# Form Factor Calculation of $e^+e^- \to t\bar{t} \to b\bar{b}W^-W^+ \to b\bar{b}q\bar{q}l(v)$

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December 9, 2024

# Outline

- Theoretical principles
- Data Distribution
- Reconstruction of Top

- Theoretical principles

# Theoretical principles

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- If both y/Z and tt are on-shell, EW coupling can be described by four Form Factors:

$$\Gamma^{\mu}_{vt\bar{t}} = \frac{g}{2}\,\bar{u}(p_t)\left[\gamma^{\mu}\{A_v + \delta A_v - (B_v + \delta B_v)\gamma_5\} + \frac{(p_t - p_{\bar{t}})^{\mu}}{2m_t}(\delta C_v - \delta D_v\gamma_5)\right]v(p_{\bar{t}}), \quad (1)$$

where $g$ denotes the $SU(2)$ gauge coupling constant, $v = \gamma, Z$, and

$$A_{\gamma} = \frac{4}{3}\sin\theta_W, \quad B_{\gamma} = 0, \quad A_Z = \frac{1}{2\cos\theta_W}\Big(1 - \frac{8}{3}\sin^2\theta_W\Big), \quad B_Z = \frac{1}{2\cos\theta_W}$$

  Where $cos\theta_W = \frac{M_W}{M_Z}$, meaning that $A_v$ and $B_v$ are both constants determined within the Standard Model .Among the above non-SM form factors, $\delta A_{\gamma,Z}$, $\delta B_{\gamma,Z}$, $\delta C_{\gamma,Z}$ describe CP -conserving while $\delta D_{\gamma,Z}$ parameterizes CP violating interactions.
- We can obtain the total **cross-section** $\sigma$ and **the four-momentum of the final-state particles**. Subsequently, the double distribution of the angle and the rescaled energy of the lepton can be discretized to obtain $\sigma(x, cos\theta)$. Differentiating this then allows us to extract $\frac{d^2\sigma}{dxdcos\theta}$.

# Theoretical principles

- On the other hand, theoretically:

$$\frac{d^2\sigma}{dx_f d\cos\theta_f} = \frac{3\pi\beta\alpha_{\mathrm{EM}}^2}{2s} B_f \left[ \Theta_0^f(x_f) + \cos\theta_f\, \Theta_1^f(x_f) + \cos^2\theta_f\, \Theta_2^f(x_f) \right],$$

where $\beta$ is the top velocity,s is the centre-of-mass energy squared, $\alpha_{EM}$ is the fine structure constant and $B_f$ denotes the appropriate branching fraction(about 0.44);they are all constant. The energy dependence is specified by the functions $\theta_i^f(x_f)$ here f we can choose lepton;they can be parameterized both by production and decay form factors.

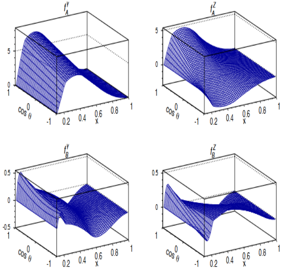
- Expanding the form factor gives another form:

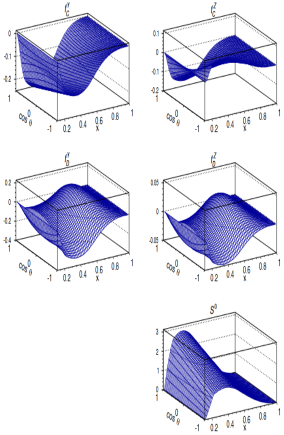
$$\frac{d^2\sigma}{dxdcos\theta} = \frac{3\pi\beta\alpha_{EM}^2}{2s} B_l S(x,cos\theta), S(x,cos\theta) = S^0(x,\theta) + \sum_{i=1}^{8} \delta_i f_i(x,cos\theta) + \sum_{i=1}^{8}\sum_{j=1}^{8} \delta_i g_{ij}$$

Where $\delta_i$ is the form factor $\delta A_\gamma, \delta A_Z, \delta B_\gamma, \delta B_Z$....;$S_0$ is the standard-model contribution. Considering that the contribution from the non-SM part is relatively small compared to the SM part, we can neglect the second-order terms and higher-order terms. On the other hand, from a theoretical perspective, the off-diagonal elements $\delta_i\delta_j (i \neq j)$ indeed tend to be close to zero in most cases.

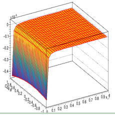
# Theoretical principles

- For $l^-, \sqrt{s} = 365GeV$ with no incoming beam polarization:





I think Multiplying $S_0$ by the constant factor should give the plot shown in Mustapha's slide.



# Statistical Uncertainty

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- For a given model, using MadGraph to generate the signal, we can obtain $\frac{d^2\sigma}{dxdcos\theta}$ through the observables.Then, by fixing the other seven form factors sequentially, we determine the remaining form factor such that the error (which can be described using a norm) between both sides of the equation is minimized.
  In Other word it means in principle, all eight form factors and their uncertainties can therefore be determined simultaneously, under the condition that the nine functions are linearly independent by maximize numerically **a global likelihood** $L = e^{-u} \prod_{N}^{k=1} p(k)$ where k is the final state particle and p(k) is its likelihood with $(x_k, cos\theta_k)$
  And the covariance matrix of form factors is :

$$V_{ij} = L \int d\Omega \frac{f_i \times f_j}{S_0}$$

  The diagonal element is the variance, and the reciprocal of the standard deviation is the uncertainty.

$$p(k) = \frac{1}{\sigma_{\text{tot}}} \frac{\mathrm{d}^2\sigma}{\mathrm{d}x \,\mathrm{d}\cos\theta}(x_k, \cos\theta_k), \text{with } \sigma_{\text{tot}} = \int \frac{\mathrm{d}^2\sigma}{\mathrm{d}x\,\mathrm{d}\cos\theta} \mathrm{d}x\,\mathrm{d}\cos\theta.$$

# forward-backward asymmetry

- Through figuring out $\frac{d^2\sigma}{dx dcos\theta}$ of every final state particle we can further obtain another observable, the **forward-backward asymmetry**(here $cos\theta$ is about top)

$$A^t_{FB} = \frac{N(cos\theta > 0) - N(cos\theta > 0)}{N(cos\theta > 0) + N(cos\theta > 0)}$$

, and compare it with the theoretical calculated value to validate a certain process.

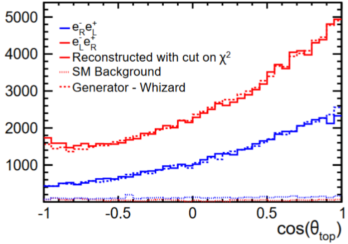


Figure: ILC Result

# Second Section

- Theoretical principles
- Data Distribution

# Data Source

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- Just Signal Process Data (Produced by Mustapha)
- With no incoming beam polarization
- Based on my observation that the masses of the four jets are much higher than that of a single b-quark, I believe this dataset has undergone parton showering. However, since QCD processes cannot be accurately calculated at this stage, I wonder if it would be more appropriate to use data before parton showering by turning off the parton shower in the Monte Carlo simulation?

# Data Distribution

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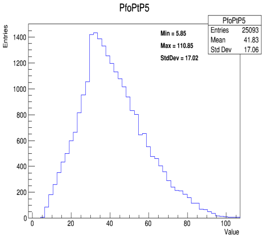
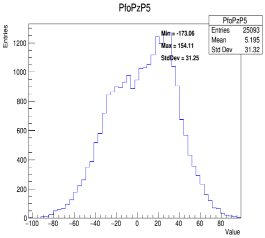
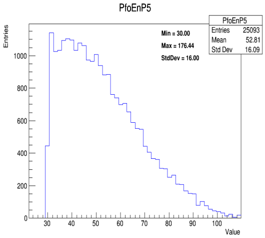
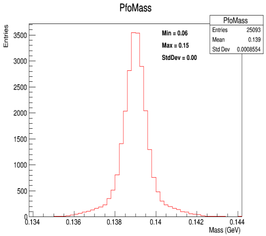
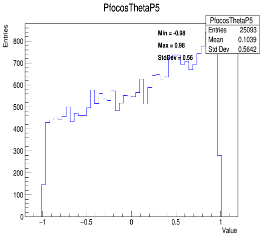
- Pt,Pz,En,Mass distribution of Pfo particle ($\mu$)



# Data Distribution

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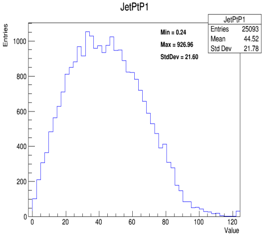
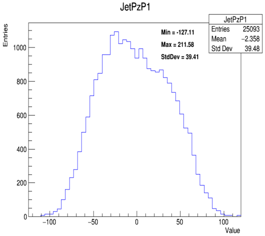
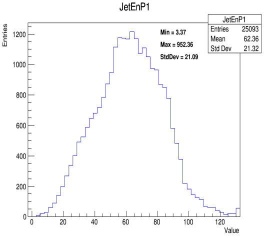
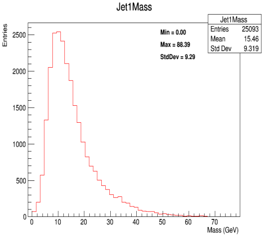
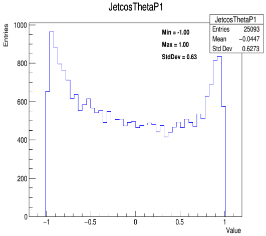
- Pt,Pz,En,Mass distribution of Jet1



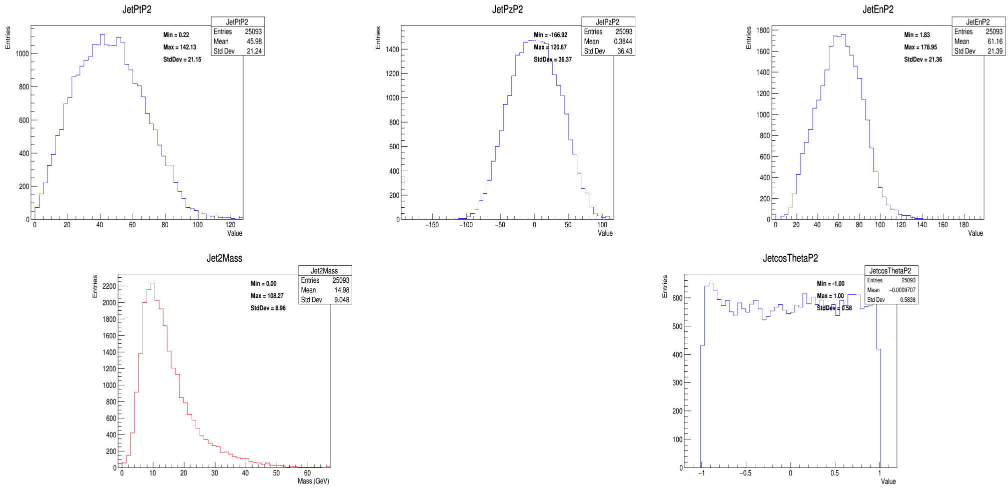



# Data Distribution

- Pt,Pz,En,Mass distribution of Jet2



# Data Distribution

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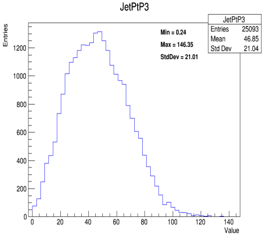
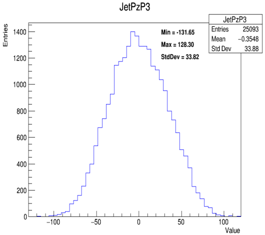
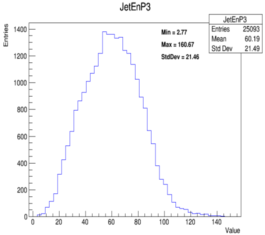
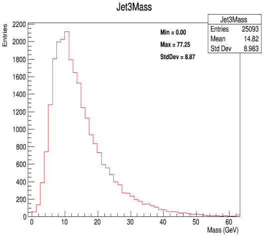
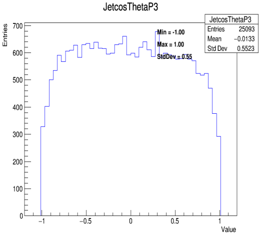
- Pt,Pz,En,Mass distribution of Jet3



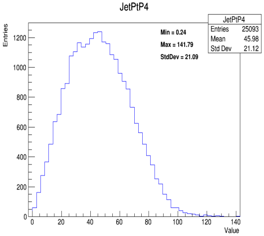



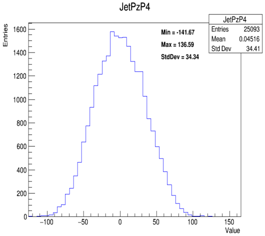



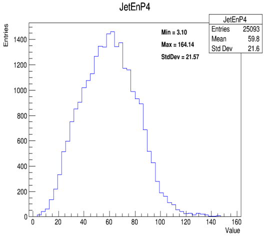



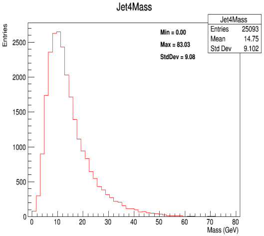
# Data Distribution

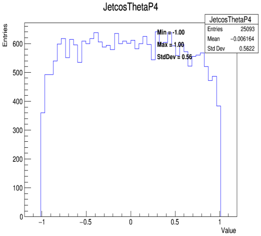
- Pt,Pz,En,Mass distribution of Jet4



- Theoretical principles
- Data Distribution
- Reconstruction of Top

# Reconstruction of Top



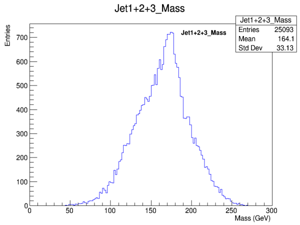
- Approach: Select three out of the existing four jets to reconstruct their parent particle, the top quark. The combination that gives a mass closest to 173 GeV, calculated by summing their four-momenta, is considered the correct one.

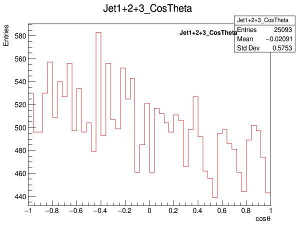
$$M_t = \sqrt{(E_b + E_q + E_{\bar{q}})^2 - (p_{xb} + p_{xq} + p_{x\bar{q}})^2 - (p_{yb} + p_{yq} + p_{y\bar{q}})^2 - (p_{zb} + p_{zq} + p_{z\bar{q}})^2}$$

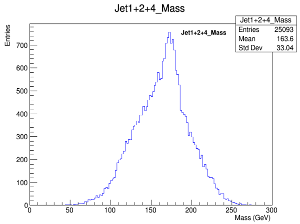
- At this point, the sum of the four-momenta of two of the quarks should correspond to the four-momentum of a W boson.

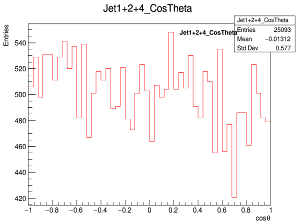
$$M_W = \sqrt{(E_q + E_{\bar{q}})^2 - (p_{xq} + p_{x\bar{q}})^2 - (p_{yq} + p_{y\bar{q}})^2 - (p_{zq} + p_{z\bar{q}})^2}$$

# Reconstruction of Top

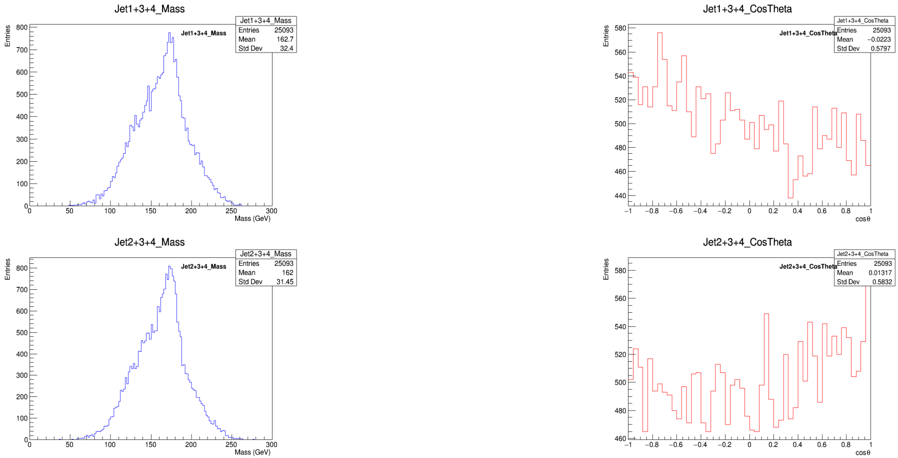


# Reconstruction of Top




- Since the charge of the top quark cannot be determined, the forward-backward asymmetry here might differ by a negative sign.

# Reconstruction of W