中国科学院高能物理研究所
Institute of High Energy Physics
Chinese Academy of Sciences

Maximizing the Azimuthal Angle Correlation in the Decay of Vector boson pairs

第四届量子场论及其应用研讨会

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1. Introduction

Introduction

- Measuring spin correlation help us understand fundamental particles and their interactions.
  - Measuring SM interactions.
  - Probing NP effects, especially CP violation.
  - Studying quantum information (e.g. ATLAS 2406.03976v2).

- $t\bar{t}$ spin correlation is well studied. WW: more challenging.

$$\hat{\rho}_{WW} = \frac{1}{9}\mathbb{I}\otimes\mathbb{I} + \frac{1}{3}\sum_{i=1}^{8} a_i\lambda_i\otimes\mathbb{I} + \frac{1}{3}\sum_{j=1}^{8} b_j\mathbb{I}\otimes\lambda_j + \sum_{i=1}^{8}\sum_{j=1}^{8} c_{ij}\lambda_i\otimes\lambda_j$$

64 spin correlation coefficients of W pair

  - Single-W particle spin density matrix can be fully measured.

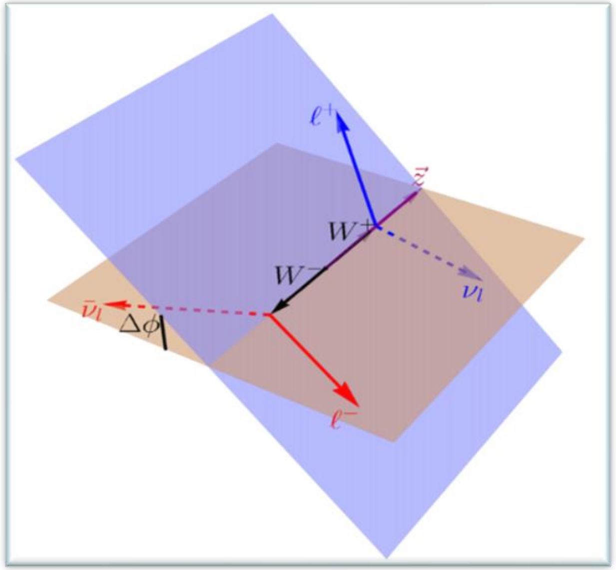
LEP: Eur. Phys. J. C 40, 333 (2005). Physics Letters B 557, 147 (2003)
Physics Letters B 585, 223 (2004)

  - Only a small part of VV's joint spin density matrix are measured.($V = W,Z$)

Eur. Phys. J. C 63, 611 (2009). . Phys. Lett. B 843, 137895 (2023).

- Single observable is useful. Azimuthal angle correlation probes part of spin correlation.

- We propose a method to maximize the azimuthal angle correlation of W boson pair.



2. Azimuthal Angle Correlation

The decay plane correlation function for the azimuthal angle

- The azimuthal angle distribution of decay particles of W boson pair is well known:

K. Hagiwara, R. D. Peccei, D. Zeppenfeld,
and K. Hikasa, Nucl. Phys. B 282, 253 (1987).

$$\boxed{\frac{1}{\sigma}\frac{d\sigma}{d\Delta\phi} = \frac{1}{2\pi} + A\cos(\Delta\phi) + \tilde{A}\sin(\Delta\phi) + B\cos(2\Delta\phi) + \tilde{B}\sin(2\Delta\phi)}$$

- The angular correlations of the decay particles are determined by the spin correlations of the W boson pair.

$$\left\{\begin{aligned} A &= -\frac{9\pi}{64}\left(C^{d}_{ii} - C^{d}_{ij}z_i z_j\right), \\ \tilde{A} &= \frac{9\pi}{64}\epsilon_{ijk}C^{d}_{ij}z_k, \\ B &= \frac{1}{4\pi}\left(C^{q}_{ij,kl}z_i z_j z_k z_l + 2C^{q}_{ij,ji} - 4C^{q}_{ij,ji}z_i z_i\right), \\ \tilde{B} &= \frac{1}{2\pi}\left(\epsilon_{ijk}C^{q}_{ij,ii}z_k - \epsilon_{ijk}C^{q}_{ii,ij}z_k\right). \end{aligned}\right.$$

$$C^{d}_{ij} = Tr\left(\rho\hat{S}^{+}_{i} \otimes \hat{S}^{-}_{j}\right)$$

$$C^{q}_{ij,kl} = Tr\left(\rho\hat{S}^{+}_{\{ij\}} \otimes \hat{S}^{-}_{\{kl\}}\right)$$

$$\hat{S}^{+}_{\{ij\}} = \hat{S}_i\hat{S}_{\mathrm{j}} + \hat{S}_{\mathrm{j}}\hat{S}_i$$

$$\rho_{\alpha_1\alpha_2,\beta_1\beta_2} \propto M^{*}_{\alpha_1\,\beta_1}M_{\alpha_2\,\beta_2}$$

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CP even

CP odd

CP properties of coefficients:

	C^d_{ij}	$C^q_{ij,kl}$
CP conservation	$C^d_{ij} = C^d_{ji}$	$C^q_{ij,kl} = C^q_{kl,ij}$
CP violation	$C^d_{ij} = -C^d_{ji}$	$C^q_{ij,kl} = -C^q_{kl,ij}$

$\vec{z}$ is used to define $\Delta\phi$

$$A = -\frac{9\pi}{64}\left(C^d_{ii} - C^d_{ij} z_i z_j\right),$$

→ Symmetric part of C^d_{ij}

$$\tilde{A} = \frac{9\pi}{64}\epsilon_{ijk} C^d_{ij} z_k,$$

→ Anti-symmetric part of C^d_{ij}

$$B = \frac{1}{4\pi}\left(C^q_{ij,kl} z_i z_j z_k z_l + 2C^q_{ij,ji} - 4C^q_{ij,ji} z_i z_i\right),$$

→ Symmetric part of $C^q_{ij,kl}$

$$\tilde{B} = \frac{1}{2\pi}\left(\epsilon_{ijk} C^q_{ij,ii} z_k - \epsilon_{ijk} C^q_{ii,ij} z_k\right).$$

→ Anti-symmetric part of $C^q_{ij,kl}$

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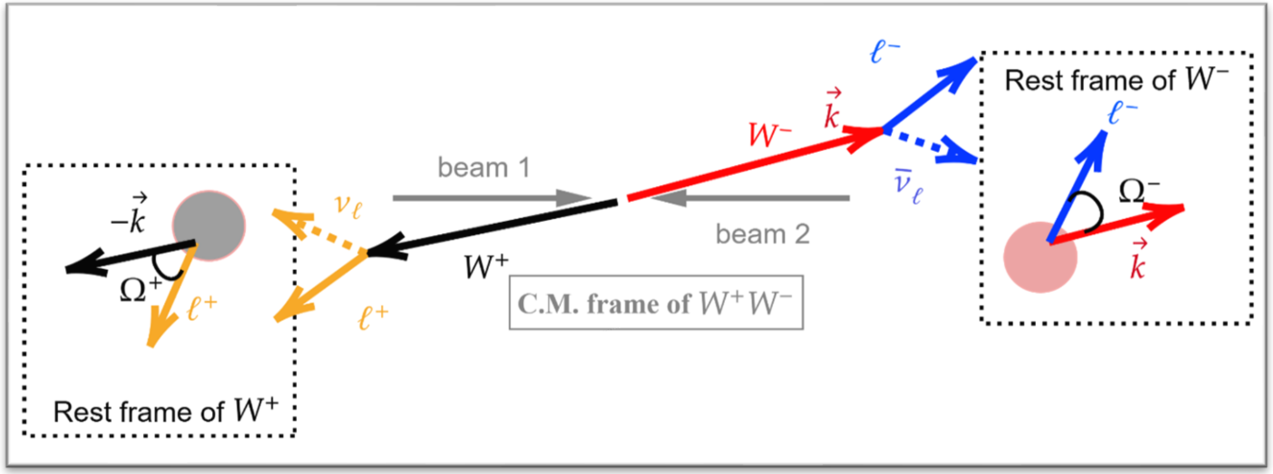
$$\frac{1}{\sigma}\frac{d\sigma}{d\Delta\phi}=\frac{1}{2\pi}\boxed{\begin{array}{c}+A\cos(\Delta\phi)\\ +B\cos(2\Delta\phi)\\ \\ \text{CP even}\end{array}}\ \boxed{\begin{array}{c}+\tilde{A}\sin(\Delta\phi)\\ +\tilde{B}\sin(2\Delta\phi)\\ \\ \text{CP odd}\end{array}}$$

In the rest frame of $W^{\pm}$

$$\ell^{\pm}:\ \mathbf{l}^{\pm}=\left(cos\phi^{\pm}sin\theta^{\pm},sin\phi^{\pm}sin\theta^{\pm},cos\theta^{\pm}\right)$$

$$\Delta\phi=\phi^{+}-\phi^{-}$$



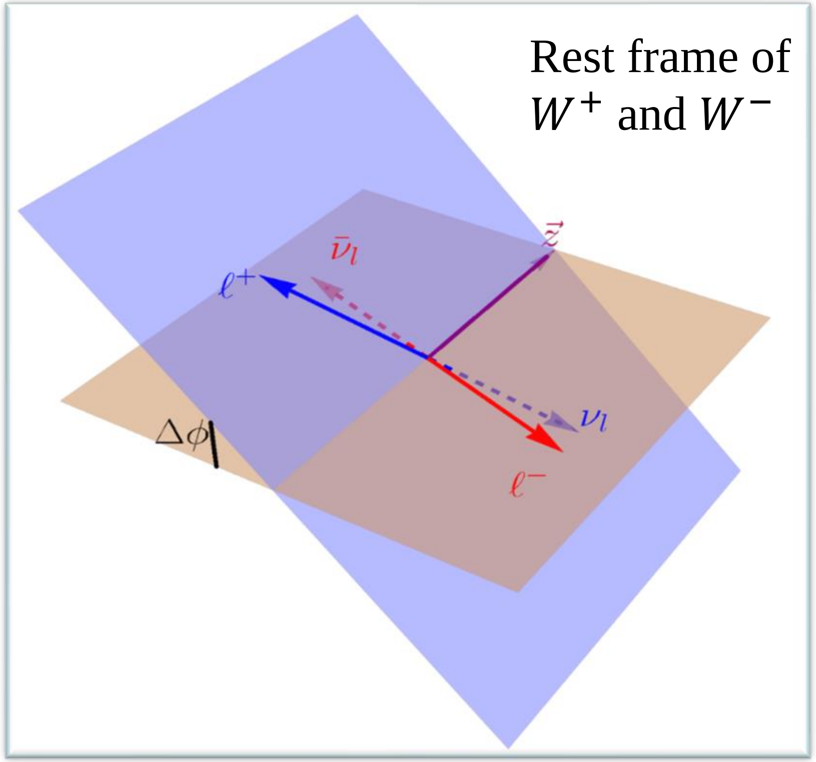
$\vec{z}$ is used to define $\Delta\phi$

$$\left\{\begin{aligned}
A &= -\frac{9\pi}{64}\left(C^{d}_{ii}-C^{d}_{ij}z_iz_j\right),\\
\tilde{A} &= \frac{9\pi}{64}\epsilon_{ijk}C^{d}_{ij}z_k,\\
B &= \frac{1}{4\pi}\left(C^{q}_{ij,kl}z_iz_jz_kz_l+2C^{q}_{ij,ji}-4C^{q}_{ij,ji}z_iz_i\right),\\
\tilde{B} &= \frac{1}{2\pi}\left(\epsilon_{ijk}C^{q}_{ij,ii}z_k-\epsilon_{ijk}C^{q}_{ii,ij}z_k\right).
\end{aligned}\right.$$

2. Azimuthal Angle Correlation

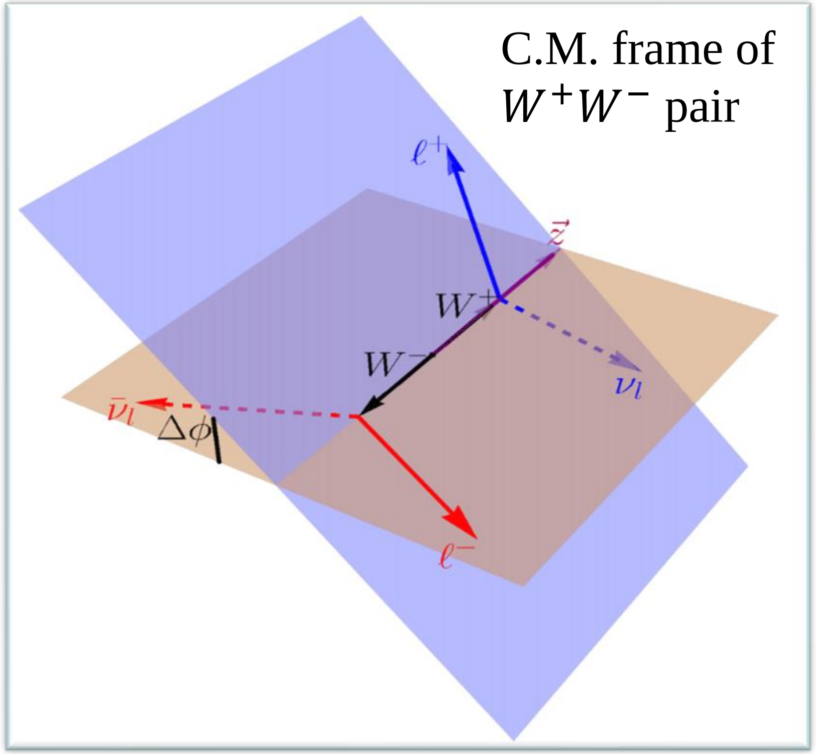
Spin correlation observable---the azimuthal angle difference:

- $\ell^{\pm}$: $\mathbf{l}^{\pm} = \left(cos\phi^{\pm}sin\theta^{\pm}, sin\phi^{\pm}sin\theta^{\pm}, cos\theta^{\pm}\right)$
- $\Delta\phi = \phi^{+} - \phi^{-}$ defined in the rest frame of $W^{\pm}$ respectively.
- A reference axis z is required to define the azimuthal angle $\phi^{\pm}$
- It is not associated with the selection of the xy axis.


Rest frame of W^{+} and W^{-}

Choosing $\vec{z}$ as the moving direction of W^{+} in W pair c.m. frame

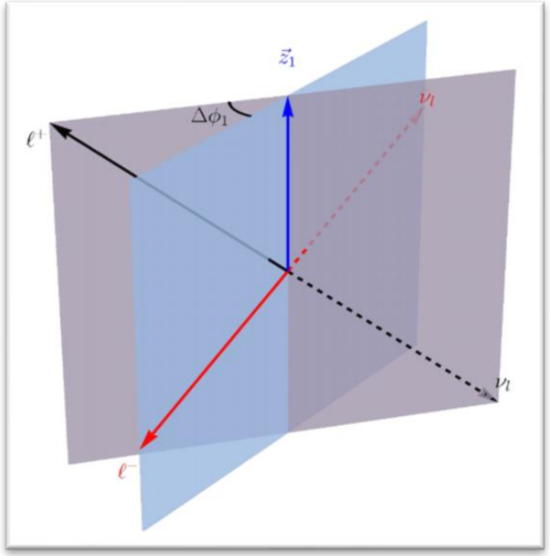
P. Achard et al. (L3), Eur. Phys. J. C 40, 333 (2005)


C.M. frame of $W^{+}W^{-}$ pair

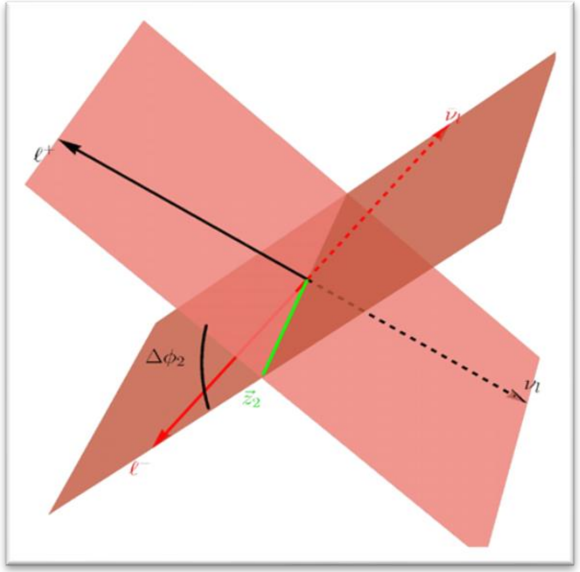
2. Azimuthal Angle Correlation

The decay plane correlation function for the dihedral angle

- In the rest frame of W^+ and W^-, We can choose any direction as the reference direction $\vec{z}$ to define the azimuthal angle.



In the rest frame of W^+ and W^-, change $\vec{z}$ direction
$\Delta\phi$ change!



- The azimuthal angle correlation is basis-dependent.
- $\frac{1}{\sigma}\frac{d\sigma}{d\Delta\phi} = \frac{1}{2\pi} + A\cos(\Delta\phi) + \tilde{A}\sin(\Delta\phi) + B\cos(2\Delta\phi) + \tilde{B}\sin(2\Delta\phi)$
- $A = -\frac{9\pi}{64}\left(C_{ii}^{d} - C_{ij}^{d} z_i z_j\right), \tilde{A} = \frac{9\pi}{64}\epsilon_{ijk} C_{ij}^{d} z_k, \ldots$

- We can choose a $\vec{z}$ direction to maximize azimuthal angle correlation.

3. Measuring Azimuthal Angle Distribution in SM

SM

- In SM (CP even) , azimuthal angle distribution is :

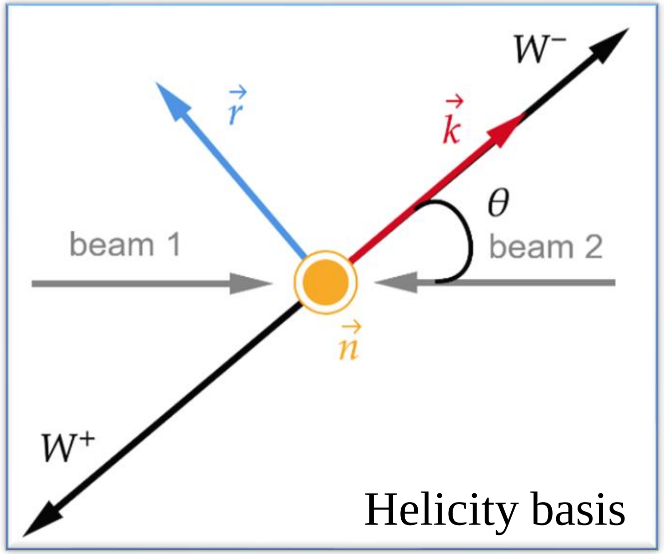
$$\frac{1}{\sigma}\frac{d\sigma}{d\Delta\phi} \approx \frac{1}{2\pi} + A\cos(\Delta\phi)$$

- Choose an optimal axis to maximize coefficient

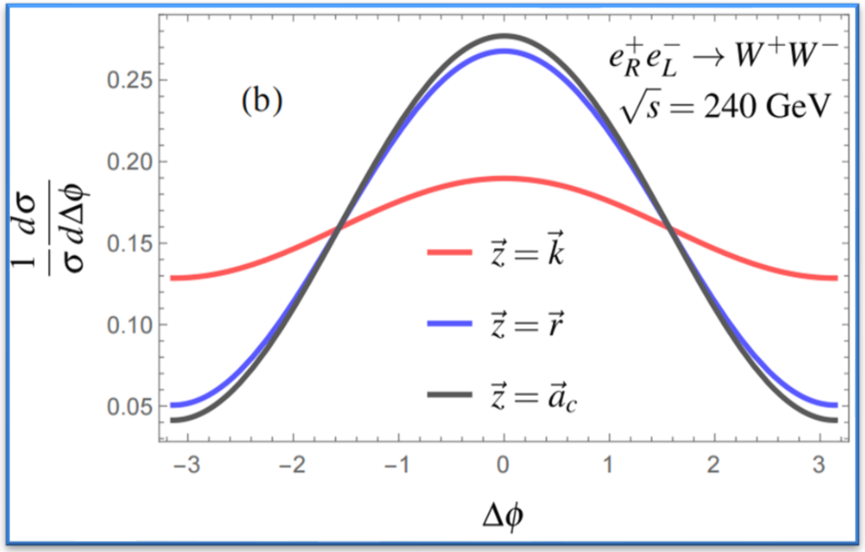
$$A = -\frac{9\pi}{64}\left(C^d_{ii} - C^d_{ij} z_i z_j\right),$$

$$\Rightarrow \quad |A| = \frac{9\pi}{64}\left|Tr(C^d - \vec{z}\cdot C^d \cdot \vec{z})\right|$$

- Maximized direction $\vec{a}_c$ is the eigenvector of C^d with the smallest eigenvalue



Helicity basis



- ✓ $\vec{a}_c$ yields a spin correlation nearly **4 times lager than** $\vec{k}$ direction.
- ✓ Maximal axis is closed to $\vec{r}$ direction.

3. CP Violating Effects

CP violating gauge interactions

- The observable $\Delta\phi$ is also sensitive to CP violating effect.

$$\boldsymbol{\Delta\phi} \xrightarrow{\textbf{CP}} -\boldsymbol{\Delta\phi}$$

- For the CP odd interaction , azimuthal angle distribution is :

$$\frac{1}{\sigma}\frac{d\sigma}{d\Delta\phi} \approx \frac{1}{2\pi} \underbrace{+ A\cos(\Delta\phi)}_{\text{SM}} \quad \underbrace{\boxed{+ \tilde{A}\sin(\Delta\phi)}}_{\text{Interference}}$$

- Asymmetry:

$$\epsilon \equiv \frac{\mathrm{N}(\Delta\phi > 0) - \mathrm{N}(\Delta\phi < 0)}{\mathrm{N}(\Delta\phi > 0) + \mathrm{N}(\Delta\phi < 0)} \quad \Longrightarrow \quad \boxed{\epsilon = 4\tilde{A}}$$

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Maximize $\tilde{A}$ coefficient to better measure the $sin(\Delta\phi)$ distribution caused by CP violating effects

$$|\tilde{A}| = \frac{9\pi}{64}\left|\epsilon_{ijk}C_{ij}^{d}z_k\right|$$

Maximized direction: $(\vec{a}_s)_k = \epsilon_{ijk}C_{ij}^{d}$

- Asymmetry:

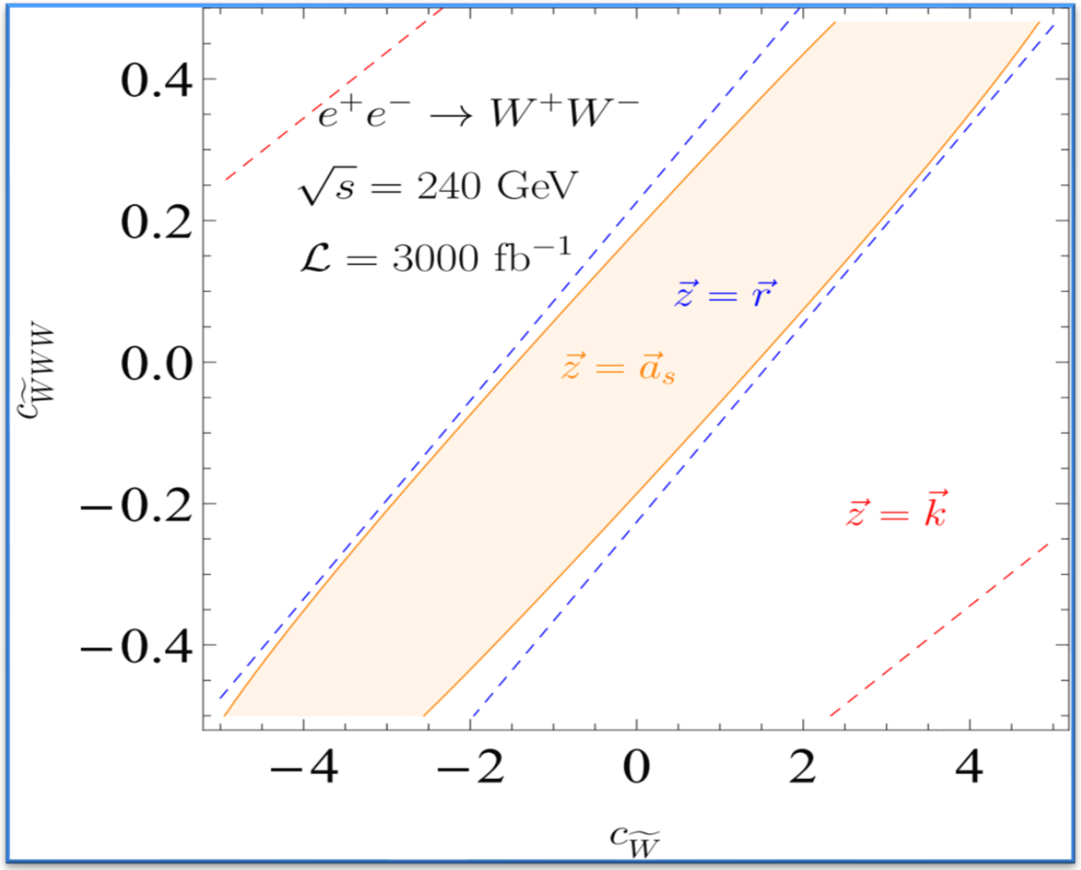
$$\epsilon \equiv \frac{\mathrm{N}(\Delta\phi > 0) - \mathrm{N}(\Delta\phi < 0)}{\mathrm{N}(\Delta\phi > 0) + \mathrm{N}(\Delta\phi < 0)}$$

$$\boxed{\epsilon = 4\tilde{A}}$$

3. CP Violating Effects

CP violating gauge interactions

The allowed parameter region at 2σ $(\Lambda = 1\text{TeV})$



- There are only two CP-odd dim-6 operators that modify electroweak vector boson self interactions:

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + c_{\widetilde{W}WW}\frac{\mathcal{O}_{\widetilde{W}WW}}{\Lambda^2} + c_{\widetilde{W}}\frac{\mathcal{O}_{\widetilde{W}}}{\Lambda^2} + \cdots$$

$$\mathcal{O}_{\widetilde{W}WW} = \mathbf{Tr}\left[\widetilde{W}_{\mu\nu}W^{\nu\rho}W_{\rho}^{\mu}\right],$$

$$\mathcal{O}_{\widetilde{W}} = \left(D_\mu\Phi\right)^\dagger \widetilde{W}^{\mu\nu}\left(D_\nu\Phi\right)$$

- $\sigma = \sigma_{SM} + \mathcal{O}(c_i^2)$,
- $\epsilon = \mathcal{O}(c_i)$, $c_i \in \{c_{\widetilde{W}}, c_{\widetilde{W}WW}\}$

✓ $\vec{z} = \vec{a}_s$ shows much stronger constraints on new physics, compared to $\vec{z} = \vec{k}$.

✓ The allowed parameter region at 2σ between $\vec{z} = \vec{r}$ and $\vec{z} = \vec{a}_s$ is very closed.

Summary

- Measuring the spin correlation of W pair is challenging, so optimization is important.
- The azimuthal angle correlation characterizes one entry from the spin correlation matrix.
- In our work we show that this correlation largely depend on the reference axis choice.
- We present the optimal basis to achieve the azimuthal angle correlation.
    - Better measurement of spin correlation of W boson pair.
    - Giving better constraints on CP violating interactions.

Thanks!

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2. Azimuthal Angle Correlation

Spin correlation :

- One can obtain the polarizations and spin correlations from the production spin density matrix as well as the joint angular distributions of the decay products

Production spin density matrix

$$\hat{\rho}_{WW} = \frac{1}{9}\mathbb{I}\otimes\mathbb{I} + \frac{1}{3}\sum_{i=1}^{8} a_i\lambda_i\otimes\mathbb{I} + \frac{1}{3}\sum_{j=1}^{8} b_j\mathbb{I}\otimes\lambda_j + \sum_{i=1}^{8}\sum_{j=1}^{8} c_{ij}\lambda_i\otimes\lambda_j$$

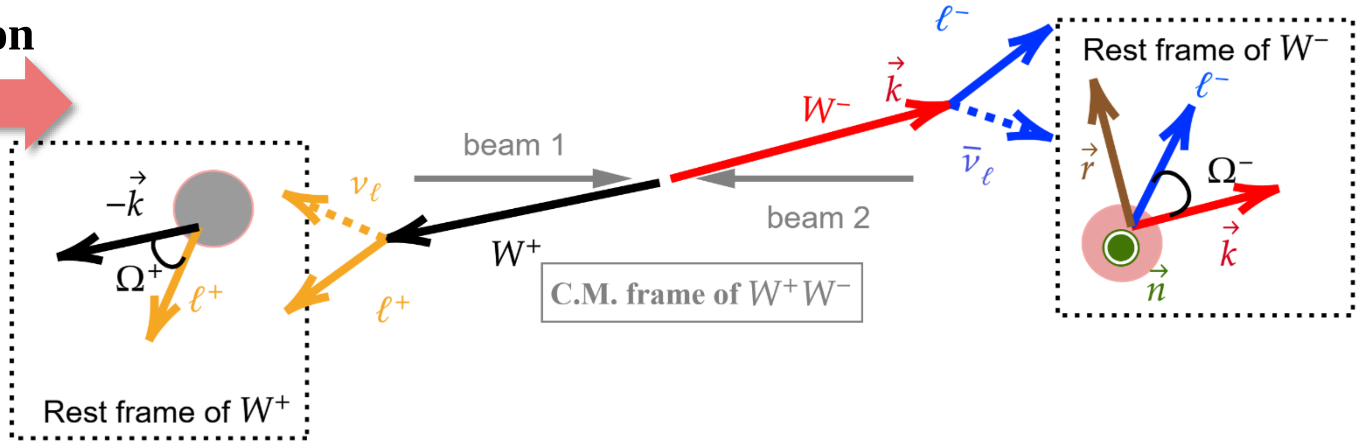
64 spin correlation coefficients of W pair

$$\hat{\rho}_{WW} = \frac{\hat{I}_9}{9} + \frac{1}{3}d_i^+\hat{S}_i^+\otimes\hat{I}_3 + \frac{1}{3}d_i^-\hat{I}_3\otimes\hat{S}_i^- + \frac{1}{3}q_{ij}^+\hat{S}_{\{ij\}}^+\otimes\hat{I}_3 + \frac{1}{3}q_{ij}^-\hat{I}_3\otimes\hat{S}_{\{ij\}}^- + C_{ij}^d\hat{S}_i^+\otimes\hat{S}_j^- + C_{ij,kl}^q\hat{S}_{\{ij\}}^+\otimes\hat{S}_{\{kl\}}^- + C_{ij,k}^{dq}\hat{S}_i^+\otimes\hat{S}_{\{jk\}}^- + C_{i,jk}^{qd}\hat{S}_{\{ij\}}^+\otimes\hat{S}_k^-$$

$$\hat{S}_{\{ij\}}^+ = \hat{S}_i\hat{S}_j + \hat{S}_j\hat{S}_i$$

SM prediction

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega^+ d\Omega^-} = \left(\frac{3}{4\pi}\right)^2\left[\frac{1}{9} + \frac{1}{3}\left(d_i^+ 1_i^+ - d_i^- 1_i^-\right) + \frac{1}{3}\left(q_{ij}^+ 1_i^+ 1_j^+ - q_{ij}^- 1_i^- 1_j^-\right) - C_{ij}^d 1_i^+ 1_j^- + C_{ij,kl}^q 1_i^+ 1_j^+ 1_k^- 1_l^- + C_{i,jk}^{dq} 1_i^+ 1_j^- 1_k^- + C_{ij,k}^{qd} 1_i^+ 1_j^+ 1_k^-\right]$$



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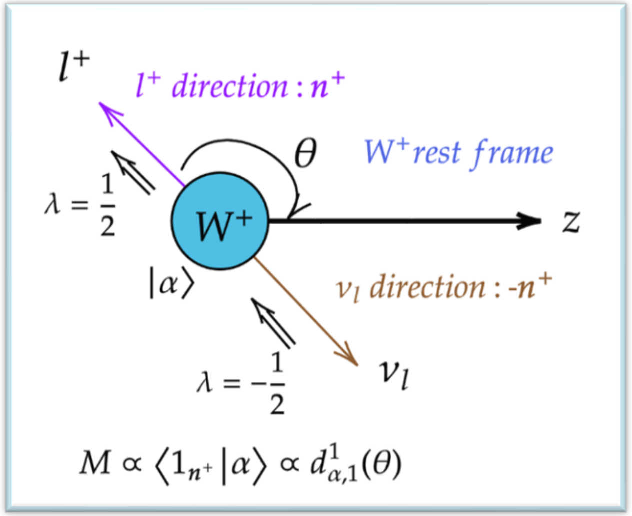
Decay Products as Spin Analyzer:

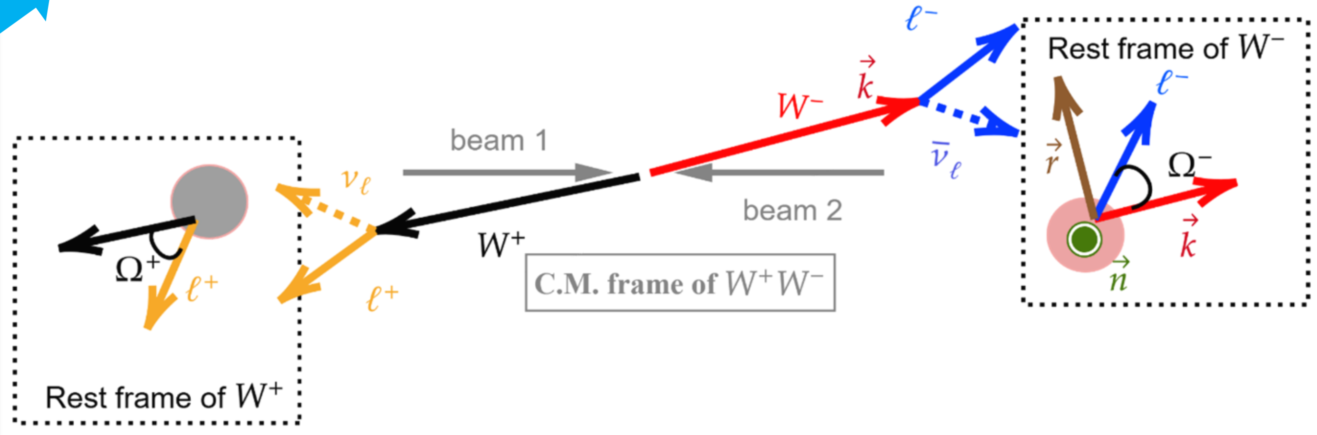
- Using decay angle distribution to recover the spin information of mother particle:



$$M \propto \langle 1_{n^+}|\alpha\rangle \propto d^1_{\alpha,1}(\theta)$$

Decay of a W boson is equivalent to a measurement of its spin along the axis of the emitted lepton.

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega^+ d\Omega^-} = \left(\frac{3}{4\pi}\right)^2 \left[\frac{1}{9} + \frac{1}{3}\left(d_i^+ 1_i^+ - d_i^- 1_i^-\right) + \frac{1}{3}\left(q_{ij}^+ 1_i^+ 1_j^+ - q_{ij}^- 1_i^- 1_j^-\right) - C_{ij}^d 1_i^+ 1_j^- + C_{ij,kl}^q 1_i^+ 1_j^+ 1_k^- 1_l^- + C_{i,jk}^{dq} 1_i^+ 1_j^- 1_k^- + C_{ij,k}^{qd} 1_i^+ 1_j^+ 1_k^-\right]$$



$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega^+ d\Omega^-} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}[\rho_{W^+W^-}(\Pi_+ \otimes \Pi_-)]$$

Backup Slides

Spin density matrix:

- At a give phase space point, the spin density matrix of $W^+ W^-$ produced from $e^+ e^-$ annihilation is：

$$\rho_{\alpha_1\alpha_2,\beta_1\beta_2} = \frac{R_{\alpha_1\alpha_2,\beta_1\beta_2}}{\mathrm{Tr}(R)}$$

- the unnormalized density matrix R is defined as:

$$R_{\alpha_1\alpha_2,\beta_1\beta_2} = \overline{\sum_{Spins}} M^*_{\alpha_1\,\beta_1} M_{\alpha_2\,\beta_2}$$

$$M_{\alpha\beta} \equiv \langle \mathrm{V}(k_1,\alpha)\overline{\mathrm{V}}(k_2,\beta)|\mathcal{T}|a(p_1)b(p_2)\rangle$$

- $\mathcal{T}$: transition matrix.
- $N_{a,b}$: the number of degrees of freedom of the respective initial state particle $e^+$and e^-,
- $p(k)$: the momentum of the initial(final) state,
- V is the W boson, and $\alpha(\beta)$is the spin indices of $W^-(W^+)$

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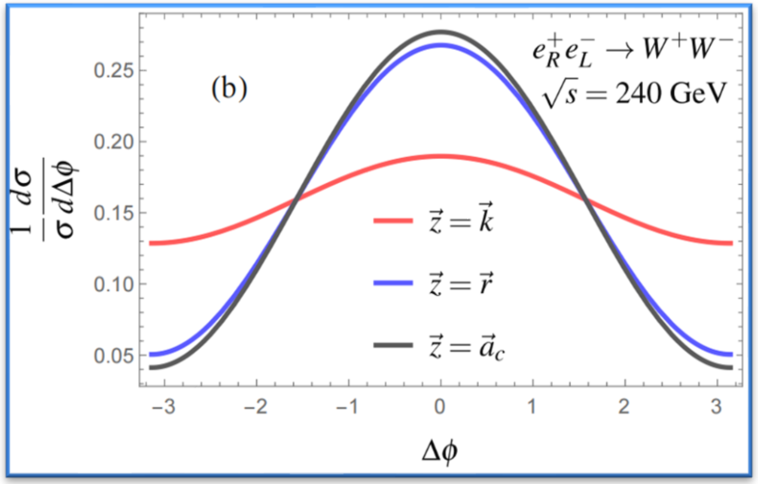
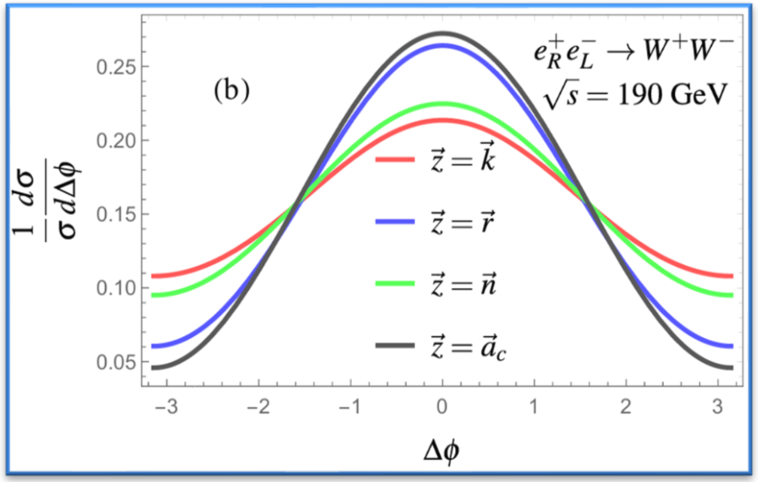
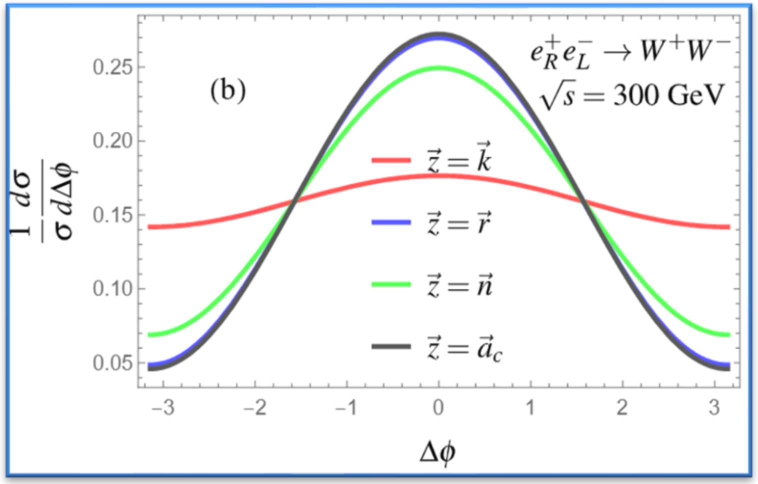
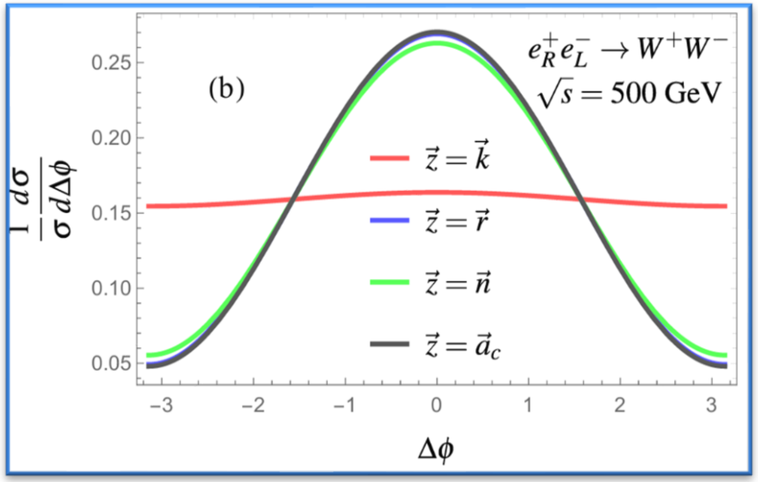
WW azimuthal angle correlation with different energy:









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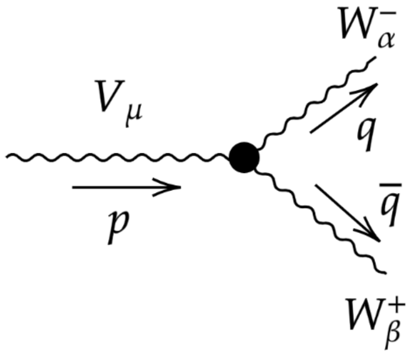
TGCs:



$$= -i\, g_{WWV}\, \Gamma_V^{\alpha\beta\mu}(q, \bar{q}, p)$$

$$\begin{aligned}\Gamma_V^{\alpha\beta\mu} \;=\; & f_1^V(q-\bar{q})^\mu g^{\alpha\beta} - \frac{f_2^V}{M_W^2}(q-\bar{q})^\mu P^\alpha P^\beta + f_3^V(P^\alpha g^{\mu\beta} - P^\beta g^{\mu\alpha}) \\ & + i f_4^V(P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha}) + i f_5^V \epsilon^{\mu\alpha\beta\rho}(q-\bar{q})_\rho \\ & - f_6^V \epsilon^{\mu\alpha\beta\rho} P_\rho - \frac{f_7^V}{m_W^2}(q-\bar{q})^\mu \epsilon^{\alpha\beta\rho\sigma} P_\rho (q-\bar{q})_\sigma\end{aligned}$$

$$\begin{aligned}f_1^\gamma &= 1 + c_{WWW}\frac{3g^2P^2}{4\Lambda^2} \\ f_1^Z &= 1 + c_W\frac{m_Z^2}{\Lambda^2} - c_{WWW}\frac{3g^2P^2}{4\Lambda^2} \\ f_2^\gamma &= f_2^Z = c_{WWW}\frac{3g^2m_W^2}{2\Lambda^2} \\ f_3^\gamma &= 2 + (c_B + c_W)\frac{m_W^2}{2\Lambda^2} + c_{WWW}\frac{3g^2m_W^2}{2\Lambda^2} \\ f_3^Z &= 2 + \left(c_W(1+\cos^2\theta_W) - c_B\sin^2\theta_W\right)\frac{m_Z^2}{2\Lambda^2} + c_{WWW}\frac{3g^2m_W^2}{2\Lambda^2} \\ f_4^V &= f_5^V = 0\end{aligned}$$

$$\boxed{\begin{aligned}f_6^\gamma &= +c_{\tilde{W}}\frac{m_W^2}{2\Lambda^2} - c_{\tilde{W}WW}\frac{3g^2m_W^2}{2\Lambda^2} \\ f_6^Z &= -c_{\tilde{W}}\tan^2\theta_W\frac{m_W^2}{2\Lambda^2} - c_{\tilde{W}WW}\frac{3g^2m_W^2}{2\Lambda^2} \\ f_7^\gamma &= f_7^Z = -c_{\tilde{W}WW}\frac{3g^2m_W^2}{4\Lambda^2}\end{aligned}}$$