

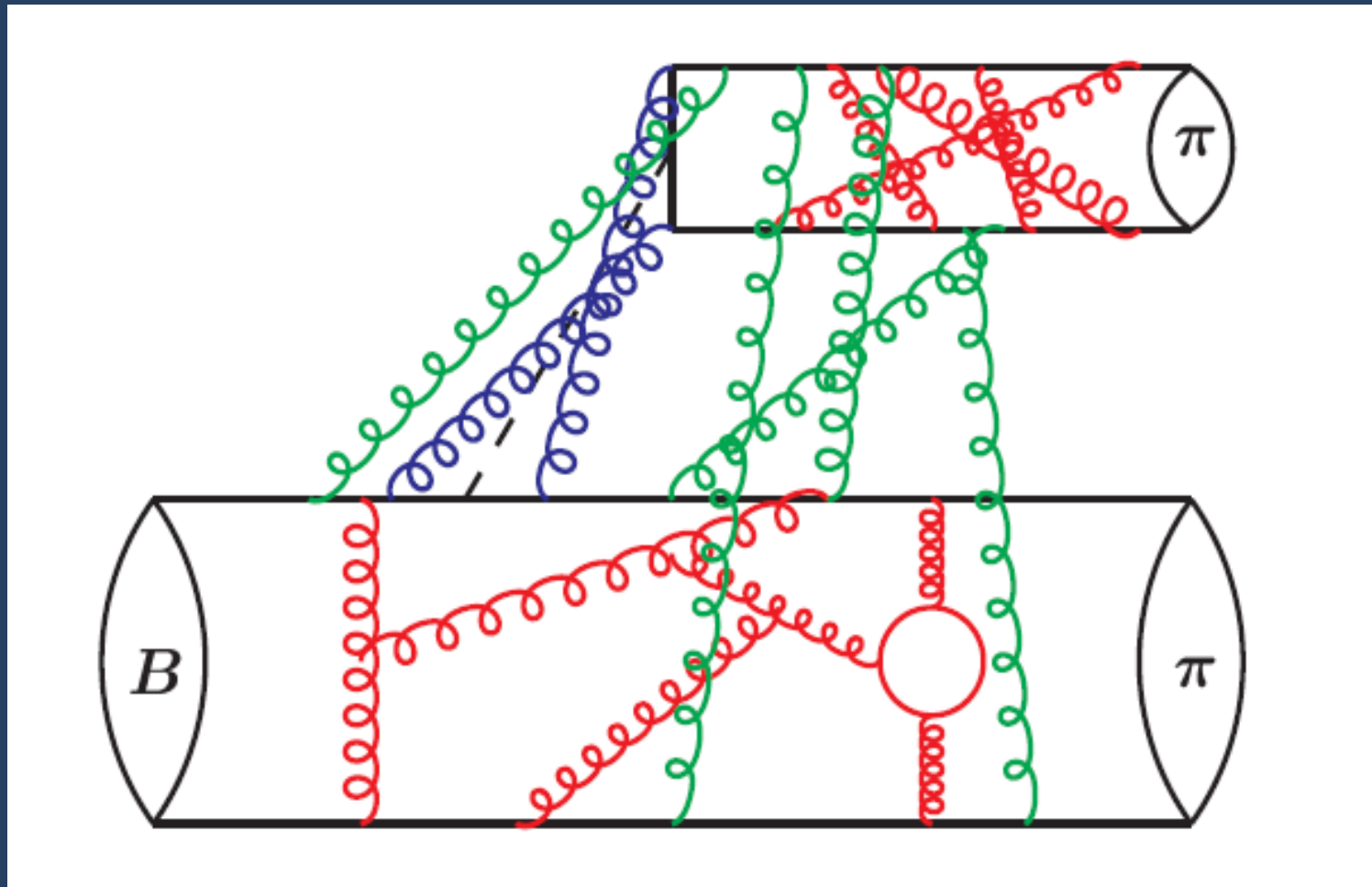
Power corrections in Heavy meson DA within the LaMET procedure

第四届量子场论及其应用研讨会

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$$\langle \pi(p') \pi(q) | Q_i | \bar{B}(p) \rangle = f^{B \rightarrow \pi}(q^2) \int_0^1 dx T_i^I(x) \phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{II}(\xi, x, y) \phi_B(\xi) \phi_\pi(x) \phi_\pi(y)$$

$B \rightarrow \pi$ form factor

Hard kernel

B-meson LCDA

QCD Factorization: BBNS, PRL 83, 1914 (1999)

For PQCD, See: Keum, Li, Sanda PRD 63,054008 (2001)

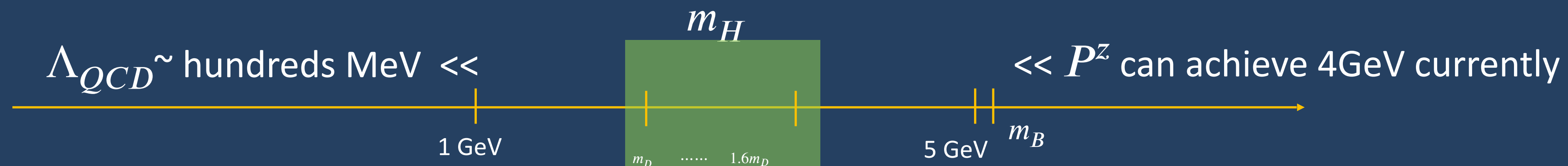
➤ Start from Quasi DA, calculable from LQCD



• A multi-scale processes:

1. LaMET requires $\Lambda_{\text{QCD}}, m_H \ll P^z$ and finally integrate out P^z ;
2. bHQET requires $\Lambda_{\text{QCD}} \ll m_H$ and integrate out m_H ;

⇒ **Hierarchy** $\Lambda_{\text{QCD}} \ll m_H \ll P^z$: A big challenge for lattice simulation but **still calculable on the lattice**



$$\tilde{\phi}_R(x, P^z) = \int_0^1 dy C(x, y, \frac{\mu}{p^z}) \phi(y, \mu)_R + O(\frac{M^2}{P^{z2}}, \frac{\Lambda_{QCD}^2}{(P^z)^2})$$

$$m_c \sim 1.76 GeV$$

$$P^z \sim \{3, 5\} GeV$$



quasi-DA 格点可算 匹配核: 微扰可算 待求LCDA

Power correction should be taken seriously when P^z is not large enough

The definition of LCDA

$$\langle 0 | \bar{q}(\frac{zn_+}{2}) \not{n}_+ \gamma_5 W_c(\frac{zn_+}{2}, -\frac{zn_+}{2}) Q(-\frac{zn_+}{2}) | H(P) \rangle = if_H P^+ \int_0^1 dx e^{-i(x-\frac{1}{2})zP^+} \underline{\phi(x, \mu)}$$

LCDA

The definition of LCDA

$$\langle 0 | \bar{q}(\frac{zn_+}{2}) \not{n}_+ \gamma_5 W_c(\frac{zn_+}{2}, -\frac{zn_+}{2}) Q(-\frac{zn_+}{2}) | H(P) \rangle = if_H P^+ \int_0^1 dx e^{-i(x-\frac{1}{2})zP^+} \phi(x, \mu)$$
$$\langle 0 | (\bar{q}(\frac{zn_z}{2}) \not{n}_z \gamma_5 W_c(\frac{zn_z}{2}, -\frac{zn_z}{2}) Q(-\frac{zn_z}{2}))_{lt} | H(P) \rangle_{non-sing} = if_H P^+ \int_0^1 dx (e^{-i(x-\frac{1}{2})zP_z})_{lt} \phi(x, \mu)$$

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The definition of LCDA

$$\langle 0 | \bar{q}(\frac{zn_+}{2}) \not{n}_+ \gamma_5 W_c(\frac{zn_+}{2}, -\frac{zn_+}{2}) Q(-\frac{zn_+}{2}) | H(P) \rangle = if_H P^+ \int_0^1 dx e^{-i(x-\frac{1}{2})zP^+} \phi(x, \mu)$$
$$\langle 0 | (\bar{q}(\frac{zn_z}{2}) \not{n}_z \gamma_5 W_c(\frac{zn_z}{2}, -\frac{zn_z}{2}) Q(-\frac{zn_z}{2}))_{lt} | H(P) \rangle_{non-sing} = if_H P^+ \int_0^1 dx (e^{-i(x-\frac{1}{2})zP_z})_{lt} \phi(x, \mu)$$

The definition of Quasi-DA

$$\langle 0 | (\bar{q}(\frac{zn_z}{2}) \not{n}_z \gamma_5 W_c(\frac{zn_z}{2}, -\frac{zn_z}{2}) Q(-\frac{zn_z}{2})) | H(P) \rangle = if_H P^+ \int_0^1 dx e^{-i(x-\frac{1}{2})zP_z} \tilde{\phi}(x, \mu)$$

Leading matching procedure within LaMET discards the singular logarithms, and left the

Leading-twist projection as it's power corrections

Leading twist projection:

$$[\bar{q}(-\frac{zn_z}{2})\not{n}\gamma_5 W_c Q(\frac{zn_z}{2})]_{lt} = \sum_n \frac{z^n}{n!} n_z^{\mu_1} \dots n_z^{\mu_n} [\bar{q}(0)\gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \dots \overleftrightarrow{D}_{\mu_n} \gamma_5 Q(0) - Trace]$$

Higher-twist operator correction $(\frac{\Lambda_{QCD}^2}{Pz^2})^n$

$$[e^{ip \cdot z}]_{lt} = \sum_{n=0}^{\infty} \frac{1}{n!} z^{\mu_1} \dots z^{\mu_n} (p_{\mu_1} \dots p_{\mu_n} - Trace)$$

Target mass correction $(\frac{m_H^2}{Pz^2})^n$

$$[e^{ip \cdot z}]_{lt} = \sum_{n=0}^{\infty} \frac{1}{n!} z^{\mu_1} \dots z^{\mu_n} (P_{\mu_1} \dots P_{\mu_n} - \text{Trace})$$

Case n=2

$$n_z^{\mu_1} n_z^{\mu_2} P_{\mu_1} P_{\mu_2} = (n_z^{\mu_1} n_z^{\mu_2} - \frac{1}{4} n_z^2 g^{\mu_1 \mu_2}) P_{\mu_1} P_{\mu_2} + \frac{1}{4} n_z^2 g^{\mu_1 \mu_2} P_{\mu_1} P_{\mu_2}$$

↓

$$[n_z^{\mu_1} n_z^{\mu_2} P_{\mu_1} P_{\mu_2}]_{lt}$$

$$[n_z^{\mu_1} n_z^{\mu_2} P_{\mu_1} P_{\mu_2}]_{lt} = n_z^{\mu_1} n_z^{\mu_2} P_{\mu_1} P_{\mu_2} \cdot (1 + \frac{1}{4} (\frac{M}{P^z})^2)$$

However the coefficients is different at different n.

$$[n_z^{\mu_1} \dots n_z^{\mu_n} P_{\mu_1} \dots P_{\mu_n}]_{lt} = n_z^{\mu_1} \dots n_z^{\mu_n} P_{\mu_1} \dots P_{\mu_n} \cdot (1 + ? (\frac{M}{P^z})^2 + ? (\frac{M}{P^z})^4 + \dots)$$

$$\psi(x) = \frac{1}{\sqrt{1+4c}} \left[\frac{f_+}{2} \psi_{lt} \left(\frac{1}{2} - \frac{1-2x}{f_+} \right) - \frac{f_-}{2} \psi_{lt} \left(\frac{1}{2} + \frac{1-2x}{f_-} \right) \right]$$

$$c = m_H^2 / (4P^z)^2 \quad f_{\pm} = \sqrt{1+4c} \pm 1$$

Phys.Rev.D 95 (2017) 9, 094514 Jian-Hui Zhang, Jiunn-Wei Chen,...

$$\psi(x, \mu) = \psi(x, \mu)_{lt} - \frac{1}{8} \frac{m_H^2}{(P^z)^2} \times \frac{d}{dx} \left((2x-1) \psi(x, \mu)_{lt} + \mathcal{O} \left(\frac{m_H^4}{(P^z)^4} \right) \right)$$

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Quasi-DA also receives contribution from higher-twist operators

$$\mathcal{O}_{twist-4} = \int_0^z \bar{q}(0) \gamma^5 \gamma^\nu \underbrace{W_c(0, z_1) D_\nu W_c(z_1, z)} Q(z) dz_1$$

$$\mathcal{O}'_{twist-4} = \int_0^z dz_1 \int_0^{z_1} dz_2 \bar{q}(0) \gamma^5 \not{n} \underbrace{W_c(0, z_2) D_\nu W_c(z_2, z_1)} \underbrace{D^\nu W_c(z_1, z)} Q(z)$$

High dimensional operator would mix with low dimension operator on Lattice.

Covariant derivatives are realized through Wilson lines, increasing uncertainty.

Thus, hard to calculate using Lattice QCD.

Cutoff regularization

$$C(x, y, \frac{\mu_F}{P^z}) = \delta(x - y) + \sum_{i=1}^{\infty} c_i(\mu_F) \alpha_s^i - \frac{\mu_F^2}{(P^z)^2} D_Q(x) + \dots$$

Since the quasi-DA is independent of the factorization scale, so the $\frac{\mu_F^2}{(P^z)^2} D_Q(x)$ term

must cancel with contribution from higher-twist operators

We could estimate the higher-twist contribution from $\frac{\mu_F^2}{(P^z)^2} D_Q(x)$.

Dimensional regularization

$$C(x, y, \frac{\mu_F}{P_z}) = \delta(x - y) + \sum_{i=1}^{\infty} c_i \alpha_s^i$$

The series $c_i \alpha_s^i$ becomes divergent, and the power dependence on μ_F disappears.

Renormalon model: Estimate the range of power dependence from divergent series.

Goal: Use DA models for D-meson to estimate the impact of power corrections $(\frac{M}{P^z})^{2n}, (\frac{\Lambda_{QCD}}{P^z})^2$ on Quasi-DA,

when using LaMET matching formula.

Models of LCDA

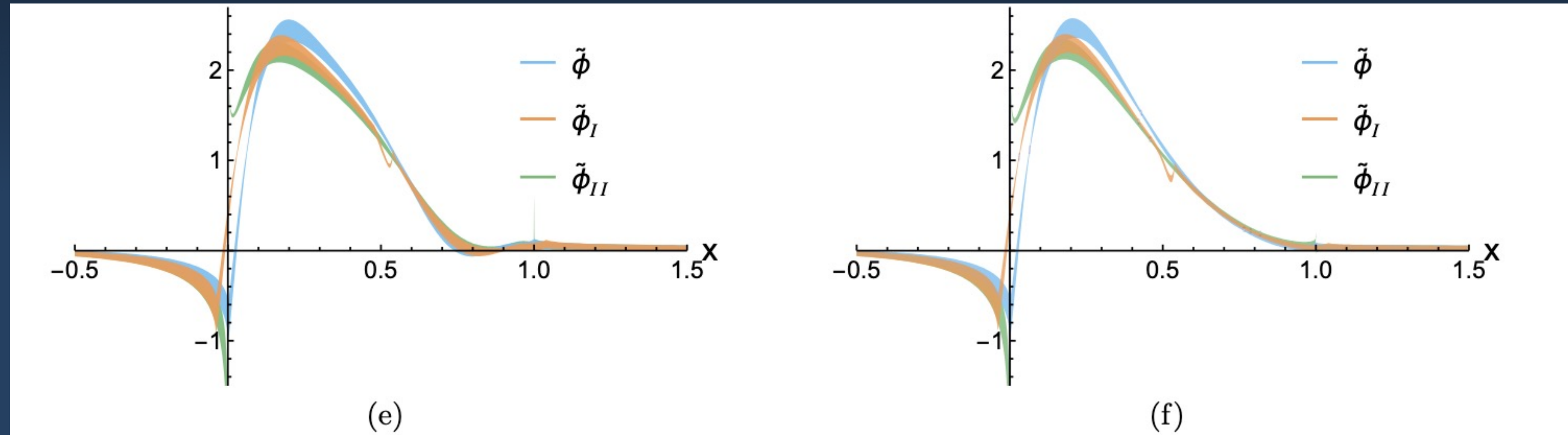
$$\phi(x, \mu) = 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n(\mu) C_n^{(3/2)}(2x-1) \right],$$

$$a_n^D(1.6\text{GeV}) = \{1, -0.659, 0.206, -0.057, 0.036, -0.004, -0.007, \dots\}$$

$$\phi(x, \mu) = a(1-x)xe^{-bx}$$

The choice of parameters:

$$m_H = 1.76 \text{ GeV}, \Lambda_{QCD} = 0.5 \text{ GeV}, z_s = 0.1672 \text{ fm}, \mu = 1.6 \sim 3 \text{ GeV}, P^z = \{3, 4, 5\} \text{ GeV}$$



Blue: Quasi-DA without mass correction;

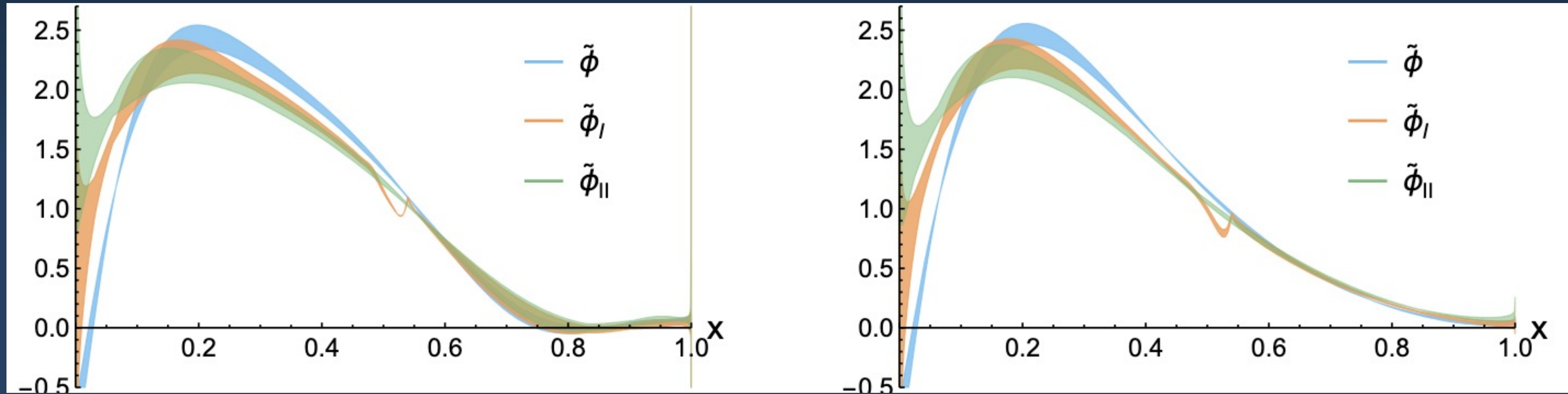
Error bands: factorization scale

Orange: Quasi-DA, $(\frac{M}{P_z})^{2n}$;

Green: Quasi-DA, $(\frac{M}{P_z})^2$.

1. *Distribution is wilder. Peak region is shifted to left.*

2. *There is a dent at $x=1/2$.*



Error bands: factorization scale and renormalon ambiguity

Blue: Quasi-DA without correction;

Orange: Quasi-DA, $(\frac{M}{P_z})^{2n}$ and $(\frac{\Lambda_{QCD}}{P_z})^2$;

Green: Quasi-DA, $(\frac{M}{P_z})^2$ and $(\frac{\Lambda_{QCD}}{P_z})^2$

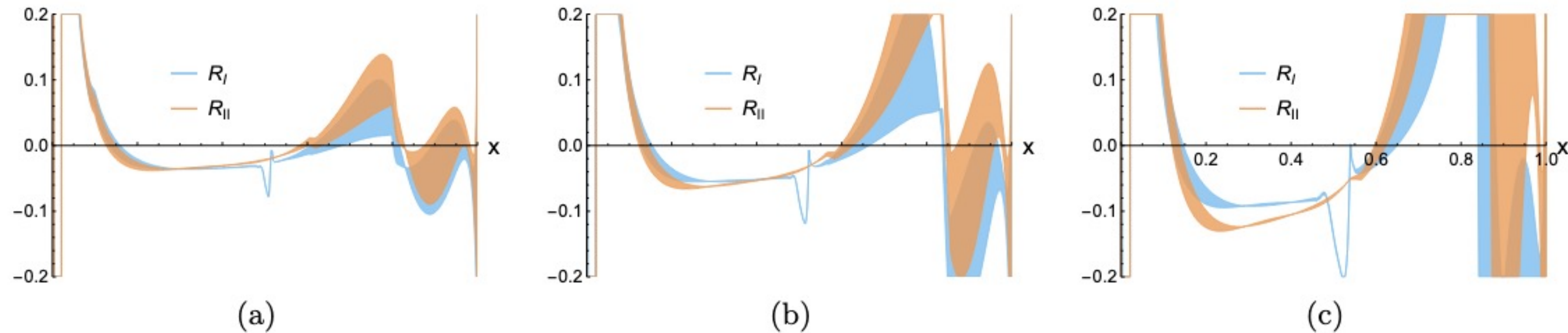


FIG. 7. Panel (a) corresponds to the ratio $R_{I/II}$ at $P^z = 5$ GeV, panel (b) to the ratio $R_{I/II}$ at $P^z = 4$ GeV, and panel (c) to the ratio $R_{I/II}$ at $P^z = 3$ GeV.

$$R_{I/II} = \frac{\tilde{\phi}_{I/II} - \tilde{\phi}}{\tilde{\phi}}$$

Error bands: factorization scale and renormalon ambiguity

For the D-meson case, in the moderate x region $x \in [0.2, 0.8]$, the corrections from mass and power are typically smaller than 20%. At peak region, mass correction dominates.

Thank you for your attention!

$$\tilde{\psi}_{lt}(x) = \frac{1}{\sqrt{1+4c}} \left[\frac{f_+}{2} \psi\left(\frac{1}{2} - \frac{1-2x}{f_+}\right) - \frac{f_-}{2} \psi\left(\frac{1}{2} + \frac{1-2x}{f_-}\right) \right]$$

$$c = m_H^2 / (4P^z)^2$$

$$f_{\pm} = \sqrt{1+4c} \pm 1$$

Taylor expansion in c of $\psi\left(\frac{1}{2} - \frac{1-2x}{f_+}\right)$ gives:

$$\psi(x, \mu) - \frac{1}{8} \frac{m_H^2}{(P^z)^2} \times \frac{d}{dx} ((2x-1)\psi(x, \mu)) + \mathcal{O}\left(\frac{m_H^4}{(P^z)^4}\right)$$

However, Taylor expansion in c of $\psi\left(\frac{1}{2} + \frac{1-2x}{f_-}\right)$ is zero

This is the non-perturbative effect in mass correction,

which cannot be seen by fixed order correction.

$$\begin{aligned} 2\delta_R\Phi(x, \mu) = & \left[\frac{\Phi'(x, \mu)}{x-1} \right. \\ & - \frac{1}{(x-1)^2} \int_{-1}^x dy \left[\frac{x-1}{y-1} + \ln \left(1 - \frac{x-1}{y-1} \right) \right] \Phi'(y, \mu) \Big] \\ & + \left[\frac{\Phi'(x, \mu)}{x+1} \right. \\ & + \frac{1}{(x+1)^2} \int_x^1 dy \left[\frac{x+1}{y+1} + \ln \left(1 - \frac{x+1}{y+1} \right) \right] \Phi'(y, \mu) \Big] \\ & + \frac{1}{4} \Phi''(x, \mu), \end{aligned}$$

The range of power correction $\Phi' = \Phi \pm \delta_R\Phi$

Borel summation

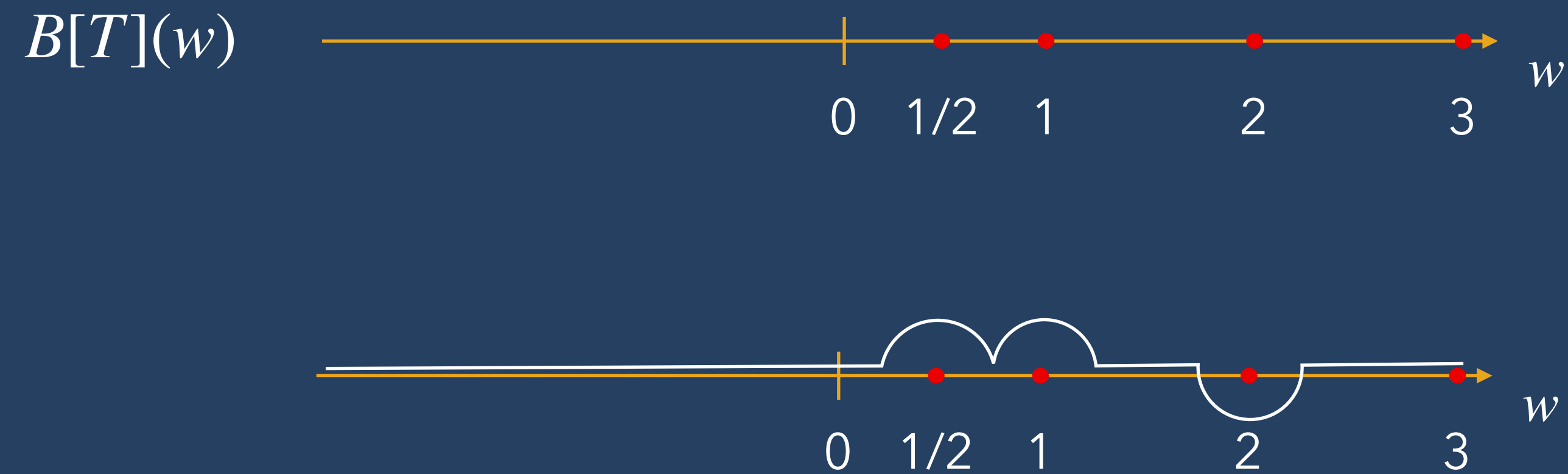
$$C(x, y, \frac{\mu_F}{P^z}) = \delta(x - y) + \sum_{i=1}^{\infty} c_i(\mu_F) \alpha_s^i$$

$$B[T](w) = \sum_{k=1}^{\infty} \frac{c_k}{k!} \left(\frac{w}{\beta_0} \right)^k.$$

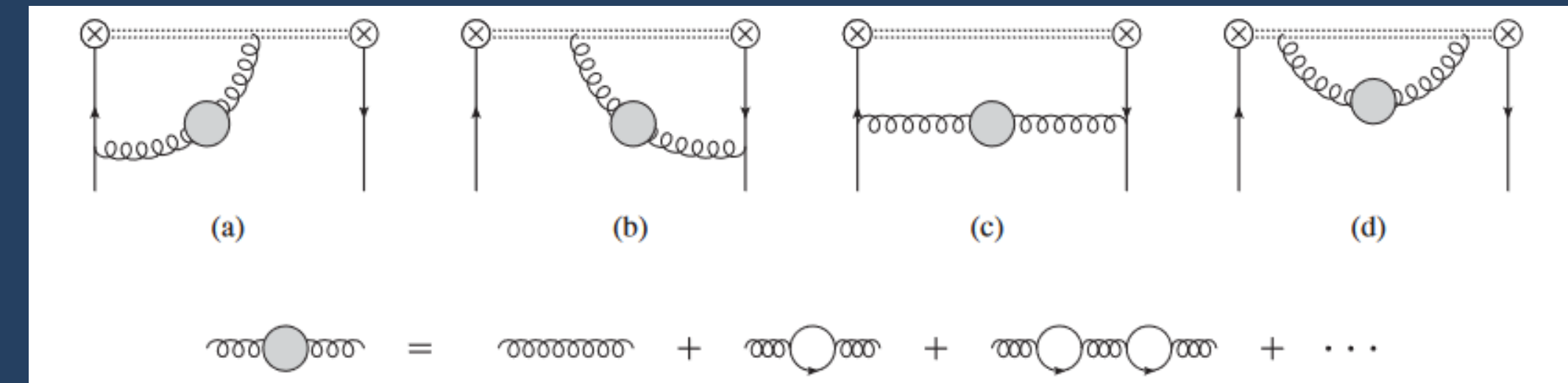
$$T(\alpha_s) = \frac{1}{\beta_0} \int_0^{\infty} dw e^{-w/(\beta_0 \alpha_s)} B[T](w)$$

The asymptotic series of $T(\alpha_s)$ is exactly $\sum_{i=1}^{\infty} c_i \alpha_s^i$

$$T(\alpha_s) = \frac{1}{\beta_0} \int_0^\infty dw e^{-w/(\beta_0 a_s)} B[T](w)$$



Diagrams contribute to these poles



Renormalon ambiguity: the choice of contour