# 射频直线加速器（五）

直线加速器空间电荷效应、加速间隙散焦及RFQ

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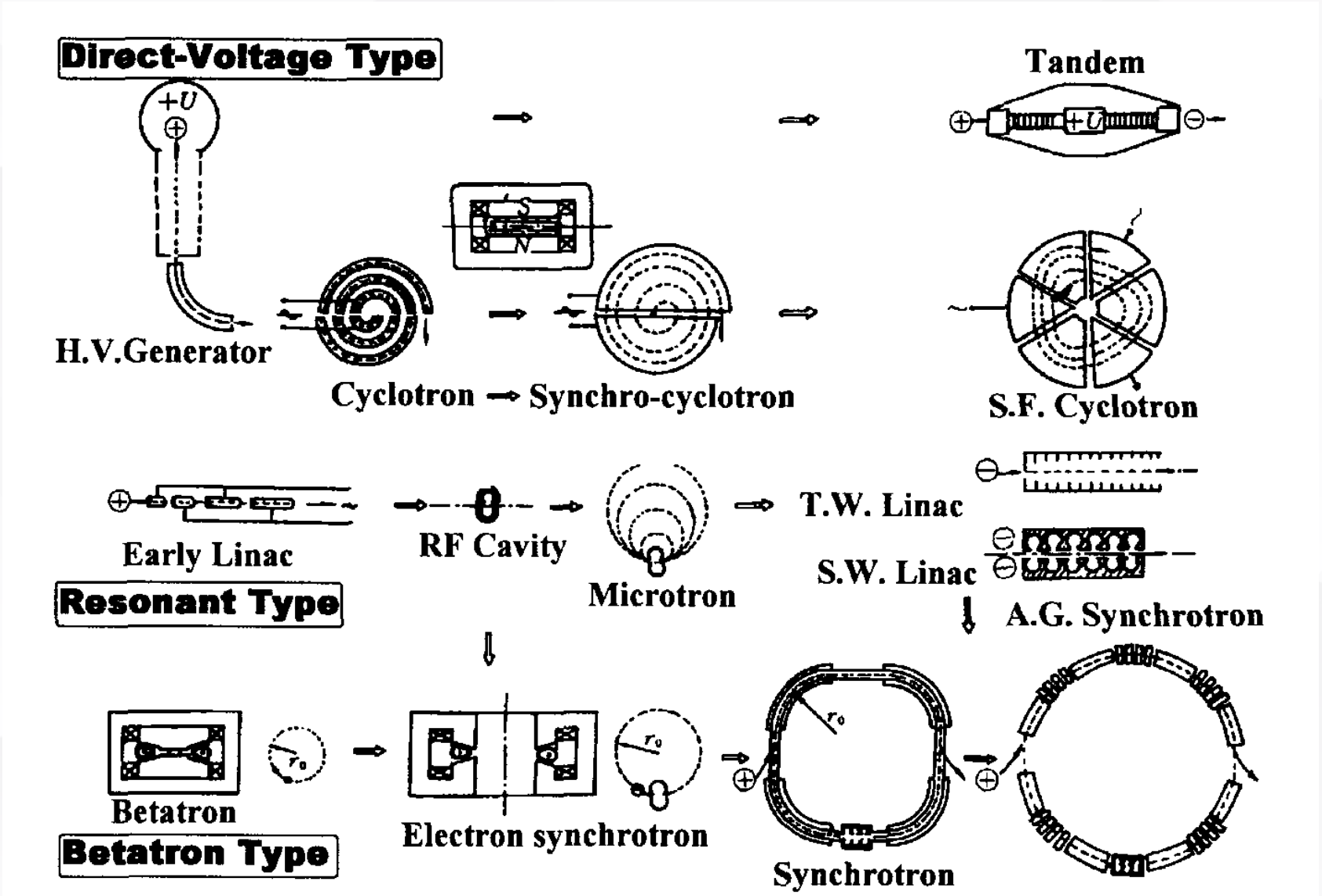
# 加速器理念衍化


Direct-Voltage Type
+U
H.V.Generator
Tandem
Cyclotron → Synchro-cyclotron
S.F. Cyclotron
Early Linac
RF Cavity
Microtron
T.W. Linac
S.W. Linac
Resonant Type
A.G. Synchrotron
Betatron
Betatron Type
Electron synchrotron
Synchrotron

# 空间电荷作用力(线性模型）及其影响

横向

当$r > a$

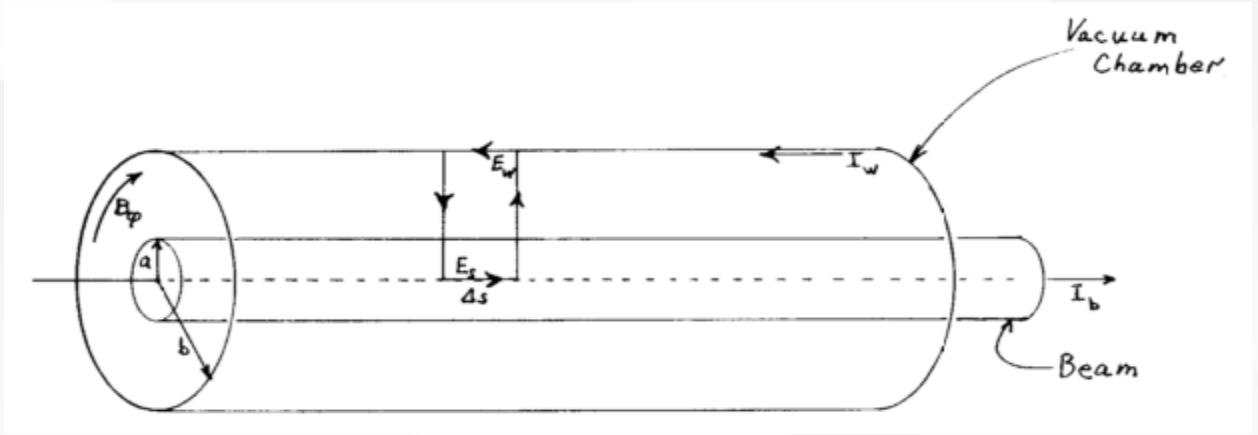
$$E = \frac{eN}{2\pi\epsilon_0 r}$$

$$B = \frac{eNv}{2\pi\epsilon_0 rc^2}$$

当$r < a$

$$E = \frac{eN}{2\pi\epsilon_0 a^2} r$$

$$B = \frac{eNv}{2\pi\epsilon_0 a^2 c^2} r$$




$$x'' + \left[K(s) - \frac{2r_0 N}{(v/c)^2 \gamma^3 a^2}\right] x = 0$$

其中 $r_0 = \frac{e^2}{4\pi\epsilon_0 mc^2}$

$$I = eNv$$

纵向

$$E_s = \frac{g}{4\pi\varepsilon_0 \gamma^2 \beta c} \frac{\partial \lambda}{\partial t} + E_w$$

$$g = 1 + 2\ln\left[\frac{b}{a}\right]$$

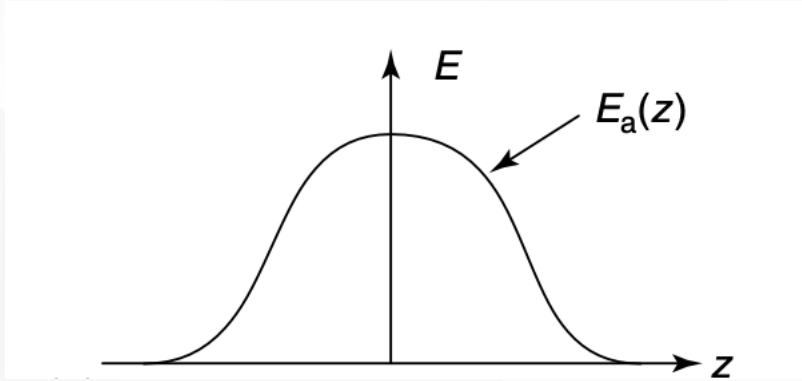
$\varepsilon_0$，真空介电常数

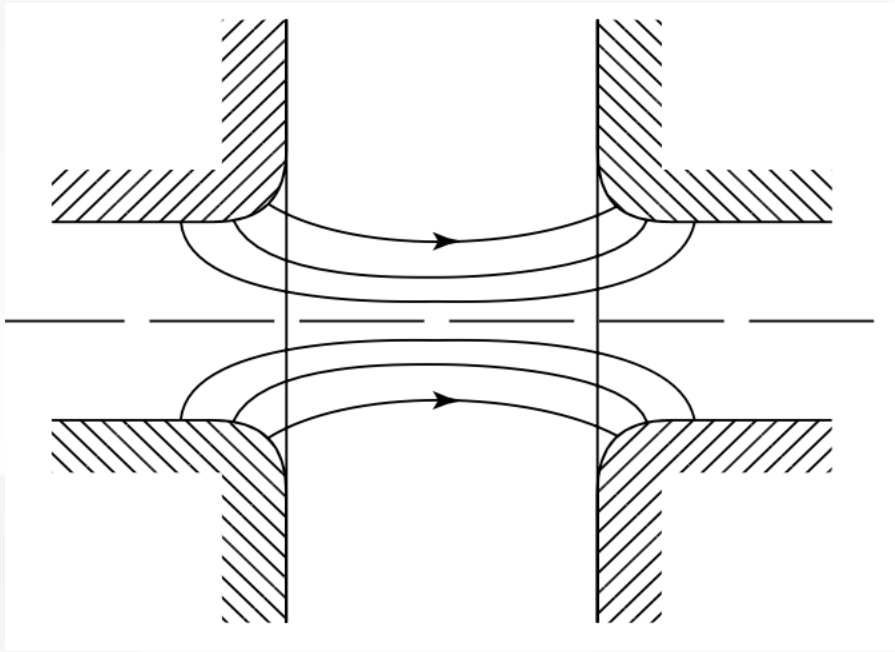
$\lambda$，线电荷密度，C/m

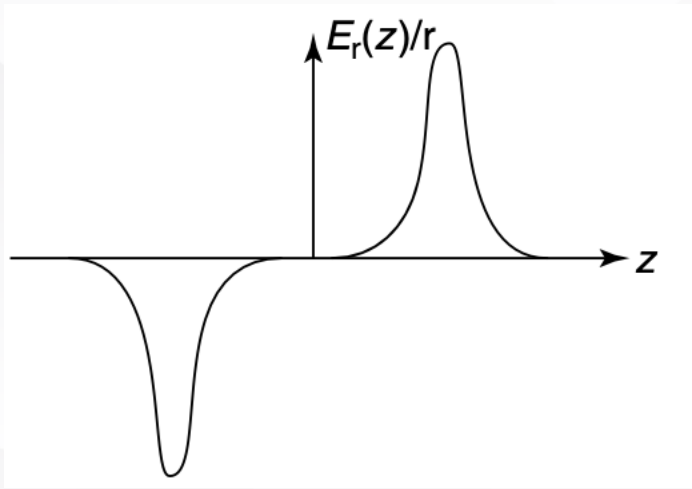
# 加速间隙上的电磁场变化对束流的影响

$$E_z(r,z,t) = E_a(z)\cos(\omega t + \phi)$$

$$\frac{\partial^2 V}{\partial^2 x} + \frac{\partial^2 V}{\partial^2 y} + \frac{\partial^2 V}{\partial^2 z} = 0$$





# 在旁轴近似下的电磁场关系

$$\nabla \cdot \mathbf{E} = 0 \qquad \frac{1}{r}\frac{\partial (rE_r)}{\partial r} + \frac{\partial E_z}{\partial z} = 0$$

当r比较小时，假设$E_z$与r无关

$$rE_r = -\frac{\partial E_z}{\partial z}\int_0^r r\,dr$$

$$E_r = -\frac{\partial E_z}{\partial z}\frac{r}{2}$$

$$(\nabla \times \mathbf{E})_\theta = -\frac{\partial B_\theta}{\partial t} \qquad \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -\frac{\partial B_\theta}{\partial t}$$

$$\frac{\partial E_r}{\partial r} = -\frac{1}{2}\frac{\partial E_z}{\partial z}$$

$$(\nabla \times \mathbf{B})_r = \frac{1}{c^2}\frac{\partial E_r}{\partial t} \qquad -\frac{\partial B_\theta}{\partial z} = \frac{1}{c^2}\frac{\partial E_r}{\partial t}$$

$$\frac{\partial B_\theta}{\partial z} = \frac{r}{2c^2}\frac{\partial}{\partial z}\frac{\partial E_z}{\partial t}$$

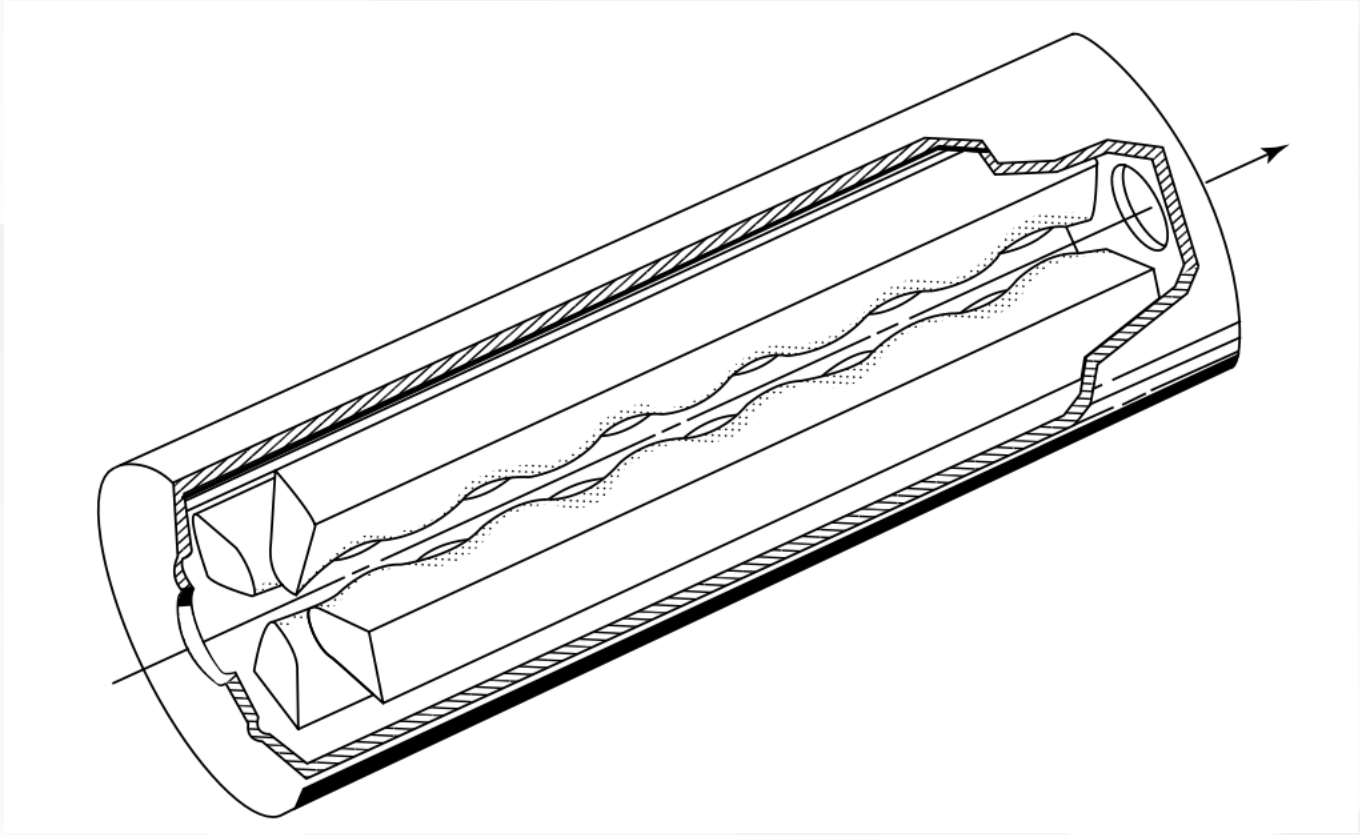
积分

$$B_\theta = \frac{r}{2c^2}\frac{\partial E_z}{\partial t}$$

$$(\nabla \times \mathbf{B})_z = \frac{1}{c^2}\frac{\partial E_z}{\partial t} \qquad \frac{1}{r}\frac{\partial (rB_\theta)}{\partial r} = \frac{1}{c^2}\frac{\partial E_z}{\partial t}$$

$$\frac{\partial B_\theta}{\partial r} = \frac{1}{2c^2}\frac{\partial E_z}{\partial t}$$

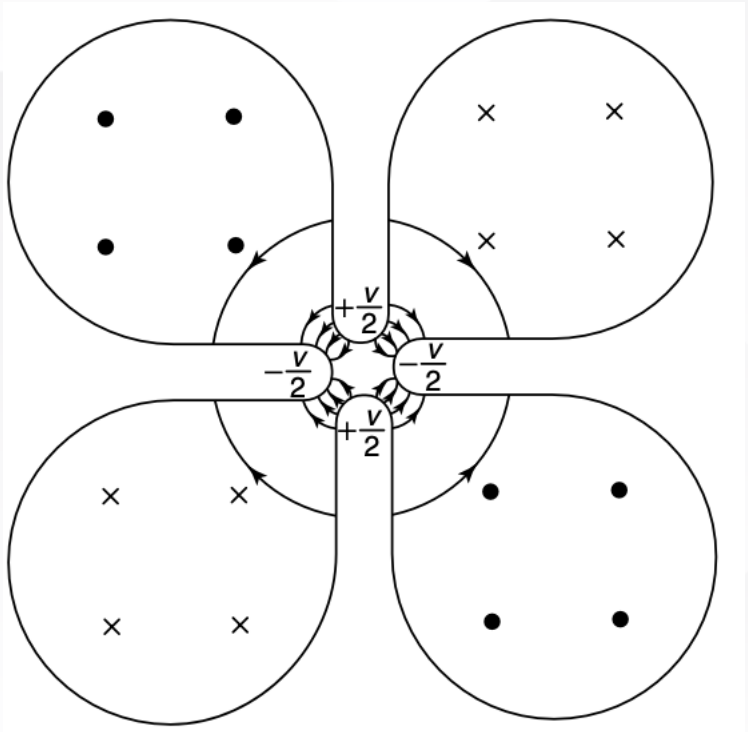
# RFQ（Radiofrequency Quadrupole Linac)



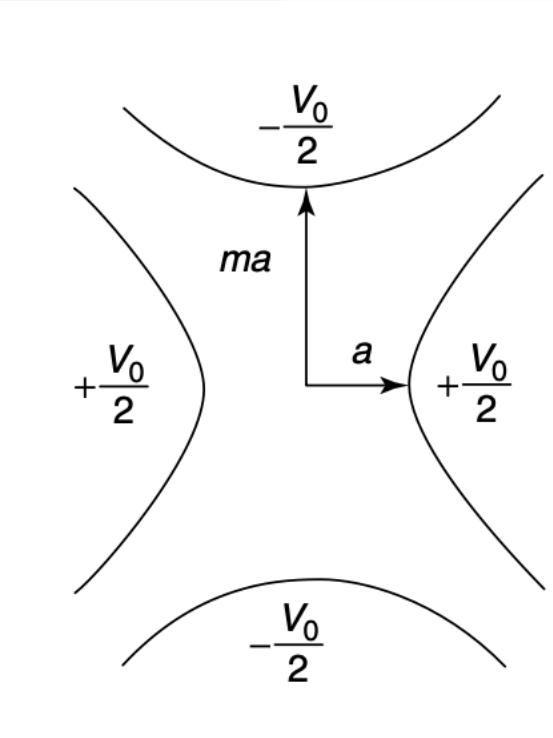
**通过横向非对称四个电极在纵向上交替调节的方式同时实现（1979）：**

- 横向的聚焦
- 纵向的聚束与加速

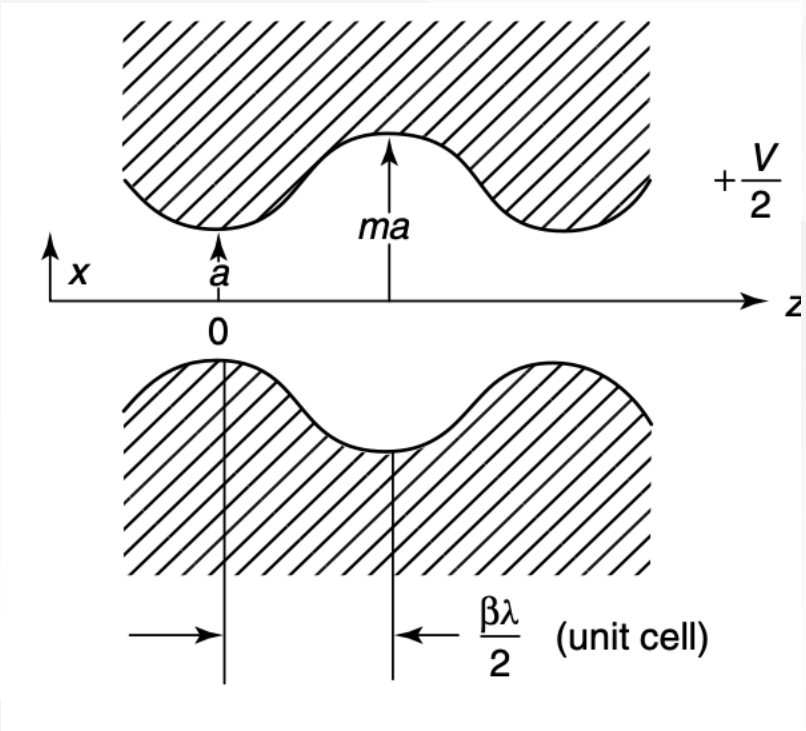
# RFQ四个电极




电极附近的场分布

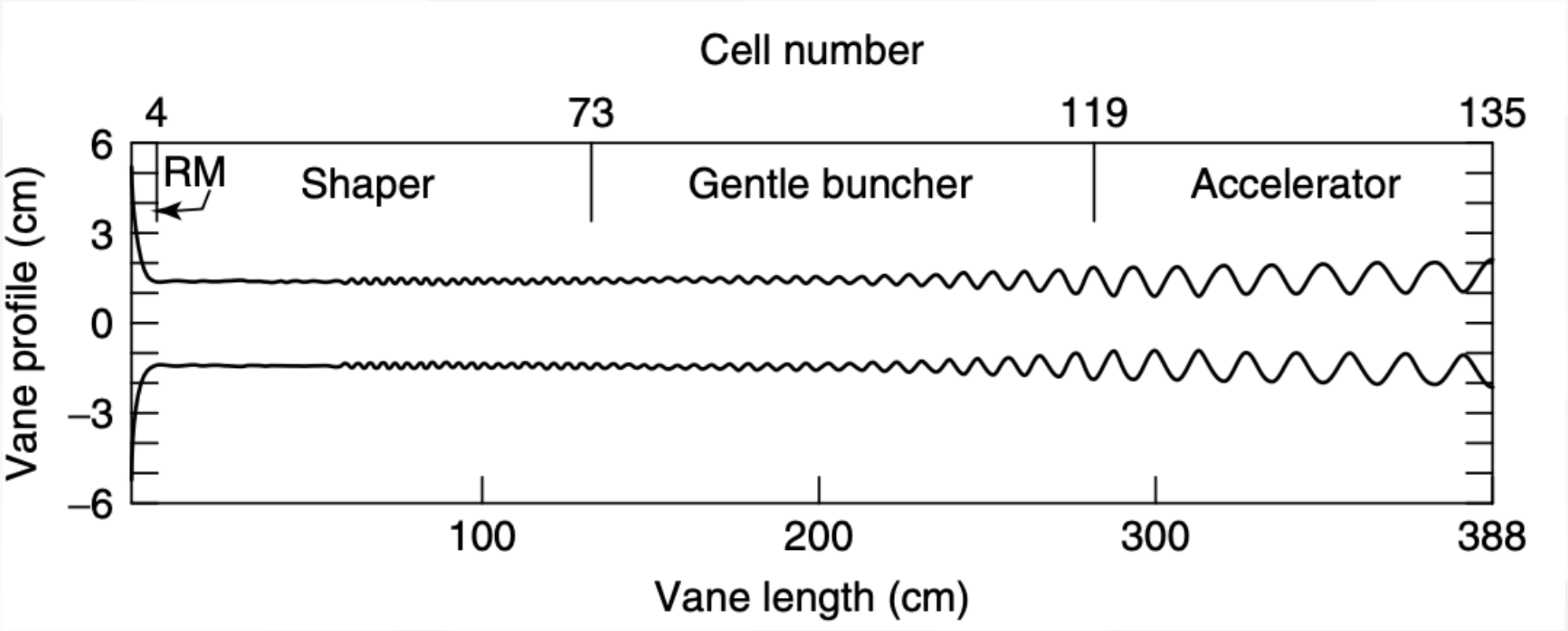



非对称的电极



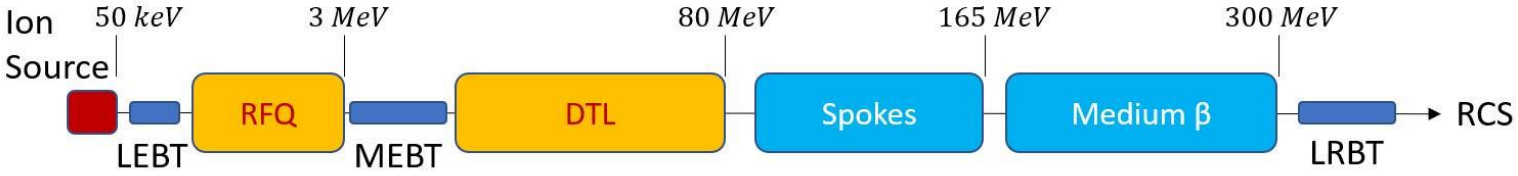

纵向上的加速单元设计

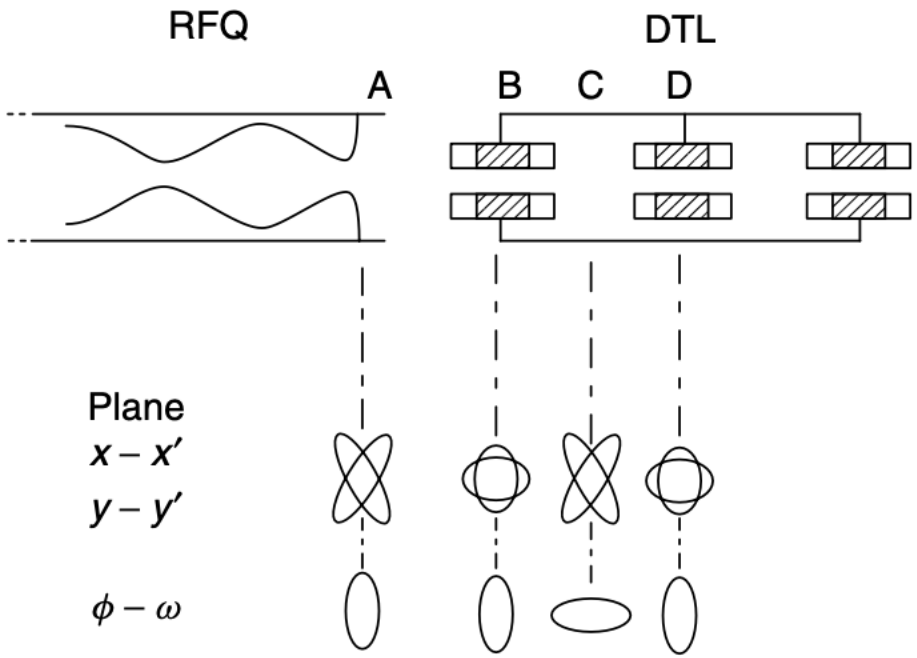
# 典型RFQ的纵向分区设计



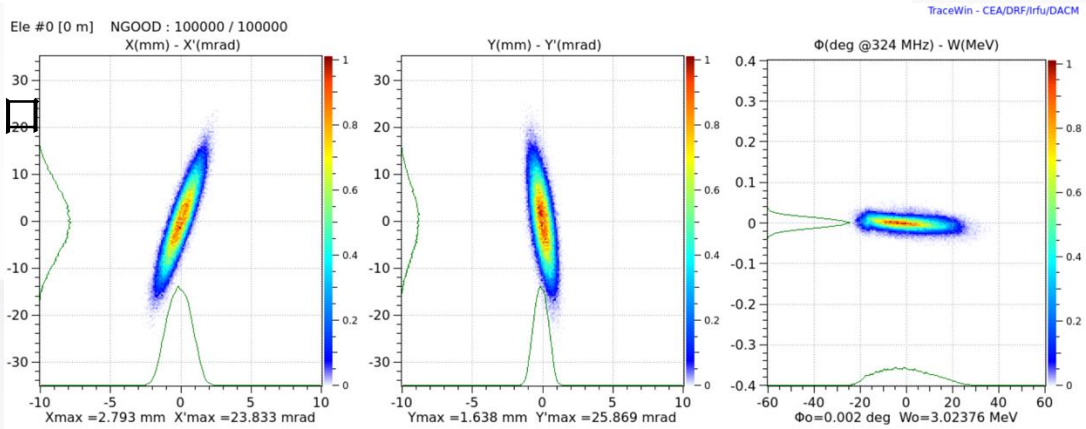

- 匹配段
- 初始调节段
- 弱聚束段
- 加速段

# RFQ与DTL之间的匹配

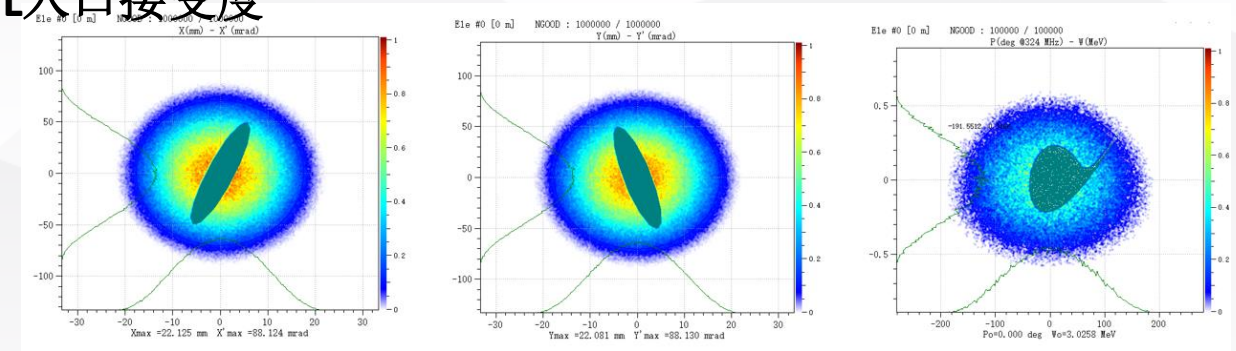



RFQ出口




DTL入口接受度



# 关于期末考试的一些建议及讨论