



$$\langle \mathcal{E}(\vec{n}_3)\mathcal{E}(\vec{n}_4)\rangle_{p_1,\,p_2} \equiv \langle \Omega | \mathbb{A}^\rho(-p_1)\mathbb{A}^\sigma(-p_2)\mathcal{E}(\vec{n}_3)\mathcal{E}(\vec{n}_4)\mathbb{A}^\mu(p_1)\mathbb{A}^\nu(p_2)|\Omega\rangle$$

Lorentz Invariant: $p_1 \cdot p_2, \quad n_3 \cdot n_4, \quad p_i \cdot n_j$

Utilizing the fact that Lorentz Transformations act on the celestial sphere as Conformal Transformations, the result depends on the cross-ratios:

$$u = \frac{(p_1 \cdot p_2)(n_3 \cdot n_4)}{(p_1 \cdot n_3)(p_2 \cdot n_4)} = \frac{2(1-\cos\chi)}{(1-\cos\theta_a)(1+\cos\theta_b)} = 1 - 2e^{\Delta Y}\cos\phi + e^{2\Delta Y}$$

$$v = \frac{(p_1 \cdot n_4)(p_2 \cdot n_4)}{(p_1 \cdot n_3)(p_2 \cdot n_4)} = \frac{(1+\cos\theta_a)(1-\cos\theta_b)}{(1-\cos\theta_a)(1+\cos\theta_b)} = e^{2\Delta Y},$$

However, the correspondence is disrupted by the energies of the initial gluons, necessitating the inclusion of an additional variable： $\omega = \frac{p_1 \cdot n_3}{p_2 \cdot n_3} = \frac{1-\cos\theta_a}{1+\cos\theta_a} = e^{-Y-\Delta Y}.$

Celestial Block

Solving the Casimir Equation

$$\langle \mathcal{E}(\vec{n}_3)\mathcal{E}(\vec{n}_4)\rangle_{p_1,\,p_2} = \left(\frac{2\omega}{1+\omega}\right)^3\left(\frac{2}{1+v\omega}\right)^3 \frac{d^2\Sigma}{d\Omega_a d\Omega_b} = \frac{G(u,v,\omega)}{(p_1\cdot p_2)^{\delta_{12}}(n_3\cdot n_4)^{\delta_{34}}}$$

$$\delta_{12} = 2\times(1-1) = 0$$
$$\delta_{34} = 4-1 = 3$$

restore Lorentz symmetry

light–ray transform

Casimir Operator: $C|\mathcal{O}\rangle = -\frac{1}{2}L_{ab}L^{ab}|\mathcal{O}\rangle = (\delta(\delta-2)+j^2)|\mathcal{O}\rangle$ $\boxed{\delta:\text{celestial dimension, } j:\text{transverse spin}}$

Acting on correlators: $C_{(3\,4)}\langle \mathcal{E}(\vec{n}_3)\mathcal{E}(\vec{n}_4)\rangle_{p_1\,p_2} = \frac{\mathcal{D}_{u,v,\omega}\,G(u,v,\omega)}{(p_1\cdot p_2)^{\delta_{12}}(n_3\cdot n_4)^{\delta_{34}}} = \frac{(\delta(\delta-2)+j^2)\,G(u,v,\omega)}{(p_1\cdot p_2)^{\delta_{12}}(n_3\cdot n_4)^{\delta_{34}}}$

The differential operator is simplest to write in the coordinates $z,\ \bar{z}$ defined by $u = z\,\bar{z}$, $v = (1-z)(1-\bar{z})$

$$\mathcal{D}(z,\bar{z},\omega) = 2\left(z^2(1-z)\frac{\partial^2}{\partial z^2} - z^2\frac{\partial}{\partial z} + \omega z^2\frac{\partial^2}{\partial\omega\,\partial z} + (z\to\bar{z})\right)$$

extra term compared to 2d conformal block

In Mellin space, $\tilde{G}(z,\bar{z},\gamma) = \int_0^\infty G(z,\bar{z},\omega)\,\omega^{-\gamma-1}\,d\omega$ satisfies:

$$\left(z^2(1-z)\frac{\partial^2}{\partial z^2} + (\gamma-1)\,z^2\frac{\partial}{\partial z} + (z\to\bar{z})\right)\tilde{G}(z,\bar{z},\gamma) = \left(h(h-1)+\bar{h}(\bar{h}-1)\right)\tilde{G}(z,\bar{z},\gamma)$$

conformal weight

$$h = \frac{\delta+j}{2}$$
$$\bar{h} = \frac{\delta-j}{2}$$

The solution is:

$$\tilde{G}(z,\,\bar{z},\,\gamma) = z^{\bar{h}}\,{}_2F_1\left(\bar{h},\,\bar{h}-\gamma,\,2\bar{h},\,z\right)\times\bar{z}^{h}\,{}_2F_1\left(h,\,h-\gamma,\,2h,\,\bar{z}\right) + (z\leftrightarrow\bar{z})$$

Celestial Block Decomposition

Applied to the Result of the HEEC

$$\langle \mathcal{E}(\vec{n}_3)\mathcal{E}(\vec{n}_4)\rangle_{p_1,p_2} = \frac{1}{z^3\bar{z}^3}\sum_{\delta,j}\left[\int \frac{d\gamma}{2\pi i}\omega^\gamma c_{\delta,j}(\gamma) G^{(\gamma)}_{\delta,j}(z,\bar{z}) + g_{\delta,j}G^{(0)}_{\delta,j}(z,\bar{z})\right]$$

terms non–vanishing in the large ω limit:

- at most $\mathcal{O}(\omega^0)$
- expanded by the 2d conformal block

twist–2

$$= -\frac{27g^6}{65536\pi^5 z^3\bar{z}^3}\left[\int \frac{d\gamma}{2\pi i}\omega^\gamma \frac{224}{75}\gamma(\gamma^4+55\gamma^2+304) G^{(\gamma)}_{4,0}(z,\bar{z}) - \frac{1792}{5}G^{(0)}_{4,0}(z,\bar{z})\right.$$
$$+\int \frac{d\gamma}{2\pi i}\omega^\gamma \frac{16\gamma(122\gamma^6+8897\gamma^4+157493\gamma^2+612168)}{11025} G^{(\gamma)}_{6,0}(z,\bar{z}) - \frac{10624}{35}G^{(0)}_{6,0}(z,\bar{z})$$
$$\left.+\int \frac{d\gamma}{2\pi i}\omega^\gamma \frac{4\gamma(641\gamma^6+50456\gamma^4+875819\gamma^2+2994204)}{55125} G^{(\gamma)}_{6,2}(z,\bar{z}) - \frac{12352}{175}G^{(0)}_{6,2}(z,\bar{z}) + \cdots\right]$$

twist–4

higher twist

light–ray OPE [Hofman, Maldacena, 2008[: $\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) = \sum_i \left(\mathbb{O}_{i,J=3,j=0}(\vec{n}) + \mathbb{O}_{i,J=3,j=2}(\vec{n}) + \mathbb{O}_{i,J=3,j=4}(\vec{n})\right)$

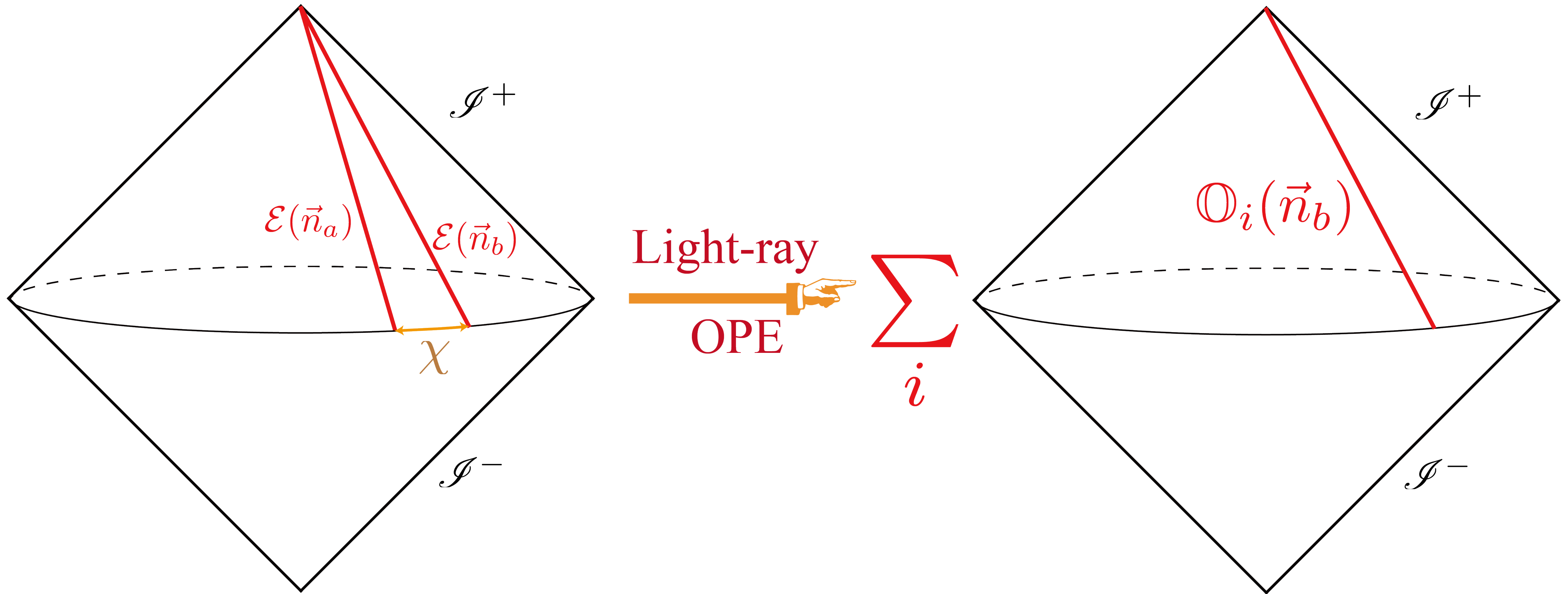
$$+\sum_{n,i}\mathcal{D}_{2n}\mathbb{O}_{i,J=3+2n,j=4}(\vec{n})$$

twist

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} r^{\Delta-J}\int_0^\infty dt\, O^{\mu_1\cdots\mu_J}(t, r\vec{n})\bar{n}_{\mu_1}\cdots\bar{n}_{\mu_J}$$

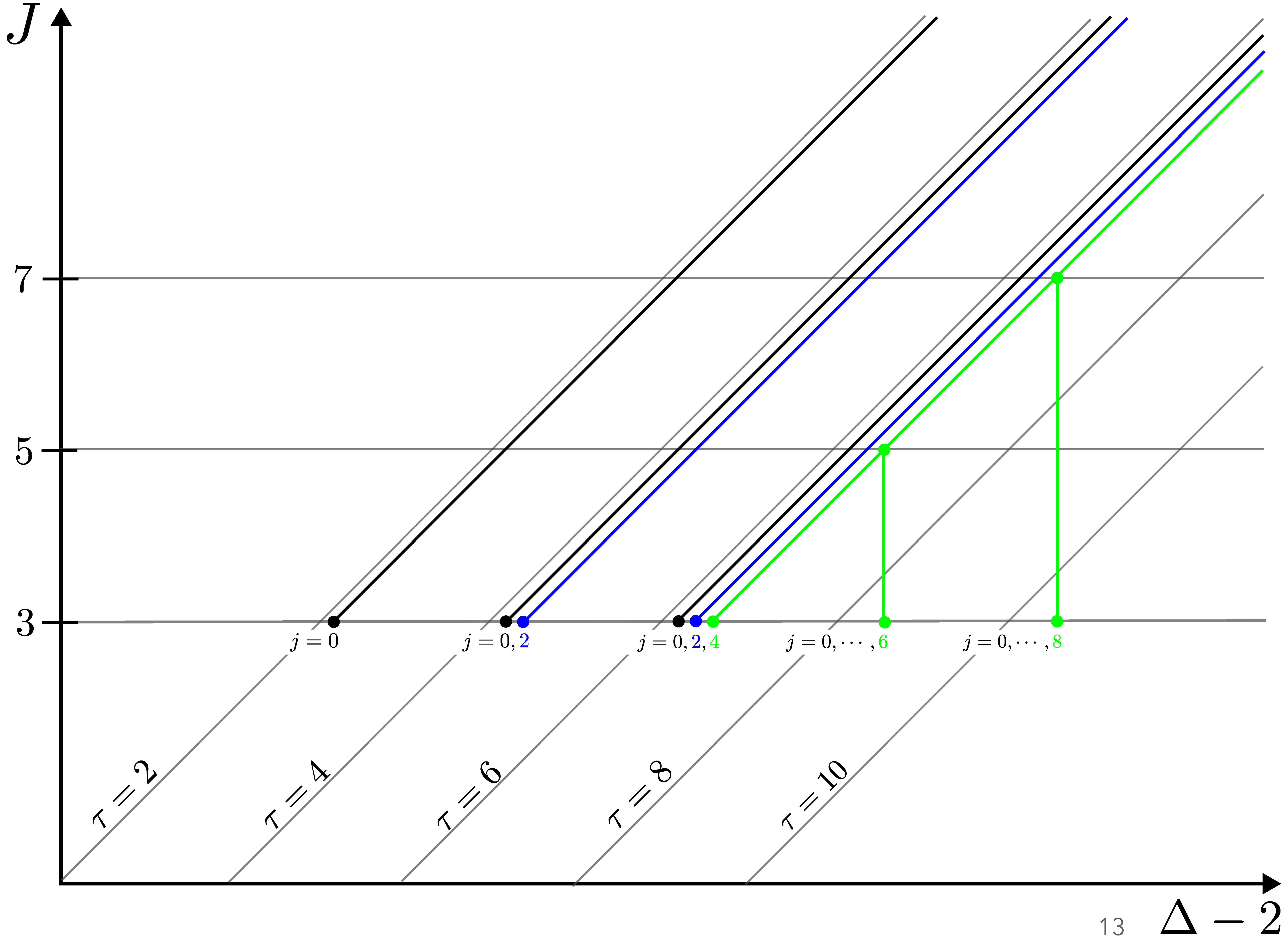
Generalization of the energy flow operator

[Kravchuk, Simmons–Duffin, 2018]



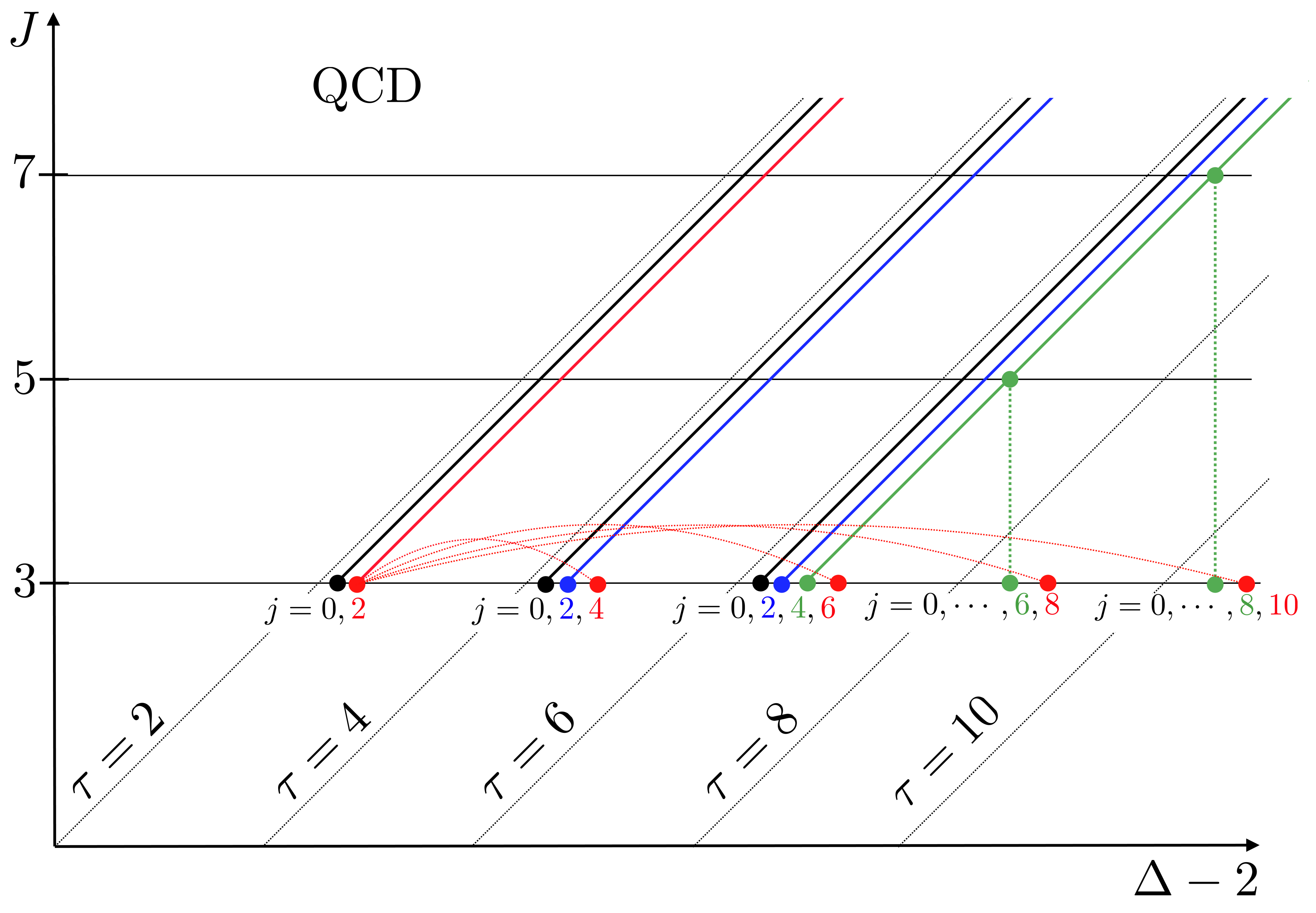
Contributing Operators

Chew–Frautschi Plot



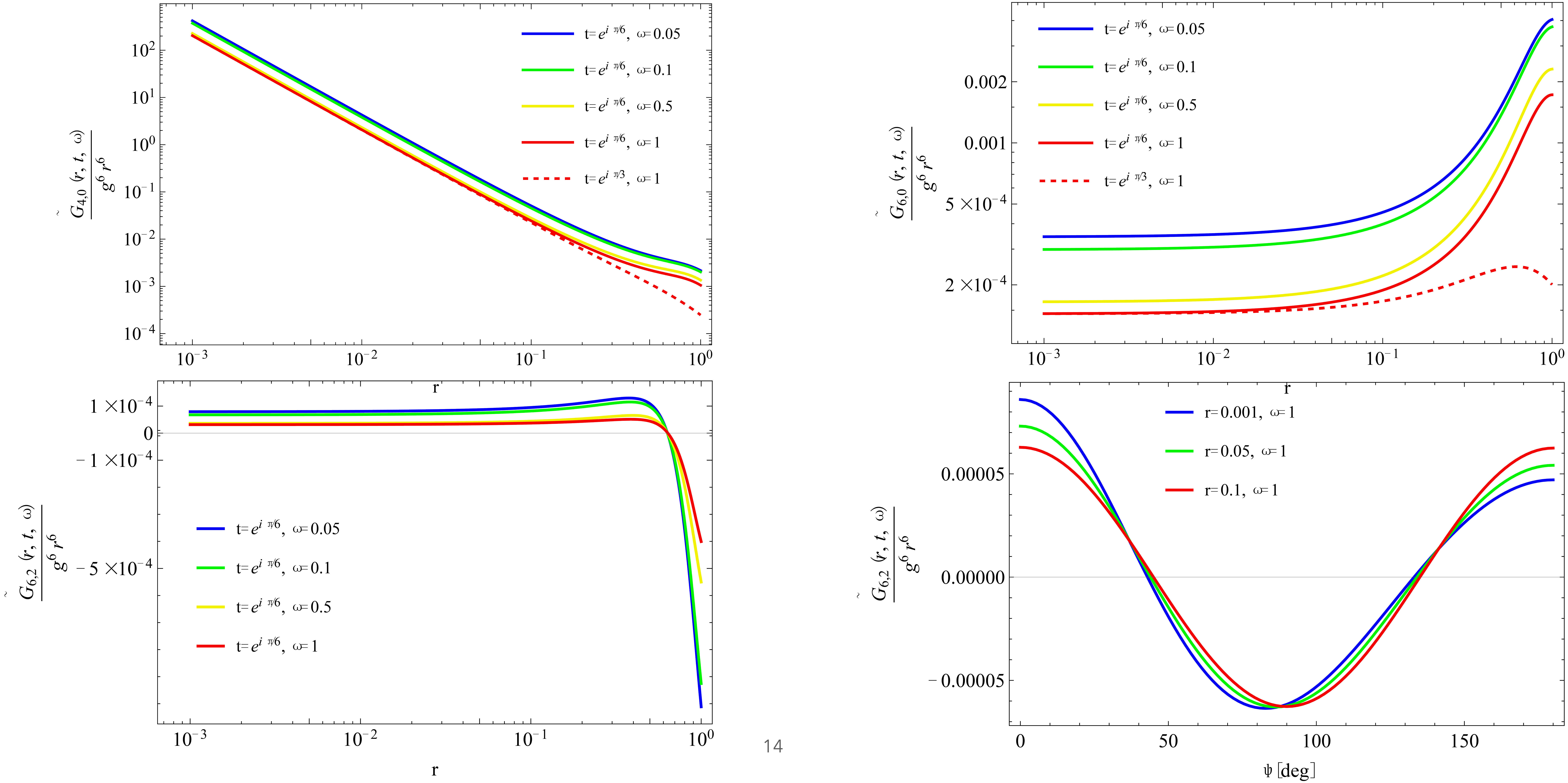
$$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)=\sum_i\left(\mathbb{O}_{i,J=3,j=0}(\vec{n})+\mathbb{O}_{i,J=3,j=2}(\vec{n})+\mathbb{O}_{i,J=3,j=4}(\vec{n})\right)+\sum_{n,i}\mathcal{D}_{2n}\mathbb{O}_{i,J=3+2n,j=4}(\vec{n})$$

In contrast, the Chew–Frautschi plot for the squeezed EEEC in QCD is:



Celestial Block Dependence

A Window to Twist-4 Operators

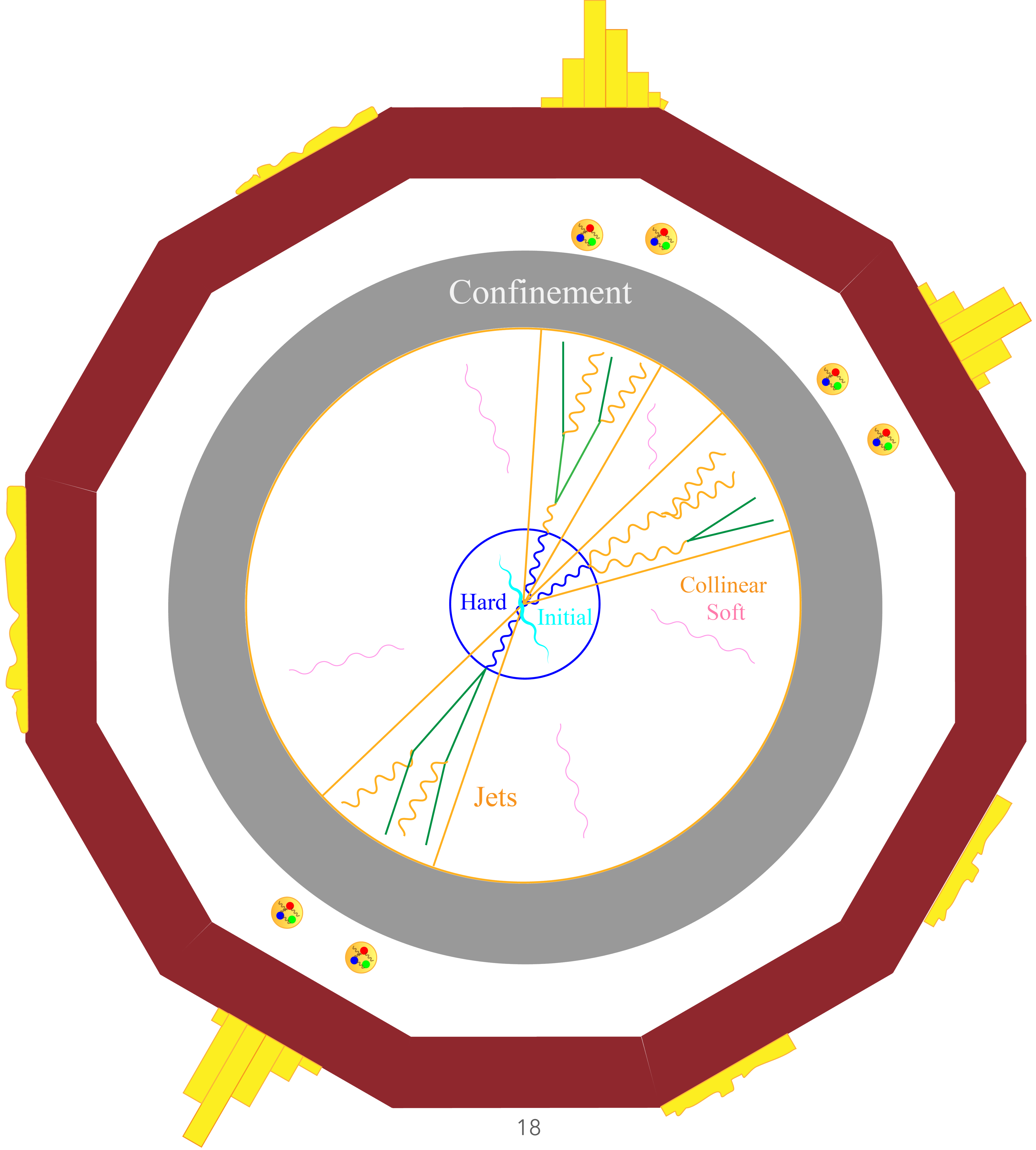


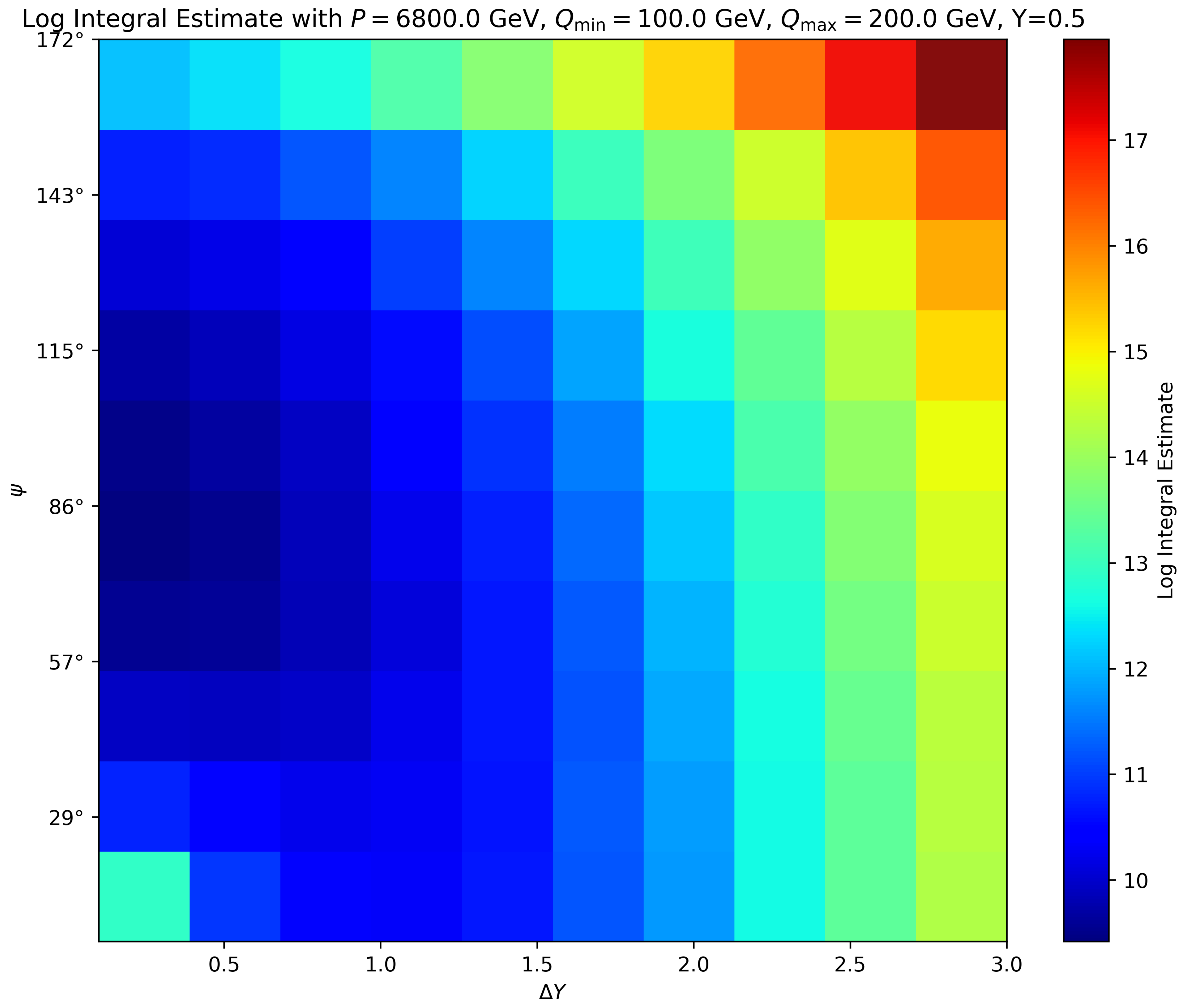
Summary

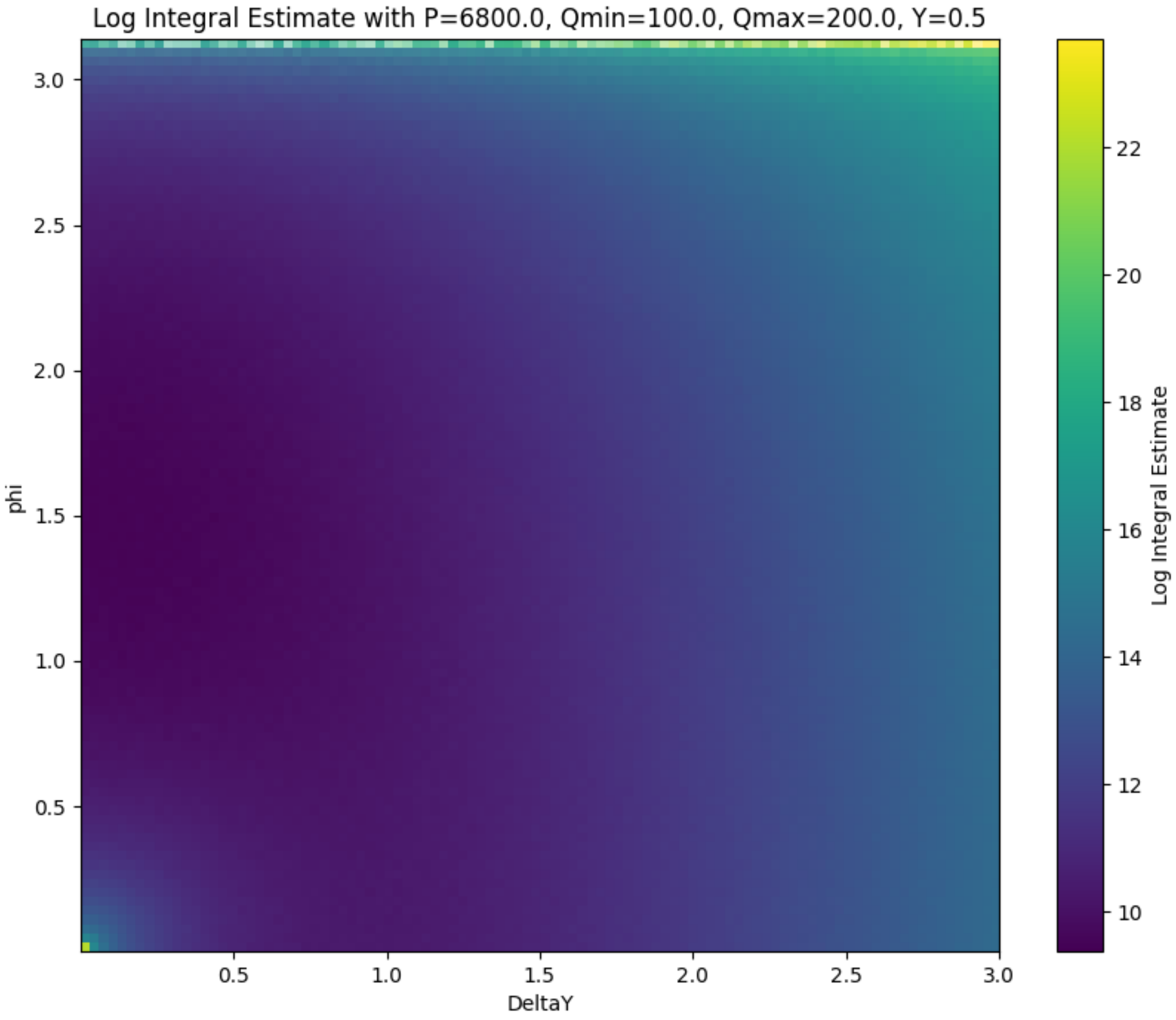
- For hadron colliders, we investigate the full-angle dependent energy-energy correlators, achieving:
  - The first leading-order (LO) analytical result in pure gluon scatterings.
  - Factorization formalisms in several limits, with the exception of the Regge limit.
- From a formal theoretical perspective:
  - We compute the celestial block, which serves as a tool to parameterize EECs with respect to various scattering processes in hadron colliders.
  - This celestial block decomposition is applied to our previous results, providing an independent consistency check.
- Potential directions for further research:
  - Exploring the Regge limit in the context of energy
  - Investigating the properties of twist-4 operators.


Thanks

Backup


Confinement
Hard
Initial
Collinear
Soft
Jets


Log Integral Estimate with $P = 6800.0$ GeV, $Q_{\min} = 100.0$ GeV, $Q_{\max} = 200.0$ GeV, Y=0.5
172°
143°
115°
86°
57°
29°
ψ
0.5
1.0
1.5
2.0
2.5
3.0
ΔY
17
16
15
14
13
12
11
10
Log Integral Estimate


Log Integral Estimate with P=6800.0, Qmin=100.0, Qmax=200.0, Y=0.5
3.0
2.5
2.0
1.5
1.0
0.5
phi
0.5
1.0
1.5
2.0
2.5
3.0
DeltaY
22
20
18
16
14
12
10
Log Integral Estimate


relative error
10^{-5}
10^{-9}
0.00
0.02
0.04
0.06
0.08
0.1
r
Laurent Expansion up to r^1
Block Expansion up to leading twist
Block Expansion up to next–to leading twist
$\omega=1$
$t=e^{i\frac{\pi}{6}}$
relative error
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
0.00
0.02
0.04
0.06
0.08
0.10
r
Laurent Expansion up to r^1
Block Expansion up to leading twist
Block Expansion up to next–to leading twist
$\omega=1$
$t=e^{i\frac{\pi}{3}}$