第六届粒子物理天问论坛

Form factors and the light-cone distribution amplitudes of light pseudoscalar mesons

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Outline

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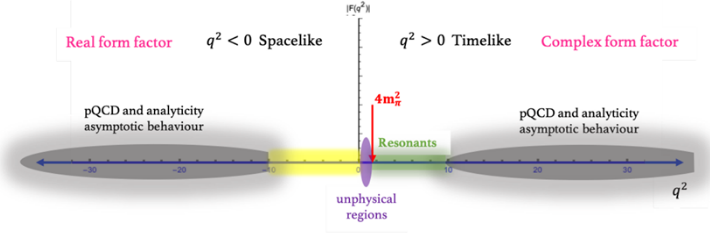
Measurements of F_π in different energy regions

- Spacelike data is available in the narrow region $q^2 \in [-2.5, -0.25]\text{GeV}^2$
  Jefferson Lab 2006,2008, $\cdots$, NA7 1996, CLEO 2005
- Timelike data is dominated by the resonant states, have not extend to large momentum transfers (perturbative QCD available)



Whole region of momentum transfers for electromagnetic form factor

- Mismatch between the QCD based calculation and the available data
- could be restored by employing the dispersion relation
- pQCD prediction at large $|q^2|$ is indispensable

Outline

Electromagnetic form factor of Pion

- † Dispersion Relation
- † Three scale factorization
- † Electromagnetic form factor of π from pQCD
- † Intrinsic transversal momentum distribution functions

Dispersion Relation

- spacelike data is available in the narrow region $q^2 \in [-2.5, -0.25]\mathrm{GeV}^2$
- the mismatch destroys the direct extracting programme from $F_\pi(q^2 < 0)$
- timelike data $F_\pi(q^2 > 0)$ provides another opportunity

Standard dispersion relation:

$$\mathcal{F}_\pi^{pQCD}(q^2) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\mathcal{F}_P(s)}{s - q^2 - i\epsilon}, \quad q^2 > s_0$$

modulus squared dispersion integral:

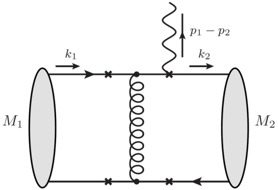
[S. Cheng, A. Khodjamirian and A. V. Rusov, PRD 102 (2020) 074022
J. Chai, S. Cheng and J. Hua, EPJC 83 (2023) no.7, 556.]

$$\mathcal{F}_\pi^{\mathrm{pQCD}}(\mathbf{q^2}) = \exp\left[\frac{\mathbf{q^2}\sqrt{\mathbf{s_0} - \mathbf{q^2}}}{\mathbf{2\pi}}\int_{\mathbf{4m_\pi^2}}^{\infty} \mathbf{ds}\frac{\ln|\mathcal{F}_\pi(\mathbf{s})|^2}{\mathbf{s}\sqrt{\mathbf{s} - \mathbf{s_0}}\,(\mathbf{s} - \mathbf{q^2})}\right].$$

$$|\mathcal{F}_\pi(s)|^2 = \Theta(s_{\max} - s)\,|\mathcal{F}_{\pi,\mathrm{Inter.}}^{\mathrm{data}}(s)|^2 + \Theta(s - s_{\max})\,|\mathcal{F}_\pi^{\mathrm{pQCD}}(s)|^2$$

Three scale factorization

- end-point singularities appear in exclusive QCD processes
  $m_{1,2}^2 \ll Q^2$, light-cone coordinate $p_2 = (\frac{Q}{\sqrt{2}}, 0, 0_{\mathrm{T}}), p_3 = (0, \frac{Q}{\sqrt{2}}, 0_{\mathrm{T}})$,
  (anti-)valence quarks: $k_2 = x_2 p_2, \bar{k}_2 = \bar{x}_2 p_2$

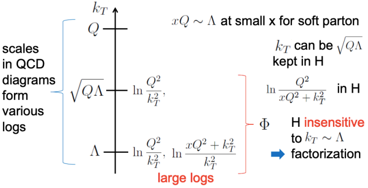


$\phi \propto u(1-u), \quad m_0^\pi \phi^{P,\sigma} \propto m_0^\pi$

$\propto \sum_t \int du_1\, du_2\, \kappa_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{u_1 u_2 Q^2 u_2 Q^2}$

- pick up k_T in the internal propagators
  $\mathcal{M} \propto \sum_{t=2,3,4} \int du_1\, du_2\, dk_{1\,T} dk_{2\,T} K_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{u_1 u_2 Q^2 - (k_{1\mathrm{T}} - k_{2\mathrm{T}})^2}$
- end-point singularity at leading and subleading powers
  $\mathcal{H} \propto \frac{\alpha_s(\mu)}{u_1 u_2 Q^2 - k_T^2} \sim \frac{\alpha_s(\mu)}{u_1 u_2 Q^2} - \frac{\alpha_s(\mu) k_T^2}{(u_1 u_2 Q^2)^2} + \cdots$
- the power suppressed TMD terms becomes important at the end-points

k_T Factorization Soft+colinear divergence appears double logarithmic term $\alpha_s ln^2(Q/k_T)$



borrowed from H.N Li

consider contribution from the iTMD

$$\frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}} \phi^p(u,\mu) = \int \frac{d^2\vec{k}_T}{16\pi^3} \phi^p_{2p}(u,\vec{k}_T) + \int \frac{d^2\vec{k}_{T1}}{16\pi^3} \frac{d^2\vec{k}_{T2}}{4\pi^2} \phi^p_{3p}(u,\vec{k}_{T1},\vec{k}_{T2}).$$

$$\psi^p_{2p}(u,\vec{k}_T) = \frac{f_\pi m_0^P}{2\sqrt{6}} \phi^p_{2p}(u,\mu) \Sigma(u,\vec{k}_T),$$

$$\psi^p_{3p}(u,\vec{k}_{1T},\vec{k}_{2T}) = \frac{f_\pi m_0^P}{2\sqrt{6}} \eta_{3\pi} \phi^p_{3p}(u,\mu) \Sigma'(\alpha_i,\vec{k}_{1T},\vec{k}_{2T}).$$

Intrinsic transversal momentum distribution functions

Two-particlie Fock state

$$\Sigma\left(u,\mathbf{k}_T\right)=16\pi^2\beta^2 g(u)\,\mathrm{Exp}\left[-\beta^2 k_T^2 g(u)\right],\, g(u)=1/(u\bar{u})$$

$$\int \frac{d^2 k_\perp}{16\pi^3}\Sigma\left(u,\mathbf{k}_T\right)=1$$

$$\beta_\pi^2=\frac{1}{8\pi^2 f_\pi^2\left(1+a_2^\pi+a_4^\pi+\cdots\right)}$$

$$\psi\left(u,\mathbf{b}_T\right)=\frac{f_\pi}{2\sqrt{6}}\varphi(u,\mu)\hat{\Sigma}\left(u,\mathbf{b}_T\right),\,\hat{\Sigma}\left(u,\mathbf{b}_T\right)=4\pi\,\mathrm{Exp}\left[-\frac{b_T^2}{4\beta^2 g(u)}\right]$$

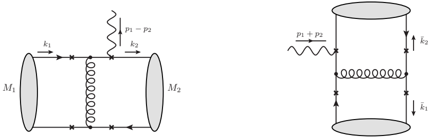
Three-particle Fock state

$$\psi_{3p}\left(u,\mathbf{k}_{1\,T},\mathbf{k}_{2\,T}\right)=\frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\varphi_{3p}(u,\mu)\int_0^u d\alpha_1\int_0^{\bar{u}} d\alpha_2\frac{\Sigma'\left(\alpha_i,\mathbf{k}_{1\,T},\mathbf{k}_{2\,T}\right)}{1-\alpha_1-\alpha_2}$$

three-particle iTMD Gaussian function is:

$$\Sigma'\left(\alpha_i,\mathbf{k}_{1\,T},\mathbf{k}_{2\,T}\right)=\frac{64\pi^3\beta'^4}{\alpha_1\alpha_2(1-\alpha_1-\alpha_2)}\,\mathrm{Exp}\left[-\beta'^2\left(\frac{k_{1\,T}^2}{\alpha_1}+\frac{k_{2\,T}^2}{\alpha_2}+\frac{(k_{1\,T}+k_{2\,T})^2}{1-\alpha_1-\alpha_2}\right)\right]$$

$$\hat{\Sigma}'\left(u,\mathbf{b}_1,\mathbf{b}_2\right)=4\pi\,Exp[-\frac{2\alpha_3(b_1^2+b_2^2)+(\alpha_1+\alpha_2)(b_1-b_2)^2}{16\beta'^2}]$$



(a) (b)

LO feynman diagrams of spacelike(left) and timelike(right) form factors

Invariant amplitudes of EM current can be written as:

$$\langle \pi^-(p_2)|j^{\rm em}_{\mu,q}|\pi^-(p_1)\rangle = \langle \pi^-(p_2)|\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right)|\pi^-(p_1)\rangle \equiv e_q(p_1+p_2)\mathcal{F}_\pi(Q^2)\,,$$

Separating the short and long distance interactions， written in the factorizable form：

$$\langle \pi^-(p_2)|J^{\rm e.m.}_\mu|\pi^-(p_1)\rangle =$$

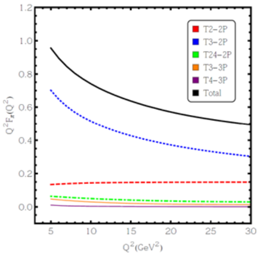
$$\oint dz_1\, dz_2\, H^{ijkl}_{\gamma\beta\alpha\delta}(z_2,z_1)\langle \pi^-(p_2)\Big|\left\{\bar{d}_\gamma(z_2)\exp\left(ig_s\int_0^{z_2} d\sigma_{\nu'}A_{\nu'}(\sigma)\right)u_\beta(0)\right\}_{kj}\Big|0\rangle_{\mu_t}$$
$$\langle 0\Big|\left\{\bar{u}_\alpha(0)\exp\left(ig_s\int_{z_1}^0 d\sigma_\nu A_\nu(\sigma)\right)d_\delta(z_1)\right\}_{il}\Big|\pi^-(p_1)\rangle_{\mu_t}$$

Different twists from different spin structures:

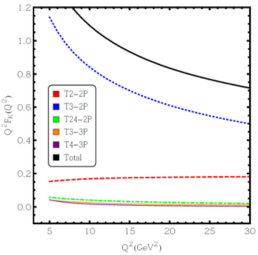
$$\mathbf{4q_{1\alpha}q_{2\delta}} = \left\{\mathbf{q_1 q_2} + \gamma_5\left(\mathbf{q_2}\gamma_5\mathbf{q_1}\right) + \gamma^\rho\left(\mathbf{q_2}\gamma_\rho\mathbf{q_1}\right) + \gamma_5\gamma^\rho\left(\mathbf{q_1}\gamma_\rho\gamma_5\mathbf{q_1}\right) + \frac{\mathbf{1}}{\mathbf{2}}\sigma^{\rho\tau}\left(\mathbf{q_2}\sigma_{\rho\tau}\mathbf{q_1}\right)\right\}_{\delta\alpha}\,.$$

Pion LCDAs

- Three sources of high twist LCDAs
  - † ”bad” components in WFs in particular of those with ” wrong” spin projection
  - † transversal motion of $q(\bar{q})$ in the leading twist components given by the integrals with additional factors of $k_\perp^2$
  - † higher Fock states with additional g and $q\bar{q}$ pairs
- higher twist contributions to exclusive QCD processes are commonly power suppressed $\mathcal{O}(1/Q)$
- but twist 3 contribution are dominate in the π, K evolved processes due to chiral enhancement $\mathcal{O}(m_0/(x_i Q))$



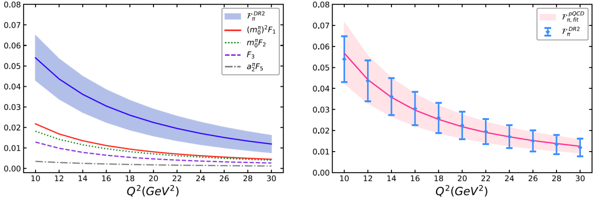
[SC, 1905.05059]

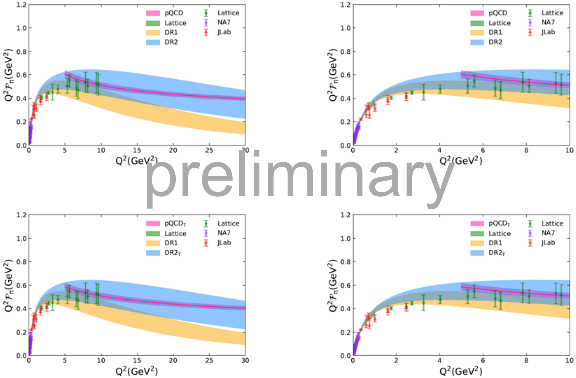


Fit

$$\chi^2 = \sum_{i=1}^{11} \frac{\left[\mathcal{F}_\pi^{\rm DR2}(Q_i^2) - \mathcal{F}_\pi^{\rm pQCD}(Q_i^2)\right]^2}{\left[\delta\mathcal{F}_\pi^{\rm DR2}(Q_i^2)\right]^2}$$

$$\begin{aligned}\mathcal{F}_\pi^{\rm em}(Q^2) &= (m_0^\pi)^2\mathcal{F}_{\pi,1}^{\rm em}(Q^2) + m_0^\pi\mathcal{F}_{\pi,2}^{\rm em}(Q^2) + \mathcal{F}_{\pi,3}^{\rm em}(Q^2) \\ &\quad + m_0^\pi a_2^\pi\mathcal{F}_{\pi,4}^{\rm pQCD}(Q^2) + a_2\pi\mathcal{F}_{\pi,5}^{\rm em}(Q^2) + (a_2^\pi)^2\mathcal{F}_{\pi,6}^{\rm em}(Q^2)\end{aligned}$$

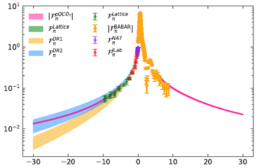


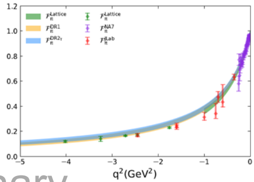


Set	IA	IB	II	III
m_0^π(GeV)	1.37 ± 0.10	1.30 ± 0.100	1.56 ± 0.06	1.85 ± 0.07

Electromagnetic form factor of Pion

- the precise pQCD calculation
- modular dispersion relation with e^+e^- annihilation data
- a comprehensive description of $F_\pi(q^2)$ in the whole kinematics



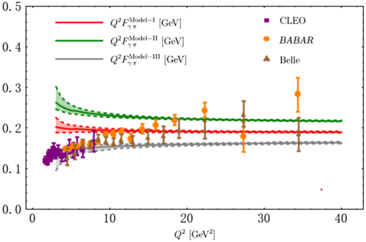


preliminary

- the slight derivation is still there despite its sensitive to iTMD in the small q^2
- form factor of K meson is being studied

Transition form factor of π

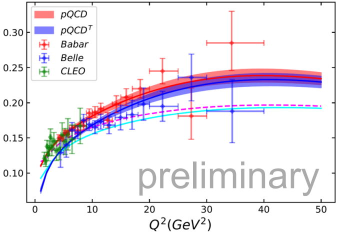
$F_{\pi\gamma\gamma^\star}$ is the theoretically most clean observable $\propto a_n^\pi$



- Model-I [Brodsky, Teramond 0707.3859, RQCD 1903.08038]
- Model-II [SC, Khodjamirian, Rosov 2007.05550]
- Model-III [Mikhailov, Pimikov, Stefanis 1604.06391]

† NLO pQCD calculation with the iTMD contribution, modification in the small and intermediate regions is significant

† TFF of $\eta^{(\prime)}$ are being studied

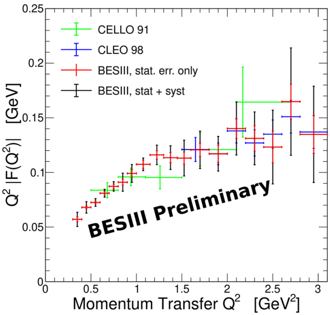


Outline

Form factors and the light-cone distribution amplitudes of light pseudoscalar mesons:

- **Electromagnetic form factor of π and K**
  - † in the light-cone dominated processes, hadron structure is well studied in terms of LCDAs
  - † a comprehensive studies of $F_\pi(q^2)$ with pQCD calculation and modular dispersion relation
  - † help to reveal inner structure of pion (moments, iTMD)

- **Transition form factor π and $\eta^{(\prime)}$**
  settle down the " fat pion" issue in $F_{\pi\gamma\gamma^\star}$



[Christoph 1810.00654[hep-ex]]

Thank you for your patience...