Probing Dark Matter with Gravitational-Wave Interferometers in Space

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中国科学院大学

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w/ Li, and Wu, arXiv:2112.14041; Yu, Yao, Wu, arXiv:2307.09197;

Zhang, arXiv:2403.13882; Yao, arXiv:2404.10494; Yu, Cao, Wu, arXiv:2404.04333

Contents

Evidence for Dark Matter
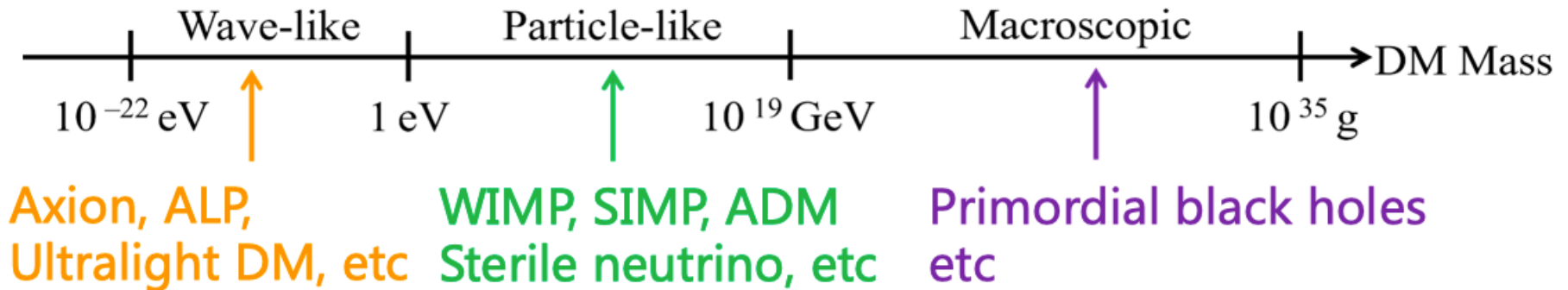

NGC3198

stars+gas
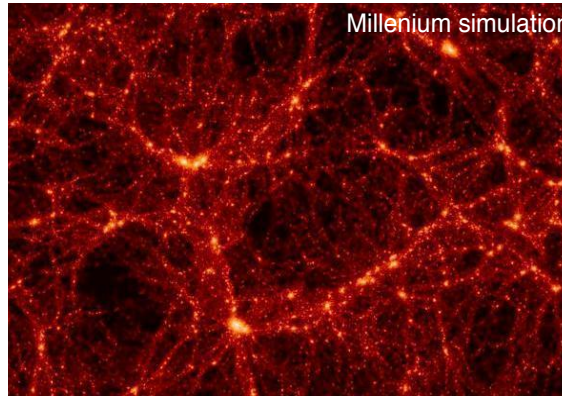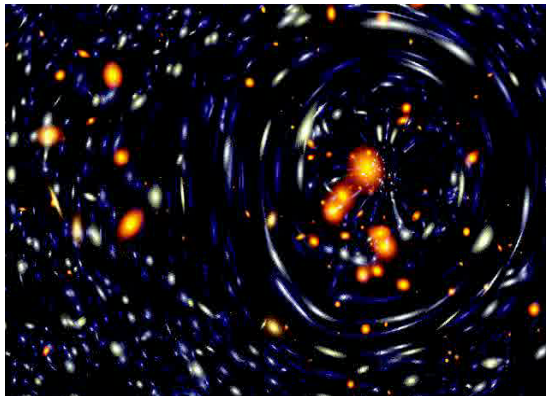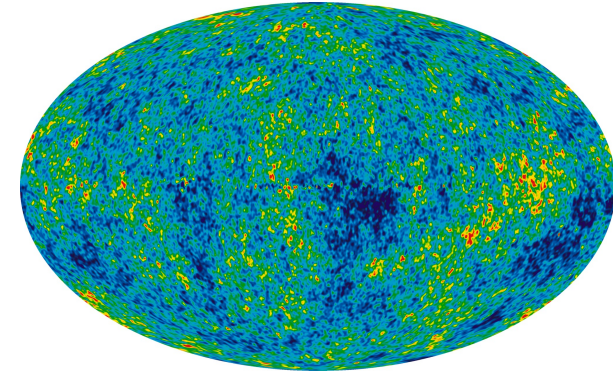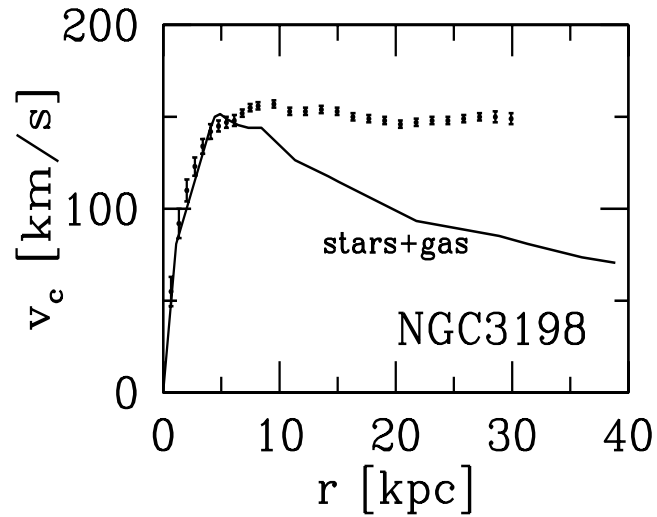
Rotation curve

Gravitational lensing

Bullet cluster

➤ Large-scale structure

➤ Anisotropy of CMB

Dark Matter Candidates

- Primordial black holes
- Super heavy particles
- Asymmetric DM
- Hidden sector DM
- ......

- Weakly-interacting (WIMP)
- Strongly-interacting (SIMP)
- Sterile neutrino
- Axion (ALP), Ultralight DM

Wave-like   Particle-like   Macroscopic → DM Mass

10^{-22} eV   1 eV 10^{19} GeV 10^{35} g

Axion, ALP, Ultralight DM, etc   WIMP, SIMP, ADM Sterile neutrino, etc   Primordial black holes etc
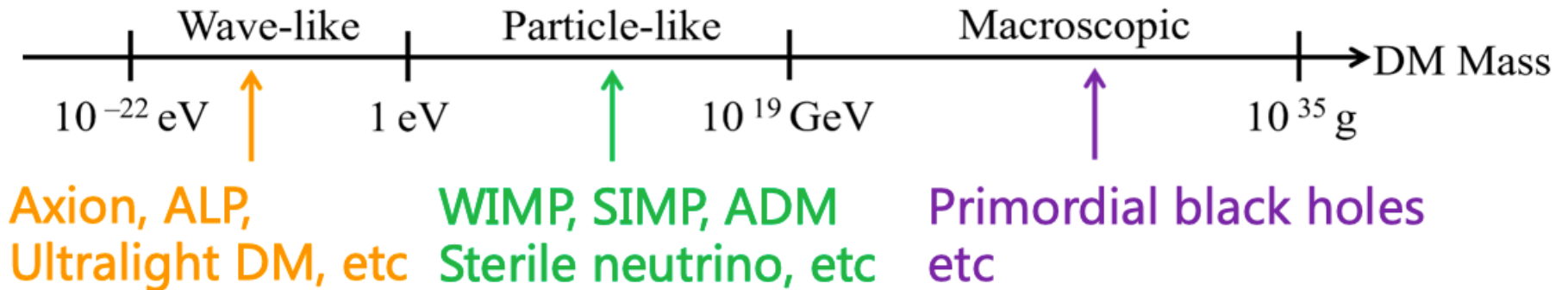
Dark Matter Candidates

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- **Weakly-interacting (WIMP)**
- Strongly-interacting (SIMP)
- Sterile neutrino
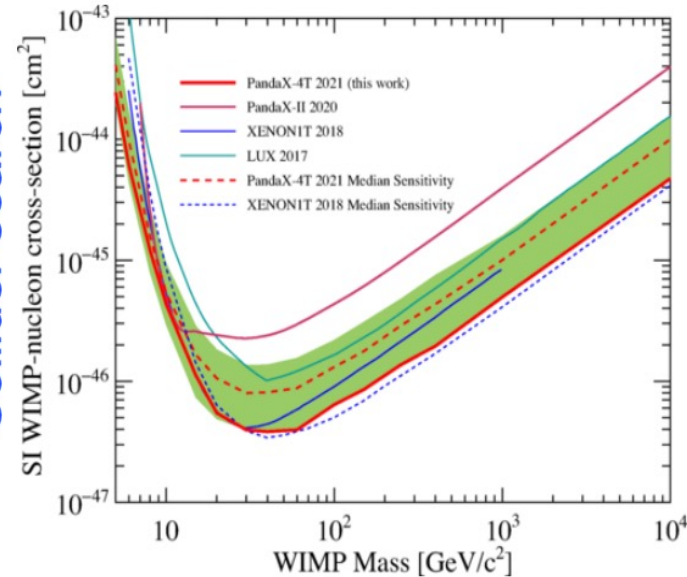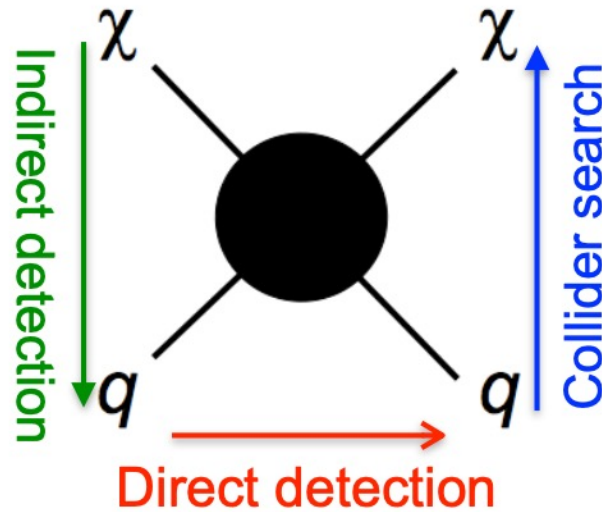- **Axion (ALP), Ultralight DM**

Wave-like | Particle-like | Macroscopic → DM Mass

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Axion, ALP, Ultralight DM, etc

WIMP, SIMP, ADM Sterile neutrino, etc

Primordial black holes etc

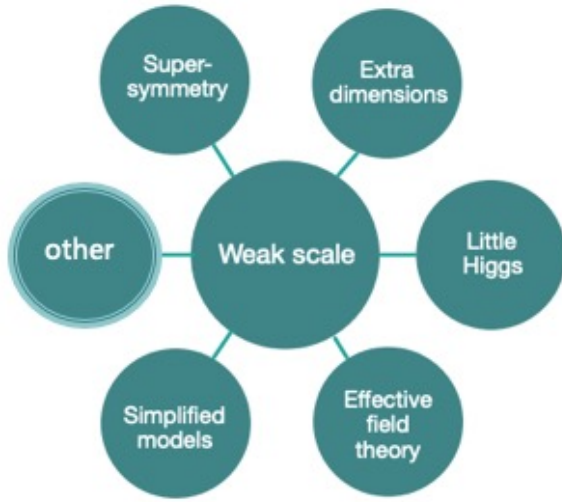# WIMP

➢ Mass ~ 100 GeV

➢ Coupling constant ~ 0.5

➢ Relic abundance ~ 0.3

➢ Thermal history

    1. Equilibrium XX <> ff

    2. Equilibrium XX > ff

    3. Freeze out

➢ Cold dark matter

➢ The & Exp attractive

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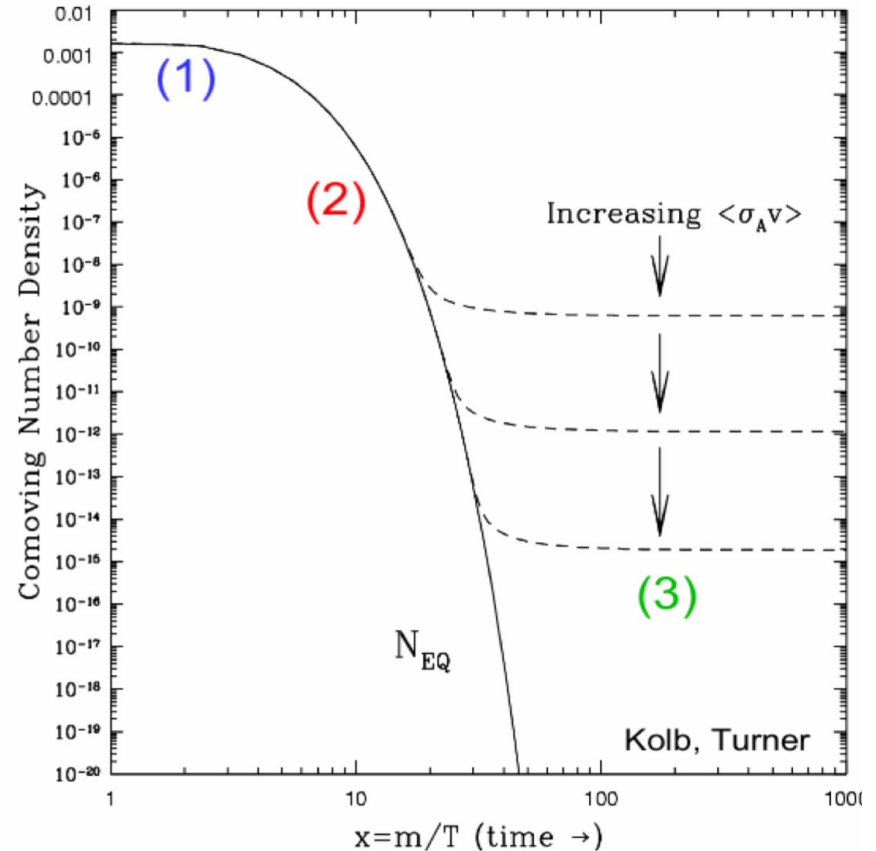
WIMP

➤ Theories: supersymmetry, extra dimensions, ······

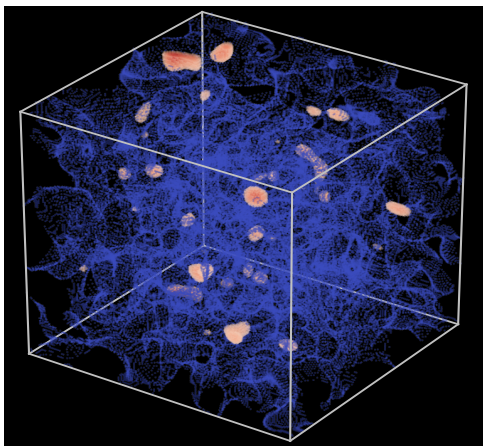➤ Direct, indirect and collider searches

➤ Impressive progress

Ultralight Dark Matter

➤ Mass < 1 eV, QCD axion, axion-like-particles, bosonic
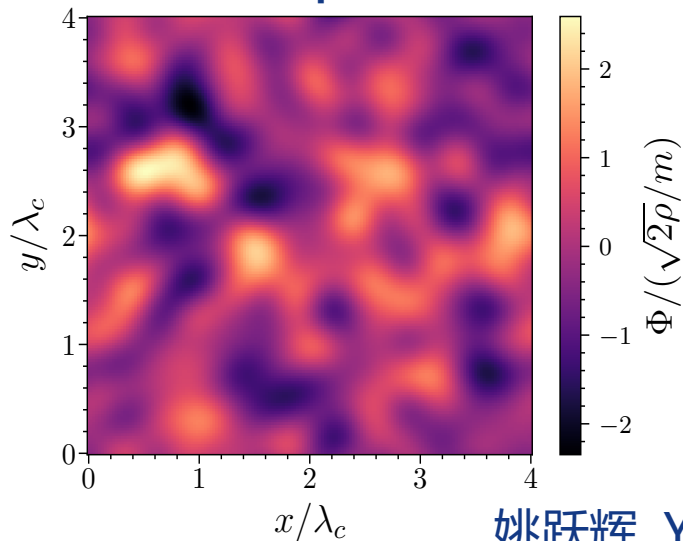
➤ Number density is very large, behaves as classical wave

$$\Phi(x) = \sum_{\mathbf{v}} \frac{\sqrt{2\rho/N}}{m} \, e^{i(\omega t - \mathbf{k}\cdot\mathbf{x} + \theta_{\mathbf{v}})},$$

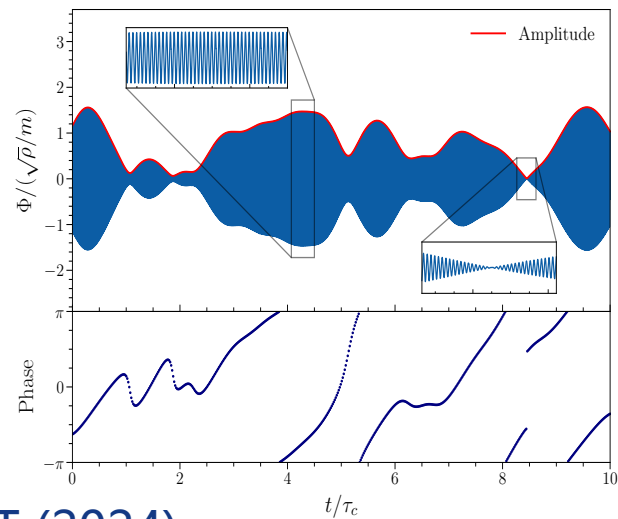at a specific point, ~ monochromatic within a small duration

3D snapshot

2D snapshot

at some point

姚跃辉, YT (2024)

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Ultralight Dark Matter

➢ Scalar ϕ

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - C\frac{\phi}{M_P}\mathcal{O}_{\mathrm{SM}}, \quad \phi(t,\vec{x}) = \phi_{\vec{k}}e^{i(\omega t - \vec{k}\cdot\vec{x} + \theta_0)},$$

➢ Interaction depending on the underlying theory, e.g.

$$C\frac{\phi}{M_P}m_\psi\overline{\psi}\psi \Rightarrow m_\psi \to \left(1 + C\frac{\phi}{M_P}\right)m_\psi, \quad S = -\int m(\phi)\sqrt{-\eta_{\mu\nu}dx^\mu dx^\nu}.$$

$$\delta x^i(t,\vec{x}) = \mathcal{M}_s\hat{k}^i e^{im_\phi(t - v\hat{k}\cdot\vec{x})}, \mathcal{M}_s \propto \phi_{\vec{k}}|\vec{k}|/m_\phi^2$$

➢ Vector A_μ

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_A^2 A^\nu A_\nu - \epsilon_D e J_D^\nu A_\nu, \quad \vec{A}(t,\vec{x}) = |\vec{A}|\hat{e}_A e^{i(\omega t - \vec{k}\cdot\vec{x})},$$

$$\delta x^i(t,\vec{x}) = \mathcal{M}_v\hat{e}_A^i e^{im_A(t - v\hat{k}\cdot\vec{x})}, \mathcal{M}_v \propto \epsilon_D e q_{D,j}|\vec{A}|/m_A M_j$$

➢ DM property

$$\phi_{\vec{k}} = \frac{\sqrt{2\rho_{\mathrm{DM}}}}{m_\phi}, \qquad |\vec{A}| = \frac{\sqrt{2\rho_{\mathrm{DM}}}}{m_A}, \quad v \sim 10^{-3}, \quad \vec{k} \approx m_\phi\vec{v} \text{ and } \omega \approx m_\phi$$

Physical Effects

- ➤ Atomic physics
  - ➤ Arvanitaki, Huang & Tilburg (2014), Graham, Kaplan, Mardon, Rajendran & Terrano (2015), Safronova, Budker, DeMille, Kimball, Derevianko & Clark (2018),
 - ➤ Stadnik (2022), ⋯⋯

- ➤ Astrophysical physics
  - ➤ Pierce, Riles & Zhao (2018), Morisaki & Suyama (2019), Guo, Riles, Yang & Zhao 2019 , Grote & Stadnik (2019),
  - ➤ An, Huang, Liu & Xue (2021), Chen, Shu, Xue, Yuan & Zhao (2019), Xia, Xu & Zhou (2020), Sun, Yang & Zhang (2021), Wu, Chen, & Huang (2023),
  - ➤ Liu, Lou & Ren (2021), Luu, Liu, Ren, Broadhurst, Yang, Wang & Xie (2023), ⋯⋯

- ➤ Underground searches
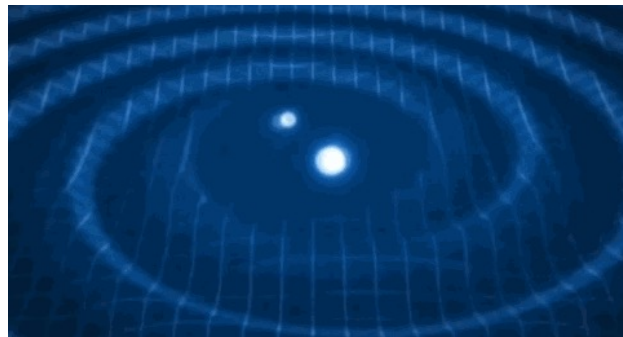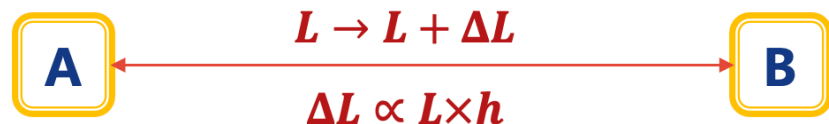 - ➤ Dark Matter Experiments, PandaX, XENONnT, ⋯

Space-based GW Interferometers
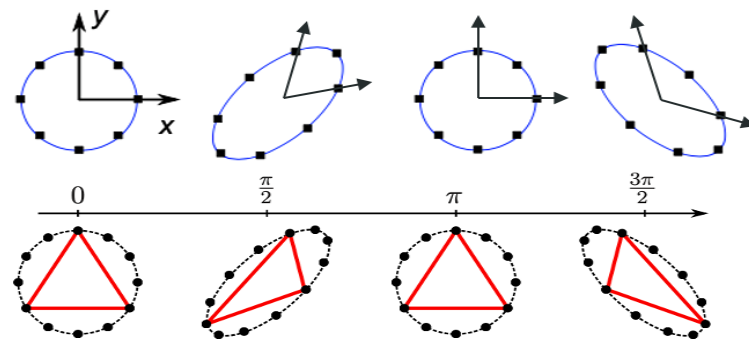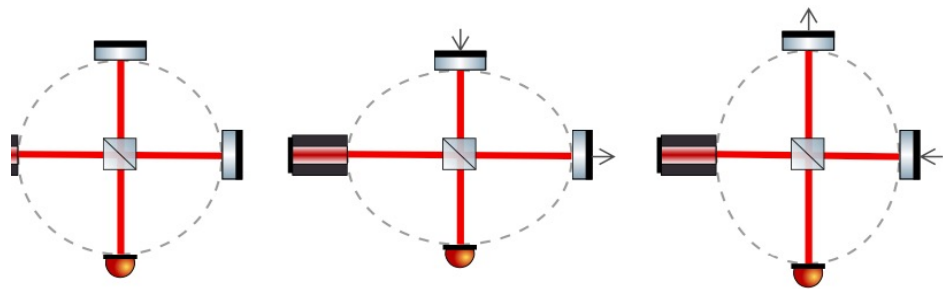
➤ LISA, Taiji, Tianqin, DECIGO, BBO, LISAMax, ASTROD-GW, μAres
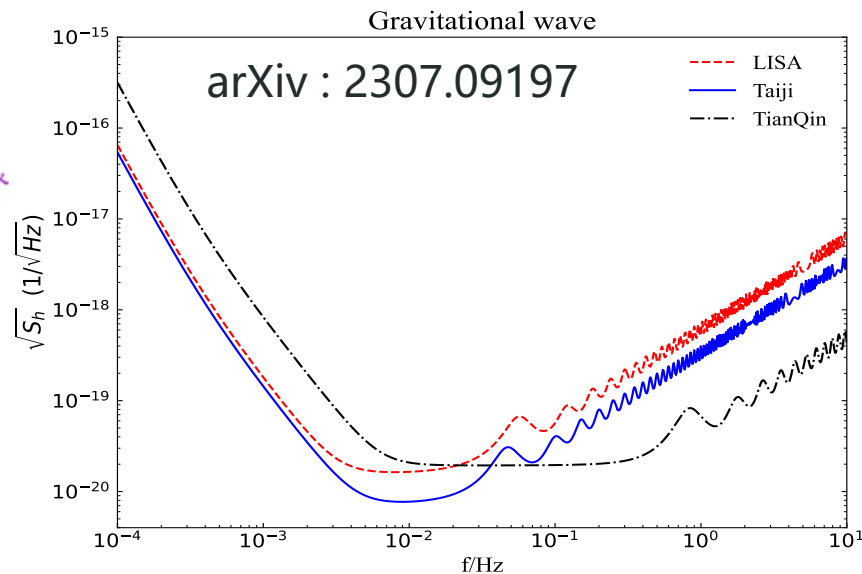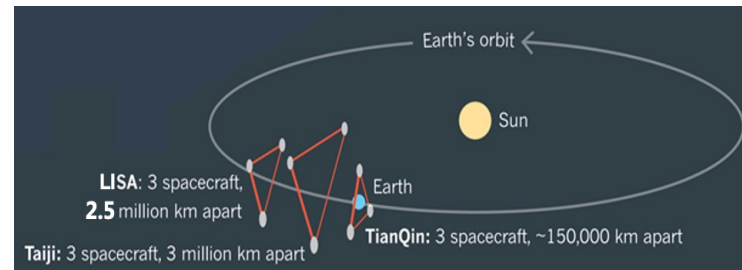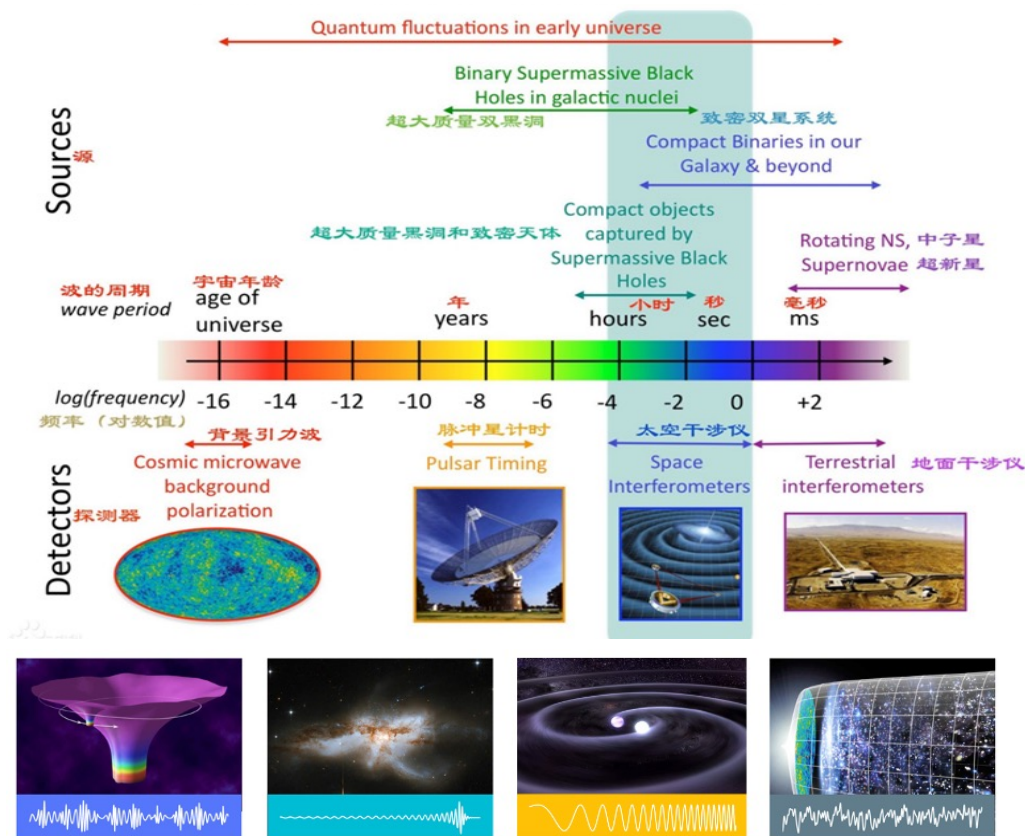
Signal Response

➤ Gravitational wave can change the structure of spacetime, and the physical distance between objects

$$L \rightarrow L + \Delta L$$

$$\Delta L \propto L \times h$$

➤ One can measure the phase by laser

➤ Response $\dfrac{\delta v(t)}{v_0} \equiv y_{BA} = -\dfrac{1}{2}\dfrac{n_i n_j}{1+\vec{k}\cdot\vec{n}}\left[h_{ij}\left(t - \dfrac{\vec{k}\cdot\vec{x}_B}{c}\right) - h_{ij}\left(t - \dfrac{\vec{k}\cdot\vec{x}_A + L}{c}\right)\right]$$
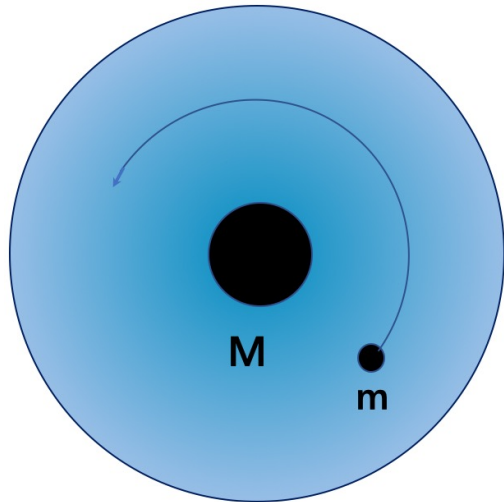
DM Spikes around Black Holes

- ➤ WIMP DM particles accretion around BH → DM spike

- ➤ NFW profile → spiky density profile   **Gondolo & Silk (1999)**

$$\rho(r) \propto r^{-\gamma}, 0 \le \gamma \le 2 \implies \rho_{\mathrm{spike}}(r) = \rho_{\mathrm{sp}}\left(\frac{r_{\mathrm{sp}}}{r}\right)^{\alpha}, \quad \alpha = \frac{9-2\gamma}{4-\gamma}$$

- ➤ Dynamical friction → Gravitational wave

Extreme-mass-ratio Inspiral (EMRI)

章子昌, Tang, 2403.18529 (PRD)

$$m\frac{d^2\mathbf{x}}{dt^2} = \mathbf{F}_{\mathrm{G}} + \mathbf{F}_{\mathrm{DF}}$$

$$h_{ij} \sim \frac{G}{d}\frac{d^2 Q_{ij}}{dt^2},$$

$$Q_{ij} \sim m\left(x_i x_j - \frac{1}{3}x^2\delta_{ij}\right)$$

M

m

13   **Eda, Itoh, Kuroyanagi & Silk, Yue & Cao, Yue, Han & Chen , Gong, Bertone et al … …**

DM Spikes around Black Holes

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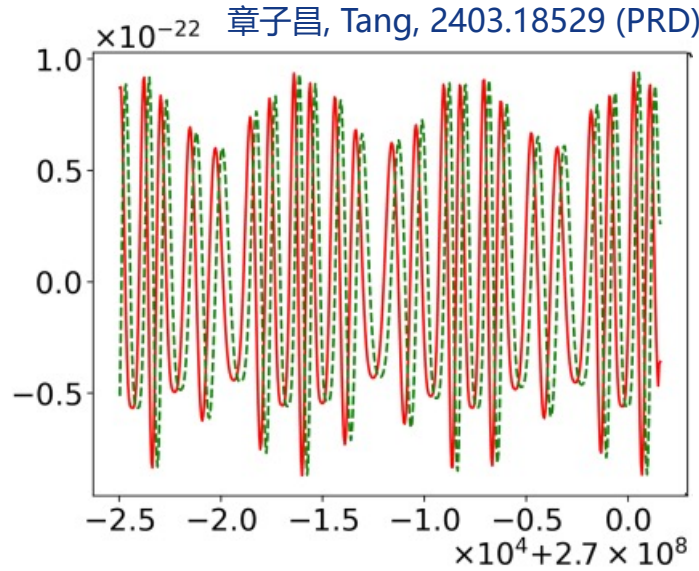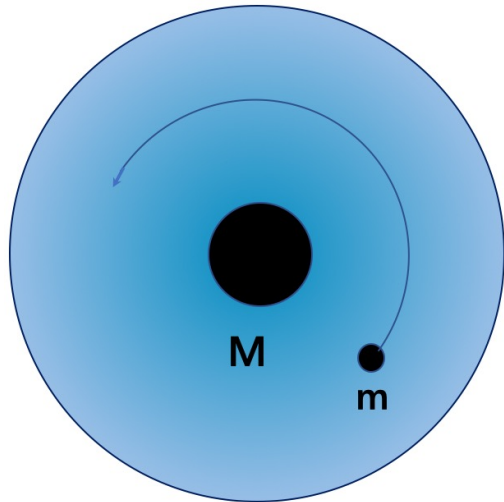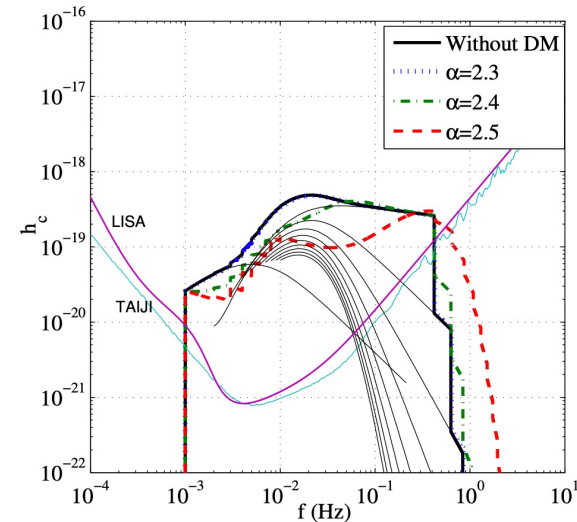➤ Dynamical friction → Gravitational wave

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黎根亮, Tang, Wu, 2112.14041(SCPMA)

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14 **Eda, Itoh, Kuroyanagi & Silk, Yue & Cao, Yue, Han & Chen , Gong, Bertone et al ... ...**
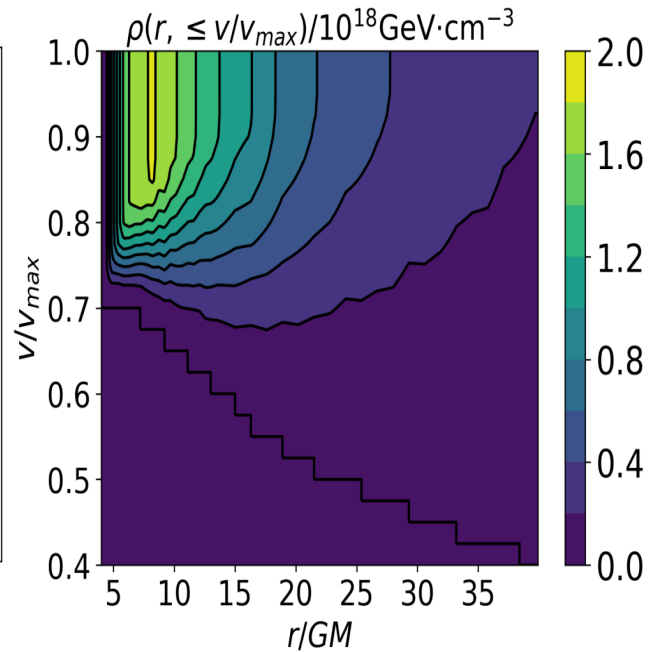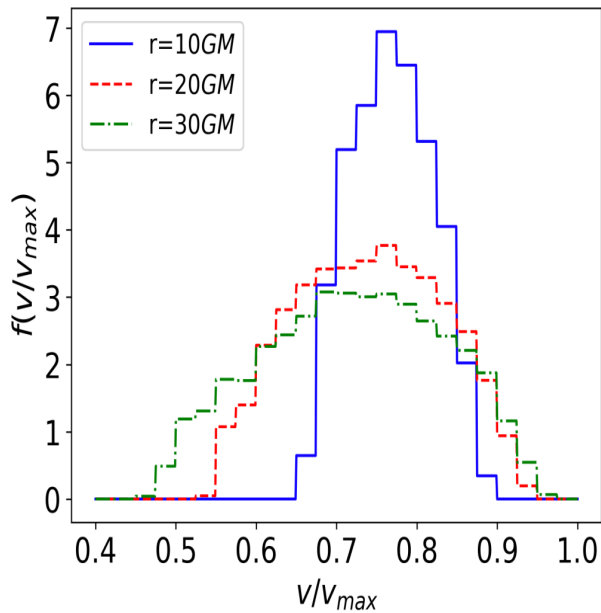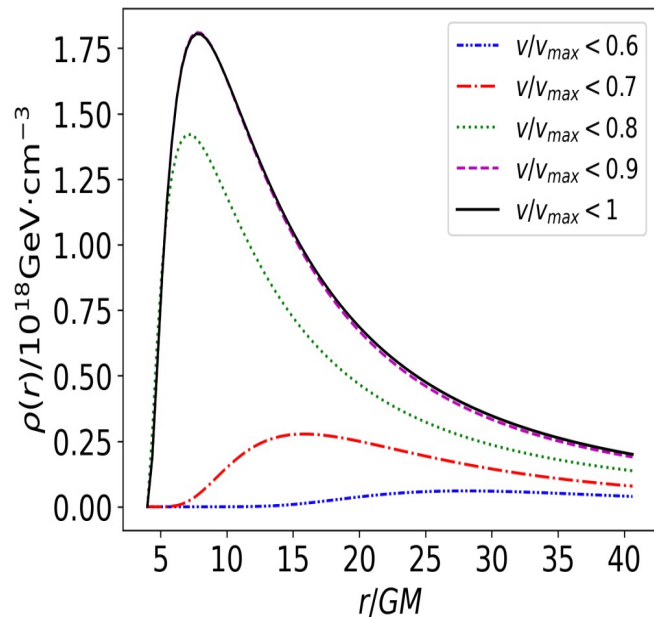
Density Distribution

➢ Relativistic treatment (Will et al)

$$\rho(r, \alpha = 1) = \frac{\kappa}{(r/GM)^\omega} \left(1 - \frac{4GM}{r}\right)^\eta.$$

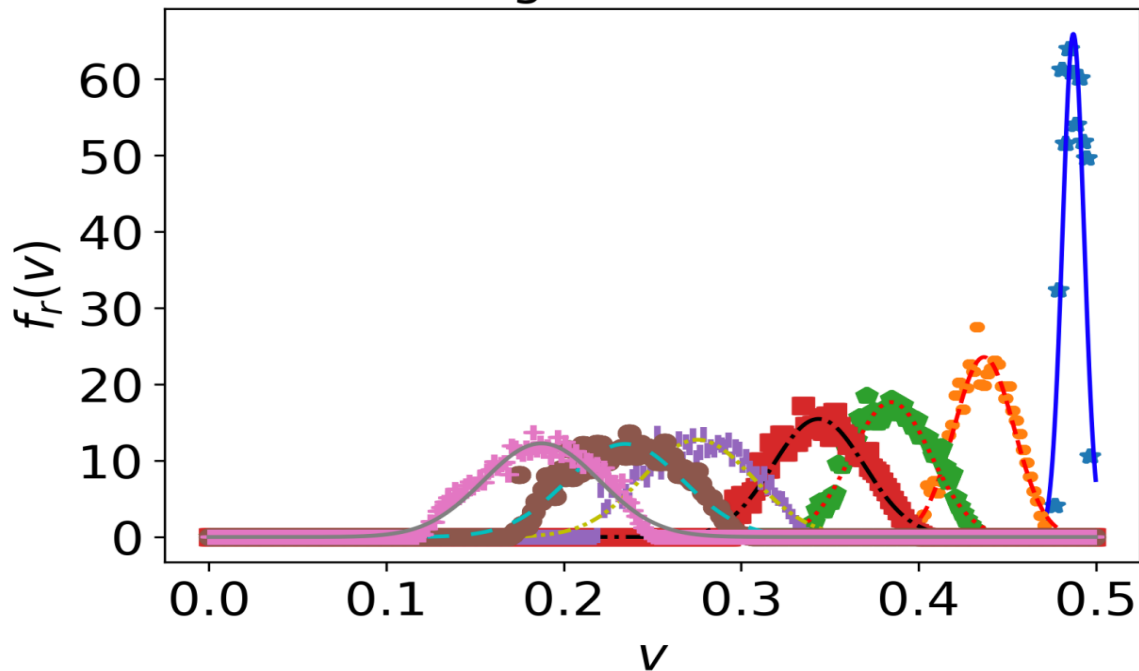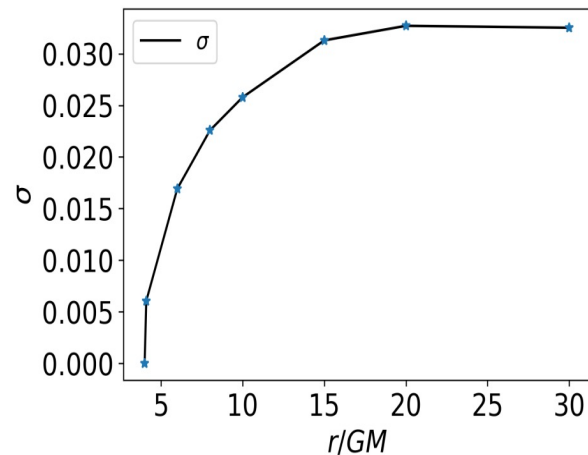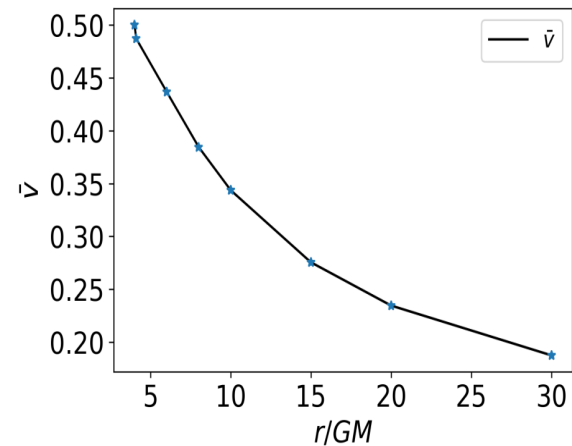| Halo parameters	Index	Fitting parameters
(M_{halo}, r_s)	(γ)	(κ, η, ω)
$(10^{12} M_\odot, 20 \text{ kpc})$	-	$(5.33 \times 10^{20} \text{ GeV/cm}^3, 1.99, 2.07)$
$(4.5 \times 10^8 M_\odot, 1.85 \text{ kpc})$	-	$(6.15 \times 10^{24} \text{ GeV/cm}^3, 2.03, 2.11)$
$(7.3 \times 10^8 M_\odot, 1.85 \text{ kpc})$	7/4	$(5.83 \times 10^{26} \text{ GeV/cm}^3, 2.04, 2.16)$

章子昌, Tang, 2403.18529 (PRD)

Velocity Distribution

➢ Fitted with Gaussian distribution

$$f_r(v) = \frac{1}{\sqrt{2\pi}\sigma(r)} \exp\left(-\frac{(v-\bar{v}(r))^2}{2\sigma^2(r)}\right),$$

章子昌, Tang, 2403.18529 (PRD)
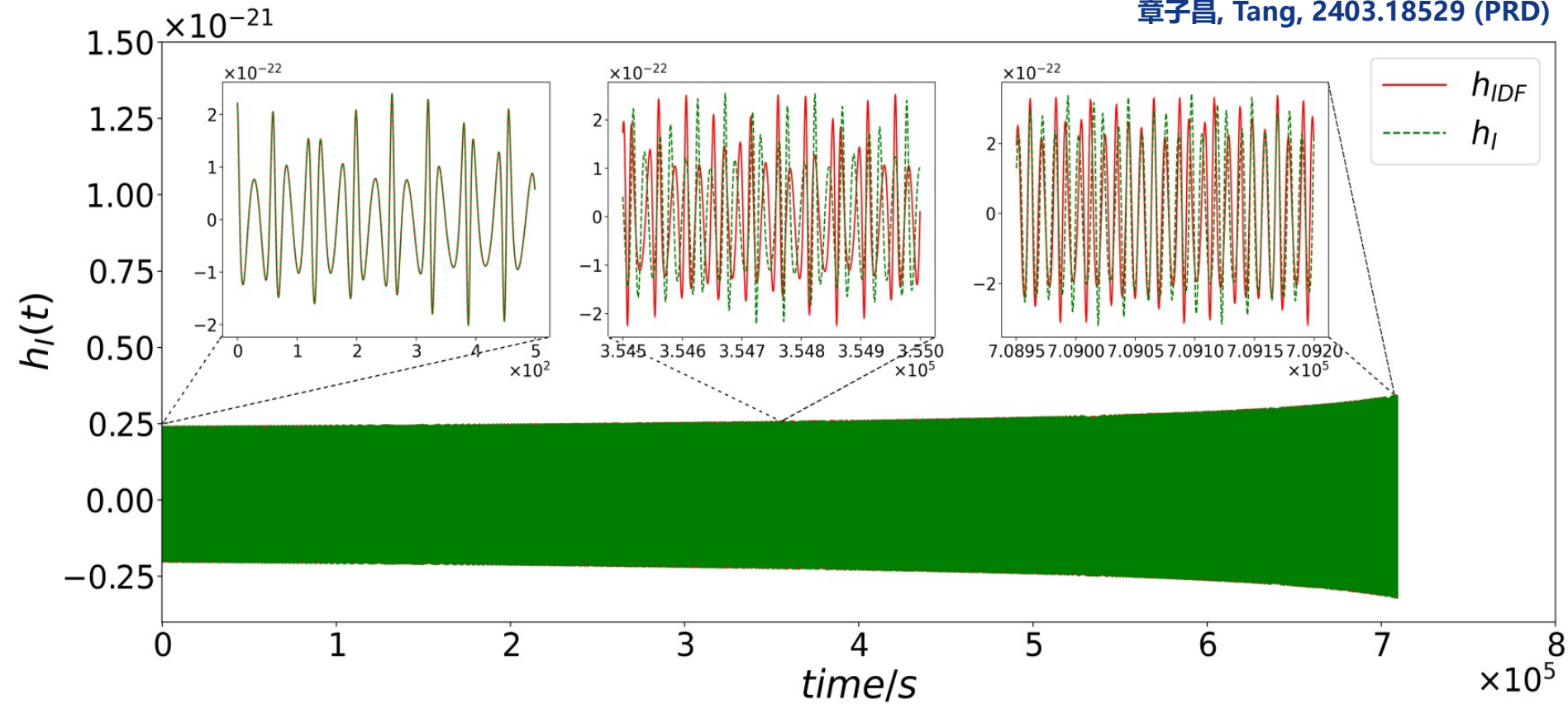

Fitting of distribution

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Effects on EMRI
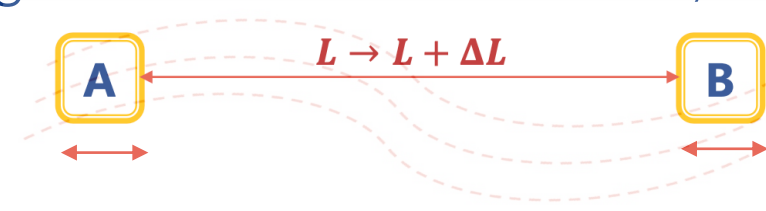
➤ Phase shift of the waveform of GW

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Ultralight/Wave DM - Signal Response

➢ DM couples to SM particles, inducing oscillations of test mass, effectively changing the length
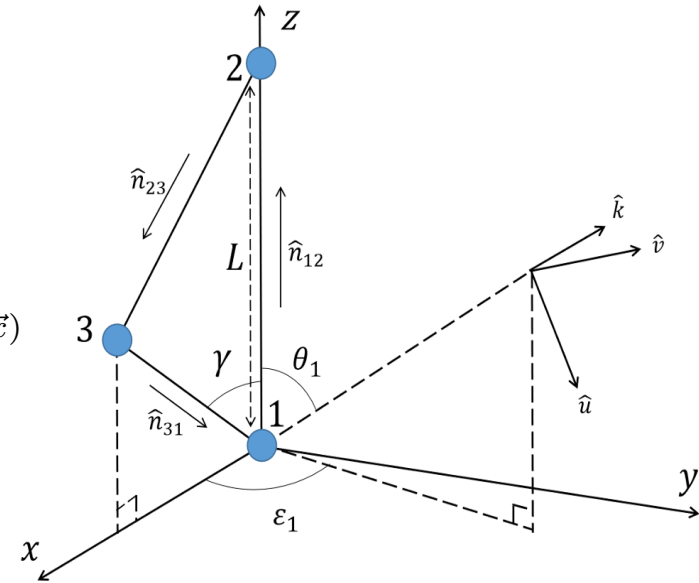
➢ One-way Doppler shift

$$\delta t_{rs} = -\hat{n}_{rs} \cdot [\delta\vec{x}(t, \vec{x}_r) - \delta\vec{x}(t - L, \vec{x}_s)],$$

$$\frac{\delta\nu_{rs}}{\nu_0} = \frac{\nu_{rs} - \nu_0}{\nu_0} = -\frac{d\,\delta t_{rs}}{dt}.$$

➢ Fractional frequency change

$$y_{rs}(t) \equiv \frac{\delta\nu_{rs}}{\nu_0} = \mu_{rs}\,[h(t, \vec{x}_r) - h(t - L, \vec{x}_s)],\ h(t, \vec{x}) \propto e^{im(t - v\hat{k}\cdot\vec{x})}$$

$$\mu_{rs} = \begin{cases} \hat{k} \cdot \hat{n}_{rs} & \text{for scalar field,} \\ \hat{e}_A \cdot \hat{n}_{rs} & \text{for vector field,} \\ \dfrac{\hat{n}_{rs}^i \hat{n}_{rs}^j e_{ij}(\hat{k}, \psi)}{2(1 + \hat{n}_{rs} \cdot \hat{k})} & \text{for gravitational wave,} \end{cases}$$

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Time-Delay Interferometry
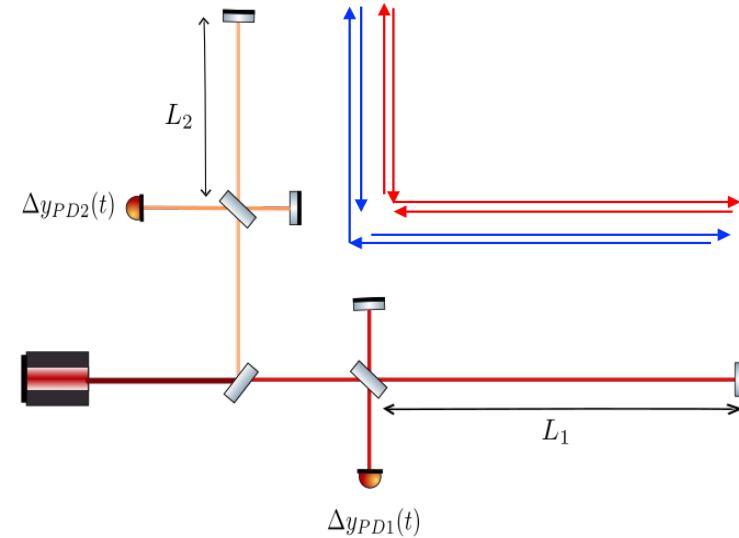
➤ The arm lengths are not equal

➤ Laser frequency noises dominate

$$X(t) \equiv [\Delta y_{PD1}(t) - \Delta y_{PD2}(t)] - [\Delta y_{PD1}(t - T_2) - \Delta y_{PD2}(t - T_1)]$$
$$= [H_1(t) - H_2(t) + p(t - T_1) - p(t - T_2)]$$
$$- [H_1(t - T_2) - H_2(t - T_1) + p(t - T_1) - p(t - T_2)]$$
$$= H_1(t) - H_2(t) - H_1(t - T_2) + H_2(t - T_1),$$

➤ Michelson interferometry

$$X(t) \equiv [\Delta y_{PD2}(t - T_1) + \Delta y_{PD1}(t)]$$
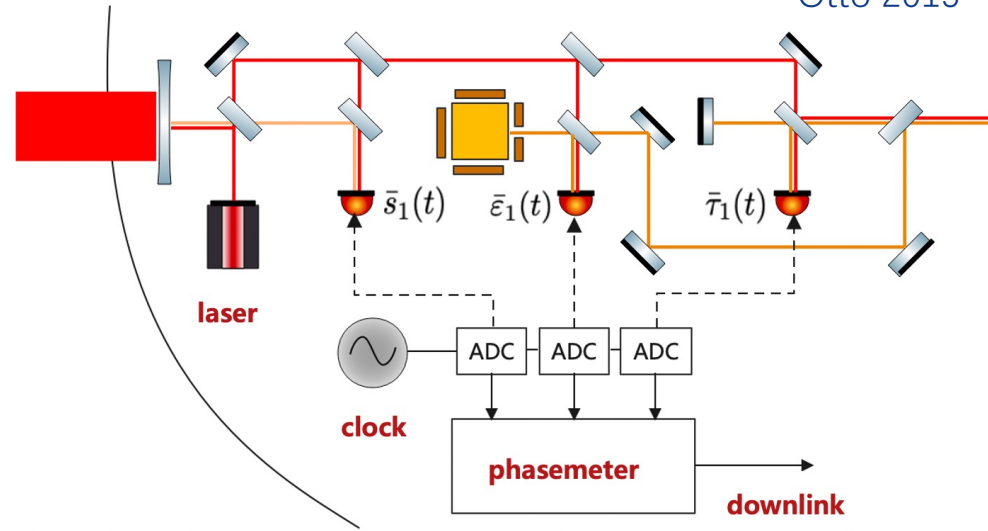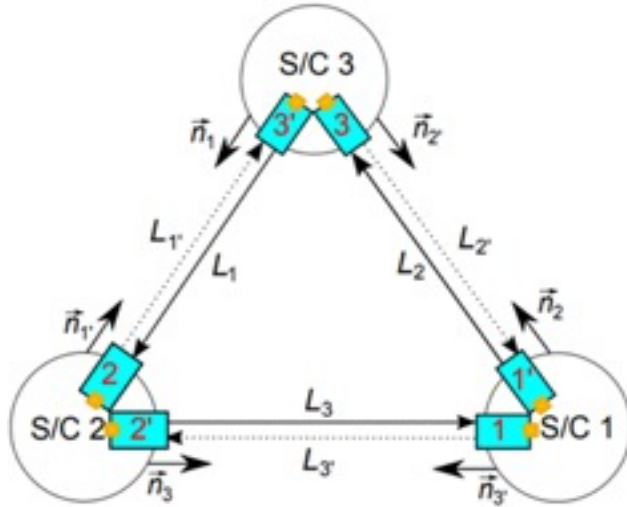$$- [\Delta y_{PD1}(t - T_2) + \Delta y_{PD2}(t)]$$

➤ TDI-virtual equal-arm interference

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$$\Delta y_{PD1}(t) = H_1(t) + p(t - T_1) - p(t),$$
$$\Delta y_{PD2}(t) = H_2(t) + p(t - T_2) - p(t),$$

$$T_1 = 2L_1 \qquad T_2 = 2L_2$$

Time-Delay Interferometry

➤ Input for TDI

$$\eta_{i'} \equiv s_{i'} + \frac{\varepsilon_{i'} - \tau_{i'}}{2} + D_{i+1'}\frac{\varepsilon_{i-1} - \tau_{i-1}}{2} + \frac{\tau_i - \tau_{i'}}{2}$$

$$\eta_i \equiv s_i + \frac{\varepsilon_i - \tau_i}{2} + D_{i-1}\frac{\varepsilon_{i+1'} - \tau_{i+1'}}{2} - D_{i-1}\frac{\tau_{i+1} - \tau_{i+1'}}{2}$$

$$\eta_{1'} \sim D_{2'}p_3 - p_1, \quad \eta_1 \sim D_3 p_2 - p_1,$$
$$\eta_{2'} \sim D_{3'}p_1 - p_2, \quad \eta_2 \sim D_1 p_3 - p_2,$$
$$\eta_{3'} \sim D_{1'}p_2 - p_3, \quad \eta_3 \sim D_2 p_1 - p_3.$$

➤ TDI cancels laser frequency noise

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Time–Delay Interferometry

➢ There are multiple combinations

➢ Michelson channels

$$X(t) = (\eta_{2':322'} + \eta_{1:22'} + \eta_{3:2'} + \eta_{1'}) - (\eta_{3:2'3'3} + \eta_{1':3'3} + \eta_{2':3} + \eta_1),$$
$$Y(t) = (\eta_{3':133'} + \eta_{2:33'} + \eta_{1:3'} + \eta_{2'}) - (\eta_{1:3'1'1} + \eta_{2':1'1} + \eta_{3':1} + \eta_2),$$
$$Z(t) = (\eta_{1':211'} + \eta_{3:11'} + \eta_{2:1'} + \eta_{3'}) - (\eta_{2:1'2'2} + \eta_{3':2'2} + \eta_{1':2} + \eta_3).$$
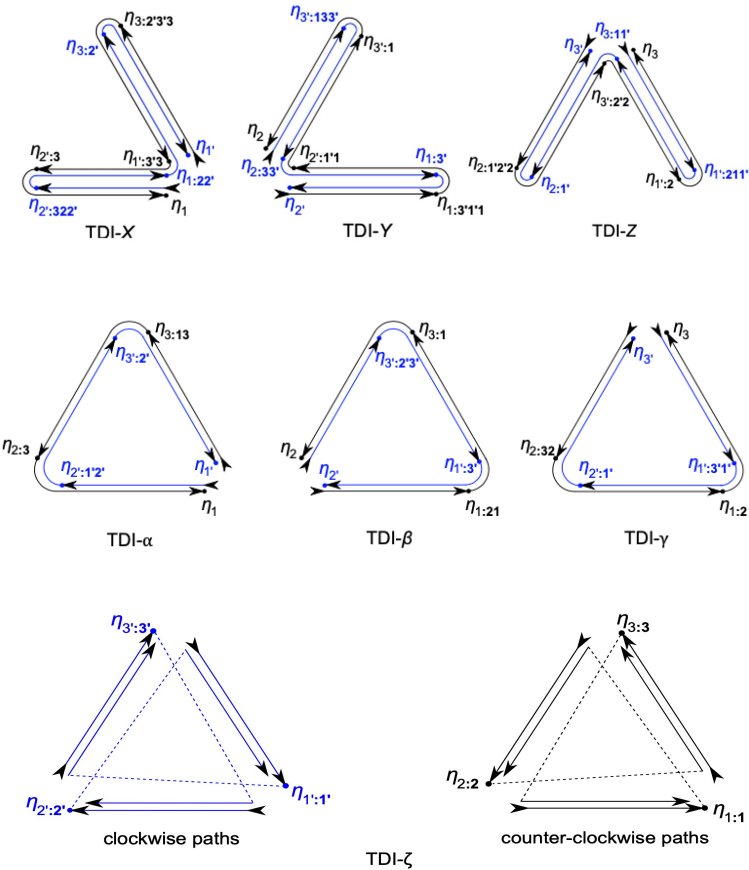
➢ Sagnac channels

$$\alpha(t) = (\eta_{2':1'2'} + \eta_{3':2'} + \eta_{1'}) - (\eta_{3:13} + \eta_{2:3} + \eta_1),$$
$$\beta(t) = (\eta_{3':2'3'} + \eta_{1':3'} + \eta_{2'}) - (\eta_{1:21} + \eta_{3:1} + \eta_2),$$
$$\gamma(t) = (\eta_{1':3'1'} + \eta_{2':1'} + \eta_{3'}) - (\eta_{2:32} + \eta_{1:2} + \eta_3).$$

➢ ζ channel

$$\zeta(t) = (\eta_{1':1'} + \eta_{2':2'} + \eta_{3':3'}) - (\eta_{1:1} + \eta_{2:2} + \eta_{3:3}).$$

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Transfer Functions

➢ Fourier transform

$$h(t) = \frac{\sqrt{T}}{2\pi} \int_0^\infty \tilde{h}(\omega) e^{i\omega t} d\omega$$

➢ One-way single link

$$y_{rs}(t) = \mu_{rs} \, \frac{\sqrt{T}}{2\pi} \int_0^\infty d\omega \, \tilde{h}(\omega) e^{i\omega t} \left[e^{-i\vec{k}\cdot\vec{x}_r} - e^{-i(\tau + \vec{k}\cdot\vec{x}_s)} \right],$$

$$\tilde{y}_{rs}(\omega) = \mu_{rs} \, \tilde{h}(\omega) \left[e^{-i(\vec{k}\cdot\vec{x}_r)} - e^{-i(\tau + \vec{k}\cdot\vec{x}_s)} \right].$$
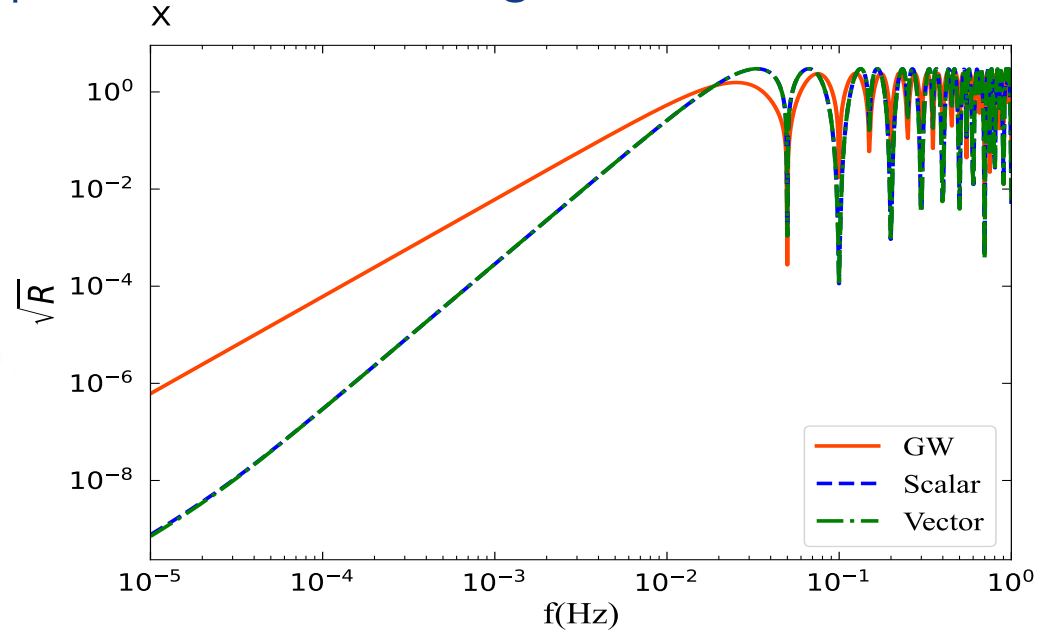
➢ Transfer function, sky and polarization averaged

$$R(\omega) = \overline{\left| \frac{\tilde{y}_{rs}(\omega)}{\tilde{h}(\omega)} \right|^2},$$

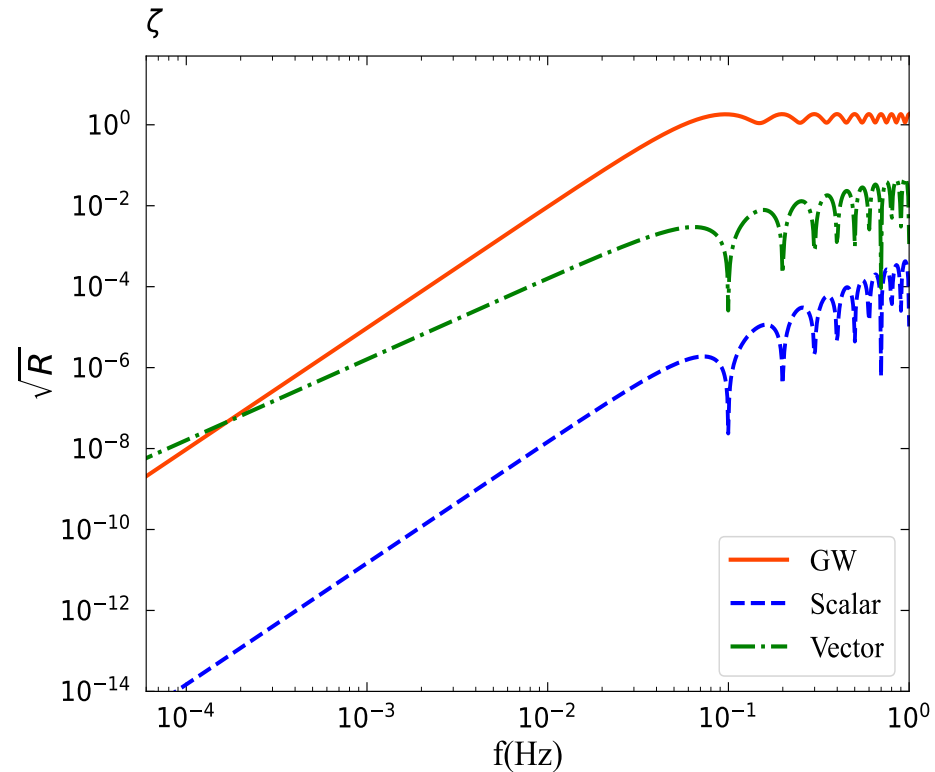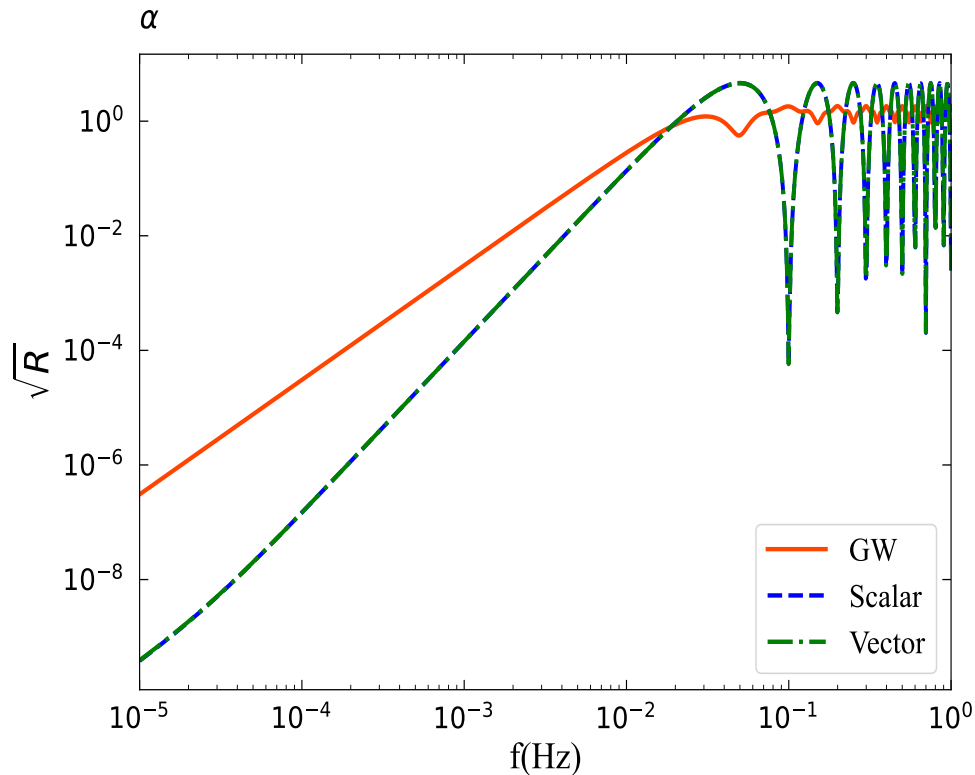$$I_s \equiv \frac{1}{4\pi} \int_{-1}^1 d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \cdots,$$

$$I_v \equiv \frac{1}{16\pi^2} \int_{-1}^1 d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \int_{-1}^1 d\cos\theta_2 \int_0^{2\pi} d\epsilon_2 \cdots$$

$$I_{GW} \equiv \frac{1}{8\pi^2} \int_{-1}^1 d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \int_0^{2\pi} d\psi \cdots.$$

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Transfer Functions

➢ Different channels have different transfer functions

➢ DM is also different from gravitational wave, velocity effect, ⋯

Sensitivity

➢ Defined by $S_O(f) = \dfrac{N_O(f)}{R_O(f)}, \quad N_X = 16\sin^2(\tau)\{[3+\cos(2\tau)]\,S_{acc} + S_{oms}\}, \quad \tau = 2\pi f L$
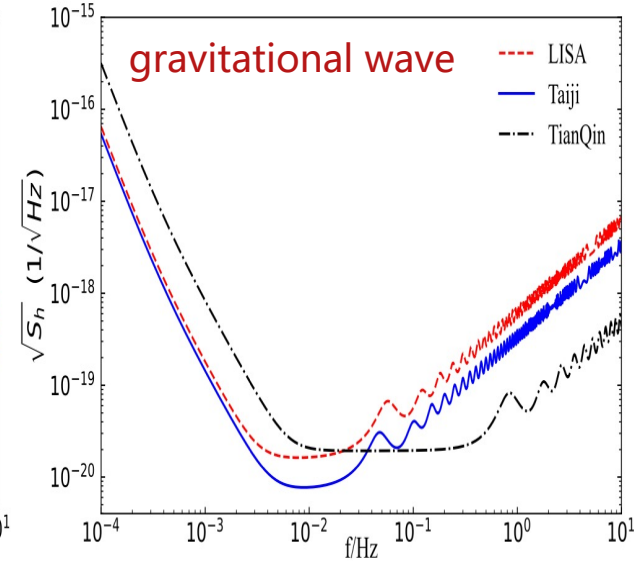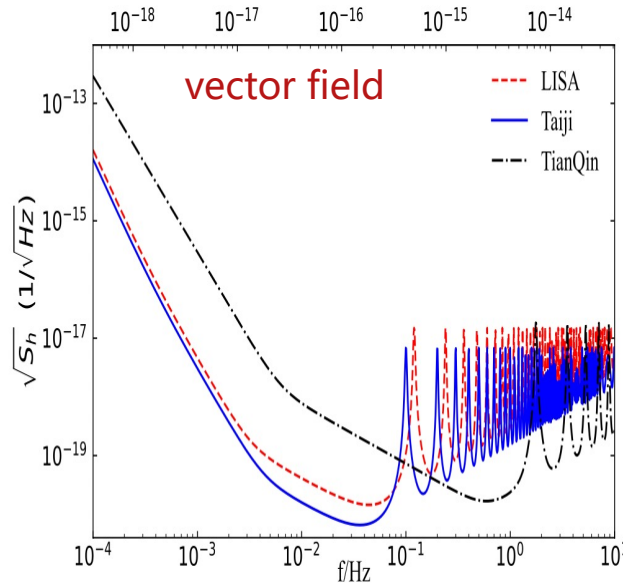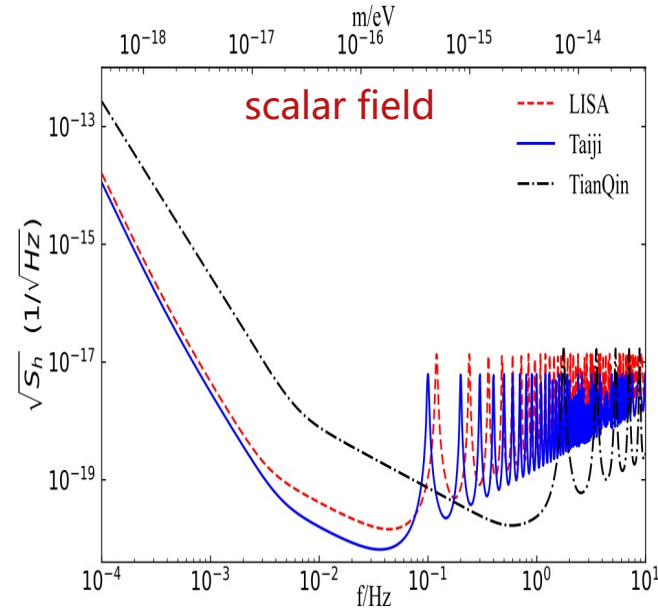
$$S_{oms}(f) = \left(s_{oms}\,\frac{2\pi f}{c}\right)^2\left[1+\left(\frac{2\times10^{-3}\text{ Hz}}{f}\right)^4\right]\frac{1}{\text{Hz}},$$

$$S_{acc}(f) = \left(\frac{s_{acc}}{2\pi f c}\right)^2\left[1+\left(\frac{0.4\times10^{-3}\text{ Hz}}{f}\right)^2\right]\left[1+\left(\frac{f}{8\times10^{-3}\text{ Hz}}\right)^4\right]\frac{1}{\text{Hz}},$$

LISA : $s_{oms} = 15\times10^{-12}$ m, $s_{acc} = 3\times10^{-15}$ m/s^2,

Taiji : $s_{oms} = 8\times10^{-12}$ m, $s_{acc} = 3\times10^{-15}$ m/s^2,
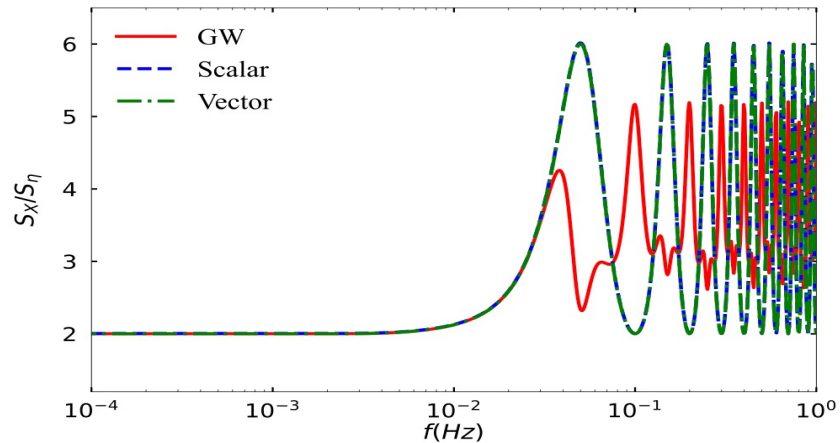
TianQin : $s_{oms} = 1\times10^{-12}$ m, $s_{acc} = 1\times10^{-15}$ m/s^2.

Sensitivity

➤ Optimal channels $A = \frac{1}{\sqrt{2}}[Z - X], E = \frac{1}{\sqrt{6}}[X - 2Y + Z], T = \frac{1}{\sqrt{3}}[X + Y + Z]. \quad \frac{1}{S_\eta} = \frac{1}{S_A} + \frac{1}{S_E} + \frac{1}{S_T}$

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Sensitivity on scalar DM

➤ Strong sector $\delta\mathcal{L} = \dfrac{\phi}{M_P}\left[-\dfrac{d_g \beta_3}{2g_3} F^A_{\mu\nu} F^{A\mu\nu} - \sum_{i=u,d} \left(d_{m_i} + \gamma_{m_i} d_g\right) m_i \bar{\psi}_i \psi_i \right]$ Damour & Donoghue

$$d_{\hat{m}} \equiv \frac{d_{m_d} m_d + d_{m_u} m_u}{m_d + m_u},$$
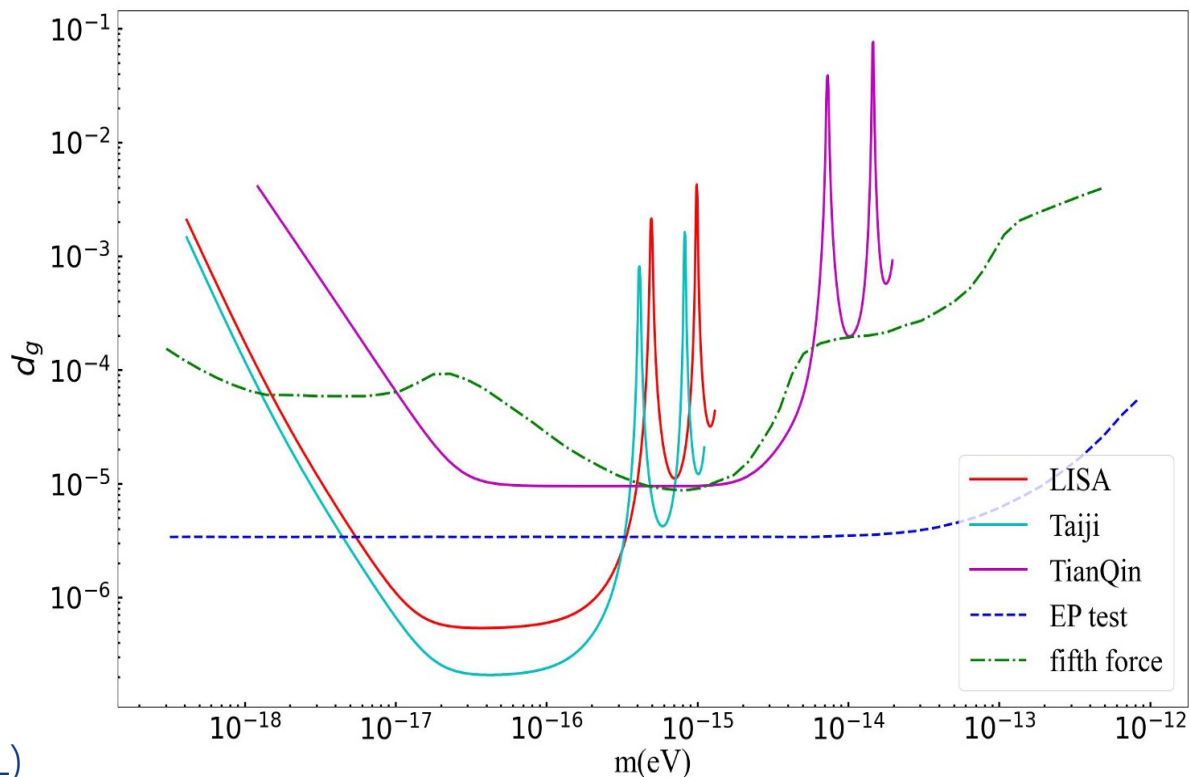
$$d_g^* \approx d_g + 0.093(d_{\hat{m}} - d_g).$$

assuming $d_m = 0$ and $d_g^* \approx 0.9 d_g$.

➤ Equivalence principle is violated.

➤ MICROSCOPE
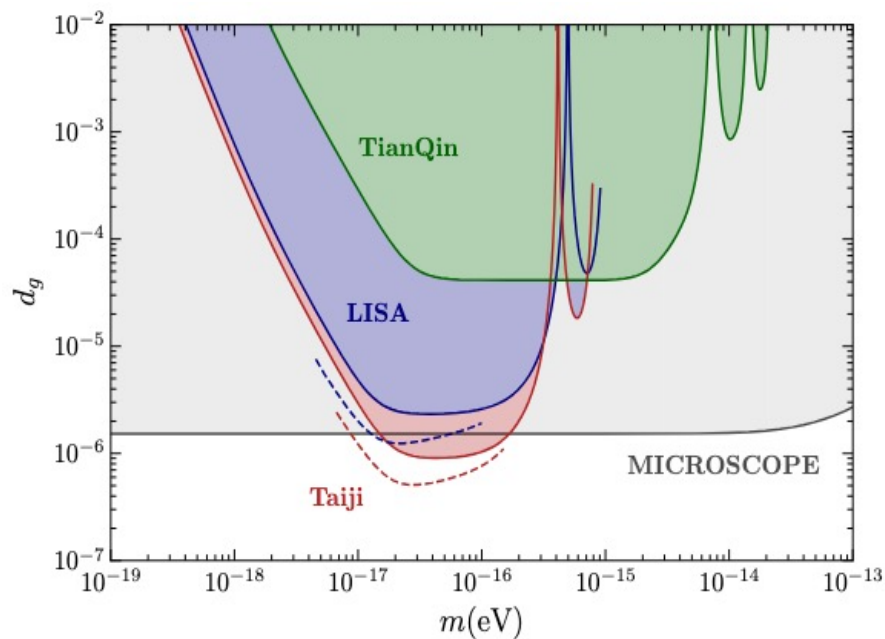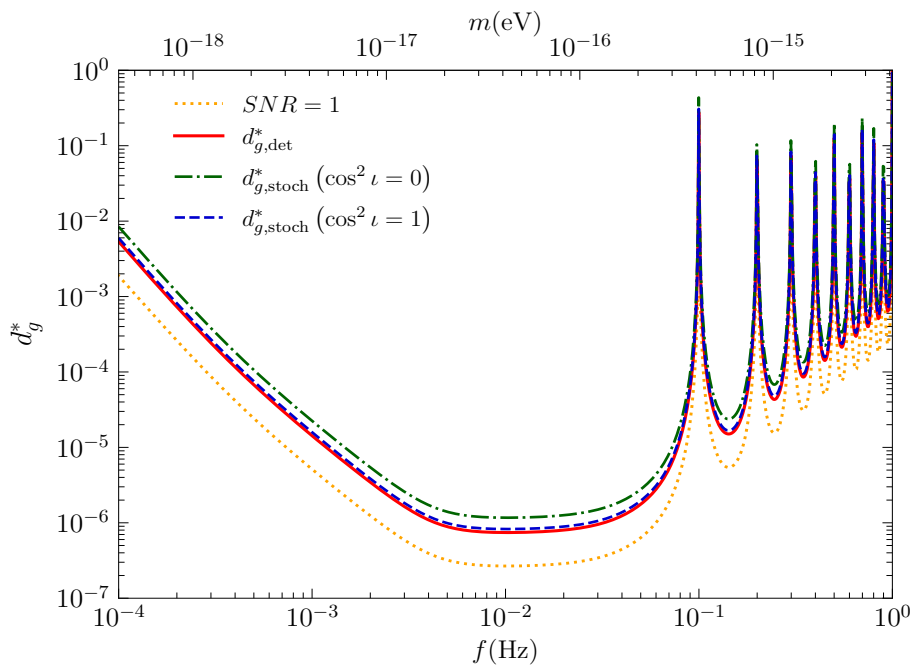
➤ Pulsars Shao, Wex, Kramer, arXiv:1805.08408(PRL)

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Yu, Yao, Tang, Wu, 2307.09197(PRD)

Statistical Effects

➢ Velocity distributions, likelihood analysis

$$S_O(\lambda_{\min}) = \left(\frac{\ln \alpha}{\ln \gamma} - 1\right) N_O, \quad \lambda_{\min}^2 = \frac{N_O}{\Gamma_O S_\Phi}\left(\frac{\ln \alpha}{\ln \gamma} - 1\right).$$

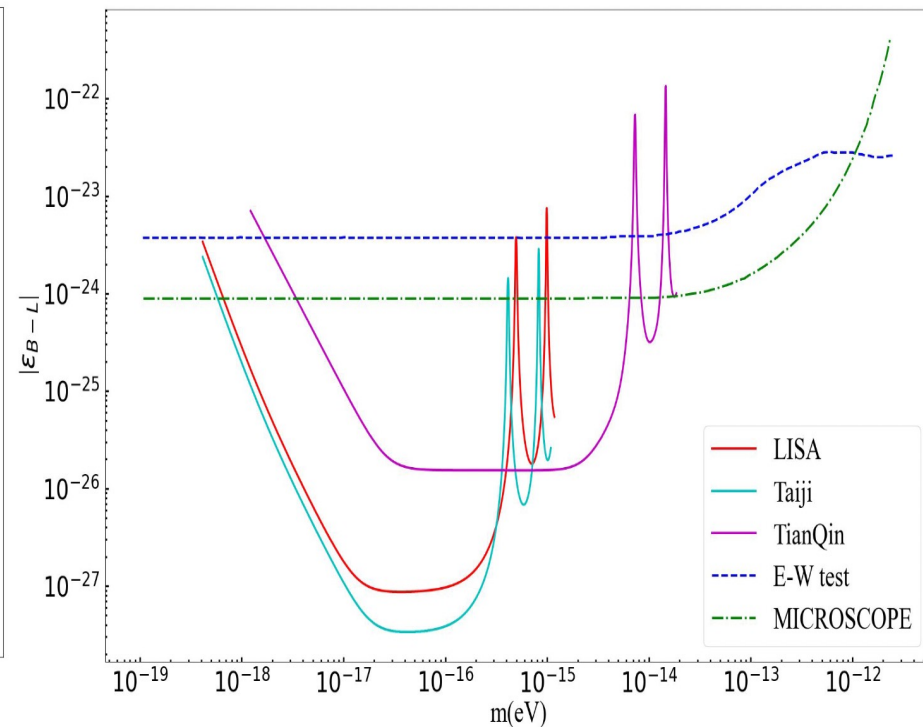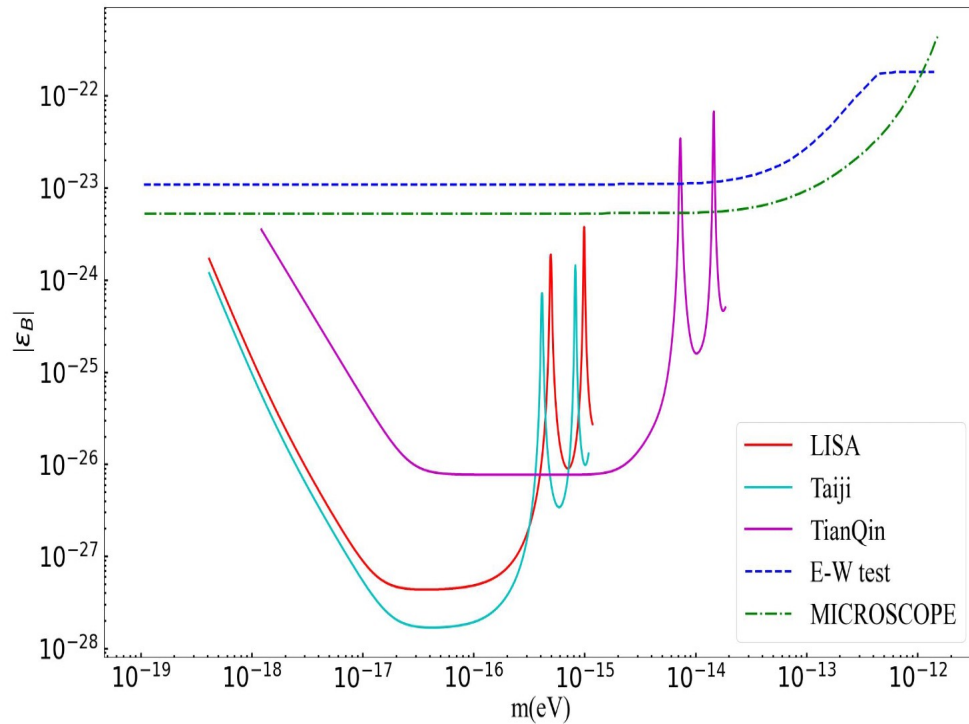$$\gamma = \int_{P_*(\alpha)}^{\infty} dP \, \mathcal{L}_{\text{stoch}}\left(\tilde{d}\Big| S_O(\lambda_{\min}), N_O\right),$$ detection probability

$$\alpha = \int_{P_*}^{\infty} dP \, \mathcal{L}_{\text{stoch}}\left(P\Big| \lambda = 0, N_O\right).$$ false alarm rate

Sensitivity on vector DM

➤ For example, vector fields couple to baryon number *B*, or *B-L*, effectively neutron number. Sensitivity on ratio $\epsilon_D = e_D/e$

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Detecting Ultralight Dark Matter Gravitationally

➢ Can we probe DM with gravitational interaction in solar system

➢ What if dark matter has only gravitational interaction?

➢ Motivations from theories of gravity, particle physics, …

➢ Viable scenarios in cosmology

  ➢ Axion or axion-like particle, misalignment

  ➢ Dark photon, vacuum fluctuation
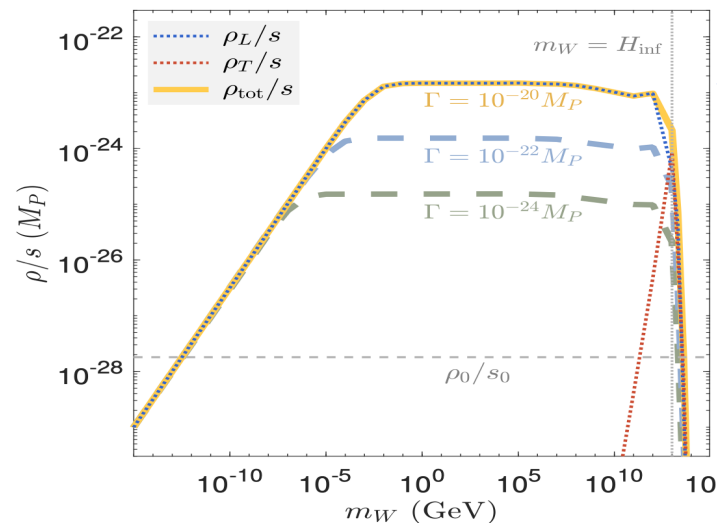
  王清扬, Tang, Wu, 2203.15452(PRD)

  Graham, Mardon, Rajendran,1504.02102(PRD)

  Ema, Nakayama, Tang, 1903.10973(JHEP),

  Ahmed, Grzadkowski, Socha, 2005.01766(JHEP)

  Kolb, Long, 2009.03828(JHEP), …

Detecting Ultralight Dark Matter Gravitationally

➢ Metric perturbation in solar system

$$ds^2 = -(1 + 2\Psi)dt^2 + (1 - 2\Phi)\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j$$

➢ Einstein equations

$$\partial_i \partial^i \Phi = 4\pi G\, T_{00},$$

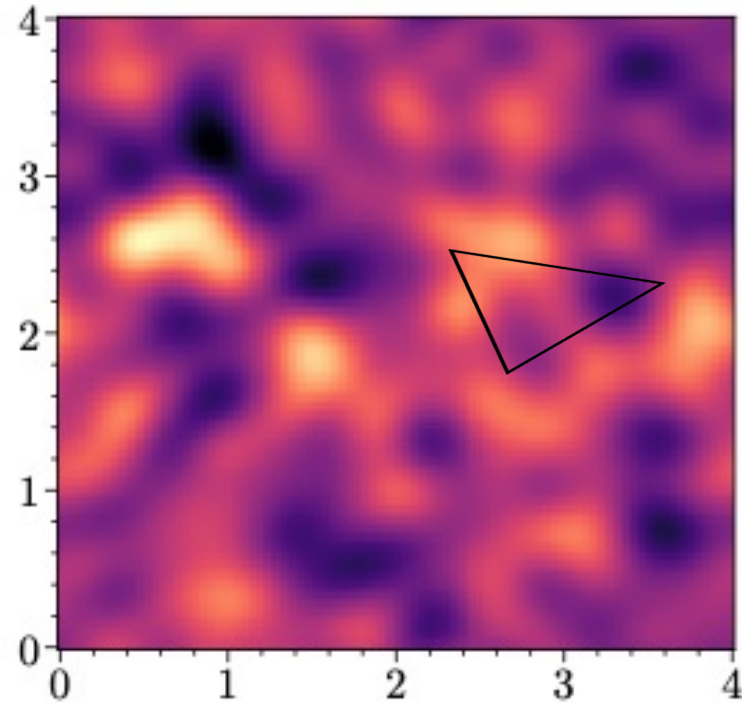$$3\ddot{\Phi} + \partial_i \partial^i (\Psi - \Phi) = 4\pi G\, T_k^k,$$

$$\ddot{h}_{ij} = 16\pi G \left(T_{ij} - \frac{1}{3}\delta_{ij}T_k^k \right),$$

$$\Psi^j \simeq \Phi^j \simeq \pi G \frac{\rho}{m^2} = \frac{7 \times 10^{-26}\rho}{0.4\,\mathrm{GeV/cm^3}} \left(\frac{10^{-18}\mathrm{eV}}{m} \right)^2,$$

$$h_{ij}^v \propto h_0 \simeq \frac{8}{3}\pi G \frac{\rho}{m^2} = \frac{2 \times 10^{-25}\rho}{0.4\,\mathrm{GeV/cm^3}} \left(\frac{10^{-18}\mathrm{eV}}{m} \right)^2,$$
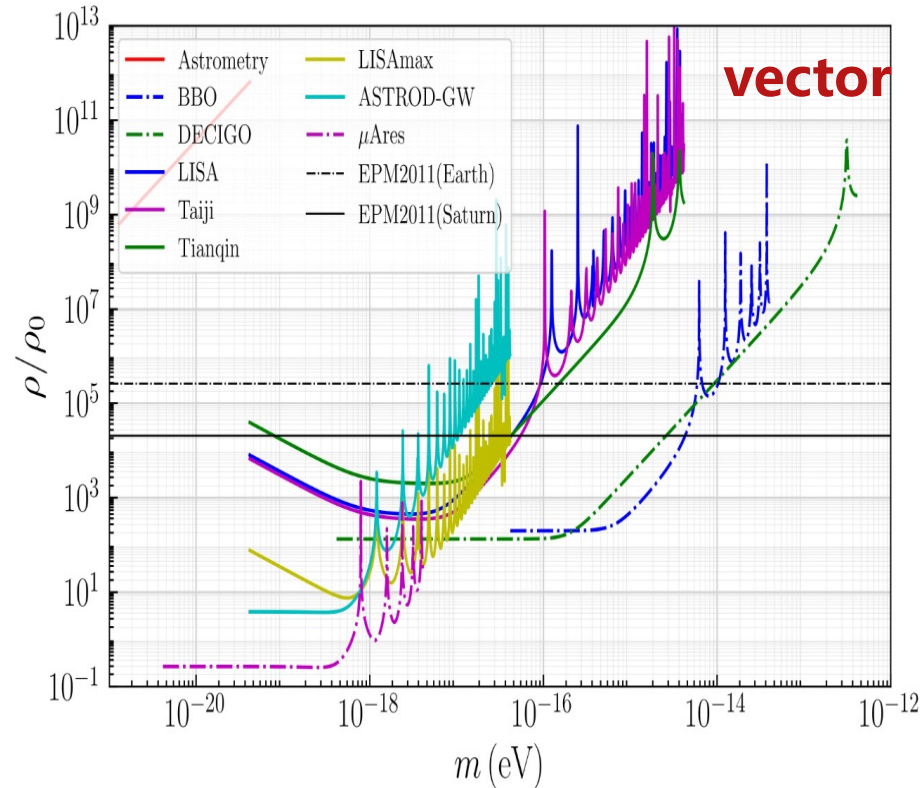
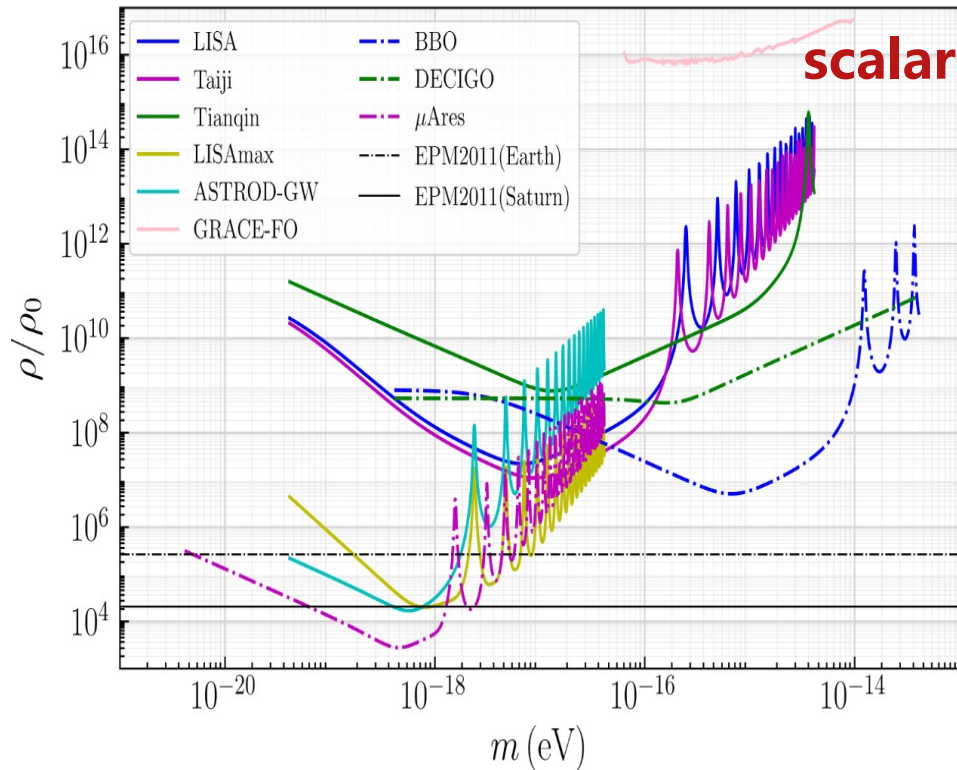$$h_{ij}^s \simeq h_0 v^2/2$$

➢ Tensor perturbation for scalar and

tensor ULDM is suppressed.

Detecting Ultralight Dark Matter Gravitationally

➤ Metric perturbation in solar system

$$ds^2 = -(1 + 2\Psi)dt^2 + (1 - 2\Phi)\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j$$

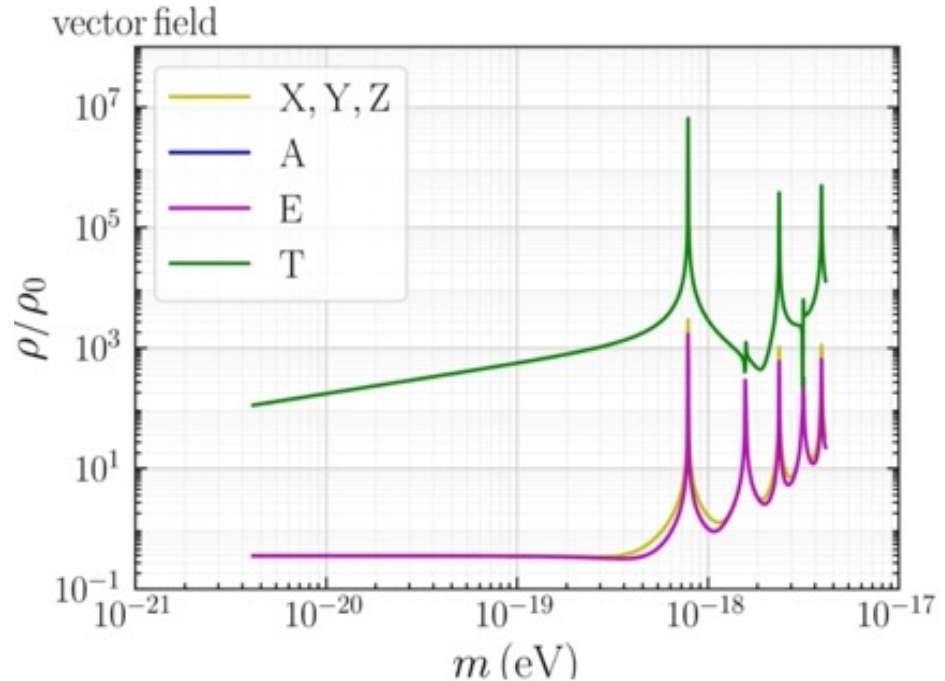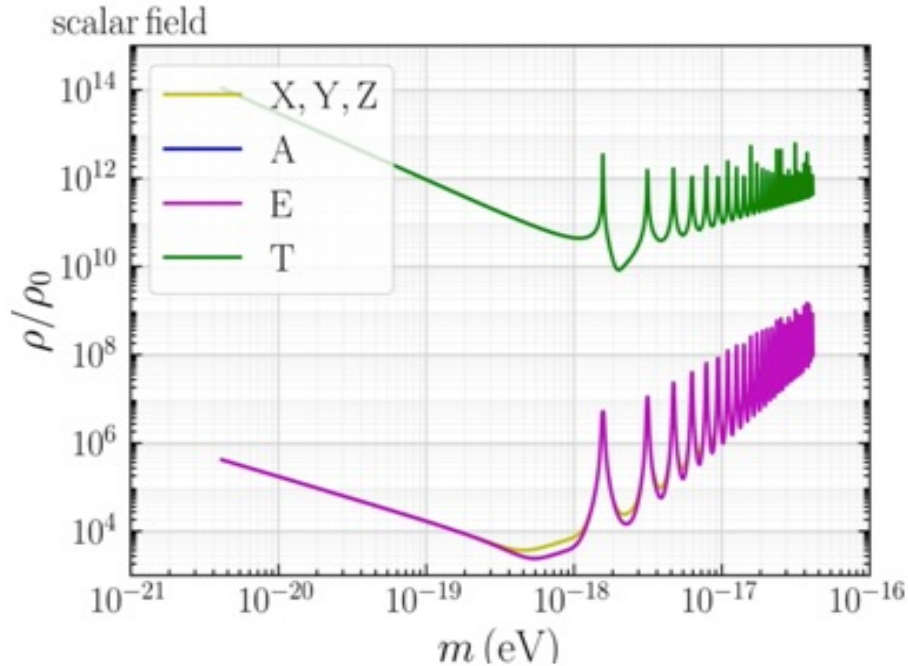**Yu, Cao, Tang, Wu, 2404.04333 (PRD)**

Detecting Ultralight Dark Matter Gravitationally

➢ Metric perturbation in solar system

$$ds^2 = -(1 + 2\Psi)dt^2 + (1 - 2\Phi)\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j$$

➢ Different channels have different sensitivities

Yu, Cao, Tang, Wu, 2404.04333 (PRD)

Summary

➢ We discuss how gravitational-wave detectors in space may help to understand the nature of dark matter.

➢ Weakly-interacting massive particles

  ➢ can form spikes around black holes and affect the orbiting compact objects, imprinting on the waveform of GW.

  ➢ Relativistic effect, velocity distribution, …

➢ Ultralight bosonic, wave like

  ➢ can couple to normal matter and induce the oscillation of test masses, leading to signals in detectors and very good sensitivities.

  ➢ Metric perturbation by vector ULDM may be detectable in future.

Thanks!