



# Global analysis of fragmentation functions with high-precision data from the LHC

高俊

INPAC, Shanghai Jiao Tong University

based on 2305.14620 with ChongYang Liu, XiaoMin Shen, Bin Zhou  
2401.02781, 2407.04424 with ChongYang Liu, XiaoMin Shen, HongXi Xing, YuXiang Zhao  
and 2407.10059 with Bin Zhou

The 29th LHC Mini-Workshop

福州 2024/12/14





# Outline

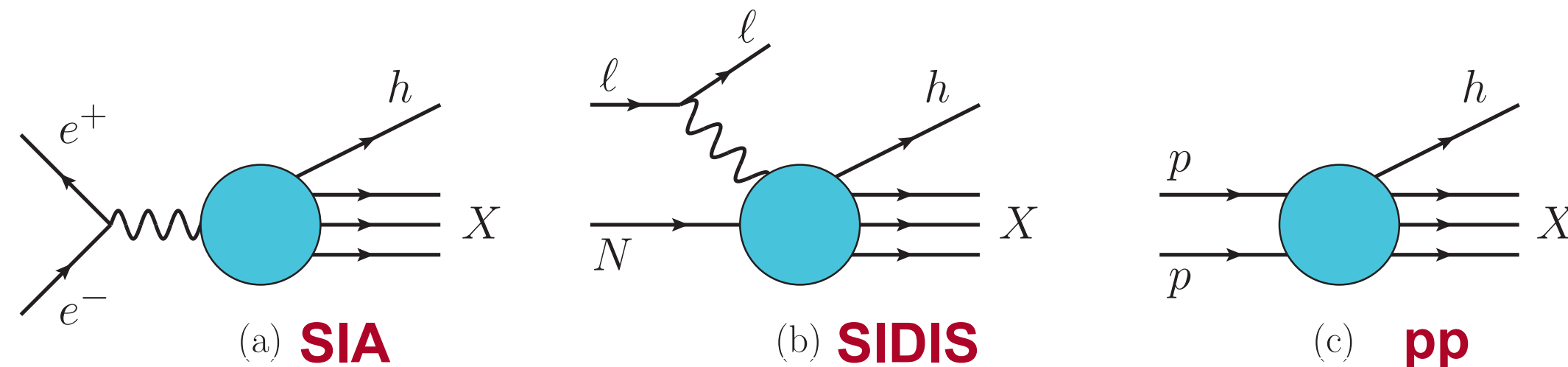
- ◆ 1. Introduction
- ◆ 2. Automation of fragmentation calculations at next-to-leading order
- ◆ 3. Global analysis of FFs to light charged hadrons
- ◆ 4. Outlook and summary

# Single inclusive hadron production

- ◆ In its simplest form, fragmentation functions (FFs) describe number density of the identified hadron wrt the fraction of momentum of the initial parton it carries, as measured in single inclusive hadron production, e.g., from single-inclusive annihilation (SIA), semi-inclusive DIS (SIDIS), pp collisions

single inclusive hadron production/observable

[1607.02521]



$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma^{e^+e^- \rightarrow hX}}{dz} = F^h(z, Q^2), \quad z = \frac{2E_h}{\sqrt{s}}$$

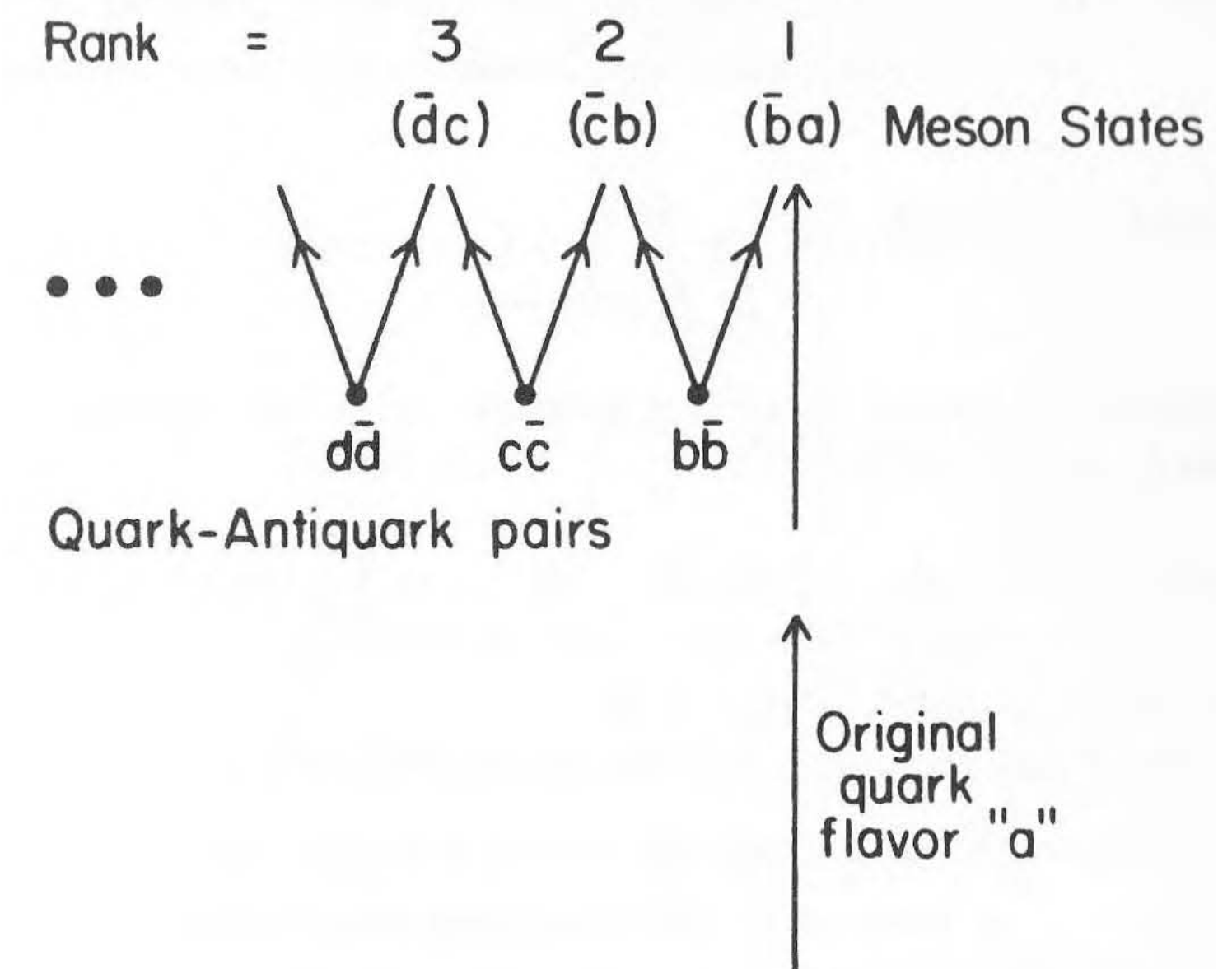
exp. definition of unpolarized collinear FFs

other forms: polarized FFs, TMD FFs, di-hadron FFs

parton model

[Field&Feynman]

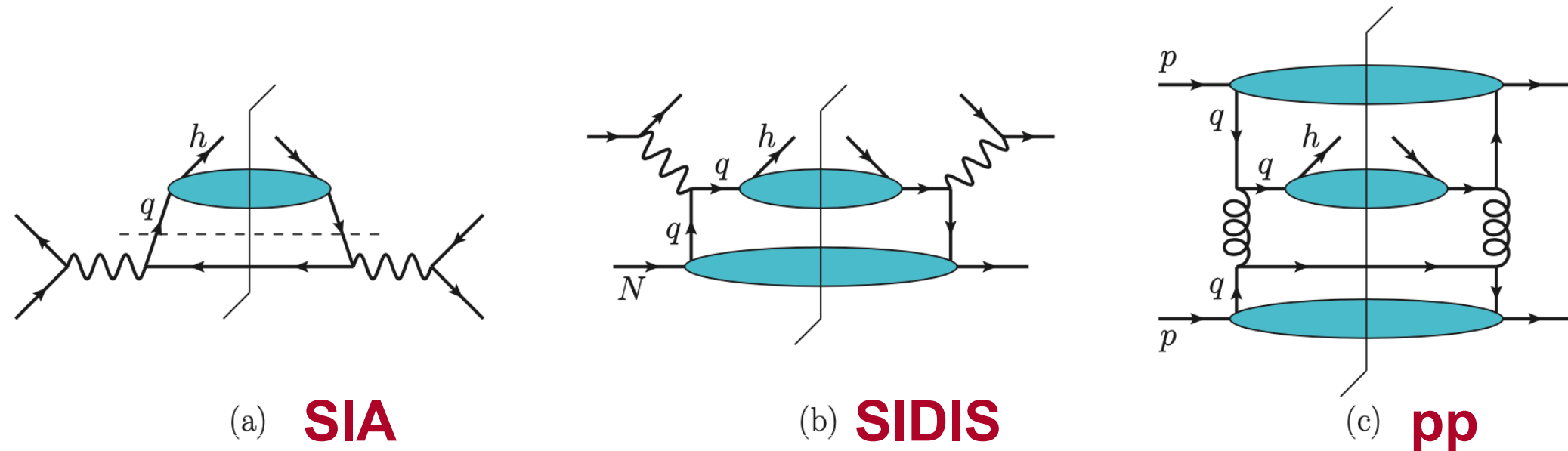
Hierarchy of Final Mesons



quark decaying functions to mesons via creation of quark-antiquark pairs in cascade

# QCD collinear factorization

- QCD collinear factorization ensures universal separation of long-distance and short-distance contributions in high energy scatterings involving initial/final state hadrons, and enables predictions on cross sections



- coefficient functions, hard scattering; infrared (IR) safe, calculable in pQCD, independent of the hadron
- FFs/PDFs, reveal inner structure of hadrons or parton-hadron transition; NP origin, universal, e.g. DIS vs. pp collisions; fitted from data
- runnings of FFs/PDFs with scales  $\mu_D/\mu_f$  are governed by the DGLAP equation

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma^{e^+e^- \rightarrow hX}}{dz} = \sum_q e_q^2 (2F_1^h(z, Q^2) + F_L^h(z, Q^2))$$

$$2F_1^h(z, Q^2) = \sum_q e_q^2 \left( D_1^{h/q}(z, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} (C_1^q \otimes D_1^{h/q} + C_1^g \otimes D_1^{h/g}) (z, Q^2) \right)$$

$$\frac{d^3\sigma^{\ell p \rightarrow \ell h X}}{dx dy dz} = \frac{2\pi\alpha_{\text{em}}^2}{Q^2} \left( \frac{1 + (1-y)^2}{y} 2F_1^h(x, z, Q^2) + \frac{2(1-y)}{y} F_L^h(x, z, Q^2) \right)$$

$$2F_1^h(x, z, Q^2) = \sum_q e_q^2 \left( f_1^{q/p} D_1^{h/q} + \frac{\alpha_s(Q^2)}{2\pi} \left( f_1^{q/p} \otimes C_1^{qq} \otimes D_1^{h/q} + f_1^{q/p} \otimes C_1^{qg} \otimes D_1^{h/g} + f_1^{g/p} \otimes C_1^{gq} \otimes D_1^{h/q} \right) \right),$$

## unpolarized collinear FFs, operator definition

$$D_1^{h/q}(z) = \frac{z}{4} \int \frac{d\xi^+}{2\pi} e^{ik^-\xi^+} \text{Tr} \left[ \langle 0 | \mathcal{W}(\infty^+, \xi^+) \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X \rangle \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}(0^+, \infty^+) | 0 \rangle \gamma^- \right].$$

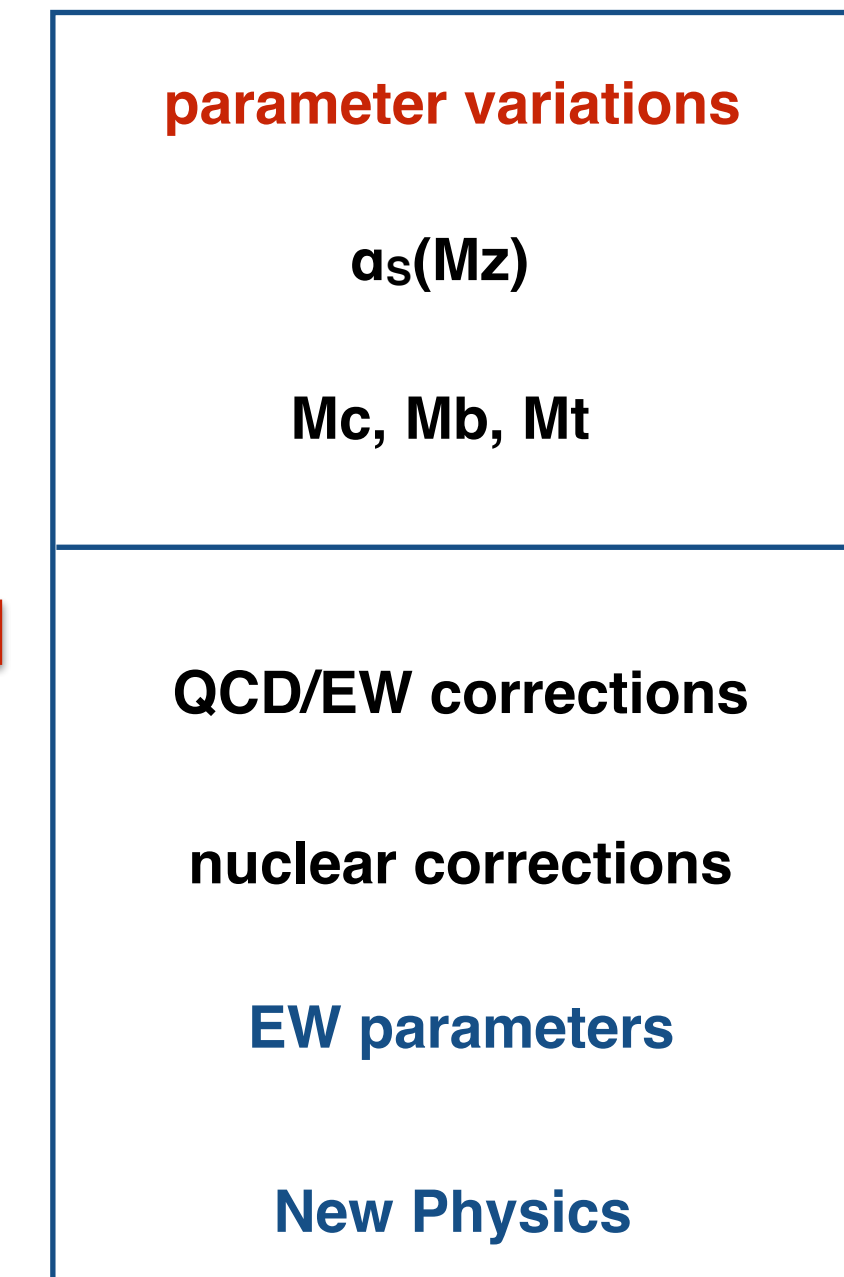
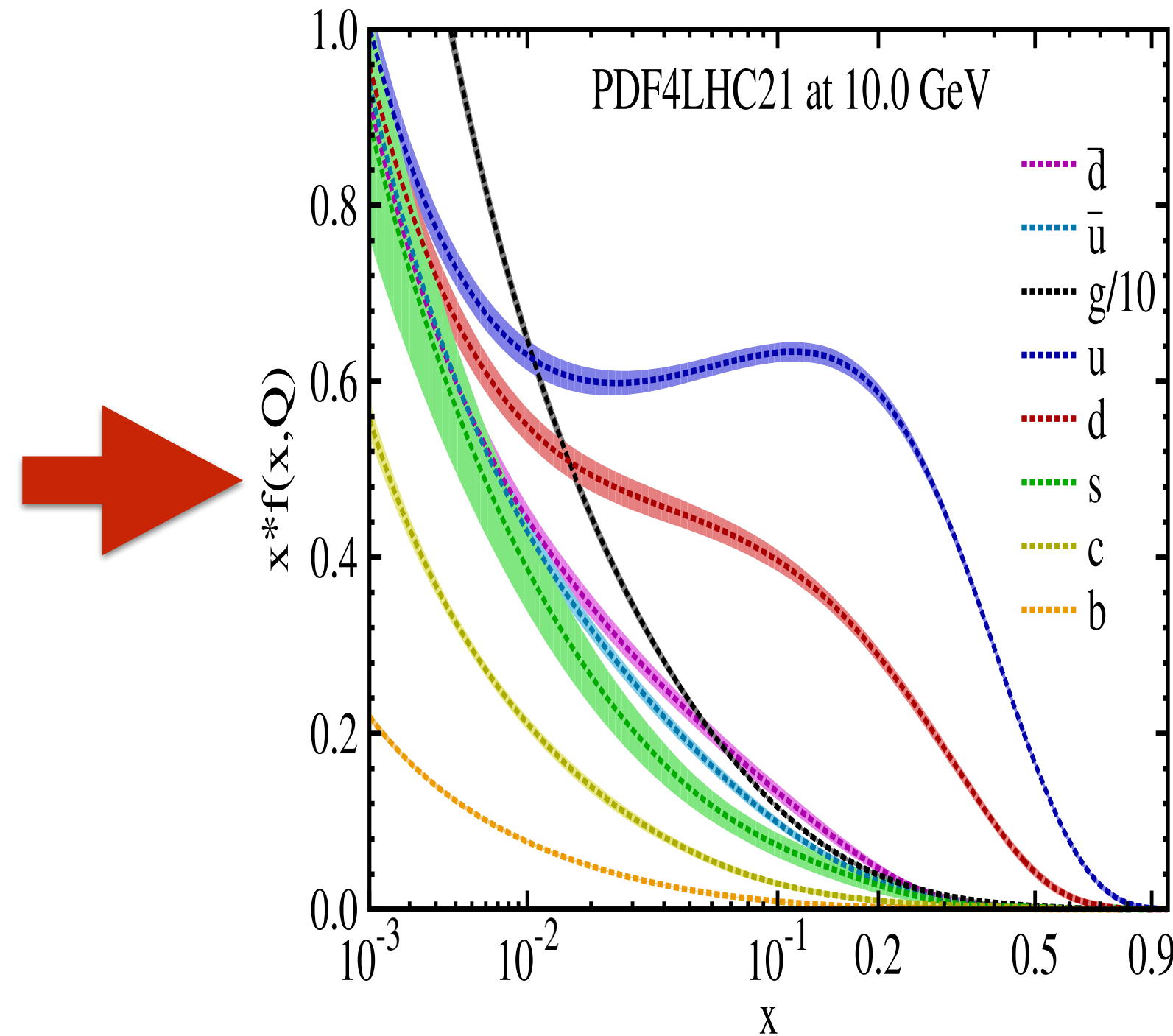
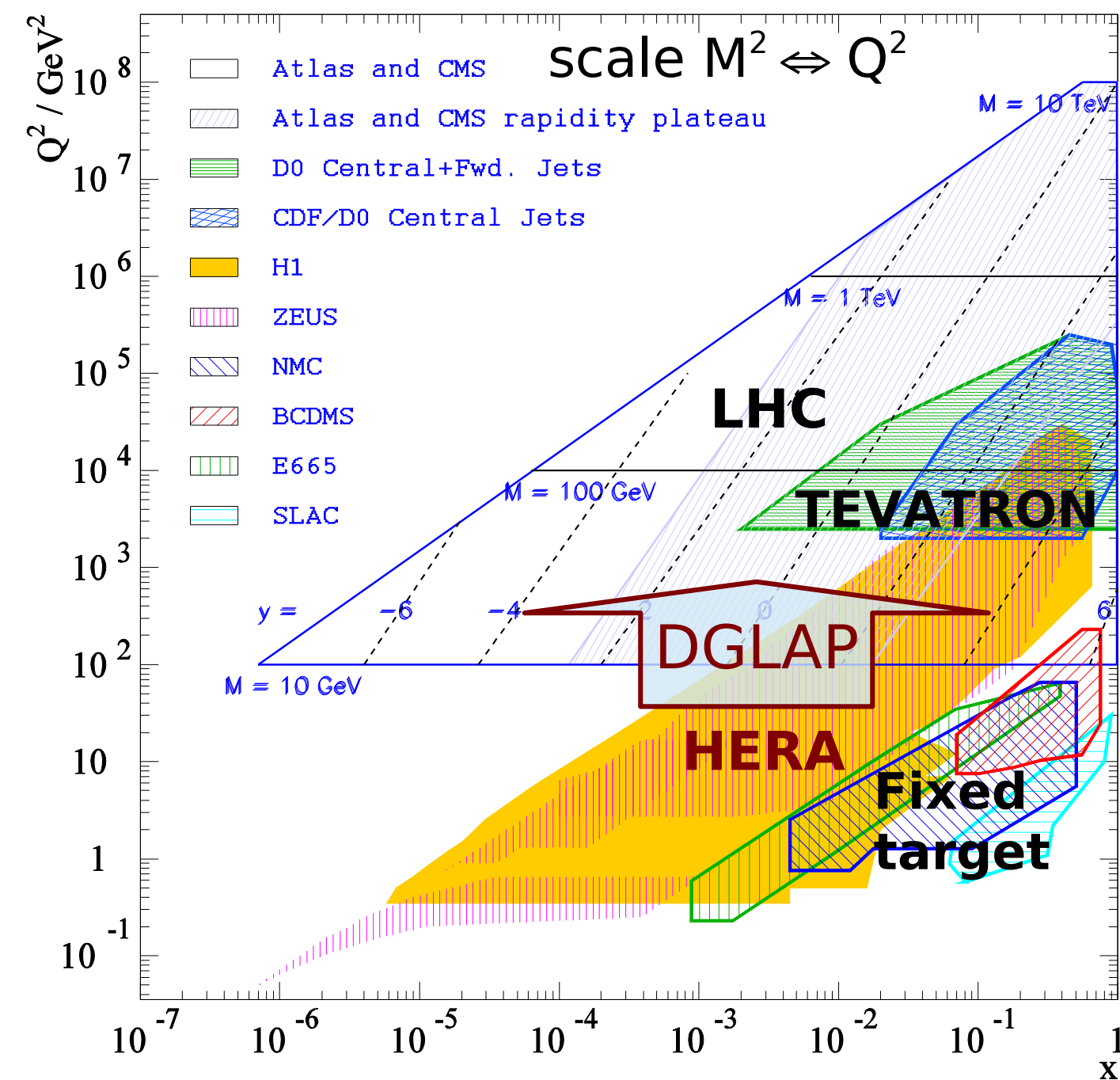
$$\frac{d}{d \ln \mu^2} D_1^{h/i}(z, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \sum_j \int_z^1 \frac{du}{u} P_{ji}(u, \alpha_s(\mu^2)) D_1^{h/j} \left( \frac{z}{u}, \mu^2 \right)$$

[Collins, Soper, Sterman]



# Global analysis of PDFs

- Proton PDFs are usually extracted from global analysis on variety of data, e.g., DIS, Drell-Yan, jets and top quark productions at fixed-target and collider experiments, with increasing weight from LHC, together with SM QCD parameters [see 1709.04922, 1905.06957 for recent reviews]

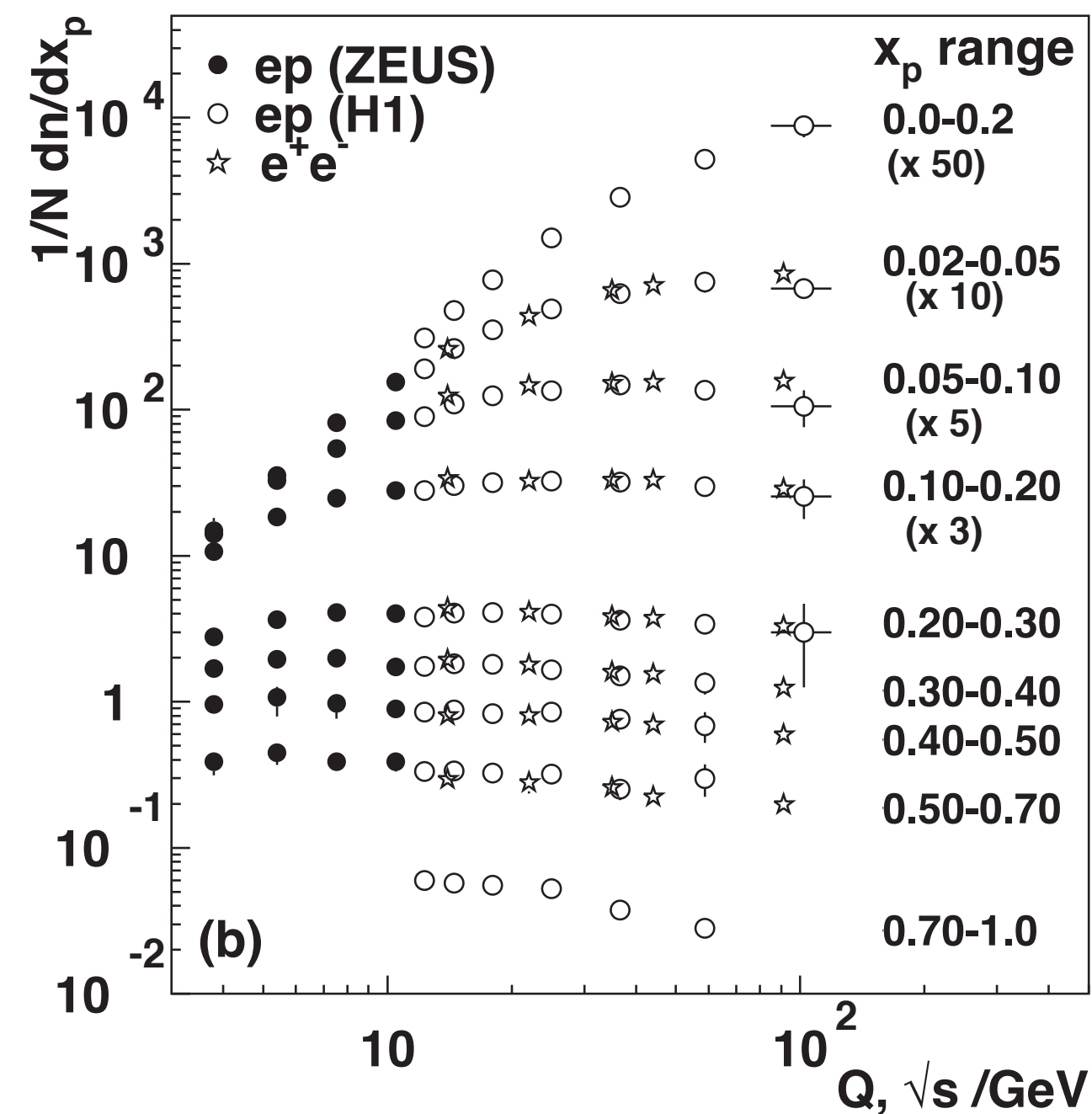


- Modern PDF analyses are carried out at NNLO/aN3LO in QCD as demanded by the LHC precision programs; diversity of the PDF analyses/groups are important to avoid theoretical/experimental bias
- analysis groups [CT, MSHT, NNPDF, ABM, HERAPDF, ATLASpdf, CJ, JAM...] use different heavy-quark schemes, selections of data, and methodologies; extensive benchmark exercises exist, especially within the PDF4LHC group

# Global analysis of FFs

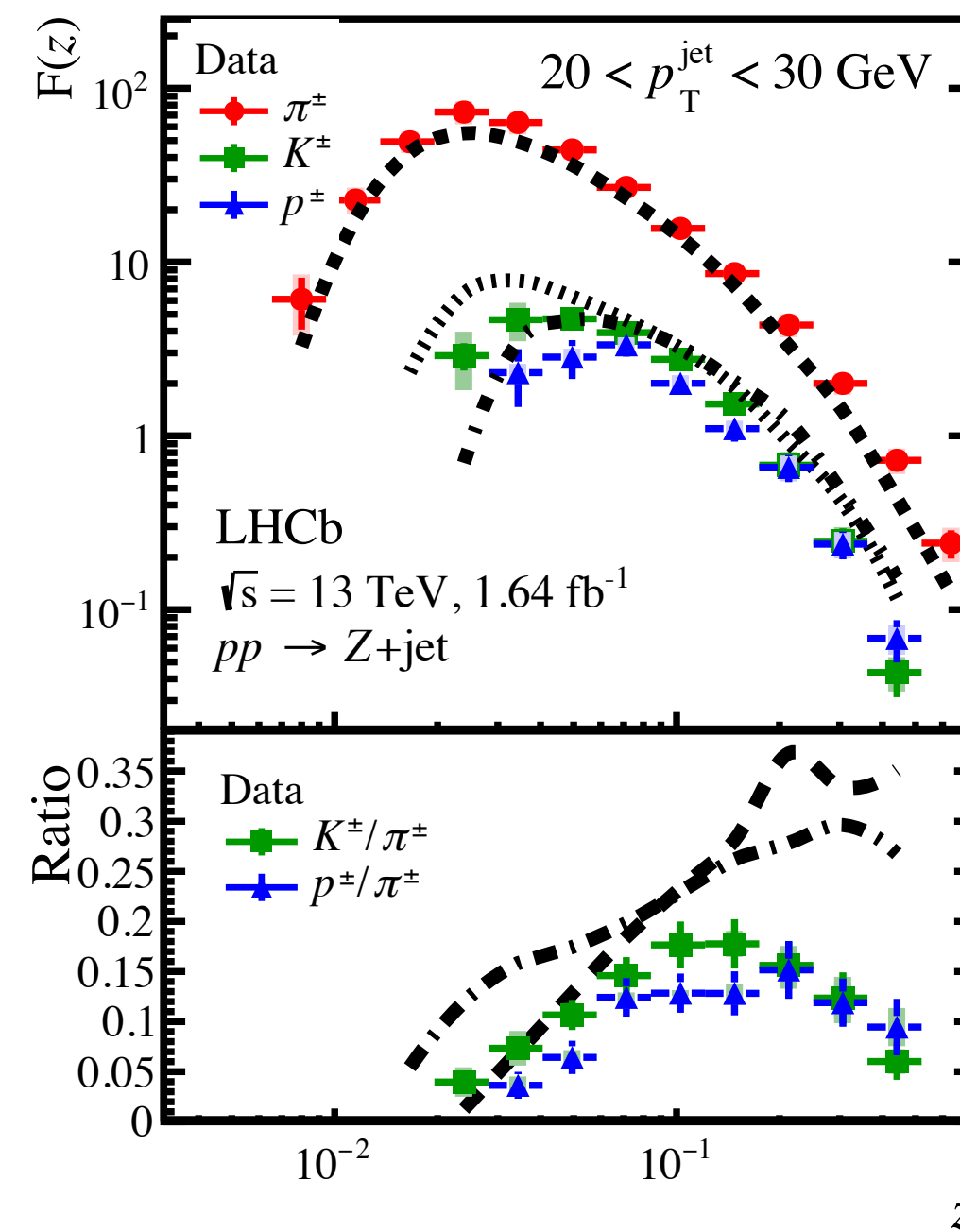
- Measurements are available from colliders SLAC, LEP, HERA, RHIC, LHC and fixed-target HERMES, COMPASS experiments for various charged hadrons as well as neutral hadrons; many groups provide phenomenological FFs from global analysis at NLO/NNLO in QCD

single incl. production of unidentified charged hadrons (SIA & SIDIS)



[Particle data group]

jet fragmentation to light charged hadrons (LHCb)



[2208.11691]

global analysis

- major groups/families include BKK, AKK, HKNS, DSS, NNFF, MAPFF, JAM, SAK etc.
- mostly done at NLO in QCD since exact NNLO coefficient functions only known recently for SIDIS
- different determination can be quite different due to selection of data sets as well as theory treatments, not converge as well as the case of PDF fits

[1607.02521 for a review]

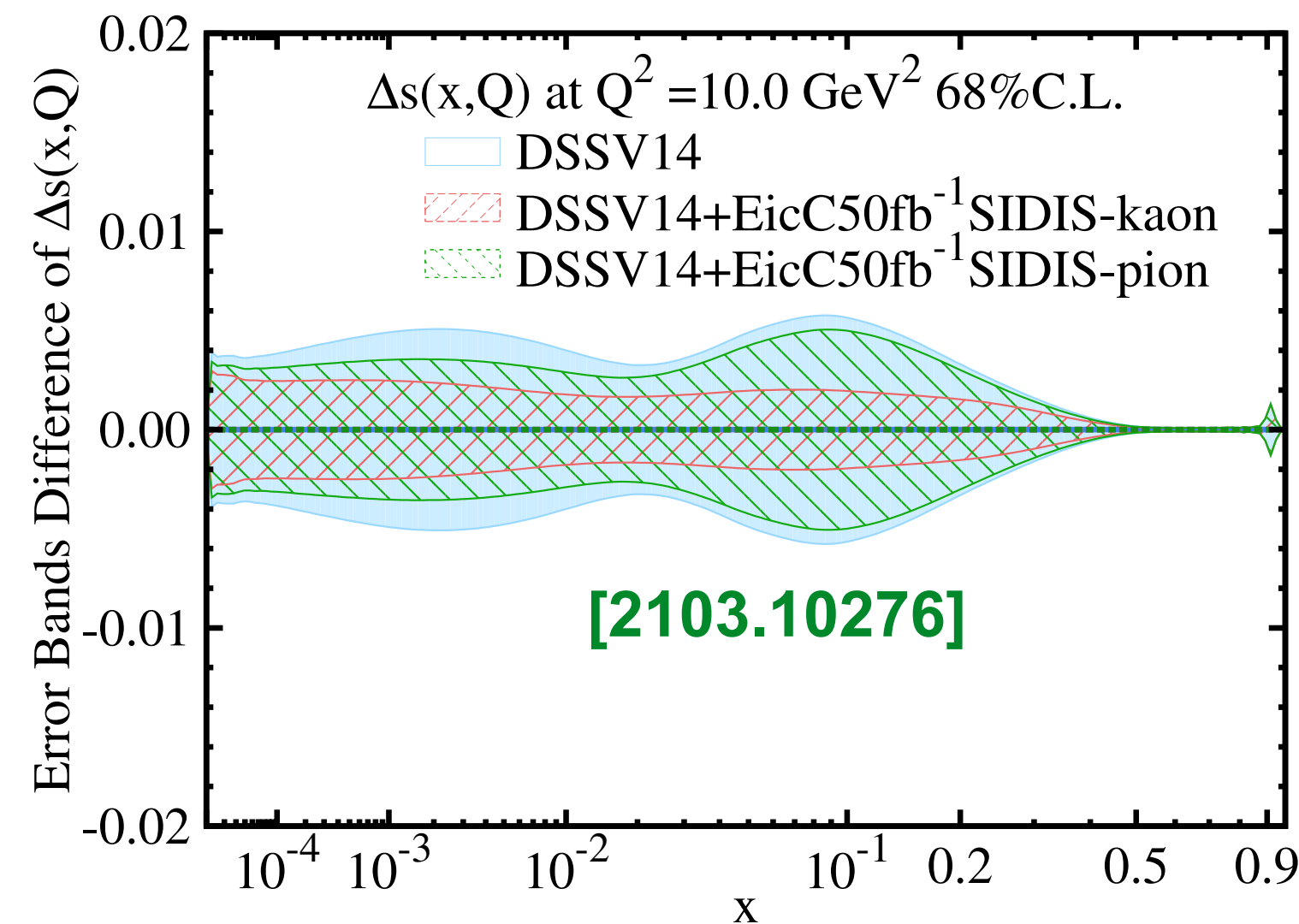
$$z = \frac{\mathbf{p}_{\text{had}} \cdot \mathbf{p}_{\text{jet}}}{|\mathbf{p}_{\text{jet}}|^2}, \quad F(z) = \frac{1}{N_{Z+\text{jet}}} \frac{dN_{\text{had}}(z)}{dz}$$



# Phenomenology implications

- FFs have wide applications in various programs of nuclear and particle physics, e.g., in studies of nucleon structure including helicity PDFs, jet flavor tagging, jet transportation properties in QGP matters, etc.

helicity PDFs of strange quark

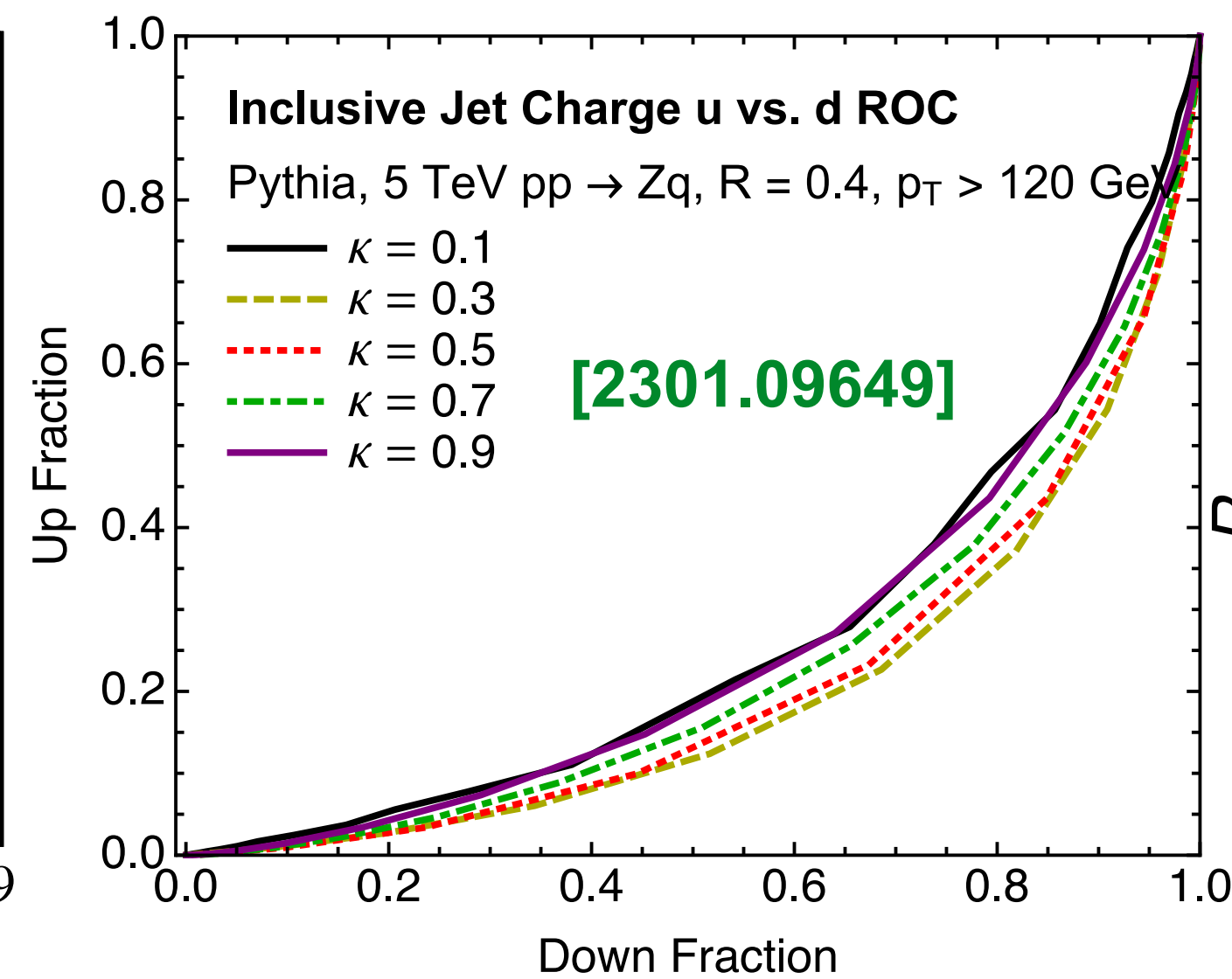


impact of SIDIS pseudo-data

$$F_1^h(x, Q^2, z) = \frac{1}{2} \sum_q e_q^2 [q(x, Q^2) D^{q \rightarrow h}(Q^2, z) + \Delta \bar{q}(x, Q^2) D^{\bar{q} \rightarrow h}(Q^2, z)],$$

$$g_1^h(x, Q^2, z) = \frac{1}{2} \sum_q e_q^2 [\Delta q(x, Q^2) D^{q \rightarrow h}(Q^2, z) + \Delta \bar{q}(x, Q^2) D^{\bar{q} \rightarrow h}(Q^2, z)].$$

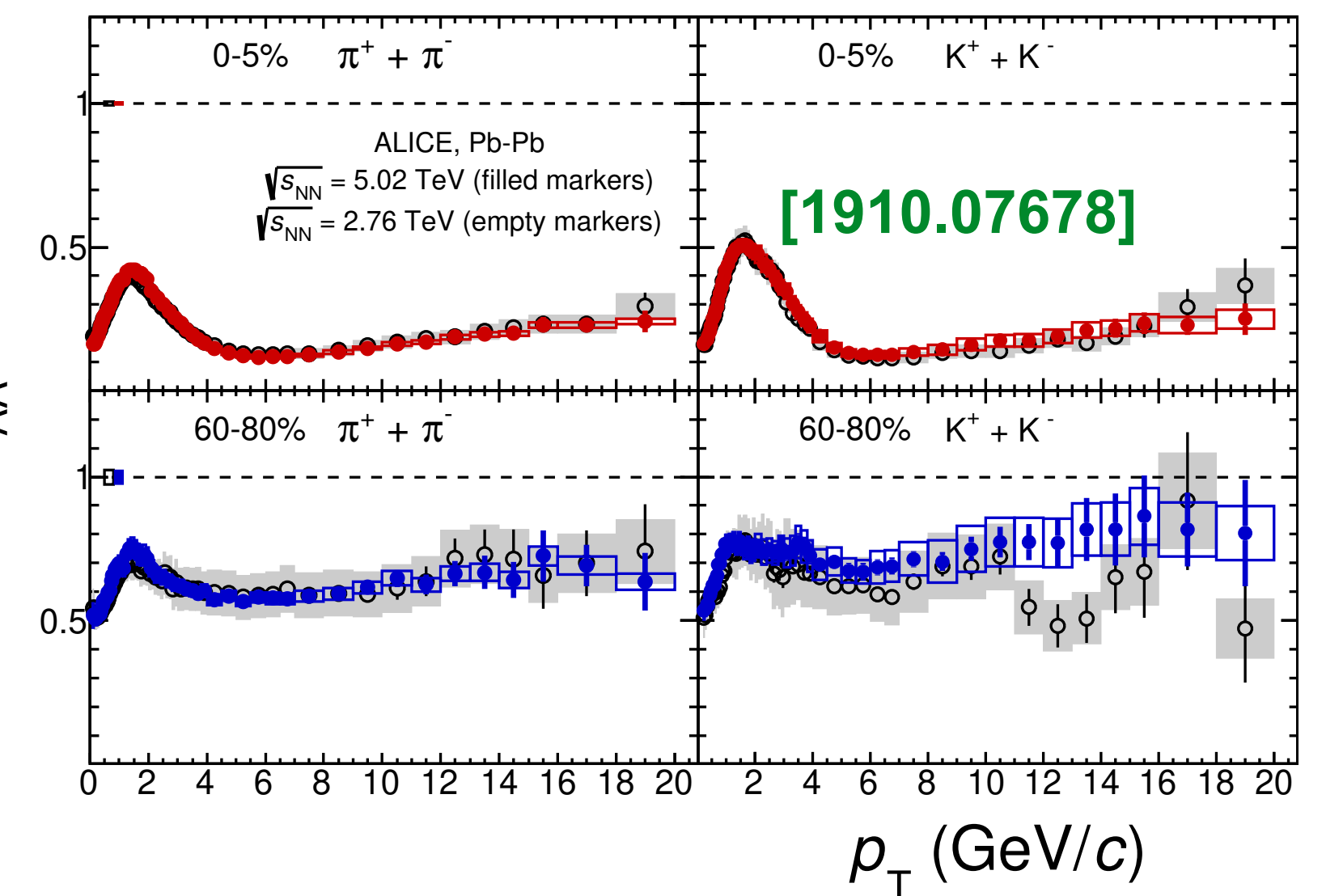
u/d quark jet separation



jet charge from constituent tracks

$$Q_J = \frac{1}{(p_{TJ})^\kappa} \sum_{i \in \text{Tracks}} q_i \times (p_{T,i})^\kappa$$

hadron production: Pb-Pb vs pp collisions



hadron suppression factor

$$R_{AA}(p_T) = \frac{dN^{AA}/dp_T}{\langle N_{\text{coll}} \rangle dN^{PP}/dp_T}$$

- ◆ 1. Introduction
- ◆ 2. Automation of fragmentation calculations at next-to-leading order
- ◆ 3. Global analysis of FFs to light charged hadrons
- ◆ 4. Outlook and summary



# Hadron production cross sections at NLO

- Generic algorithms on NLO calculations of jet production cross sections have been developed for long times, based on local subtraction or phase-space slicing method; especially automation of NLO jet cross sections exists, e.g., in MG5 and Sherpa

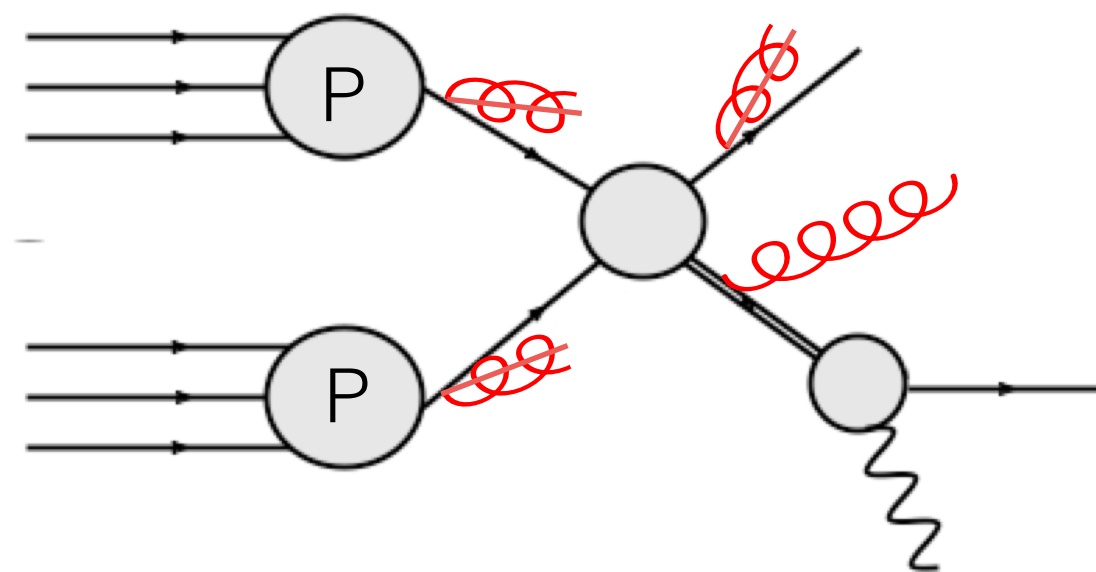
## local subtraction

$$\frac{d\sigma}{dF} = \int dPS_m \left[ |M|_{B,m}^2 + |M|_{V,m}^2 + |\tilde{\mathcal{I}}|_m^2 \right] \delta(\hat{F}(p_m; f_m) - F) \\ + \int dPS_{m+1} \left[ |M|_{R,m+1}^2 \delta(\hat{F}(p_{m+1}; f_{m+1}) - F) - |\mathcal{I}|_{m+1}^2 \delta(\hat{F}(\tilde{p}_m; \tilde{f}_m) - F) \right]$$

## phase-space slicing

$$\frac{d\sigma}{dF} = \int dPS_m \left[ |M|_{B,m}^2 + |M|_{V,m}^2 \right] \delta(\hat{F}(p_m; f_m) - F) \\ + \int dPS_{m+1} (\Theta(C - \lambda) + \Theta(\lambda - C)) \left[ |M|_{R,m+1}^2 \delta(\hat{F}(p_{m+1}; f_{m+1}) - F) \right]$$

## QCD radiations



- FKS subtraction (jet), **[Frixione, Kunszt, Signer]**, as implemented in Madgraph5
- Dipole subtraction (jet & hadron), **[Catani, Seymour]**, as implemented in MCFM
- Two-cutoff slicing (jet & hadron), **[Harris, Owens]**
- Antenna subtraction (jet & hadron), **[2406.09925]**; alternative subtraction **[2403.14574]**; semi-analytical calculations, **[1903.01529, BigkT]**

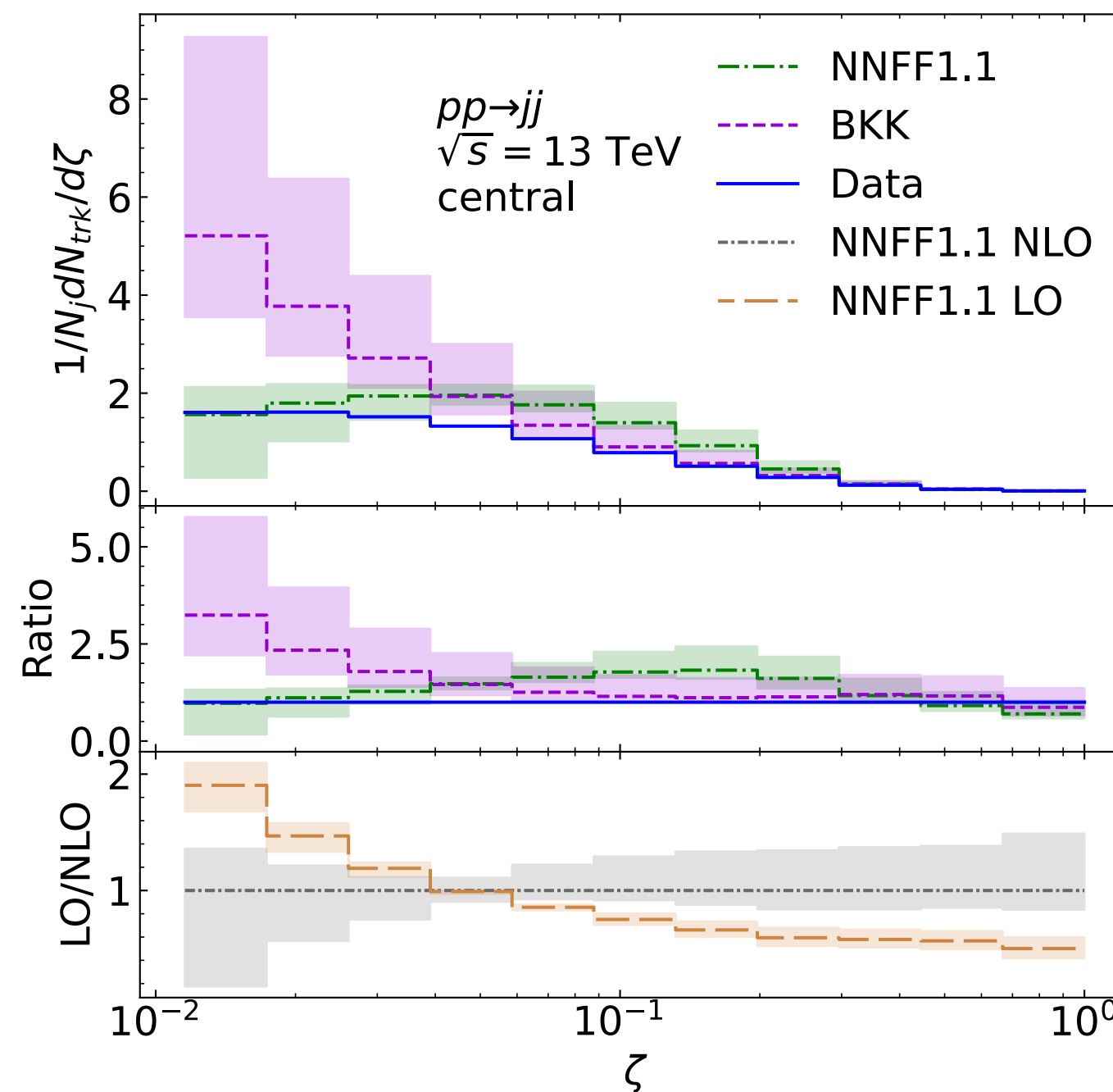
public tools and automation on the hadron cross sections calculations are very much limited!!!

# FMNLO (fragmentation at NLO in QCD)

- ◆ FMNLO is a program for automated and fast calculations of fragmentation cross sections of arbitrary processes. It is based on a hybrid scheme of phase-space slicing method and local subtraction method, accurate to NLO in QCD

- ❖ automation of fragmentation calculations for arbitrary hard processes up to NLO, within SM and BSMs via MG5\_aMC@NLO
- ❖ fast convolution algorithms of partonic cross sections with FFs without repeating the time consuming MC integrations
- ❖ future goal/generalizations: transverse observables, **NNLO corrections**

## QCD inclusive dijets at LHC



上海交通大学 INPAC 粒子与核物理研究所 上海市粒子物理和宇宙学重点实验室

Home Release Tutorial

## FMNLO

FMNLO is a framework to combine general-purpose Monte Carlo generator and parton fragmentation functions(FFs). It is based on a hybrid scheme of phase-space slicing method and local subtraction method, and accurate to next-to-leading order in QCD.

### News

2024.07: 🎉 [FMNLOv2.1](#) NNLO calculations are available for limited cases, SIA, decay of the Higgs boson to gluons, and SIDIS.

2024.07: 🎉 [FMNLOv2.0](#) include a **SIDIS** module for calculations of SIDIS at NLO.

2023.05: 🎉 [FMNLOv1.0](#) first release of **FMNLO** interfaced with **MG5\_aMC@NLO**.

### Publications

2024.07: 🎉 arXiv:[2407.10059](#), Towards ultimate fragmentation functions at future lepton colliders.

[JG, Liu, Shen, Zhou, 2305.14620 (JHEP)]

<https://fmnlo.sjtu.edu.cn/~fmnlo/>



- ◆ 1. Introduction
- ◆ 2. Automation of fragmentation calculations at next-to-leading order
- ◆ 3. Global analysis of FFs to light charged hadrons
- ◆ 4. Outlook and summary

# Overview of the NPC23 analysis of FFs

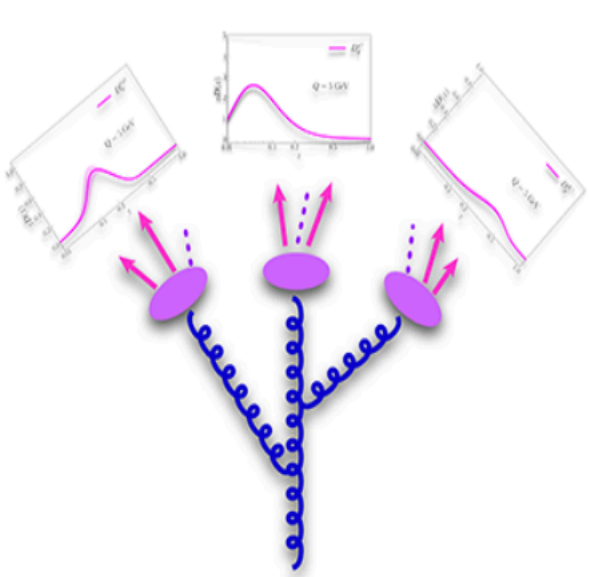
- Establishing a new framework on global analysis of fragmentation functions to identified charged hadrons, including charged pion, kaon and proton, using most recent data from SIA, SIDIS, and pp collisions

parametrization of FFs to charged pion/kaon/  
proton at an initial scale ( $Q=5$  GeV):

$$zD_i^h(z, Q_0) = z^{\alpha_i^h} (1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^m a_{i,n}^h (\sqrt{z})^n\right)$$

parton-to- $\pi^+$	avored	$\alpha$	$\beta$	$a_0$	$a_1$	$a_2$	d.o.f.
$u$	Y						5
$d \simeq u$	Y	-	-		-	-	1
$\bar{u} = d$	N					x	4
$s = \bar{s} \simeq \bar{u}$	N	-				x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
$g$	N		F				4

- a **joint determination** of FFs to charged pion, kaon and proton (via ratios or sum) at NLO in QCD (63 parameters) including estimation of uncertainties with Hessian sets
- apply a **strong selection criteria** on the kinematics of fragmentation processes to ensure validity of LT factorization and perturbative calculations ( $E_h/p_{T,h} > 4$  GeV, and  $z > 0.01$ )
- including **theory uncertainties** (residual scale variations) into the covariance matrix
- use fast interpolation techniques as in FMNLO for calculations of cross sections which largely increase efficiency of the global fit



EDITORS' SUGGESTION Global analysis of fragmentation functions to charged hadrons with high-precision data from the LHC  
12 December, 2024

This paper presents a global analysis of fragmentation functions for light charged hadrons, which describe the production of these states from partons. The fit includes for the first time jet data from proton-proton collisions at the LHC. The authors find good agreement with data, but note significant differences with previous work and the need for careful experimental definitions for future efforts.

Jun Gao *et al.*  
[Phys. Rev. D 110, 114019 \(2024\)](#)

[JG, Liu, Shen, Xing, Zhao, 2401.02781 (PRL), 2407.04424 (PRD Editors' suggestion)]



# Selection of data

- For the first time the jet fragmentation data from LHC have been incorporated into the global analysis of FFs to light charged hadrons, including from processes of incl. jet, dijet, Z or photon tagged jet productions, due to the development of FMNLO

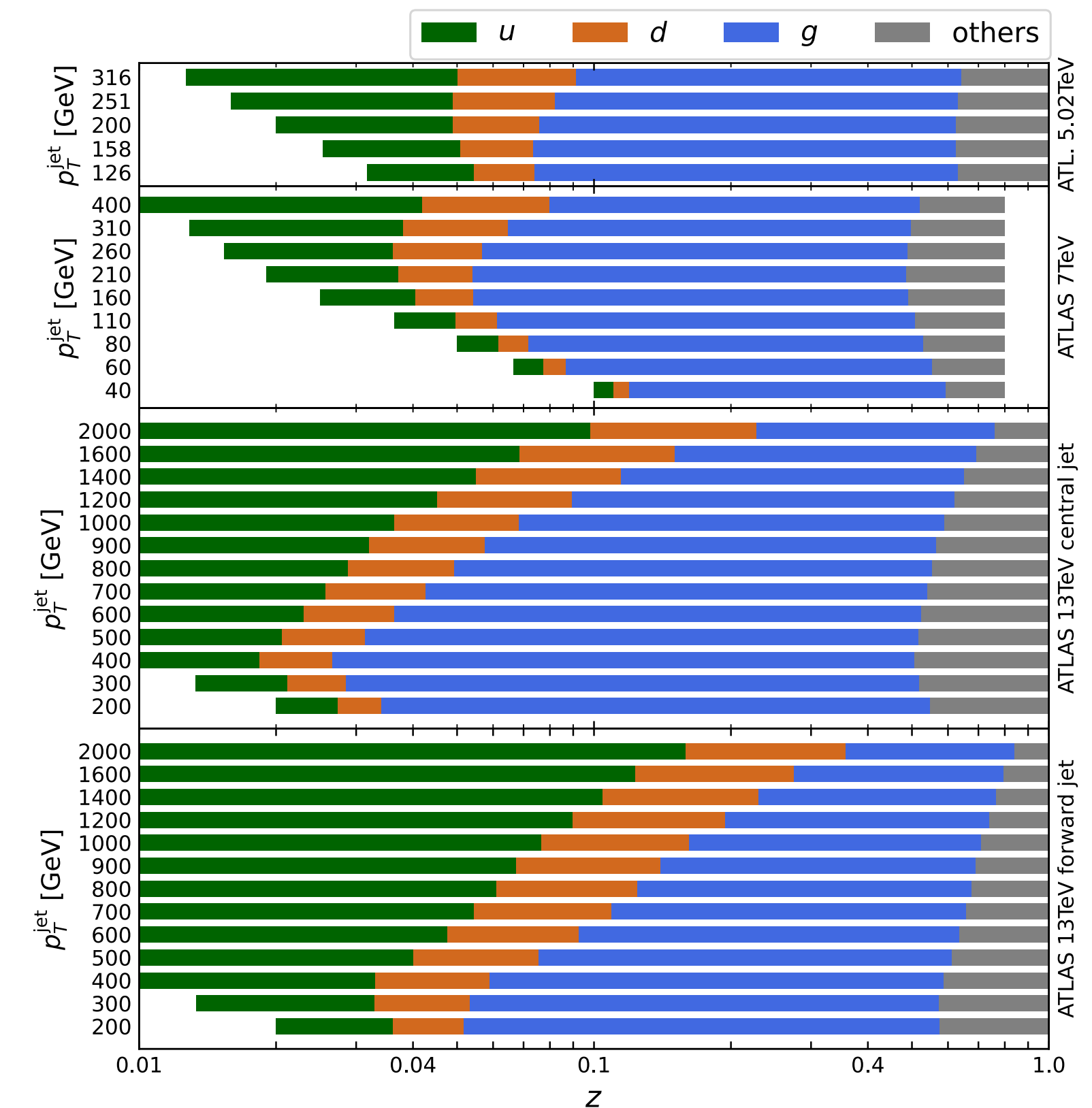
## LHC measurements for hadron inside jet measurements (jet fragmentation)

exp.	$\sqrt{s}$ (TeV)	luminosity	hadrons	final states	$R_j$	cuts for jets/hadron	observable	$N_{\text{pt}}$
ATLAS[60]	5.02	25 pb <sup>-1</sup>	$h^\pm$	$\gamma + j$	0.4	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{dp_{T,h}}$	6
CMS[61]	5.02	27.4 pb <sup>-1</sup>	$h^\pm$	$\gamma + j$	0.3	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	4
ATLAS[62]	5.02	260 pb <sup>-1</sup>	$h^\pm$	$Z + h$	no jet	$\Delta\phi_{h,Z} > \frac{3}{4}\pi$	$\frac{1}{n_Z} \frac{dN_{\text{ch}}}{dp_{T,h}}$	9
CMS[63]	5.02	320 pb <sup>-1</sup>	$h^\pm$	$Z + h$	no jet	$\Delta\phi_{h,Z} > \frac{7}{8}\pi$	$\frac{1}{n_Z} \frac{dN_{\text{ch}}}{dp_{T,h}}$	11
LHCb[64]	13	1.64 fb <sup>-1</sup>	$\pi^\pm, K^\pm, p/\bar{p}$	$Z + j$	0.5	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{n_Z} \frac{dN_{\text{ch}}}{d\xi}$	20
ATLAS[65]	5.02	25 pb <sup>-1</sup>	$h^\pm$	inc. jet	0.4	-	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	63
ATLAS[66]	7	36 pb <sup>-1</sup>	$h^\pm$	inc. jet	0.6	$\Delta R_{h,j} < R_j$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	103
ATLAS[67]	13	33 fb <sup>-1</sup>	$h^\pm$	dijet	0.4	$p_T^{\text{lead}}/p_T^{\text{sublead}} < 1.5$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	280

❖ LHC measurements on hadron inside jet provide essential inputs for u/d/g flavor separation with wide kinematic coverages, both in energy scale Q and in momentum fraction z

❖ In dijets or inclusive jets production, low p<sub>T</sub> and central (high p<sub>T</sub> and forward) jets are mostly initiated by g(u-quark); Z or photon tagged jets are more likely from u/d quarks

## kinematic/flavor coverage (LO) for ATLAS jet fragmentation



# Quality of the fit

- ◆ A best-fit with good agreements to the global data sets (1370 points in total) are found,  $\chi^2/N$  well below 1; individual agreements to the 138 sub-datasets are also tested, motivating usage of a tolerance  $\Delta\chi^2 \sim 2$  in determination of Hessian uncertainties

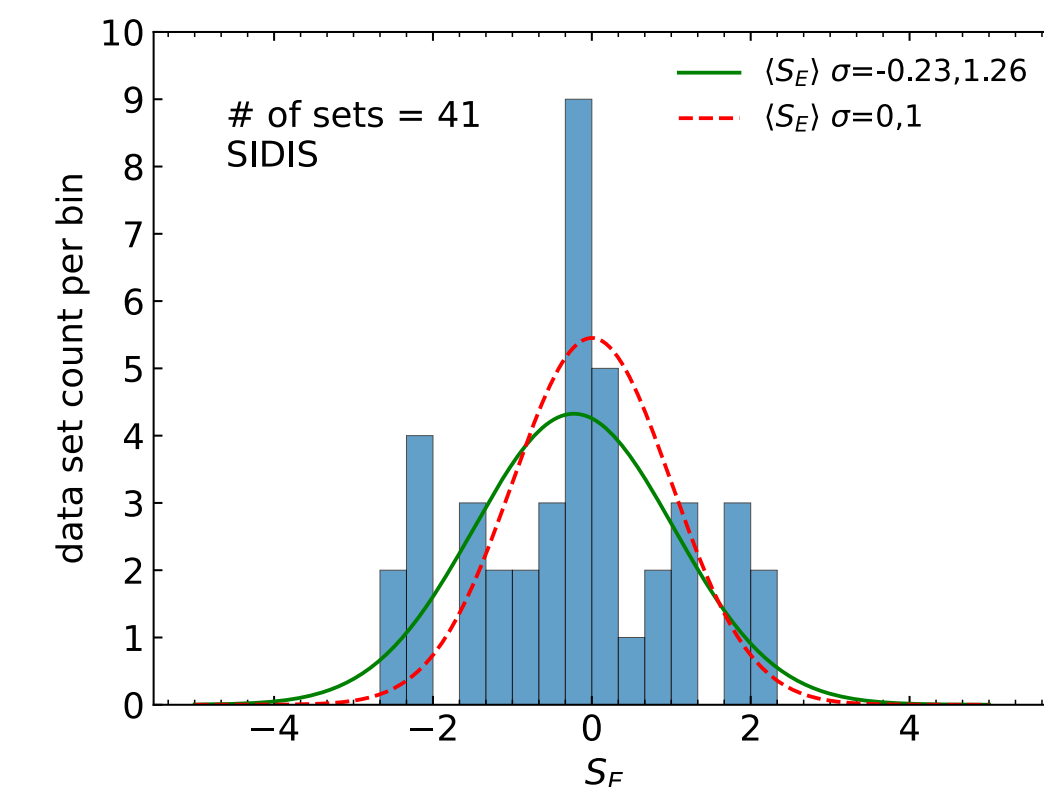
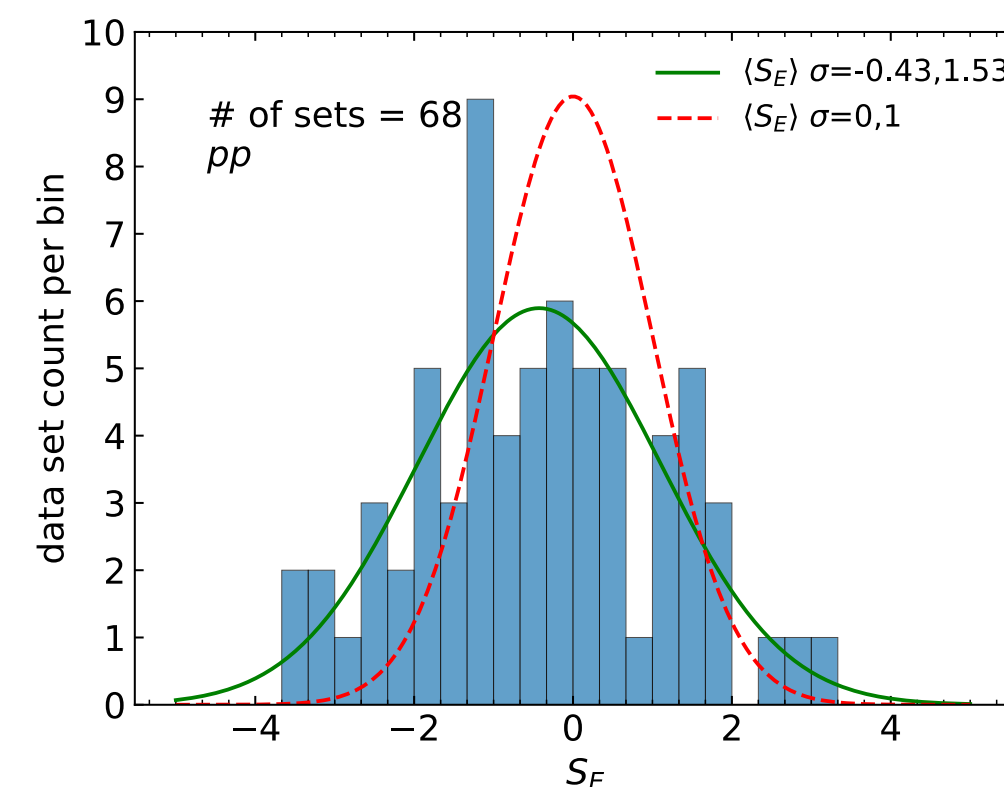
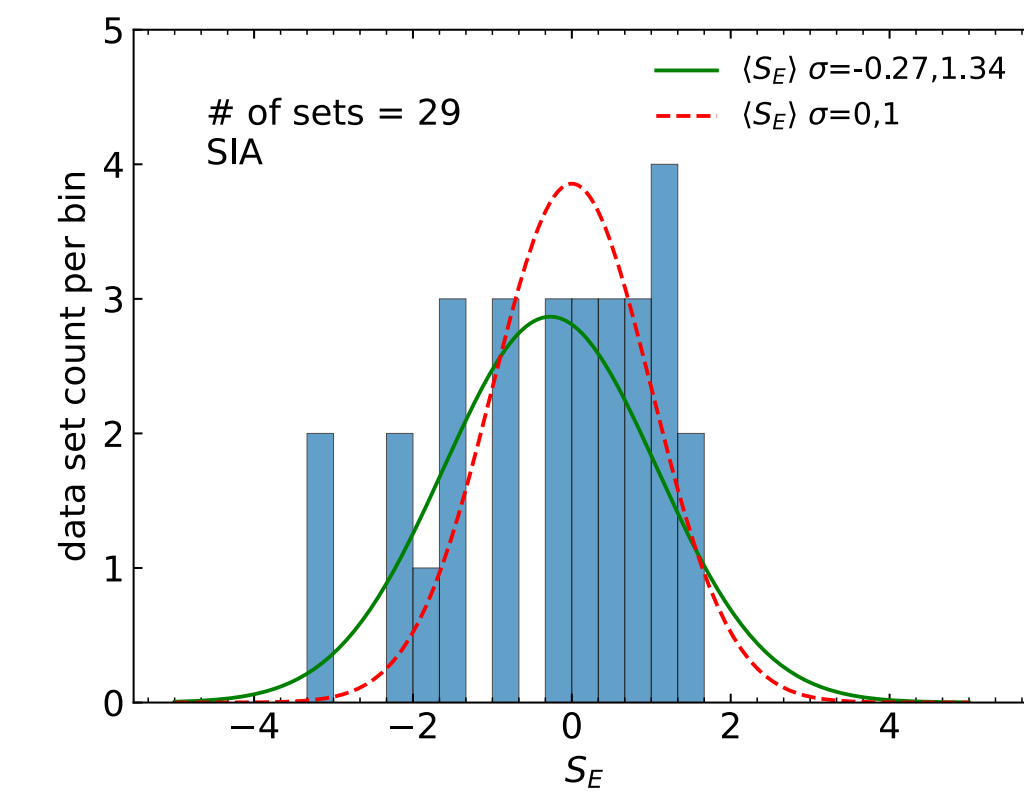
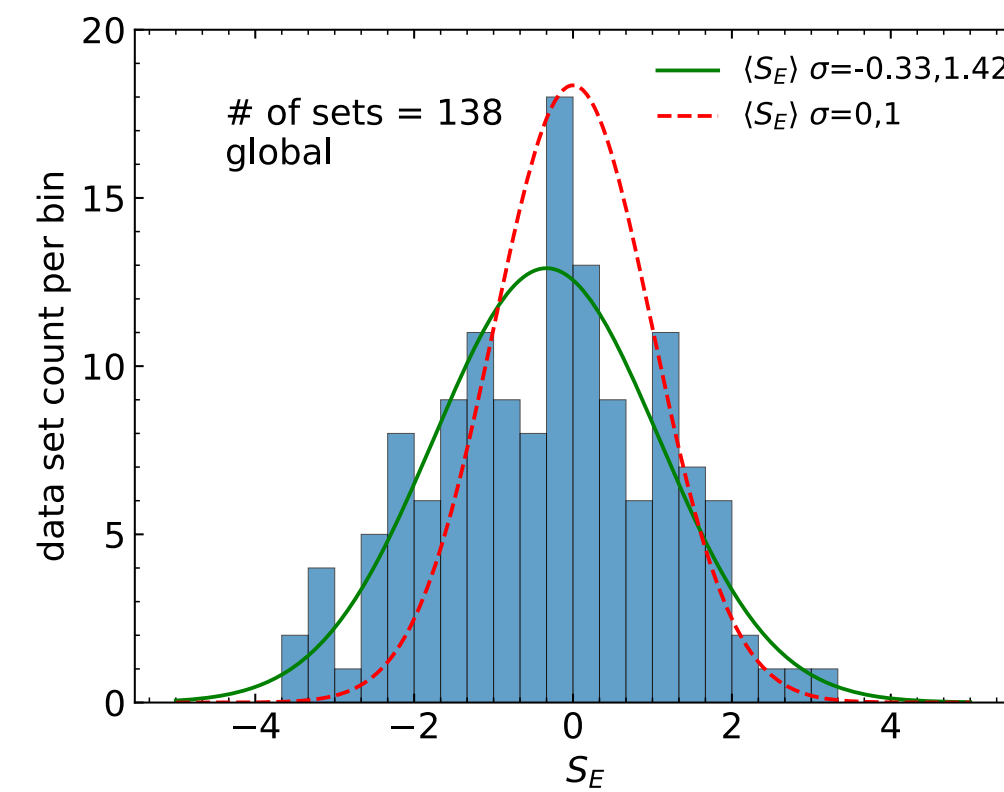
[CTEQ-TEA]

overall agreement:  $\chi^2$  breakdown to sub-groups for the best-fit

Experiments	$N_{pt}$	$\chi^2$	$\chi^2/N_{pt}$
ATLAS jets <sup>†</sup>	446	350.8	0.79
ATLAS Z/ $\gamma$ +jet <sup>†</sup>	15	31.8	2.12
CMS Z/ $\gamma$ +jet <sup>†</sup>	15	17.3	1.15
LHCb Z+jet	20	30.6	1.53
ALICE inc. hadron	147	150.6	1.02
STAR inc. hadron	60	42.2	0.70
<b>pp sum</b>	<b>703</b>	<b>623.3</b>	<b>0.89</b>
TASSO	8	7.0	0.88
TPC	12	11.6	0.97
OPAL	20	16.3	0.81
OPAL (202 GeV) <sup>†</sup>	17	24.2	1.42
ALEPH	42	31.4	0.75
DELPHI	78	36.4	0.47
DELPHI (189 GeV)	9	15.3	1.70
SLD	198	211.6	1.07
<b>SIA sum</b>	<b>384</b>	<b>353.8</b>	<b>0.92</b>
H1 <sup>†</sup>	16	12.5	0.78
H1 (asy.) <sup>†</sup>	14	12.2	0.87
ZEUS <sup>†</sup>	32	65.5	2.05
COMPASS (06I)	124	107.3	0.87
COMPASS (16p)	97	56.8	0.59
<b>SIDIS sum</b>	<b>283</b>	<b>254.4</b>	<b>0.90</b>
<b>Global total</b>	<b>1370</b>	<b>1231.5</b>	<b>0.90</b>

individual agreement: distributions of the effective Gaussian variable

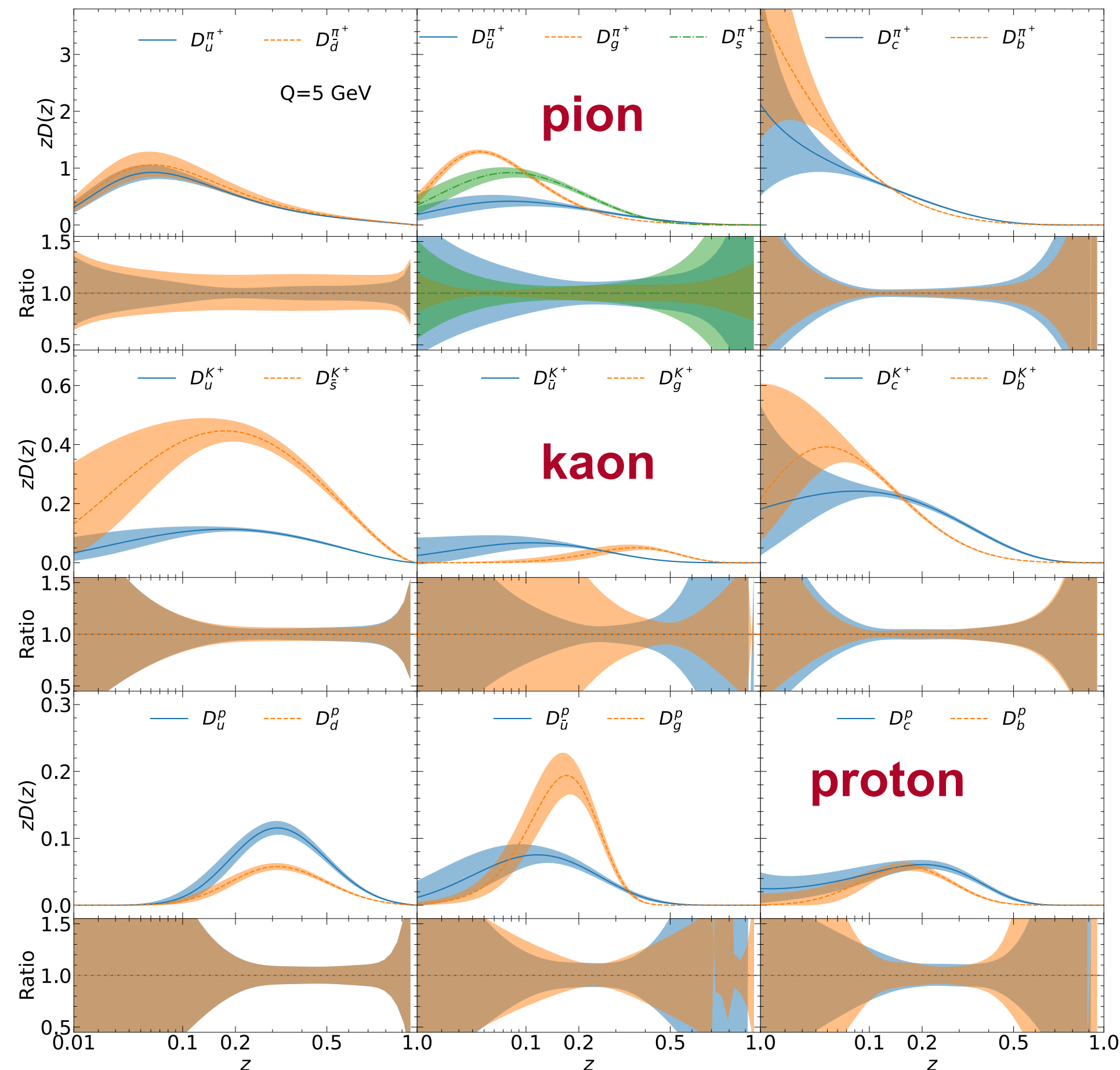
$$S_E = \frac{(18N_{pt})^{3/2}}{18N_{pt} + 1} \left\{ \frac{6}{6 - \ln(\chi^2/N_{pt})} - \frac{9N_{pt} - 1}{9N_{pt}} \right\}$$





# NPC23 FFs to charged hadrons

- ◆ We arrive at a best-fit of the charged pion, kaon and proton FFs together with 126 Hessian error FFs, two for each of the eigenvector direction; FFs are generally well constrained in the region with  $z \sim 0.1-0.7$



FFs (positively charged) vs. momentum fraction

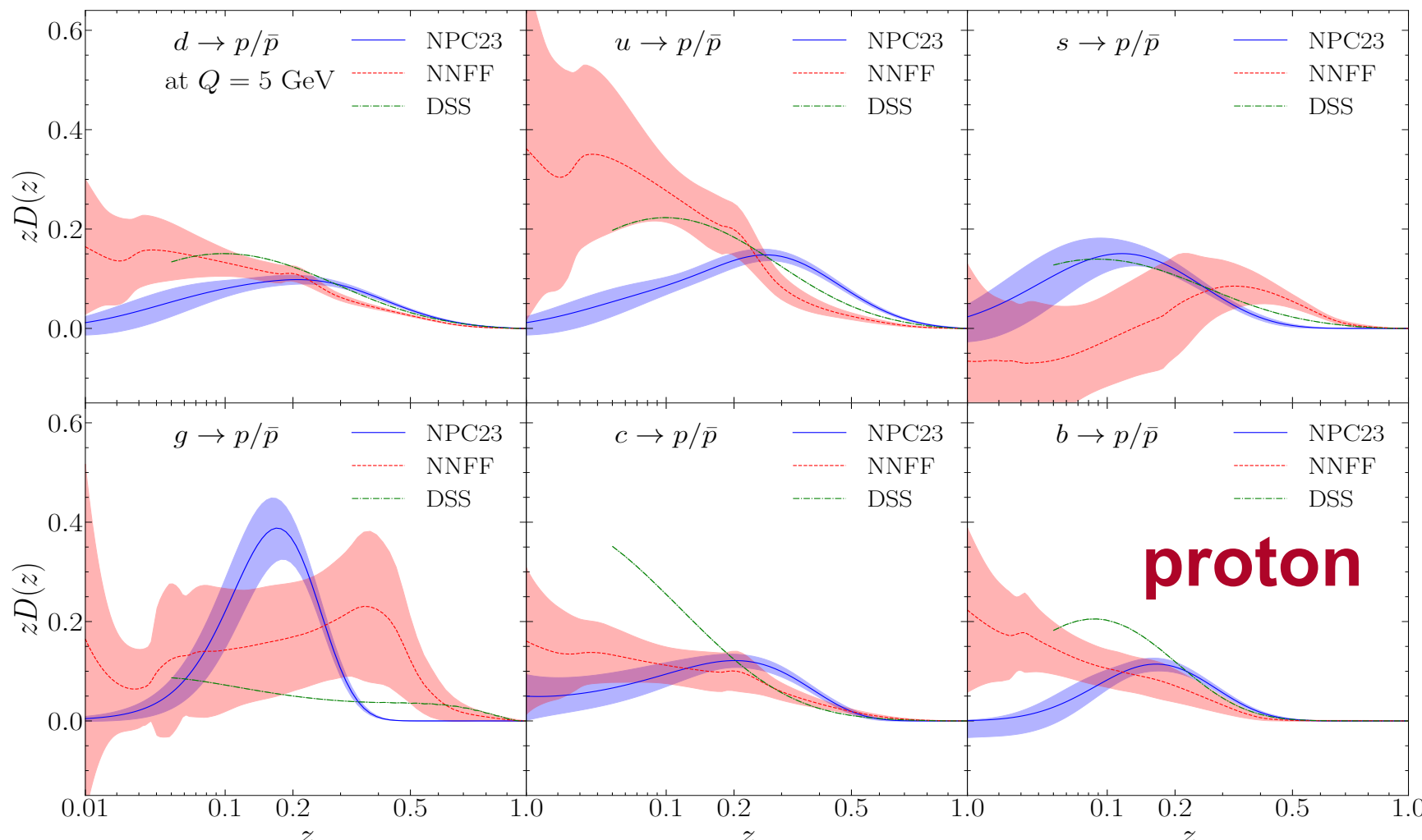
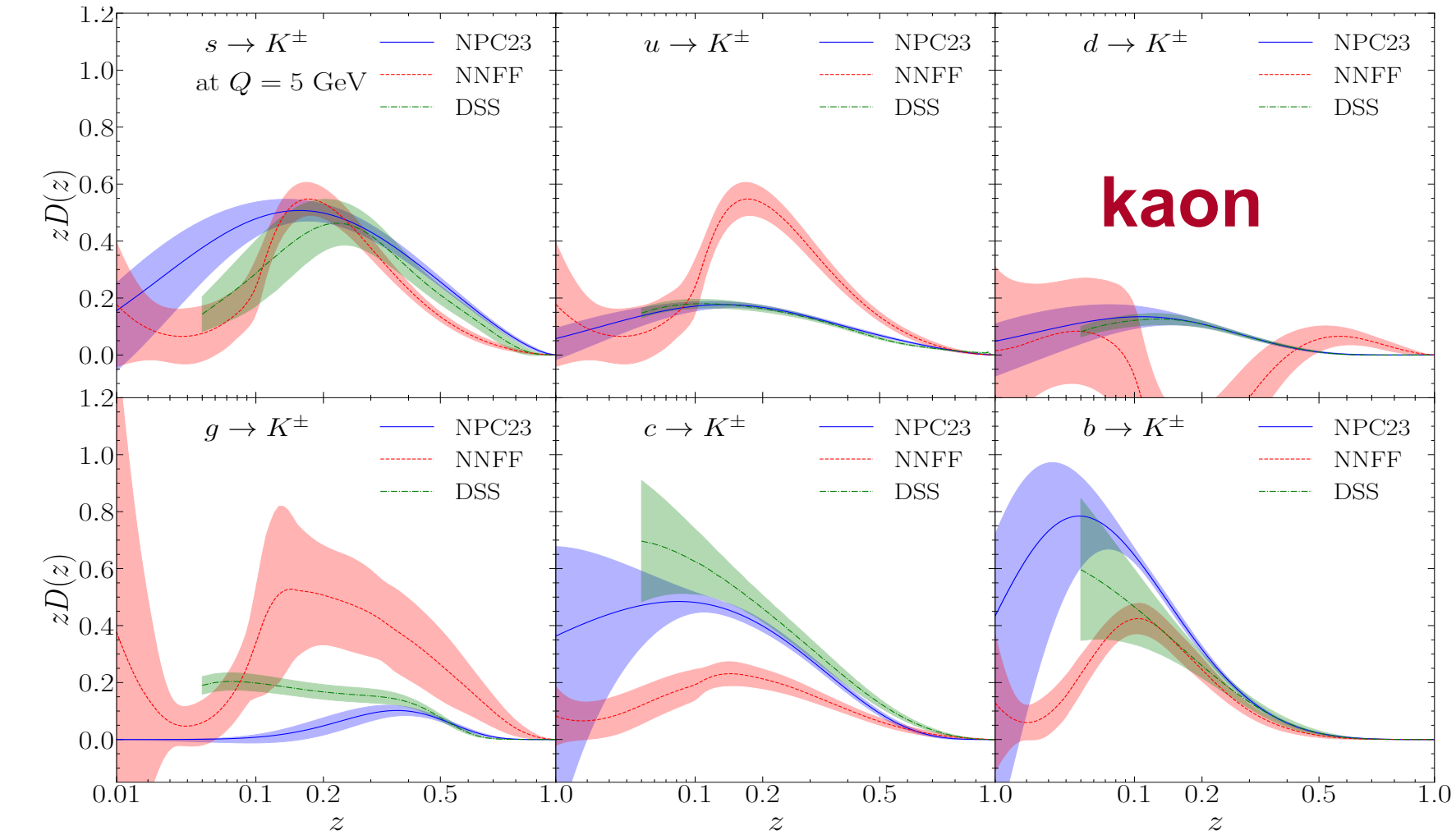
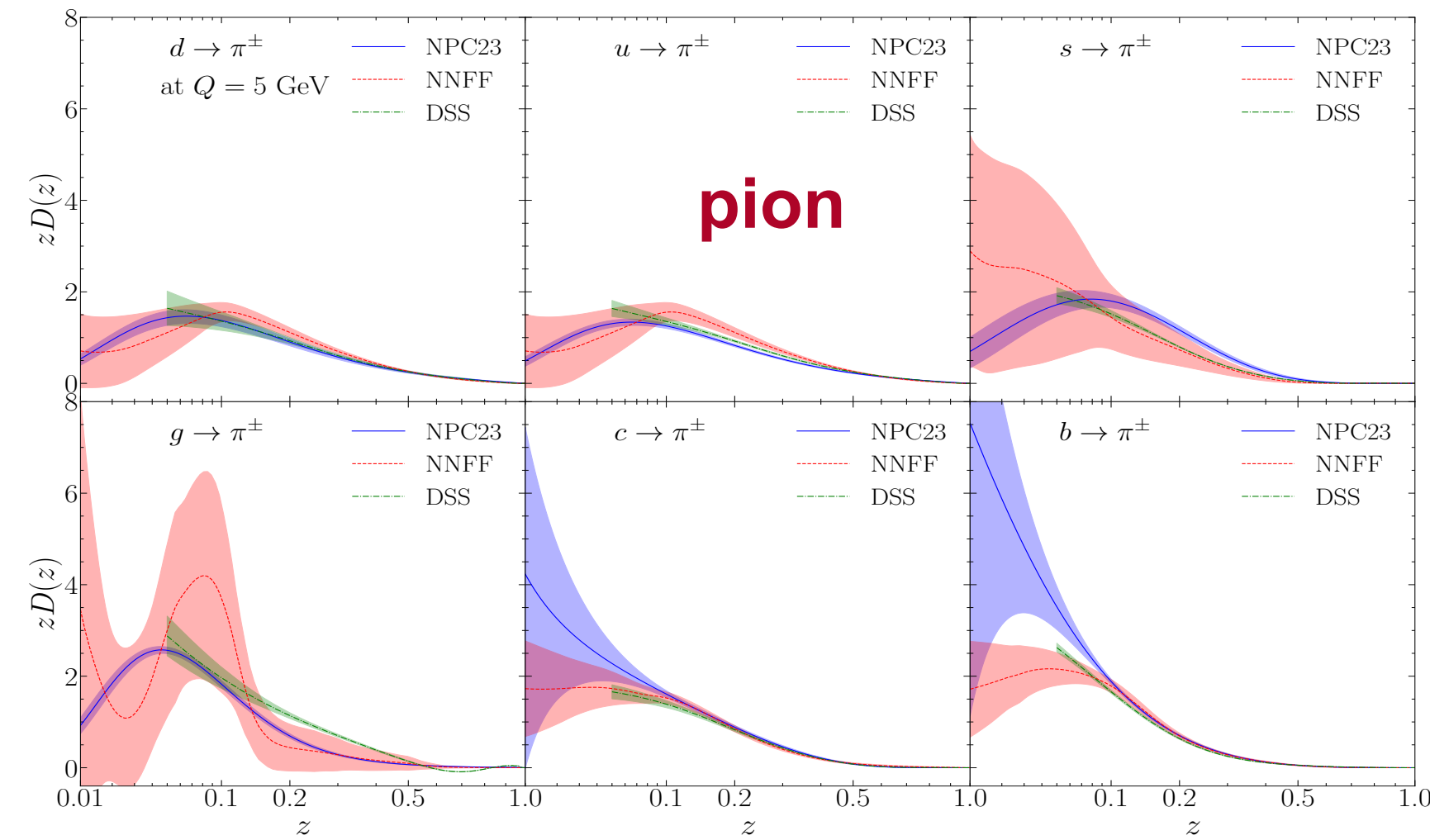
- ◆ our results show an uncertainty of 3%, 4% and 8% for FFs of gluon to pion at  $z=0.05, 0.1$  and  $0.3$ , respectively
  - ◆ similarly an uncertainty of 4%, 4% and 7% for FFs of u-quark to pion, kaon and proton at  $z=0.3$ , respectively
  - ◆ FFs of heavy-quarks are well constrained for  $z$  between  $0.1 \sim 0.5$  due to the tagged SIA events of Z-pole measurements
- ◆ high precision of gluon FFs is mostly due to the data of jet fragmentation from the LHC
  - ◆ a preference for larger FFs of s quark to pion due to pulls from SIA data

# Comparison to other determinations

- Our new extractions on FFs are compared to previous determinations from other groups (e.g., DSS and NNFF) for the charge-summed pion, kaon and proton; discrepancies are found and further investigations will be needed

FFs (charge-summed) vs. momentum fraction

[DSS21, DSS17, DSS07, NNFF1.0]



- We find general agreement between ours with DSS for FFs of u and d quarks to pion, and of u, d and s quark to kaon

- however, discrepancies are found for FFs to protons and for FFs of gluon to all three charged hadrons

- NNFFs show larger uncertainties in general and can become negative in some kinematic regions

- future benchmark works involving different groups will be needed for investigation on discrepancies

# Test on momentum sum rule

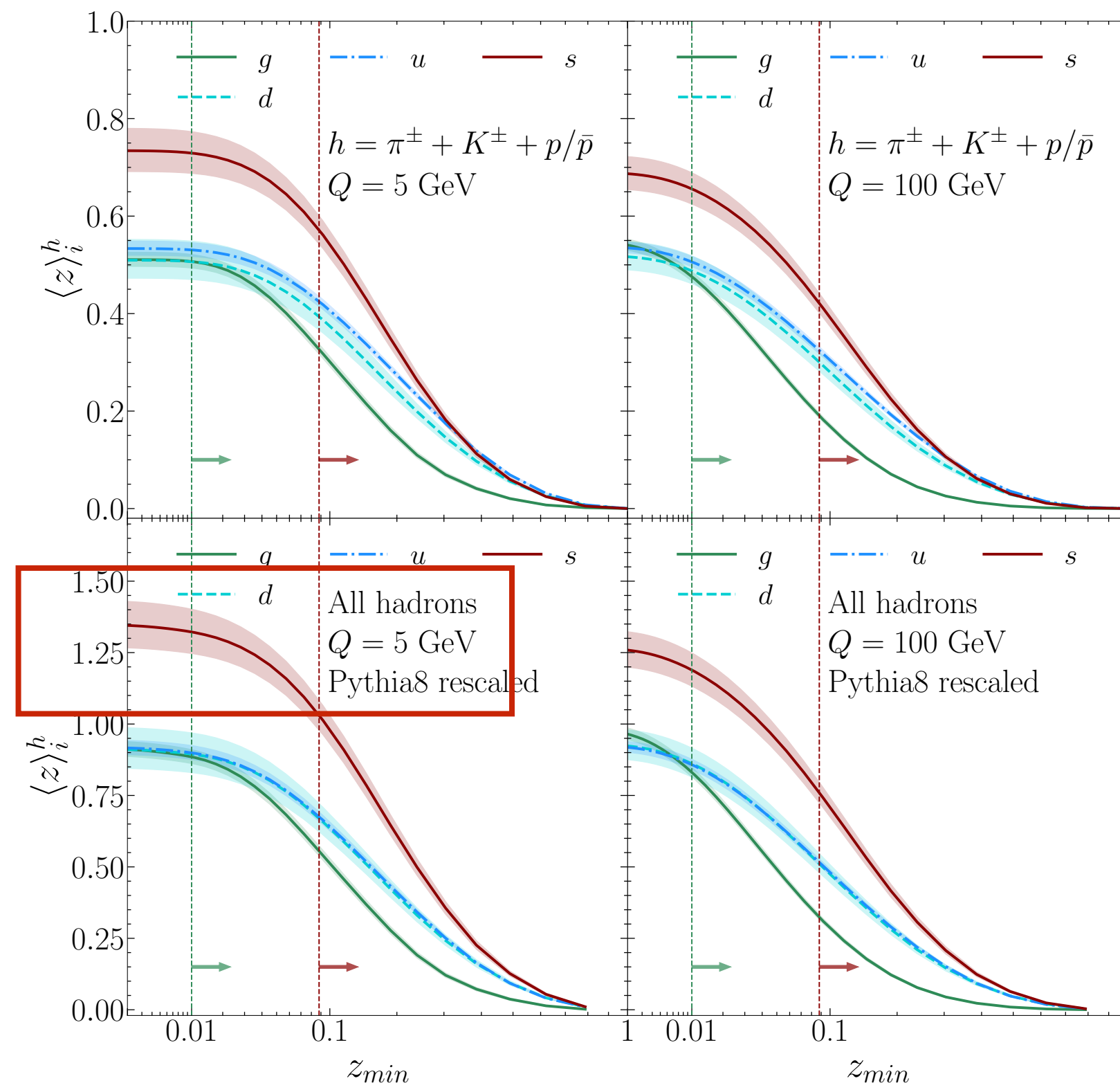
- FFs have the interpretation of number densities of hadrons and satisfy various fundamental sum rules as derived from first principle, including momentum sum rule, charge sum rule, etc. [Collins, Rogers, 2023]; momentum sum rules are tested with the extracted FFs

momentum sum rule: 
$$\sum_h \int_0^1 dz z D_i^h(z, Q) = 1$$

with finite cutoff: 
$$\langle z \rangle_i^h = \int_{z_{min}}^1 dz z D_i^h(z, Q)$$

mom.	$g(z > 0.01)$	$u(z > 0.01)$	$d(z > 0.01)$	$s(z > 0.088)$
$\pi^+$	$0.200^{+0.008}_{-0.008}$	$0.262^{+0.017}_{-0.016}$	$0.128^{+0.020}_{-0.019}$	$0.161^{+0.013}_{-0.013}$
$K^+$	$0.018^{+0.004}_{-0.003}$	$0.058^{+0.005}_{-0.004}$	$0.019^{+0.004}_{-0.004}$	$0.015^{+0.002}_{-0.002}$
$p$	$0.035^{+0.006}_{-0.005}$	$0.044^{+0.004}_{-0.004}$	$0.022^{+0.002}_{-0.002}$	$0.015^{+0.002}_{-0.002}$
$\pi^-$	$0.200^{+0.008}_{-0.008}$	$0.128^{+0.020}_{-0.019}$	$0.299^{+0.054}_{-0.049}$	$0.161^{+0.013}_{-0.013}$
$K^-$	$0.018^{+0.004}_{-0.003}$	$0.019^{+0.004}_{-0.004}$	$0.019^{+0.004}_{-0.004}$	$0.205^{+0.014}_{-0.013}$
$\bar{p}$	$0.035^{+0.006}_{-0.005}$	$0.019^{+0.003}_{-0.003}$	$0.019^{+0.003}_{-0.003}$	$0.015^{+0.002}_{-0.002}$
<b>Sum</b>	$0.507^{+0.014}_{-0.013}$	$0.531^{+0.015}_{-0.013}$	$0.506^{+0.042}_{-0.037}$	$0.572^{+0.029}_{-0.028}$

momentum carried by individual/all light charged hadrons at Q=5 GeV



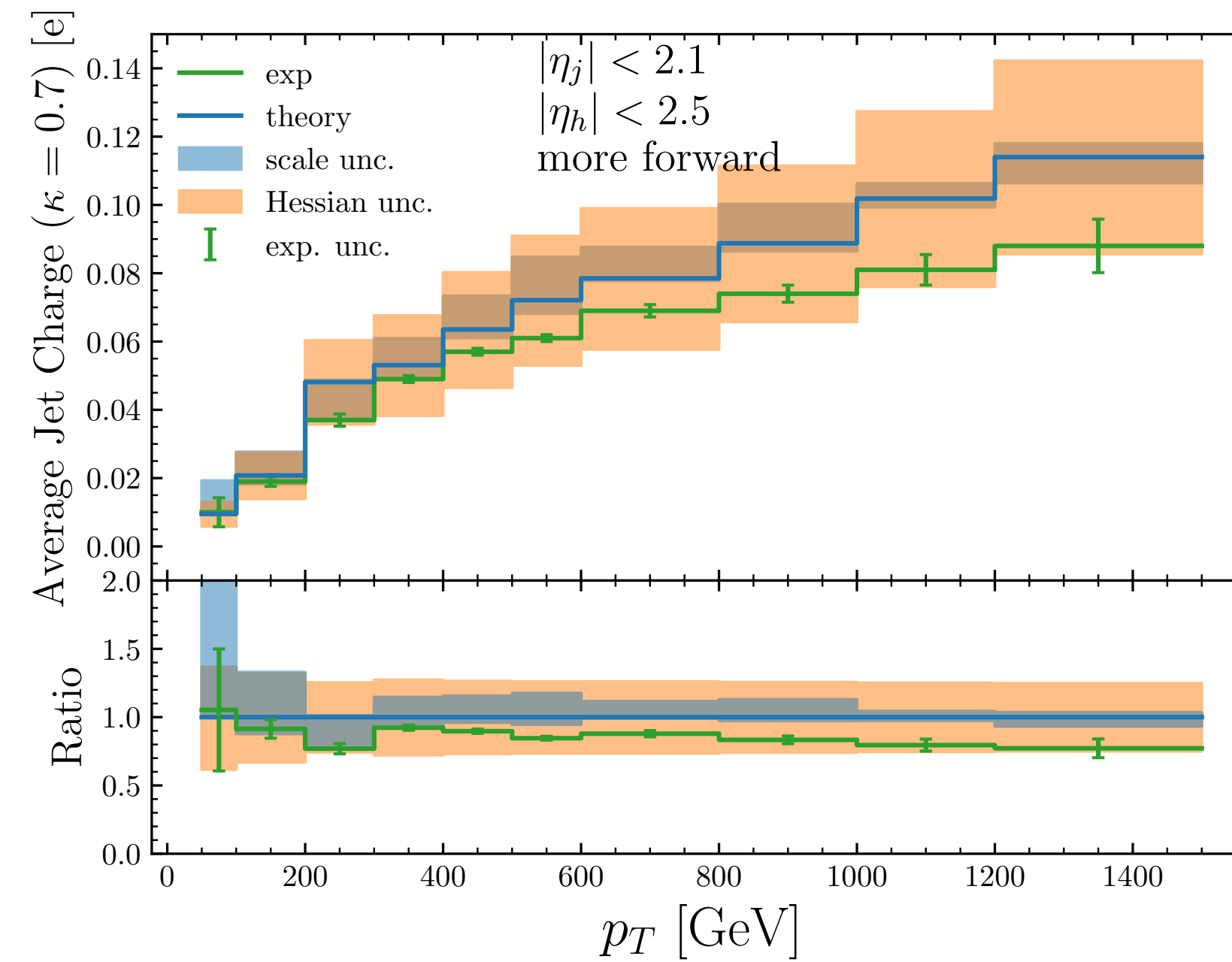
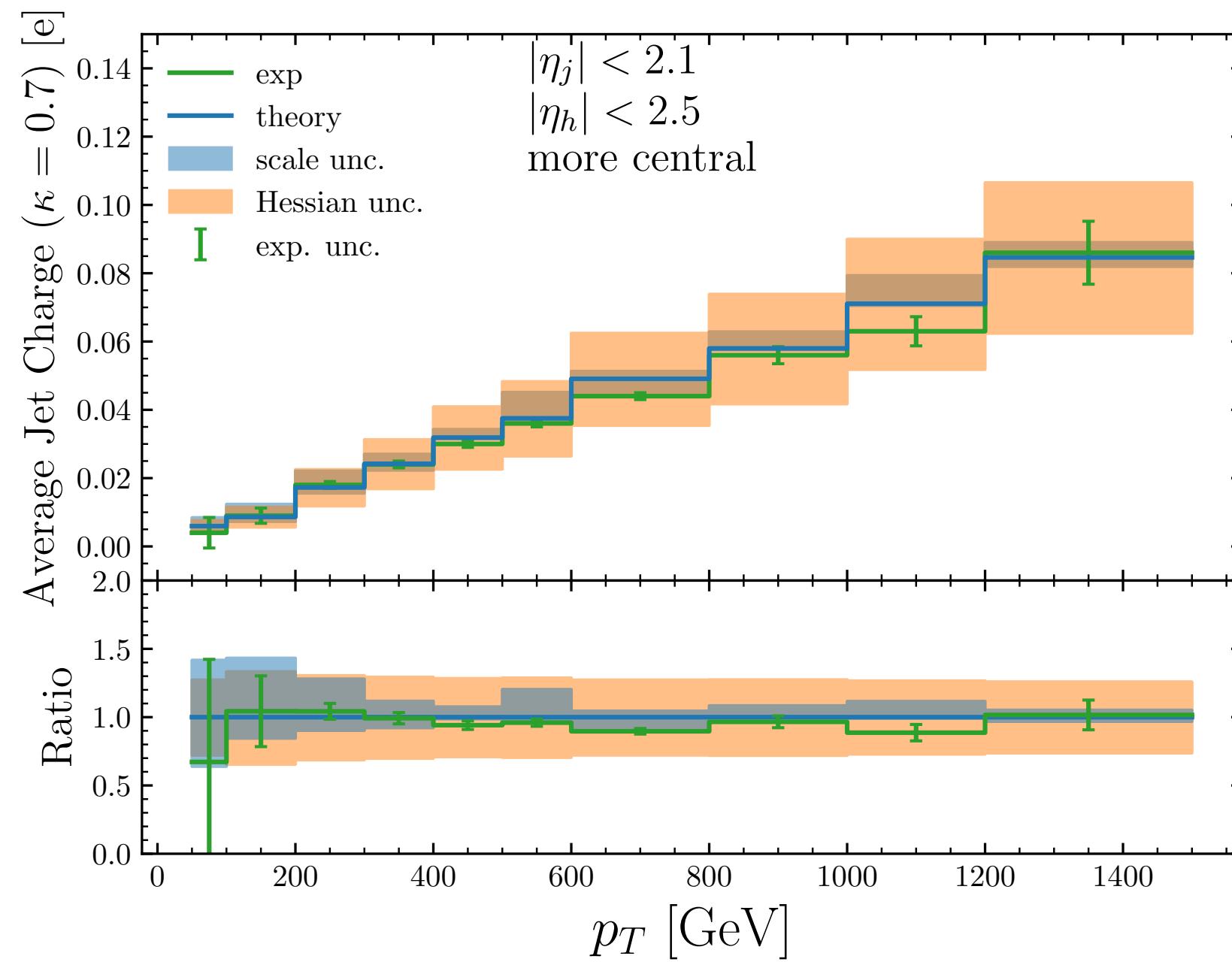
total momentum vs. cutoff: light charged hadron; all hadrons (scaled from PYTHIA8)



# Average Jet charge at the LHC

- Our NLO predictions on average jet charge agree well with the ATLAS measurements; larger uncertainties from FFs comparing to experimental precision indicate potential strong constraints from data on jet charge

average jet charge for forward/central jet at LHC 8 TeV



[ATLAS 8 TeV]

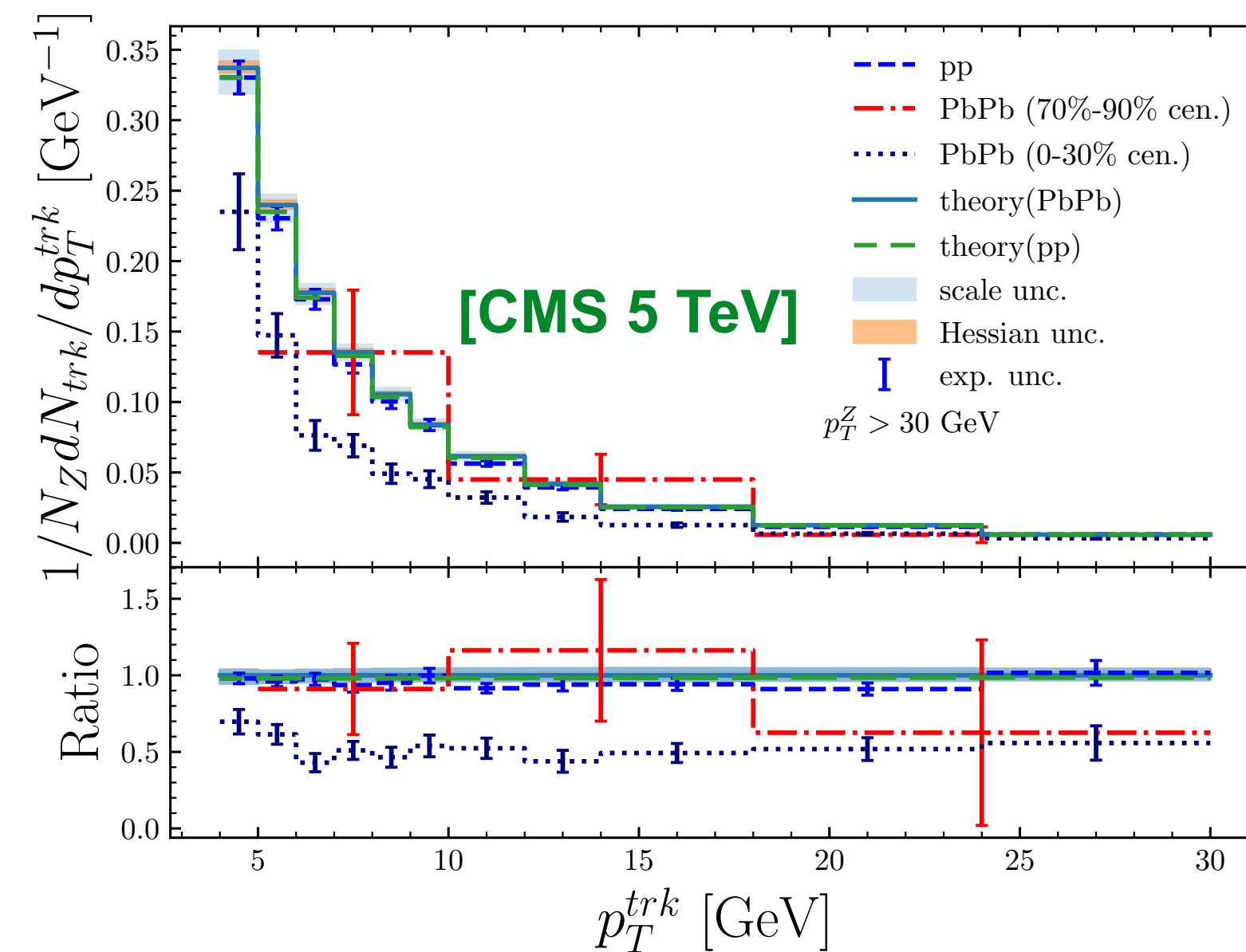
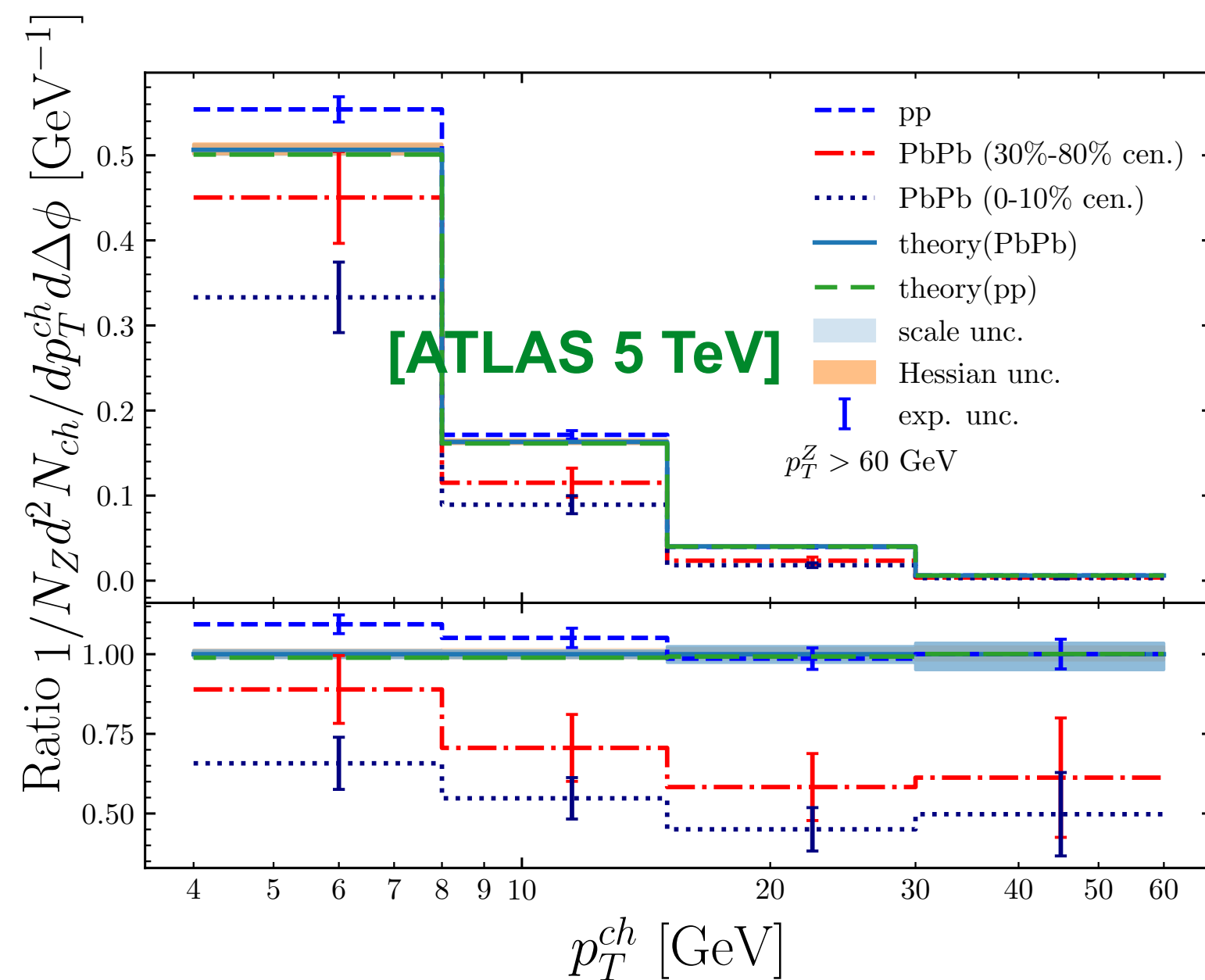
$$Q_J = \sum_{i \in J} \left( \frac{p_{T,i}}{p_{T,J}} \right)^\kappa Q_i$$

$$\langle Q_J \rangle = \int_{z_0}^1 dz_h z_h^\kappa \frac{1}{\sigma_J} \left( \frac{d\sigma_{h^+}}{dz_h} - \frac{d\sigma_{h^-}}{dz_h} \right)$$

# Reference cross sections for PbPb collisions

- ◆ We provide reference NLO predictions (w/o final state medium effects) for jet fragmentation in heavy-ion collisions using FMNLO together with NPC23 FFs; medium effects will be included in future

charged hadron multiplicities in jet fragmentation from Z+jet production in PbPb/pp



for central PbPb collisions data lying well below NLO predictions w/o medium corrections

- ◆ 1. Introduction
- ◆ 2. Automation of fragmentation calculations at next-to-leading order
- ◆ 3. Global analysis of FFs to light charged hadrons
- ◆ 4. Outlook and summary



# Opportunities at future lepton colliders

- High luminosity and high energies of future lepton colliders open new opportunities for precision determination of FFs, especially with production of the W boson pairs and the Higgs boson with hadronic decays  
[Zhou, JG, 2407.10059]

proposed hadron multiplicity measurements  
from annihilation to quarks

$e^+e^-$ annihilation							
$\sqrt{s}$ (GeV)	luminosity ( $\text{ab}^{-1}$ )			final state	kinematic cuts	hadrons	$N_{\text{pt}}$
	CEPC	FCC- $ee$	ILC				
91.2	60	150	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	132
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	65
160	4.2	-	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	168
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	83
161	-	10	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	168
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	83
240	13	5	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	186
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	92
250	-	-	2	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	186
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	92
350	-	0.2	0.2	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	98
360	0.65	-	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	98
365	-	1.5	-	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	98
500	-	-	4	$q\bar{q}$	$\cos(\theta) > 0$	$h^{+,-}$	198
				$c\bar{c}/b\bar{b}$	-	$h^\pm$	98

proposed hadron multiplicity measurements  
from decays of W or Higgs bosons

W boson decay channels							
$\sqrt{s}$ (GeV)	# events (million)			final state	kinematic cuts	hadrons	$N_{\text{pt}}$
	CEPC	FCC- $ee$	ILC				
80.419	116	68	62	$W^-W^{+*} \rightarrow W^-q\bar{q}$	-	$h^{+,-}$	120
	58	34	31	$W^-W^{+*} \rightarrow W^-c\bar{s}$			
Higgs boson decay channels							
$\sqrt{s}$ (GeV)	# events (million)			final state	kinematic cuts	hadrons	$N_{\text{pt}}$
	CEPC	FCC- $ee$	ILC				
125	0.23	0.09	0.07	$gg$	-	$h^\pm$	77
	0.08	0.03	0.02	$c\bar{c}$			
	1.53	0.59	0.47	$b\bar{b}$			

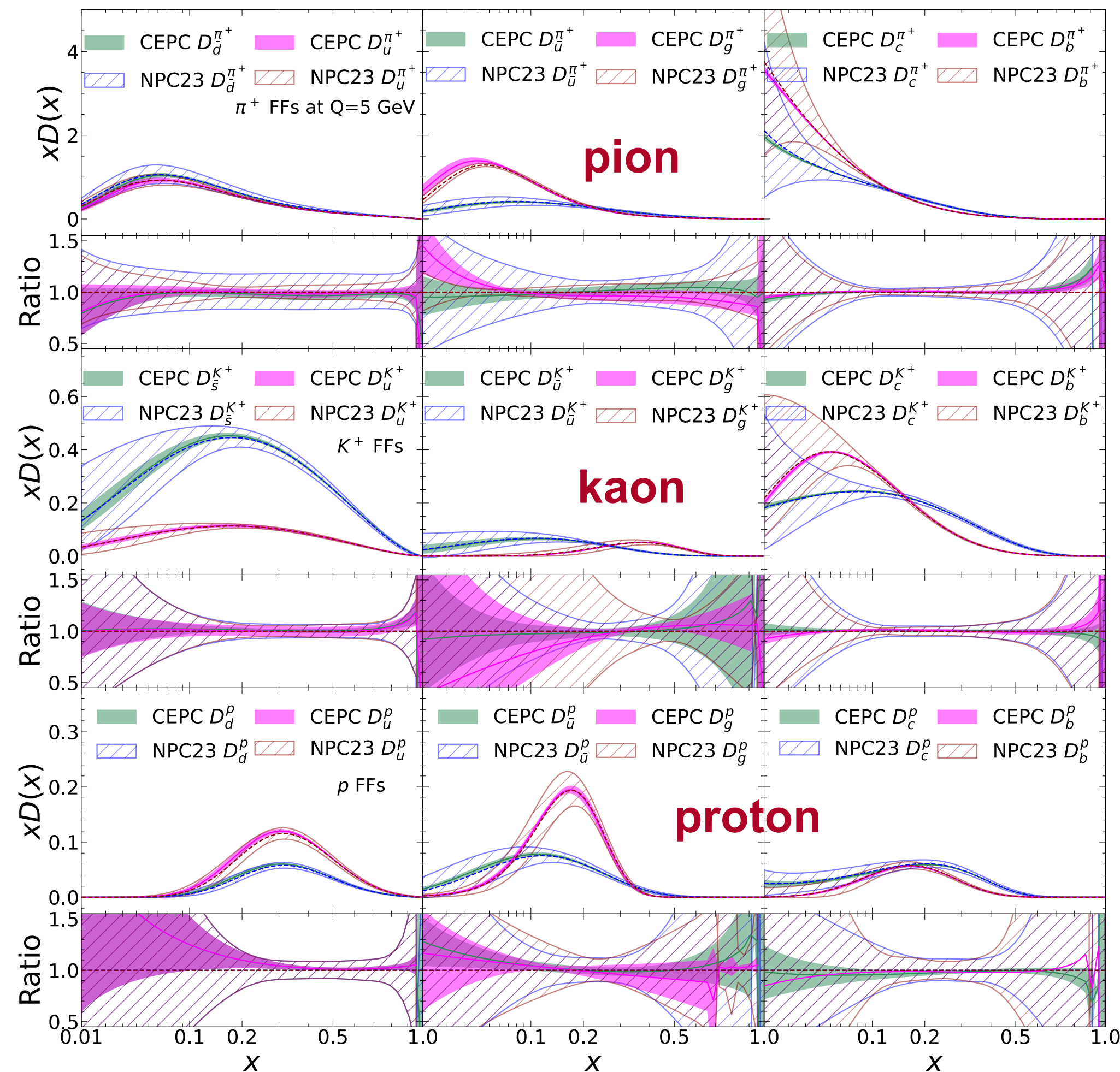
- (anti-)quark flavor separation from different energies, angular distributions, and heavy-flavor tagging
- d/s quark separation from W boson decays; probe of gluon fragmentation from Higgs boson decays

Higgs factory alone are ideal machine for fragmentation functions!

# Projection for constraints on FFs

- ◆ Pseudo-data on the proposed measurements are constructed using NPC23 FFs as the truth theory; fits to FFs at NLO in QCD are carried out with data solely from future electron-positron colliders

[Zhou, JG, 2407.10059]



FFs (positively charged) vs. momentum fraction

- ◆ assuming same (un-)correlated systematic uncertainties as in SLD; statistical errors calculated based on prescribed luminosities
- ◆ fits using the same fitting framework as NPC23 including theoretical uncertainties
- ◆ best-fit agrees well with the truth FF; uncertainties are greatly reduced taking the CEPC as an example
- ◆ W boson data are essential for quark flavor separation; similarly Higgs boson data for constraining gluon FFs
- ◆ removal of theoretical uncertainties leads to reduction of FF uncertainties by more than a factor of two in many cases
- ◆ ILC, FCC-ee and CEPC give quite similar results except in regions statistics are limited

# Summary

- ◆ Fragmentation functions (FFs) are essential non-perturbative inputs for precision calculations of hadron production cross sections in high energy scattering from first principle of QCD
- ◆ FMNLO is a program for automated and fast calculations of fragmentation processes at NLO in QCD is now publicly available, which is desirable for global analysis of FFs providing much improved efficiency and capability for arbitrary hard processes
- ◆ We perform a joint global analysis of FFs to identified charged hadrons at NLO in QCD, using most recent data from SIA, SIDIS, and pp collisions; constraints on gluon FFs are much improved and discrepancies are found wrt. previous determinations; NPC23 FFs are publicly available in lhpdf6 format
- ◆ Further analyses from the NPC group to include neutral hadrons as well as going to NNLO precision in QCD are on the way

<https://fmnlo.sjtu.edu.cn/~fmnlo/>



# Summary

- ◆ Fragmentation functions (FFs) are essential non-perturbative inputs for precision calculations of hadron production cross sections in high energy scattering from first principle of QCD
- ◆ FMNLO is a program for automated and fast calculations of fragmentation processes at NLO in QCD is now publicly available, which is desirable for global analysis of FFs providing much improved efficiency and capability for arbitrary hard processes
- ◆ We perform a joint global analysis of FFs to identified charged hadrons at NLO in QCD, using most recent data from SIA, SIDIS, and pp collisions; constraints on gluon FFs are much improved and discrepancies are found wrt. previous determinations; NPC23 FFs are publicly available in lhpdf6 format
- ◆ Further analyses from the NPC group to include neutral hadrons as well as going to NNLO precision in QCD are on the way

<https://fmlno.sjtu.edu.cn/~fmlno/>

**Thank you for your attention!**