2024年紫金山暗物质研讨会
江苏/10.11-15

Primordial gravitational atom

康召丰

Based on work 2407.13385,
ZK, Tianjun Li & Weitao Ye

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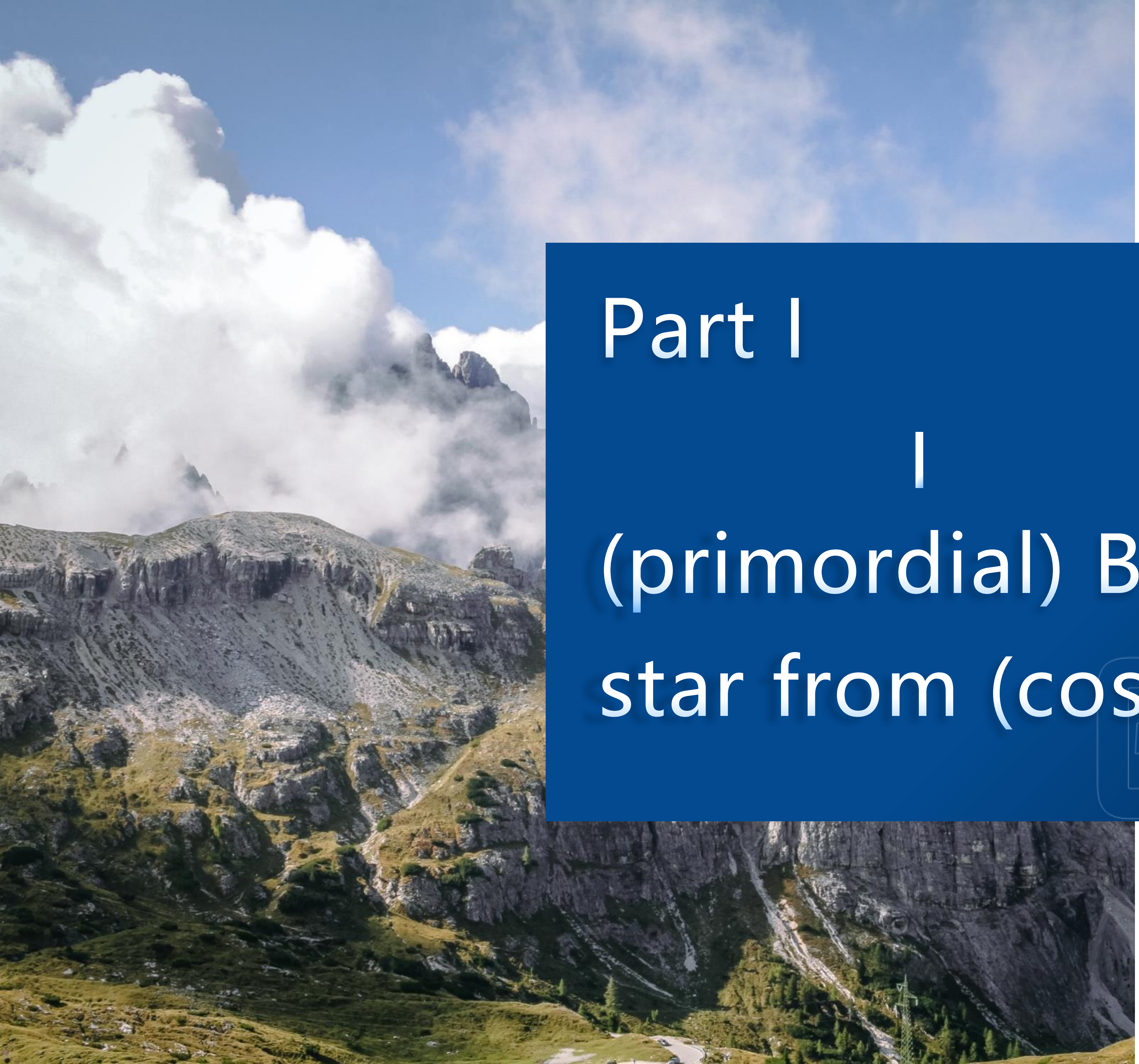


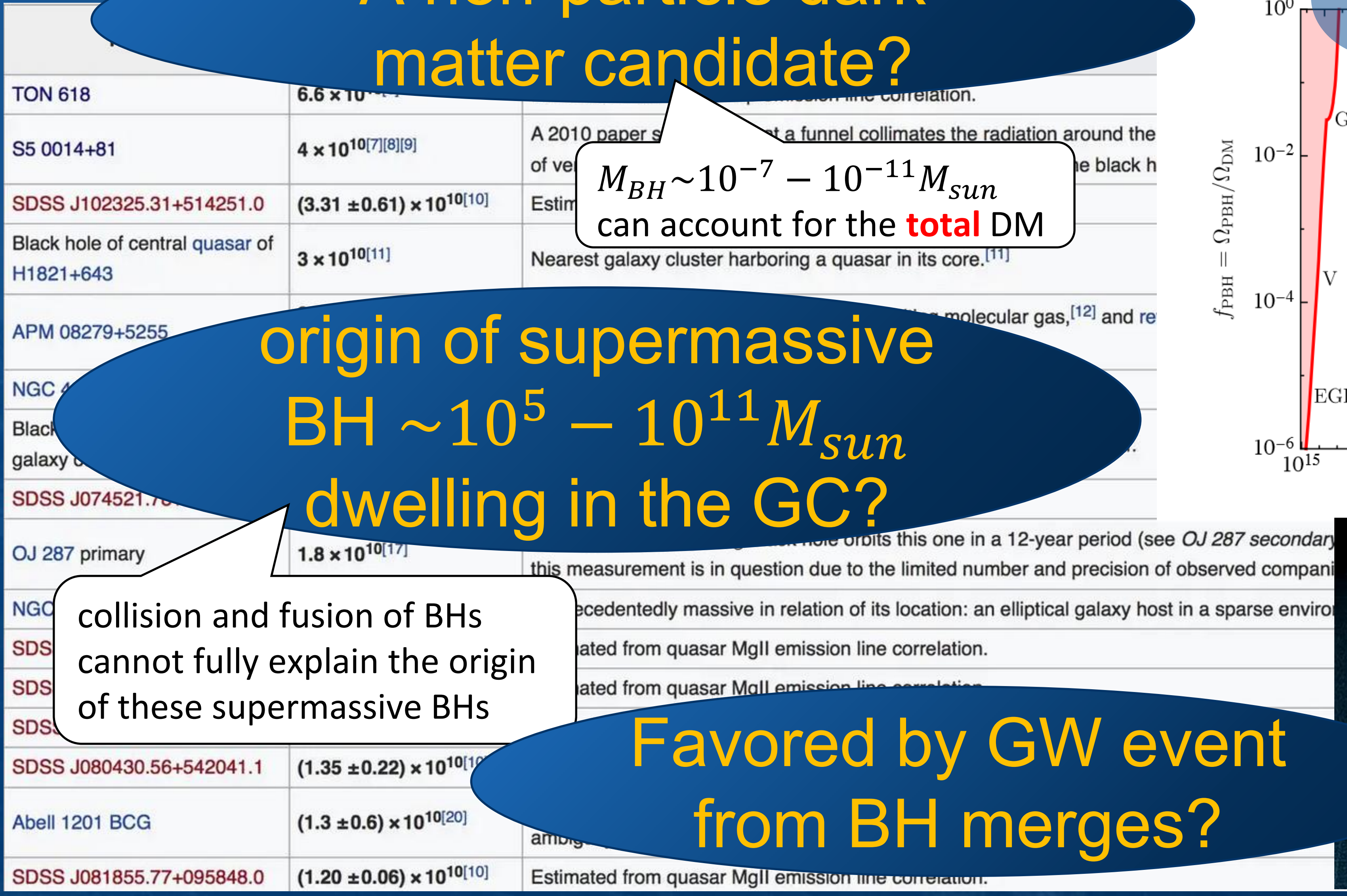
Part I

I

(primordial) Black hole: A super star from (cosmology) gravity

GEOGRAPHY

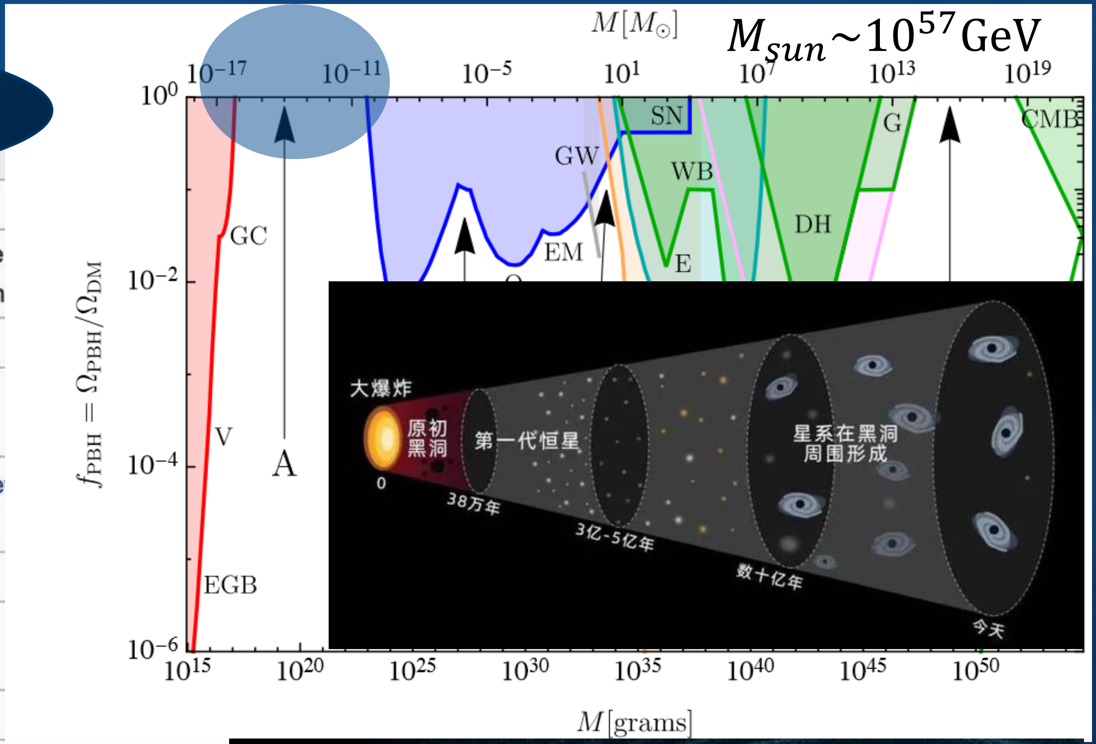
Why interest in PBH if you are a particle worker?



TON 618	6.6 × 10…	…correlation.
S5 0014+81	4 × 10^10 [7][8][9]	A 2010 paper s… a funnel collimates the radiation around the … of ve… …e black h…
SDSS J102325.31+514251.0	(3.31 ±0.61) × 10^10 [10]	Estim…
Black hole of central quasar of H1821+643	3 × 10^10 [11]	Nearest galaxy cluster harboring a quasar in its core.[11]
APM 08279+5255		…molecular gas,[12] and re…
NGC 4…		
Black… galaxy …		
SDSS J074521.7…		
OJ 287 primary	1.8 × 10^10 [17]	…orbits this one in a 12-year period (see OJ 287 secondary… this measurement is in question due to the limited number and precision of observed compani…
NGC…		…ecedentedly massive in relation of its location: an elliptical galaxy host in a sparse enviro…
SDS…		…ated from quasar MgII emission line correlation.
SDS…		…ated from quasar MgII emission line correlation.
SDS…		
SDSS J080430.56+542041.1	(1.35 ±0.22) × 10^10 [1…	
Abell 1201 BCG	(1.3 ±0.6) × 10^10 [20]	ambig…
SDSS J081855.77+095848.0	(1.20 ±0.06) × 10^10 [10]	Estimated from quasar MgII emission line correlation.

A non-particle dark matter candidate?

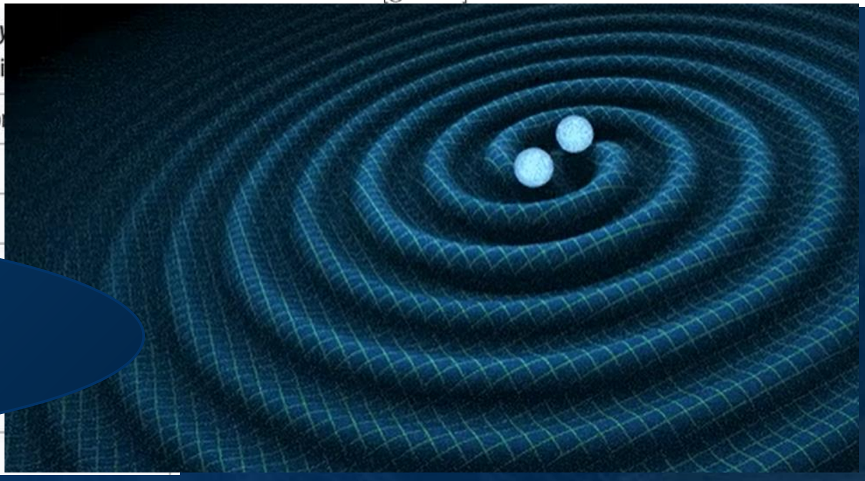
$M_{BH} \sim 10^{-7} - 10^{-11} M_{sun}$ can account for the **total** DM



origin of supermassive BH $\sim 10^{5} - 10^{11} M_{sun}$ dwelling in the GC?

collision and fusion of BHs cannot fully explain the origin of these supermassive BHs

Favored by GW event from BH merges?

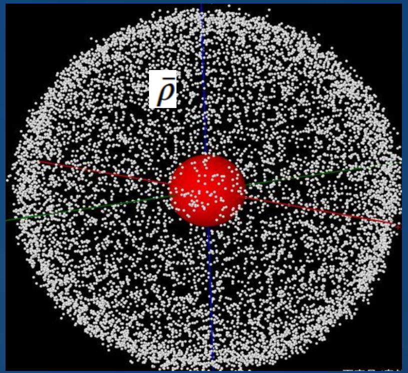


Primordial BH (PBH) zoo in the early universe

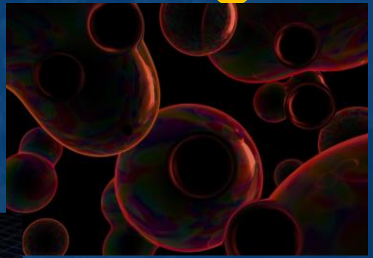
PBH is by no means a rare guest in the universe

Khlopov & Polnarev 1980

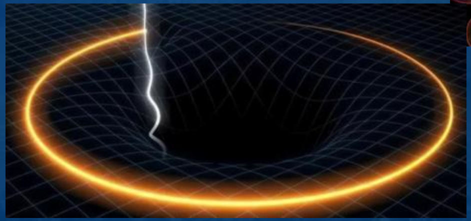
- Collapse from inhomogeneities during RD era
- Collapse from single/multi-field inflation
- Collapse from inhomogeneities during MD era



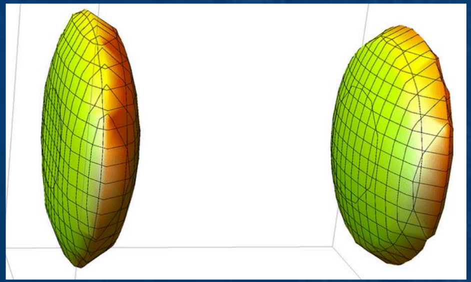
- Collapse from bubble collision



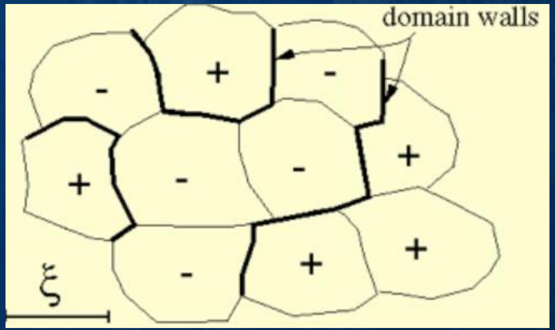
- Collapse of cosmic string loops



- Collapse of scalar condensate



- Collapse of domain walls



Primordial fluctuations as seeds of PBH

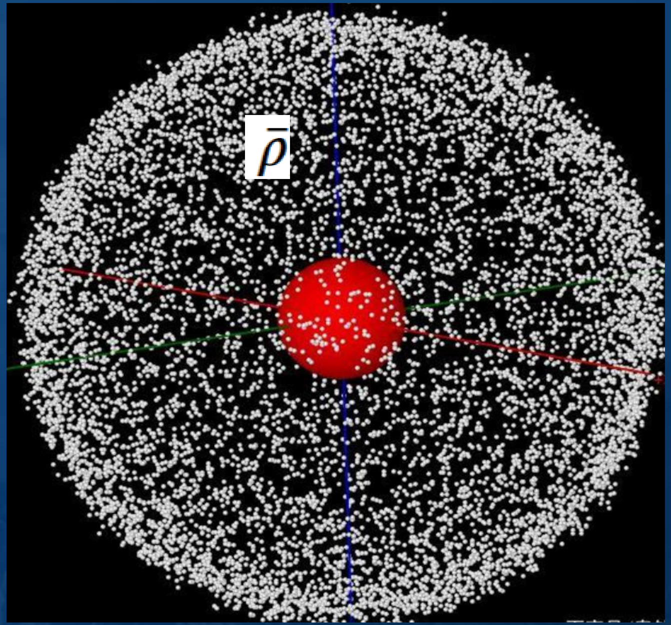
- perturbation on scale k with density contrast $\delta_k \equiv (\rho - \bar{\rho})/\bar{\rho}$

1. Before entering horizon, the physical scale of perturbation continues to expand according to $a(t)/k$

2. Entering horizon as $\frac{a(t_H(k)))}{k} = \frac{1}{H(t_H(k))}$, with density contrast δ_H following $P(\delta_H) = \frac{1}{\sqrt{2\pi}\sigma_H} \exp(-\delta_H^2/2\sigma_H^2)$

3. Perturbation gradually decouples from the Hubble flow due to gravity, making $\delta \propto \delta_H t^{2/3}$ until t_d when $\delta \sim 1$ and perturbation begins to collapse

4. PBH mass, the Hubble mass at horizon crossing: $M_{\rm BH} = \frac{M_{\rm PL}^2}{2H(t_H(k))} \propto t_H(k)$



PBH formation in the dust-like era

No pressure to against gravity, collapse to PBH is enhanced

- $\beta(k)$: probability of perturbation k collapse to a PBH

1. Overestimates the probability in the spherical limit $\beta_0 \sim (\sigma_H/\delta_c)\exp(-\delta_c^2/2\sigma_H^2)$

by Jeans criterion $\delta_c \simeq \left(\frac{3(1+w)}{5+3w}\right)\sin^2\left[\frac{\pi\sqrt{w}}{1+3w}\right]$ for $p = w\rho$, so the dust limit $w \to 0$ gives $\delta_c \to 0$, i.e., slightly overdense would collapse to BH

2. Nonspherical effects lead to $\beta_{ani} \approx 0.056\sigma_H^5$ for $\sigma_H \ll 1$

Zel'dovich approximation (1970) describes the nonlinear evolution of density perturbation + hoop conjecture for formation of BH horizon (Thorne, 1972)

Harada etc.,PRD, 2016

3. inhomogeneous effects yields a further suppression by $\beta_{\rm inhom} \sim 3.7\sigma_H^{3/2}$

PBH formation in the dust-like era

T. Harada, etc., PRD 2017

PBH born in the MD era tends to develop maximal $\tilde{a} \equiv J/M_{BH} \to 1$!

PBH born in the RD era usually has $\tilde{a} \ll 1$, but Hawking radiation may lead to sizable $\tilde{a}$

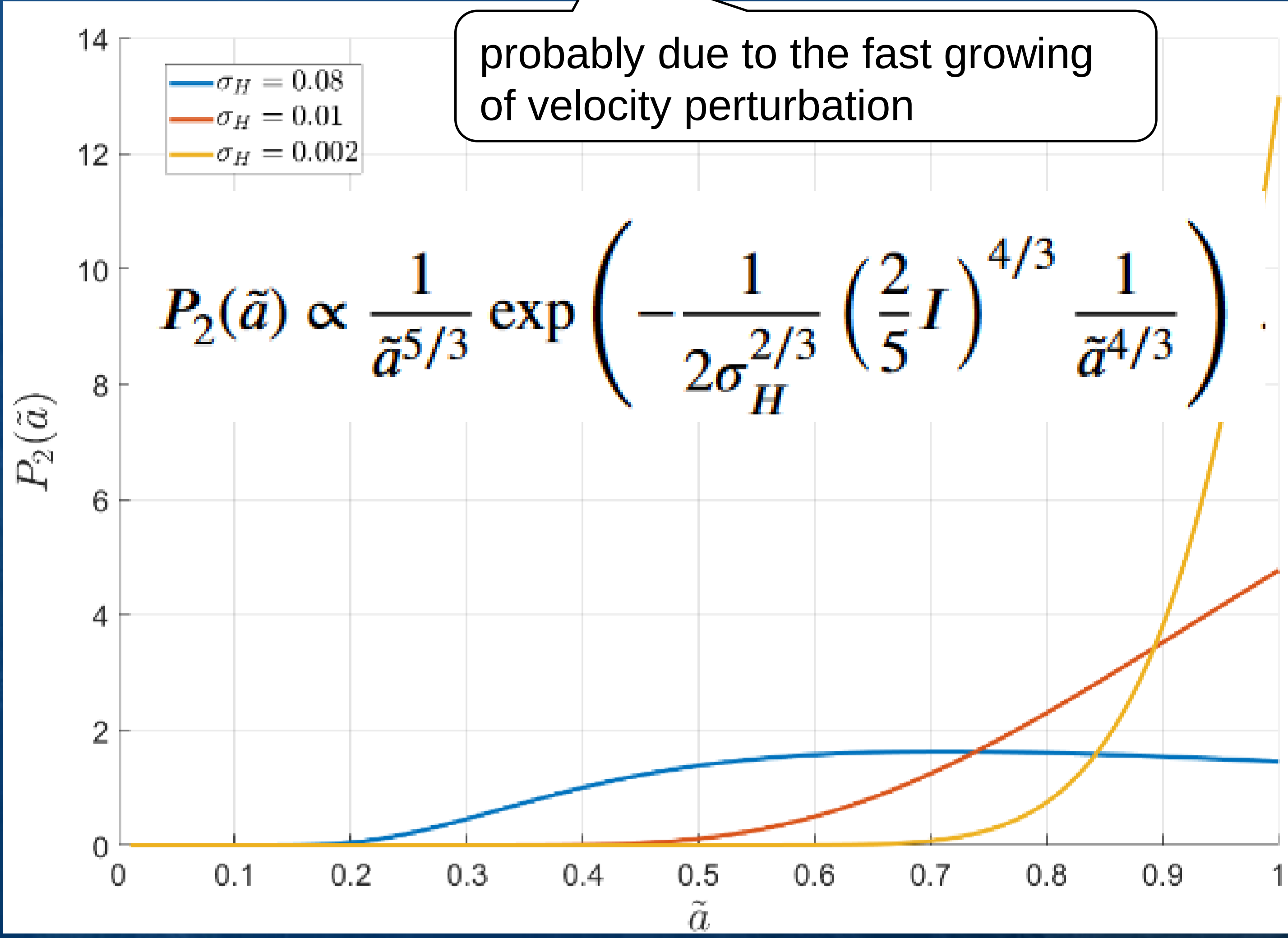
- accretion of nonrotating matter may fast reduce $\tilde{a}$

Eloy de Jong, etc, 2023

- So, a short MD era⇒ monochromatic spectrum

It picks the perturbation scale that collapse at the end of MD, with $\sigma_{end} \simeq 0.08 f_{PBH}^{4/29} \left(\frac{M_{BH}}{M_{sun}}\right)^{2/29}$

probably due to the fast growing of velocity perturbation



$$P_2(\tilde{a}) \propto \frac{1}{\tilde{a}^{5/3}} \exp\left(-\frac{1}{2\sigma_H^{2/3}}\left(\frac{2}{5}I\right)^{4/3}\frac{1}{\tilde{a}^{4/3}}\right).$$

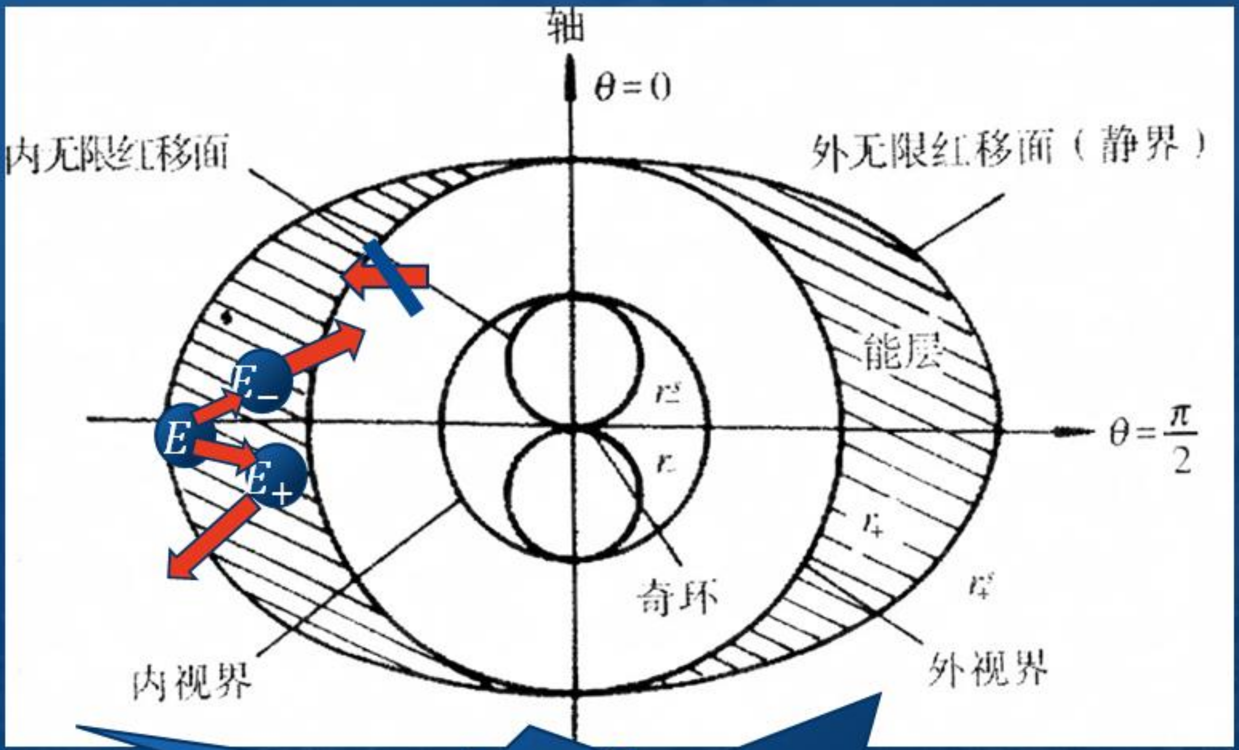
Part II

PBH meets a light bosonic field

GEOGRAPHY

Penrose process (1969)

Extracting rotating kinematic from rotating Kerr BH

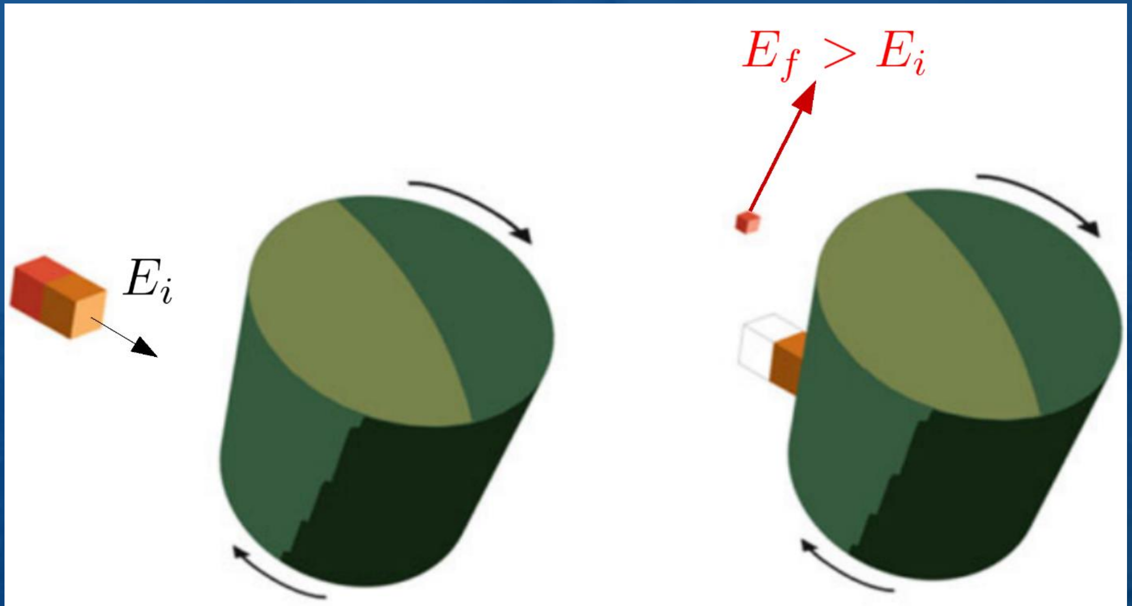


extraction upper limit: 29%

- negative energy orbital within the ergosphere allows an incoming particle with energy E to escape carrying larger energy: $E_+ = E - E_- > E$
- Both BH's angular & mass are reduced by eating the negative energy particle

More realistic: Zeldovich superradiance (1971)

Extracting rotating energy from rotating BH via wave



- If the surface rotates fast enough, the frequency of the incoming wave will change from positive to negative, obtaining energy from the rotating surface.

The underlying principle is the rotational Doppler effect

test using the sound wave (2013) . A test using EM wave? 10^9rotation/s!

- superradiant instability and black bomb

Press and Teukolsky , 1972

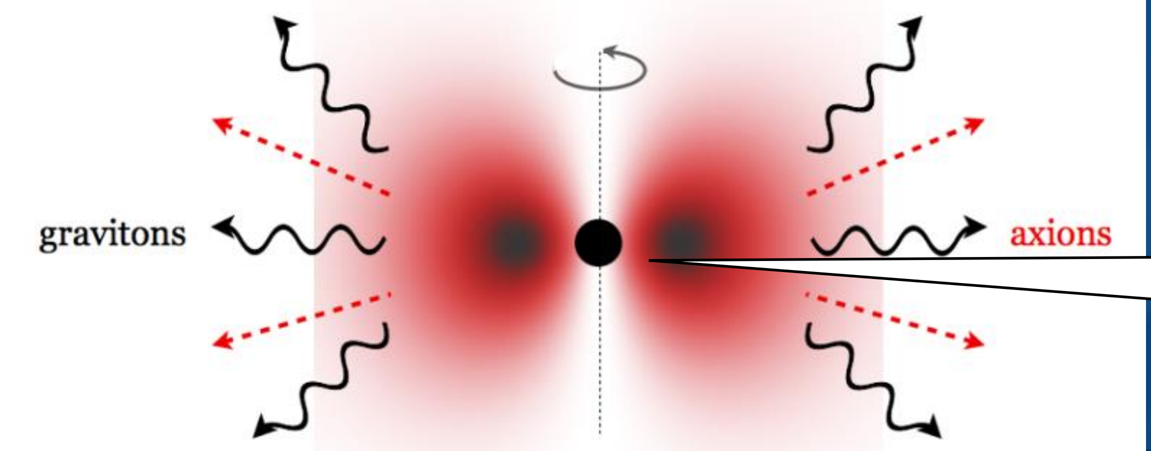
- quasi-normal bound states given a light massive bosonic wave

Th Damour, etc., 1976

Superradiance & gravitational atom

Reviewed by R. Brito and P. Pani

BH-boson gravity system ~ nucleus-electron EM system



Black denotes the Kerr BH with gravitational/horizon scale $r_{BH} = \frac{M_{BH}}{M_{PL}^2}$

1. A strong gravitational field around BH may bound a light particle

Gravitational fine structure constant $\alpha = \frac{\mu_S M_{BH}}{M_{PL}^2} \sim \frac{r_{BH}}{\lambda_S}$

2. According to QM, fermions are limited by the Pauli exclusion principle and do not form macroscopic clouds, while bosons can form BEC and accumulate in clouds

3. Bohr radius of the BH-boson system should exceed BH horizon: $r_B = \frac{\lambda_S}{\alpha} \sim \frac{r_{BH}}{\alpha^2} \gg r_{BH} \Rightarrow \alpha < 1 \Leftrightarrow \mu_S \ll \alpha \frac{M_{PL}^2}{M_{BH}}$

gravitational potential of BH is Coulomb-like at large distances, thus Hydrogen-like

Superradiance & gravitational atom

- quantify the boson cloud (a generalized BEC)

1. KG equation in the Kerr spacetime: $(\nabla^2 - \mu_S^2)\Phi = 0$

2. Solution: $\Phi_{\omega lm} = e^{-i\omega t} e^{-im\phi} S_{lm}(\theta) R_{\omega lm}(r) + c.c.$ with ω complex

3. quasi-bound states $|nlm\rangle$ at energy level $\omega_{R,nlm} = \mu_S(1 - \alpha^2/2n^2)$ with $n = 0, 1, 2 \dots, l = 0, 1, \dots n-1$, with $m = -l, -l+1, \dots l$ the boson angular momentum along the BH spin

4. $\omega_{I,nlm} = \tilde{r}_+ C_{nl} G_{lm} (m\Omega_H - \omega_R)\alpha^{4l+5} > 0$ with $\Omega_H = \frac{\tilde{a}}{2 r_{BH} \tilde{r}_+}$ the BH horizon angular velocity and $\tilde{r}_+ = 1 + \sqrt{1 - \tilde{a}^2}$ indicates a cloud with growing energy $\frac{\mathrm{d}M_C}{dt} = 2\omega_{I,nlm} M_C$

1. superradiant condition: $\tilde{a} > \frac{4\alpha/m}{4\alpha^2/m^2+1}$ && $\alpha < m/2$

2. The fastest-growing mode is $|n = l+1 = 2, l = 1, m = 1\rangle$

3. Cloud exponential growth time scale $1/\omega_{I,211} \sim \alpha^{-9} r_{BH}$

4. Cloud saturates with $M_C^{\max} \sim \alpha M_{BH} \tilde{a}$

Superradiance & gravitational atom

Cloud decay via GW emission

1. condensation oscillates as $\Phi_{\omega lm} \propto \sin \omega_R t$ with angular frequency $\omega_R \simeq \mu_S$, thus emitting GW at frequency $2\mu_S$

2. Cloud energy M_C decays as $\frac{\mathrm{d}M_C}{dt} = -\frac{M_{PL}^2}{M_{\mathrm{BH}}^2}\frac{d\tilde{E}}{dt} M_C^2$ with $\frac{d\tilde{E}}{dt}(\alpha)$, with solution $M_C(t) \approx M_C^{\max}/(1 + (t - t_0)/\tau_{GW})$

3. characteristic time for the consumption of the scalar cloud through GW emission is estimated as

$$\tau_{\mathrm{GW}} \sim 6 \times 10^{-12} \mathrm{years} \frac{M_{\mathrm{BH}}}{M_{\mathrm{solar}}} \alpha^{-15} \tilde{a}^{-1}.$$

$\frac{\mathrm{d}M_C}{dt}$ is also the power of radiated GWs

$\frac{d\tilde{E}}{dt}$ can be determined numerically, or analytically for small α

Huang Yang and Fa Peng Huang. PRD, 2023

Which is much longer than the cloud growing time scale τ_{grow}

$$\tau_{\mathrm{grow}} \sim 7 \times 10^{-12} \mathrm{years} \frac{M_{\mathrm{BH}}}{M_{\mathrm{solar}}} \alpha^{-9} \tilde{a}^{-1},$$

Part III

|

SGWB from primordial gravitational atoms (PGA)

Till now, almost all studies are interested in GA with an astrophysical BH core, since PBH was believed to have a small dimensionless spin

GEOGRAPHY

Superradiance & gravitational atom

SGWB from the PGA

1. Initial GW produced at z with uniform frequency $\frac{\mu_S}{\pi}$, redshifted to $f = \frac{\mu_S/\pi}{1+z}$

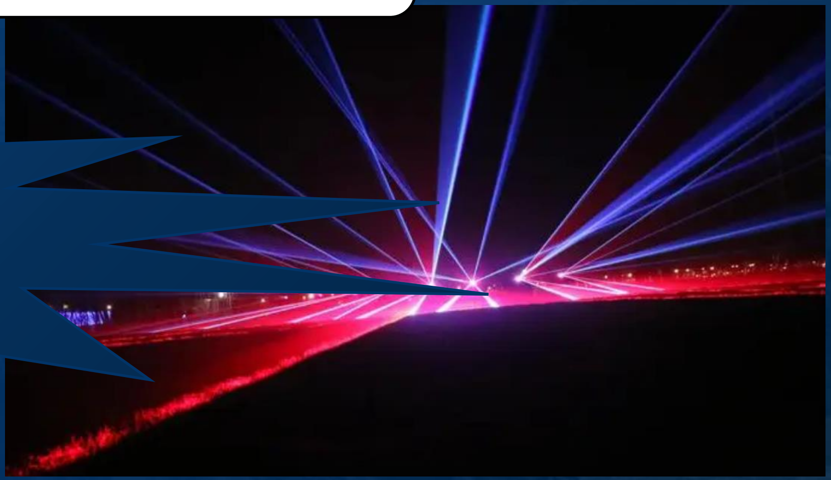
2. isotropically and homogeneously distributed monochromatic PBHs

$$\Omega_{\mathrm{GW}}(f) = \theta\left(\frac{1}{2} - \alpha\right) \frac{M_{\mathrm{PL}}^2}{M_{\mathrm{BH}}^3} \frac{\Omega_{\mathrm{DM}} f_{\mathrm{PBH}}}{|\dot{z}|} \frac{d\tilde{E}}{dt} \int_{4\alpha/(4\alpha^2+1)}^{1} P(\tilde{a}) d\tilde{a}\, M_C^2[t(f)],$$

Random PBH spin direction and GW phase, the GWs emitted by clouds overlap to form an isotropic and homogeneous SGWB

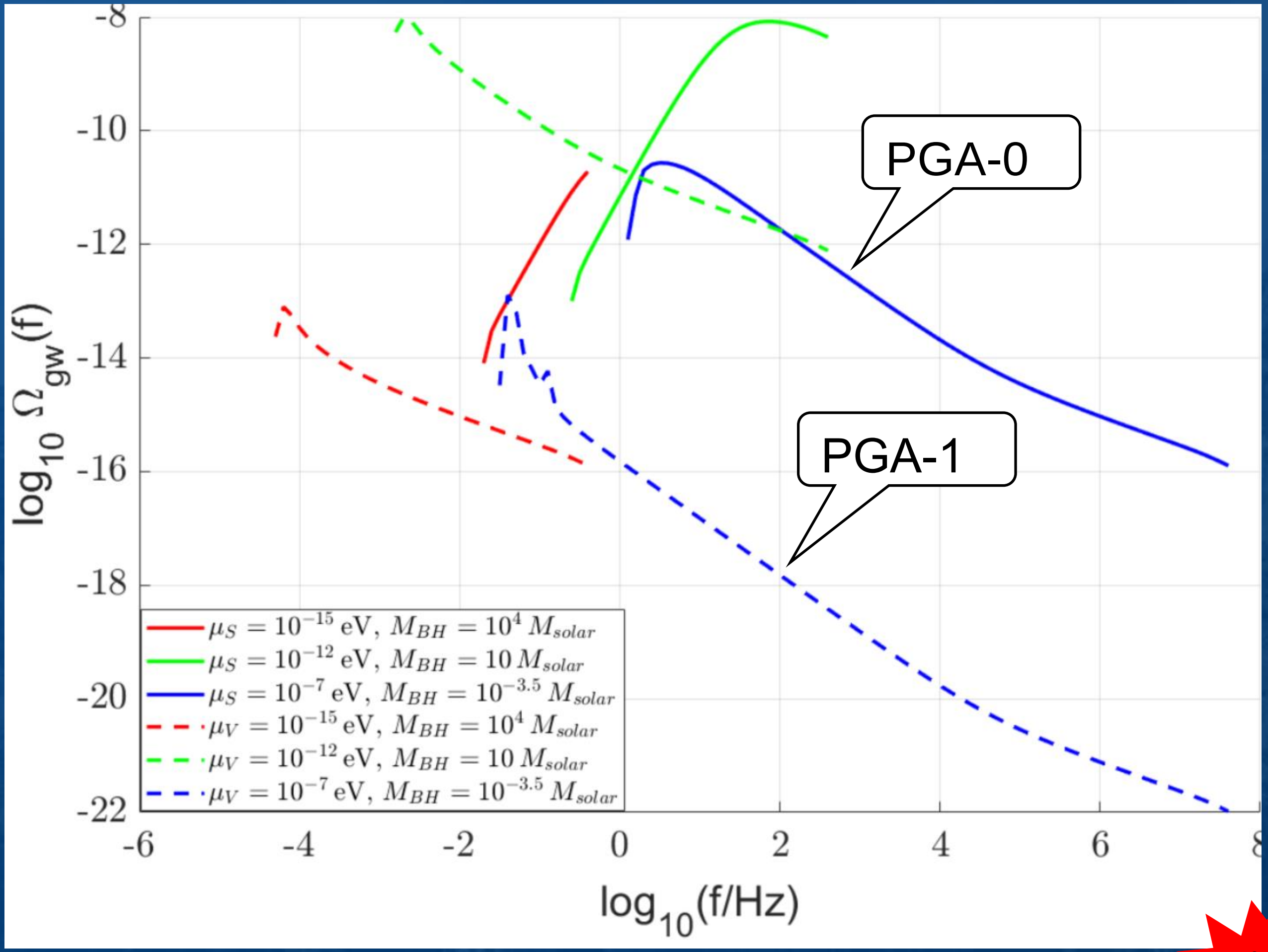
$P(\tilde{a})$ the PBH spin distribution function, taking two examples

BEC cloud M_C^2 as a giant GW laser, radiating coherent GWs



Superradiance & gravitational atom

- SGWB from the PGA: spectrum shape analysis



1. Shape of GW produced in the RD

$$\Omega_{GW}(f) \propto \frac{M_C^2}{|\dot{z}|} \propto \frac{f^3}{(1+\frac{\tilde{\tau}}{\tau_{GW}}\frac{f^2}{\mu^2})^2}.$$

2. $f_{peak} \sim \mu_S \sqrt{3\tau_{GW}/\tilde{\tau}}$ with $\tilde{\tau} \sim 10^{12} year$

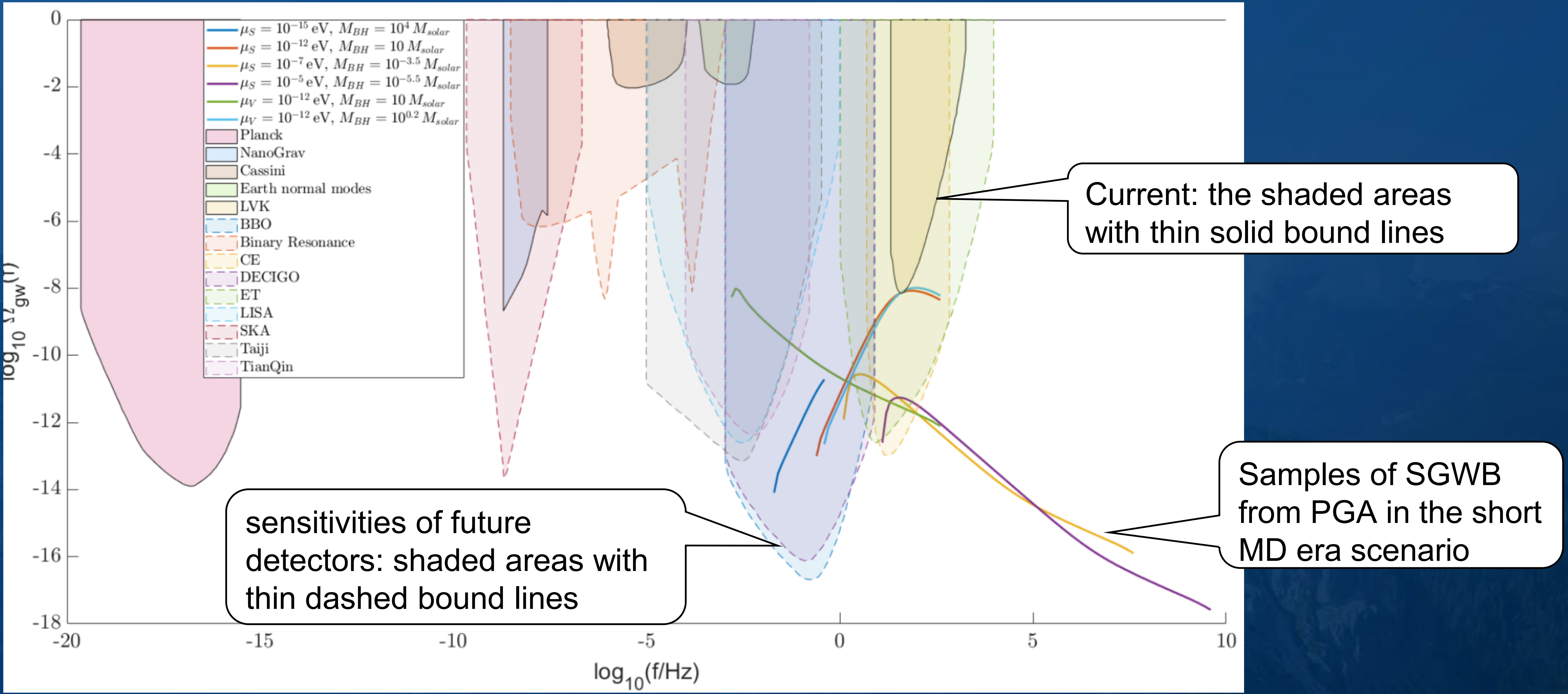
Quite different between PGA-0 and -1

3. Rising & falling power

$$\Omega_{GW} \propto \begin{cases} f^3 & t_0 < t(f) \lesssim 3\tau_{GW} \\ f_{peak}^4 f^{-1} & 3\tau_{GW} \lesssim t(f) < t_{eq}. \end{cases}$$

Superradiance & gravitational atom

- SGWB from the PGA: current constraints and prospects

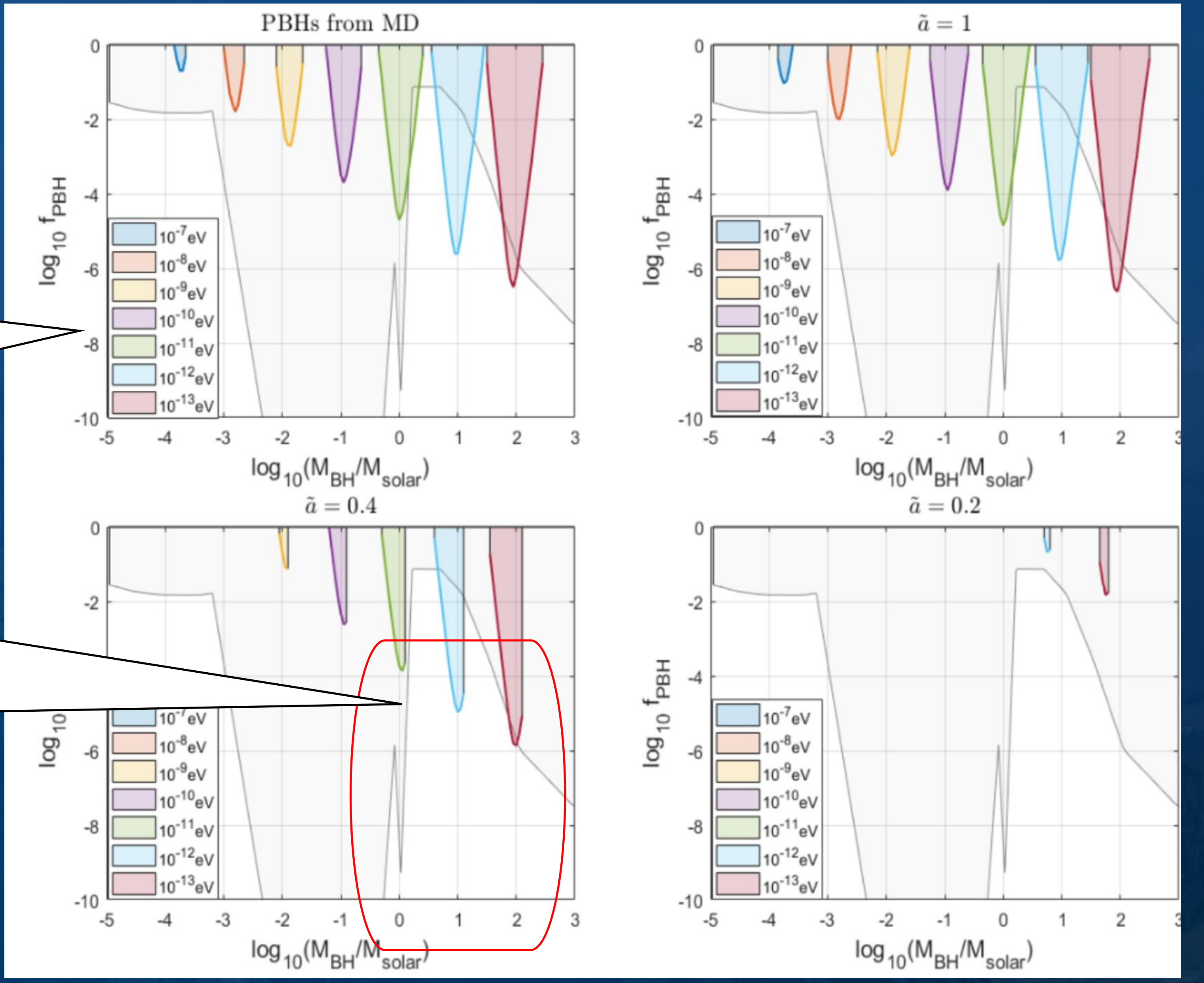


Superradiance & gravitational atom

- SGWB from the PGA-0: new constraints on PBH

The first taking spin distribution from short MD scenario; while for others we take several fixed spin for demonstration

For $M_{BH} \sim 1 - 100\ M_{sun}$, which subject to existing relatively weak constraints, the SGWB signal is already able to yield stronger constraints provided the scalar clouds with $\mu_S \sim 10^{-11} - 10^{-13}\text{eV}$

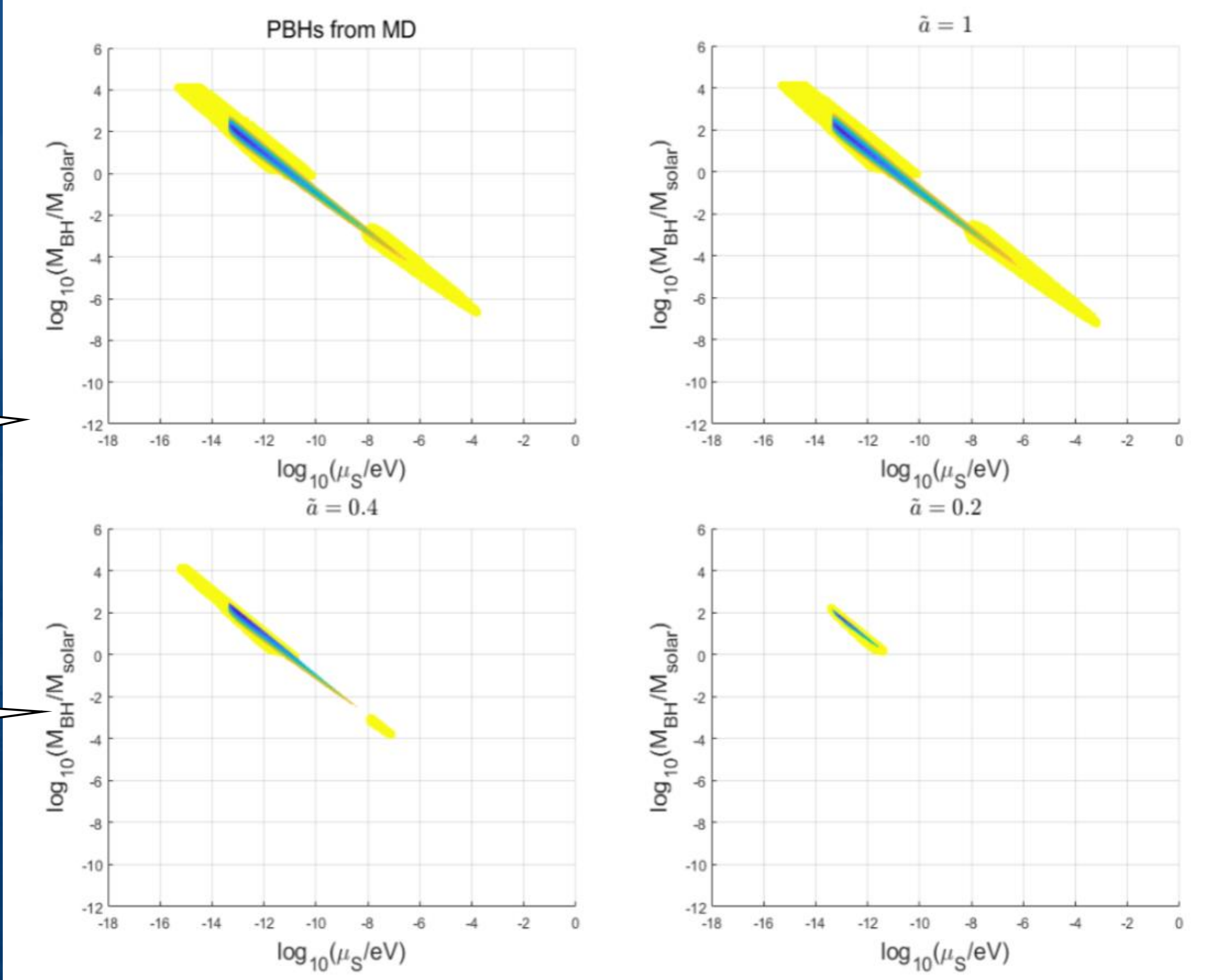


Superradiance & gravitational atom

- SGWB from the PGA: constraints & prospects in the $\mu_S - M_{BH}$ plane, a narrow stripe

Green: the current bound;
yellow: the future prospect.
One needs a $\tilde{a} \gtrsim 0.1$

For the PGA-1, the situation can be significantly improved



Thank you for your attention!!!

Conclusion & outlook

- Both PBH and light bosonic fields may appear in the early universe, and we proposed the scenario of primordial gravitational atom
- How to analytically calculate the PBH spin produced in the MD era including accretion effect ???
- Other scenarios of PBH? More solid estimate the SGWB with a general setup, and many other works can be explored

Penrose process (1969)

- perturbation on scale k with density contrast $\delta_k \equiv (\rho - \bar{\rho})/\bar{\rho}$

$$ds^2 = (1 - \frac{2GMr}{c^2r^2 + a^2cos^2\theta})c^2dt^2 - \frac{c^2r^2 + a^2cos^2\theta}{c^2r^2 + a^2 - 2GMr}dr^2$$
$$- (r^2 + \frac{a^2}{c^2}cos^2\theta)d\theta^2 - [(r^2 + \frac{a^2}{c^2})sin^2\theta$$
$$+ \frac{2GMra^2sin^4\theta}{c^4r^2 + c^2a^2cos^2\theta}]d\varphi^2 + 2\frac{2GMrasin^2\theta}{c^2r^2 + a^2cos^2\theta}dtd\varphi$$

- $M_H(k)$: PBH mass

Khlopov & Polnarev 1980