Subtraction of the $t\bar{t}$ contribution in $tW\bar{b}$ production at one-loop level

[LD, H. T. Li, Z.-Y. Li, J. Wang arXiv: 2411.07455]

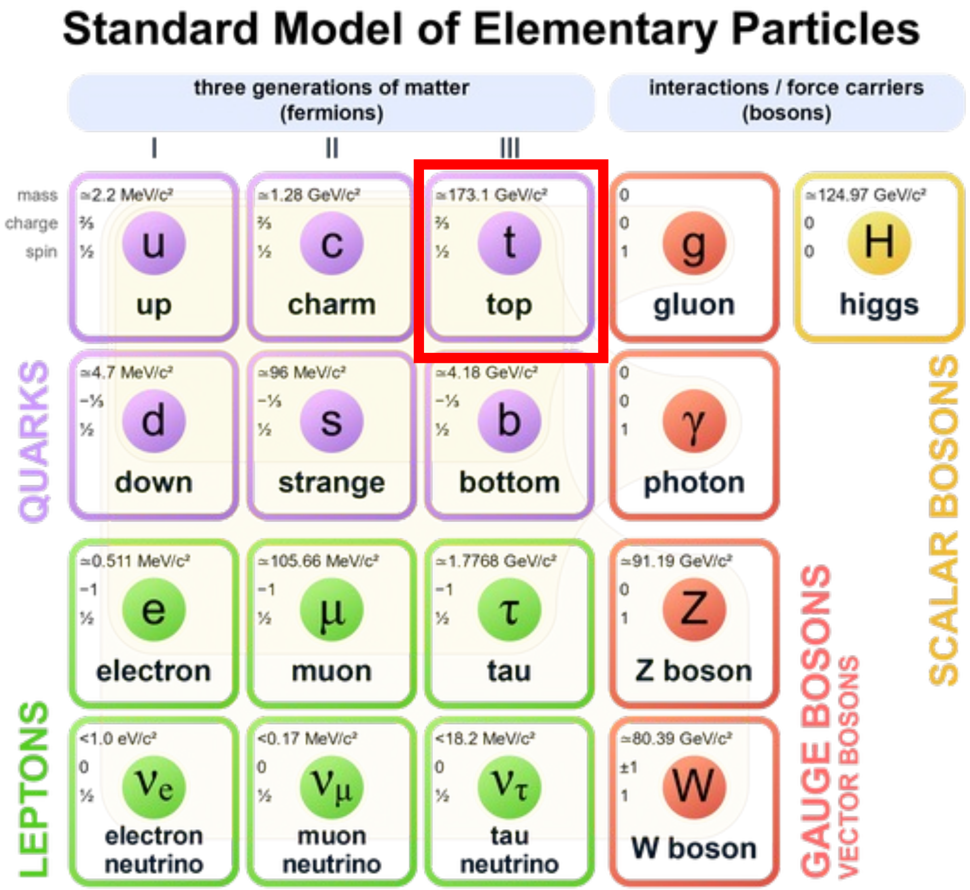
董亮

山东大学

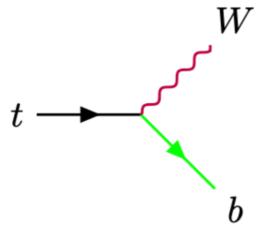
第四届量子场论及其应用研讨会

2024年11月16日至20日，广州

Introduction to tW



Top quark is the heaviest elementary particle in the Standard Model (SM). It decays to Wb before hadronization.



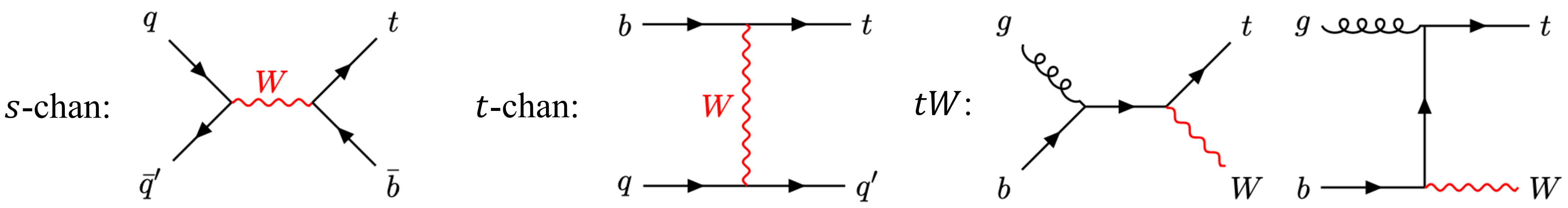
N3LO QCD decay width is about 1.321GeV.

[L. B. Chen, H. T. Li, Z. Li, J. Wang, Y. Wang, Q. Wu 2309.00762]

[L. Chen, X. Chen, X. Guan, Y.-Q. Ma 2309.01937]

Introduction to tW

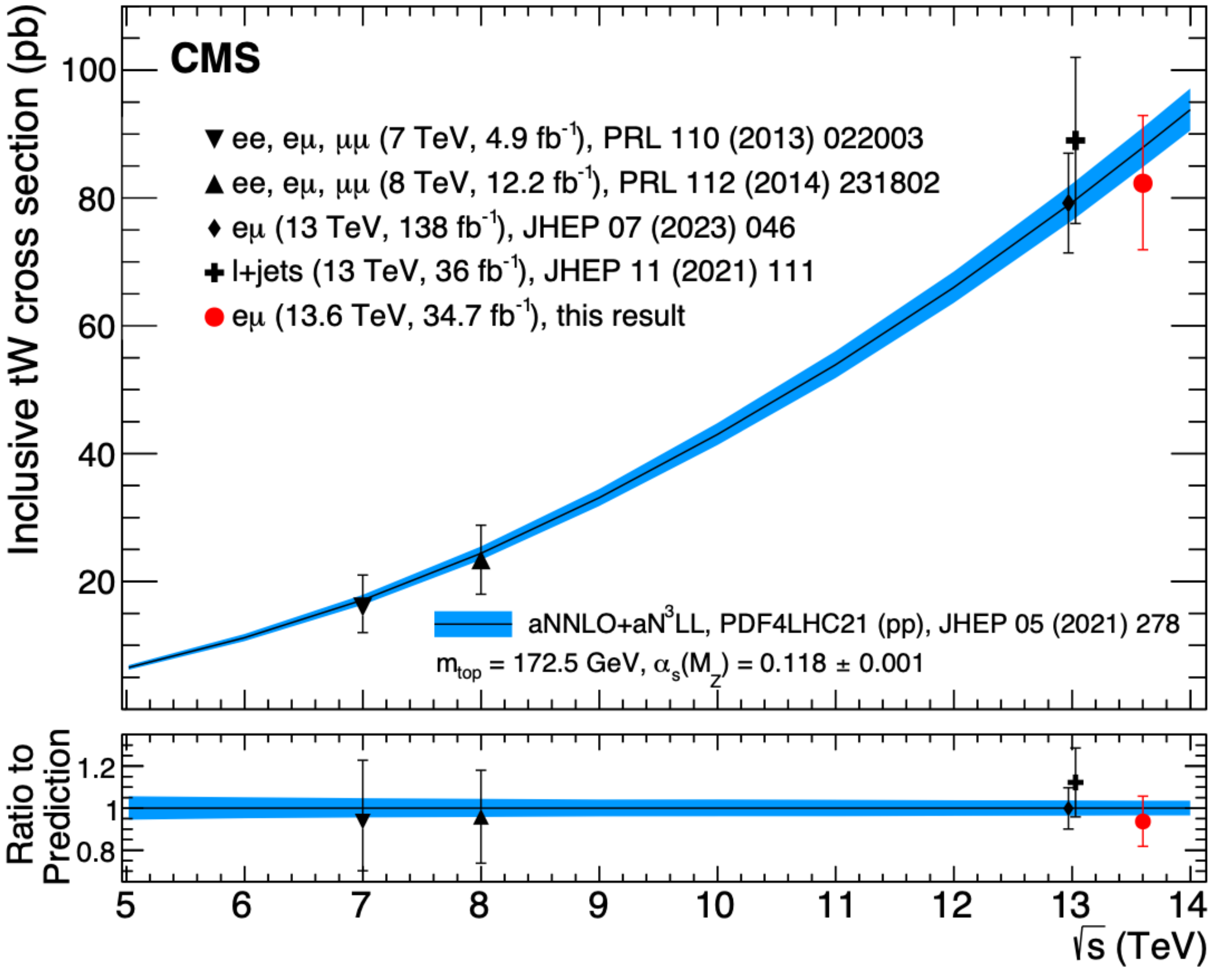
Single top quark production. Three modes: s-channel, t-channel and tW associated production.



At the LHC, production cross section at 13 TeV : $\sigma_t(\sim 130\text{ pb}) > \sigma_{tW}(\sim 79\text{ pb}) > \sigma_s(\sim 8\text{ pb})$.

[CMS 1812.10514] [CMS 2208.00924] [CMS 2209.08990]

Measurements and predictions of tW



[CMS 2409.06444]

Agreement with the SM prediction.

The complete NNLO QCD correction for tW production is not available!

- NNLO N-jettiness soft function

  [H. T. Li, J. Wang 1611.02749]
  [H. T. Li, J. Wang 1804.06358]

- squared one-loop and two-loop QCD amplitudes

  [L.-B. Chen, LD, H. T. Li, Z. Li, J. Wang, Y. Wang 2204.13500]
  [L.-B. Chen, LD, H. T. Li, Z. Li, J. Wang, Y. Wang 2208.08786]
  [L.-B. Chen, LD, H. T. Li, Z. Li, J. Wang, Y. Wang 2212.07190]

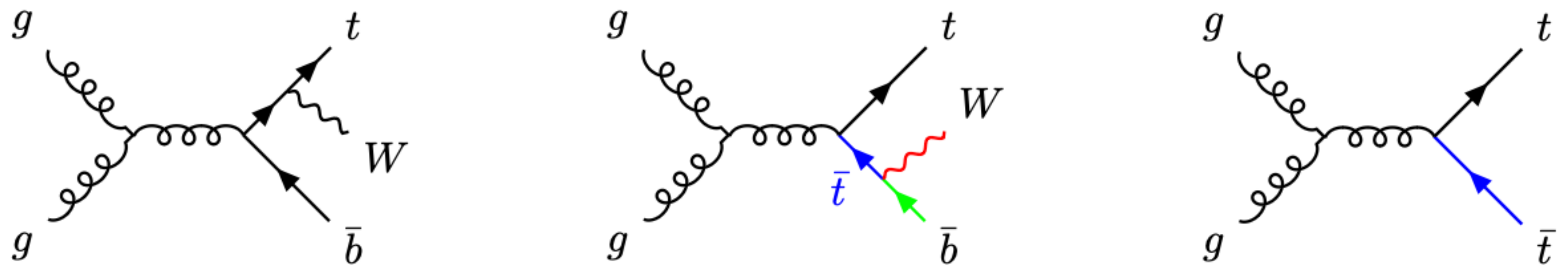
- virtual-real correction?

  Difficulty:

  The tW process *interferes* with the $t\bar{t}$ process.

Interferences between tW and $t\bar{t}$

The interference starts at the NLO real correction of tW:



no on-shell $\bar{t}$ potentially on-shell $\bar{t}$ exactly on-shell $\bar{t}$ in $t\bar{t}$ production

The resonant divergence emerges:

$$m_{W\bar{b}} \to m_t \quad \Longrightarrow \quad \frac{1}{m_{W\bar{b}}^2 - m_t^2} \to \infty$$

The resonant divergence comes from on-shell anti-top. To extract the events for tW production, a subtraction of the resonance contribution is required.

Resonance subtraction at tree level

SHANDONG UNIVERSITY

Leading order squared amplitude of $A(p_1) + B(p_2) \to \bar{b}(p_3) + W(p_4) + t(p_5)$:

$$|\mathcal{M}_{tW\bar{b}}|^2_{\mathrm{LO}} = |\mathcal{M}^{(0)}_{1t} + \mathcal{M}^{(0)}_{2t}|^2 = \underbrace{|\mathcal{M}^{(0)}_{1t}|^2}_{\text{non-resonance, } tW\bar{b}} + \underbrace{2\,\mathbf{Re}[\mathcal{M}^{(0)}_{1t}\mathcal{M}^{(0)*}_{2t}]}_{\text{interference}} + \underbrace{|\mathcal{M}^{(0)}_{2t}|^2}_{\text{resonance, } t\bar{t}(\to W\bar{b})}$$

diagram removal (DR) schemes : Remove all the contribution from resonant diagrams,

[S. Frixione, E. Laenen, P. Motylinski, B. Webber, C. D. White 0805.3067]

$$\left(|\mathcal{M}_{tW\bar{b}}|^2_{\mathrm{LO}}\right)_{\mathrm{DR1}} = |\mathcal{M}^{(0)}_{1t}|^2$$

gauge-dependent

or reserve the interference term,

[W. Hollik, J. M. Lindert, D. Pagani 1207.1071]

$$\left(|\mathcal{M}_{tW\bar{b}}|^2_{\mathrm{LO}}\right)_{\mathrm{DR2}} = |\mathcal{M}^{(0)}_{1t}|^2 + 2\,\mathbf{Re}[\mathcal{M}^{(0)}_{1t}\mathcal{M}^{(0)*}_{2t}]$$

Resonance subtraction at tree level

Subtraction scheme: Construct a resonance subtraction term

gauge invariant

$$\left(|\mathcal{M}_{tW\bar{b}}|^2_{\mathrm{LO}}\right)_{\mathrm{Sub}} = |\mathcal{M}^{(0)}_{1t} + \mathcal{M}^{(0)}_{2t}|^2 - \mathcal{R}_{\mathrm{LO}}.$$

$\mathcal{R}_{LO}$: Cancel the value of $\left|\mathcal{M}^{(0)}_{1t} + \mathcal{M}^{(0)}_{2t}\right|^2$ near the on-shell region and rapidly decline in the off-shell region.

For calculating $\mathcal{R}_{LO}$, we should use the kinematics for $t\bar{t}$ to preserve gauge invariance and multiply a pre-factor to suppress the contribution in the off-shell region.

$$\mathcal{R}_{\mathrm{LO}} = S(\{p_i\}, \{\tilde{p}_i\}) \cdot R_{\mathrm{LO}}(\{\tilde{p}_i\})$$

$\{p_i\}$ is the momentum set for $tW\bar{b}$ production, while $\{\tilde{p}_i\}$ is that for $t\bar{t}$ process, obtained by reshuffling $\{p_i\}$ and satisfying $(\tilde{p}_W + \tilde{p}_{\bar{b}})^2 = m_t^2$.

Resonance subtraction at tree level

diagram subtraction (DS) scheme

[S. Frixione, E. Laenen, P. Motylinski, B. Webber, C. D. White 0805.3067]

$$S_1 = \frac{(m_t\Gamma_t)^2}{\Delta^2 + (m_t\Gamma_t)^2} \qquad (R_{\rm LO})_1 = \widetilde{|\mathcal{M}_{2t}^{(0)}|^2} \equiv |\mathcal{M}_{2t}^{(0)}|^2\Big|_{\{p_i\}\to\{\tilde{p}_i\}}$$

$$(|\mathcal{M}_{tW\bar{b}}|^2_{\rm LO})_{\rm DS} = \Big[|\mathcal{M}_{1t}^{(0)} + \mathcal{M}_{2t}^{(0)}|^2 - S_1\cdot(R_{\rm LO})_1\Big]_{\rm Reg}$$

"Reg" represents the replacement

$$\frac{1}{p_{Wb}^2 - m_t^2} \to \frac{1}{p_{Wb}^2 - m_t^2 + i m_t\Gamma_t}$$

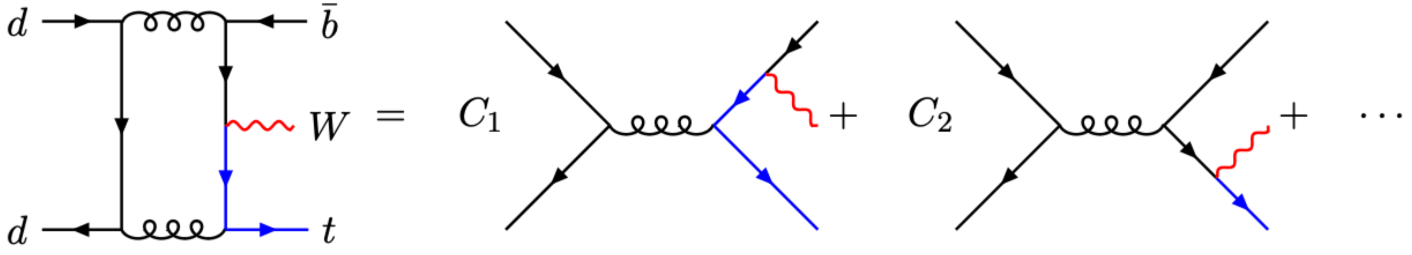
to regulate the divergences.

Resonance subtraction at tree level

SHANDONG UNIVERSITY

However, the DR and DS schemes can't be generalized to higher orders.

One can't distinguish non-resonant and resonant diagrams:



A new subtraction scheme at one-loop level is desired.

power subtraction (PS) scheme:

Observation:

resonance contributions in $|\mathcal{M}|^2 \sim 1/\Delta^2$ $\qquad\qquad$ $\Delta \equiv m_{W\bar{b}}^2 - m_t^2$

Idea:

Expand $|\mathcal{M}|^2$ (rather than diagrams) at $\Delta \to 0$, extract and then subtract the most singular term, $1/\Delta^2$ term.

Resonance subtraction at tree level

Expand the squared amplitude around $\Delta = 0$

$$|\mathcal{M}_{tW\bar{b}}|^2_{\rm LO} = \frac{B^{(2)}}{\Delta^2} + \frac{B^{(1)}}{\Delta} + B^{(0)} + \cdots .$$

Thus, the resonance subtraction term is

$$\mathcal{R}_{\rm LO} = S_2 \cdot R_{\rm LO} = \frac{\widetilde{\Delta}^2}{\Delta^2} \cdot \frac{\widetilde{B^{(2)}}}{\widetilde{\Delta}^2} = \frac{\widetilde{B^{(2)}}}{\Delta^2} .$$

The new suppression factor $S_2 = \widetilde{\Delta}^2/\Delta^2$ is introduced to cancel the divergence $1/\widetilde{\Delta}^2$.

Therefore, squared amplitude:

$$\left(|\mathcal{M}_{tW\bar{b}}|^2_{\rm LO}\right)_{\rm PS} = |\mathcal{M}^{(0)}_{1t} + \mathcal{M}^{(0)}_{2t}|^2 - \frac{\widetilde{B^{(2)}}}{\Delta^2} . \tag{4.1}$$

Resonance subtraction at one-loop level

SHANDONG UNIVERSITY

Expand the squared amplitude around $\Delta = 0$

$$V_{\rm Ren} \equiv 2\,\mathbf{Re}[\mathcal{M}^{(0)*}\mathcal{M}^{(1)}_{\rm Ren}] = \frac{C^{(2)}}{\Delta^2} + \frac{C^{(1)}}{\Delta} + C^{(0)} + \cdots$$

Expansion coefficients, $C^{(i)}$, have IR divergences. How to cancel? Dipole subtraction formalism.

[S. Catani, M. H. Seymour 9605323]

$V_{\rm Ren} + \mathcal{I}_{\rm NLO}$ is free of IR divergences.

[S. Catani, S. Dittmaier, M. H. Seymour, Zoltan Trocsanyi 0201036]

Then similar expansion of the integrated dipole term:

$$\mathcal{I}_{\rm NLO} = \frac{I^{(2)}}{\Delta^2} + \frac{I^{(1)}}{\Delta} + I^{(0)} + \cdots$$

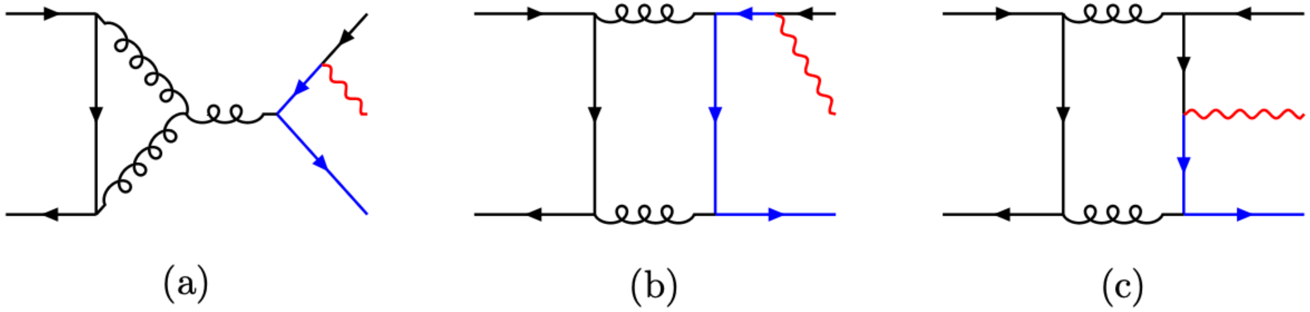
Therefore, squared amplitude:

$$\left(|\mathcal{M}_{tW\bar{b}}|^2_{\rm V+I}\right)_{\rm PS} = V_{\rm Ren} + \mathcal{I}_{\rm NLO} - \frac{\widetilde{C^{(2)}} + \widetilde{I^{(2)}}}{\Delta^2}. \qquad (4.2)$$

Resonance subtraction at one-loop level

SHANDONG UNIVERSITY

One-loop diagrams with a resonance:



The $\bar{t}$ propagator existing inside the loop will result in $\log\Delta$, hence we have

$$\widetilde{C^{(2)}} = C^{(2)}_{\text{no-log}}|_{\{p_i\}\to\{\tilde{p}_i\}} + \log\Delta \times C^{(2)}_{\text{log}}|_{\{p_i\}\to\{\tilde{p}_i\}}.$$

An example: pp $\rightarrow d\bar{d} \rightarrow \bar{b}Wt$

Input:

$V_{tb} = 1$

$\alpha = 1/132.16656$

$m_W = 80.377\text{GeV}$

$\sin^2\theta_W = 0.22305189$

PDF: CT18LO, CT18NLO

$m_t = 172.5\text{GeV}$

$\Gamma_t = 1.3\text{GeV}$

$\mu = m_t$

On-shell renormalization: masses, fields

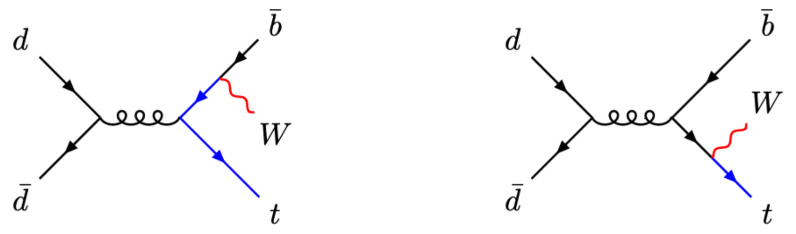
The full amplitude is produced by **OpenLoops**.

PS scheme: $|m_{W\bar{b}} - m_t|/m_t > \delta$ for numerical stability

$d(p_1)\bar{d}(p_2) \to \bar{b}(p_3)W(p_4)t(p_5)$

SHANDONG UNIVERSITY

Born cross section:



- Subtraction term

$$\frac{\widetilde{B^{(2)}}}{\Delta^2} = \frac{16\pi^3\alpha\alpha_s^2 C_A C_F}{9m_W^2\tilde{s}_{12}^2\sin^2\theta_W\Delta^2}(m_t^2m_W^2 + m_t^4 - 2m_W^4)$$
$$\left[2m_t^2\tilde{s}_{12} - 2m_W^2(\tilde{s}_{12} + 2\tilde{s}_{23} + 2\tilde{s}_{24}) + 2m_W^4 + \tilde{s}_{12}^2 + 2(\tilde{s}_{23} + \tilde{s}_{24})(\tilde{s}_{12} + \tilde{s}_{23} + \tilde{s}_{24})\right]$$

with $\tilde{s}_{ij} = (\tilde{p}_i + \sigma_{ij}\tilde{p}_j)^2$.

- Numerical results

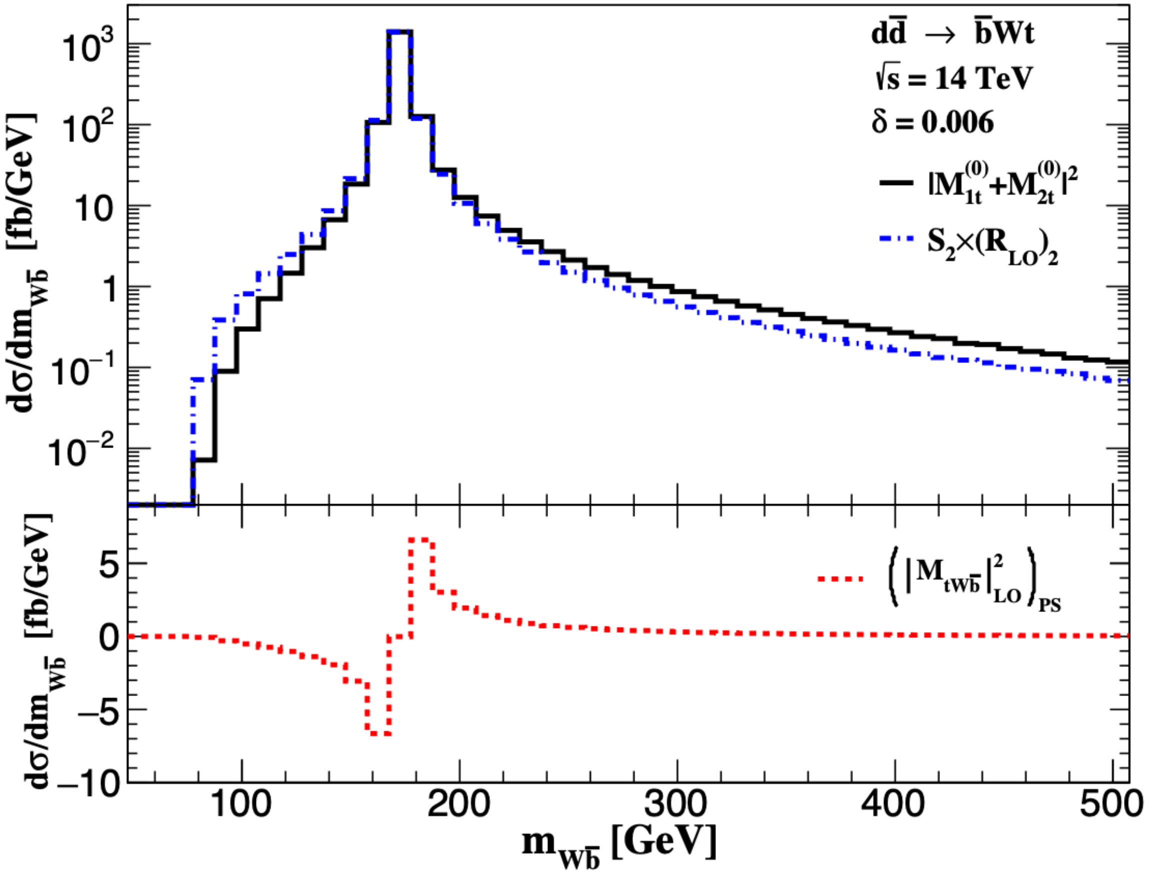
Schemes	DR1	DR2	DS	PS
$\sigma_{\rm LO}$(fb)	63.56(1)	−45.03(1)	56.7(2)	56.1(1)

✓ DR1, DS and PS have similar values.
✓ The interference is negative and relatively large.
✓ $(\sigma_{\rm LO})_{\rm PS}$ is stable in the interval for $10^{-2} < \delta < 10^{-6}$.

$d(p_1)\bar{d}(p_2) \to \bar{b}(p_3)W(p_4)t(p_5)$

Born cross section:

- $m_{W\bar{b}}$ distribution



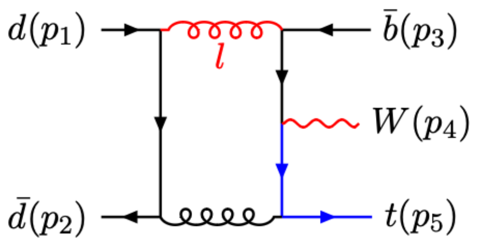
- Resonance is subtracted in the bin of 167.5 GeV $< m_{W\bar{b}} <$ 177.5 GeV (5 orders of magnitude).
- The resonance-subtracted distribution has an explicit form of $1/\Delta$.
- The 11 bins around the resonance peak, 117.5 GeV $< m_{W\bar{b}} <$ 227.5 GeV, contribute less than 1% .

$d(p_1)\bar{d}(p_2) \to \bar{b}(p_3)W(p_4)t(p_5)$

SHANDONG UNIVERSITY

One-loop correction:

- Extract the $1/\Delta^2$ term
  - ➢ Expand the full amplitude



A pentagon can be reduced to a linear combination of five box integrals:

[Z. Bern, L. Dixon, D. A. Kosower 9306240]

$$I_5 = \sum_{i=1}^{5} c_i I_4^{(i)} + \mathcal{O}(\epsilon),$$

Only the following integral contains $\mathcal{O}(1/\Delta)$ contribution,

$$\int d^D l \frac{1}{(l^2 + i0)[(l - p_3)^2 + i0][(l - p_3 - p_4)^2 - m_t^2 + i0][(l - p_1)^2 + i0]}$$

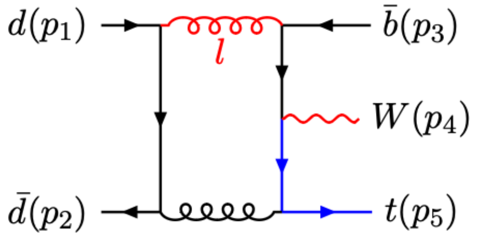
with $D = 4 - 2\varepsilon$.

$d(p_1)\bar{d}(p_2) \to \bar{b}(p_3)W(p_4)t(p_5)$

One-loop correction:

- Extract the $1/\Delta^2$ term
  - Soft gluon approximation

[M. Beneke, A. P. Chapovsky, A. Signer, G. Zanderighi 0312331]



$$\mathcal{M}_{l\to 0} = -\frac{\alpha_s}{2\pi}(2\,p_1\cdot p_3)\Delta\,\underbrace{\mathbf{T}_1\cdot\mathbf{T}_3}_{\text{color operators}}\,\underbrace{|a_1,a_2,a_3,a_5\rangle}_{\text{LO amplitude}}\,I_s$$

[S. Catani, M. H. Seymour 9605323]

$$I_s = \frac{\mu^{2\epsilon}}{i\pi^{2-\epsilon}r_\Gamma}\int d^D l\,\frac{1}{(l^2+i0)(-2l\cdot p_3+i0)(-2l\cdot p_{34}+\Delta+i0)(-2l\cdot p_1+i0)}$$

$$l\sim m_t(\lambda^2,\lambda^2,\lambda^2),\quad \lambda^2\sim\Delta/m_t^2$$

$d(p_1)\bar{d}(p_2) \to \bar{b}(p_3)W(p_4)t(p_5)$

SHANDONG UNIVERSITY

Subtraction term:

- Infrared divergences

$$
\begin{aligned}
C^{(2)} =& \frac{\alpha_s}{2\pi} B^{(2)} \Bigg\{ -\frac{3C_F}{\epsilon^2} + \frac{1}{\epsilon}\Bigg[(C_A - 2C_F)\log\left(\frac{\mu^2}{s_{12}}\right) - 2(C_A - 2C_F)\log\left(\frac{\mu^2}{-s_{13}}\right) \\
& + (C_A - 4C_F)\log\left(\frac{\mu^2}{-s_{23}}\right) + (C_A - 4C_F)\log\left(\frac{m_t\mu}{-s_{15}+m_t^2}\right) \\
& - 2(C_A - 2C_F)\log\left(\frac{m_t\mu}{-s_{25}+m_t^2}\right) + (C_A - 2C_F)\log\left(\frac{m_t\mu}{s_{35}-m_t^2}\right) - \frac{11}{2}C_F \Bigg]\Bigg\} \\
& + \cdots,
\end{aligned}
$$

No $\log\Delta$ due to color conservation and soft-collinear divergences:

$$
\begin{aligned}
& \frac{\alpha_s}{2\pi\epsilon^2}\left(\frac{-\Delta - i0}{\mu m_t}\right)^{-2\epsilon}\left[2\mathbf{T}_1\cdot\mathbf{T}_3 + 2\mathbf{T}_2\cdot\mathbf{T}_3 + \mathbf{T}_3\cdot\mathbf{T}_5 - \mathbf{T}_1\cdot\mathbf{T}_{\bar{t},f} - \mathbf{T}_2\cdot\mathbf{T}_{\bar{t},f} - \mathbf{T}_3\cdot\mathbf{T}_{\bar{t},i}\right] \\
=& \frac{\alpha_s}{2\pi\epsilon^2}\left(\frac{-\Delta - i0}{\mu m_t}\right)^{-2\epsilon}\left[\mathbf{T}_1\cdot\mathbf{T}_3 + \mathbf{T}_2\cdot\mathbf{T}_3 + \mathbf{T}_3\cdot\mathbf{T}_5 + \mathbf{T}_3\cdot\mathbf{T}_{\bar{t},f}\right] = 0
\end{aligned}
$$

$d(p_1)\bar{d}(p_2) \to \bar{b}(p_3)W(p_4)t(p_5)$

SHANDONG UNIVERSITY

Subtraction term:

- $\log\Delta$ in finite term

$$2\log\left(\frac{m_t\mu}{|\Delta|}\right)\cdot\frac{\alpha_s}{2\pi}B^{(2)}\Bigg[(C_A-2C_F)\ \left(\log\left(\frac{m_t^2}{s_{35}-m_t^2}\right)-\frac{1+\beta_t^2}{2\beta_t}\log\left(\frac{1-\beta_t}{1+\beta_t}\right)\right)$$
$$-2(C_A-2C_F)\left(\log\left(\frac{m_t^2}{-s_{13}}\right)-\log\left(\frac{m_t^2}{2p_1\cdot p_{\bar{t}}}\right)\right)$$
$$+(C_A-4C_F)\left(\log\left(\frac{m_t^2}{-s_{23}}\right)-\log\left(\frac{m_t^2}{2p_2\cdot p_{\bar{t}}}\right)\right)$$
$$+2C_F\log\left(\frac{m_t^2}{2p_{\bar{t}}\cdot p_3}\right)+2C_F\Bigg]$$

This term comes from the difference for IR divergences of off-shell and on-shell anti-top.

$$I^{\rm div}_{d\bar{d}\to\bar{b}Wt}=\frac{d_2}{\epsilon^2}+\frac{d_1}{\epsilon}+\frac{d_s}{\epsilon}\left(\frac{-\Delta-i0}{\mu\,m_t}\right)^{-2\epsilon}=\frac{d_2}{\epsilon^2}+\frac{d_1+d_s}{\epsilon}-2d_s\log\left(\frac{-\Delta-i0}{\mu\,m_t}\right)$$

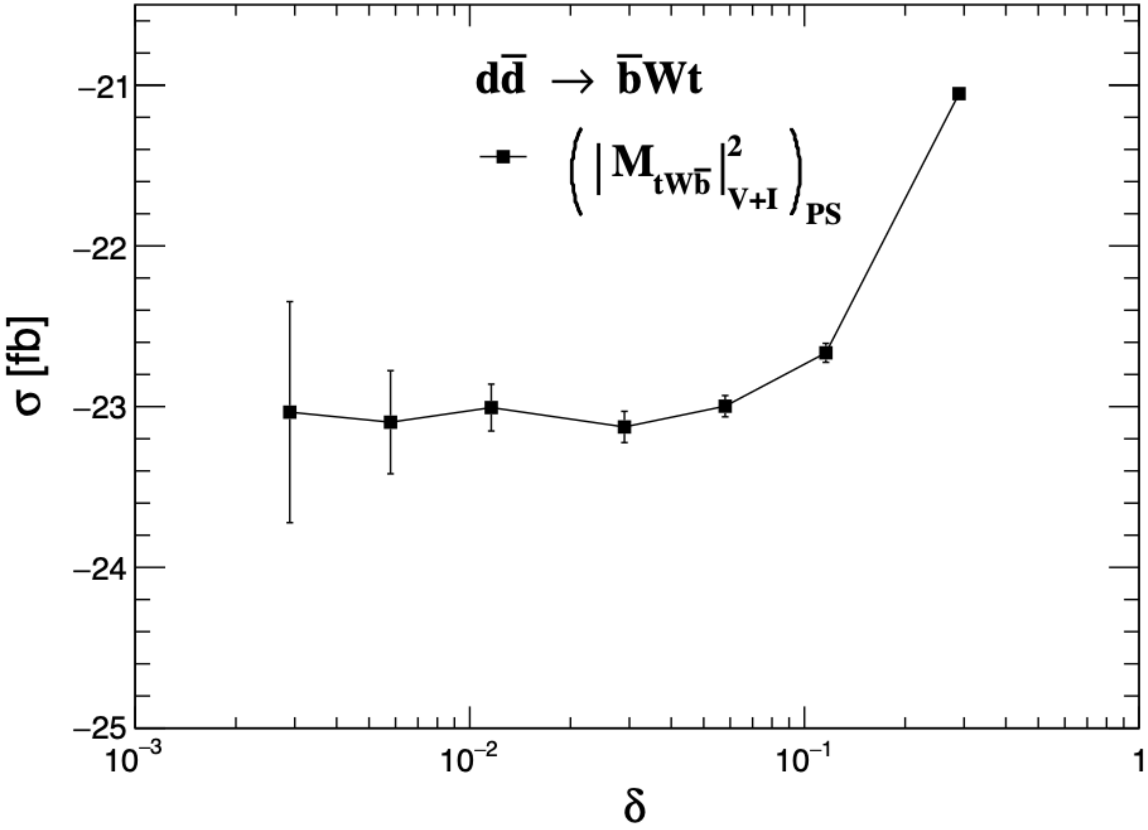
$$I^{\rm div}_{d\bar{d}\to\bar{t}(\to\bar{b}W)t}=\frac{d_2}{\epsilon^2}+\frac{d_1}{\epsilon}$$

$d(p_1)\bar{d}(p_2) \to \bar{b}(p_3)W(p_4)t(p_5)$

SHANDONG UNIVERSITY

Subtracted one-loop correction:

- Cross section
  - Different values of δ



  - Compare with LO

$\delta = 0.006$

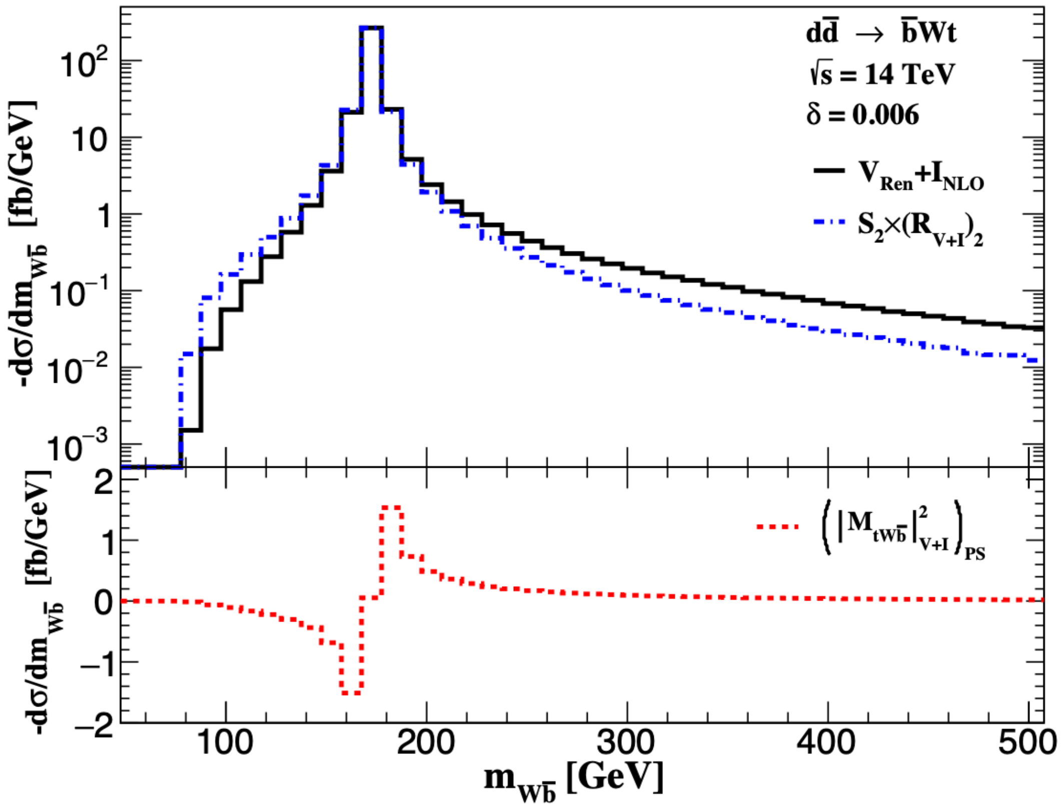
Scheme	σ_{LO}(fb)	σ_{V+I}(fb)	σ_{V+I}/σ_{LO}
PS	56.1(1)	-23.1(3)	-0.41

$d(p_1)\bar{d}(p_2) \rightarrow \bar{b}(p_3)W(p_4)t(p_5)$

SHANDONG UNIVERSITY

Subtracted one-loop correction:

- $m_{W\bar{b}}$ distribution



- Resonance is subtracted in the bin of 167.5 GeV $< m_{W\bar{b}} <$ 177.5 GeV (4 orders of magnitude).
- The resonance-subtracted distribution has an explicit form of $1/\Delta$.
- The 9 bins around the resonance peak, 127.5 GeV $< m_{W\bar{b}} <$ 217.5 GeV, contribute about 10% .

Summary and outlook

SHANDONG UNIVERSITY

- The tW process interferes with the $t\bar{t}$ process. The significant resonance contribution makes the perturbative expansion in couplings unreliable.

- A new scheme of subtracting $\bar{t}$ resonance is introduced, which is implemented by extracting the $1/\Delta^2$ term in power expansion around $\Delta = 0$.

- This scheme is demonstrated by calculating one-loop corrections to $d\bar{d} \to \bar{b}Wt$.

- The power subtraction scheme can be applied to one-loop corrections to other processes with resonances. For example, $gg \to \bar{b}Wt$.

Thanks for your attention!