Improved constraints on the Higgs potential

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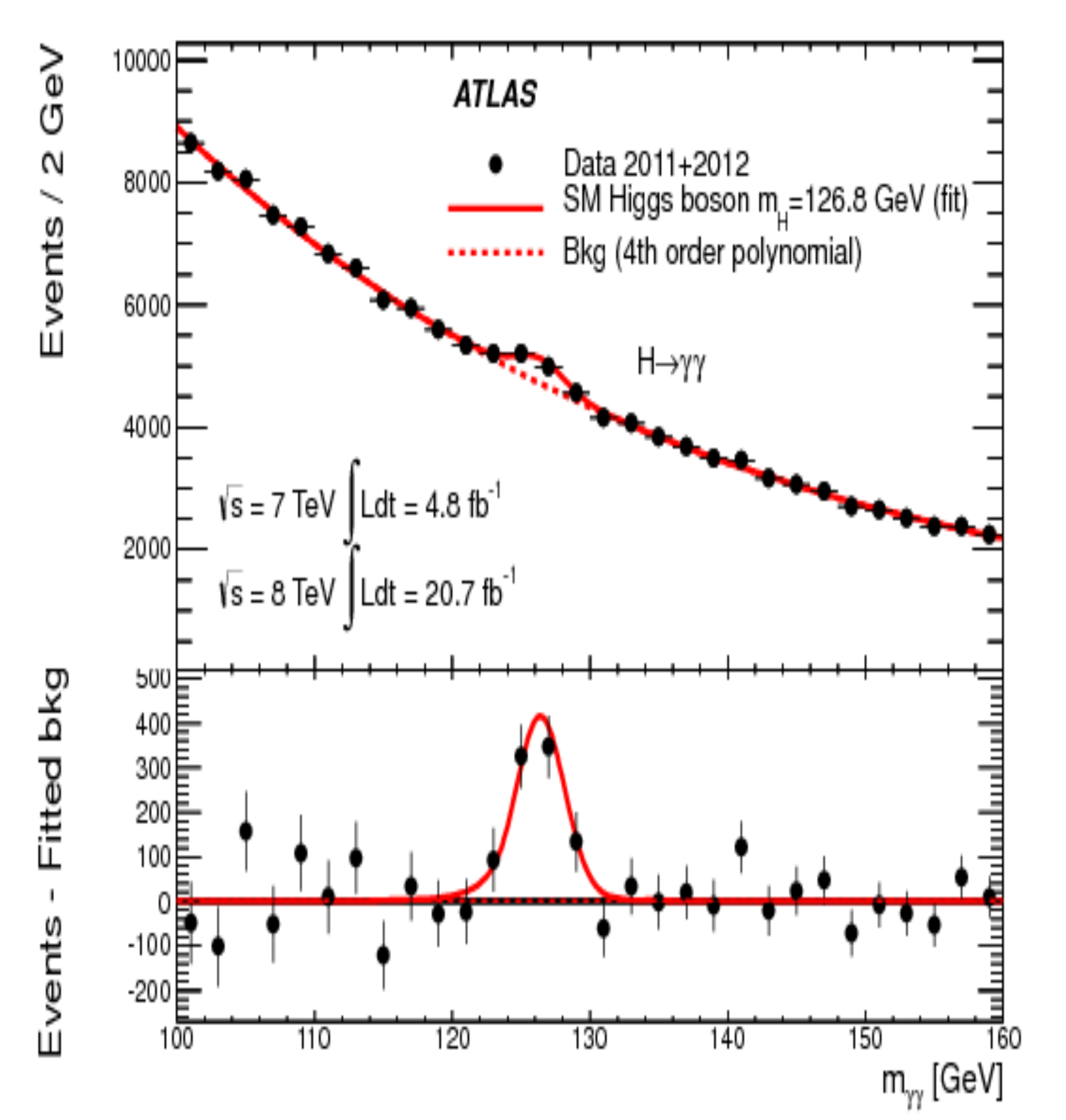
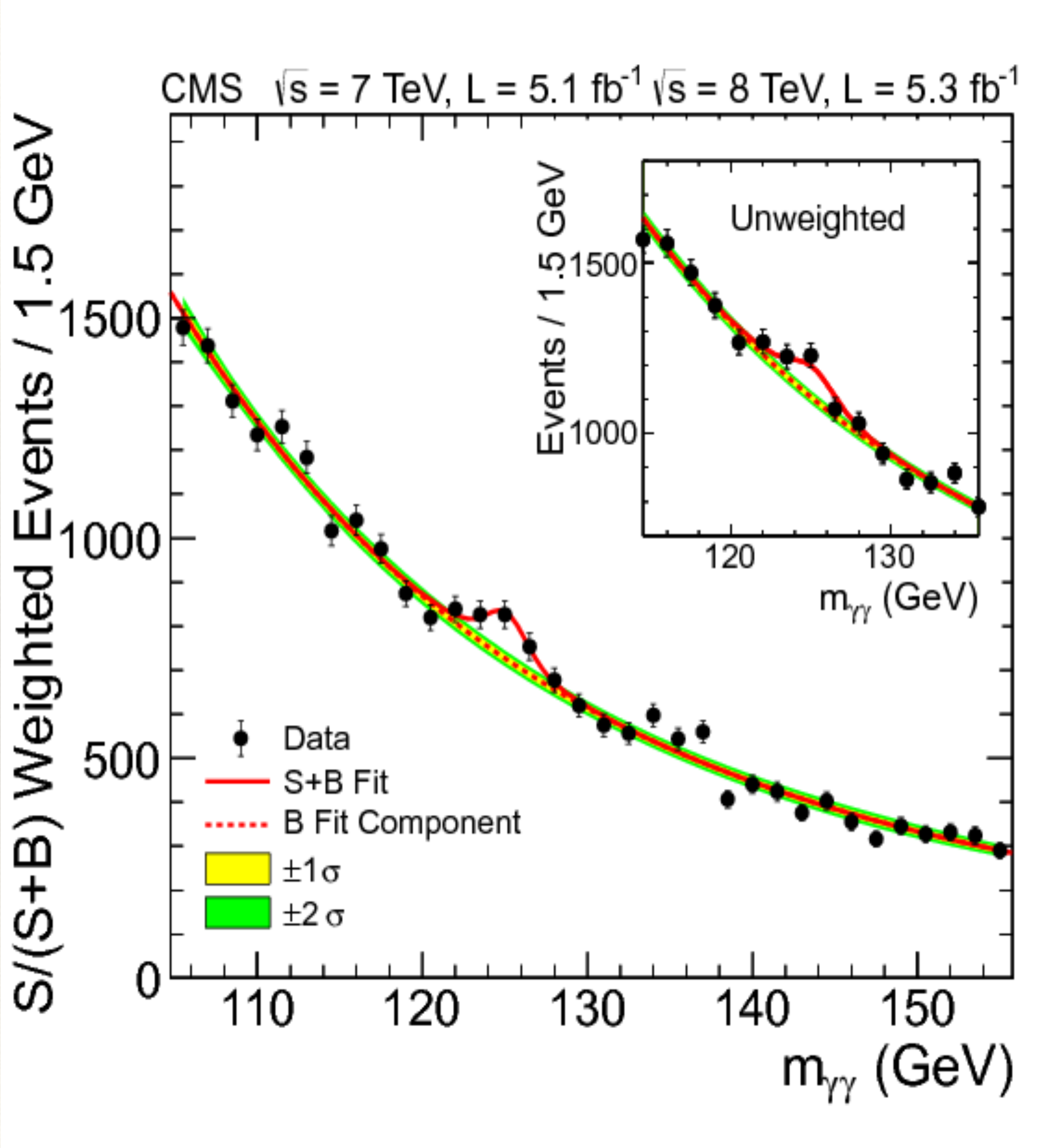
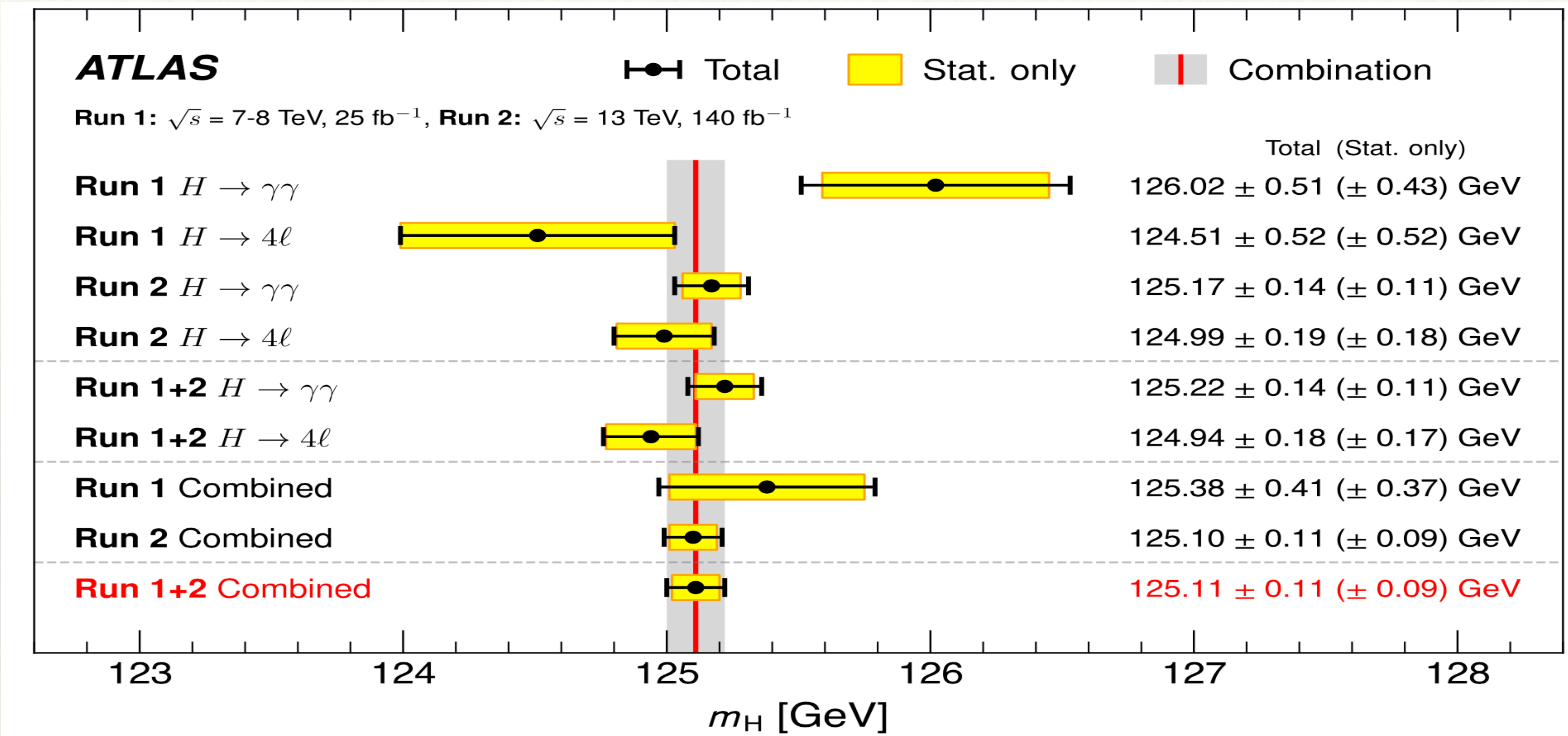
Dec 14, 2024

Discovery in 2012

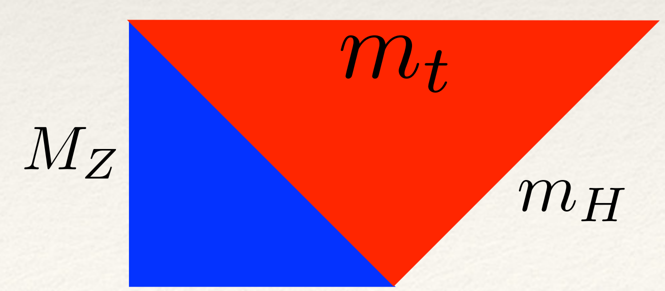


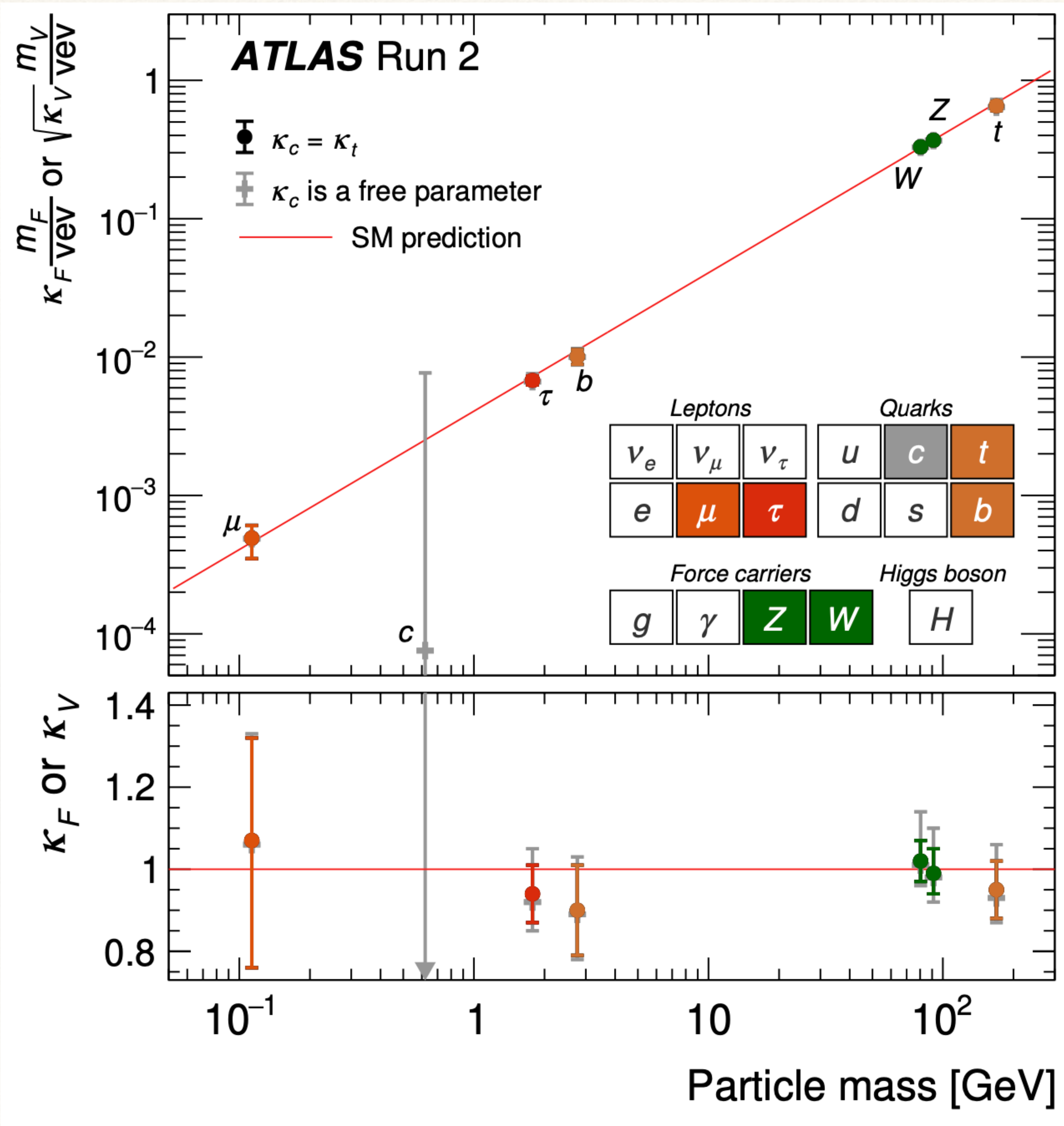


How precise is our understanding?



$$\frac{m_t}{m_H} = \frac{m_H}{M_Z} \approx \sqrt{2} \pm 0.04$$





So far, the H(125) properties are consistent with the SM expectation!

Its couplings with SM particles are proportional to their masses; Higgs mechanism

existence of a "fifth force" different from gravity

Nature 607(2022)52

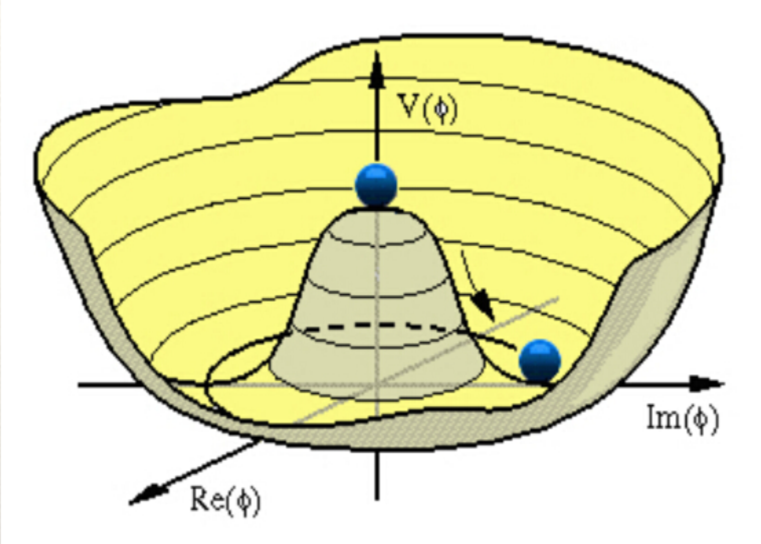
Higgs self-coupling in the SM

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$V(h) = \tfrac{1}{2}m_h^2 h^2 + \sqrt{\tfrac{\lambda}{2}}m_H h^3 + \tfrac{1}{4}\lambda h^4$$

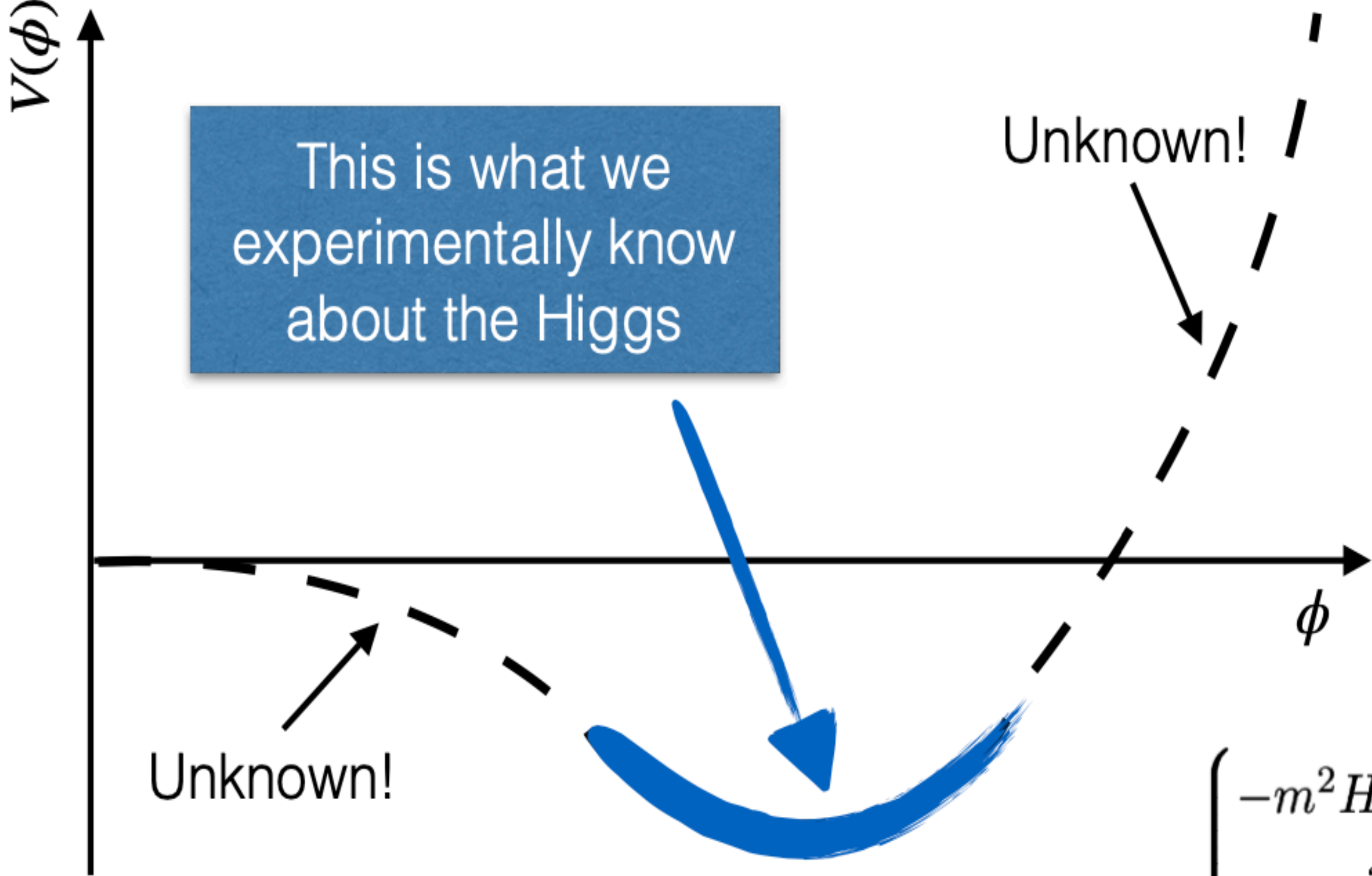
$$\lambda = m_H^2/2v^2$$



In some new physics models, the trilinear Higgs self-coupling may change by O(100)%, while the couplings with gauge bosons and fermions are still in agreement with SM.

S.Kanemura, et al, PLB558,157

We need to measure the trilinear self coupling directly.

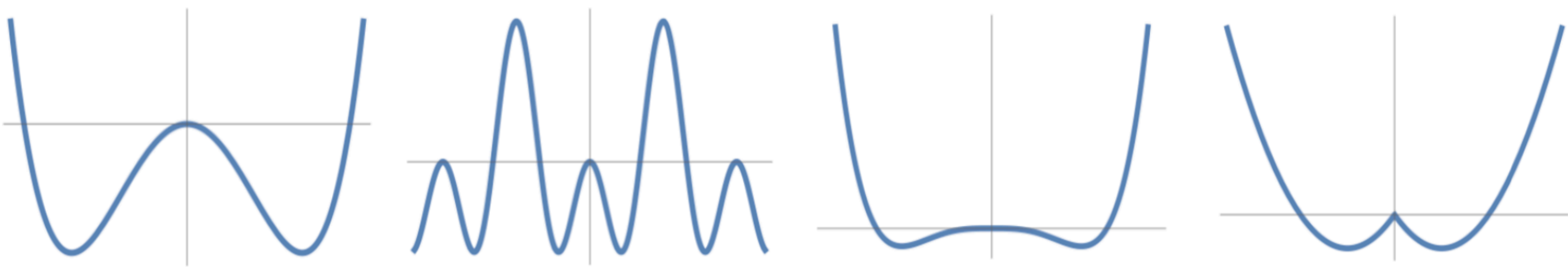


Different potential shapes could explain the same physics we see now!

$$
V(H) \simeq \begin{cases} -m^2 H^\dagger H + \lambda (H^\dagger H)^2 + \frac{c_6 \lambda}{\Lambda^2} (H^\dagger H)^3, & \text{Elementary Higgs} \\ -a \sin^2(\sqrt{H^\dagger H}/f) + b \sin^4(\sqrt{H^\dagger H}/f), & \text{Nambu-Goldstone Higgs} \\ \lambda (H^\dagger H)^2 + \epsilon (H^\dagger H)^2 \log \frac{H^\dagger H}{\mu^2}, & \text{Coleman-Weinberg Higgs} \\ -\kappa^3 \sqrt{H^\dagger H} + m^2 H^\dagger H, & \text{Tadpole-induced Higgs} \end{cases}
$$

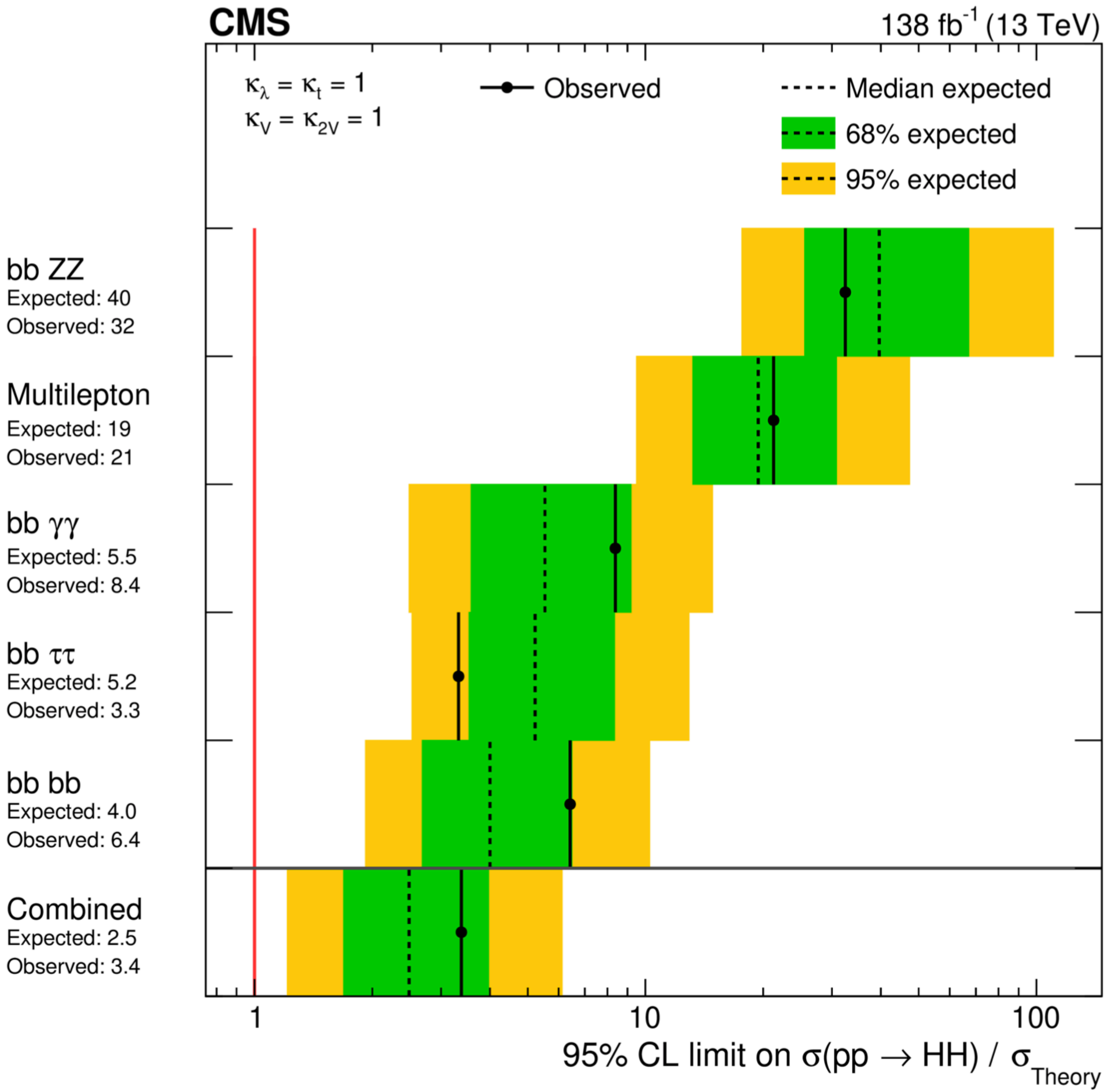
Agrawal, Saha, L.X.Xu, J.-H. Yu, C.-P. Yuan

arxiv:1907.02078



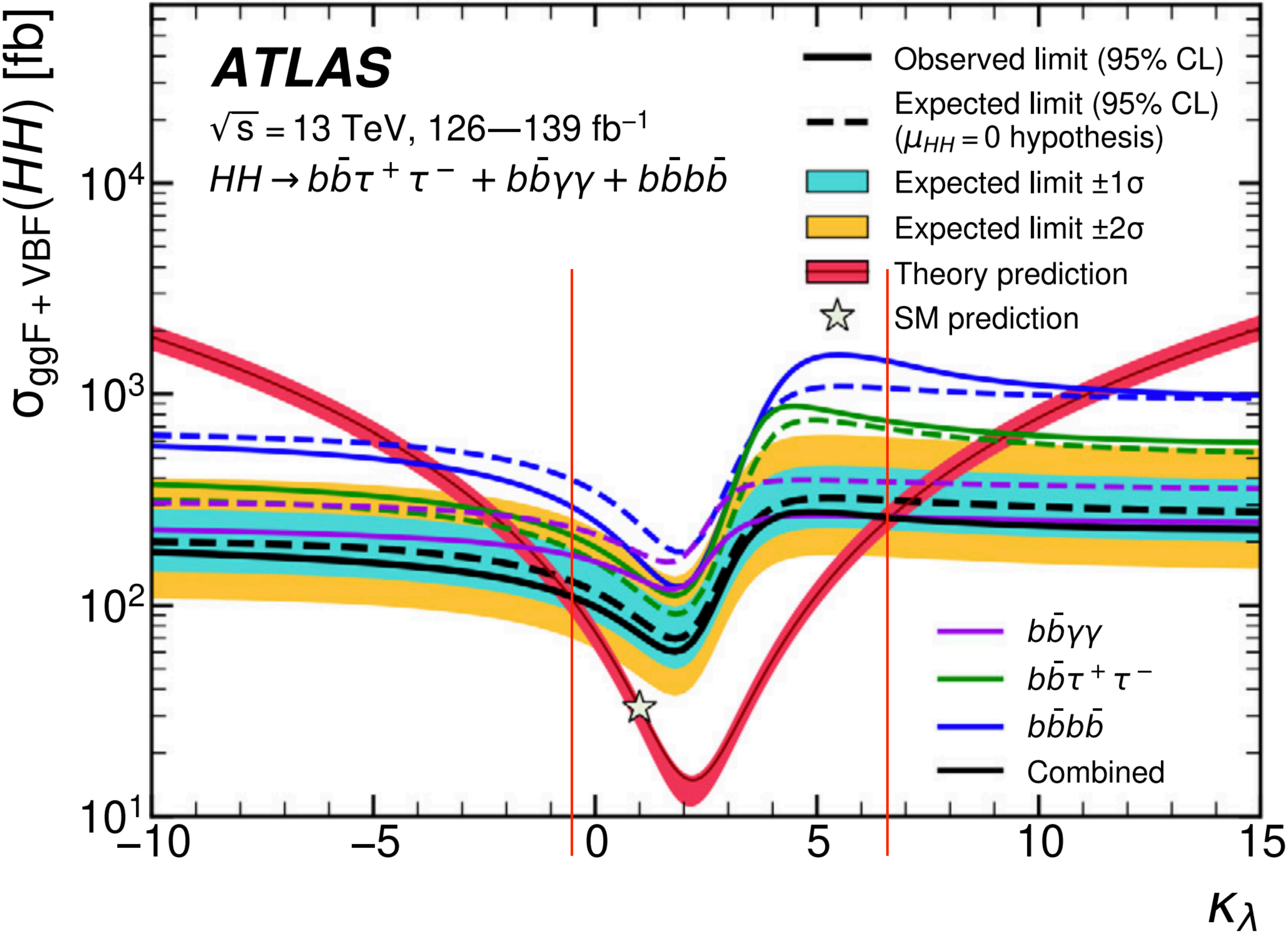
Landau-Ginzburg Higgs **Nambu-Goldstone Higgs** **Coleman-Weinberg Higgs** **Tadpole-Induced Higgs**

From Marco Valente

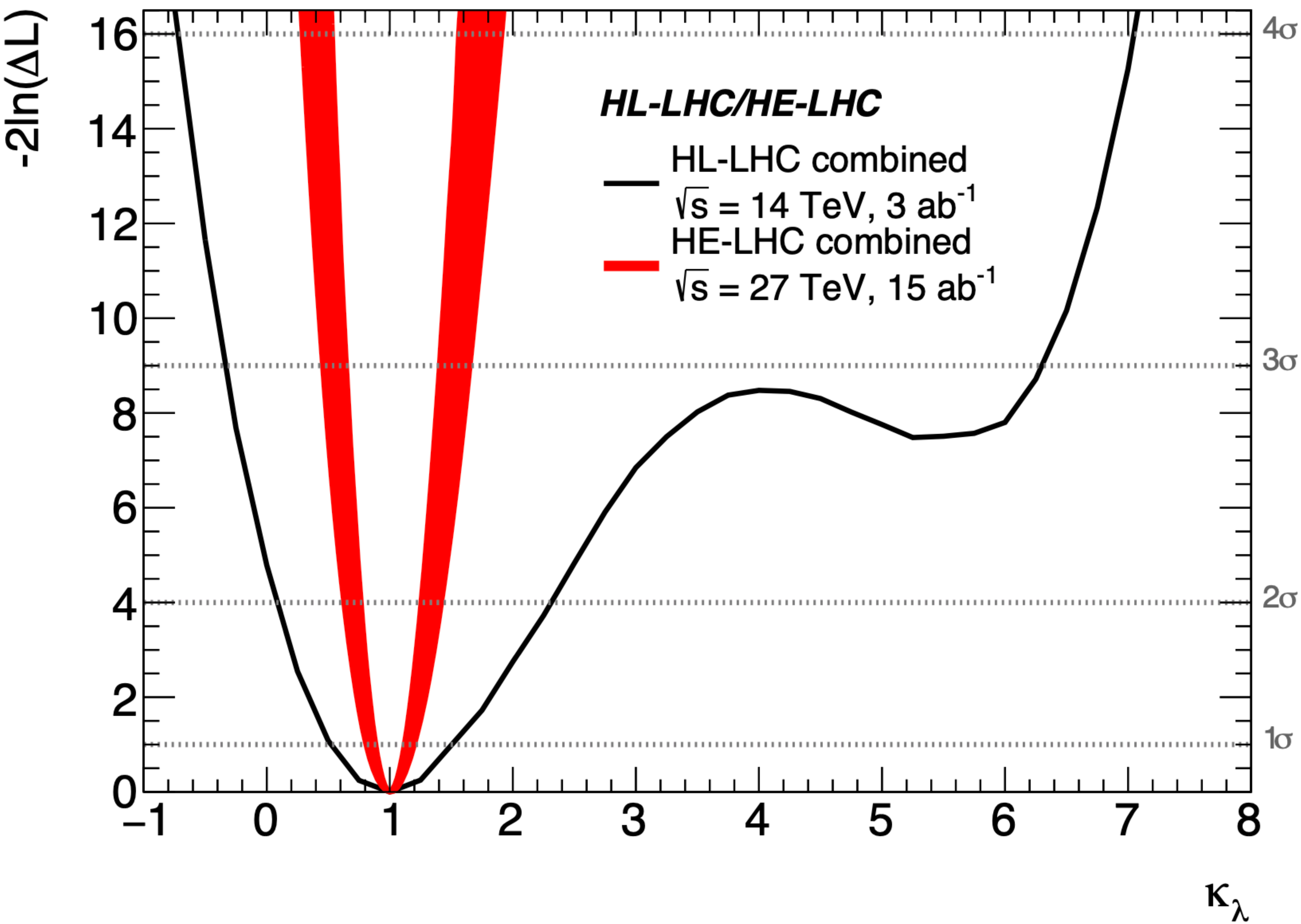


$$-1.2 < \kappa_\lambda < 6.5$$

Nature 607(2022)60



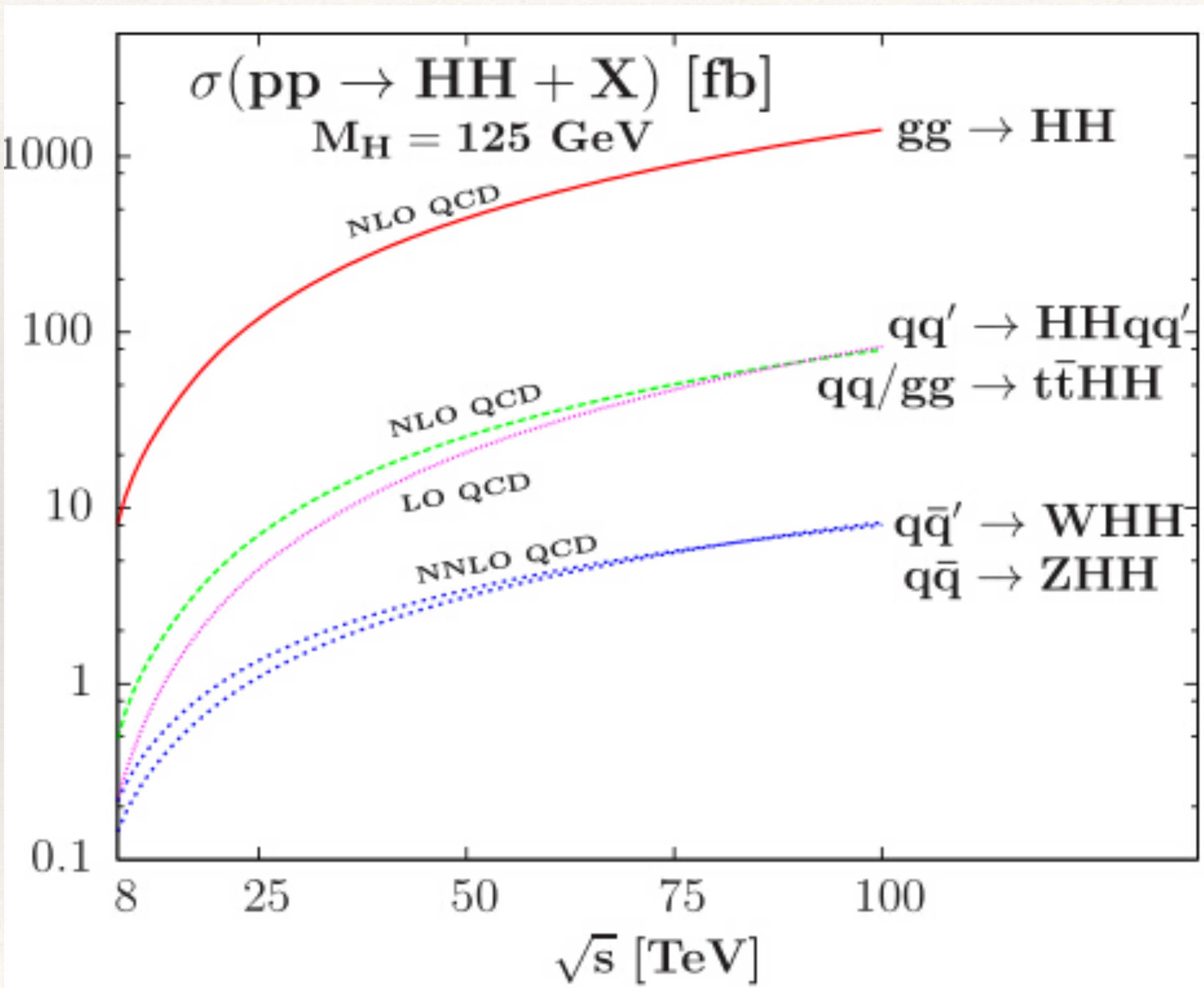
Physics Letters B 843 (2023) 137745

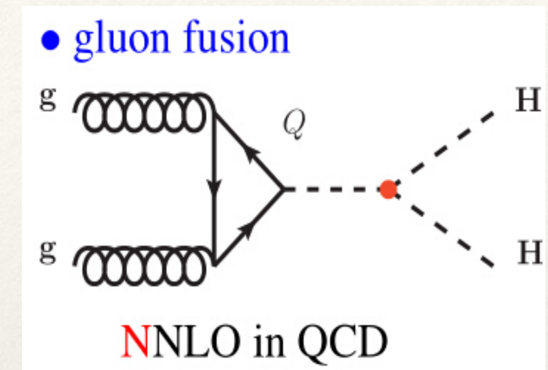


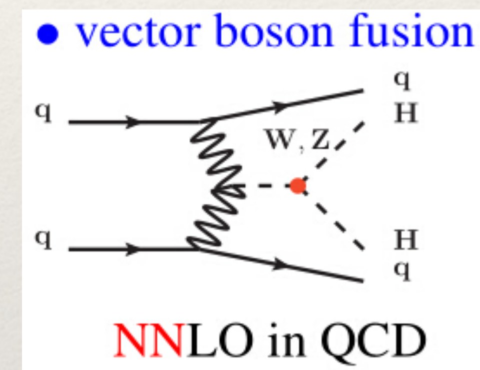
Expected accuracy:
~50% at HL-LHC
~10% at HE-LHC

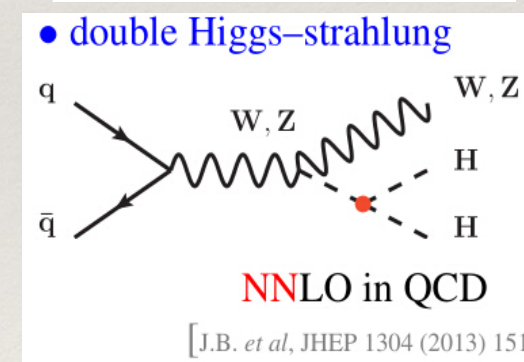
1902.00134

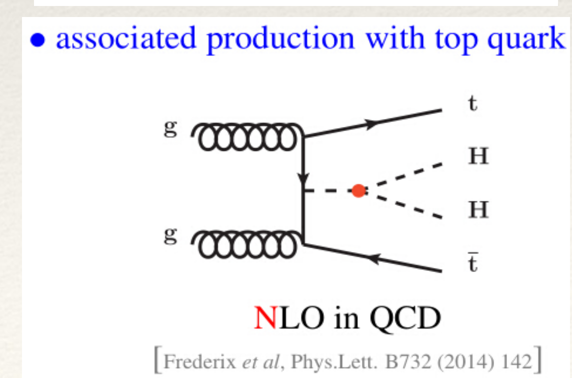
Higgs pair production at the LHC





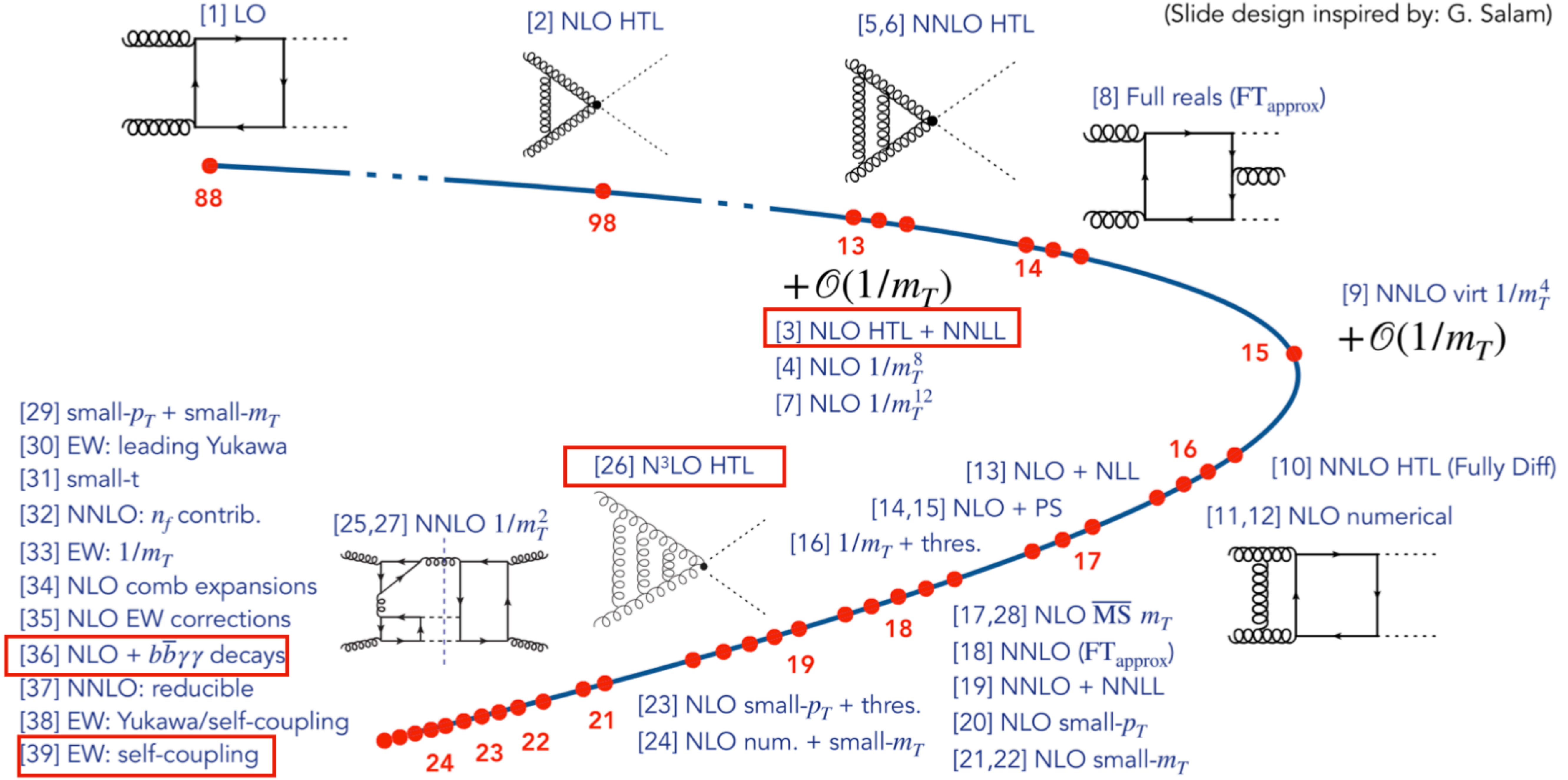






Perturbative corrections

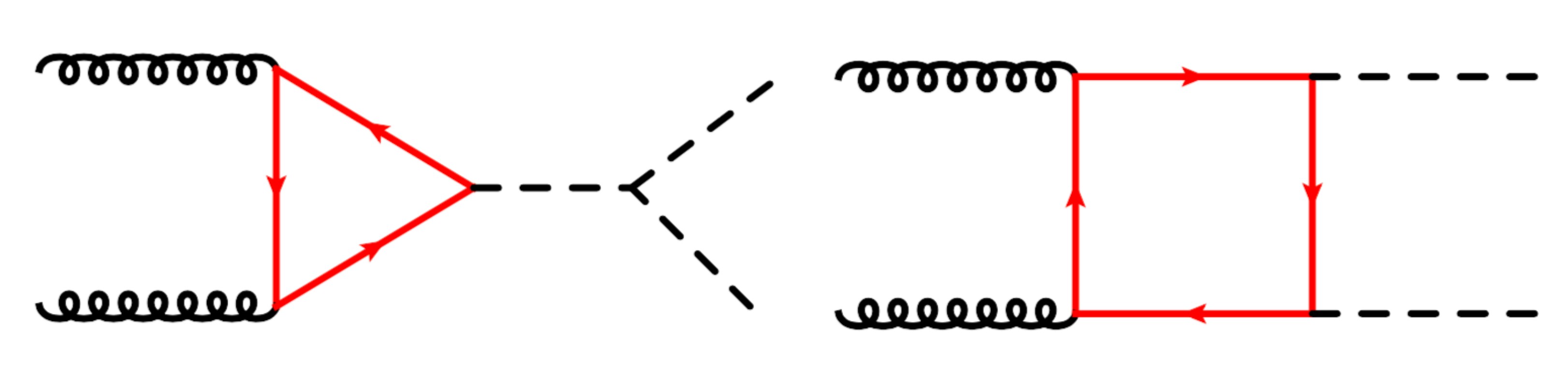
(Slide design inspired by: G. Salam)



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagnaschi, Degrassi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23; [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, SPJ, Kerner, Stone, Vestner; [39] Li, Si, Wang, Zhang, Zhao 24

taken from Spira's talk at Higgs 2024

pp → HH as a function of κ

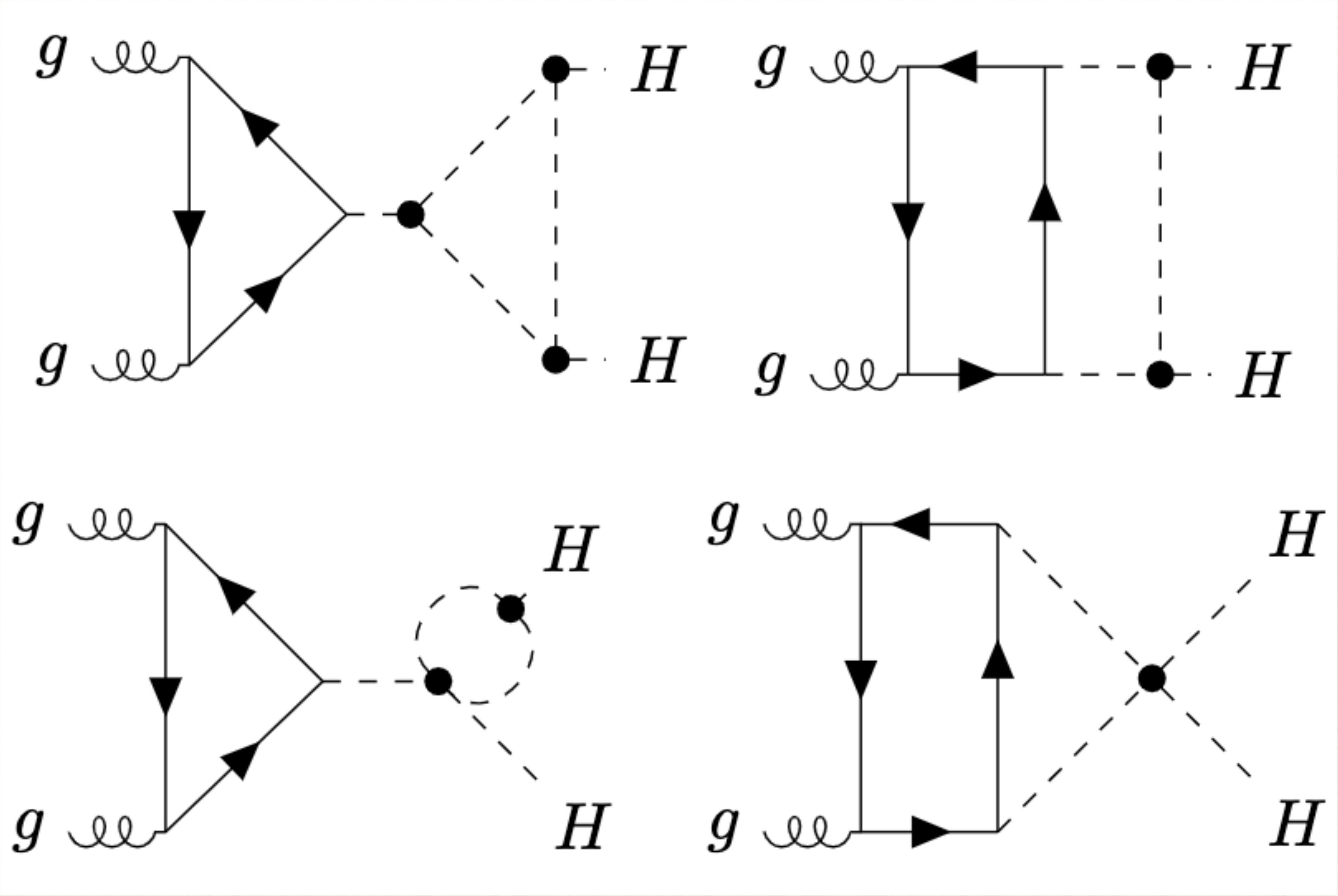


$$\sigma_{HH} = A + B\kappa + C\kappa^2$$

computation	A [fb]	A/A(LO)	B [fb]	B/B(LO)	C [fb]	C/C(LO)
LO m_t fin	35.0		-23.0		4.73	
NLO m_t fin	62.6	1.79	-44.4	1.93	9.64	2.04
NLO m_t fin × NNLO SM FTApprox	70.0	2.00	-49.6	2.16	10.8	2.28
NNLO + NNLL $m_t \to \infty$ × NNLO+NLL SM (partial m_t fin)	71.3	2.04	-47.7	2.08	9.93	2.10

1910.00012

A more realistic function form



$$\sigma_{HH} = A + B\kappa + C\kappa^2 + D\kappa^3 + E\kappa^4$$

Non-trivial task

Input parameters of conventional EW calculations:

e, m_H, m_t, m_W, m_Z

If one takes the Higgs self-coupling λ as an input, the correction would be proportional to λ.

Performing rescaling $\lambda \to \kappa\lambda$ before or after substituting $m_H^2 = 2\lambda v^2$ gives different results.

Renormalization

In the SM, the Lagrangian for the Higgs sector can be written as

$$\mathscr{L}_{\mathrm{H}} = (D_\mu \phi_0)^\dagger (D^\mu \phi_0) + \mu_0^2 (\phi_0^\dagger \phi_0) - \lambda_0 (\phi_0^\dagger \phi_0)^2$$

where ϕ_0 denotes the bare Higgs doublet and D_μ is the covariant derivative. The relations between the bare fields and couplings, and their renormalized counterparts, are given by $\phi_0 = Z_\phi^{1/2} \phi$, $\mu_0^2 = Z_{\mu^2} \mu^2$, and $\lambda_0 = Z_\lambda \lambda$.

The EW gauge symmetry is spontaneously broken once the Higgs field develops a non-vanishing vacuum expectation value v. Taking the unitary gauge, we write the Higgs field as

$$\phi = \frac{1}{\sqrt{2}} \left(0,\ Z_v v + H\right)^T$$

Our strategy is equivalent to the application of HEFT in Higgs boson pair production.

Renormalization

The renormalized Lagrangian in the κ framework after EW gauge symmetry breaking:

$$\mathscr{L}_{\rm H}^{\kappa} = \frac{1}{2} Z_\phi (\partial_\mu H)^2 - \left(-\frac{1}{2} Z_{\mu^2} Z_\phi Z_v^2 \mu^2 v^2 + \frac{1}{4} Z_\lambda Z_\phi^2 Z_v^4 \lambda v^4 \right) - (Z_\lambda Z_\phi^2 Z_v^3 \lambda v^3 - Z_{\mu^2} Z_\phi Z_v \mu^2 v) H$$
$$- \left(\frac{3}{2} Z_\lambda Z_\phi^2 Z_v^2 \lambda v^2 - \frac{1}{2} Z_{\mu^2} Z_\phi \mu^2 \right) H^2 - Z_{\kappa_{3\rm H}} Z_\lambda Z_\phi^2 Z_v \lambda_{3\rm H} v H^3 - \frac{1}{4} Z_{\kappa_{4\rm H}} Z_\lambda Z_\phi^2 \lambda_{4\rm H} H^4 + \cdots$$

The linear term is

$$(\mu^2 v - \lambda v^3) H + [(\delta Z_{\mu^2} + \delta Z_\phi + \delta Z_v)\mu^2 v - (\delta Z_\lambda + 2\delta Z_\phi + 3\delta Z_v)\lambda v^3] H$$

We choose the renormalization scheme in which there is no tadpole contributions.

$\mu^2 = \lambda v^2$ and $(\delta Z_{\mu^2} - \delta Z_\lambda - \delta Z_\phi - 2\delta Z_v)\mu^2 v + T = 0$ with T the one-loop diagrams.

$$T = \frac{3\lambda_{3\rm H} v}{16\pi^2} m_H^2 \left(\frac{1}{\epsilon} + \ln \frac{\mu_R^2}{m_H^2} + 1 \right)$$

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Renormalization

The quadratic term is

$$\frac{1}{2}(\partial_\mu H)^2 - \mu^2 H^2 + \frac{1}{2}\delta Z_\phi(\partial_\mu H)^2 - \left(\frac{3}{2}\delta Z_\lambda + \frac{5}{2}\delta Z_\phi - \frac{1}{2}\delta Z_{\mu^2} + 3\delta Z_v\right)\mu^2 H^2$$
$$\equiv \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}m_H^2 H^2 + \frac{1}{2}\delta Z_\phi(\partial_\mu H)^2 - \frac{1}{2}(\delta Z_{m_H^2} + \delta Z_\phi)m_H^2 H^2$$

We choose the on-shell renormalization scheme.

$$\delta Z_{m_H^2} = \frac{3\lambda_{4\mathrm{H}}}{16\pi^2}\left(\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{m_H^2} + 1\right) + \frac{9\lambda_{3\mathrm{H}}^2 v^2}{m_H^2}\frac{1}{8\pi^2}\left(\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{m_H^2} + 2 - \frac{\pi}{\sqrt{3}}\right)$$
$$\delta Z_\phi = \frac{9\lambda_{3\mathrm{H}}^2 v^2}{8\pi^2}\frac{\sqrt{3} - 2\pi/3}{\sqrt{3}m_H^2}$$

Since we focus on the corrections induced by the Higgs self-couplings, we can simply take $\delta Z_v + \delta Z_\phi/2 = 0$

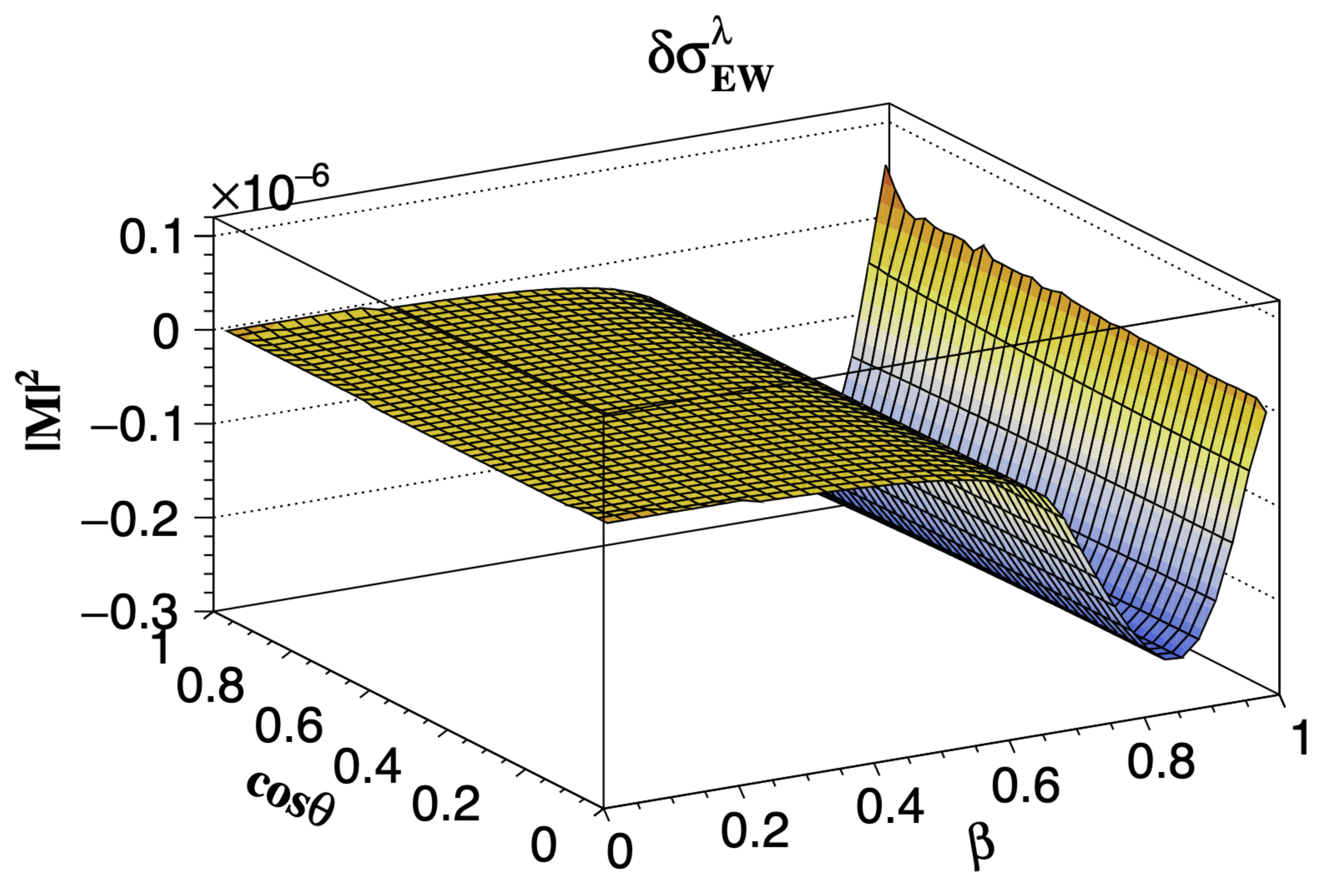
H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Renormalization

The result of one-particle reducible diagrams and counter-terms:

$$\mathcal{M}^{\rm LO}_{gg\to H^*\to HH} \times \left\{ \frac{3}{16\pi^2}\frac{1}{\epsilon}\left(-2\lambda_{\rm 4H} - \lambda_{\rm 3H} + 6\lambda_{\rm 3H}^2\frac{v^2}{m_H^2}\right) + \delta Z_{\kappa_{\rm 3H}} \right.$$
$$+ \frac{3}{16\pi^2}\ln\frac{\mu_R^2}{m_H^2}\left[-2\lambda_{\rm 4H} - \lambda_{\rm 3H} + 6\lambda_{\rm 3H}^2\frac{v^2}{m_H^2}\right]$$
$$- \frac{9\lambda_{\rm 3H}^2}{8\pi^2}\frac{v^2}{s-m_H^2}\left[\beta\left(\ln\left(\frac{1-\beta}{1+\beta}\right) + i\pi\right) + \frac{s}{m_H^2}\left(1 - \frac{2\pi}{3\sqrt{3}}\right) + \frac{5\pi}{3\sqrt{3}} - 1\right]$$
$$+ \frac{3\lambda_{\rm 3H}^2}{16\pi^2}\frac{v^2}{m_H^2}(21 - 4\sqrt{3}\pi) - \frac{9\lambda_{\rm 3H}^2 v^2}{4\pi^2}C_0[m_H^2, m_H^2, s, m_H^2, m_H^2, m_H^2]$$
$$\left. - \frac{3\lambda_{\rm 4H}}{16\pi^2}\left[\beta\left(\ln\left(\frac{1-\beta}{1+\beta}\right) + i\pi\right) + 5 - \frac{2\pi}{\sqrt{3}}\right] - \frac{3\lambda_{\rm 3H}}{16\pi^2}\right\},$$

Squared matrix elements



H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Updated function forms

The λ dependent correction is

$$\delta\sigma^{\kappa_\lambda}_{\mathrm{ggF,EW}} = (0.075\kappa^4_{\lambda_{3H}} - 0.158\kappa^3_{\lambda_{3H}} - 0.006\kappa^2_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.058\kappa^2_{\lambda_{3H}} + 0.070\kappa_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.149\kappa_{\lambda_{4H}}) \ \ \mathrm{fb}$$

$$\delta\sigma^{\kappa_\lambda}_{\mathrm{VBF,EW}} = (0.0215\kappa^4_{\lambda_{3H}} - 0.0324\kappa^3_{\lambda_{3H}} - 0.0019\kappa^2_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.0043\kappa^2_{\lambda_{3H}} + 0.0151\kappa_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.0211\kappa_{\lambda_{4H}}) \ \ \mathrm{fb}$$

$\kappa_{\lambda_{3H}}$	$\kappa_{\lambda_{4H}}$	ggF			VBF		
		$\sigma^{\kappa_\lambda}_{\mathrm{LO}}$	$\sigma^{\kappa_\lambda}_{\mathrm{NNLO-FT}}$	$\delta\sigma^{\kappa_\lambda}_{\mathrm{EW}}$	$\sigma^{\kappa_\lambda}_{\mathrm{LO}}$	$\sigma^{\kappa_\lambda}_{\mathrm{NNNLO}}$	$\delta\sigma^{\kappa_\lambda}_{\mathrm{EW}}$
1	1	16.7	31.2	-0.225	1.71	1.69	-2.30×10^{-2}
3	1	8.59	18.4	1.28	3.59	3.53	8.35×10^{-1}
6	1	67.3	161	60.6	25.1	24.6	20.7
1	3	16.7	31.2	-0.393	1.71	1.69	-3.89×10^{-2}
1	6	16.7	31.2	-0.646	1.71	1.69	-6.27×10^{-2}
3	3	8.59	18.4	1.30	3.59	3.53	8.50×10^{-1}
6	6	67.3	161	61.0	25.1	24.6	20.7

The QCD corrections are significant in ggF, but not sensitive to κ_{3H}.

The EW corrections are 91% (82%) in ggF (VBF) for $\kappa_{3H} = 6$.

The dependence on λ_{4H} is weak.

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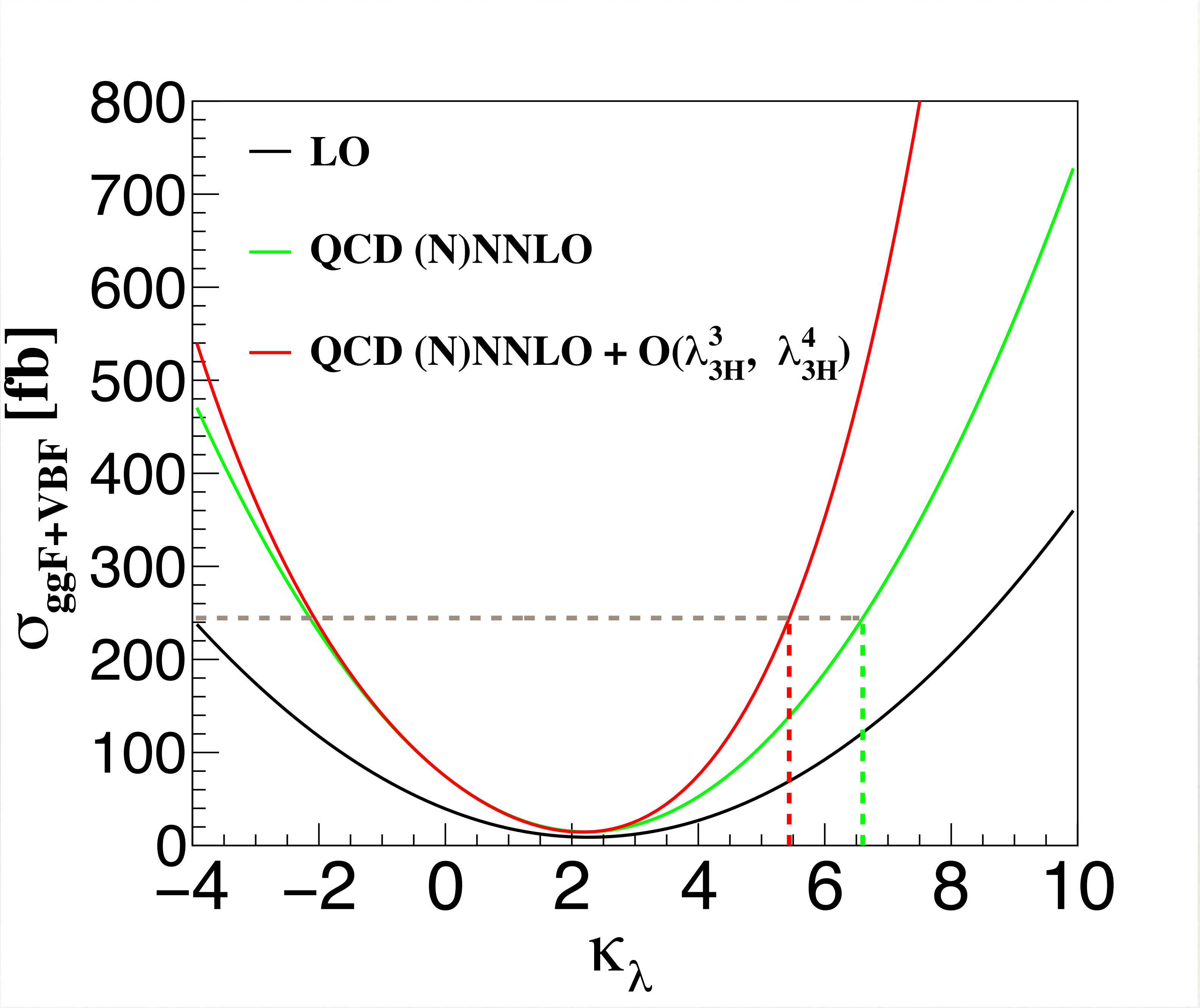
More stringent constraint



ATLAS (CMS) limit

6.6 (6.49)

5.4 (5.37)

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Summary

The SM is a master piece in human history. It has been tested by a lot of experiments at very high precision level.

However, the Higgs sector still needs more precise comparison between theories and experiments.

Higher-order quantum corrections provide more precise estimate of the dependence on Higgs self-couplings.

Thanks a lot for your attention!